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RESEARCH MEMORANDUM

A LIMIT PRESSURE COEFFICIENT AND AN ESTIMATION OF LIMIT FORCES ON AIRFOILS AT SUPERSONIC SPEEDS

By

John P. Mayer

Langley Aeronautical Laboratory
Langley Field, Va.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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RESEARCH MEMORANDUM

A LIMIT PRESSURE COEFFICIENT AND AN ESTIMATION OF LIMIT FORCES ON AIRFOILS AT SUPERSONIC SPEEDS

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SUMMARY

The results of an estimation of the limit forces on airfoils at supersonic speeds are presented. The value of the maximum lift coefficient obtained from this estimation decreased from about 1.13 to 0.96 in a Mach number range from 1.4 to 3.0. Computed values of the forces on two-dimensional wings are in good agreement with three-dimensional wind-tunnel data at high angles of attack where detached shock waves are present.

The limit pressure coefficient attainable on an airfoil is shown to be equal to about 70 percent of the pressure coefficient for a vacuum over a wide range of Mach numbers. This result is based on the analysis of a large amount of experimental data.

Practical implications of the maximum lift coefficients at supersonic speeds are given as load-factor Mach number boundaries.

INTRODUCTION

In the design of supersonic airplanes and guided missiles a knowledge of the maximum loads that might be imposed is required. There has been a great deal of investigation of the aerodynamic characteristics of airfoils and wings in supersonic flow at low angles of attack where shock waves are attached. However, there is only a limited amount of information available for the case where shock waves are detached from the airfoil as would normally occur at high angles of attack. The problem of the detached shock wave has been treated theoretically in reference 1 for symmetrical bodies at zero angle of attack while reference 2 gives results of supersonic wind-tunnel tests of a variety of wings which extend to angles and speeds beyond the point of shock detachment.

Since the maximum loads that can be imposed on aircraft at supersonic speeds may be associated with high angle-of-attack conditions where shock waves are detached from the nose of the airfoil, a brief study was initiated to derive a method by which the maximum lifts and limit pressures attainable on an airfoil in supersonic flow could be estimated. A limit pressure coefficient has been established from the compilation of a large amount of experimental data on maximum negative pressures and the empirical limit pressure curve is presented. The present paper presents the results of a simplified method for obtaining limit forces on two-dimensional wings at supersonic speeds and comparisons are made with existing experimental data.

SYMBOLS

A	aspect ratio
c	section chord
c_c	section chord-force coefficient (Chord force/qc)
c_{d_p}	section pressure drag coefficient (Pressure drag/qc)
c_l	section lift coefficient (Lift/qc)
c_n	section normal-force coefficient (Normal force/qc)
d	section drag
l	section lift
M	Mach number
n	load factor
p	static pressure
P	pressure coefficient $\left(\frac{p - p_o}{q_o}\right)$
q	dynamic pressure $\left(\frac{1}{2}\rho V^2\right)$
S	wing area
t	maximum airfoil thickness
t/c	thickness ratio
V	stream velocity
W	airplane weight
x	longitudinal distance along chord

y lateral distance from chord

α angle of attack

γ ratio of specific heats taken as 1.40 $\left(c_p/c_v\right)$

θ angle between shock wave and free-stream direction

ρ stream density

ϕ angle between airfoil surface and free-stream direction

Subscripts:

1 local Mach number of 1.0

D shock detachment

L limit

max maximum

o free stream

U ultimate or vacuum

LIMIT NEGATIVE PRESSURE COEFFICIENT

Although the absolute limit to the minimum pressure attainable on an airfoil is an absolute vacuum, experiments have indicated that there is some higher physical limit to the value obtainable. Various investigators have sought a value of the limit pressure for some time. Reference 3 presents one such limit-pressure-coefficient curve obtained from limited flight test data and certain theoretical considerations. Other limit-pressure-coefficient curves have been used and they have usually been expressed as constant local Mach numbers, constant pressure ratios, or faired curves of experimental data.

Over a period of several years, a large amount of flight and wind-tunnel experimental data on maximum negative pressure coefficients attainable on various aerodynamic bodies has been collected and an envelope or limit-pressure curve has been established. This curve of limit negative pressure coefficient P_L is presented as the dash line in figure 1. The test points shown on the limit curve are the maximum values of negative pressure coefficient obtained at each Mach number, and the curve represents the envelope of hundreds of test points. No attempt has been made to reference the experimental points because of the many sources used. From these data the equation of the limit curve was found to be

$$P_L M_o^2 = -1 \tag{1}$$

For the limit pressure coefficient given by equation (1) it can be shown that the ratio of the limit pressure coefficient to the ultimate or vacuum pressure coefficient $\frac{P_L}{P_U}$ is equal to $\gamma/2$. Also, the ratio of the limit static pressure to the free-stream static pressure $\frac{p_L}{p_o}$ is equal to $1 - \frac{\gamma}{2}$. Thus, the empirical pressure coefficient obtained corresponds to 70 percent of the pressure coefficient for a vacuum or to a static pressure ratio of 0.30.

