TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 24.

DEVELOPMENT OF THE INFLOW THEORY OF THE PROPELLER.

By

A. Betz

Resumé translated from the German by
Paris Office, N.A.C.A.

November, 1920.

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The following resumé of Mr. Betz' paper on the "Development of the Inflow Theory of the Propeller" was made by the technical staff of the Paris Office of the National Advisory Committee for Aeronautics. The problem is discussed in a very interesting way, and it was felt that considerable interest would be evidenced in the value of the conclusions arrived at by Mr. Betz.

R E S U M E

v - the speed of advance of a propeller.

$\omega = 2\pi n$ - speed of rotation.

v' - the additional axial velocity of the air after its passage in the plane of rotation of the propeller at the radial distance r.

ω' - the additional speed of rotation of the air after its passage in the plane of rotation of the propeller at the radial distance r.

S - thrust of the propeller.

R - its radius.

M - the propeller torque.

δ - the density of the air.

The units employed are the kilogram, meter and second.

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*Zeitschrift für Flugtechnik und Motorluftschiffahrt., Apr. 30, 1920, p 105-110

We know that the theory of inflow of the propeller assumes that in a space limited by two concentric cylinders having as axis the axis of the propeller, the axial and tangential velocity of the air is constant. If, as is often done, we consider only the suctional axial velocity of the air, it can easily be shown that maximum efficiency is obtained on condition of having constant speed along the radius of the propeller.

But it is not so if we introduce the suctional tangential velocity.

The author, assuming that the stream does not contract and neglecting the effect of rotation after the passage in the plane of rotation of the propeller, shows that the axial velocity in the plane of rotation of the propeller is equal to $v'/2$ and that the speed of rotation in this same plane is $\omega'/2$.

The loss of energy, E_1, due to the fact that the axial velocity in the plane of rotation of the propeller is $v + \frac{v'}{2}$, is:

$$E_1 = \pi\rho\int_0^R \frac{v'}{2}\, dS \qquad (1)$$

But

$$S = 2\pi\rho\int_0^R r\left(v + \frac{v'}{2}\right) v'\, dr \qquad (2)$$

whence

$$E_1 = \pi\rho\int_0^R r\left(v + \frac{v'}{2}\right) v'^2 dr \qquad (3)$$

The loss of energy E_2 due to the fact that the speed of rotation in the plane of the propeller is $\omega'/2$, has the value

$$E_2 = \int_0^R \frac{\omega'}{2}\, dM \qquad (4)$$

But

$$M = 2\pi\rho\int_0^R r^3\left(v + \frac{v'}{2}\right)\omega'\, dr \qquad (5)$$

whence

$$E_2 = \pi\rho\int_0^R r^3\left(v + \frac{v'}{2}\right)\omega'^2 dr \qquad (6)$$

On the other hand, between the values of v' and ' there exists the following relation

$$\frac{\omega'}{\omega} = 1 - \sqrt{1 - \frac{v'(2v + v')}{r^2\,\omega^2}} \tag{7}$$

In the expression of S, E_1 and E_2, we introduce the values:

$$x = \frac{\omega\, r}{v} \tag{8}$$

$$y = \frac{v'}{v} \tag{9}$$

and

$$\lambda = \frac{v}{R\,\omega} \tag{10}$$

We then get the total loss of energy:

$$E = E_1 + E_2 = F\rho v^3 \lambda^2 \int_0^{x = 1/\lambda} (2+y)x \left[x^2-y-x \sqrt{x^2-y(2+y)} \right] dx \tag{11}$$

and the thrust

$$S = F\rho v^2 \lambda^2 \int_0^{x = 1/\lambda} xy(2 + y)dx \tag{12}$$

The author seeks to determine the function $y = f(x)$ such that for given values of S, F, v, and n, the total loss of energy shall be minimum. The calculation of the variations gives the following relations between y and x, satisfying these conditions:

$$\left[x^2-2(\theta + 1)(y+1) \right] \sqrt{x^2-y(2+y)} + x(2y^2+5y+2-x^2) = 0 \tag{13}$$

where θ is a constant corresponding in each particular case to a set of values of S, F, v, and n, and is determined by introducing into equation (12) the value of $y = f(\theta, x)$ deduced from equation (13).

Fig. 1, Pl.B.19, gives the curves $y = f(x)$, that is, the distribution of the additional axial velocity along the radius for various values of θ.

