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PRELIMINARY RESULTS OF STABILITY CALCULATIONS FOR THE BENDING OF BOX BEAMS WITH LONGITUDINALLY STIFFENED COVERS CONNECTED BY POSTS

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SUMMARY

The preliminary results of a computational program are presented which give numerical values of the stiffnesses required of posts and longitudinal stiffeners along the row of posts to achieve desired buckling-stress values in the covers of a box beam subjected to bending. The validity of a short-cut solution to the stability equation derived in NACA TN 2760 is also shown.

INTRODUCTION

The idea of using a systematic arrangement of posts as stabilizing members between the tension and compression covers of box-beam structures has raised the possibility of weight reduction as well as simplified construction of wing and tail structures. The design conditions under which this type of construction would be structurally favorable, however, have not been established. Before such a determination can be made, research is needed into the contributions that post members make to the strength of box beams. A theoretical investigation of this problem, reference 1, presented charts for the required stiffness of posts at various spacings to achieve a desired buckling stress in the otherwise unstiffened covers of a box beam subjected to bending. The analysis of reference 1 has recently been extended to include the effect of stiffeners placed on both covers along longitudinal rows of posts and is presented in reference 2. A single solution of the stability criterion resulting from this analysis requires extensive calculations, but the criterion is in a convenient form for solution by high-speed computing machines.

The present paper gives the preliminary results of a computational program using equation (24) of reference 2. The purpose of the computations is to determine the range of post and stiffener stiffnesses required to achieve desired buckling stress values in the covers of a box-beam structure subjected to bending. The results are presented in chart form and cover a limited but useful range of the structural parameters.

SYMBOLS

l	length of cover bay between post supports
b	width of cover bay between longitudinal lines of support
β	aspect ratio of cover bay, l/b
m	number of bays in length of box beam
n	number of bays in width of box beam
L	length of beam between ribs, ml
B	width of beam between shear webs, nb
t_C	thickness of compression cover
t_T	thickness of tension cover
E	Young's modulus of elasticity
μ	Poisson's ratio
D_C	flexural stiffness of compression cover, $Et_C^3/12(1 - \mu^2)$
D_T	flexural stiffness of tension cover, $Et_T^3/12(1 - \mu^2)$
EI	flexural stiffness of longitudinal stiffener (may be taken about the plane of attachment to the cover for the range of proportions in this paper)
γ_C	flexural-stiffness ratio of stiffener to cover bay on compression side of beam, EI/bD_C
γ_T	flexural-stiffness ratio of stiffener to cover bay on tension side of beam, EI/bD_T
A	cross-sectional area of longitudinal stiffener
δ_C	area ratio of stiffener to cover bay on compression side of beam, A/bt_C

δ_T area ratio of stiffener to cover bay on tension side of beam, A/bt_T

p number of buckles occurring across width of beam

q number of buckles occurring along the length of the beam

λ wave length of buckle (distance between nodes), L/q

F spring stiffness of post support, force per unit extension

S post-support-stiffness parameter, $Fb^2/\pi^4 D_C$

N_C compressive load per unit width of cover

N_T tensile load per unit width of cover

k_C compressive-load coefficient, $N_C b^2/\pi^2 D_C$

k_T tensile-load coefficient, $N_T b^2/\pi^2 D_T$

r,s integers

DESCRIPTION OF STRUCTURAL CALCULATIONS

The calculations presented in this paper apply to a box beam similar to the configuration shown in figure 1. The beam is composed of relatively thick cover sheets with an arbitrary number of shear webs which are assumed to provide simple support to the covers. A single longitudinal stiffener is located on each cover running down the center line of each cell of the beam, and vertical post members of equal stiffness connect the stiffeners at frequent equally spaced intervals. A structure is thus established in which the tension cover of the beam helps stabilize the compression cover through the medium of lightweight members known as posts. For a certain intermediate range of wing depths, a combination of stiffeners and posts of this kind may provide a more efficient means of stabilizing the covers than either stiffeners of large moment of inertia or full-depth webs.

Stability Equation

The purpose of the present calculations is to obtain numerical values of the stiffnesses required of posts and stiffeners to achieve desired buckling-stress values in the covers of beam configurations similar to that shown in figure 1, when subjected to a bending moment.

The calculations were based on the following stability equation which was derived in reference 2:

$$\frac{1}{\beta^3 S} + \sum_{s=-\infty}^{\infty} \frac{1}{\left(2s + \frac{q}{m}\right)^2 \left[\left(2s + \frac{q}{m}\right)^2 \gamma_C - \beta^2 \delta_C k_C\right] + \cfrac{1}{\sum\limits_{r=-\infty}^{\infty} \cfrac{1}{\left[\left(2s + \frac{q}{m}\right)^2 + \left(2r + \frac{p}{n}\right)^2 \beta^2\right]^2 - \left(2s + \frac{q}{m}\right)^2 \beta^2 k_C}}} +$$

$$\frac{D_C}{D_T} \sum_{s=-\infty}^{\infty} \frac{1}{\left(2s + \frac{q}{m}\right)^2 \left[\left(2s + \frac{q}{m}\right)^2 \gamma_T + \beta^2 \delta_T k_T\right] + \cfrac{1}{\sum\limits_{r=-\infty}^{\infty} \cfrac{1}{\left[\left(2s + \frac{q}{m}\right)^2 + \left(2r + \frac{p}{n}\right)^2 \beta^2\right]^2 + \left(2s + \frac{q}{m}\right)^2 \beta^2 k_T}}} = 0 \qquad (1)$$

This equation may be solved either for an infinitely long beam or for a beam whose length between ribs is specified. Solutions for the long-beam case, which are conservative when applied to beams with finite rib spacings, are presented in this paper. The degree of conservatism may be expected to be slight, however, when the rib spacing is greater than about three times the shear-web spacing.

