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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 949

SIMPLY SUPPORTED LONG RECTANGULAR PLATE UNDER COMBINED AXIAL LOAD AND NORMAL PRESSURE

By Samuel Levy, Daniel Goldenberg,
and George Zibritosky
National Bureau of Standards

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SUMMARY

A solution is presented for the load-strain curve of a simply supported rectangular plate having a width-length ratio of 1:4 under combined normal pressure and axial load. The calculations are carried to axial loads considerably in excess of those required to buckle the plate.

Normal pressure was found to make the buckling load larger than its value for zero normal pressure; the theoretical buckling load was larger by a factor of 3.1 for one combination of axial load and normal pressure. Normal pressure caused a decrease in effective width at loads below the normal buckling load and an increase in effective width for loads somewhat greater than the normal buckling load; however, normal pressure caused less than 1 percent increase in effective width for average compressive strains greater than six times the buckling strain for zero normal pressure.

For some combinations of normal pressure and axial load the plate can be in equilibrium in more than one buckle pattern. Under such circumstances it is possible for the plate to be either buckled or unbuckled depending on the previous history of loading.

The results indicate it to be conservative design in the elastic range to neglect the effect of lateral pressure on the sheet buckling load and on the load carried by the sheet after buckling.

INTRODUCTION

The sheet in airplane wings, fuselages, and hull bottoms constructed of sheet metal reinforced by stringers frequently is subjected to normal pressure as well as forces in the plane of the sheet. It is important, therefore, to determine the effect of normal pressure on the load-strain curve of a long rectangular plate which approximates the sheet between stringers.

Experimental results on the effect of normal pressure on the critical compressive stress of curved sheet are given by Rafel (reference 1). These results indicate that for the two specimens tested normal pressure can more than double the critical compressive stress.

A general solution for the deflection and stress distribution in flat sheet subjected to normal pressure and axial force is given in reference 2. This general solution will be used in the present paper to determine the effective width of a simply supported flat rectangular plate subjected to combined axial compression and normal pressure up to axial loads considerably exceeding the normal buckling load. A ratio of width to length of plate of 1:4 was chosen, since this is typical of both hull-bottom plating and monocoque wings.

SYMBOLS

The symbols have the following significance (see fig. 1):

a length of plate

$b = \frac{a}{4}$ width of plate

h thickness of plate

w deflection of plate

x,y coordinate axes with origin at corner of plate

E Young's modulus

$\mu = \sqrt{0.1} = 0.316$, Poisson's ratio

$D = Eh^3/12(1-\mu^2)$, flexural rigidity of plate

p uniform normal pressure on plate

e average compressive strain at edges $y = 0$ and b

P axial load on plate

FUNDAMENTAL EQUATIONS

An initially flat rectangular plate of uniform thickness will be considered. The plate is simply supported on all four edges. The loading consists of a uniform normal pressure combined with axial loading in the direction of the longer side of the rectangle.

DEFLECTION EQUATIONS

By use of the method outlined on page 3 of reference 2, it can be shown that, if the lateral deflection of the plate is approximated by an expression having four undetermined constants,

$$w = w_{1,1} \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} + w_{3,1} \sin\frac{3\pi x}{a} \sin\frac{\pi y}{b}$$

$$+ w_{5,1} \sin\frac{5\pi x}{a} \sin\frac{\pi y}{b} + w_{7,1} \sin\frac{7\pi x}{a} \sin\frac{\pi y}{b} \qquad (1)$$

the following relations hold:

$$0 = -0.26628\frac{pb^4}{Eh} + 1.6725h^2 w_{1,1} - 0.1013\frac{Pb}{Eh} w_{1,1} + 1.004w_{1,1}^3$$

$$- 3w_{1,1}^2 w_{3,1} + 4.093w_{3,1}^2 w_{1,1} + 3.222w_{3,1}^2 w_{5,1}$$

$$- 3.0625w_{3,1}^2 w_{7,1} + 4.326w_{5,1}^2 w_{1,1} + 4.680w_{7,1}^2 w_{1,1} \qquad (2)$$

$$- 6.045w_{1,1}w_{3,1}w_{5,1} - 6.205w_{1,1}w_{5,1}w_{7,1} + 7.07w_{3,1}w_{5,1}w_{7,1}$$

$$0 = -0.08876\frac{pb^4}{Eh} + 3.6169h^2 w_{3,1} - 0.9119\frac{Pb}{Eh} w_{3,1} - w_{1,1}^3$$

$$+ 4.093w_{1,1}^2 w_{3,1} - 3.022w_{1,1}^2 w_{5,1} + 1.316w_{3,1}^3$$

$$+ 5.766w_{5,1}^2 w_{3,1} + 4.992w_{5,1}^2 w_{7,1} + 7.294w_{7,1}^2 w_{3,1}$$

$$+ 6.443w_{1,1}w_{3,1}w_{5,1} - 6.125w_{1,1}w_{3,1}w_{7,1} + 7.07w_{1,1}w_{5,1}w_{7,1} \qquad (3)$$

$$0 = -0.05326\frac{pb^4}{Eh} + 9.7280h^2w_{5,1} - 2.5330\frac{Pb}{Eh}w_{5,1} - 3.022w_{1,1}^2w_{3,1}$$