MAXIMUM POSITIVE PRESSURE COEFFICIENT

The method for computing the limit forces and therefore the maximum loads at supersonic speeds used in the present paper is based on the limit negative pressure coefficient just obtained (equation (1)) and the maximum positive pressure coefficient behind a normal shock wave. The maximum possible positive pressure is equal to the total head. In supersonic flow the maximum positive pressure coefficient for this condition behind a normal shock is

$$P_{max} = \frac{1}{\frac{1}{2}\gamma M_o^2}\left\{\left[\frac{\gamma + 1}{2\gamma M_o^2 - (\gamma - 1)}\right]^{\frac{1}{\gamma-1}}\left[\left(\frac{\gamma + 1}{2}\right)M_o^2\right]^{\frac{\gamma}{\gamma-1}} - 1\right\} \tag{2}$$

The maximum positive pressure coefficient behind a normal shock wave is shown in figure 1 by the part of the line designated P_{max} above a Mach number of 1.0.

In addition to the limit pressure coefficient and the maximum positive pressure coefficient for supersonic flow, certain other quantities such as the maximum positive pressure coefficient for subsonic flow P_{max}, the pressure coefficient for a vacuum P_U, the pressure coefficient for local sonic velocity P_1, and the pressure coefficient for shock detachment P_D are shown in figure 1 as a matter of interest. The equations from which these quantities were obtained are listed in the appendix.

LIMIT FORCE COEFFICIENTS AT SUPERSONIC SPEEDS

Method of estimating limit forces.- The maximum and limit pressure coefficients developed previously are used to estimate the limit forces on an airfoil in supersonic flow. If it is assumed as a first approximation that, at high angles of attack, the shock is normal in front of the wing, that the pressure coefficients at every point on the upper surface of the airfoil have reached the limit negative pressure coefficient, and that the average normal force on the lower surface is proportional to the projected surface perpendicular to the free-stream direction, the following equations are obtained:

$$c_{n_L} = P_{max} \int \sin \phi \, d\frac{x}{c} - P_L \tag{3}$$

$$c_{c_L} = P_{max} \int \sin \phi \, d\frac{y}{c} \tag{4}$$

$$c_{l_L} = c_{n_L} \cos \alpha - c_{c_L} \sin \alpha \tag{5}$$

$$c_{d_{p_L}} = c_{n_L} \sin \alpha + c_{c_L} \cos \alpha \tag{6}$$

For thin airfoils, these equations may be given as:

$$c_{n_L} = P_{max} \sin \alpha - P_L \tag{7}$$

$$c_{l_L} = P_{max} \sin \alpha \cos \alpha - P_L \cos \alpha \tag{8}$$

$$c_{d_{p_L}} = P_{max} \sin^2 \alpha - P_L \sin \alpha \tag{9}$$

The maximum lift coefficient may be found by differentiating the equation for the limit lift coefficient (equation (8)) with respect to the angle of attack and equating to zero. The maximum lift coefficient obtained as a result of this differentiation is

$$c_{l_{max}} = P_{max} \sin \alpha_{max} \cos \alpha_{max} - P_L \cos \alpha_{max} \tag{10}$$

where

$$\alpha_{max} = \sin^{-1} \frac{1}{4P_{max}} \left(P_L + \sqrt{P_L^2 + 8P_{max}^2}\right)$$

and where P_L and P_{max} are the limit and maximum pressure coefficients given by equations (1) and (2), respectively.

Comparisons with experiment.- A comparison between the calculated limit lift and drag coefficients based on the previous assumptions and the experimental values of lift and drag from the supersonic wind-tunnel tests of reference 2 is shown in figure 2. Wind-tunnel results are shown for a rectangular wing with a circular-arc airfoil section at Mach numbers of 1.55, 1.90, and 2.32. It can be seen that, although the equations from which the calculated lift coefficients were obtained are based on a two-dimensional analysis, the results agree fairly well with the three-dimensional wind-tunnel data. It should be noted that the limit coefficients are based on maximum pressures behind a normal shock wave and that at the low angles of attack where attached oblique shock waves are present or where the pressures on the upper surface have not reached the limit pressure the calculated limit lift coefficients would differ from the measured lift coefficients. As the angle of attack is increased it can be noted in figure 2 that the measured lift coefficient increases until it reaches the calculated lift coefficient and then follows the calculated lift coefficient to the extent of the test data.

Shown in figure 2, in addition to the calculated force coefficients at high angles of attack, are the theoretical angle of attack where the shock detaches from the leading edge of the airfoil, the theoretical lift and drag curves for the airfoil up to the point of shock detachment, and the linearized three-dimensional lift curves for the particular rectangular wings tested in reference 2.

In the wind-tunnel tests of the small wing models at maximum lift at supersonic speeds, models with various plan forms and airfoil shapes were tested. The tests showed no appreciable difference in the maximum lift coefficient with wing shape. Although, in figure 2, comparisons are shown only for rectangular wings, the agreement is equally as good for the other plan forms tested. The other wings tested consisted of triangular, sweptback, and trapezoidal plan forms with aspect ratios up to 4.06 and had various airfoil sections. Presented in figure 3 are comparisons of the experimental maximum lift coefficients and drag coefficients at maximum lift for all of the models tested in the supersonic tunnel with maximum lift and drag coefficients calculated from equations (9) and (10). It can be noted that, within the experimental error of the tests, the measured and calculated maximum lift coefficients agree well with each other. The experimental accuracy of the maximum lift coefficient in the wind-tunnel tests was stated to be about ±0.05. The measured drag coefficients at maximum lift are somewhat higher than the calculated pressure drag coefficients. The wind-tunnel-measured total drag coefficients were estimated to be from 4 to 6 percent high, so the calculated drag would be closer to the tunnel-corrected total drag.

