NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4334

LAG IN PRESSURE SYSTEMS AT EXTREMELY LOW PRESSURES

By William T. Davis

Langley Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

A theoretical formula for determining time lags in pressure-measuring systems at all pressures, including extremely low pressures where molecular flow occurs, is derived and shown to be accurate to within 10 percent for pressures down to approximately 0.2 millimeter of mercury (0.556 lb/sq ft) for nearly linear pressure changes.

INTRODUCTION

Because of the low pressure encountered by aircraft at high altitudes and the extremely low test-section static pressures at which high-speed wind tunnels now operate, it is desired to extend the equations now used for determining time lags in pressure systems (refs. 1 and 2) to the slip and molecular flow regions where the mean free path is of the same order of magnitude as the diameter of pressure-measuring tubing. The purpose of the present investigation is to derive a theoretical equation for time lag and to provide an experimental check of the validity of the derivation.

SYMBOLS

d tube inside diameter, ft except when specified in inches

f fraction of gas molecules striking walls which are diffusely reflected

g acceleration due to gravity, ft/sec^2

K exponent in the relation $\frac{p_a}{p_b} = \left(\frac{\rho_a}{\rho_b}\right)^K$ where p_a, p_b, ρ_a, ρ_b are pressures and densities at two different conditions

l	tube length, ft
p	pressure
p_m	mean pressure in tube, lb/sq ft or mm Hg
p_1	applied pressure, lb/sq ft
p_2	pressure at instrument, lb/sq ft
Q	gas-flow rate, or volume-flow rate times pressure, ft-lb/sec
r	tube radius, ft
t	time, sec
V	volume of instrument plus one-half tubing volume, cu ft
Δp	difference between applied pressure and pressure at instrument, $p_1 - p_2$, lb/sq ft
λ	time constant at measured pressure, sec
λ_o	time constant at reference conditions, a temperature of 24° C and a pressure of 760 millimeters of mercury (2,116 lb/sq ft), sec
μ	viscosity, lb/sec-ft
ρ_2	weight density at p_2, lb/cu ft

BASIC THEORY

A theoretical equation is derived in references 1 and 2 for determining lag in tubing leading into an instrument volume at pressures outside the slip and molecular flow regions. The time constant, defined as $\lambda = \frac{\Delta p}{dp_2/dt}$, is given in equation (6) of reference 1 (with modifications of symbols and minor substitutions) as

$$\lambda = \frac{8V\mu l}{\pi r^4 K p_m g} \tag{1}$$

For air undergoing isothermal change, $K = 1$.

In reference 3, which is concerned with the flow of gases in tubing at extremely low pressures, the following theoretical equation (applicable at all pressures) is derived for the gas flow rate:

$$Q = \frac{\pi g r^4}{16\mu l}\left(p_1{}^2 - p_2{}^2\right)\left[1 + 4\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{2}{f} - 1\right)X\right] \tag{2}$$

where

$$X = \frac{\mu}{p_m r\left(\frac{\rho_2 g}{p_2}\right)^{1/2}}$$

$$p_1{}^2 - p_2{}^2 = \left(p_1 + p_2\right)\left(p_1 - p_2\right) = 2p_m\left(p_1 - p_2\right)$$

The mass rate of flow into the instrument $Q\rho_2/p_2$ is also the rate of change of mass of air $\frac{d}{dt}\left(V\rho_2\right)$.

Therefore,

$$\frac{Q}{p_2} = \frac{V}{\rho_2}\frac{d\rho_2}{dt} = \frac{V}{p_2}\frac{dp_2}{dt}$$

or

$$Q = V\frac{dp_2}{dt} \tag{3}$$

Simplifying equation (2) and setting it equal to equation (3) gives

$$Q = \frac{\pi g r^4}{16\mu l}2p_m\left(p_1 - p_2\right)\left[1 + 4\left(\frac{\pi}{\frac{2\rho_2 g}{p_2}}\right)^{1/2}\left(\frac{2}{f} - 1\right)\frac{\mu}{p_m r}\right] = V\frac{dp_2}{dt}$$

The time constant is

$$\lambda = \frac{p_1 - p_2}{\frac{dp_2}{dt}} = \frac{8V\mu l}{\pi g r^3\left[p_m r + 4\left(\frac{2}{f} - 1\right)\mu\left(\frac{\pi}{\frac{2\rho_2 g}{p_2}}\right)^{1/2}\right]} \tag{4}$$

At a temperature of 24° C and a pressure of 760 millimeters of mercury (2,116 lb/sq ft)

$$\mu = 1.25 \times 10^{-5} \text{ lb/sec-ft}$$

$$g = 32.2 \text{ ft/sec}^2$$

$$\frac{\rho_2}{p_2} = 3.55 \times 10^{-5} \text{ ft}^{-1}$$

According to reference 3, f varies between 0.77 and 0.84. A value of 0.8 was used here. Substituting these values into equation (4) gives

$$\lambda = \frac{9.89 \times 10^{-7} V l}{r^3\left(p_m r + 0.00278\right)} \tag{5}$$

At mean pressures greater than 5 millimeters of mercury (13.9 lb/sq ft) and values of r greater than 1/32 inch, the second term in the denominator of equation (5) is small compared with the first, and equation (5) reduces to equation (1) where the time constant is inversely proportional to pressure for a given tube length and diameter. This fact may be noted in figure 1, where λ/λ_o (the ratio of time constant at test conditions to time constant at reference conditions) is presented as a function of mean pressure. As the mean pressure decreases to values below 0.1 millimeter of mercury (0.278 lb/sq ft), the first term in the denominator becomes small compared with the second, and λ approaches a point where it tends to become independent of pressure.

