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RESEARCH MEMORANDUM

ANALYTICAL INVESTIGATION OF PROPELLER EFFICIENCY AT HIGH-SUBSONIC FLIGHT SPEEDS NEAR MACH NUMBER UNITY

By Jean Gilman, Jr., John L. Crigler,
and F. Edward McLean

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NATIONAL ADVISORY COMMITTEE
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SUMMARY

An investigation has been made to ascertain the probable maximum efficiency levels of single-rotating propellers operating at high-subsonic flight speeds near Mach number unity. Use was made of preliminary airfoil data in the transonic Mach number range obtained from special propeller tests, augmented by calculations of airfoil data in the supersonic range, to determine the compressibility losses. The calculations of the propeller efficiency included both induced and compressibility effects. The method of analysis used was found to be very useful in studying propeller efficiency in the transonic flight speed range.

The results of the study indicate that beyond the flight Mach number where the compressibility loss can be delayed or minimized by operation at high advance ratios, the compressibility loss is minimized by propeller operation at lower values of the advance ratio with the blade sections operating at supersonic speeds at or near the optimum helix angles. Compressibility losses are greatly reduced by using very thin airfoil sections (thickness ratios of the order of 3 percent for the outer radii). In the flight Mach number region from about 0.9 to unity, the maximum profile efficiency of propellers of conventional thickness is of the order of 70 percent but can be increased to the order of 80 percent by using thin blade sections.

Propeller efficiencies calculated with airfoil data from special propeller tests are in very good agreement with experimental efficiencies.

A brief performance analysis indicates that a six-blade 16.7-foot-diameter propeller which operates at 76-percent efficiency in absorbing 8000 horsepower at an altitude of 40,000 feet at a flight Mach number of 0.90 can be made to operate with about 84-percent efficiency at a cruising Mach number of 0.75 at the same altitude.

INTRODUCTION

Tests have recently been conducted in the Langley 8-foot high-speed tunnel (reference 1) to study the effects of changes in operating advance ratio, blade plan form, blade-section thickness, section camber, and other variables on the propeller characteristics in the transonic-speed range. The results of these tests showed that propeller efficiencies of the order of 75 percent or greater are possible at very high-subsonic Mach numbers. These tests indicated, however, that after the critical speed of the blade sections has been reached the efficiency is critically dependent on the blade-section-thickness ratio and on the operating advance ratio. To cover experimentally the complete range of advance ratio, thickness ratio, and the other design variables and to determine the exact magnitude of the effect of these variables on the propeller efficiency in the transonic region would require a prohibitively long test program.

An analytical method is needed, therefore, to aid in the determination of the effects of the various design variables on the propeller characteristics in the transonic-speed range. The method presented in reference 2, which evaluated the profile drag losses and the induced losses separately, is followed in this paper. The method as presented offers the general prediction of propeller performance, and can be used to supplement the experimental data.

In the analysis of the drag losses, airfoil data in the transonic-speed range are required. These data are very scarce at present but are currently being made available through special propeller tests (reference 3). Maximum lift-drag ratios from preliminary results of these tests are used in the present investigation to calculate propeller profile efficiencies at flight Mach numbers up to unity for several values of advance ratio. The calculations are made for two series of propellers, one series having a blade-section-thickness distribution which may be considered as representative of current design practice, and another series having much thinner blade sections.

In addition to the profile drag losses, the determination of propeller efficiency requires consideration of the induced losses. At low-subsonic velocities, existing vortex theory enables the determination of induced losses with good accuracy. In the pure supersonic range, it appears that the propeller efficiency losses can be determined by methods analogous to those by which are determined the drag losses of wings of finite aspect ratio in supersonic flow. At flight Mach numbers in the transonic range, however, flow of both types may occur simultaneously. At high forward speeds, below but near sonic velocity, the blade-section resultant velocities may be entirely supersonic, but the flow field of the propeller will not be represented by either the incompressible-flow case or the pure supersonic case. In the present paper, for lack of better information, the conventional vortex theory is assumed to apply up to sonic flight velocity regardless of the resultant Mach number variation along the blades.

The minimum induced loss for a given operating condition is obtained very readily from charts in references 2 and 4. The use of these charts requires that the speed, power, and density be stated. In this paper, the induced losses are evaluated for examples that may be considered as typical of design requirements for flight at transonic speeds. The induced losses are then combined with the drag losses to determine the over-all efficiency as a function of the advance ratio.

Comparisons of calculated results with experimental results from reference 1 are included herein and are found to be in very good agreement.

SYMBOLS

B	number of propeller blades
b	blade-section chord, feet
c_d	section drag coefficient
c_l	section lift coefficient
D	propeller diameter, feet
D	drag, pounds
h	maximum thickness of blade section, feet
J	advance ratio (V/nD)
L	lift, pounds
M	flight Mach number
M_x	blade-section resultant Mach number $\left(M\sqrt{1 + \left(\frac{\pi x}{J}\right)^2}\right)$
M_t	resultant tip Mach number

n	propeller rotational speed, revolutions per second
P	power absorbed by propeller, foot-pounds per second
R	radius to propeller tip, feet
r	radius to propeller element, feet
T	propeller thrust, pounds
V	free-stream velocity, feet per second
W	true resultant velocity, feet per second
W_o	geometric resultant velocity, feet per second
x	radius ratio (r/R)
x_o	radius ratio at spinner juncture
$\gamma = \tan^{-1}\frac{D}{L}$	
η	propeller efficiency $\left(\eta_i\eta_o\right)$
η'	element efficiency
η_i	induced efficiency (neglecting drag)
η'_i	element induced efficiency
η_o	profile efficiency (including drag loss only)
ρ	mass density of air, slugs per cubic foot
σ	section solidity $(Bb/2\pi r)$
ϕ	aerodynamic helix angle, degrees
ϕ_o	geometric helix angle, degrees

METHOD

A study of propellers is considerably simplified through separation of the induced and drag losses. The determination of induced losses of optimum propellers is conveniently accomplished through the charts of reference 2. These charts show the induced efficiency plotted against the power disk-loading coefficient as functions of J and σc_l. This power coefficient is readily evaluated from the desired speed, power, and density conditions. In evaluating the power disk-loading coefficient, the power lost in propeller profile drag in the torque direction is deducted.

The drag loss is calculated in terms of profile efficiency, which is the efficiency including drag effects only. A method of calculating the profile efficiency is derived in the appendix. The method is based upon propellers having the optimum $bc_l/bc_{l_{0.7R}}$ distribution for single-rotating propellers. This distribution, which varies with J and, to some extent, with numbers of blades, is obtained readily from charts in reference 5. The calculations herein are made with the distribution for four-blade propellers; results obtained using other numbers of blades, however, would not vary appreciably from the results given. The spinner radius ratio x_o is 0.3.

The over-all propeller efficiency is the product of the induced and profile efficiencies.

AIRFOIL DATA

Airfoil data in the subsonic region below a Mach number of about 0.8 are readily available for a number of airfoils. In the supersonic region above a Mach number of about 1.2, airfoil data can be calculated with good accuracy for thin airfoils with sharp leading edges. Two-dimensional data in the transonic range are not currently available. Even if the data were available, it is not certain that they would apply without corrections in propeller calculations.

Tests to obtain airfoil characteristics have recently been made in the Langley 16-foot high-speed tunnel, during which the chordwise pressure distribution was measured at resultant Mach numbers up to 1.2 for several stations along the blades of operating propellers. The technique and some of the preliminary results are described in reference 3.

Figures 1 to 3, which give the drag-lift ratio (or $\tan \gamma$) as a function of the lift coefficient with section Mach number as parameter, show the preliminary cross-faired results of some of the integrations of the pressure distributions. The radial stations and airfoil sections are designated in the figures. The results shown, which were obtained from tests of the NACA 10-(3)(08)-03 propeller, include the usual subsonic corrections for obtaining the angles of attack. A friction drag coefficient of 0.004 was arbitrarily added to the pressure drag coefficient to obtain the total drag. While this value may not be exact, the friction drag is small as compared to the pressure drag in the transonic range, and the results of reference 6 show that the friction drag coefficient is relatively independent of Mach number.

The data shown in figures 1 to 3 should be particularly applicable in propeller calculations since most of the special effects, such as tip relief, experienced by propeller blades are inherently included. The figures show that near-maximum lift-drag ratios are obtainable for a fairly large range of operating lift coefficients in the transonic region. For example, in figure 3, at $M_x = 1.00$ and 1.05, the lift-drag ratio varies only slightly from its maximum value through a range of lift coefficients from 0.35 to the highest experimental values obtained in the tests (about 0.50). The upper limit was not defined by the tests due to power limitations, but no apparent tendency toward reductions in the lift-drag ratios is evident at the upper limits of these tests.

At a given value of J and with a given blade-load distribution, maximum profile efficiency occurs when all sections operate at their maximum L/D ratio. Figures 4 and 5 show the variation of maximum L/D (shown as $(D/L)_{min}$ for convenience in plotting) with section Mach number for various thickness ratios.

The curves of figure 4 cover the thickness range for the series of propellers which have the blade-thickness distribution considered as representative of current design (hereinafter termed the "thick" propeller). The curves in figure 5 cover the thickness range for the series of "thin" propellers. Both figures designate the radial station to which a particular value of h/b applies.

In the transonic and supersonic Mach number region, the maximum lift-drag ratio drops off very rapidly with increasing thickness ratio, as may be seen in figures 4 and 5. In view of this rapid drop, it is desirable from the aerodynamic standpoint to keep the blade sections as thin as possible.

RESULTS AND DISCUSSION

The results of this investigation are arranged as follows: The variation with Mach number of the profile efficiency of optimum single-rotating propellers as calculated with the given airfoil characteristics is shown. A discussion with example charts of the induced efficiency is given, followed by a comparison of the over-all efficiency with wind-tunnel test results. A brief analysis of propeller size and performance is presented.

