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RESEARCH MEMORANDUM

EFFECT OF DOWNWASH ON THE ESTIMATED ELEVATOR DEFLECTION REQUIRED FOR TRIM OF THE XS-1 AIRPLANE AT SUPERSONIC SPEEDS

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EFFECT OF DOWNWASH ON THE ESTIMATED ELEVATOR DEFLECTION REQUIRED FOR TRIM OF THE XS-1 AIRPLANE AT SUPERSONIC SPEEDS

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SUMMARY

This report contains the results of an investigation to determine from linearized theory, which has recently become available, the downwash at supersonic speeds at the tail of the XS-1 airplane and the effect of the downwash on the elevator deflection required for trim. The results are presented in the form of curves showing the variation of downwash angle with angle of attack $\frac{d\epsilon}{d\alpha}$ and elevator deflection required for trim plotted against Mach number.

The average value across the span of the horizontal tail (neglecting the fuselage) of $\frac{d\epsilon}{d\alpha}$ is about 0.5 at a Mach number of 1.1 and decreases rapidly to a value of about 0.08 at a Mach number of 1.4. The value of $\frac{d\epsilon}{d\alpha}$ then gradually decreases to 0 at a Mach number of about 1.9 with the possibility of a very slight amount of upwash in the Mach number range from 1.9 to 2.2. Above a Mach number of 2.2 the Mach cones from the wing tips are outboard of the tail surfaces and $\frac{d\epsilon}{d\alpha}$ is the same as if the tail were in two-dimensional flow $\left(\text{that is, } \frac{d\epsilon}{d\alpha} = 0\right)$.

The calculations indicate that increasing up-elevator deflection is required with increasing Mach number (unstable variation) in level flight between Mach numbers of 1.1 and 1.6. A slight reduction in up-elevator deflection occurs between Mach numbers of 1.6 and 2.0. The stabilizer angle has a similar variation, that is, unstable up to a Mach number of about 1.6 and then becoming slightly stable up to a Mach number of 2.0. The reduction of downwash with increasing Mach number is not the main cause of the increase in up-elevator deflection. The main reasons for this trend are that the pitching-moment coefficients due to the wing camber, the wing lift, and the lift of the stabilizer are all in a nose-down direction, and as the Mach number increases, these pitching-moment coefficients apparently decrease less rapidly than the elevator effectiveness.

INTRODUCTION

Any information that can be used to predict the stability and control changes of an airplane at supersonic speeds is urgently needed at the present time. This paper presents the variation of downwash with angle of attack at supersonic speeds for the XS-1 airplane. This variation was obtained by applying several simplifying assumptions to Lagerstrom's linearized-theory calculations for the downwash of three-dimensional lifting wings at supersonic speeds. Several curves showing the estimated variation of elevator deflection required for trim with and without the effect of downwash are presented to give an indication of the effect of downwash on the longitudinal stability and control of the airplane.

SYMBOLS

α	angle of attack
A	aspect ratio
$\bar{c}$	mean aerodynamic chord
C_L	lift coefficient (L/qS)
$C_{L_\alpha} = \frac{dC_L}{d\alpha}$	
$C_{L_\delta} = \frac{\partial C_L}{\partial \delta}$	
C_{m_o}	pitching-moment coefficient of the wing-fuselage combination about its aerodynamic center $\left(\frac{M_o}{qS\bar{c}}\right)$
$\frac{d\epsilon}{d\alpha}$	variation of downwash with angle of attack
i_t	stabilizer incidence, degrees
δ_e	elevator deflection, degrees (measured relative to stabilizer)
l	tail length (measured from c.g. of airplane to hinge line of elevator)
M	Mach number

q	dynamic pressure $\left(\frac{1}{2}\rho V^2\right)$
S	surface area, square feet
x	distance from center of gravity to aerodynamic center of wing-fuselage combination (positive for aerodynamic center ahead of c.g.)