EQUIPMENT AND METHOD

NACA pressure cells with 3-inch beryllium-copper diaphragms activating a mechanical-optical recorder were used to obtain plots of pressure as a function of time measured at the ends of several sections of copper tubing (fig. 2). A mirror coupled to the diaphragm deflects a light beam on the film in the recorder. This deflection is proportional to the change in pressure. The cells have a full-scale range of 5 millimeters of mercury (13.9 lb/sq ft) with an accuracy of 0.5 percent of full scale. One end of the tubing was connected to a vacuum chamber where the applied pressure was measured by a pressure cell with a negligible length of tubing. Also connected to the chamber were a pump and a needle valve to admit air from the atmosphere. The pressure-measuring instrument was a pressure cell connected to the other end of the tube.

Tests were made to obtain time lags for tubing of different lengths and inside diameters. In addition, measurements were made with pressure increasing and decreasing. The sequence of the tests and the tube configurations are given in the following table:

Test	Tube length, ft	Tube inside diameter, in.	Method
1	4	0.1225	Pressure increasing
2	4	.1225	Pressure increasing
3	4	.1225	Pressure increasing
4	4	.1225	Pressure decreasing
5	2	.1225	Pressure increasing
6	2	.0597	Pressure increasing
7	2	.0597	Pressure decreasing

Tests involving increasing pressure were made by first pumping the system to below 0.02 millimeter of mercury (0.0556 lb/sq ft) and then allowing air to enter the chamber through the valve. The pressure increased at an almost constant rate over the range recorded. A sample plot of recorded data gathered by use of the increasing pressure method in test 3 is shown in figure 3. Data gathered in test 7, in which the pressure in the chamber was decreased by use of the pump, are shown in figure 4.

The time constant λ was obtained by measuring both the pressure difference at the two ends of the tube and the rate of pressure change, dp_2/dt, at the measuring instrument (fig. 5):

$$\lambda = \frac{\Delta p}{dp_2/dt}$$

RESULTS AND SOURCES OF ERROR

Plots of λ/λ_o, both experimental and theoretical, are shown in figures 6 and 7. Plots of λ are shown in figure 8. The experimental points fall within 5 percent of the theoretical curves for pressures greater than 0.7 millimeter of mercury (1.95 lb/sq ft). The large errors existing for pressures below 0.7 millimeter of mercury in all tests except tests 4 and 7 are due to the transient effects as the applied pressure suddenly begins to increase. An example of these effects may be seen in figure 3. The rise of applied pressure p_1 begins approximately 2 seconds before that of the pressure p_2 at the instrument, and the transient effects last approximately 8 seconds in this test. The data symbols in this region in the figures are filled to indicate transient errors.

An attempt was made in test 4 of the 0.1225-inch inside-diameter tubing and in test 7 of the 0.0597-inch inside-diameter tubing to minimize the transient effects. The tests were begun with the system at an initial pressure of approximately 20 millimeters of mercury (55.6 lb/sq ft). The applied pressure was varied by decreasing the pressure in the vacuum chamber with the pump. It is believed that no more than a negligible part of the transient effects was still present as the pressure decreased to a value within the range of the pressure cells.

It may be noted from the plots of λ in figure 8 that most of the difference between experiment and theory existing in the tests with increasing pressure is not present in those with decreasing pressure. The difference that is still present increases as pressure decreases and reaches approximately 10 percent as the mean pressure falls to 0.3 millimeter of mercury (0.834 lb/sq ft). Most of this difference is believed to be due to inaccuracy in the measurement of Δp and in the measurement of the slope of the plot of p_2 against time. Both the slope dp_2/dt and Δp are small at the low pressures where the error is appreciable, and small errors in the measurement of these quantities could account for the error in the values for λ.

The pressure cell indicated a pressure of -0.025 millimeter of mercury (0.0695 lb/sq ft) in the vacuum chamber as it was evacuated, and this value indicates a shift in the zero point of the pressure cell subsequent to its calibration. This shift would appear as a very noticeable error in the value of Δp when Δp is small, and is in the correct direction to account for some of the error existing in λ. The calibrations were not corrected for the shift in the zero reading, because the pressure cells are only accurate to this order of magnitude.

Assume an error in Δp of 0.03 millimeter of mercury (0.0834 lb/sq ft) in order to realize the effect of an error of this magnitude in the calibration. This error in Δp would produce an error in the values obtained for λ of 4 to 6 percent at a mean pressure of 0.3 millimeter of mercury (0.834 lb/sq ft).

The effect of inaccuracy in values used for the tube diameter in the formulas in this report should be kept in mind. Equation (4) will reduce to an equation in which the diameter appears to the fourth power at the higher pressures and to the third power at the lower pressures. The diameters given by the manufacturer for the tubing used in this investigation contained errors of 2 percent and 4.5 percent. The mean effective diameters were used in the calculations. These values were obtained by measuring the volume of water required to fill the tubing.

Since the mean pressure was used in calculating the lags, the determination of the applied pressures from measured values may require successive approximations.