Propeller Efficiency

Profile efficiency.- The variation of profile efficiency with flight Mach number at various values of the advance ratio is shown for the thick and thin propellers in figures 6 and 7, respectively. The effect of increased advance ratio at constant forward speed, of course, is to reduce the blade-section Mach numbers. Because of this effect, increasing the advance ratio increases the flight Mach number at which the initial onset of the adverse compressibility effect occurs, as shown by the figures. This profile-efficiency drop tends to become more abrupt as the flight Mach number and advance ratio increase. At higher Mach numbers the curves converge and cross over. Highest profile efficiency is then obtained at the lower advance ratios. This tendency for the curves to cross over so that higher efficiency is obtained at the lower values of J is borne out experimentally by the results of reference 7. The propellers operating at the lower advance ratios at the higher subsonic Mach numbers are essentially supersonic propellers since the resultant velocities over most of the blade elements are fully supersonic. Although slightly higher maximum L/D ratios may be attainable at higher advance ratios, higher efficiency is obtained at lower advance ratios because the blade elements operate at helix angles nearer optimum values.

The Mach number at which the profile efficiency for the higher advance ratios drops below that for the lower advance ratios depends upon the blade-section airfoil characteristics. For example, the cross-over region for the thick propellers occurs at $M \approx 0.88$. For the thin propellers, this cross-over region is near $M = 0.95$.

In the flight Mach number region where compressibility effects occur, the profile efficiency of the thin propellers is in general much higher than that of the thick propellers. For example, in the flight Mach number region from about 0.9 to unity, the maximum profile efficiency of the thick propellers is of the order of 70 percent. In the same Mach number region the maximum profile efficiency of the thin propellers is in excess of 80 percent.

Inspection of the results in figures 6 and 7 shows that, below the cross-over Mach number region, the gain in profile efficiency due to operation at increased values of J is less for the thin propellers than for the thick propellers.

Induced efficiency.- Propeller induced efficiency is a function of the propeller diameter, the advance ratio, the blade loading, and the number of blades. The relationship between these quantities for optimum induced efficiency is presented in reference 2. The induced efficiency of propellers for high forward speeds will be illustrated for three examples:

(1) 4000 horsepower at an altitude of 40,000 feet at $M = 0.7$

(2) 6000 horsepower at an altitude of 40,000 feet at $M = 0.8$

(3) 8000 horsepower at an altitude of 40,000 feet at $M = 0.9$

Plots showing combinations of the propeller diameter, blade loading, and advance ratio which will meet these three design conditions are given in figures 8 to 10. These plots, which show propeller diameter as a function of J with blade loading $\sigma c_{l_{0.7R}}$ as parameter, are readily obtainable for other than these design conditions from the data in reference 2. In order to determine the induced efficiency it is also necessary to specify the number of blades. For illustrative purposes, induced efficiencies using four blades are given in the figures. At a given loading and diameter, the efficiency of two-blade or three-blade propellers would be slightly lower while the efficiency with greater numbers of blades would be higher than the values shown.

Figures 8 to 10 indicate the large variety of propellers which are capable of absorbing the given power at the specified altitude and airplane velocity. For a given loading, the diameter is required to increase as J is increased because of the reduced resultant velocities at the blade sections.

Although all points on the charts would satisfy the design conditions, there are limitations which greatly restrict the choice of the variables. For example, $\sigma c_{l_{0.7R}}$ must be chosen with structural, vibrational, and weight considerations in mind. The propeller diameter is often restricted by other than aerodynamic considerations. As a matter of fact, where a large amount of power is to be absorbed, considerations such as those mentioned will sharply narrow the designer's choice of variables.

Illustrative examples in the present paper are based on blade-load distributions giving optimum induced efficiency, but the actual blade-load distribution can vary considerably from the optimum distribution without undue penalty to the induced efficiency. (See references 2 and 8.) Increases in the induced losses due to nonoptimum loadings are in general small as compared to the much larger compressibility drag losses at high-subsonic Mach numbers.

Comparison of calculated over-all efficiencies with experimental values.- Very little experimental data exist for propellers operating near sonic forward velocity with blade sections in the thickness range for which airfoil data are currently available. A few tests have recently been made, however, with the NACA 4-(0)(03)-045 propeller in the Langley 8-foot high-speed tunnel (reference 1). The thickness ratio of this propeller varies from about 0.06 at the 0.45R to 0.03 at the 0.7R to 0.02 at the 0.95R. Thus maximum L/D values are available for the entire blade.

Figure 11 gives a comparison of experimental and calculated efficiencies. The agreement is within 1 percent except at $M = 0.9$ in spite of the fact that the blade-load distribution was nonoptimum and that maximum L/D all along the blades as assumed for the calculations was not obtained in the propeller tests. As airfoil data in the appropriate Mach number range become available, it should be possible to realize the calculated values experimentally, since it will then be possible to incorporate the proper pitch distribution for maximum L/D all along the blade for a given J and M.

The good agreement between experimental and calculated efficiencies indicates quite strongly that the lift-drag ratios obtained from the results of the special propeller tests and used in the calculations are of the correct order of magnitude.

Propeller Size and Over-All Efficiency

The profile efficiency curves of the thin propellers show that through the Mach number range from approximately 0.55 to 0.95, the compressibility loss is reduced by operation at the higher advance ratios. The diameter and blade-loading curves in figures 8 to 10, however, show that larger propellers are required at the higher advance ratios than at lower values.

Figure 12 shows the variation in propeller diameter requirements for the following design specifications together with the over-all propeller efficiency:

(1) 4000 horsepower at an altitude of 40,000 feet at $M = 0.70$

(2) 8000 horsepower at an altitude of 40,000 feet at $M = 0.90$

The solidity per blade at 0.7R is 0.04. The lift coefficient at 0.7R is taken to be 0.5, giving a loading per blade of 0.02. Actually, of course, the c_l for maximum L/D varies with Mach number; this variation is not considered in the present analysis.

The flight Mach numbers for both examples of figure 12 are in the Mach number region where increased profile efficiency is obtained at high values of J (fig. 7). With constant loading per blade as assumed, however, the induced efficiency variation is such that at $M = 0.7$ maximum over-all efficiency occurs at $J \approx 3.0$. At $M = 0.9$ maximum efficiency occurs at $J \approx 4.0$. In both cases the variation of diameter required with J is very large. For example, in figure 12(b) ($M = 0.9$) operation at $J = 6.0$ with a four-blade propeller requires a 32.5-foot diameter. For the same conditions, operation at $J = 2.0$ requires only a 15-foot diameter. The charts show that with $M = 0.9$ (fig. 12(b)) operation at $J = 2.0$ results in a negligibly small reduction in the over-all efficiency from its peak value at $J \approx 4.0$. At $M = 0.7$ (fig. 12(a)) the loss in efficiency at $J = 2.0$ from its optimum value at $J \approx 3.0$ is of the order of 5 percent.

Increasing the number of blades at a given loading per blade and constant J results in substantial reductions in diameter. The accompanying decrease in efficiency is relatively small, but increasing the number of blades increases propeller weight and aggravates other mechanical problems.

Figure 13 shows the efficiency variation with J where the diameter is arbitrarily fixed at 18 feet and $\sigma c_{l_{0.7R}}$ is the variable. This example is for 8000 horsepower at an altitude of 40,000 feet with $M = 0.9$. The propeller efficiencies are for a six-blade propeller. In this example increasing the value of J from 2.0 to 4.5 requires an increase in the element-load coefficient $\sigma c_{l_{0.7R}}$ from 0.06 to 0.18. For constant c_l this would require a threefold increase in the blade width, assuming no change in the number of blades. Consideration of other numbers of blades would lead to some variation of the efficiencies shown but there would be a negligible effect on the element-loading parameter.

In figure 13, the efficiency drops about 5 percent on going from $J = 2.0$ to $J = 4.5$, even though the flight Mach number is in the range where η_o increases with increasing J. This efficiency drop is due to the increased induced loss, as may be seen in figure 10.

Propeller Performance

Propeller-performance analysis includes the determination of the propeller efficiency for operating conditions other than the design condition. In the present analysis, the efficiency is evaluated from optimum induced efficiency charts and the profile efficiency curves of figure 7. Changes in load distribution due to operation away from the design condition are assumed to have only secondary effects. The permissibility of this assumption is partly borne out by the comparison in figure 11 of the experimental efficiencies of a given propeller with the calculated efficiencies of propellers with optimum distribution of blade load and lift-drag ratios.

A 16.7-foot six-blade thin propeller is selected for the following design high-speed conditions:

Flight Mach number 0.9
Altitude . . . . 40,000 feet
Power available . . . . 8000 horsepower

The propeller has the following characteristics:

Solidity per blade 0.04
$c_{l_{0.7R}}$ 0.5
η 0.76
J 3.0
Propeller speed, rpm 1047

The flight Mach number for the cruising condition is assumed to be 0.75. It is further assumed that the airplane drag coefficient at $M = 0.75$ is 90 percent of the drag coefficient at $M = 0.9$. The required thrust horsepower at $M = 0.75$ is therefore 3160 horsepower. As a first approximation, the efficiency is assumed to be 85 percent and, therefore, 3720 horsepower must be absorbed by the propeller.

The following table shows the blade-element loading required to absorb 3720 horsepower, the induced and profile efficiency, and the over-all efficiency for various advance ratios which might be used at the cruise condition ($M = 0.75$):

J	$\sigma c_{l_{0.7R}}$	η_i	η_o	η	$c_{l_{0.7R}}$	Propeller speed (rpm)
2.5	0.080	0.934	0.890	0.831	0.333	1047
3.0	.108	.910	.923	.840	.450	873
3.5	.135	.885	.936	.828	.562	748
4.0	.160	.856	.945	.810	.668	655

Because the preceding table is intended for illustrative purposes, the over-all efficiencies are considered sufficiently close to the first approximation of 85 percent to make further approximations unnecessary.

The table shows that as J is increased, the induced efficiency decreases. The profile efficiency, on the other hand, increases with increasing J because of reduced compressibility effects. The over-all efficiency for the cruise condition ranges from 0.831 at $J = 2.5$ to 0.810 at $J = 4.0$ with maximum efficiency, 0.840, occurring at $J = 3.0$.

