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RESEARCH MEMORANDUM

MOMENT OF INERTIA AND DAMPING OF FLUID IN TANKS

UNDERGOING PITCHING OSCILLATIONS

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MOMENT OF INERTIA AND DAMPING OF FLUID IN TANKS UNDERGOING PITCHING OSCILLATIONS

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SUMMARY

Studies were made of the fluid dynamics of some different-shaped fuel tanks in a horizontal flight attitude and undergoing pitching oscillations of a few degrees. The moment of inertia of the fluid was determined for the various tanks over a range of tank fullness from empty to full. For the tank-full case, comparisons of the experimental moment of inertia of the fluid with some theoretical solutions for the moment of inertia of fluid in tanks having elliptic and rectangular sections indicate that the theoretical solutions predict the experimental values for the elliptic and rectangular tanks and offer engineering approximations of the fluid moment of inertia for some tanks of other shapes. It was found that the ratio of the fluid inertia to the solid inertia for a given tank fullness increased as the tank fineness ratio increased. However, for a given tank the ratio of the fluid inertia to the solid inertia for the same fullness was essentially constant over a large range of tank fullnesses and had approximately the value indicated for the tank-full condition.

The studies of the effective fluid damping revealed that, for the case of oscillations resulting from a step release, there was little damping during the first few cycles before sloshing developed, and after sloshing developed the apparent damping became nonlinear. For the forced oscillations the damping factor was substantially larger than that for a step-release oscillation for the same initial amplitude and tended to increase with increasing frequency and amplitude.

INTRODUCTION

For some airplanes the quantity of fuel carried in wing-mounted tanks comprises a large percentage of the total mass of the wing. It has become evident that the sloshing of these large masses of fuel may cause important dynamic effects. In a recent paper (ref. 1) the case of model tanks mounted on a wing undergoing translation following instantaneous release has been treated. It was shown that a fairly complicated physical process takes place and that the effective mass and damping depend sharply on the number of cycles after the start of the motion. The present study deals with the moment of inertia and damping of large tanks undergoing pitching oscillations with amplitudes of a few degrees. The dynamic effects of the fuel for this case are of significance, for example, in the flutter phenomenon, and are particularly important in those cases where the susceptibility of a wing to flutter is critically dependent on the wing bending-torsion frequency ratio.

DESCRIPTION OF APPARATUS

Some of the tank shapes and the dynamic system used are shown in figure 1. The shapes were as follows: tank A, an actual 185-gallon tip tank with baffles; tank B, a 58-gallon circular cylinder; and tank C, a 230-gallon tank with a square profile. These tanks were oscillated about a transverse axis as shown in figure 1. In addition, a 1/3-scale model of tank B was oscillated as shown here and also about the cylindrical axis in which configuration it will be referred to as tank D. With the exception of the round profile, the tanks were mounted in a simple spring-inertia dynamic system shown here. This system could be caused to oscillate by a step release from an initial angular displacement or could be oscillated at a constant amplitude by supplying energy through a weak spring. The decaying oscillations resulting from a step release were investigated at various degrees of tank fullness for all tank shapes. Further studies were made on the 185-gallon tank oscillating at a constant amplitude. The effective inertia of the fluid was obtained from a knowledge of the stiffness and measured natural frequency of the system. A measure of the energy dissipation was obtained from a study of the rate of decay of the oscillation for the case of the step release and from the work required to maintain a constant amplitude of oscillation, as obtained from a moment-displacement diagram.

RESULTS AND DISCUSSION

A convenient place to begin the study of the inertia of fluids as a function of tank fullness is the tank-full condition. Theoretical solutions for the fluid inertia for the tank-full condition have been given in reference 2 for the two-dimensional elliptic tank and are given by

$$\frac{I_{eff}}{I_{solid}} = \left(\frac{\left(\frac{a}{b}\right)^2 - 1}{\left(\frac{a}{b}\right)^2 + 1}\right)^2 \tag{1}$$