CONCLUDING REMARKS

An analysis has been presented for the theoretical determination of time lags in pressure systems at extremely low pressures where free molecular flow occurs. Experimental data to verify the theory indicate the equation to be accurate within an error of 10 percent for mean pressures as low as 0.2 millimeter of mercury (0.556 lb/sq ft). Lower mean pressures were not obtained in this investigation.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., August 12, 1958.

REFERENCES

1. Charnley, W. J.: A Note on a Method of Correcting for Lag in Aircraft Pitot-Static Systems. R. & M. No. 2352, British A.R.C., 1950.

2. Wildhack, W. A.: Pressure Drop in Tubing in Aircraft Instrument Installations. NACA TN 593, 1937.

3. Brown, Gordon P., DiNardo, Albert, Cheng, George K., and Sherwood, Thomas K.: The Flow of Gases in Pipes at Low Pressures. Jour. Appl. Phys., vol. 17, no. 10, Oct. 1946, pp. 802-813.

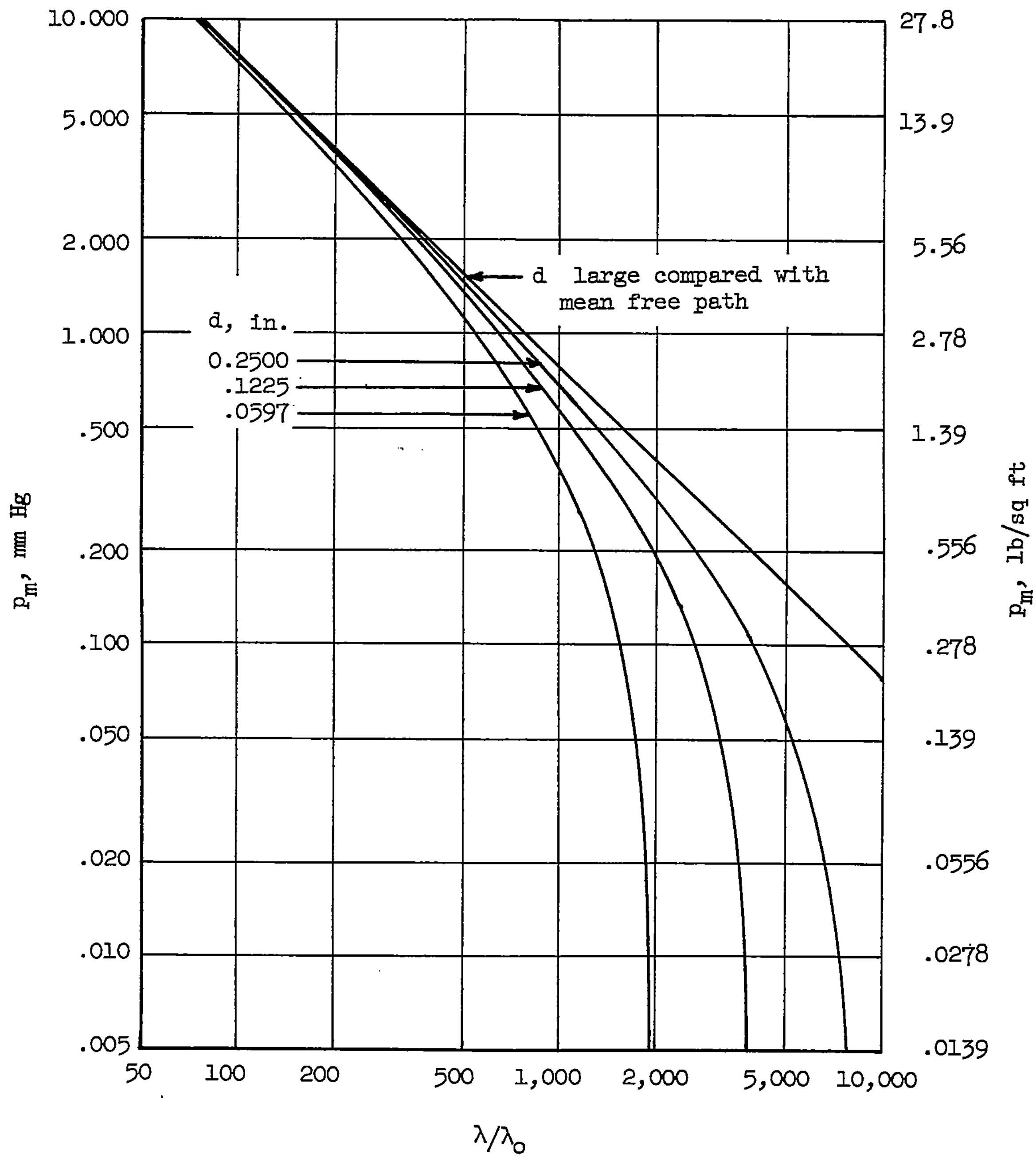

Figure 1.- Variation of mean pressure in tubing with the ratio of time constant at test conditions to time constant at reference conditions.