A column is included in the table to show the propeller speed (rpm) corresponding to the various values of J. Increased J, of course, is accomplished through decreased propeller speed, and since the resultant velocities along the blade are thereby also reduced, increased blade loading is required at the higher values of J. Had the drag coefficient of the airplane been assumed constant, the element-load coefficient $\sigma c_{l_{0.7R}}$ for cruise at $J = 3.0$ would have been the same as for $J = 3.0$ at high speed. With the assumption of a smaller drag coefficient at cruise than at high speed, the element-load coefficient at cruise for $J = 3.0$ is somewhat smaller than the original high-speed value. The propeller operation for best efficiency is thus seen to depend somewhat on the airplane characteristics.

The approximate values of the operating c_l at 0.7R are also included in the table. This column shows that operation in cruise at $J = 2.5$ leads to rather low values of the operating lift coefficient. Profile efficiencies much lower than the values shown will be obtained if the lift coefficients vary too widely from the values for maximum L/D.

APPLICATION OF METHOD

In view of the generally good agreement obtained between experimental and calculated efficiencies in the present paper, the analytical method presented herein should be quite useful in making preliminary evaluations of propeller applications. It must be realized that in analyzing off-design conditions, variations of operating lift coefficients must be in the range giving only small reductions in the lift-drag ratios. This range of operating lift coefficients appears to be reasonably large, however, for airfoils suitable for operation in the required Mach number range.

In the present analysis the propeller thrust and torque coefficients have not been calculated, only the over-all efficiency being given. It should be possible to determine the propeller force coefficients from strip theory calculations, however, when complete airfoil data become available.

CONCLUSIONS

The data presented for an analytical investigation of propeller efficiency near Mach number unity show that the method herein afforded a useful means of making preliminary evaluations of propeller efficiency in the transonic flight speed range. The calculated and experimental efficiencies were in very good agreement both as to magnitude and trends in the propeller efficiency with operating advance ratio. This good agreement indicates that the lift-drag ratios obtained from the results of the special propeller tests are of the correct order of magnitude.

The following specific conclusions were also made:

1. Beyond the flight Mach number region where the compressibility loss can be delayed or minimized by operation at high advance ratios, peak propeller efficiency is obtained at lower values of advance ratio. The Mach number region where this change-over occurs depends on the airfoil characteristics.

2. Compressibility losses can be reduced greatly by using very thin blades. In the flight Mach number region from about 0.9 to unity, the maximum profile efficiency of a propeller of approximately conventional thickness (6 percent thick at the 0.7 radius) is of the order of 70 percent. The maximum profile efficiency of a thin propeller (3 percent thick at the 0.7 radius) in the same Mach number range is of the order of 80 percent.

3. A brief performance analysis indicates that a six-blade 16.7-foot-diameter propeller which operates at 76-percent efficiency in absorbing 8000 horsepower at an altitude of 40,000 feet at a flight Mach number of 0.90 can be made to operate with about 84-percent efficiency at a cruising Mach number of 0.75 at the same altitude.

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APPENDIX

METHOD OF CALCULATING EFFICIENCY

The propeller element efficiency is

$$\eta' = \frac{V\ dT}{dP}$$

where

$$dT = \frac{1}{2}\rho W^2 Bb(c_l \cos \phi - c_d \sin \phi)dr$$

$$= BR\frac{1}{2}\rho W^2 bc_l(1 - \tan \gamma \tan \phi)\cos \phi\ dx$$

and

$$dP = 2\pi nr\frac{1}{2}\rho W^2 Bb(c_l \sin \phi + c_d \cos \phi)dr$$

$$= 2\pi nBR^2\frac{1}{2}\rho W^2 bc_l(1 + \tan \gamma \cot \phi)x \sin \phi\ dx$$

$$= 2\pi nBR^2\frac{1}{2}\rho W^2 bc_l(1 + \tan \gamma \cot \phi)x \tan \phi \cos \phi\ dx$$

The propeller efficiency is

$$\eta = \frac{TV}{P} = \frac{V}{\pi nD} \frac{\int_{x_o}^{1.0} W^2 bc_l \cos \phi(1 - \tan \gamma \tan \phi)dx}{\int_{x_o}^{1.0} W^2 bc_l \cos \phi(1 + \tan \gamma \cot \phi)x \tan \phi \, dx}$$

$$= \frac{J}{\pi} \frac{\int_{x_o}^{1.0} \left(\frac{W}{V}\right)^2 \frac{bc_l}{bc_{l_{0.7R}}} \cos \phi(1 - \tan \gamma \tan \phi)dx}{\int_{x_o}^{1.0} \left(\frac{W}{V}\right)^2 \frac{bc_l}{bc_{l_{0.7R}}} \cos \phi(1 + \tan \gamma \cot \phi)x \tan \phi \, dx} \tag{1}$$

Equation (1) can be made specific by considering the Goldstein condition. This special case is based upon the Betz minimum-energy-loss loading which is met approximately when the induced efficiency η'_i is constant along the blade. It can be shown that

$$\eta'_i = \frac{\tan \phi_o}{\tan \phi} = \frac{J}{\pi} \frac{1}{x \tan \phi}$$

For the Goldstein condition, this equation may be transposed to

$$x \tan \phi = \frac{J}{\pi \eta'_i} = \text{Constant}$$

For $x \tan \phi$ = Constant, the term in equation (1) may be removed from the integral sign giving the equation

$$\eta = \frac{J}{\pi} \frac{1}{x \tan \phi} \frac{\int_{x_o}^{1.0} \left(\frac{W}{V}\right)^2 \frac{bc_l}{bc_{l_{0.7R}}} \cos \phi(1 - \tan \gamma \tan \phi)dx}{\int_{x_o}^{1.0} \left(\frac{W}{V}\right)^2 \frac{bc_l}{bc_{l_{0.7R}}} \cos \phi(1 + \tan \gamma \cot \phi)dx} \tag{2}$$

which is, in alternative form,

$$\eta = \eta_i \frac{\displaystyle\int_{x_o}^{1.0} \left(\frac{W}{V}\right)^2 \frac{bc_l}{bc_{l_{0.7R}}} \cos \phi(1 - \tan \gamma \tan \phi)dx}{\displaystyle\int_{x_o}^{1.0} \left(\frac{W}{V}\right)^2 \frac{bc_l}{bc_{l_{0.7R}}} \cos \phi(1 + \tan \gamma \cot \phi)dx}$$

$$= \eta_i \eta_o \tag{3}$$

By means of the Goldstein conditions, the terms $\frac{bc_l}{bc_{l_{0.7R}}}$ and ϕ have a specific radial distribution which depends on the advance ratio, the number of blades, and the power disk loading as discussed in references 2 and 4.

For light loading, the quantity ϕ approaches ϕ_o and the quantity W approaches W_o in value. The condition of a light loading therefore permits the following substitutions:

$$\tan \phi_o = \frac{J}{\pi x}$$

$$\cos \phi_o = \frac{\pi n D x}{W_o}$$

$$\frac{W_o}{V} = \frac{M_x}{M}$$

With these substitutions, the equation becomes

$$\eta \approx \eta_i \frac{\displaystyle\int_{x_o}^{1.0} \frac{M_x}{M} \frac{bc_l}{bc_{l_{0.7R}}} x\left(1 - \frac{J}{\pi x} \tan \gamma\right) dx}{\displaystyle\int_{x_o}^{1.0} \frac{M_x}{M} \frac{bc_l}{bc_{l_{0.7R}}} x\left(1 + \frac{\pi x}{J} \tan \gamma\right) dx} \tag{4}$$

where

$$\frac{M_x}{M} = \sqrt{1 + \left(\frac{\pi x}{J}\right)^2}$$

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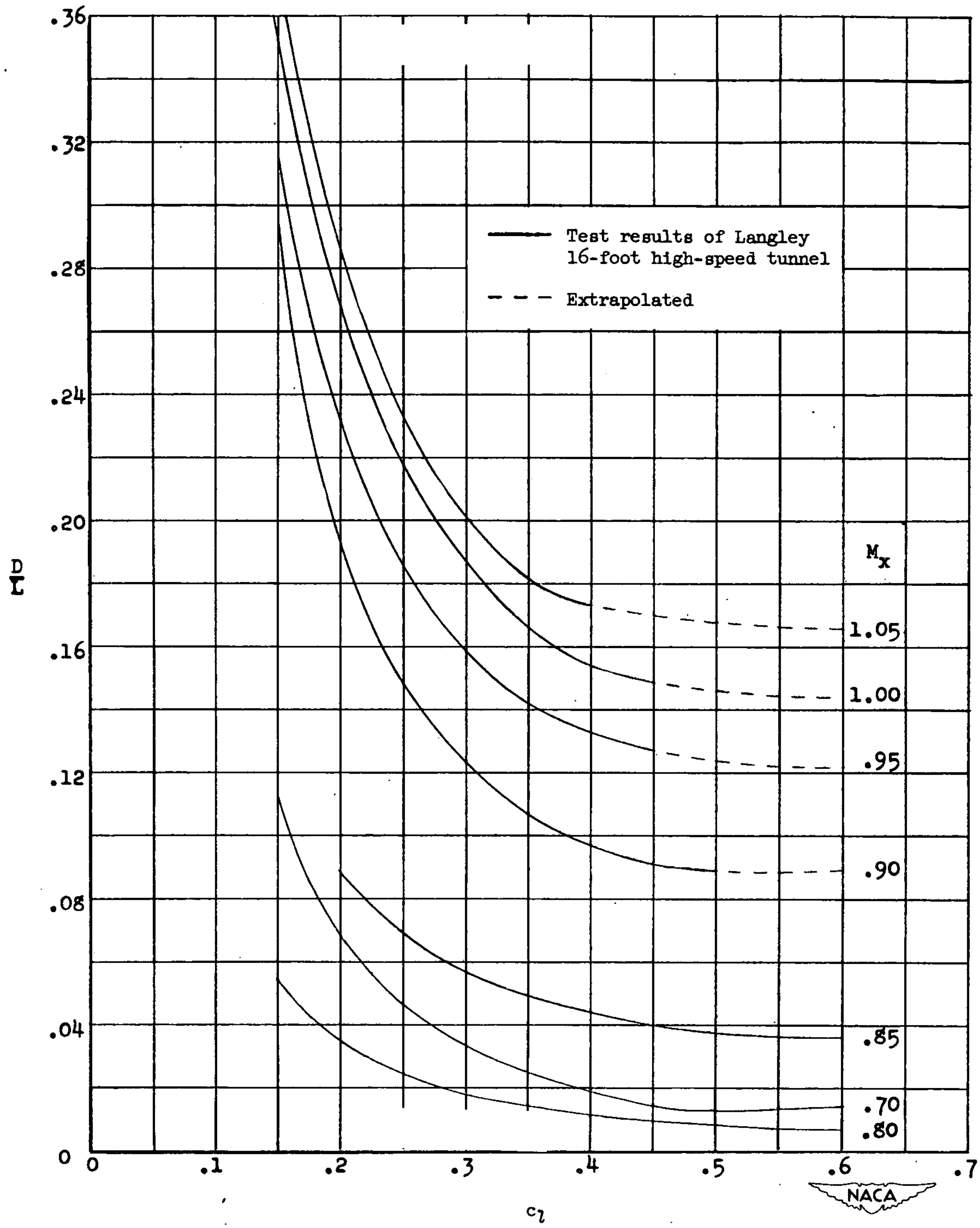