$$+ 4.326w_{1,1}^2w_{5,1} - 3.102w_{1,1}^2w_{7,1} + 3.222w_{3,1}^2w_{1,1}$$

$$+ 5.766w_{3,1}^2w_{5,1} + 3.441w_{5,1}^3 + 13.27w_{7,1}^2w_{5,1}$$

$$+ 7.07w_{1,1}w_{3,1}w_{7,1} + 9.986w_{3,1}w_{5,1}w_{7,1} \tag{4}$$

$$0 = -0.038040\frac{pb^4}{Eh} + 24.450h^2w_{7,1} - 4.9647\frac{Pb}{Eh}w_{7,1}$$

$$- 3.102w_{1,1}^2w_{5,1} + 4.680w_{1,1}^2w_{7,1} - 3.062w_{3,1}^2w_{1,1}$$

$$+ 7.295w_{3,1}^2w_{7,1} + 4.994w_{5,1}^2w_{3,1} + 13.27w_{5,1}^2w_{7,1}$$

$$+ 10.37w_{7,1}^3 + 7.07w_{1,1}w_{3,1}w_{5,1} \tag{5}$$

EFFECTIVE WIDTH

The ratio of the effective width to the initial width (defined as the ratio of the actual compressive load carried by the plate to the load the plate would have carried if the stress had been uniform and equal to the Young's modulus times the average edge strain) was computed from equation (11) of reference 2 as:

$$\frac{\text{Effective width}}{\text{Initial width}} = \frac{P}{P + \dfrac{\pi^2 Eh}{128b}\left(w_{1,1}^2 + 9w_{3,1}^2 + 25w_{5,1}^2 + 49w_{7,1}^2\right)} \tag{6}$$

The average compressive strain at the edges $y = 0$, $y = b$ was also computed from equation (11) of reference 2 as:

$$e = \frac{P}{Ebh} + \frac{\pi^2}{128b^2}\left(w_{1,1}^2 + 9w_{3,1}^2 + 25w_{5,1}^2 + 49w_{7,1}^2\right) \quad (7)$$

Equation (1) restricts the shape of the deflected surface of the plate to one sine wave across its width and a combination of four sine waves along its length. This introduces errors into the solution; however, reference 2 shows that for plate deflections less than twice the plate thickness the errors are probably less than 5 percent.

SOLUTION

The four simultaneous cubic equations, (2) to (5), were solved for the deflection coefficients $w_{1,1}$, $w_{3,1}$, $w_{5,1}$, and $w_{7,1}$, using the following steps:

1. Divide each of equations (2) to (5) by h^3.

2. Estimate values of $w_{1,1}/h$, $w_{3,1}/h$, $w_{5,1}/h$, and $w_{7,1}/h$ corresponding to chosen values of Pb/Eh^3 and pb^4/Eh^4.

3. Expand the right-hand side of each of equations (2) to (5) in a Taylor series in the neighborhood of the estimated values of $w_{1,1}/h$, $w_{3,1}/h$, $w_{5,1}/h$, and $w_{7,1}/h$, omitting terms of higher order than the first.

4. Solve the resulting linear equations for the difference between the estimated values of $w_{1,1}/h$, $w_{3,1}/h$, $w_{5,1}/h$, and $w_{7,1}/h$ and their improved values; use Crout's method (reference 3).

5. Repeat until the estimated error is less than 0.2 percent. One or two trials usually were sufficient to give a satisfactory answer.

6. If the load is constant while the deflection changes, consider one of the deflection coefficients $w_{1,1}/h$, $w_{3,1}/h$, $w_{5,1}/h$, or $w_{7,1}/h$ as independent variable in place of one of the load coefficients Pb/Eh^3 or pb^4/Eh^4. The deflection

coefficients determined by this procedure are given for $p = 0$ in table I, for $p = 2.40Eh^4/b^4$ in table II, for $p = 12.02Eh^4/b^4$ in table III, and for $p = 24.03Eh^4/b^4$ in table IV. The average compressive strain e at the edges computed from equation (7) is also given in tables I to IV.

Cubic equations like equations (2) to (5) frequently have more than one real solution. For the case where the lateral pressure is zero (table I), two solutions, one corresponding to 5 buckles and the other to 7 buckles, are given. In the other cases the pressure is not zero (tables II to IV), and only one solution is given although other solutions are possible. The single solutions given in these cases correspond to a continuous change in buckle pattern from zero axial load to the maximum axial load considered and probably correspond to the lowest equilibrium load.

The development of the buckle pattern is shown graphically in figures 2 to 5 for pressures $p = 0$, $2.40Eh^4/b^4$, $12.02Eh^4/b^4$, and $24.03Eh^4/b^4$, respectively. It is seen that the deflection of the plate at the axial center line is a single long bulge for low axial force P and gradually builds up to a regular buckle pattern at larger values of P. The shifting of the buckle pattern from 3 to 7 buckles in figures 4 and 5 is accompanied by a drop in axial load. It is significant to note that the initial general downward deflection of the sheet due to normal pressure p is almost entirely wiped out at large values of axial force P.

The axial load P given in tables I to IV is plotted against the average edge compressive strain e in figures 6 to 9. The most striking feature of figures 6 to 9 is the fact that the plate can be in equilibrium in more than one buckle pattern for a given combination of loads. For example, with a normal pressure $p = 24.03Eh^4/b^4$ (fig. 9) and an axial load $P = 10.00Eh^3/b$, the sheet can be in stable equilibrium with 1 buckle at $e = 12.5h^2/b^2$, with 3 buckles at $e = 16.1h^2/b^2$, and with 7 buckles at $e = 18.6h^2/b^2$, and the sheet is in unstable equilibrium with 3 buckles at $e = 13.6h^2/b^2$ and with 7 buckles at $e = 17.3h^2/b^2$. This anomalous condition also has been observed experimentally. Almost any condition of stable equilibrium of the sheet can be reached by a suitable history of previous loading. For example, when $P = 7Eh^3/b$ and $p = 12.02Eh^4/b^4$ in figure 8, the sheet is in stable equilibrium at $e = 8.2h^2/b^2$

for axial loads increasing from zero and at $e = 9.8h^2/b^2$ for axial loads decreasing from $9Eh^3/b$.

The axial load at which buckling occurs is $P = 3.84Eh^3/b$ when $p = 0$, $P = 4.05Eh^3/b$ when $p = 2.40Eh^4/b^4$, $P = 8.56Eh^3/b$ when $p = 12.02Eh^4/b^4$, and $P = 11.84Eh^3/b$ when $p = 24.03Eh^4/b^4$. The buckling load at the highest normal pressure is 3.1 times the buckling load with no normal pressure.