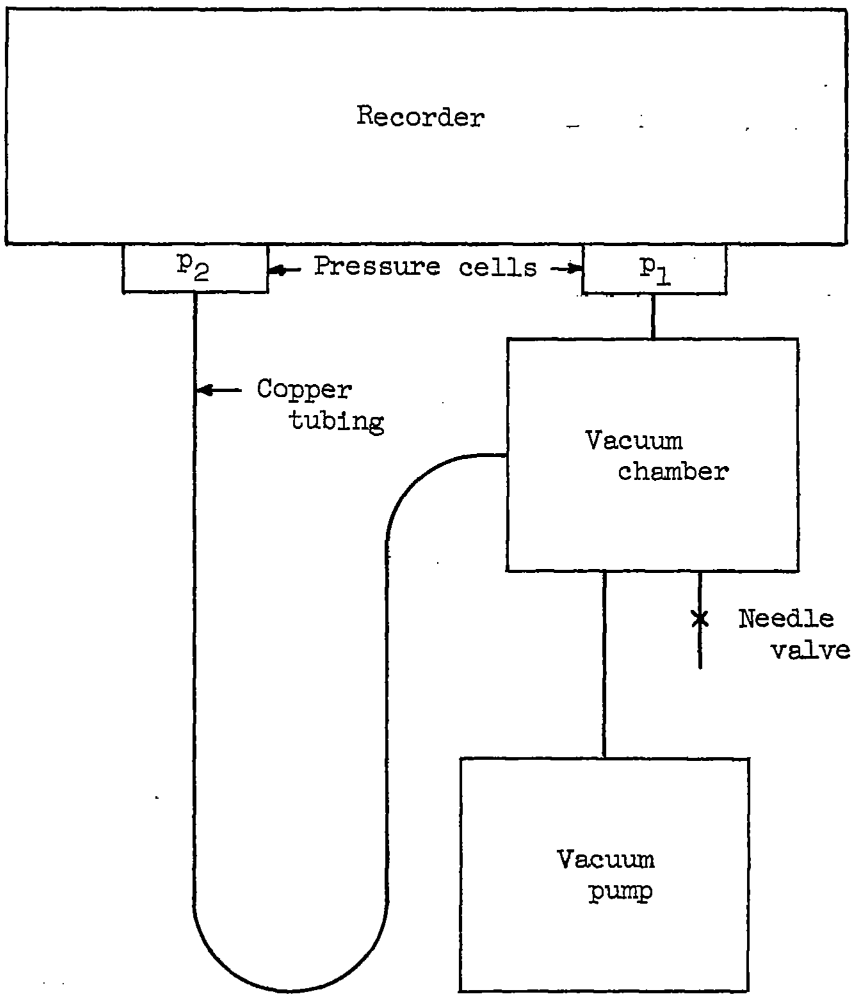

Figure 2.- Block diagram of testing apparatus.