Figure 1.- Variation of D/L with c_l for x = 0.8 station at given section Mach numbers. NACA 16-(2.95)06.95 airfoil section.

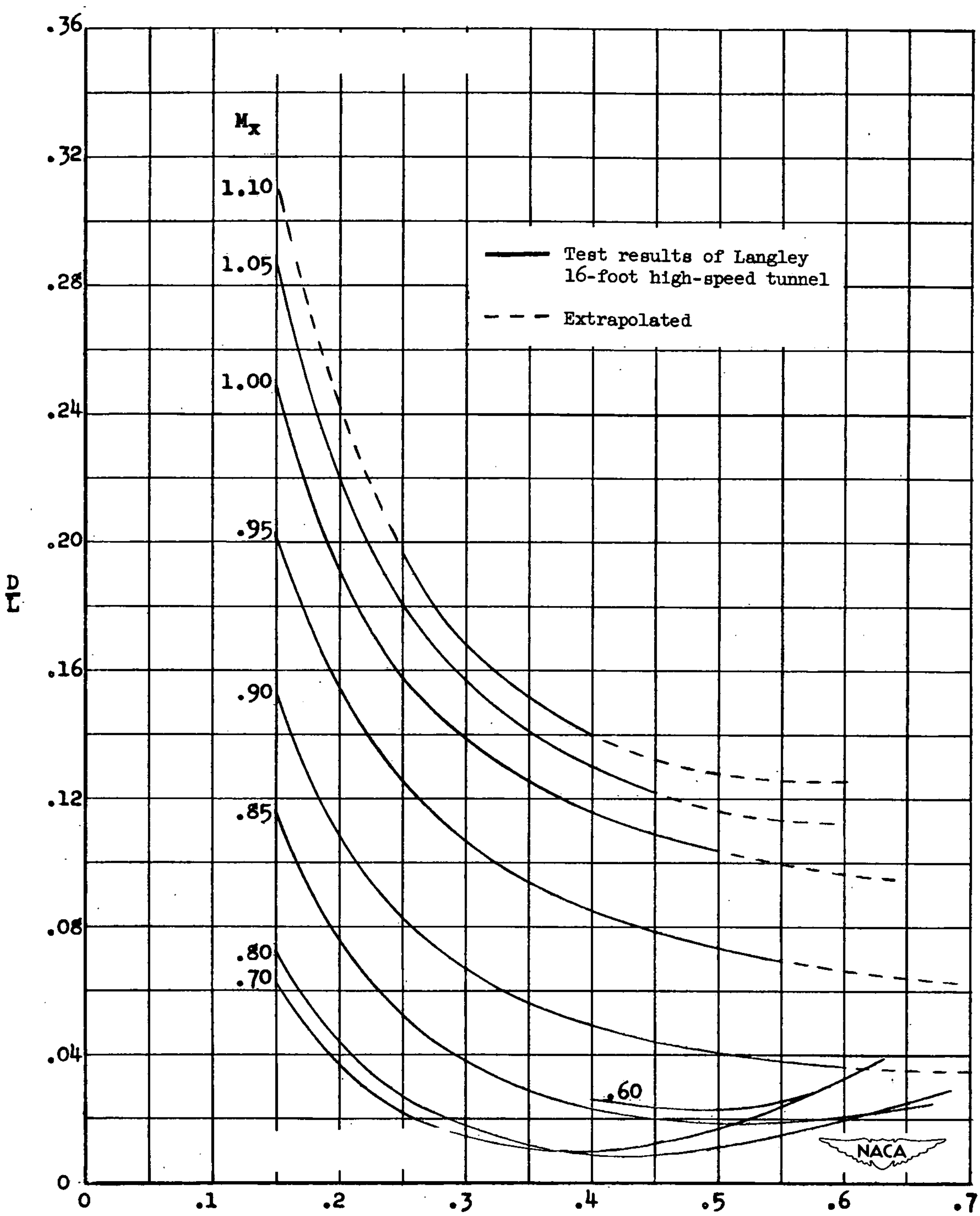


Figure 2.- Variation of D/L with c_l for x = 0.9 station at given section Mach numbers. NACA 16-(2.52)05.77 airfoil section.

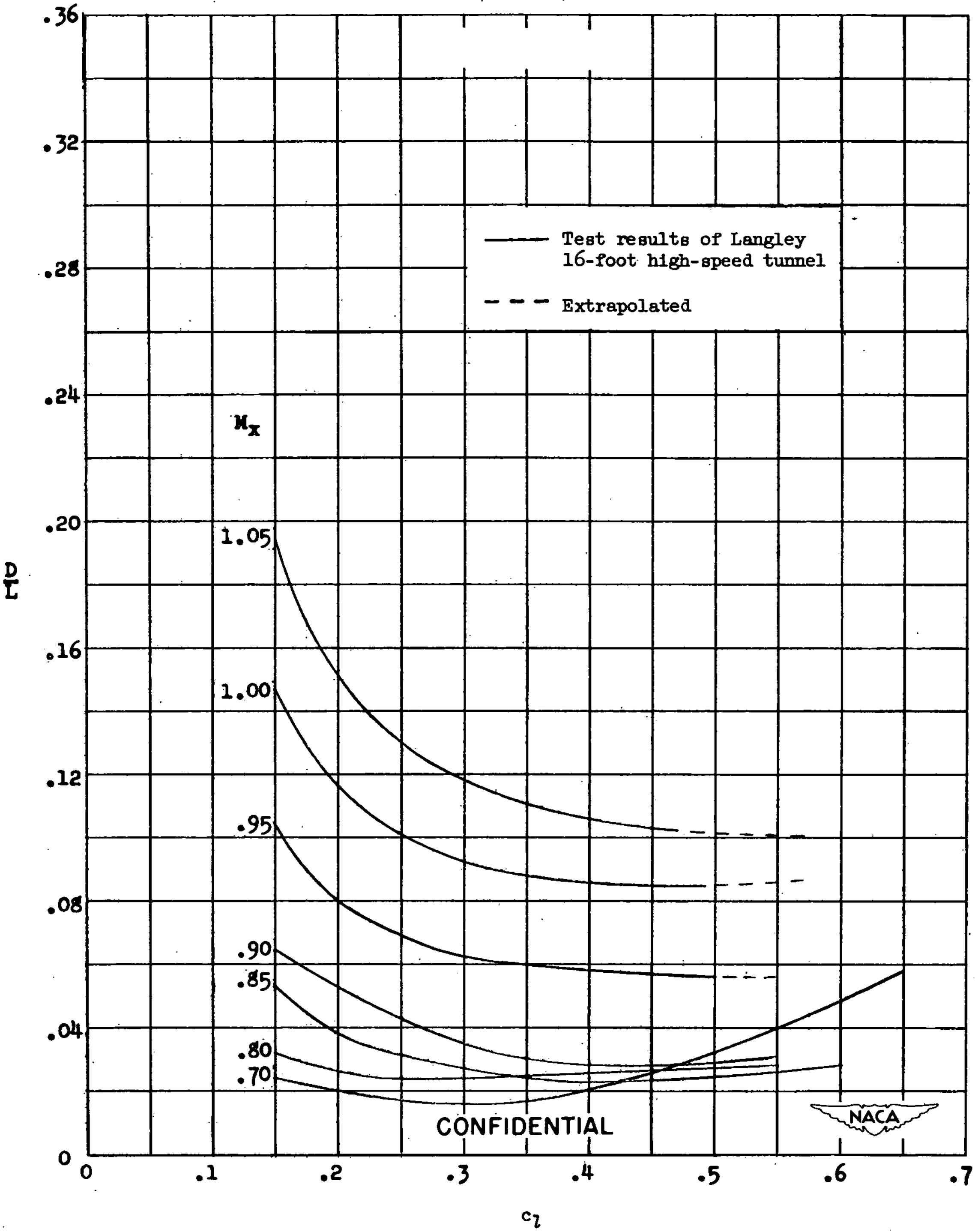


Figure 3.- Variation of D/L with c_l for x = 0.95 station at given section Mach numbers. NACA 16-(2.03)04.76 airfoil section.