APPLICATION AND DISCUSSION

Presented in figure 4 are the variations with Mach number of the maximum lift together with the normal-force coefficient, the drag coefficient, the angle of attack, and the lift-drag ratio at maximum lift, as calculated from equations (7) to (10). It can be seen that the calculated maximum lift coefficient decreases with increasing Mach number approaching a value of $C_{L_{max}} = 0.92$, and that the angle of attack for maximum lift increases with Mach number, approaching an angle of 45° as the Mach number increases. The computed drag coefficient is approximately constant at maximum lift with a value of about 0.92. The lift-drag ratio and normal-force coefficient calculated at maximum lift decrease with increasing Mach number, approaching values of $\frac{l}{d} = 1.0$ and $c_n = 1.30$, respectively.

The results given in figure 4 are for Mach numbers only above an arbitrarily selected value of $M = 1.4$ since it is believed that the estimated limit force coefficients would not be reached at low supersonic Mach numbers near 1.0. In this region the maximum lift might be determined by flow separation characteristic of subsonic flow conditions. At the higher supersonic Mach numbers, however, the maximum lift coefficient is associated with the rearward inclination of the normal-force vector at the high angles of attack and differs from the customary breakdown of lift at subsonic speeds.

An application of the maximum lift coefficient at supersonic speeds is shown in figure 5 as a normal load-factor Mach number diagram. The load factors given are based on the normal-force coefficient at maximum lift. Although the normal-force coefficient is still increasing at maximum lift and greater normal load factors could be obtained beyond maximum lift, it is felt that the occurrence of maximum lift imposes a more practical limit on the possible load factors attainable. It can be observed in figure 5 that the load factors obtainable at supersonic speeds are very high, especially at the lower altitudes. Although the high normal load factors at medium altitudes are beyond human endurance, the need for the knowledge of these load factors is important in the structural design of guided missiles where these high load factors conceivably could be reached.

CONCLUDING REMARKS

The results of a simplified method of estimating the limit forces on two-dimensional airfoils at supersonic speeds are presented and fairly good agreement is obtained with three-dimensional wind-tunnel data on wings at high angles of attack where detached shock waves are present. The value of the maximum lift coefficient obtained from this estimation decreased from about 1.13 to 0.96 in a Mach number range from 1.4 to 3.0.

A limit pressure coefficient attainable on an airfoil is shown to be equal to about 70 percent of the pressure coefficient for a vacuum over a wide range of Mach numbers. This result is based on a large amount of experimental data.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va.

APPENDIX

For subsonic flow, the maximum positive pressure coefficient is

$$P_{max} = \frac{1}{\frac{1}{2}\gamma M_o^2}\left\{\left[1 + \left(\frac{\gamma - 1}{2}\right) M_o^2\right]^{\frac{\gamma}{\gamma-1}} - 1\right\} \tag{A1}$$

The maximum positive pressure coefficient for subsonic flow is shown in figure 1 as the part of the line designated P_{max} below a Mach number of 1.0.

The ultimate or vacuum pressure coefficient is given by the equation

$$P_U = \frac{-1}{\frac{1}{2}\gamma M_o^2} \tag{A2}$$

and is shown in figure 1 by a solid line in the negative pressure-coefficient range.

The sonic curve given in figure 1 by the part of the line designated P below a Mach number of 1.0 is defined by

$$P_1 = \frac{1}{\frac{1}{2}\gamma M_o^2}\left\{\left[\frac{1 + \left(\frac{\gamma - 1}{2}\right) M_o^2}{\left(\frac{\gamma + 1}{2}\right)}\right]^{\frac{\gamma}{\gamma-1}} - 1\right\} \tag{A3}$$

For Mach numbers greater than 1.0 where shock waves are attached to the airfoil, the pressure coefficient for a local Mach number of 1.0 can be found to be

$$P_1 = \frac{4}{(\gamma + 1)M_o^2}\left(M_o^2 \sin^2\theta_1 - 1\right) \tag{A4}$$

where

$$\sin^2\theta_1 = \frac{1}{4\gamma M_o^2}\left\{(\gamma + 1)M_o^2 + (\gamma - 3) + \sqrt{(\gamma + 1)\left[(\gamma + 1)M_o^4 + 2(\gamma - 3)M_o^2 + (\gamma + 9)\right]}\right\}$$

This curve is shown in figure 1 by the part of the line designated P_1 above a Mach number of 1.0.

The pressure coefficient for shock detachment is

$$P_D = \frac{4}{(\gamma + 1)M_o^2}\left(M_o^2 \sin^2\theta_D - 1\right) \tag{A5}$$

where

$$\sin^2\theta_D = \frac{1}{4\gamma M_o^2}\left\{(\gamma + 1)M_o^2 - 4 + \sqrt{(\gamma + 1)\left[(\gamma + 1)M_o^4 + 8(\gamma - 1)M_o^2 + 16\right]}\right\}$$

The pressure coefficient for shock detachment is shown in figure 1 by a line designated P_D.

[redacted]

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3. Rhode, Richard V.: Correlation of Flight Data on Limit Pressure Coefficients and Their Relation to High-Speed Burbling and Critical Tail Loads. NACA ACR L4I27, 1944.