Subscripts:

t	tail
w	wing
e	elevator

ANALYSIS

Calculations of the variation of downwash at the tail with angle of attack were made using reference 1. Theoretical calculations based on the linearized theory of supersonic flow are presented in reference 1 for the downwash at supersonic speeds of trapezoidal wings and rectangular wings. Since no calculations were presented for a tapered wing similar to the wing of the XS-1 airplane, a rectangular wing of the same area and span was assumed in this investigation.

The data of reference 1 for the trapezoidal wings with tips cut off along the inboard edge of the Mach cones from the wing tip are more complete than those for the rectangular wings. It was found by comparing the curves of reference 1 for the case in which the tail was in the plane and infinitely far behind the wing that the downwash was almost identical for both types of wings provided the span of the trapezoidal wing was taken slightly larger than the span of the rectangular wing. For this reason the more complete data for the trapezoidal wing were used as an aid in fairing the curves used to estimate the downwash at the tail of the XS-1 airplane.

A three-view drawing is presented in figure 1 showing the pertinent dimensions and characteristics of the XS-1 airplane. Figure 2 presents the theoretical variation of $\frac{d\epsilon}{d\alpha}$ with Mach number. The values of $\frac{d\epsilon}{d\alpha}$ presented are average values over the semispan of the horizontal tail. It is expected that the actual downwash at supersonic speeds will be less then the theoretical value below a Mach number of about 1.1 and will fair into the subsonic values. Above a Mach number of 2.2 the Mach cones from the wing tips are outboard of the tail surfaces and $\frac{d\epsilon}{d\alpha}$ is the same as if the tail were in two-dimensional flow $\left(\text{that is, } \frac{d\epsilon}{d\alpha} = 0\right)$.

The elevator deflections required for trim were computed by equating the pitching moments of the airplane to zero about its center of gravity $(0.25\bar{c})$ using the following relation:

$$C_{m_o}qS\bar{c} + C_LqSx - C_{L_{\alpha_t}}\alpha_tqS_tl - C_{L_{\delta_e}}\delta_eqS_tl = 0$$

Figure 3 presents the assumed variation with Mach number of the pitching-moment coefficient C_{m_o}, the lift-curve slopes for the wing and tail $C_{L_{\alpha_w}}$ and $C_{L_{\alpha_t}}$, and the elevator effectiveness $C_{L_{\delta_e}}$. The experimental curves at subsonic speeds were arbitrarily faired into the theoretical curves at supersonic speeds as shown by the dashed lines. The experimental subsonic values were used as an aid in fairing the values near a Mach number of unity, as it is generally accepted that the linearized-supersonic-flow theory is not applicable in the low supersonic range of Mach numbers. The experimental values of $C_{L_{\alpha_w}}$ and $C_{L_{\alpha_t}}$ were obtained from reference 2. The experimental values of $C_{L_{\delta_e}}$ were obtained from reference 3. The pitching-moment coefficient at zero lift about the aerodynamic center C_{m_o} was calculated from the formula given in reference 4 which is based on the linearized theory for two-dimensional flow. The supersonic values of C_{L_α} were calculated from the following relation:

$$C_{L_\alpha} = \frac{4}{\sqrt{M^2 - 1}}\left(1 - \frac{1}{2A\sqrt{M^2 - 1}}\right)$$

The values of $C_{L_{\delta_e}}$ at supersonic speeds were calculated from reference 5; however, these values were found by comparison with unpublished experimental data to be about 50 percent too high at all Mach numbers. The values of $C_{L_{\delta_e}}$ used herein were reduced accordingly.

An average subsonic value for the aerodynamic center of the wing-fuselage combination of 5 percent of the mean aerodynamic chord obtained from wind-tunnel tests was shifted rearward to 30 percent of the mean aerodynamic chord for supersonic speeds. The rearward shift of the aerodynamic center of the wing alone is shown by theory to be somewhat less than 25 percent of the mean aerodynamic chord. The relative destabilizing effect of the fuselage decreases at supersonic speeds, however, because of the disappearance of upwash ahead of the wing. The value assumed for the aerodynamic-center location was intended to account for this effect.