Structural Parameters

The calculations were made for the following values of parameters appearing in the stability equation:

$$\beta = \frac{l}{b} = 1, \frac{1}{2} \qquad \frac{D_T}{D_C} = 1, \frac{1}{8}$$

$$k_C = \frac{N_C b^2}{\pi^2 D_C} = 4, 3 \qquad \delta_C = \frac{A}{bt_C} = 0, 0.2$$

For each of the 16 possible combinations of these parameters, a range of combinations of the post-stiffness parameter $S = \frac{Fb^2}{\pi^4 D_C}$ and stiffener-stiffness parameter $\gamma_C = \frac{EI}{bD_C}$ can be determined which satisfy the stability equation.

With respect to the stress values achieved in the beam, a value of $k_C = 4$ means that the compression cover receives enough support from the tension cover, stiffeners, and posts, to buckle as if it were a long, simply supported plate of width b and thickness t_C. Theoretically, this combination of support stiffnesses is sufficient to form a longitudinal node down the center line of the compression cover at buckling and no further increase in buckling stress is possible without adding torsional restraint. Since it is not necessarily desirable to develop a stress value corresponding to $k_C = 4$ at a given cross section of a beam, the combinations of stiffener and post stiffness required to develop 75 percent of this value, or $k_C = 3$, were also computed.

The values of the ratio β of post spacing to bay width were chosen to correspond with beam proportions considered of practical interest. The flexural-stiffness ratio of the covers D_T/D_C was assumed in these calculations to be a function only of the thickness ratio of the covers t_T/t_C. Thus, a flexural-stiffness ratio $\frac{D_T}{D_C} = \frac{1}{8}$ corresponds to a tension cover one-half the thickness of the compression cover. The parameter δ_C, which is the ratio of the cross-sectional area of the stiffener to the cross-sectional area of the compression-cover bay, determines the proportion of the total panel end load carried by the stiffener and thereby influences the effective bending stiffness of the stiffener. Since the size of stiffeners to be used in conjunction with posts is anticipated to be considerably smaller than in conventionally stiffened sheets, the values 0 and 0.2 were chosen for δ_C.

Other parameters appearing in the stability equation are p/n, q/m, δ_T, γ_T, and k_T. The values of p/n and q/m determine the mode of buckling of the beam. For the beam under consideration, $n = 2$ (two bays wide) and $p = 1$ (instability occurs with a single buckle across the width of the box). The value of q/m determines the length of the buckles in the longitudinal direction and must be varied until the natural wave length is found. The natural wave length is associated with the highest value of post stiffness required to satisfy the stability equation for a given value of stiffener stiffness.

The parameters δ_T and γ_T may be defined as

$$\delta_T = \frac{A}{bt_T} = \frac{A}{bt_C}\frac{t_C}{t_T} = \delta_C \frac{t_C}{t_T} \tag{2}$$

and

$$\gamma_T = \frac{EI}{bD_T} = \frac{EI}{bD_C}\frac{D_C}{D_T} = \gamma_C \frac{D_C}{D_T} \tag{3}$$

The value of k_T may be defined in terms of k_C through the relation between the end loads carried by each cover of the beam. The stress in a stiffener is assumed to be equal to the stress in the cover to which it is attached, and the stiffeners on the two covers are assumed to be of equal cross-sectional area. For a box beam subjected to bending moment, the load carried in each cover is the same; hence the following equation may be written:

$$nbN_T\left(1 + \frac{n - 1}{n}\delta_T\right) = nbN_C\left(1 + \frac{n - 1}{n}\delta_C\right)$$

or

$$\frac{b^2N_T}{\pi^2D_T}\frac{D_T}{D_C}\left(1 + \frac{n - 1}{n}\delta_C\frac{t_C}{t_T}\right) = \frac{b^2N_C}{\pi^2D_C}\left(1 + \frac{n - 1}{n}\delta_C\right) \tag{4}$$

When the buckling-stress coefficients k_T and k_C are substituted for b^2N_T/π^2D_T and b^2N_C/π^2D_C, respectively, and $n = 2$, the value of k_T is given by

$$k_T = \frac{D_C}{D_T}k_C\frac{1 + \frac{1}{2}\delta_C}{1 + \frac{1}{2}\delta_C\frac{t_C}{t_T}} \tag{5}$$

RESULTS AND DISCUSSION

Stiffness Charts

The results of the calculations for the structural parameters discussed in the previous section are presented in figures 2 to 5. The curves give the combinations of $\gamma_C = \frac{EI}{bD_C}$ and $S = \frac{Fb^2}{\pi^4 D_C}$ which satisfy the stability equation for the constant values of β, D_T/D_C, k_C, and δ_C listed on each figure. The calculated points used in plotting these curves are listed in table I. Also given in table I are the values of the buckle length, listed as λ/b, associated with each combination of γ_C and S.

The curves presented in figures 2 to 5 cover what is believed to be the practical range of combinations of γ_C and S. When S approaches zero, it is evident that the values of γ_C approach the required values of EI/bD_C for a stiffener on a long plate (see ref. 3). As S becomes large, the values of γ_C tend toward definite limiting values in all cases where $\beta = 1$. These limiting values, shown as dashed lines at the right margins of figures 2 and 3, are the values of EI/bD_C that would be required if nondeflecting supports were located at the post stations and can be found from the data in reference 3. The actual values of S, (if finite values exist) associated with these limiting values of EI/bD_C were not computed. For the curves associated with $\beta = \frac{1}{2}$, it may be deduced from the results of reference 1 that for a sufficiently small value of γ_C, the value of S must approach ∞.