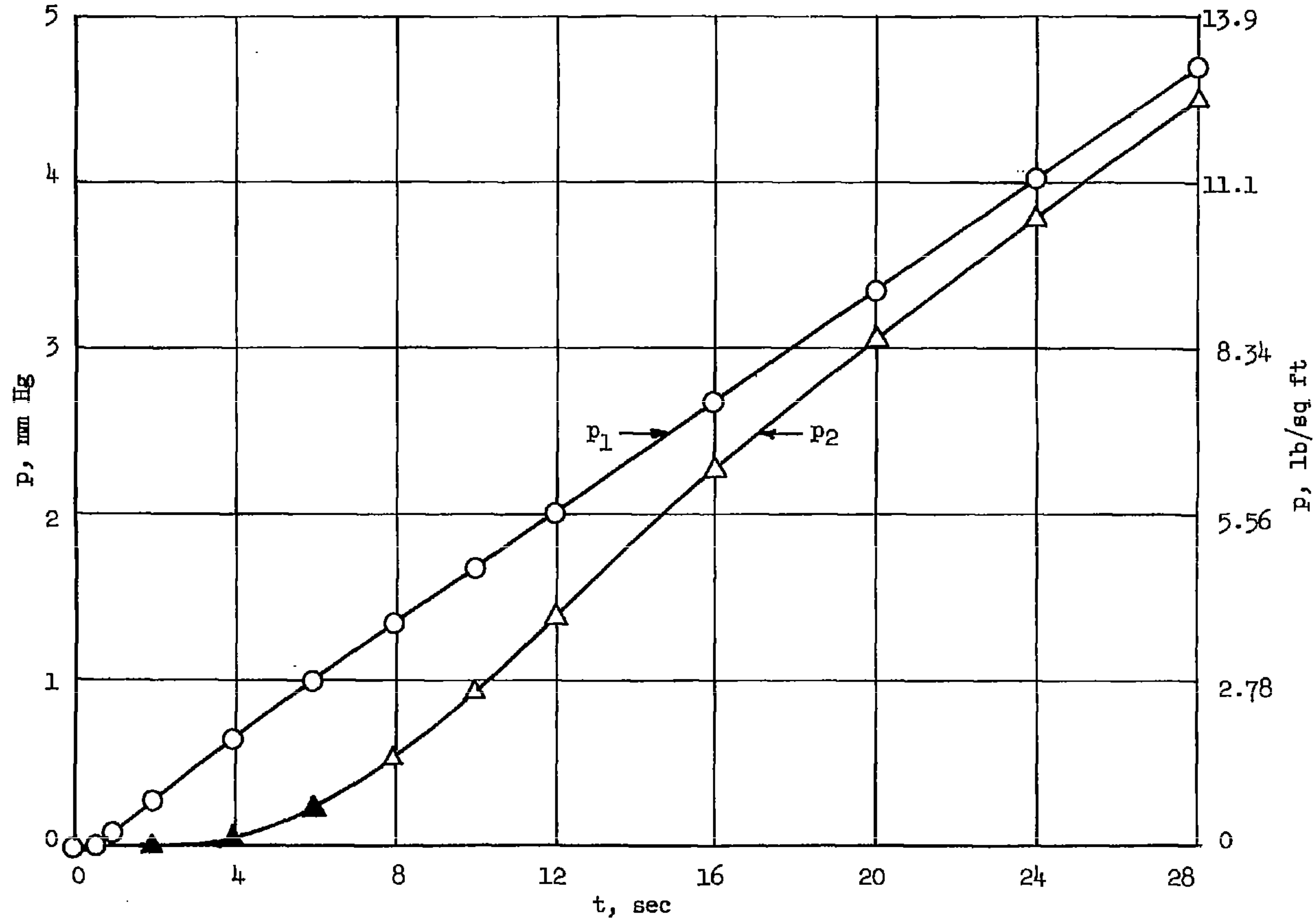

Figure 3.- Variation of pressure with time at the ends of 4 feet of copper tubing. d = 0.1225 inch. (Filled symbols indicate region of transient effects.)

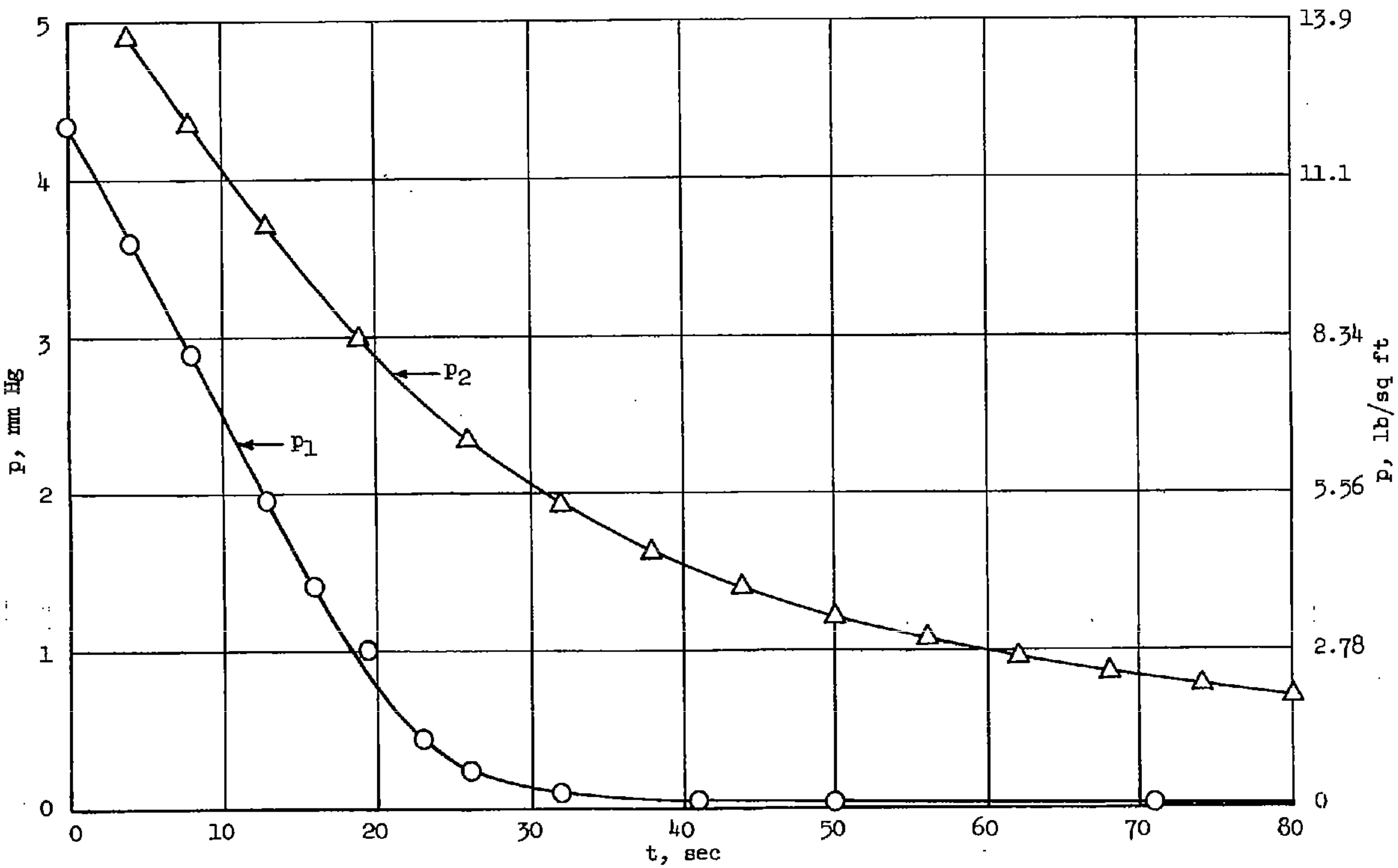

Figure 4.- Variation of pressure with time at the ends of 2 feet of copper tubing. d = 0.0597 inch.