The ratio of effective width to initial width was computed from equation (6) and tables I to IV. The results are plotted in figure 10 for $p = 0$, figure 11 for $p = 2.40Eh^4/b^4$, figure 12 for $p = 12.02Eh^4/b^4$, and figure 13 for $p = 24.03Eh^4/b^4$. Increasing normal pressure lowers the effective-width ratio for strains less than the buckling strain, $e = 3.8h^2/b^2$, and raises the effective-width ratio for strains somewhat greater than this. For strains well beyond this ($e > 16.5h^2/b^2$ when $p = 12.02Eh^4/b^4$, and $e > 21.1h^2/b^2$ when $p = 24.03Eh^4/b^4$) the normal pressure causes less than 1 percent increase as compared with the effective-width ratio found for zero normal pressure.

CONCLUSIONS

The buckling load is considerably increased by normal pressure. For the highest pressure considered, the theoretical buckling load is 3.1 times the buckling load for zero normal pressure. Normal pressure causes a decrease in effective width at strains below the normal buckling strain and an increase in effective width for strains somewhat greater than the normal buckling strain. If the buckling load is considerably exceeded, however, normal pressure causes less than 1-percent increase in effective width. For some combinations of normal pressure and axial load the sheet can be in equilibrium in more than one position. Under such circumstances it is possible for the sheet to be either unbuckled or buckled, depending on the previous history of loading.

The results indicate it to be conservative design in the elastic range to neglect the effect of lateral pressure on the sheet buckling load and on the load carried by the sheet after buckling.

National Bureau of Standards,
Washington, D. C., June 2, 1944.

REFERENCES

1. Rafel, Norman: Effect of Normal Pressure on the Critical Compressive Stress of Curved Sheet. NACA RB, Nov. 1942.

2. Levy, Samuel: Bending of Rectangular Plates with Large Deflections. NACA Rep. No. 737, 1942. (Issued also as TN No. 846, 1942.)

3. Crout, Prescott D.: A Short Method for Evaluating Determinants and Solving Systems of Linear Equations with Real or Complex Coefficients. Trans. A.I.E.E., vol. 60, 1941.

Table I.- Values of deflection coefficients for various values of axial compressive load in the x - direction, P, for simply supported rectangular plate, $a = 4b$, $\mu = 0.316$. Normal pressure, $p = 0$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{eb^2}{h^2}$
3.84	0	0	0	0	3.84
3.95	0	0	.281	0	4.10
4.05	0	0	.390	0	4.34
4.24	0	0	.546	0	4.82
4.44	0	0	.665	0	5.29
5.20	0	0	1.000	0	7.13
5.92	0	0	1.238	0	8.88
6.80	0	0	1.476	0	11.00
7.60	0	0	1.664	0	12.94
8.40	0	0	1.833	0	14.88
9.28	0	0	2.001	0	16.99
4.93	0	0	0	0	4.93
7.02	0	0	0	1.000	10.80
9.11	0	0	0	1.414	16.66
11.20	0	0	0	1.732	22.53
13.29	0	0	0	2.000	28.40

Table II.- Value of deflection coefficients for various values of axial compressive load in the x - direction, P, for simply supported rectangular plate, $a = 4b$, $\mu = 0.316$. Normal pressure, $p = 2.40Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{eb^2}{h^2}$
0	.366	.064	.015	.004	.01
.99	.388	.083	.020	.005	1.00
1.97	.417	.115	.031	.007	2.00
2.96	.448	.173	.058	.013	3.00
3.95	.484	.258	.161	.019	4.06
4.05	.482	.257	.195	.017	4.18
4.13	.470	.247	.245	.011	4.31
4.24	.349	.181	.445	-.012	4.66
4.44	.230	.123	.652	-.022	5.28
4.93	.144	.082	.902	-.023	6.51
5.92	.085	.054	1.245	-.021	8.92
6.91	.060	.042	1.509	-.019	11.30
7.90	.047	.035	1.733	-.018	13.69
8.90	.038	.030	1.934	-.016	16.11
9.87	.032	.027	2.110	-.015	18.45

Table III.- Values of deflection coefficients for various values of axial compressive load in the x - direction, P, for simply supported rectangular plate, a = 4b, μ = 0.316. Normal pressure, $p = 12.02Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{eb^2}{h^2}$
0	1.331	.388	.112	.034	0.27
.99	1.392	.382	.139	.045	1.28
1.97	1.460	.429	.173	.060	2.34
2.96	1.539	.480	.215	.081	3.42
3.95	1.620	.532	.263	.109	4.52
4.93	1.710	.579	.317	.146	5.67
5.92	1.796	.620	.373	.194	6.85
6.91	1.880	.656	.428	.255	8.08
7.90	1.928	.676	.472	.344	9.37
8.13	1.929	.675	.482	.374	9.71
8.38	1.914	.668	.488	.416	10.09
8.41	1.908	.665	.488	.424	10.14
8.46	1.899	.661	.488	.436	10.22
8.49	1.889	.657	.488	.447	10.28
8.52	1.878	.653	.487	.458	10.34
8.56	1.851	.642	.484	.480	10.43
8.55	1.787	.617	.473	.520	10.51
8.49	1.730	.596	.460	.548	10.51
8.29	1.602	.556	.422	.600	10.41
8.26	1.585	.562	.405	.608	10.38
8.24	1.578	.574	.390	.611	10.36
8.23	1.576	.583	.380	.612	10.35
8.21	1.573	.603	.360	.612	10.31
8.14	1.569	.647	.320	.606	10.21
8.03	1.556	.689	.280	.596	10.04
7.46	1.462	.800	.158	.542	9.22
6.55	1.270	.900	.024	.463	8.05
5.88	1.067	1.000	-.085	.413	7.32
5.75	.952	1.100	-.151	.412	7.34
5.92	.894	1.200	-.197	.438	7.78
6.28	.872	1.300	-.235	.482	8.49
6.77	.874	1.400	-.267	.541	9.43
7.40	.896	1.500	-.297	.619	10.64
8.24	.934	1.600	-.325	.724	12.27
8.83	.961	1.650	-.339	.799	13.42
9.15	.976	1.670	-.344	.843	14.06
9.81	.998	1.682	-.348	.941	15.43
10.00	1.001	1.670	-.345	.975	15.84
10.24	.974	1.560	-.320	1.062	16.46
10.16	.950	1.490	-.303	1.081	16.36
9.98	.917	1.400	-.281	1.094	16.08
9.73	.879	1.300	-.256	1.099	15.65
9.45	.840	1.200	-.231	1.098	15.17
9.17	.801	1.100	-.205	1.095	14.67
8.89	.762	1.000	-.179	1.089	14.17
8.64	.724	.900	-.151	1.084	13.72
8.40	.687	.800	-.123	1.079	13.30
8.19	.651	.700	-.093	1.076	12.96
8.02	.617	.600	-.062	1.077	12.69
7.89	.583	.500	-.030	1.083	12.53
7.83	.548	.400	.003	1.100	12.54
7.89	.505	.300	.034	1.135	12.83
8.00	.474	.248	.047	1.170	13.24
8.33	.417	.185	.054	1.250	14.28
8.70	.373	.151	.052	1.327	15.38
9.30	.320	.119	.046	1.438	17.13
9.87	.283	.101	.041	1.533	18.77
10.90	.235	.080	.033	1.690	21.70
11.85	.204	.067	.028	1.821	24.39
12.80	.180	.058	.024	1.943	27.07
13.82	.160	.050	.021	2.065	29.93