where a/b is the ratio of the major axis to the minor axis, I_{eff} is the fluid inertia about the fluid centroid, and I_{solid} is the solid inertia about the fluid centroid. This relation is represented in figure 2 as the lower solid curve, and, notably enough, the same curve also holds for the ellipsoid of revolution. This work has been extended in reference 3 to the case of arbitrary two-dimensional tank shapes. In reference 3, it has been shown that there is a direct analogy between the torsional modulus for solid sections and the fluid inertia of full tanks which is given by

$$\frac{I_{eff}}{I_{solid}} = 1 - \frac{\rho J_p}{I_{solid}} \tag{2}$$

where the I's are as defined in equation (1), J_p is the torsional modulus for the solid section, and ρ is the mass density of the fluid. The case of a full tank of rectangular section is shown by the upper solid curve of figure 2. The following observations may be made from these curves: For low fineness ratios, such as for a round or square tank (which has a fineness ratio of 1), there is little or no transmitted moment of inertia and the fluid moment of inertia is low as compared with the solid inertia; however, as the fineness ratio becomes large, the inertia ratio tends toward unity and, moreover, the differences due to tank shape become small as shown by the convergence of the curves. Several experimental points corresponding to the shapes investigated are shown in figure 2. Note the excellent agreement with theory for the square shape C and for the round shape D. The experimentally determined moment-of-inertia ratios for shapes A and B have been plotted at their fineness ratios which are defined as the maximum length to the maximum thickness. It may be seen that for these shapes the results fall near the theoretical curves although the theoretical curves are for rectangular and elliptic shapes. This agreement between the experimental and theoretical fluid inertias even when there is some deviation in tank shape suggests that the theoretical solutions offer a good basis for estimating the fluid inertia of a new tank configuration and, as will be noted, help in the estimation of the inertia results for the partially full tank.

Some experimental results for the partially full tank are presented in figure 3. The ratio of the effective fluid inertia to the solid inertia for the same amount of fluid is shown as a function of tank fullness. The full-tank condition for the various shapes shown here is the same as that shown in figure 2. As before, the effect of tank shape on the inertia ratio may be seen; as the fineness ratio is increased, the inertia ratio for a given tank fullness is increased. Note that, for a wide variation in tank fullness of from 10 to 100 percent, the inertia ratio varies but little. In order to illustrate the effect of frequency on the data, tank A, the 185-gallon tank, was tested over a range of frequencies from 2 to 8 cycles per second. The resulting variation is shown in the top curve of figure 3 by the shaded area. In order to indicate the effect of size, the 1/3-scale model of tank B was tested over a considerable frequency range. The results which are independent of frequency over the range tested are given by the dashed line (fig. 3) and indicate the effect of size on the moment of inertia may not be important. In light of the small variation of the curves with tank fullness, the results shown for the full-tank condition take on added significance because it appears that the inertia ratio for any tank fullness may be estimated for engineering purposes if the ratio of the fluid inertia to the solid inertia can be determined either experimentally or analytically.

In the study of damping, it was found, as in reference 1, that a complicated physical process takes place which leads to wide changes in the damping with time history, that is, with the number of cycles. Fairly complicated wave motions exist in the tank, even for small oscillations, and give rise to the possibility of a conservative transfer of energy from pitching motion of the system to wave motion of the fuel and subsequently back to pitching motion; thus, the ordinary inference of damping drawn from the ratio of successive peak amplitudes of decaying oscillations of the tank must be modified. In order to illustrate this phenomenon, a "standard-type" semilogarithmic plot of amplitude against number of cycles for the partially full 185-gallon tank as obtained from a step release is given in figure 4. During the first few cycles after release, before sloshing has developed, the rate of decay is small and indicates that there is little apparent damping. As sloshing develops, the curve becomes increasingly steep as energy is being both dissipated and transferred to other modes of fuel motion. After considerable decrease in the amplitude, the curve reverses its slope and indicates that energy is being fed back into pitching motion. It was worthy of note that this second peak was not known to become nearly as large as the initial amplitude. In view of the nonlinear nature of the effective damping it would appear that, when it is necessary to treat the damping analytically, some averaged value for the desired conditions must be used.