More detailed estimation of this quantity was not thought to be justified because accurate theoretical treatment of a wing-fuselage combination in supersonic flow is not available. The angle of attack of the wing for zero lift was assumed to be zero. The wing incidence was taken as 2.5°. The effect of the 1.0° twist of the XS-1 wing and the interference of the wing body were neglected. The angle of attack of the tail used in the pitching-moment equation includes a constant 2° downflow. It is believed that this downflow exists because of the flow around the fuselage. The 2° downflow was found from wind-tunnel data to occur at subsonic speeds. The same value has been assumed to exist at supersonic speeds since theory indicates that the angle of flow in the region of the tail is very similar at subsonic and supersonic speeds.

DISCUSSION

Figure 4 presents two pairs of computed curves of the elevator-deflection variation with Mach number. One pair of curves is for level-flight lift coefficients with and without the effect of downwash and the other pair of curves is for a constant lift coefficient of 0.27 with and without the effect of downwash. All the computed curves of elevator deflection are for a stabilizer incidence of 2.2° leading edge up, a wing loading of 80 pounds per square foot, and a pressure altitude of 49,000 feet.

The calculations indicate an unstable variation of elevator deflection with Mach number (increasing up-elevator deflection is required with increasing Mach number) in level flight between Mach numbers of 1.1 and 1.6. After a Mach number of about 1.6, there is a slight reduction in the amount of up elevator required up to a Mach number of 2.0, which is the extent of this investigation. The variation of stabilizer incidence for trim ($\delta_e = 0.0^\circ$) with Mach number is presented in figure 5 and indicates that the variation is unstable in the Mach number range from about 1.1 to 1.5 and then becomes slightly stable in the Mach number range from about 1.5 to 2.0. The calculations also show that the reduction in downwash with increasing Mach number is not the main cause of the increase in up-elevator deflection. The main reasons for this trend are that the pitching-moment coefficients due to the wing camber, the wing lift, and the lift of the stabilizer are all in a nose-down direction. As the Mach number increases these pitching-moment coefficients apparently decrease less rapidly than the elevator effectiveness.

It appears that in level flight at a pressure altitude of 49,000 feet with a wing loading of 80 pounds per square foot and a stabilizer incidence of 2.2° (leading edge up) the maximum up elevator of 11.0° will be reached at a Mach number of about 1.6. Ample stabilizer travel is available, however, to change the trim so that the elevator deflection may be reduced to zero at any desired Mach number. Under the conditions stated previously, but by use of a smaller stabilizer incidence, it appears that level flight could be maintained with the elevator travel available from a Mach number of 1.3 to 2.0.