Table IV.- Values of deflection coefficients for various values of axial compressive load in the x - direction, P, for simply-supported rectangular plate, a = 4b, μ = 0.316. Normal pressure, $p = 24.03Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{eb^2}{h^2}$
0	2.024	.560	.218	.080	0.65
.99	2.089	.600	.248	.096	1.73
1.97	2.157	.640	.281	.115	2.82
2.96	2.228	.682	.316	.138	3.93
3.95	2.302	.722	.353	.164	5.06
4.93	2.378	.762	.391	.194	6.21
5.92	2.454	.800	.430	.228	7.38
6.91	2.528	.836	.468	.267	8.58
7.90	2.599	.868	.506	.311	9.80
8.88	2.662	.898	.541	.362	11.05
9.87	2.714	.921	.574	.422	12.33
10.66	2.735	.933	.595	.482	13.40
11.84	2.674	.916	.607	.633	15.20
11.81	2.516	.867	.574	.720	15.42
11.30	2.284	.811	.501	.800	15.06
10.65	2.086	.847	.374	.839	14.41
9.93	1.962	.980	.226	.803	13.43
9.28	1.857	1.063	.133	.759	12.54
8.58	1.732	1.138	.046	.712	11.63
7.87	1.581	1.217	-.042	.666	10.77
7.32	1.414	1.318	-.131	.635	10.24
7.22	1.323	1.397	-.183	.637	10.31
7.26	1.285	1.443	-.208	.645	10.49
7.58	1.225	1.553	-.258	.683	11.27
8.24	1.200	1.673	-.303	.752	12.60
9.13	1.202	1.789	-.341	.844	14.38
10.13	1.222	1.879	-.369	.949	16.36
11.45	1.249	1.941	-.388	1.100	19.04
12.26	1.246	1.896	-.379	1.225	20.82
12.35	1.237	1.867	-.372	1.249	21.05
12.28	1.183	1.701	-.331	1.312	21.11
11.80	1.111	1.506	-.282	1.334	20.35
11.00	1.012	1.237	-.210	1.337	18.98
10.20	.910	.947	-.127	1.332	17.62
9.90	.865	.813	-.087	1.333	17.15
9.60	.802	.627	-.027	1.345	16.76
9.60	.692	.364	.054	1.416	17.31
9.87	.629	.284	.069	1.482	18.26
11.00	.488	.180	.066	1.682	21.74
12.00	.413	.143	.056	1.828	24.66
12.83	.368	.123	.050	1.938	27.05
13.82	.325	.106	.044	2.059	29.86

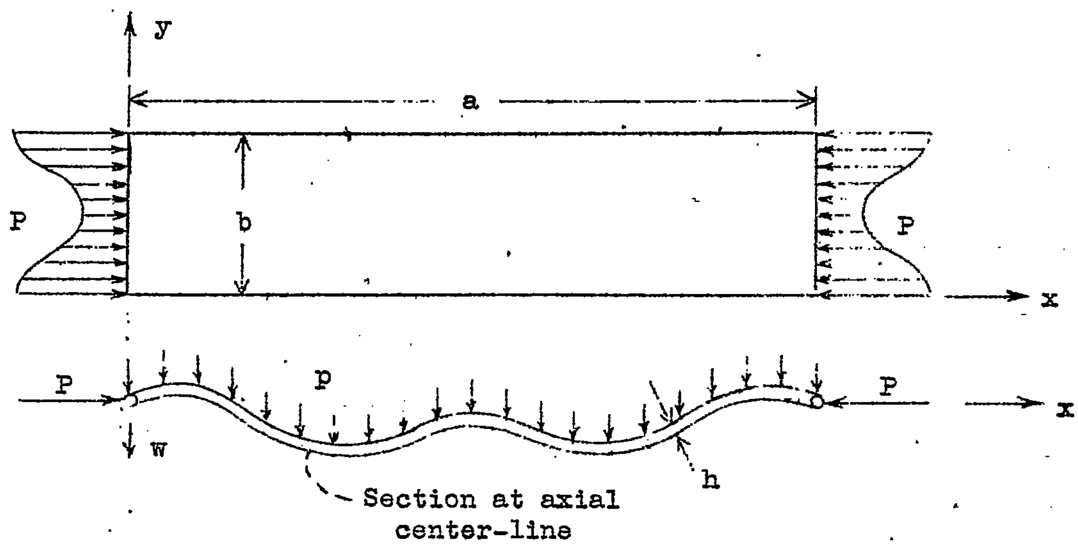
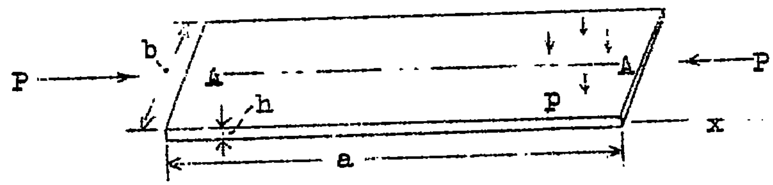


Figure 1.- Plate under axial load and normal pressure (a = 4b).