But θ, according to equation (12), is a function of the "propeller load"

$$q = S/\rho\pi R^2 v^2$$

and of $\lambda = v/R\omega$. Thus, assuming $x = 1/\lambda$, we may plot on Fig. 1 a sheaf of iso-φ curves which will enable us to estimate the iso-θ, that is, the curve $v'/v = f(r\omega/v)$, corresponding to given values of φ and λ.

For this purpose (see Fig.3, Pl.B.19) we seek on Fig. 1 the intersection of the ordinate $x = 1/\lambda$ with the given iso-φ: the iso-θ passing through this point of intersection is the desired curve giving the distribution of additional axial velocity along the radius.

Fig. 2, Pl.B.19 gives the sheaf of iso-λ in the system of axes of coordinates φ and θ which served for plotting the sheaf of iso-φ in Fig. 1.

Fig. 4, Pl.B.20, represents the variation of the value $\frac{r\omega'}{v}$ along the radius of the optimum propeller. The iso-θ corresponding to given values of φ and λ is determined in the same way as in Fig.1.

Fig. 6, Pl.B.20, gives the variation of the angle of deviation ε (see Fig. 5, Pl.B.19) as a function of x and θ: here again, the iso-θ representing this variation along the radius for given values of φ and λ, must pass through the point of intersection pf the iso-$\lambda = 1/x$ with the iso-φ.

Lastly, Fig. 7, Pl.B.20, gives the values of maximum efficiency as a function of φ and λ. It shows that for a given value of φ, maximum efficiency is reached with $\lambda = 0$, that is, with a propeller of very low power, and that, for a given value of λ, the "load of the propeller" and consequently, the thrust, reaches a maximum.

A.BETZ - DEVELOPMENT OF THE INFLOW THEORY OF PROPELLERS.

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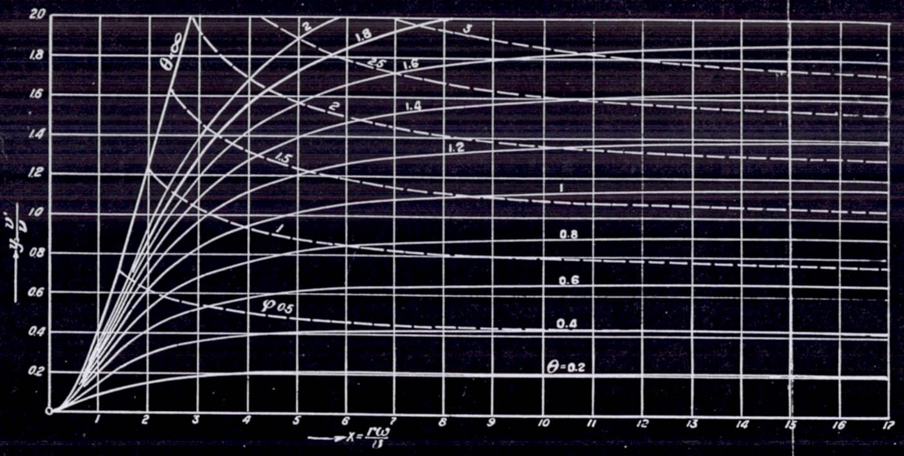

Fig.1 Optimum distribution of axial speed along the radius of the propeller.

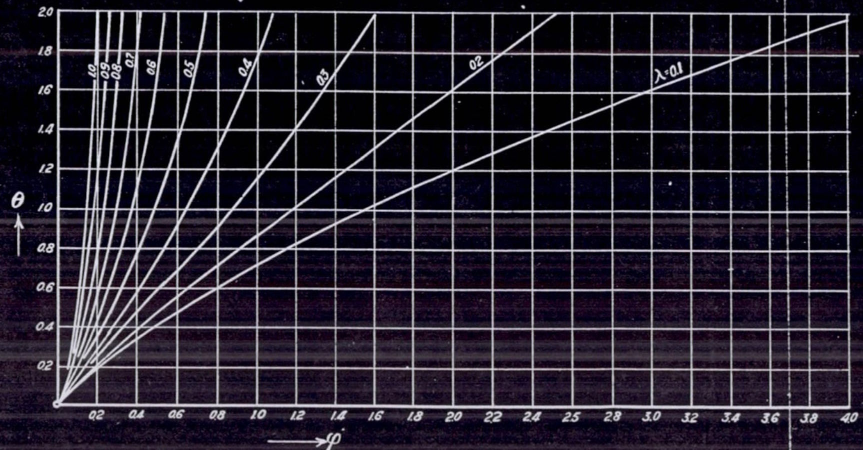

Fig.2 Values of θ in function of $\varphi = \frac{S}{\rho \pi R^2 v^2}$ and of $\lambda = \frac{v}{R\omega}$