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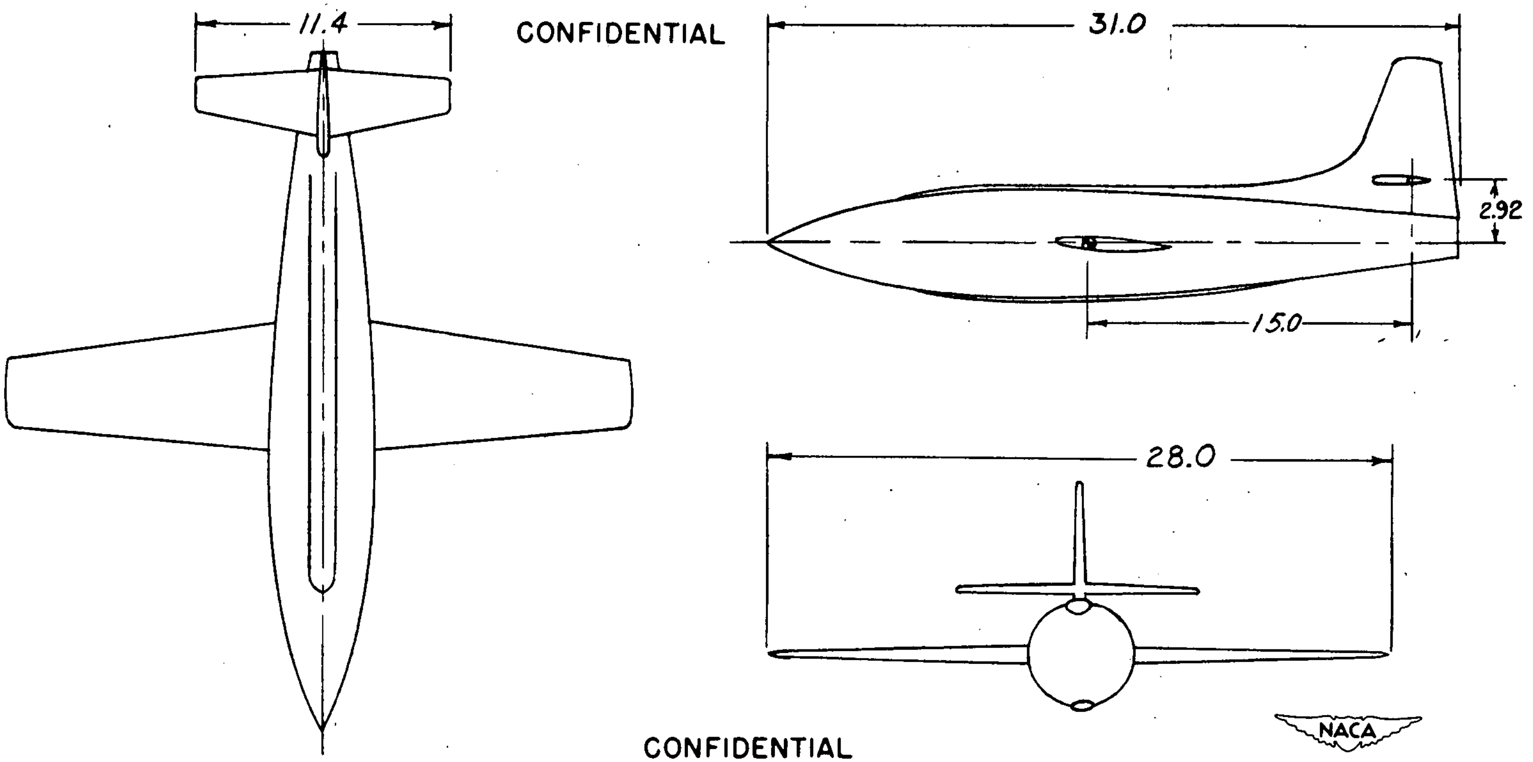

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Wing loading:	Wing:	Horizontal tail:
Take-off, lb/sq ft - - - - -103	Area, sq ft - - - - 130	Area, sq ft - - - -26
Landing, lb/sq ft - - -55	M.A.C., ft - - - - - -4.81	Elevator chord, % - 20
Center of gravity - $.25\bar{c}$	Aspect ratio - - - - 6	Aspect ratio - - - 5

Figure 1.- Three-view drawing of the XS-1 airplane with pertinent physical characteristics. All dimensions in feet.