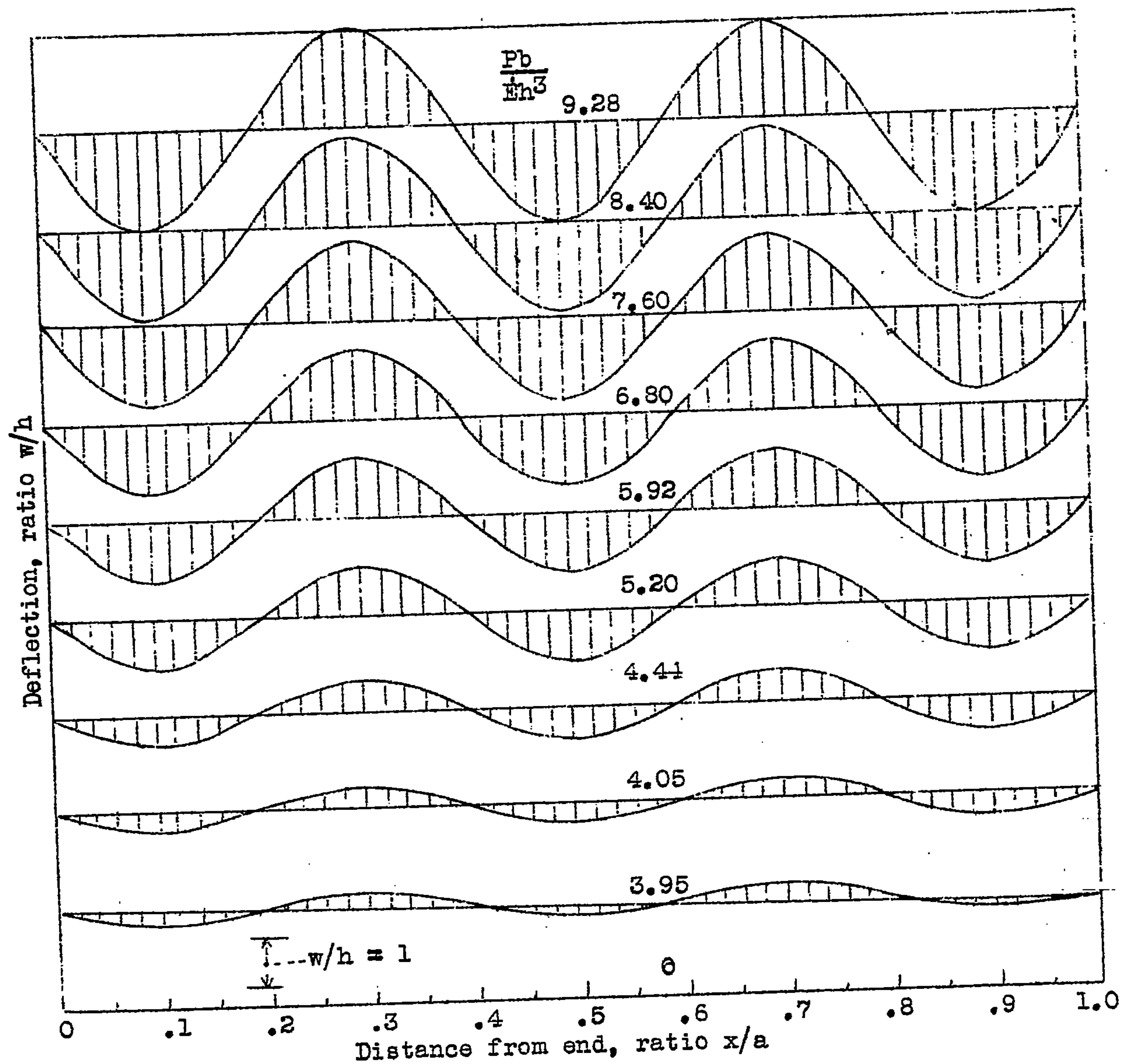


Figure 2.- Relation between deflection at midwidth, A-A, and distance from end of plate. Normal pressure, p = 0.