In figure 5 is shown the variation of the damping factor with tank fullness. The data in the upper two curves are for forced oscillations at a constant tip amplitude of 0.8 inch. The lower curve is obtained from the first few cycles after release from an initial tip amplitude of 0.8 inch before the full sloshing has developed, that is, from the solid line shown in figure 4. For the upper curves in the case of steady oscillations, where it is assumed that the violent sloshing action has become statistically independent of time, a marked difference in the size of the damping factor from that pertaining to the step release (the lower curve) may be noted. The two upper curves shown for the forced-oscillation case differ in the range of frequencies covered and thus indicate an effect of frequency. For a given percent-full condition, the frequency of the upper points is approximately 1.6 times the frequency of the lower points and the damping factor at the higher frequency is consistently larger than at the lower frequency. It is seen that the damping factor becomes small at the extremes of tank fullness and that the maximum values tend to occur at the middle of the curve. It was also noted in the course of this work that the damping factor exhibited a tendency to increase as the amplitude of oscillation was increased.

Before concluding, it may be appropriate to consider briefly the effects of viscosity. With respect to the fluid inertia, viscosity would enter primarily in the thickness of the boundary layer formed on the container walls and would tend to make the fluid in the boundary layer fully effective in contributing to the system inertia. In reference 2, it is shown that for periodic motion the thickness of the boundary layer is given by

$$\lambda = \left(\frac{2\nu}{\omega}\right)^{1/2} \tag{3}$$

where λ is the boundary-layer thickness, ν is the kinematic coefficient of viscosity, and ω is the circular frequency. With water as the fluid and the frequencies of these experiments, the boundary-layer thickness was shown not to exceed 0.028 inch and, consequently, may be considered negligible. With respect to the influence on the damping characteristics, it may be remarked that the studies of reference 1 have indicated that, within the viscosity range of current-type fuels, no influence of viscosity could be detected for the case of a tank undergoing translation. Although no data are available for the pitching case, it is felt that, in the range of turbulent damping, the damping is relatively unaffected by viscosity.

CONCLUDING REMARKS

In conclusion, it may be said that these studies indicated that some hope exists for engineering approximations of the effective-mass moment of inertia of fluids in wing-supported tanks by using as a convenient basis the tank-full condition. For the range investigated, the tank size, frequency, and amplitude of oscillation were found to have little influence on the effective inertia. With regard to the damping action of the fluid, it was found that the step-release type of motion contributes little to the damping of the system during the initial stages of the oscillation and that, after the establishment of sloshing, the apparent damping becomes nonlinear. For a forced oscillation, it was noted that considerable damping was present over a range of tank fullnesses and that the damping action appeared to be a function of the frequency and amplitude.

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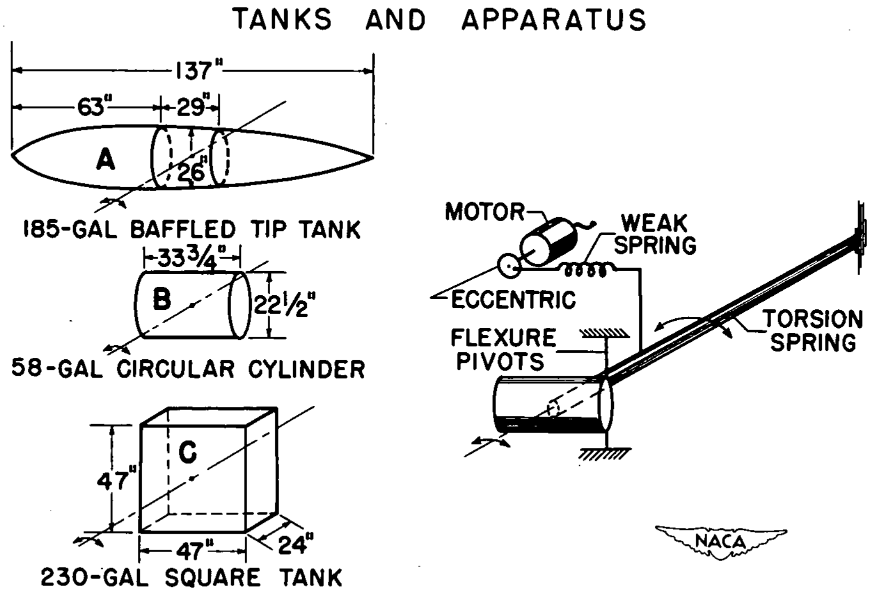

Figure 1.