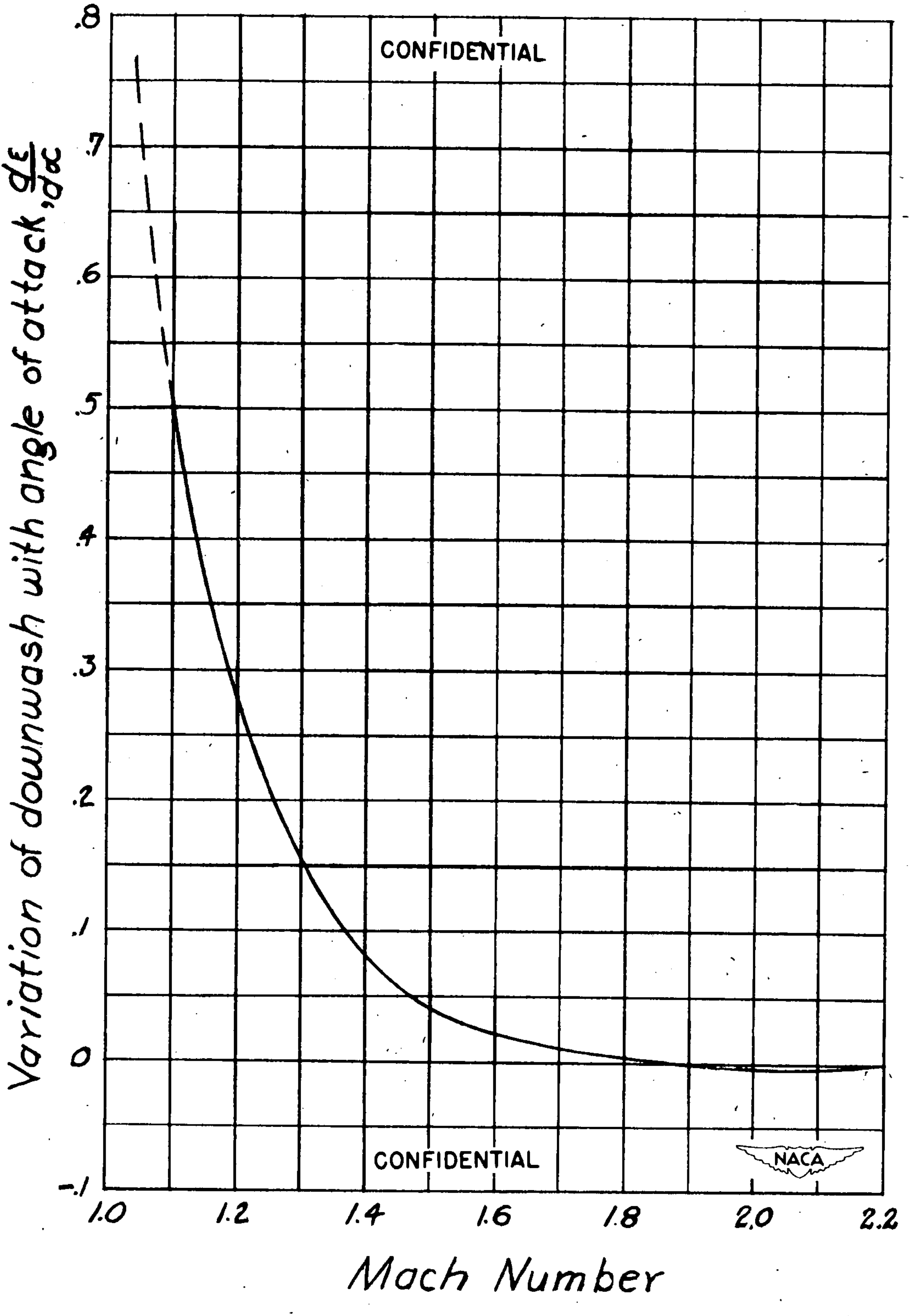

Figure 2.- Theoretical variation of the downwash parameter $\frac{d\epsilon}{d\alpha}$ with Mach number for the XS-1 airplane.