With respect to the post-stiffness parameter $S = \frac{Fb^2}{\pi^4 D_C}$, a careful interpretation must be placed upon the quantity F, which is defined simply as the spring stiffness of the post. A preliminary study indicates that if F is thought of as the axial spring stiffness of the post, the range of values for S covered in this investigation can easily be achieved by posts of relatively small cross-sectional area. In practical construction, however, the stiffness of the attachments between the post and the covers will play an important role in determining the effective value of the post stiffness and should be included in the calculation of F. It should be noted that the values of $S = \frac{Fb^2}{\pi^4 D_C}$ presented in this paper cannot be compared directly with the values of $S = \frac{FB^2}{\pi^2 D_C}$ (notation of present paper) given in figure 2 of reference 1. A comparison is obtained by dividing the values of S presented in reference 1 by the factor $4\pi^2$.

Although the number of curves presented is too limited to give a complete picture of the interaction of the various parameters, a number of interesting features are illustrated. From a comparison of the results of this paper with the results of a similar analysis for posts alone (ref. 1), it can be concluded that, for posts of the same stiffness at comparable spacings, appreciably higher buckling stresses can be developed when a relatively small longitudinal stiffener is used in conjunction with posts. Thus, for high values of structural index, a combination of posts and stiffeners should be more efficient than posts alone. Also, the presence of a stiffener tends to minimize the change in required post stiffness as the relative thickness t_C/t_T of the covers is changed. As the stiffness of the stiffeners approaches the stiffness of the compression cover $\left(\frac{EI}{bD_C} = 1\right)$, however, a variation in the relative stiffness of the covers causes an appreciable shift in the curves, as illustrated in figures 4 and 5. The influence of the parameter δ_C on the combined effective stiffness of two stiffeners working together, one in tension and the other in compression, is unpredictable when the values of the stiffness ratio EI/bD_C become small as shown by the crossing of the curves in figure 5.

Computational Procedure

The procedure used to calculate a given point listed in table I may be briefly summarized as follows:

1. Insert the desired values of β, k_C, and the other quantities into equation (1).

2. To calculate the value for S associated with a given value of γ_C, assume a value of q/m and sum the terms in the equation until the desired accuracy is obtained.

3. For the given value of γ_C, vary q/m in small steps and sum the series for each variation in q/m until a maximum value of S is obtained.

It is obvious that a complete solution, in which both the s and r summations are summed to a large number of terms, will require a large number of calculations to determine a single combination of γ_C and S. A reasonable short cut consists in limiting the s series to the term for which $s = 0$ but carrying out the r-summation to the desired accuracy, a procedure equivalent to restricting the cover deflections to a sinusoidal variation in the length direction. A single-term solution with s and r equal to zero is equivalent to restricting the deflection to a sinusoidal pattern in both the length and width directions. The results of the latter two short cuts are compared in figure 6 with a "complete" solution in which the computations were carried out to a high degree of accuracy. The comparison shown in figure 6 indicates the accuracy of the results that can be obtained by short-cut solutions. Similar results were obtained for all of the other curves shown in figures 2 to 5. Although small differences are obscured by a logarithmic scale, the complete solution and the approximation $s = 0$ give essentially the same results over a large portion of the curve, with appreciable percentage differences in the values for S occurring only in the lower range of values for γ_C. The approximate solutions, however, continue down into a range of values for γ_C for which the complete solution indicates that no actual finite values of S exist. This failing of the approximate methods of solution can yield misleading results, especially in those cases where the lower limit γ_C may be quite high, as it sometimes is with other combinations of parameters.

An interesting point is that when the s series is eliminated from the stability equation, the remaining terms represent a complete solution for a case in which the post spacing approaches zero and in effect a uniform distribution of stiffness is created. It is evident from the comparison shown in figure 6 that posts at reasonably small spacings may be considered "smeared out" for analysis purposes and that the resulting simplified solution is applicable to discrete post spacings if the number of posts per buckle length is not too small.

The curves presented in figures 2 to 5 are a composite of the results of two computational procedures. The upper portion of each curve was calculated with the approximation $s = 0$, using an IBM Card-Programmed Calculator. The solution for the lower end of each curve was coded for the National Bureau of Standards Eastern Automatic Computer (SEAC). The procedure was programmed so that a sequence of points along a given curve was obtained automatically in a continuous run on SEAC. The number of s and r terms used in the operation was adjusted until it was evident that the errors in the results would be less than 1 percent.

CONCLUDING REMARKS

The curves presented give numerical values of the stiffness required of posts working in combination with longitudinal stiffeners along the row of posts to achieve desired buckling-stress values in the covers of a box beam subjected to bending. Comparison of these results with those of a similar analysis of beams without longitudinal stiffeners shows that for the same post configuration appreciably higher buckling stresses can be developed when a small longitudinal stiffener is used in conjunction with posts.

The results indicate that a short-cut solution to the stability equation for the beam, if used with a knowledge of its inherent limitations, will give good results over a large range of the parameters investigated in this paper.

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2. Seide, Paul: Derivation of Stability Criterions for Box Beams With Longitudinally Stiffened Covers Connected by Posts. NACA TN 2760, 1952.

3. Seide, Paul, and Stein, Manuel: Compressive Buckling of Simply Supported Plates With Longitudinal Stiffeners. NACA TN 1825, 1949.