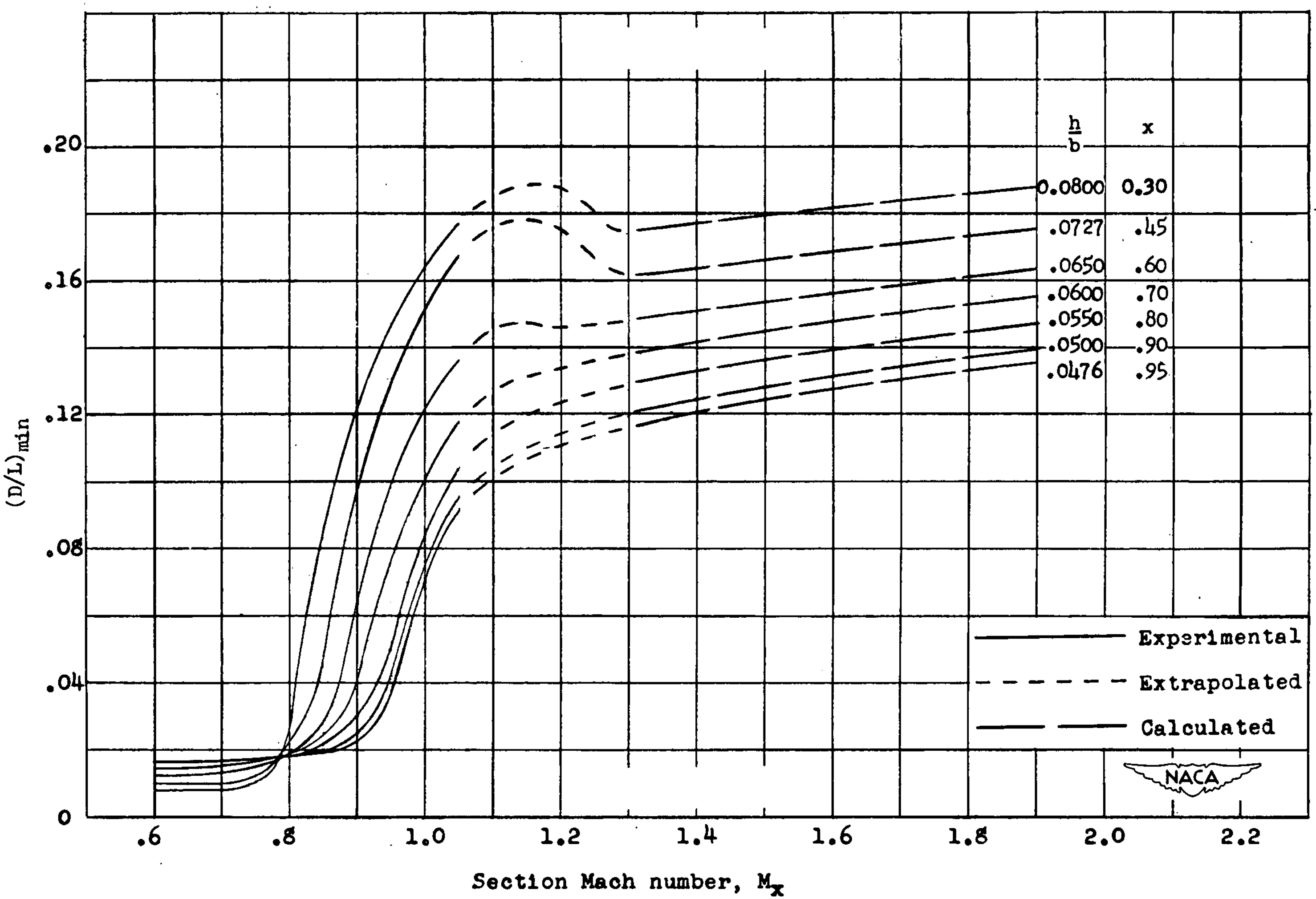


Figure 4.- Minimum drag-lift ratio $(\tan \gamma)_{min}$ against section Mach number for various thickness ratios with corresponding radial location for thick propeller.

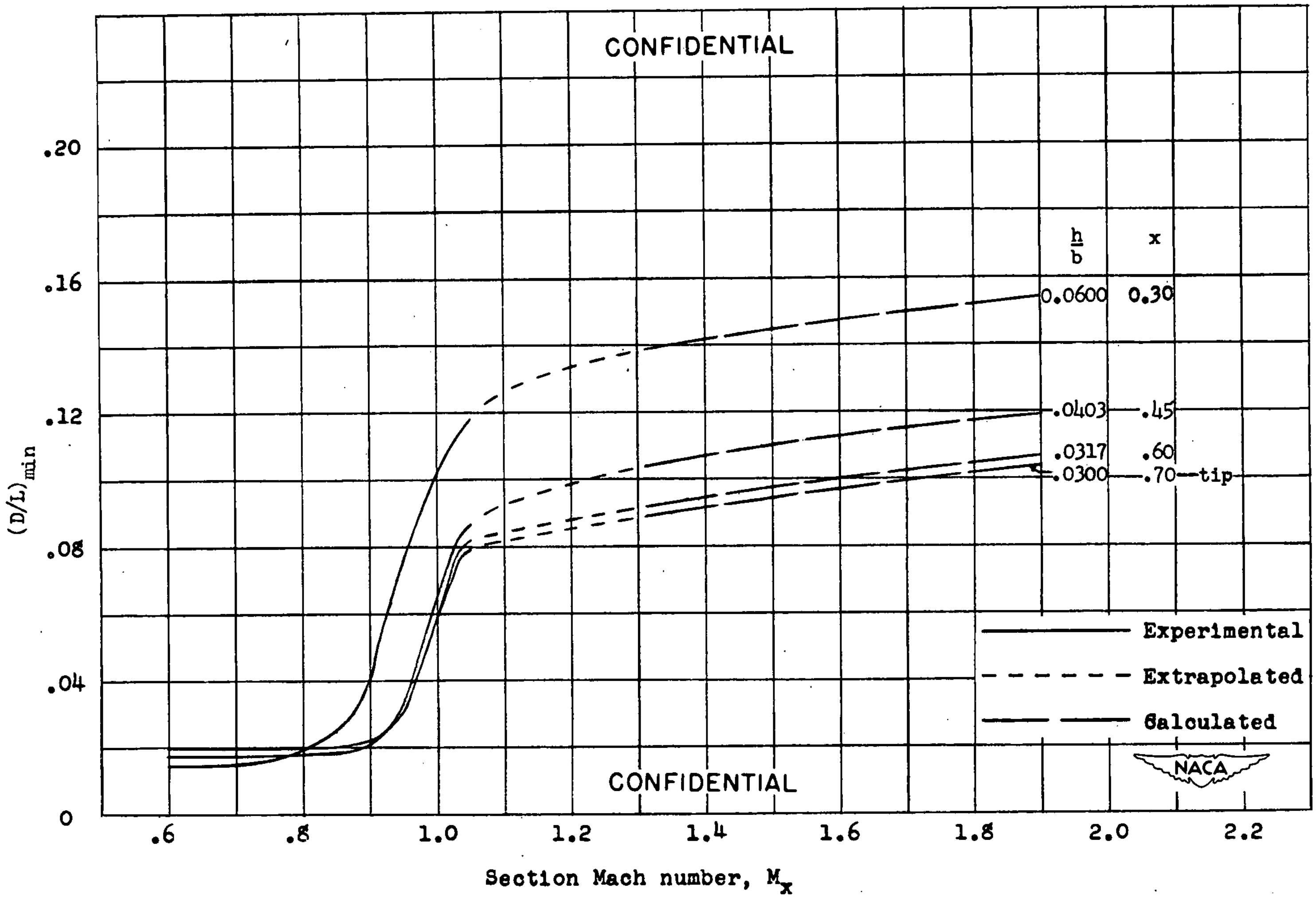


Figure 5.- Minimum drag-lift ratio $(\tan \gamma)_{min}$ against section Mach number for various thickness ratios with corresponding radial location for thin propeller.