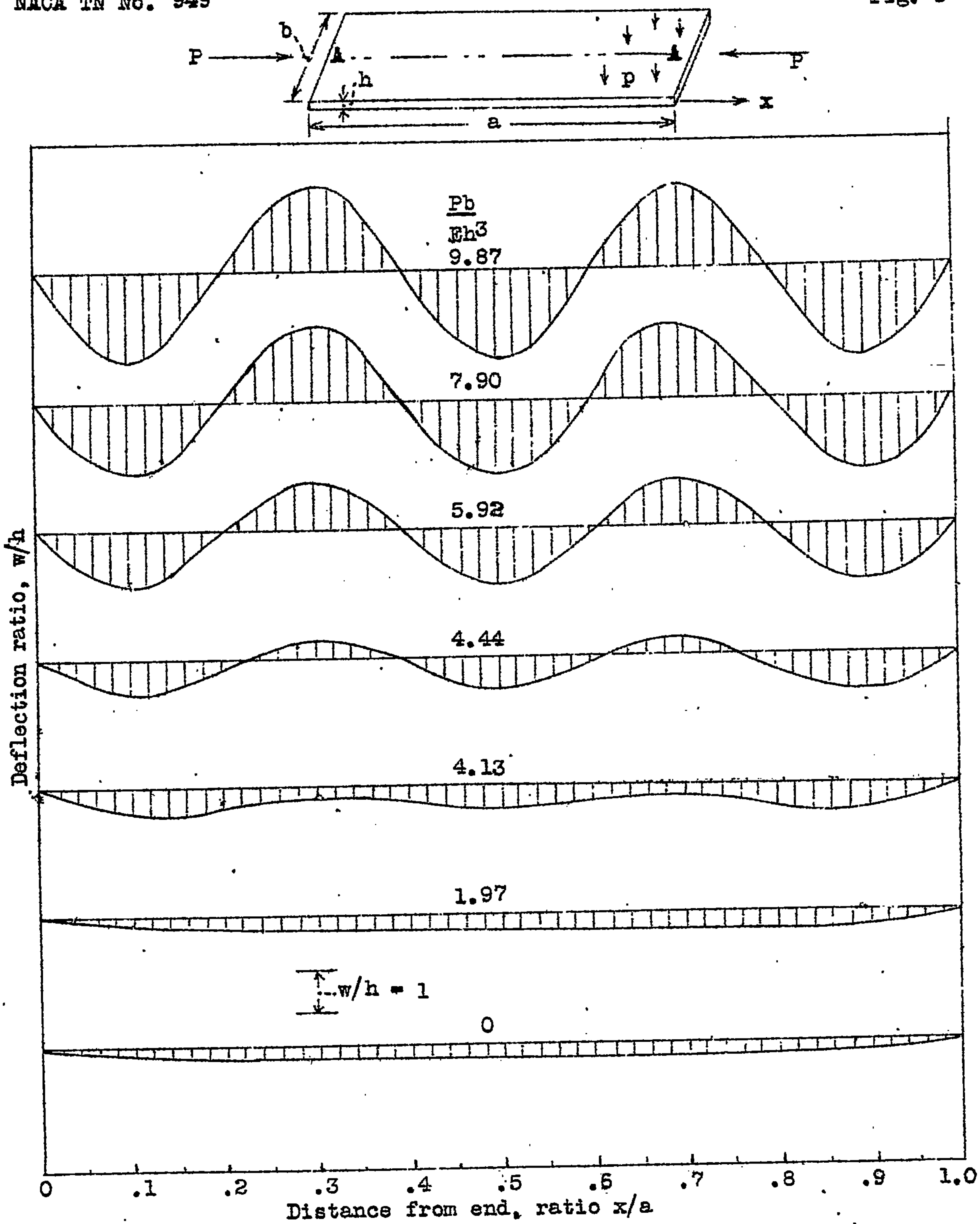


Figure 3.- Relation between deflection at midwidth, A-A, and distance from end of plate. Pressure, $p = 2.40Eh^4/b^4$.