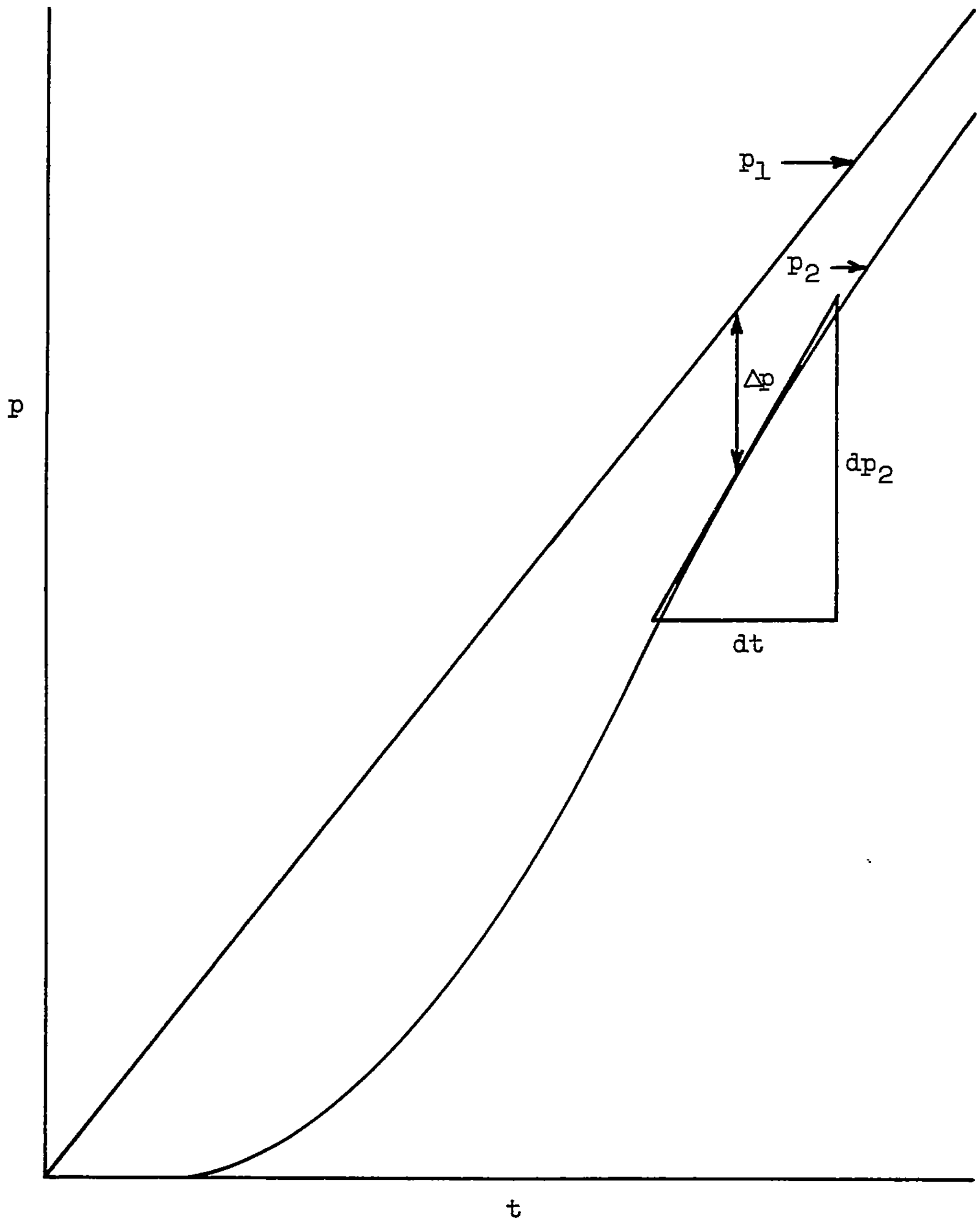

Figure 5.- Method of measuring time constant λ.