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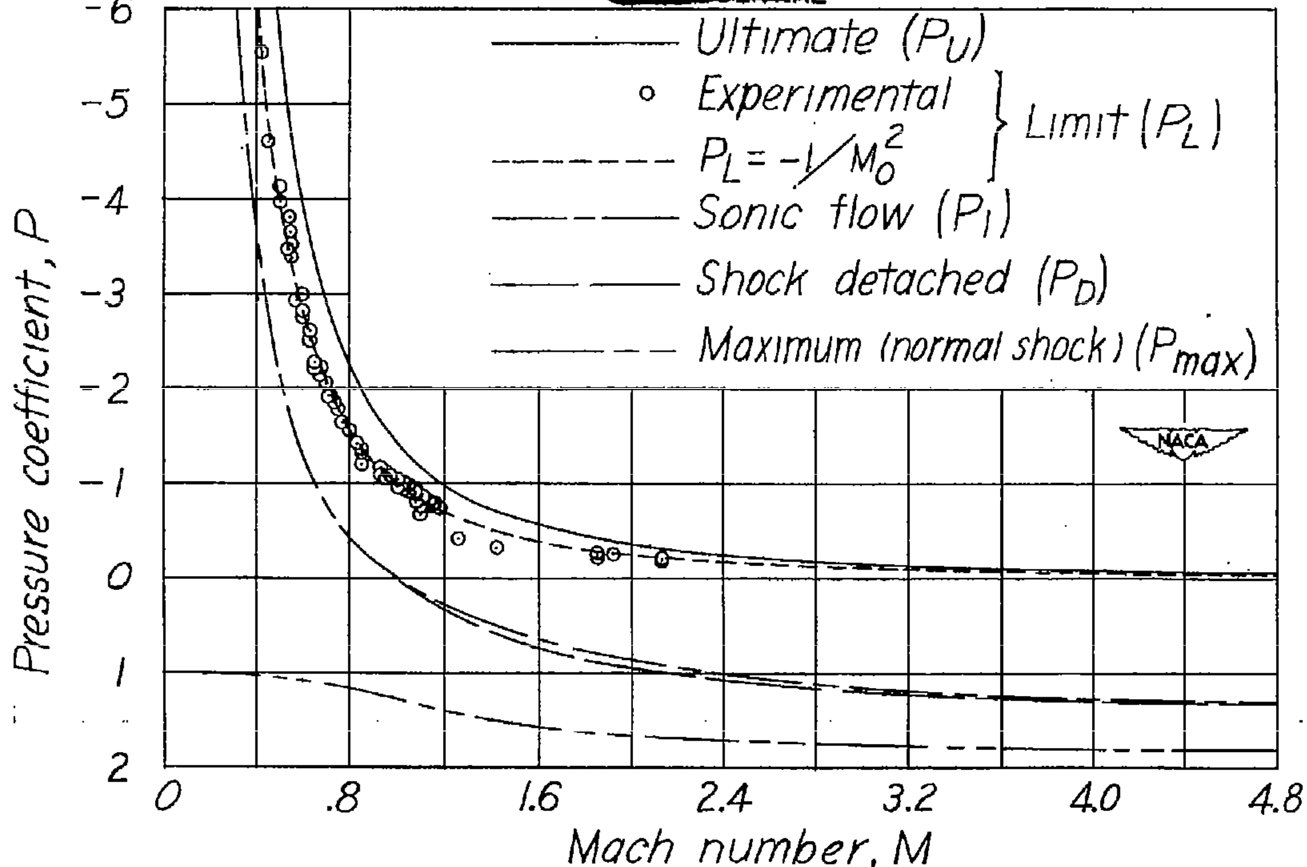

Figure 1.- Variation of pressure coefficient with Mach number for limit, maximum, and ultimate pressure and for sonic flow and shock detachment.

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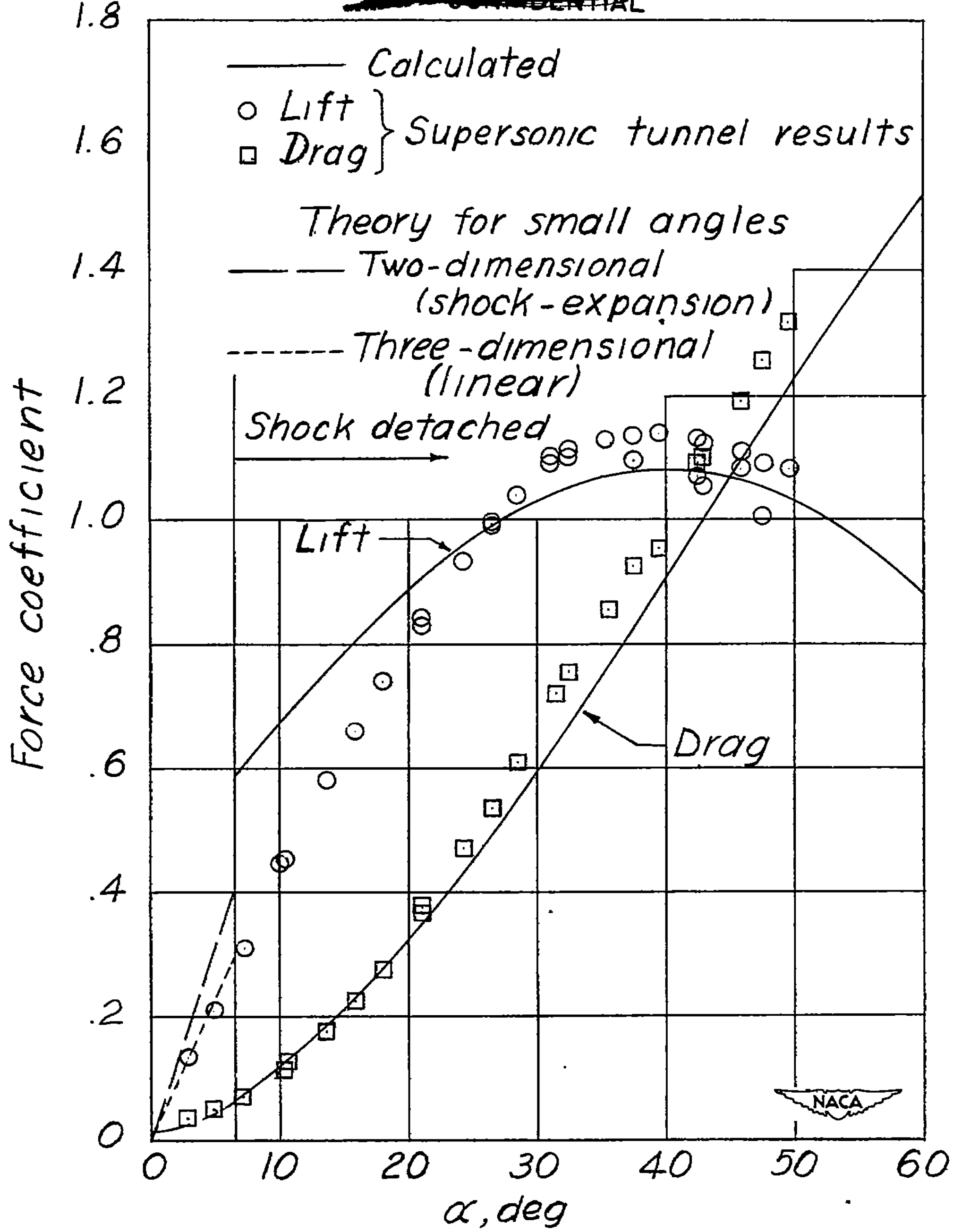