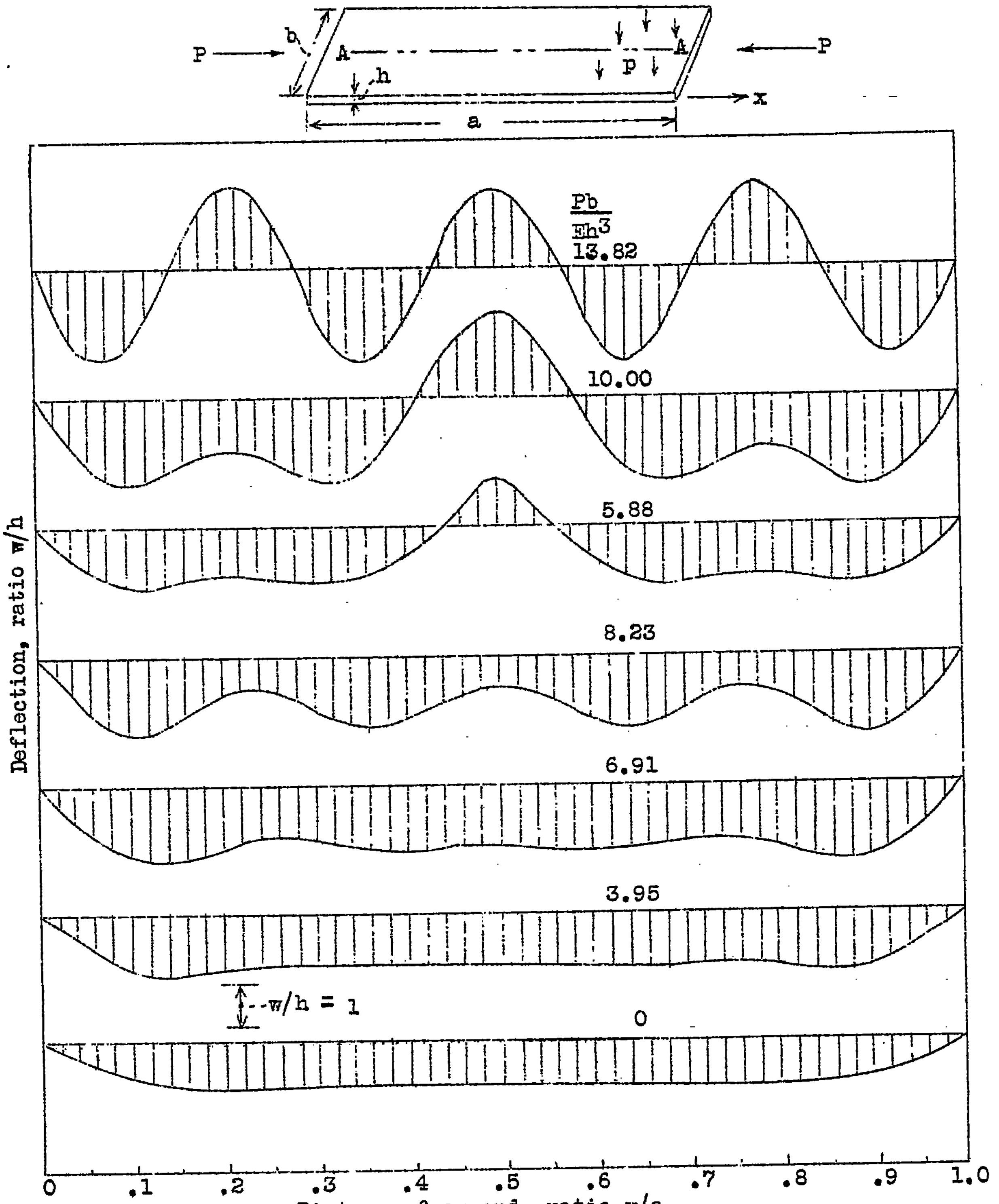


Figure 4.- Relation between deflection at midwidth, A-A, and distance from end of plate. Pressure, $p = 12.02Eh^4/b^4$.

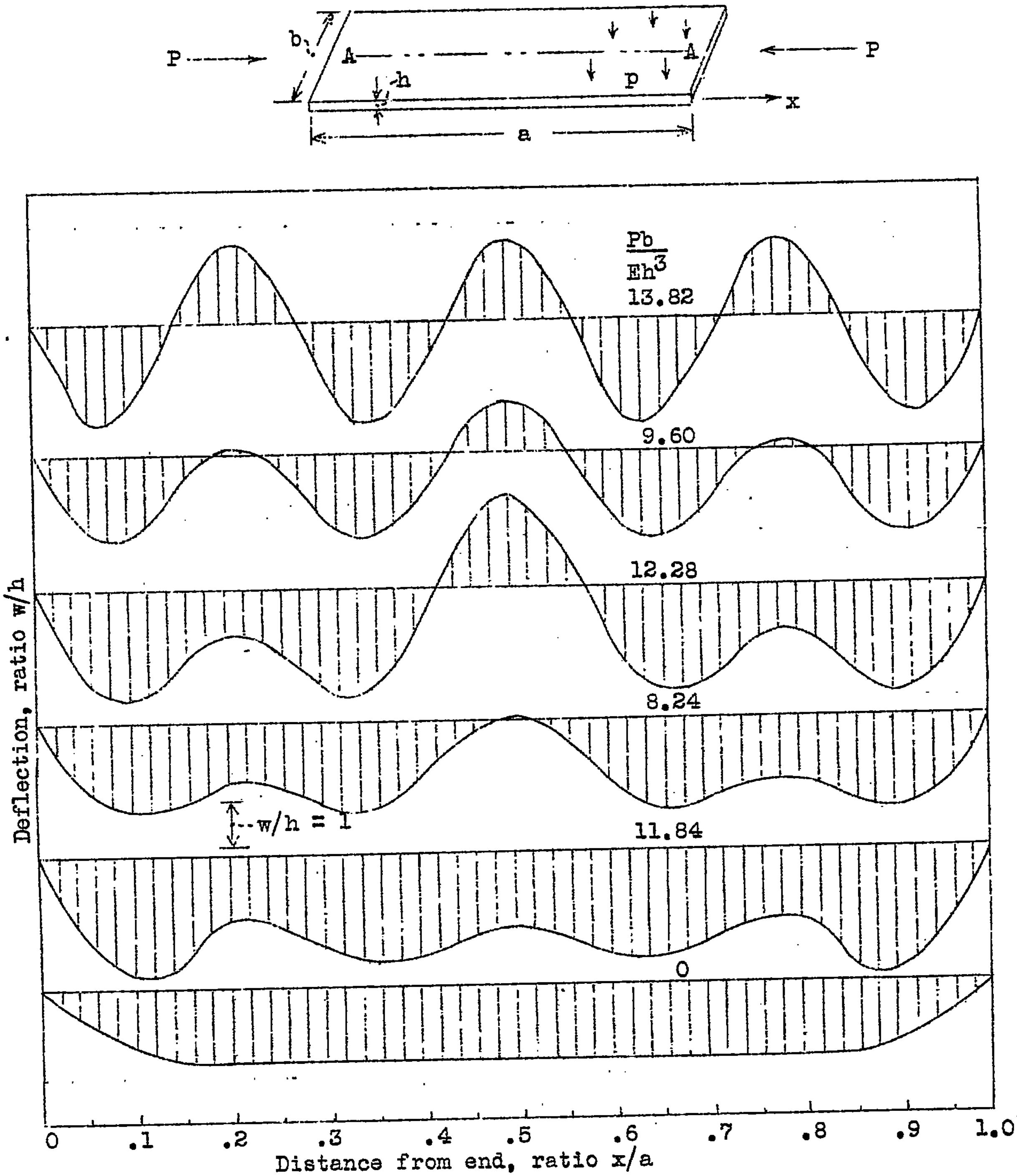


Figure 5.- Relation between deflection at midwidth and distance from end of plate. Pressure, $p = 24.03Eh^4/b^4$.