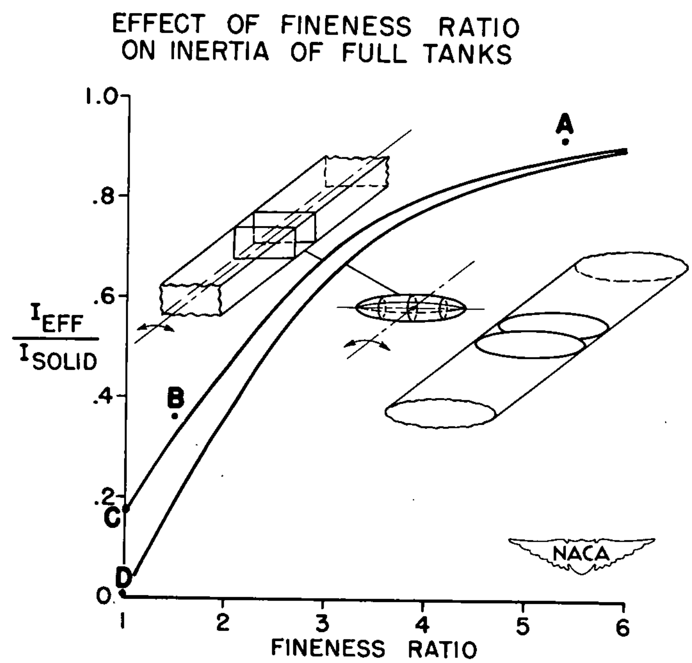

Figure 2.

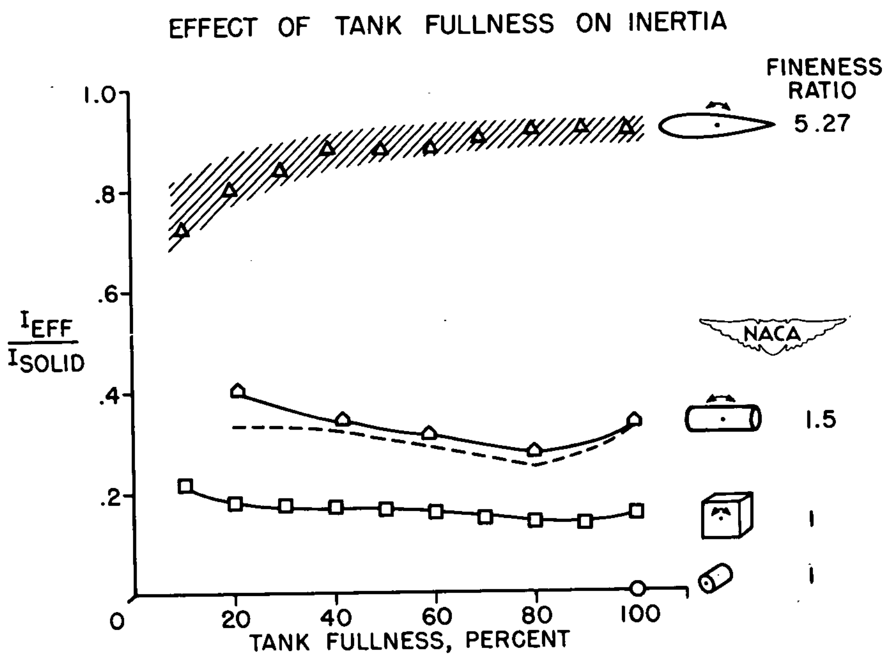

Figure 3.