TABLE I. TABLE OF VALUES COMPUTED FROM STABILITY EQUATION

		$\beta = 1$						$\beta = \frac{1}{2}$					
		$\frac{D_T}{D_C} = 1$			$\frac{D_T}{D_C} = \frac{1}{8}$			$\frac{D_T}{D_C} = 1$			$\frac{D_T}{D_C} = \frac{1}{8}$		
k_C	δ_C	γ_C	$\frac{\lambda}{b}$	S	γ_C	$\frac{\lambda}{b}$	S	γ_C	$\frac{\lambda}{b}$	S	γ_C	$\frac{\lambda}{b}$	S
4	0	48.70	5.24	0	48.70	5.24	0	48.70	5.24	0	48.70	5.24	0
		40	4.80	.0146	40	4.80	.0150	40	4.80	.0073	40	4.80	.0075
		30	4.15	.0457	30	4.12	.0480	20	3.39	.0585	30	4.12	.0240
		20	3.33	.1163	20	3.57	.1268	10	2.40	.1931	20	3.48	.0634
		10	2.38	.3922	10	2.63	.4779	5	1.61	.5939	15	3.12	.1119
		8	2.08	.5692	8	2.38	.7315	3	1.28	1.3892	8	2.38	.3540
		5	1.52	1.4138	6	2.17	1.3462	2	1.19	2.6860	4	2.08	1.7111
		4.5	1.28	1.9987	5	2.08	2.1728	1	1.16	11.223	3	2.08	9.2765
	.2	73.41	5.83	0	73.41	5.83	0	73.41	5.83	0	73.41	5.83	0
		60	5.37	.0158	60	5.31	.0151	60	5.37	.0079	60	5.31	.0075
		40	4.38	.0607	40	4.41	.0635	40	4.38	.0304	40	4.41	.0317
		30	3.85	.1134	30	3.57	.1249	20	3.13	.1153	30	3.86	.0596
		15	2.78	.3651	15	2.78	.4023	10	2.17	.3331	20	3.35	.1261
		8	1.92	.9820	10	2.38	.7783	5	1.52	.9554	10	2.38	.3780
		6	1.56	1.6956	7	2.00	1.4626	3	1.25	2.1248	5	1.79	1.1808
		5.3	1.28	2.4726	6	1.79	2.0244	2	1.19	4.0366	3	1.67	3.4142
								1	1.16	19.403	2	1.67	26.413
3	0	24.34	4.42	0	24.34	4.42	0	24.34	4.42	0	24.34	4.42	0
		20	4.08	.0143	20	4.08	.0146	20	4.08	.0072	20	4.08	.0073
		10	2.94	.1077	10	2.94	.1185	15	3.56	.0215	10	2.98	.0590
		6	2.27	.2524	6	2.44	.3002	10	2.91	.0536	8	2.72	.0898
		4	1.92	.4561	4	2.13	.5989	8	2.64	.0796	6	2.38	.1472
		3	1.72	.6908	3	1.96	1.0240	5	2.19	.1791	4	2.17	.2821
		2	1.35	1.4714	2.2	1.75	2.2217	3	1.72	.3116	3	2.00	.4463
		1.9	1.22	1.8183	1.9	1.61	4.1418	1.5	1.43	.6523	2	1.85	.8999
								.8	1.28	1.0971	1.5	1.79	1.6885
								.4	1.19	1.6982	1	1.79	11.704
	.2	37.61	4.95	0	37.61	4.95	0	37.61	4.95	0	37.61	4.95	0
		30	4.48	.0168	30	4.48	.0170	30	4.48	.0084	30	4.48	.0085
		20	3.71	.0631	20	3.73	.0657	20	3.71	.0315	20	3.73	.0328
		10	2.68	.2218	10	2.78	.2423	10	2.68	.1108	10	2.78	.1202
		8	2.38	.3108	6	2.22	.5315	8	2.44	.1534	6	2.27	.2569
		4	1.79	.8265	4	1.92	.9954	5	2.00	.2867	3	1.79	.6565
		3	1.56	1.3148	3	1.67	1.6946	3	1.67	.5250	1.5	1.61	1.6741
		2.5	1.22	2.2160	2.5	1.30	2.9409	2	1.47	.8034	1	1.56	3.1797
								1	1.28	1.4772	.7	1.52	6.9441
								.5	1.19	2.2834	.5	1.52	39.361