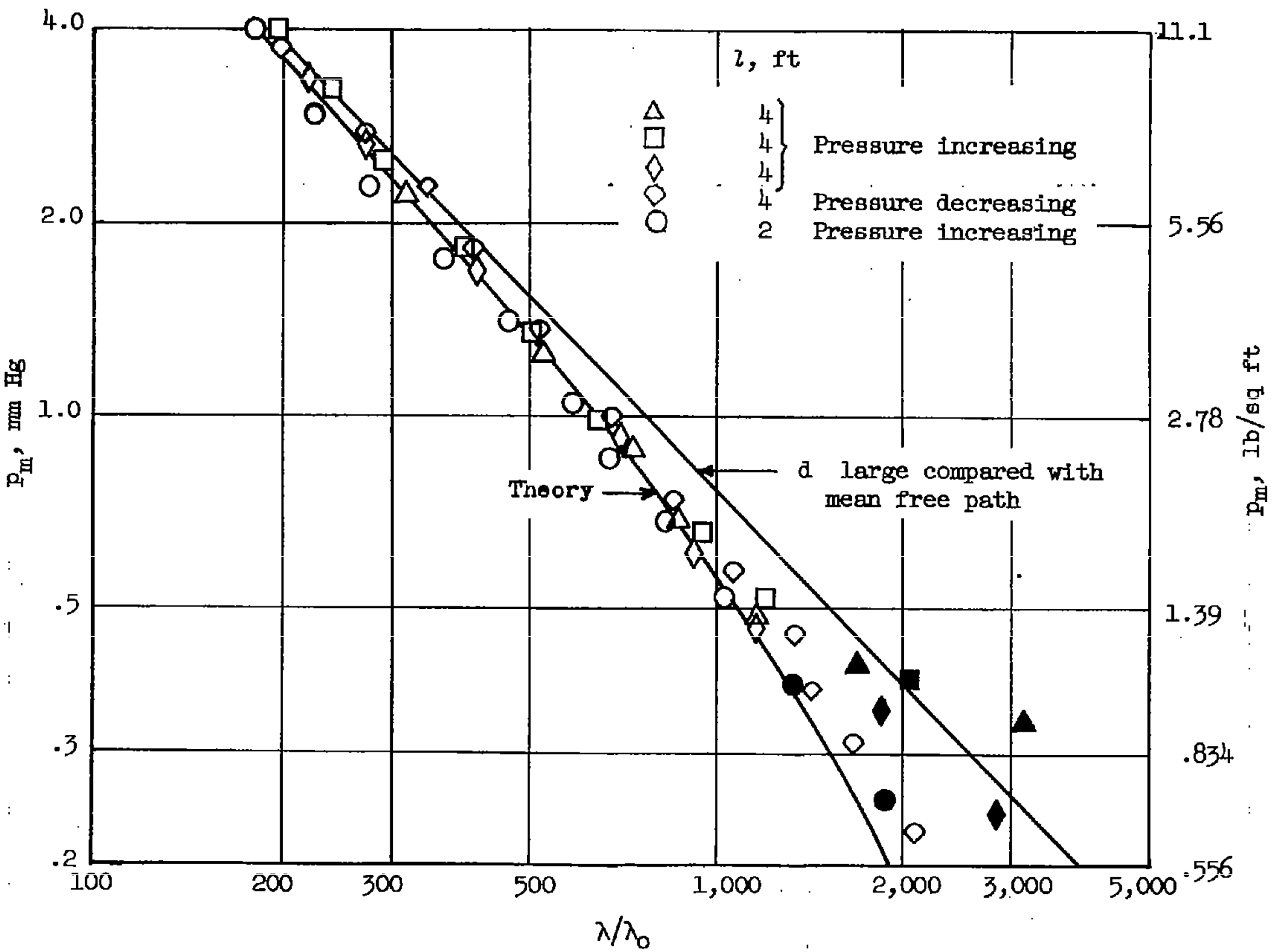

Figure 6.- Variation of mean pressure with the ratio of time constant at test conditions to time constant at reference conditions for copper tubing. d = 0.1225 inch. (Filled symbols indicate region of transient effects.)