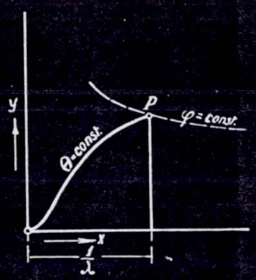

Fig.3 Utilisation diagram of Fig.2,4 and 6

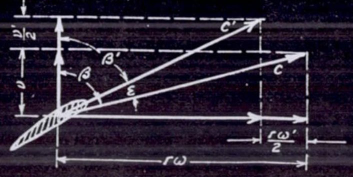

Fig.5 Functioning of an element of propeller

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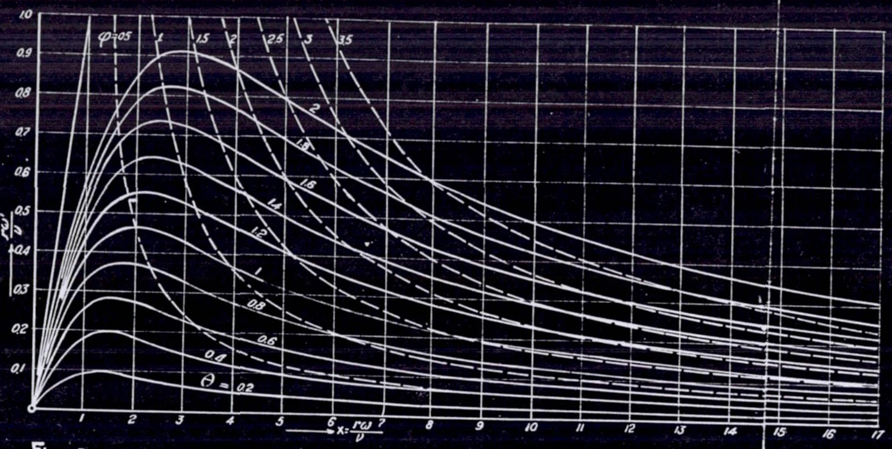

Fig. 4 Optimum distribution of tangential speed along the radius.

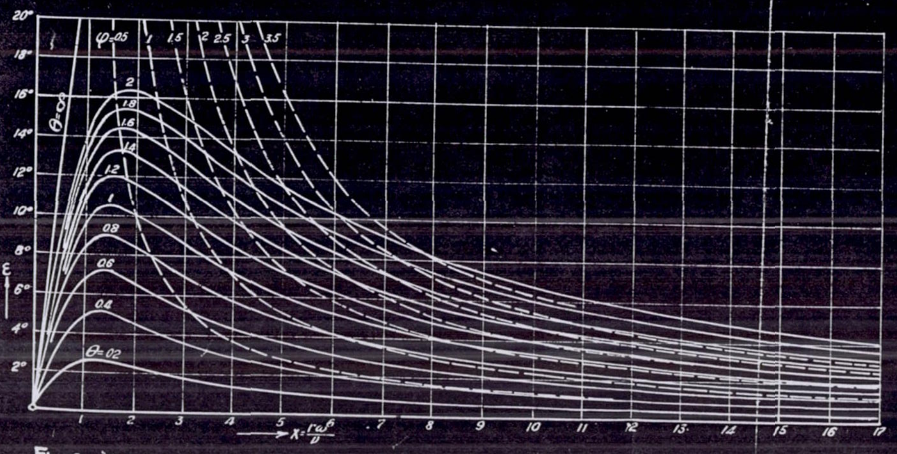

Fig. 6 Angle of deviation of air filet for the optimum distribution of thrust.

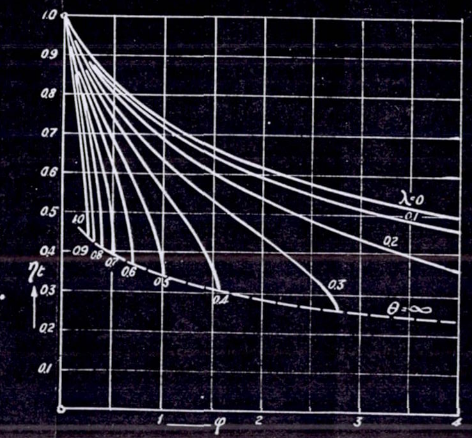

Fig 7 Maximum theoretic efficiency in function of unit load φ and of λ