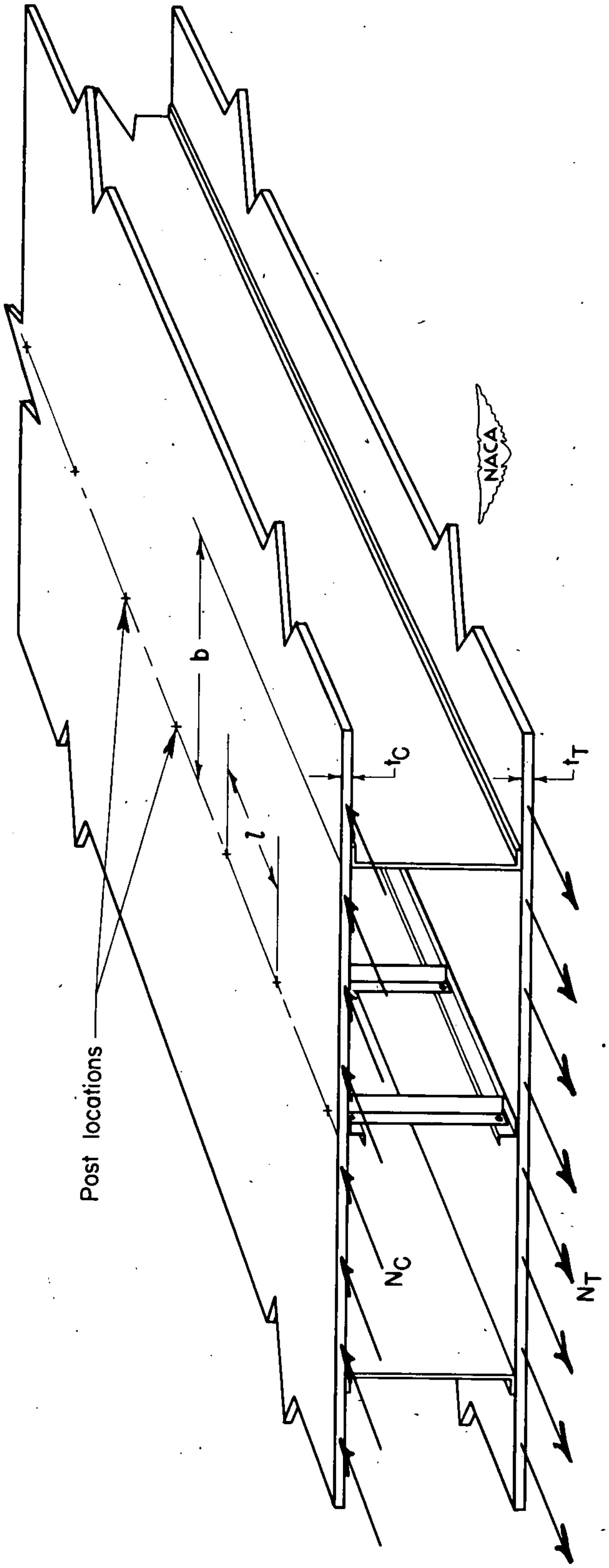

Figure 1.- Portion of box beam stabilized by longitudinal stiffeners and posts.

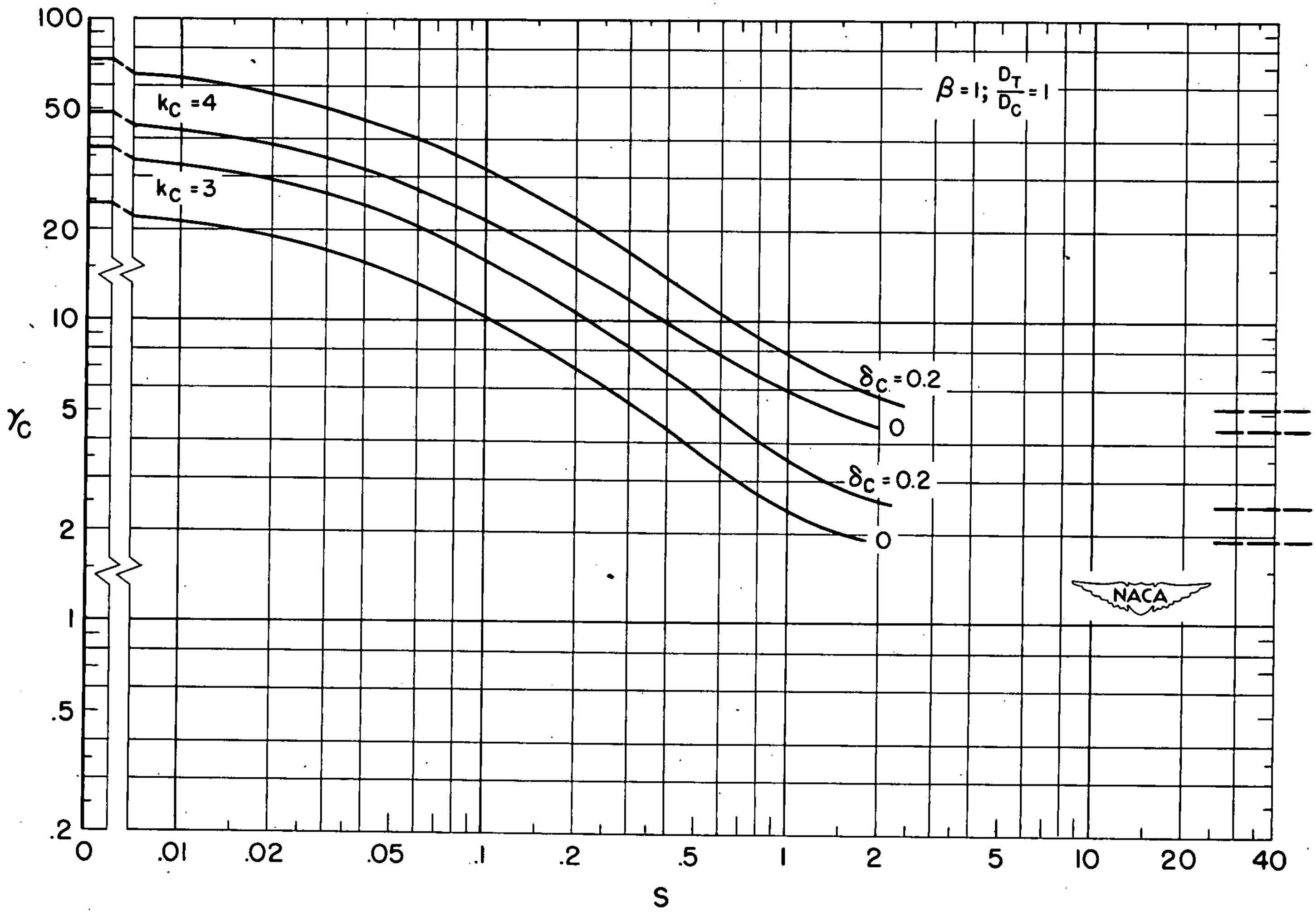

Figure 2.- Combinations of stiffener stiffness and post stiffness required to stabilize covers of box beam with $\beta = 1$ and $\frac{D_T}{D_C} = 1$.