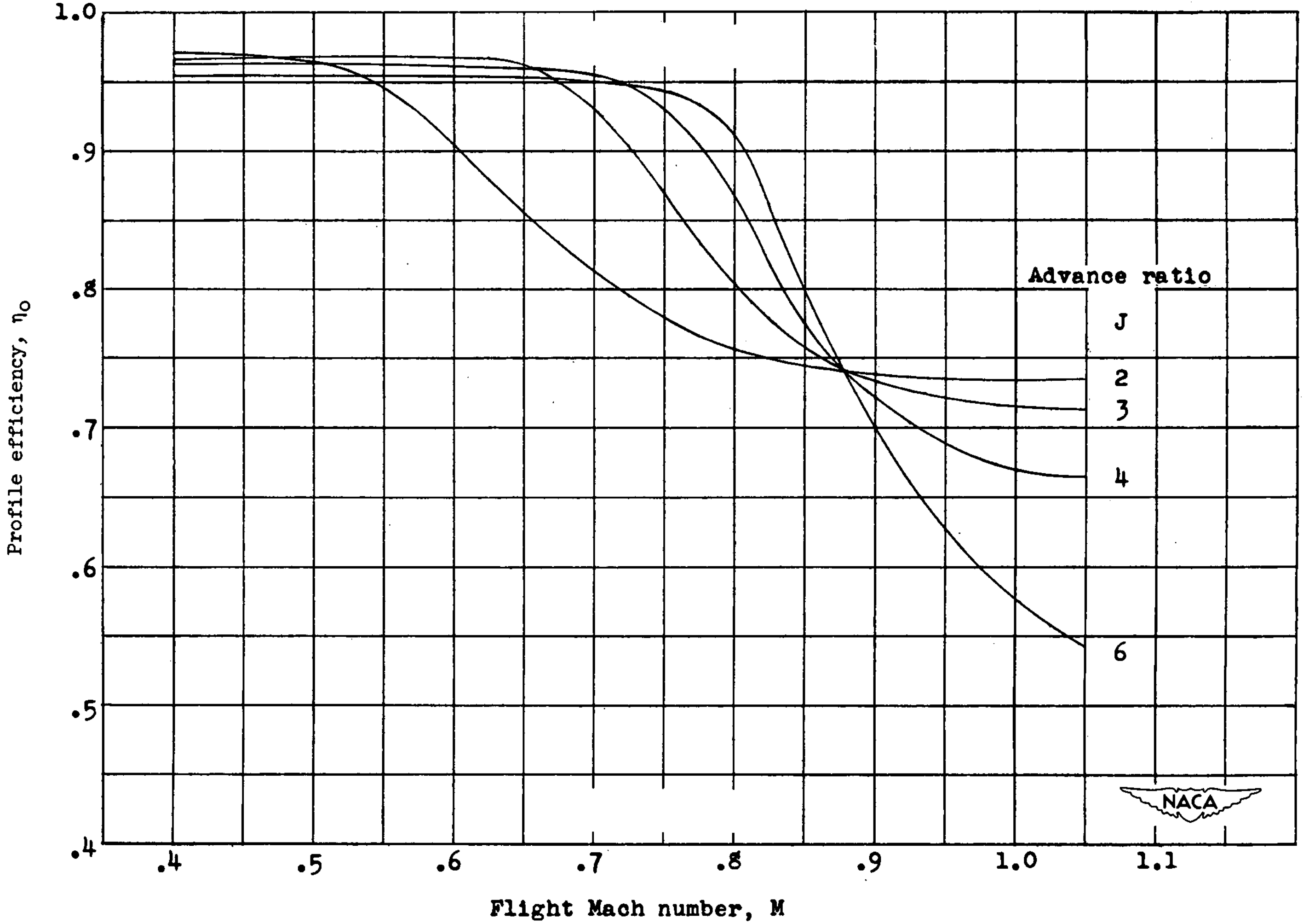


Figure 6.- Peak profile efficiency against flight Mach number for various advance ratios. (Thick propeller.)

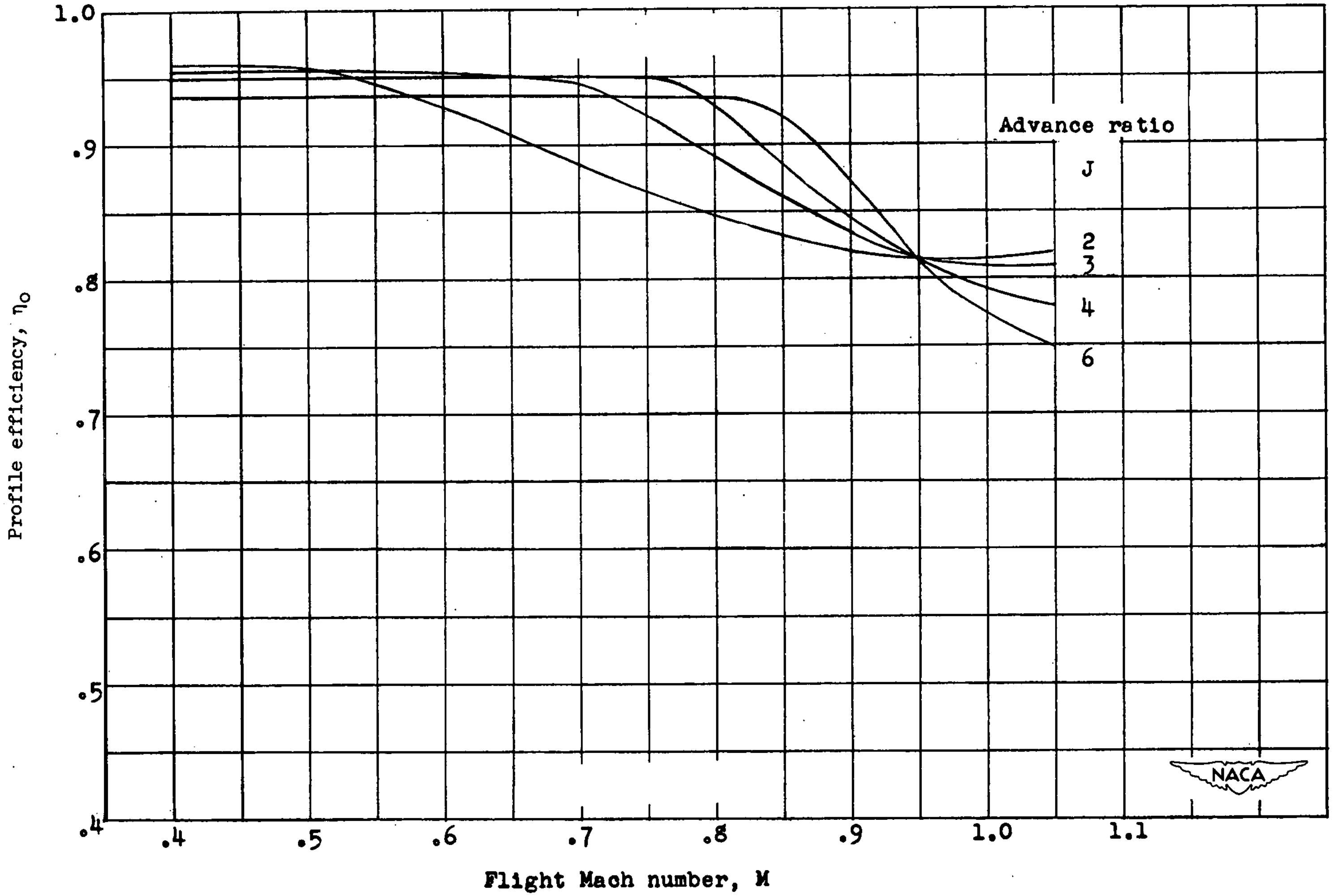


Figure 7.- Peak profile efficiency against flight Mach number for various advance ratios. (Thin propeller.)

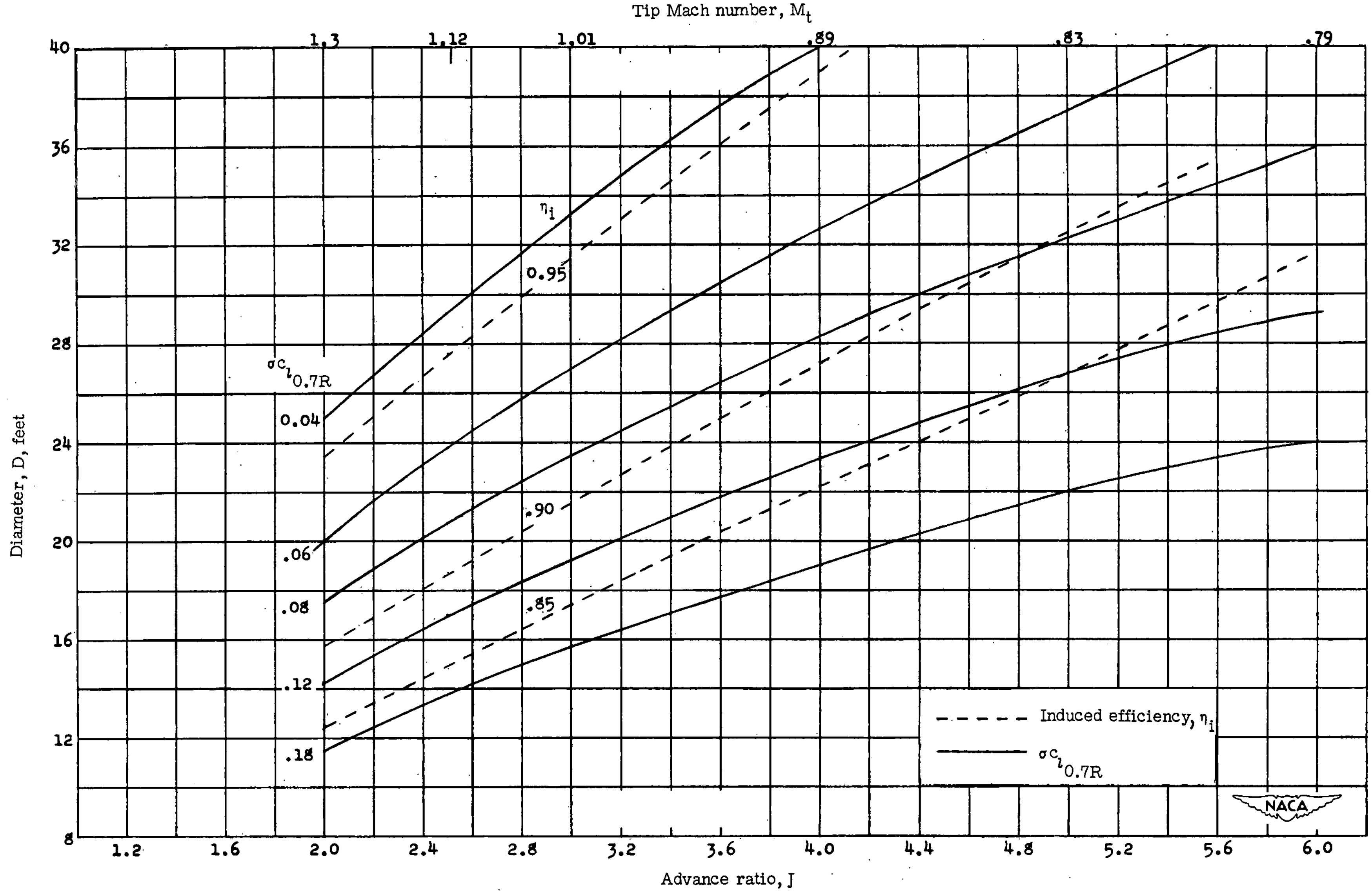


Figure 8.- Diameter requirements for single rotating propellers. Altitude = 40,000 feet; M = 0.7; P = 4000 horsepower.