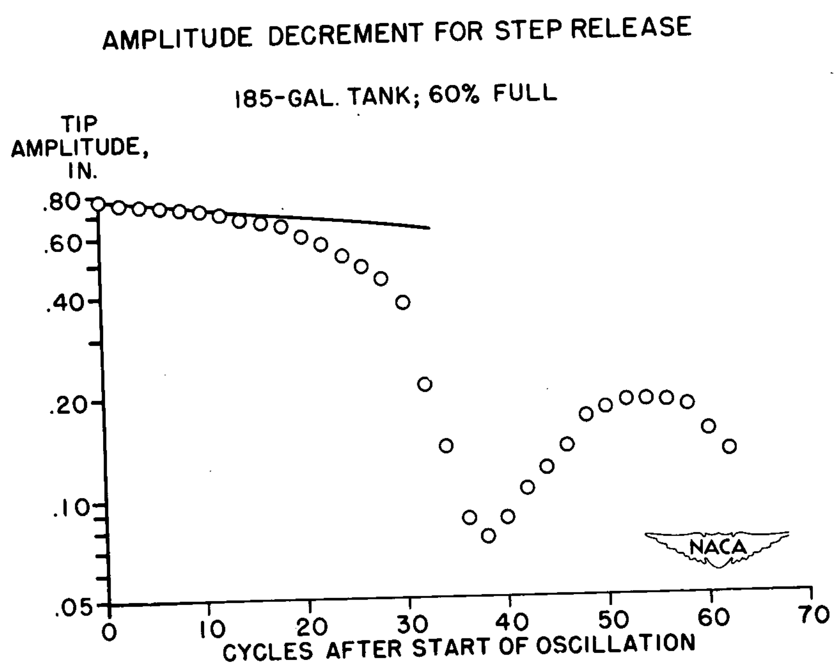

Figure 4.

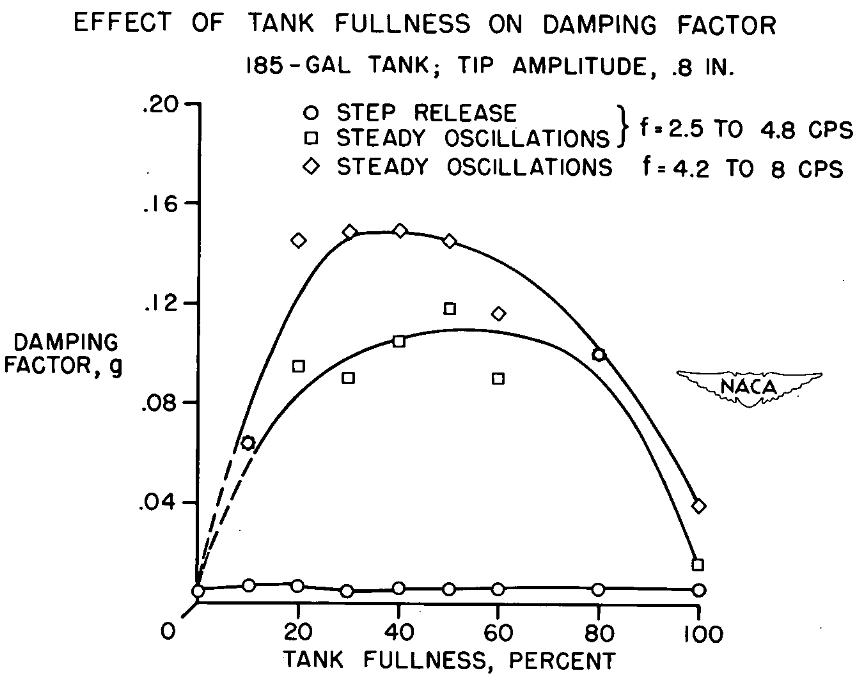
EFFECT OF TANK FULLNESS ON DAMPING FACTOR
185-GAL TANK; TIP AMPLITUDE, .8 IN.
O STEP RELEASE
□ STEADY OSCILLATIONS
f = 2.5 TO 4.8 CPS
◇ STEADY OSCILLATIONS f = 4.2 TO 8 CPS
DAMPING FACTOR, g
.20
.16
.12
.08
.04
0
20
40
60
80
100
TANK FULLNESS, PERCENT
NACA

Figure 5.