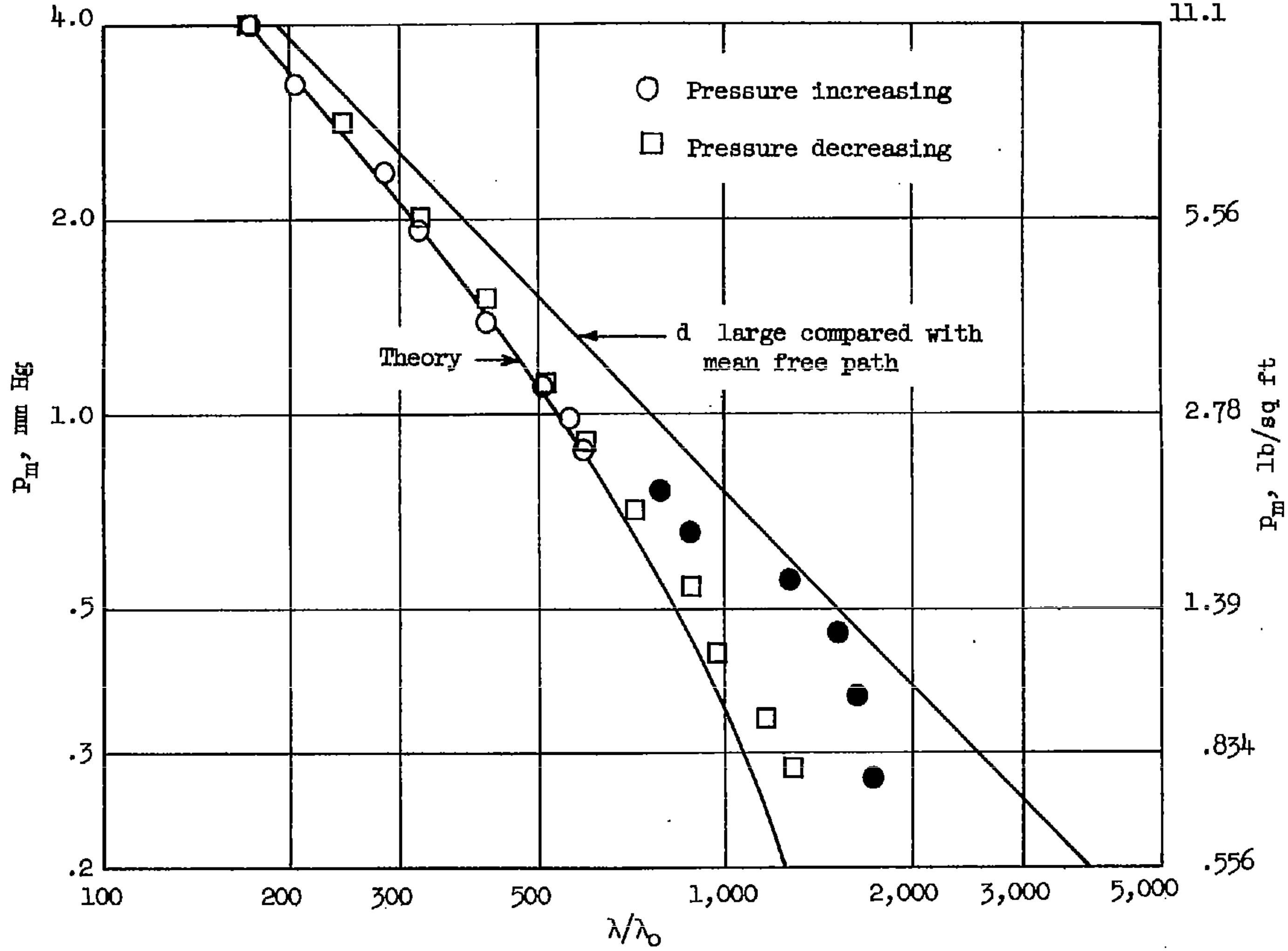

Figure 7.- Variation of mean pressure with the ratio of time constant at test conditions to time constant at reference conditions for 2 feet of copper tubing. d = 0.0597 inch. (Filled symbold indicate region of transient effects.)

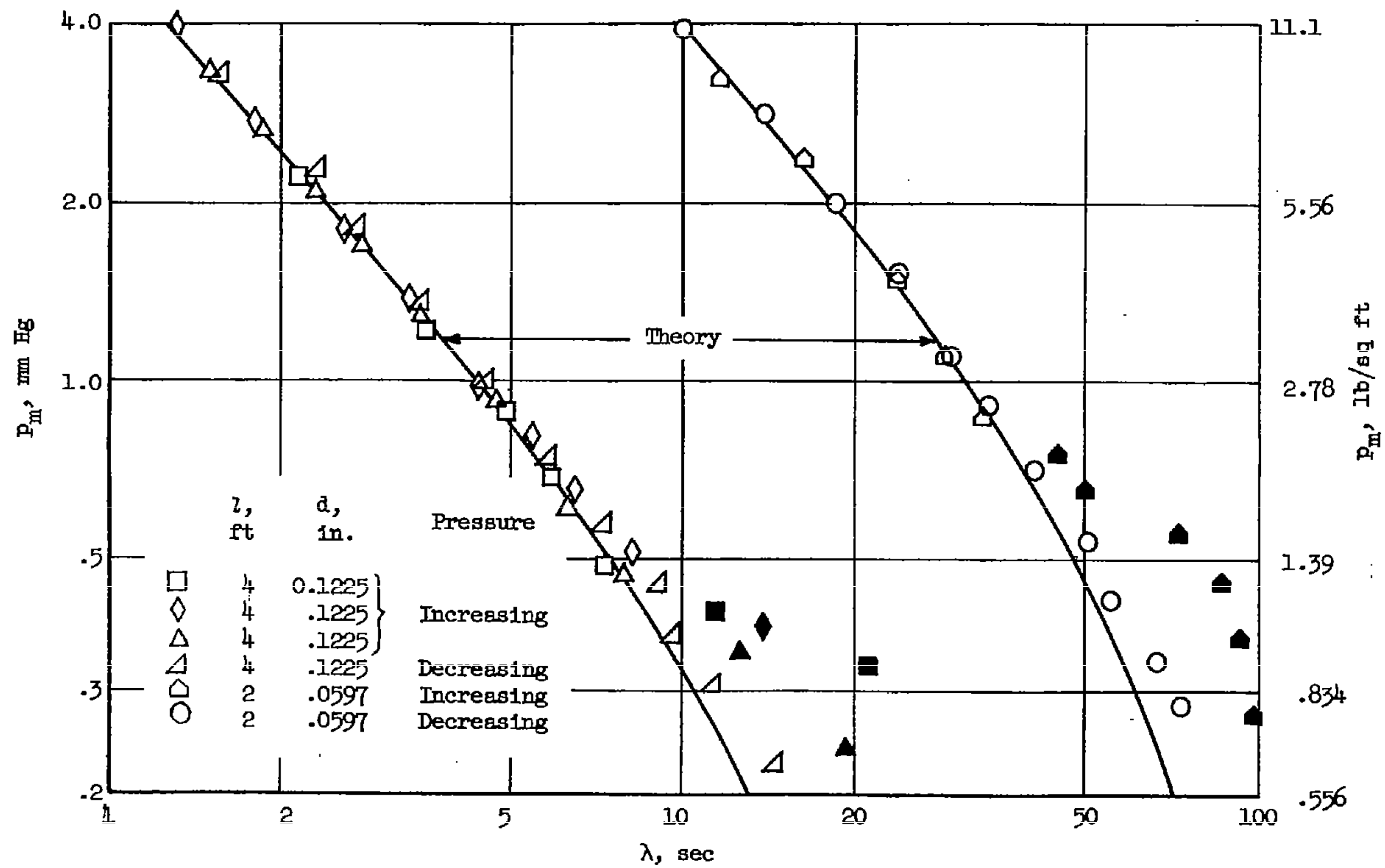

Figure 8.- Variation of mean pressure in copper tubing with time constant. (Filled symbols indicate region of transient effects.)