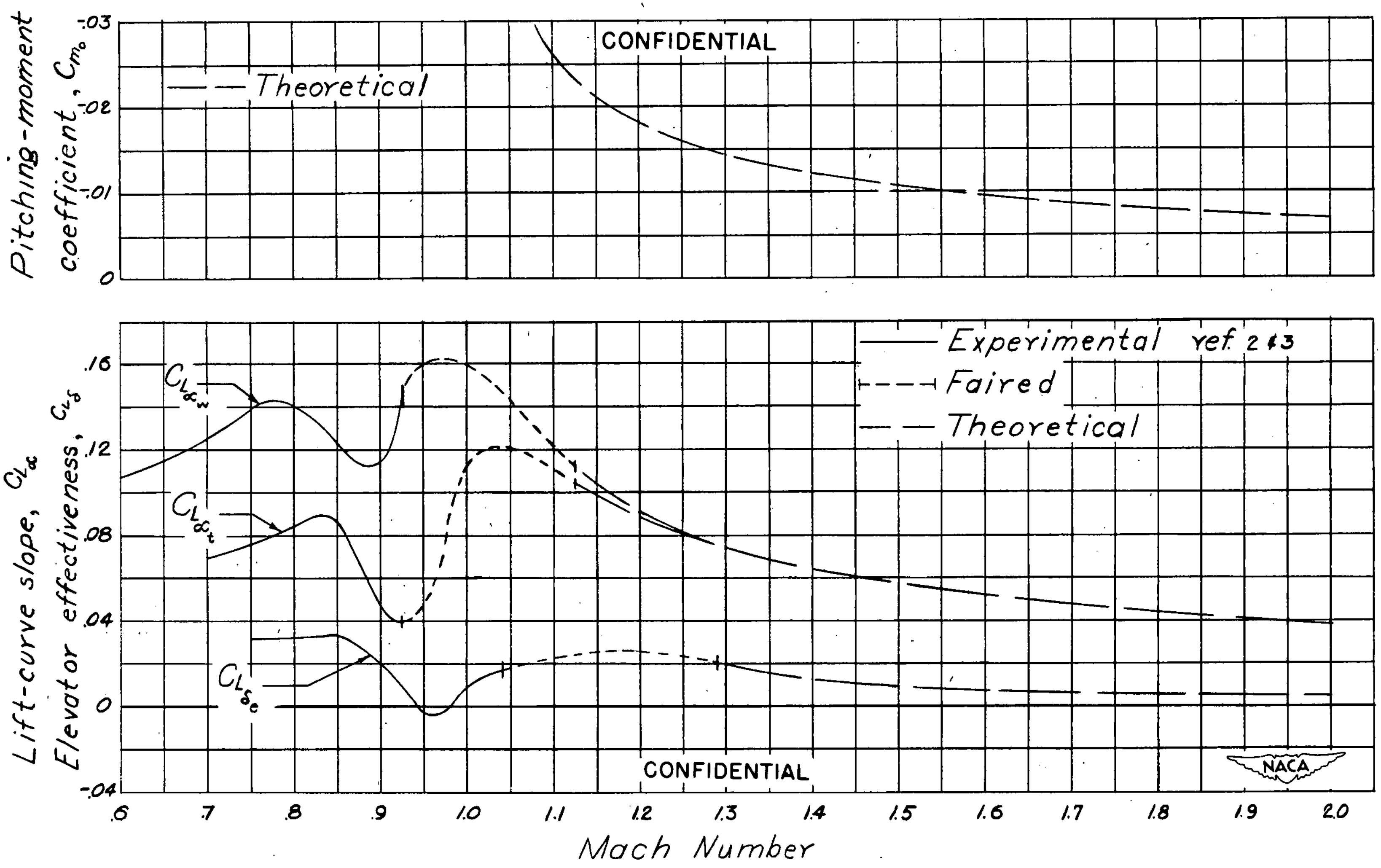

Figure 3.- Variation with Mach number of the lift-curve slopes of the wing and tail, elevator effectiveness, and the pitching-moment coefficient about the aerodynamic center for the XS-1 airplane.