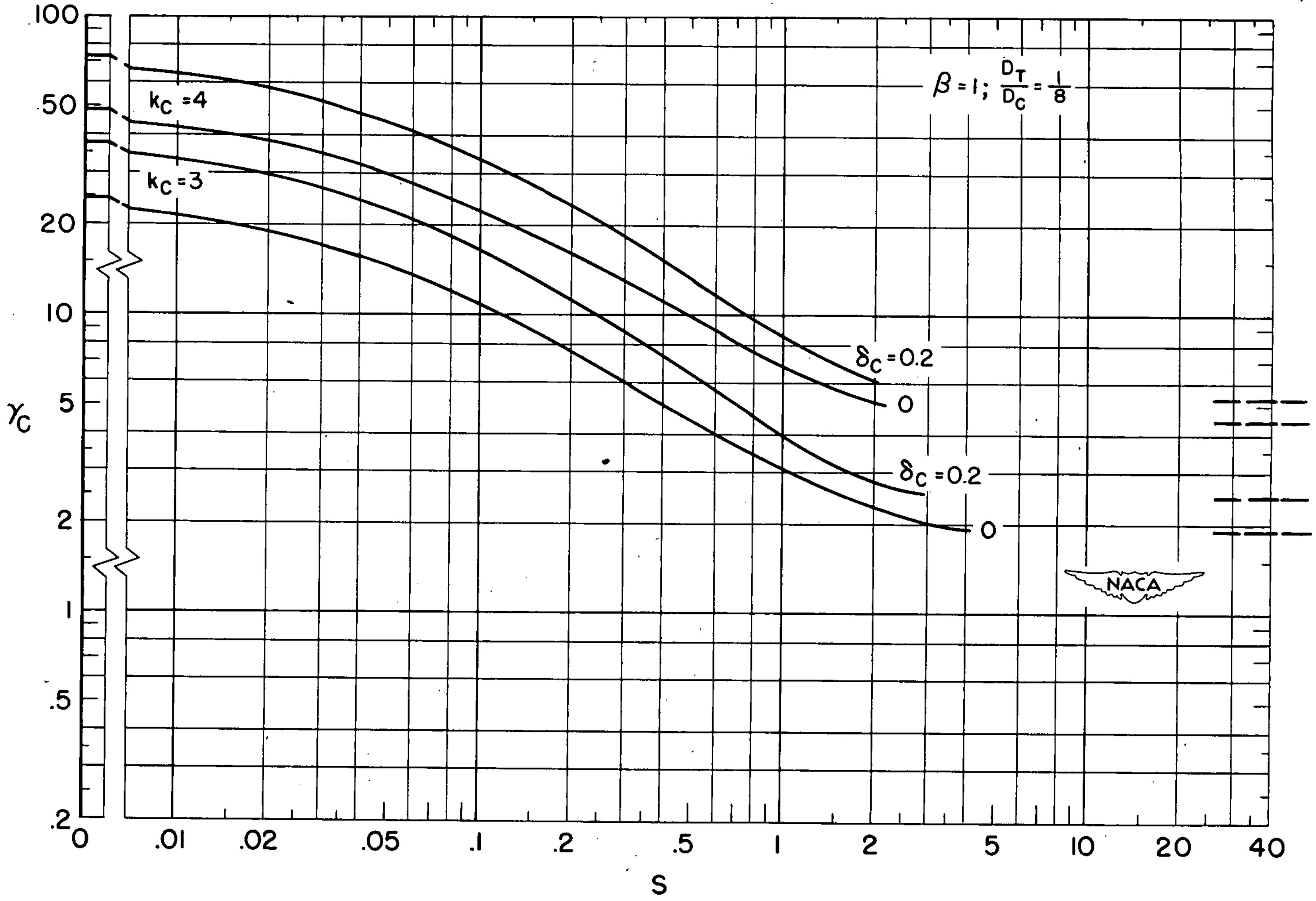

Figure 3.- Combinations of stiffener stiffness and post stiffness required to stabilize covers of box beam with $\beta = 1$ and $\frac{D_T}{D_C} = \frac{1}{8}$.