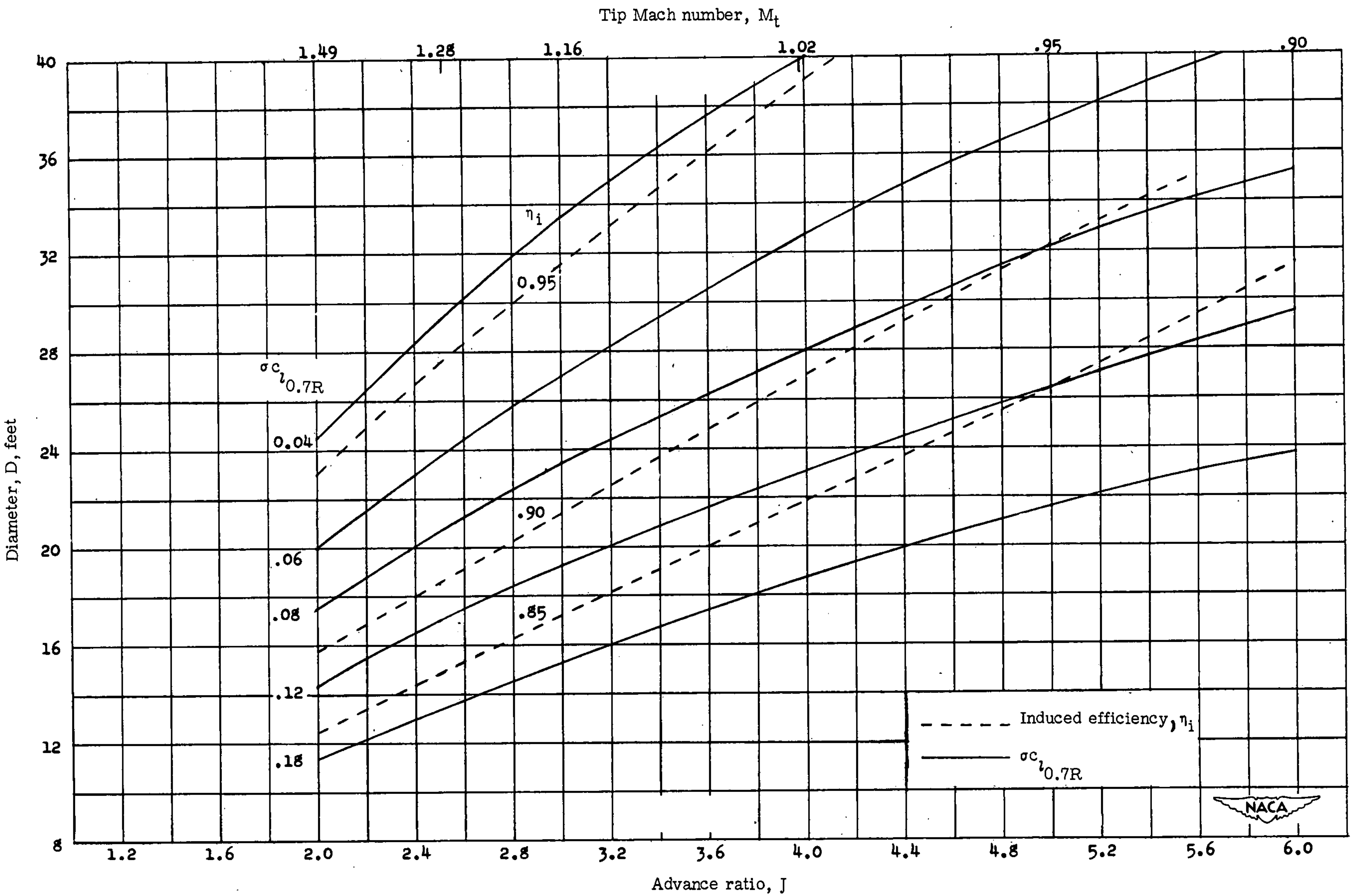


Figure 9.- Diameter requirements for single rotating propellers. Altitude = 40,000 feet; M = 0.8; P = 6000 horsepower.

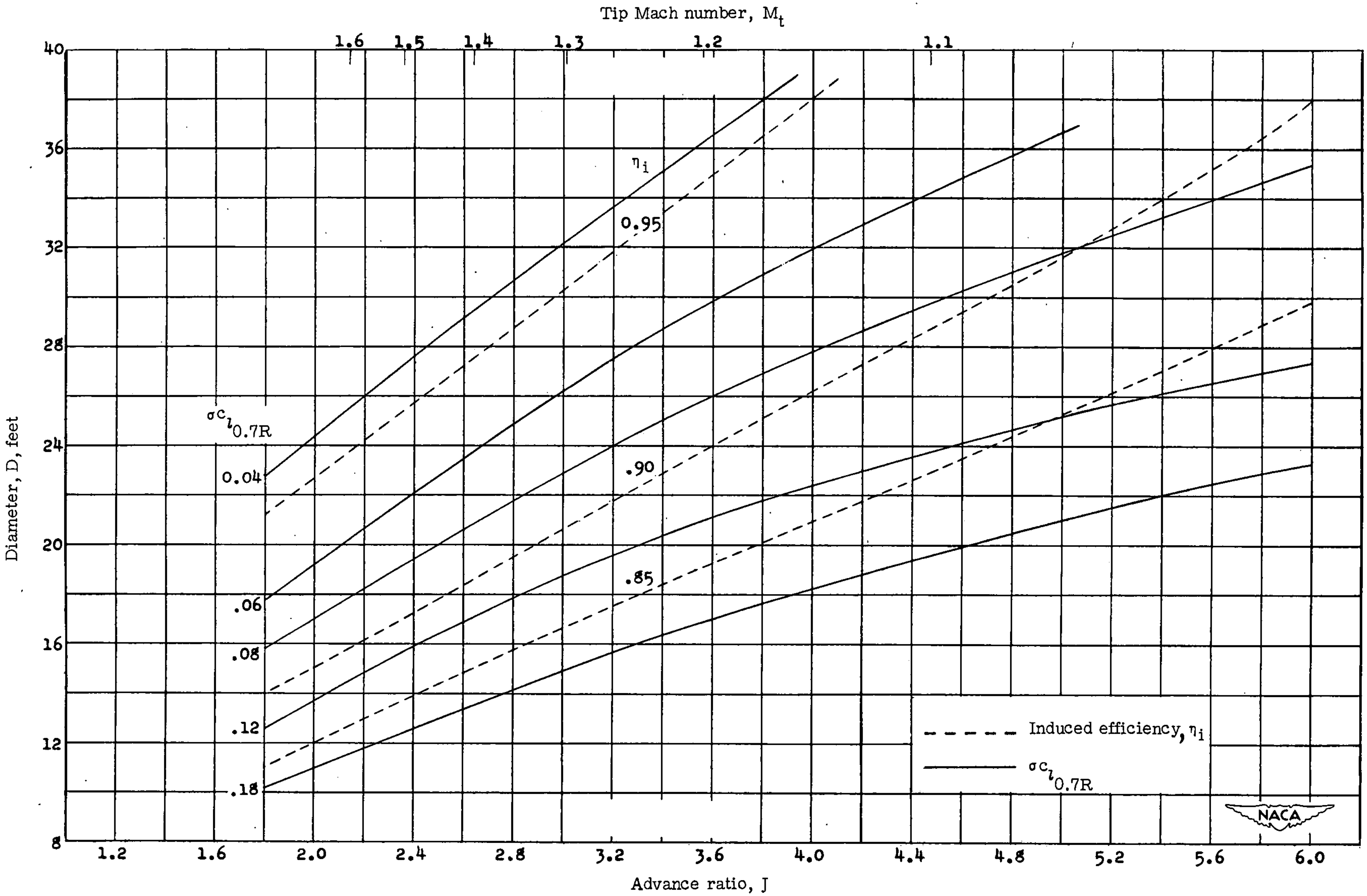


Figure 10.- Diameter requirements for single rotating propellers. Altitude = 40,000 feet; M = 0.9; P = 8000 horsepower.