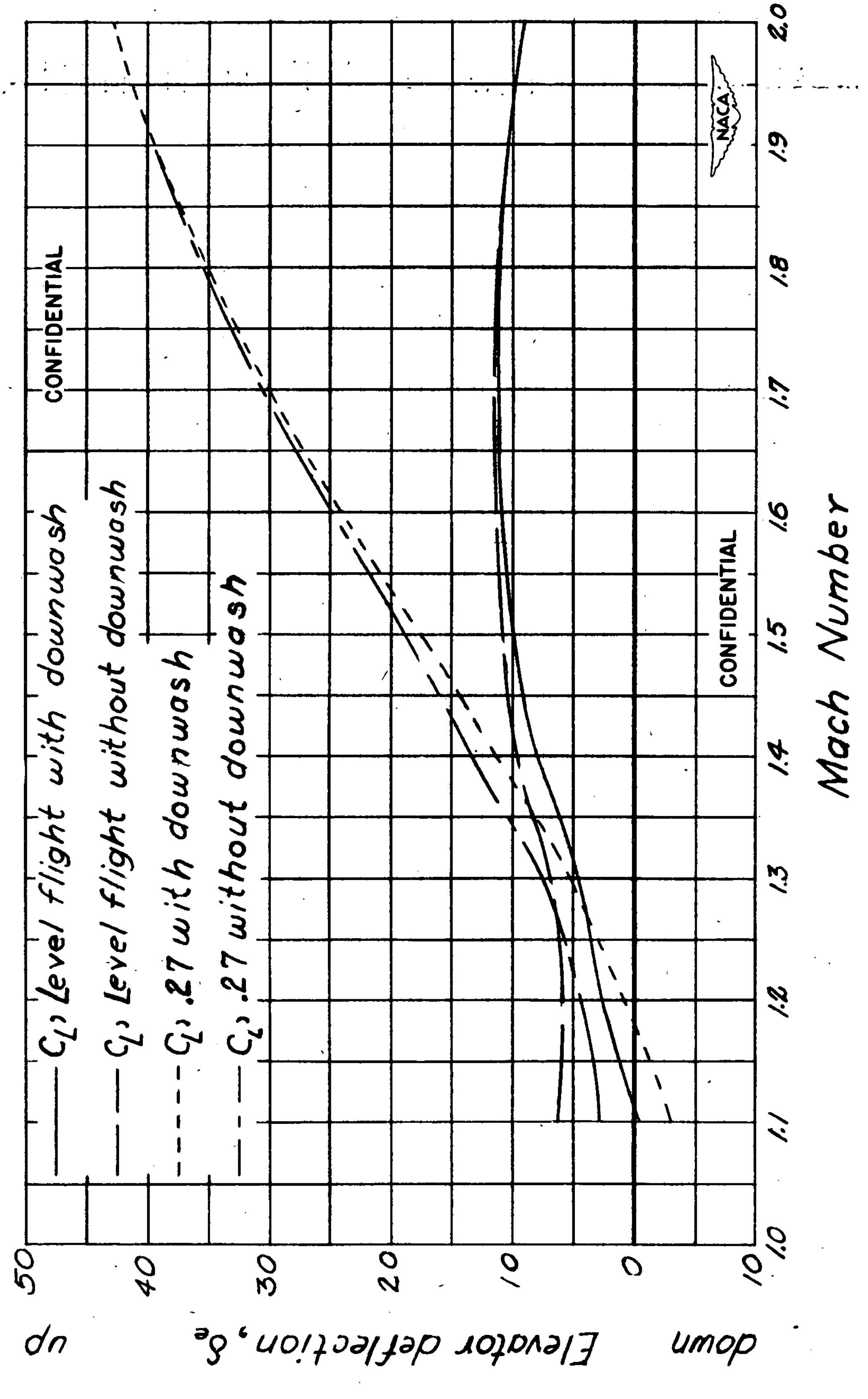

Figure 4.- Variation with Mach number of elevator deflection required for trim. Stabilizer incidence 2.2° leading edge up; pressure altitude, 49,000 feet; wing loading, 80 pounds per square foot.