(a) $M = 1.55$; $A = 1.74$; $\frac{t}{c} = 0.06$.

Figure 2.– Comparison between calculated and experimental force coefficients for rectangular wings with biconvex airfoil sections.

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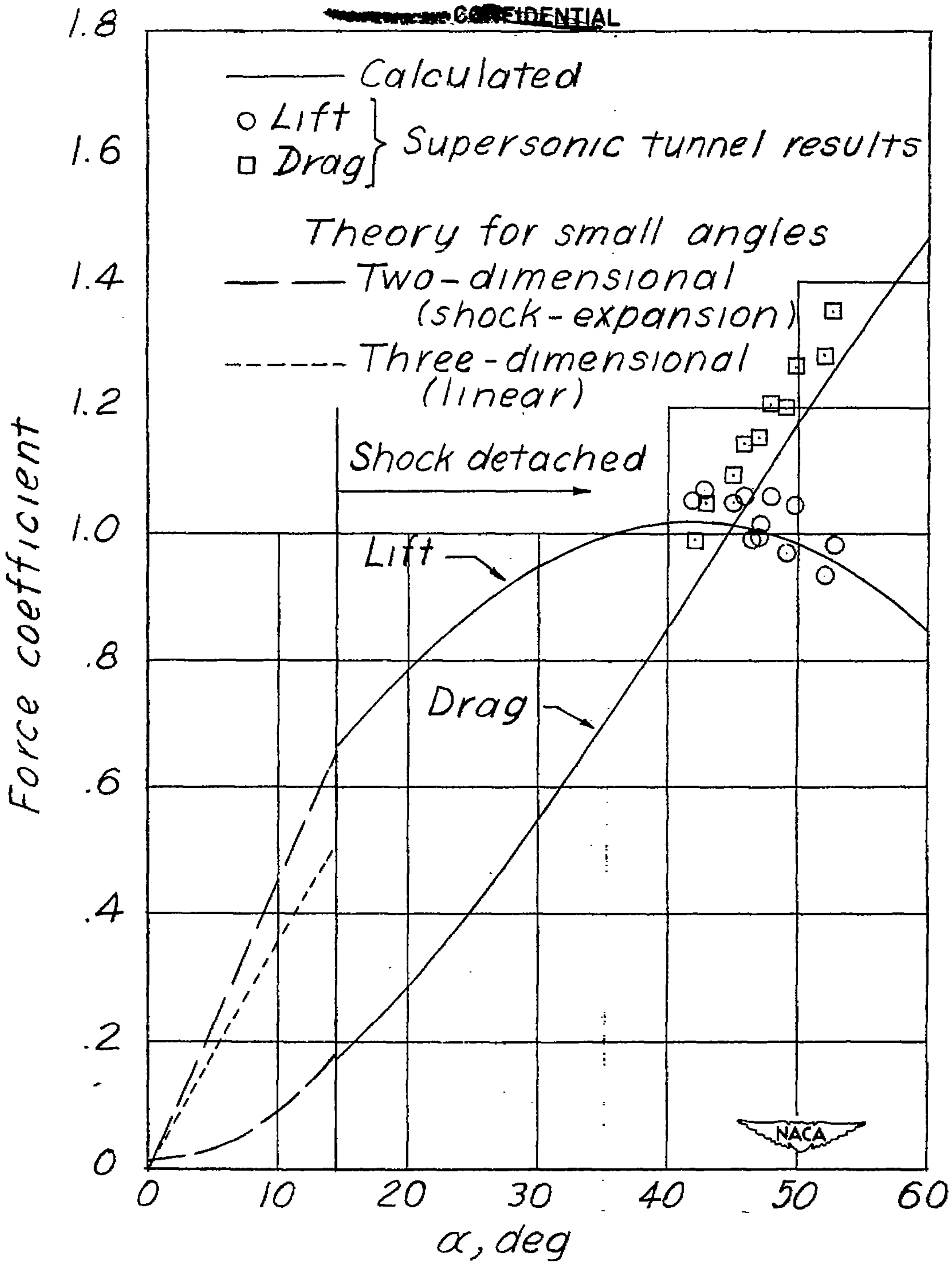

(b) $M = 1.90$; $A = 1.74$; $\frac{t}{c} = 0.06$.