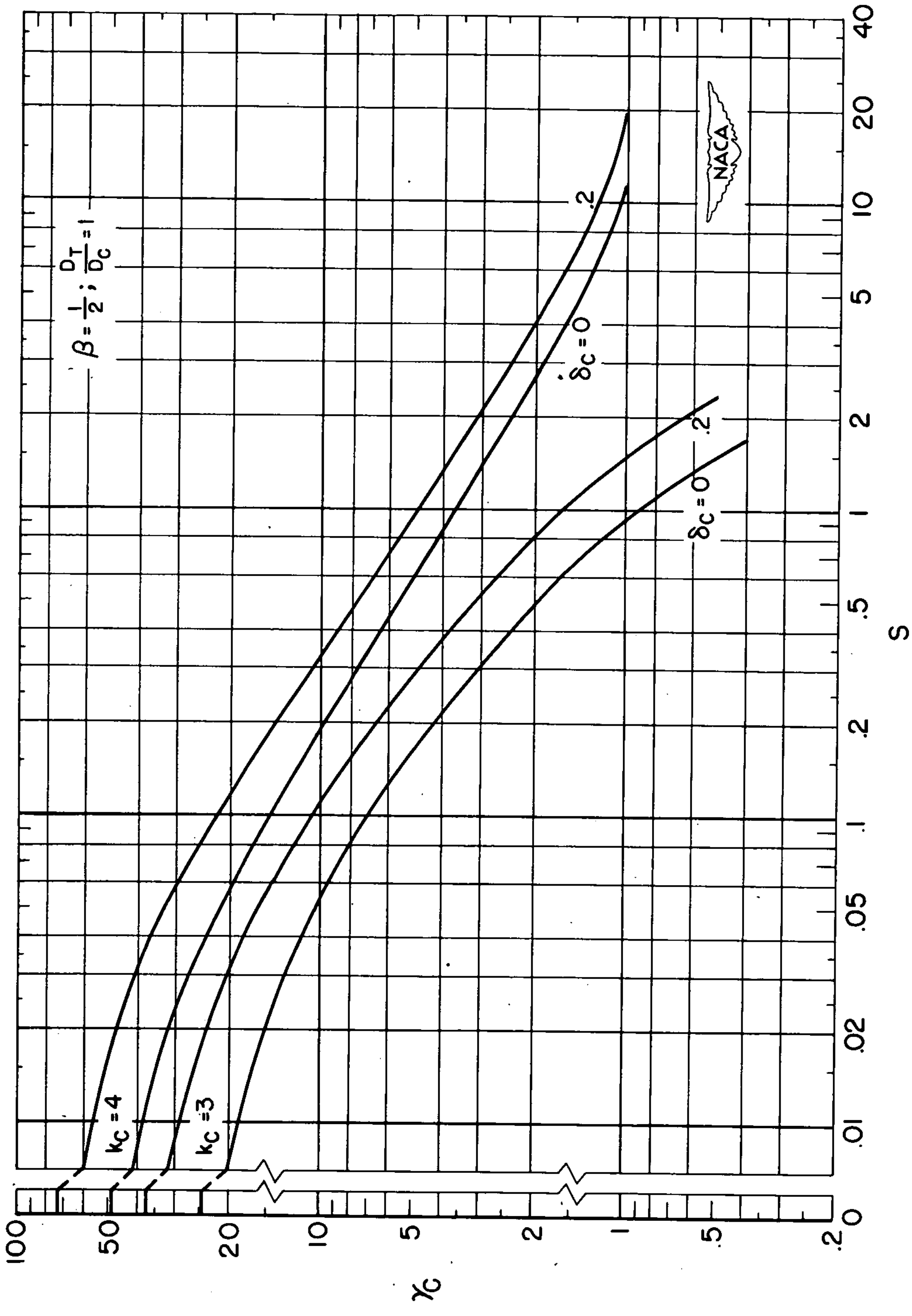

Figure 4.- Combinations of stiffener stiffness and post stiffness required to stabilize covers of box beam with $\beta = \frac{1}{2}$ and $\frac{D_T}{D_C} = 1$.