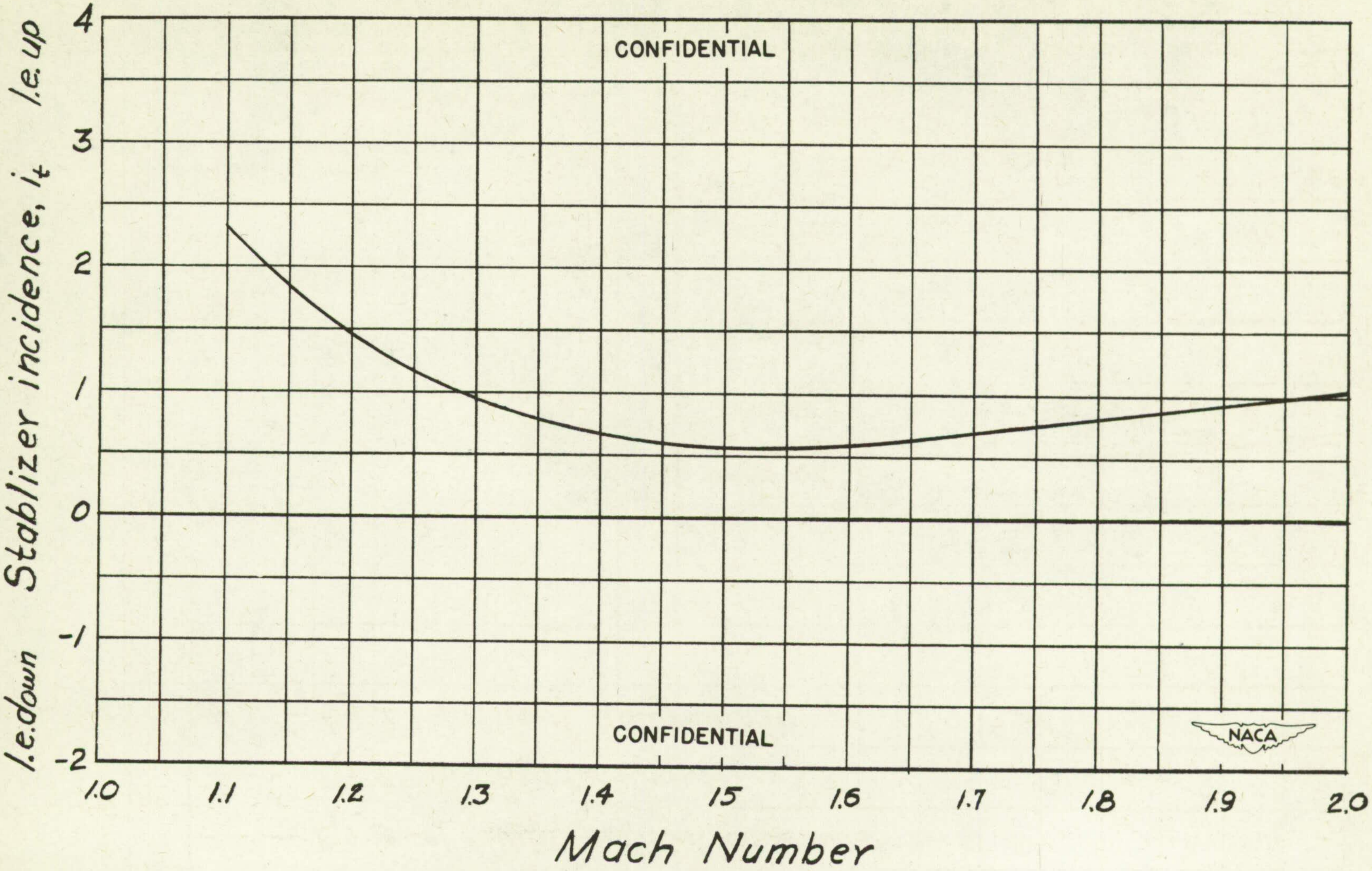

Figure 5.- Variation with Mach number of stabilizer incidence required for trim in level flight. Elevator deflection, 0.0°; pressure altitude, 49,000 feet; wing loading, 80 pounds per square foot.