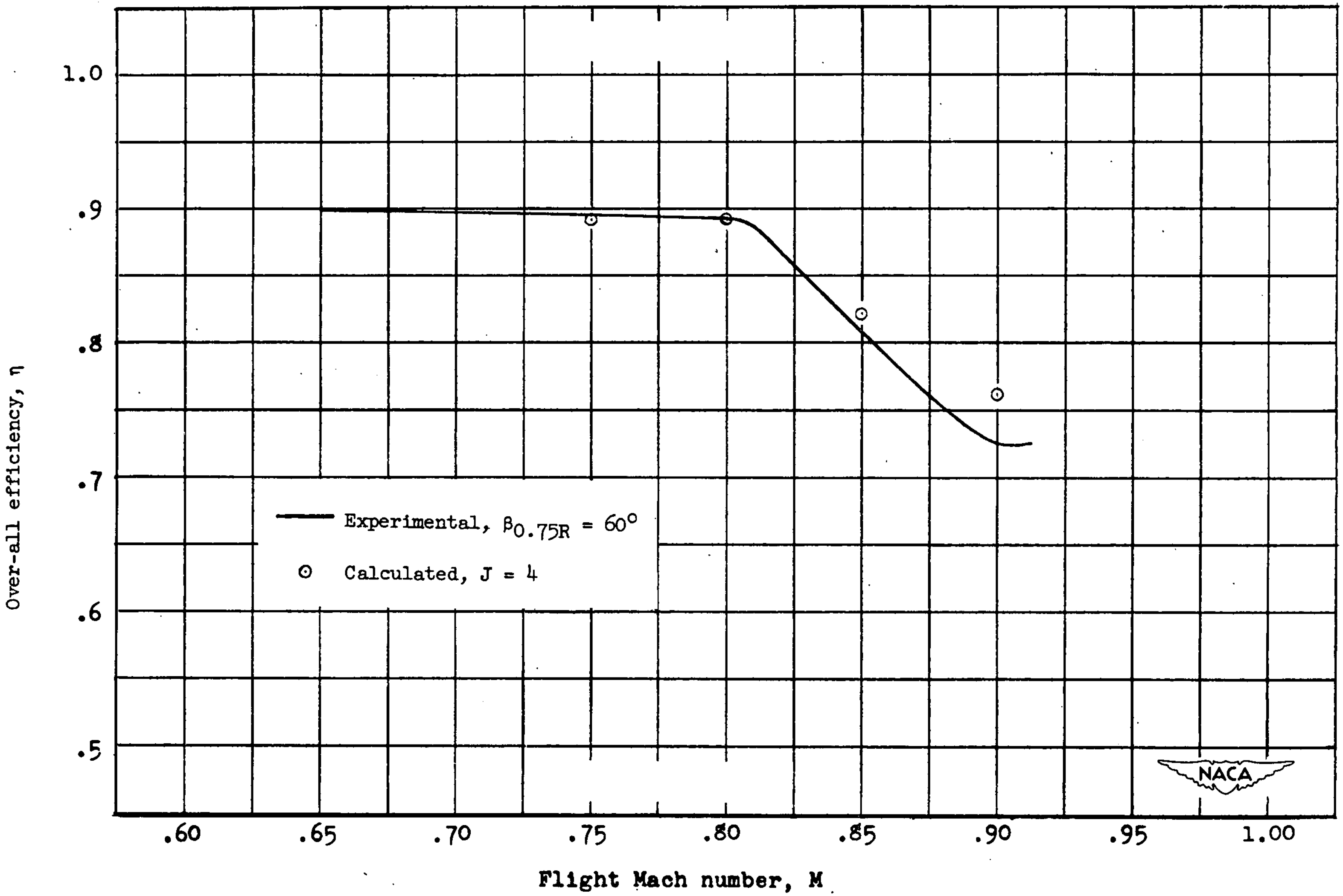
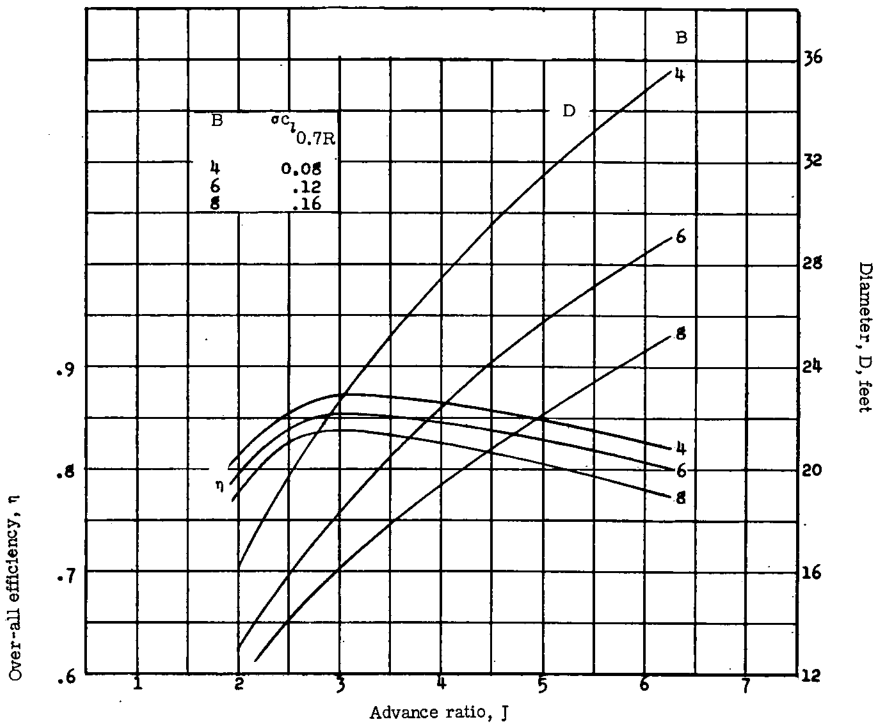
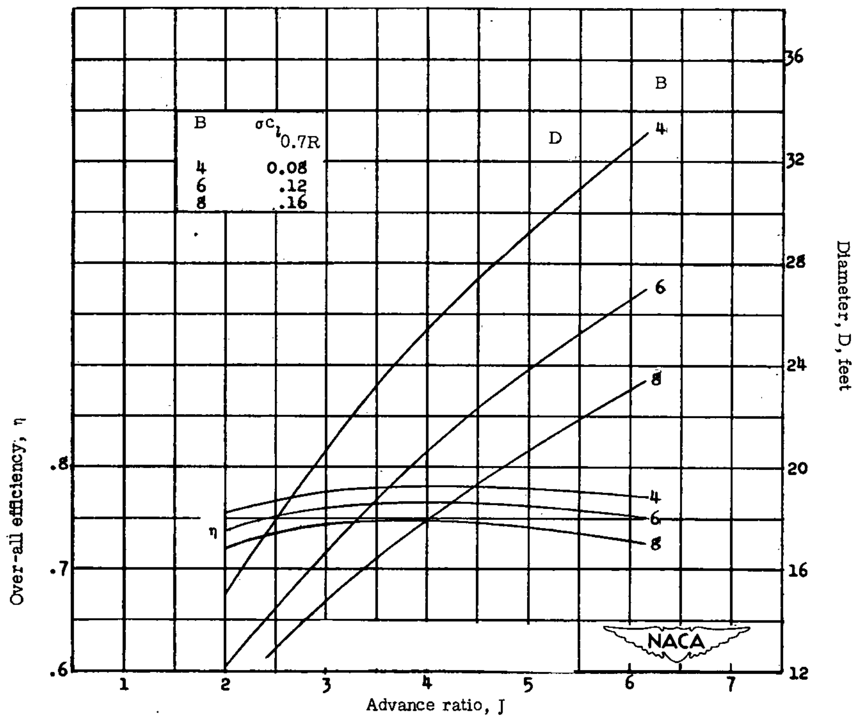


Figure 11.- Comparison of experimental and calculated efficiency. NACA 4-(0)(03)-045 propeller.



(a) Altitude = 40,000 feet; P = 4000 horsepower; M = 0.7.



(b) Altitude = 40,000 feet; P = 8000 horsepower; M = 0.9.

Figure 12.- Over-all efficiency and diameter of four-blade, six-blade, and eight-blade propellers against advance ratio for various blade loadings.

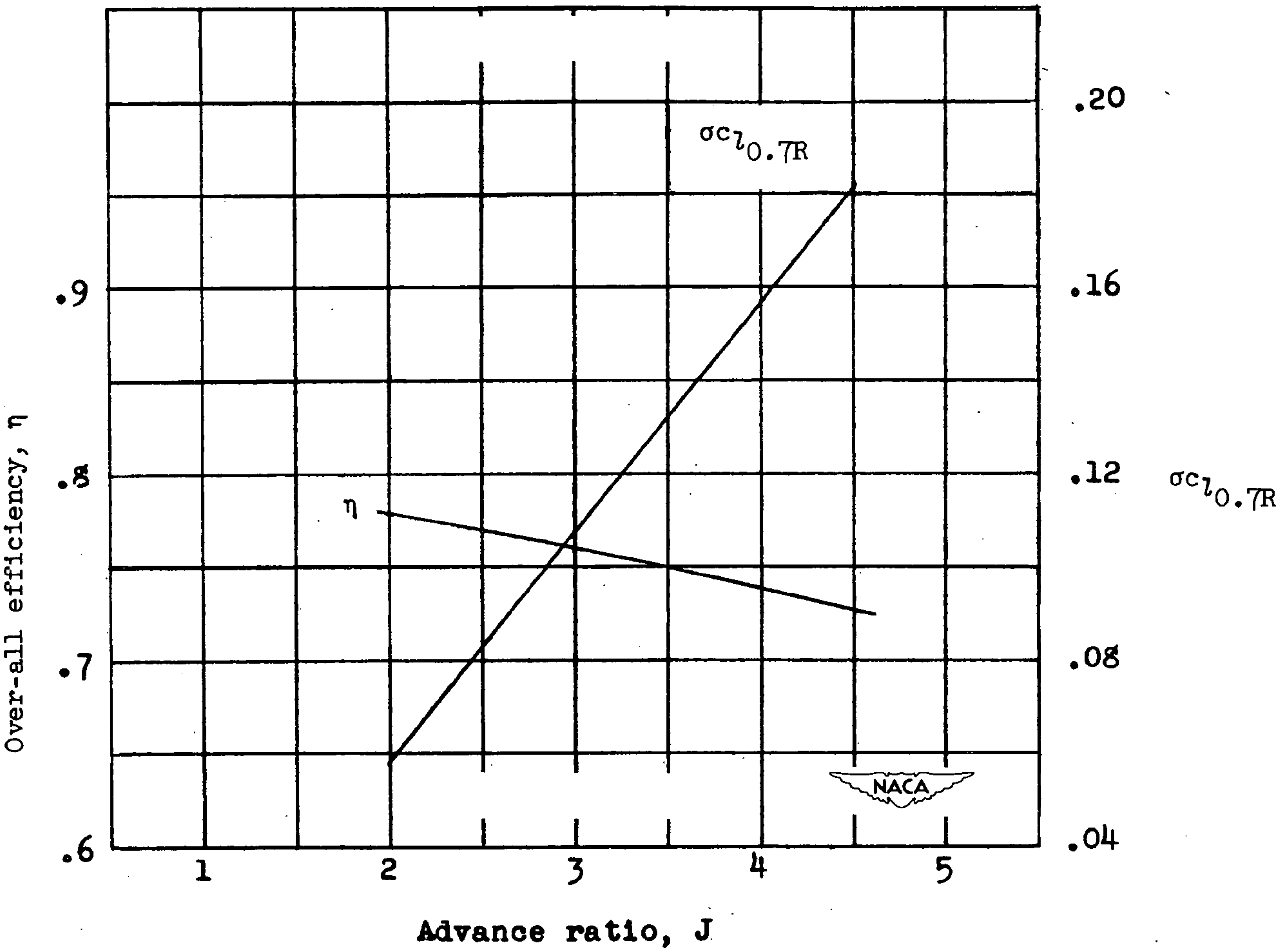


Figure 13.- Variation with advance ratio of over-all efficiency and loading at constant diameter. Altitude = 40,000 feet; M = 0.9; P = 8000 horsepower; B = 6; D = 18 feet.