Figure 2 .- Continued.

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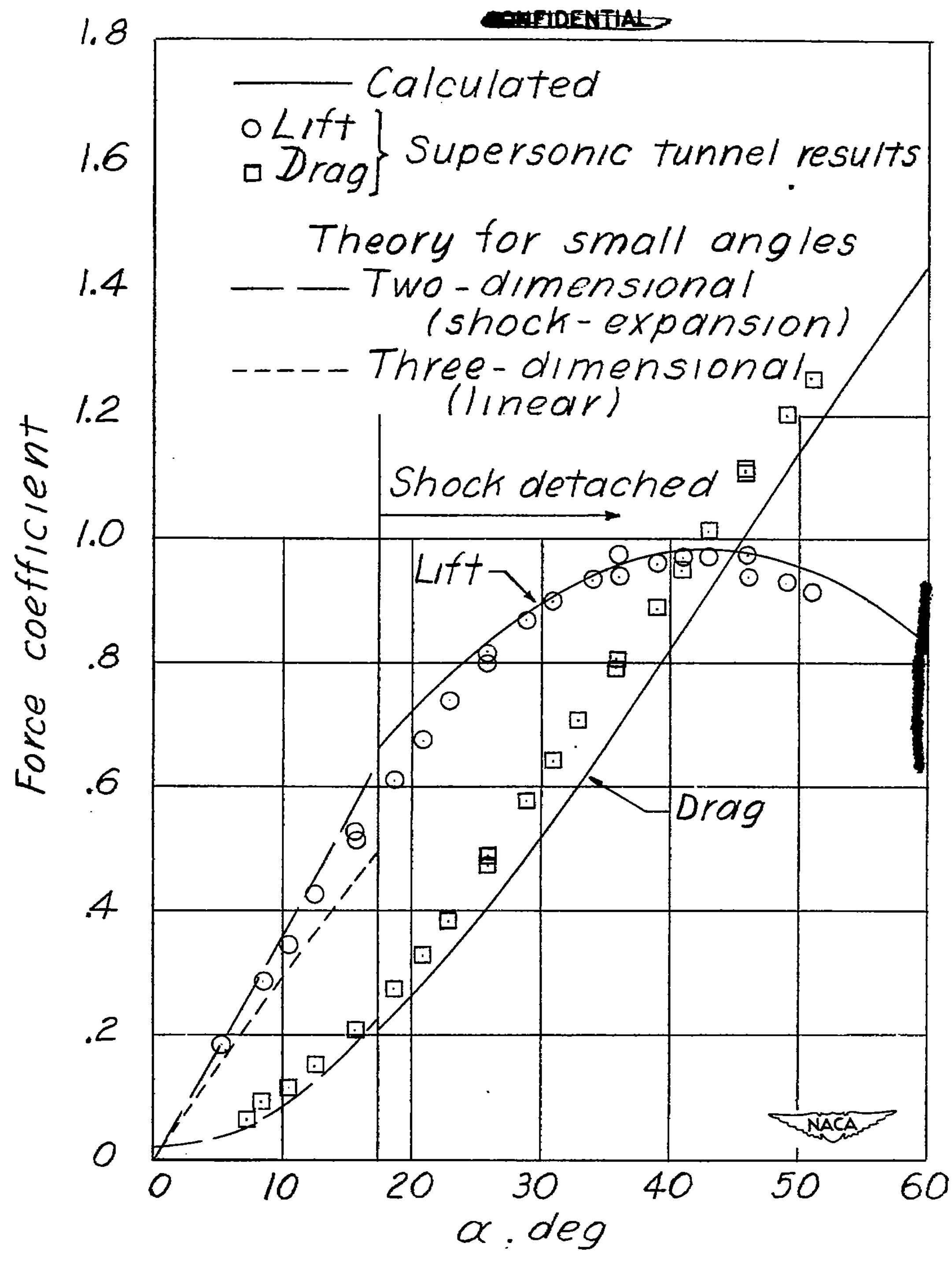

(c) $M = 2.32$; $A = 1.99$; $\frac{t}{c} = 0.09$.

Figure 2.- Concluded.