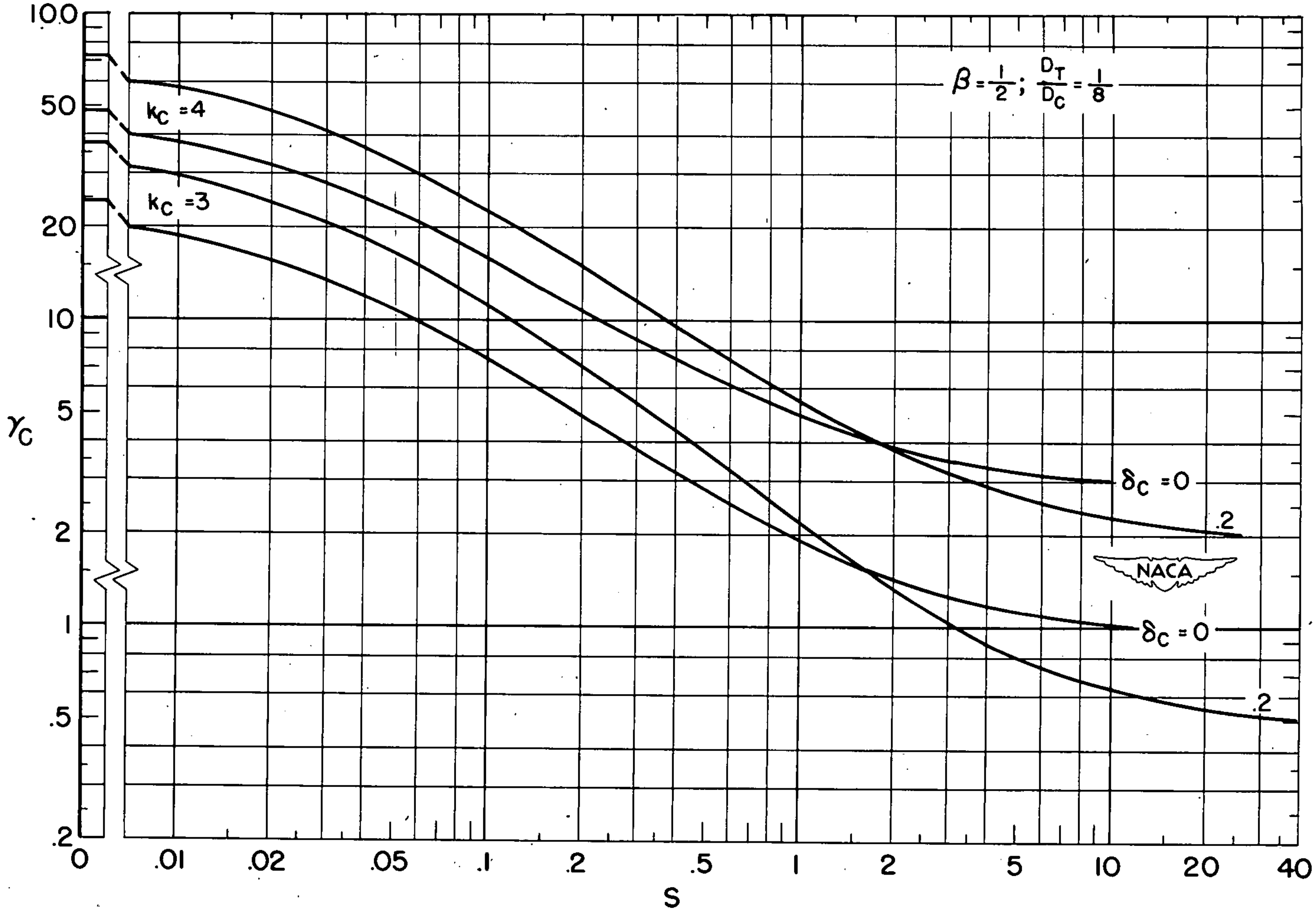

Figure 5.- Combinations of stiffener stiffness and post stiffness required to stabilize covers of box beam with $\beta = \frac{1}{2}$ and $\frac{D_T}{D_C} = \frac{1}{8}$.

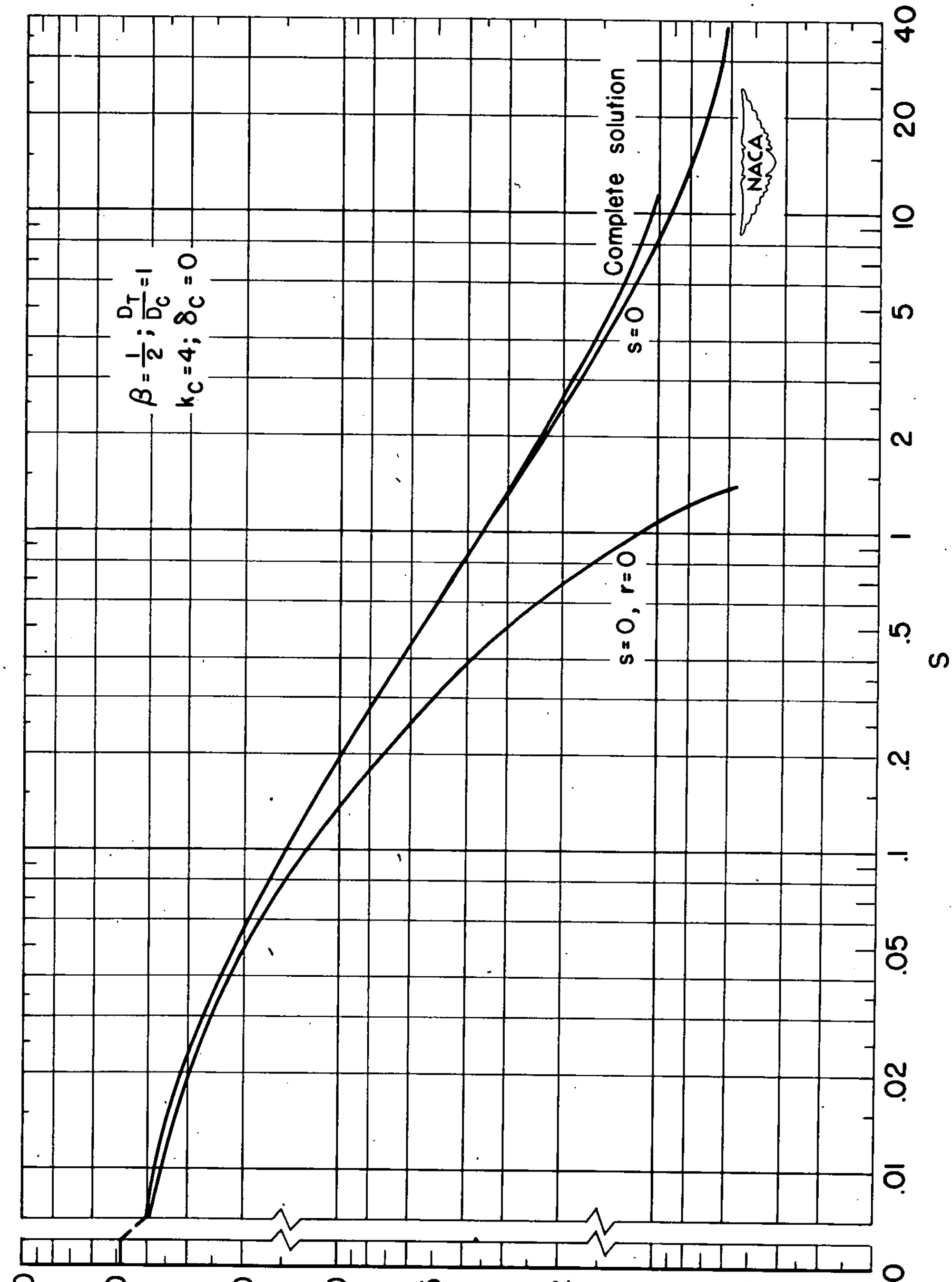

Figure 6.- Comparison of results of approximate and complete solutions.