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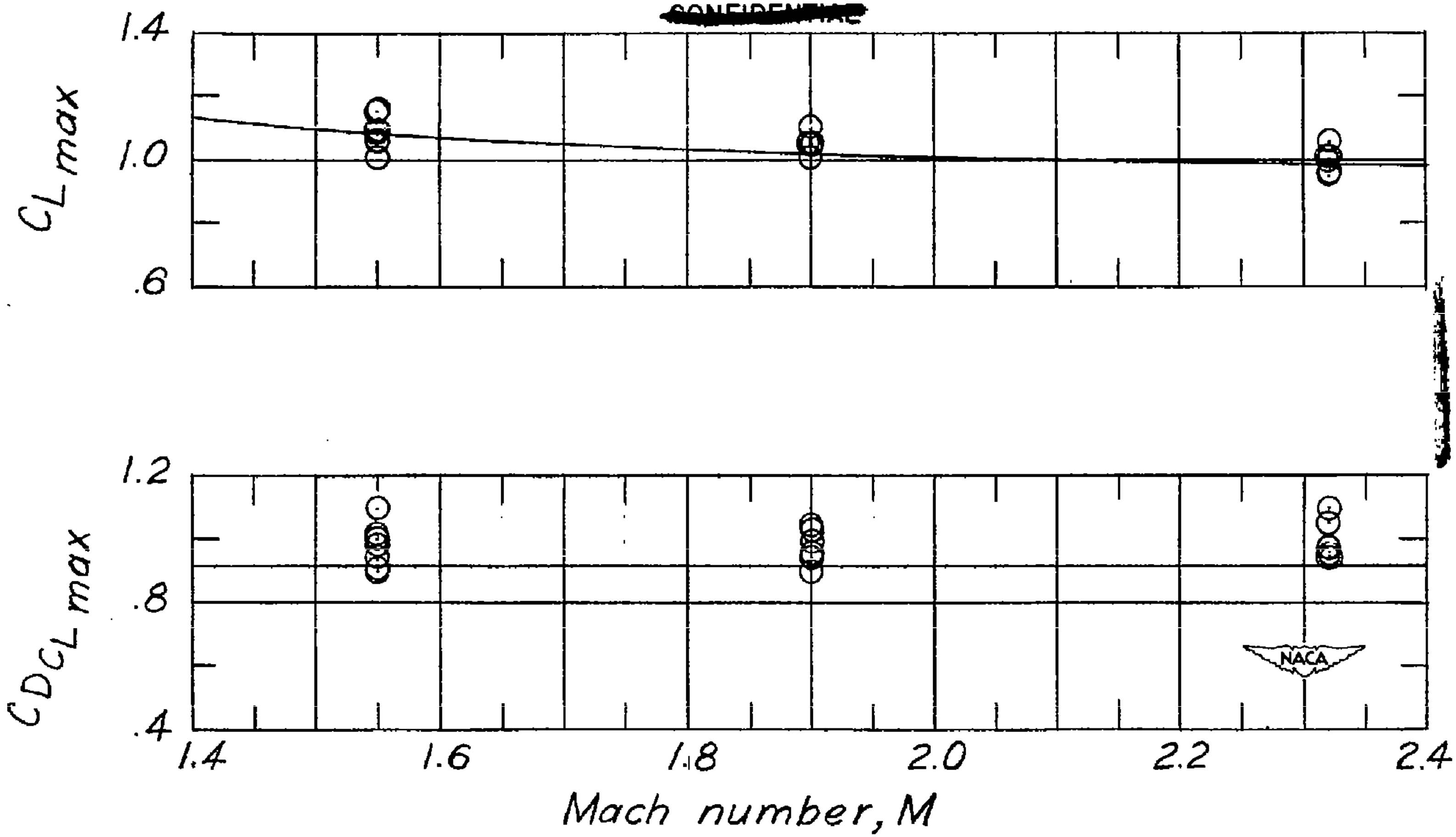

Figure 3.- Comparison between calculated and experimental lift and drag coefficients at maximum lift.

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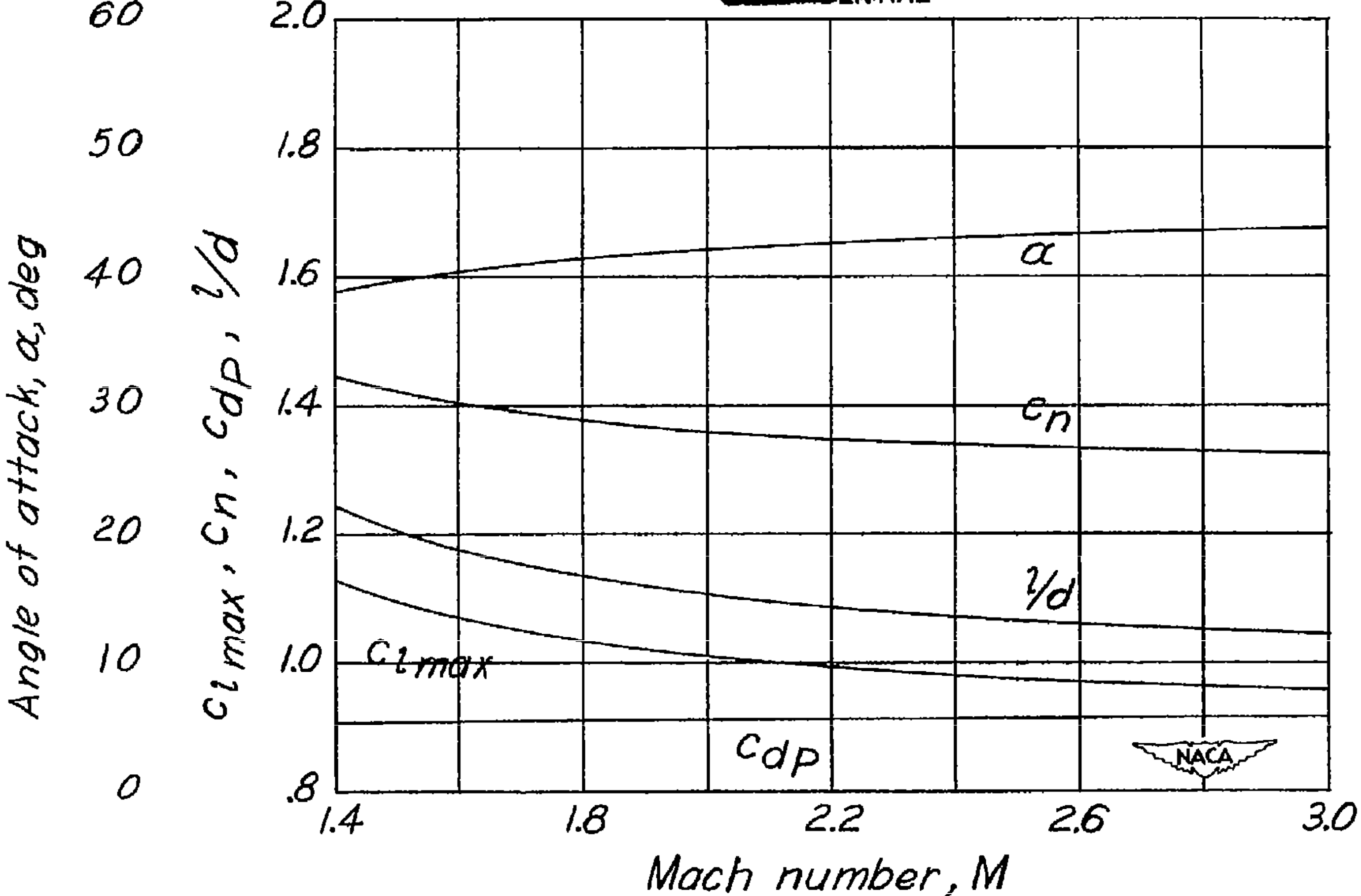

Figure 4.- Variation of calculated force coefficients, angle of attack, and lift-drag ratio at maximum lift with Mach number at supersonic speeds.

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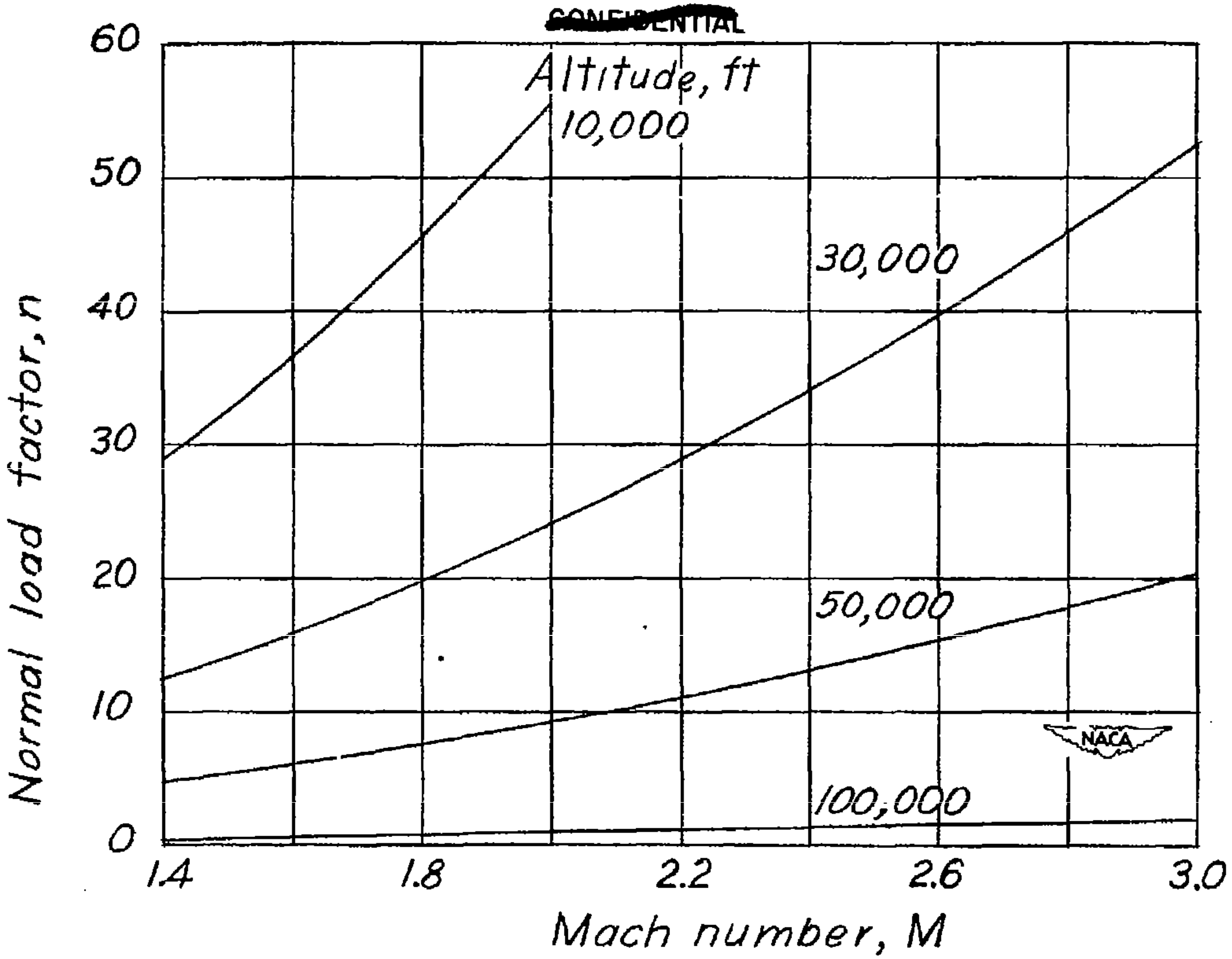

Figure 5.- Maximum lift-load factor boundaries for supersonic ai[redacted] $(\frac{W}{S} = 100)$

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