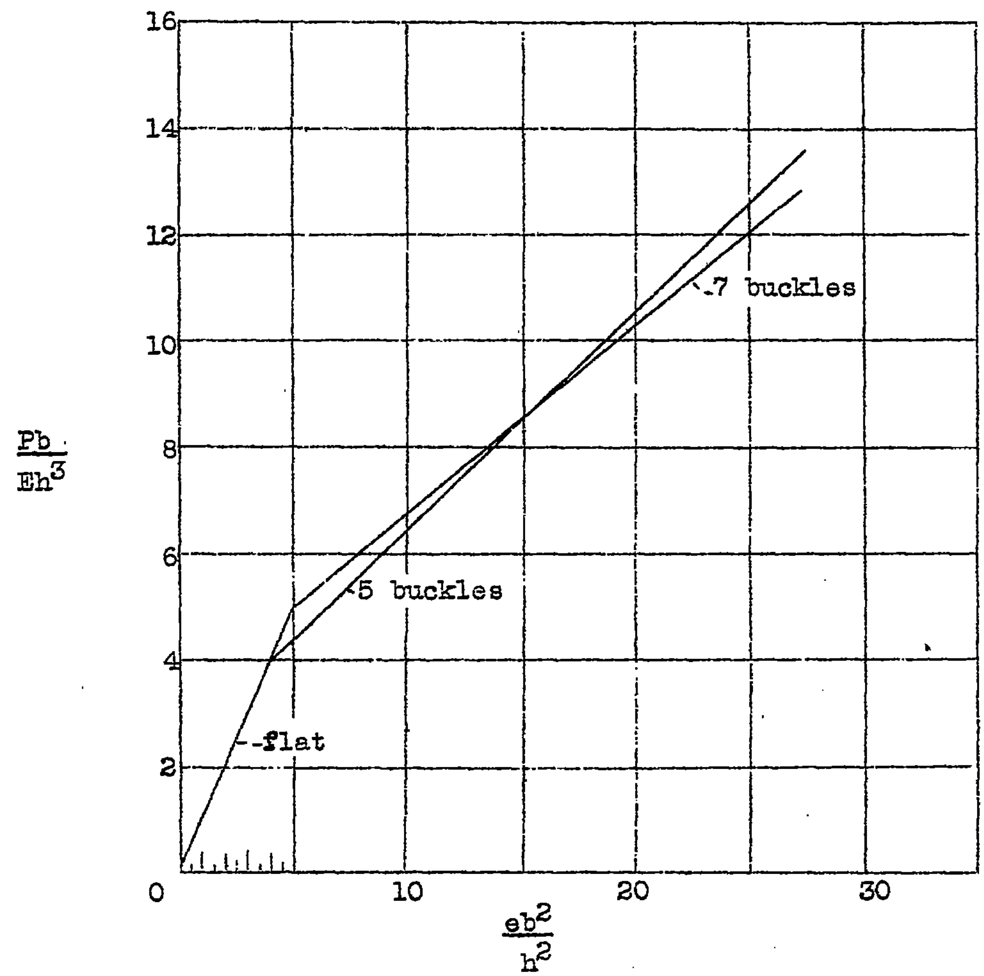


Figure 6.- Axial load P as a function of average edge strain e when normal pressure, $p = 0$, (b = plate width, h = plate thickness).

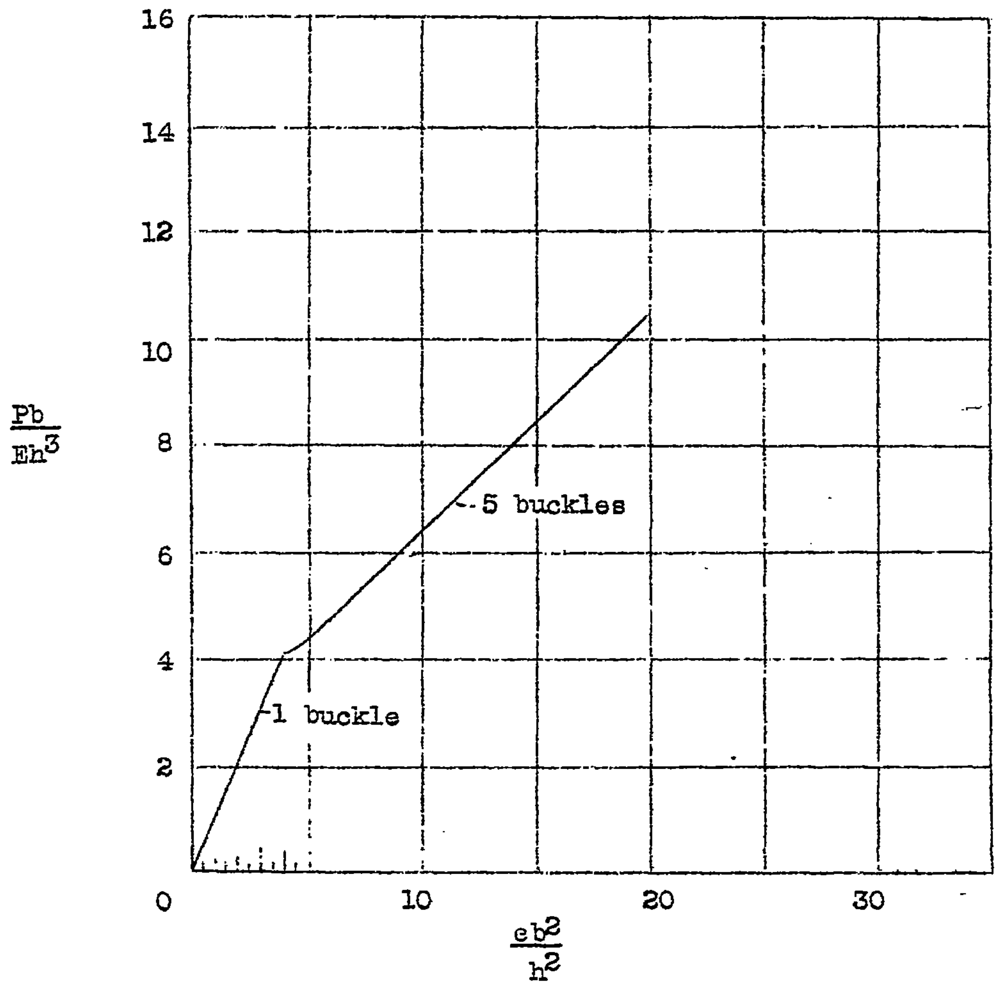


Figure 7.- Axial load P as a function of average edge strain e. Normal pressure, $p = 2.40Eh^4/b^4$, (b = plate width, h = plate thickness).

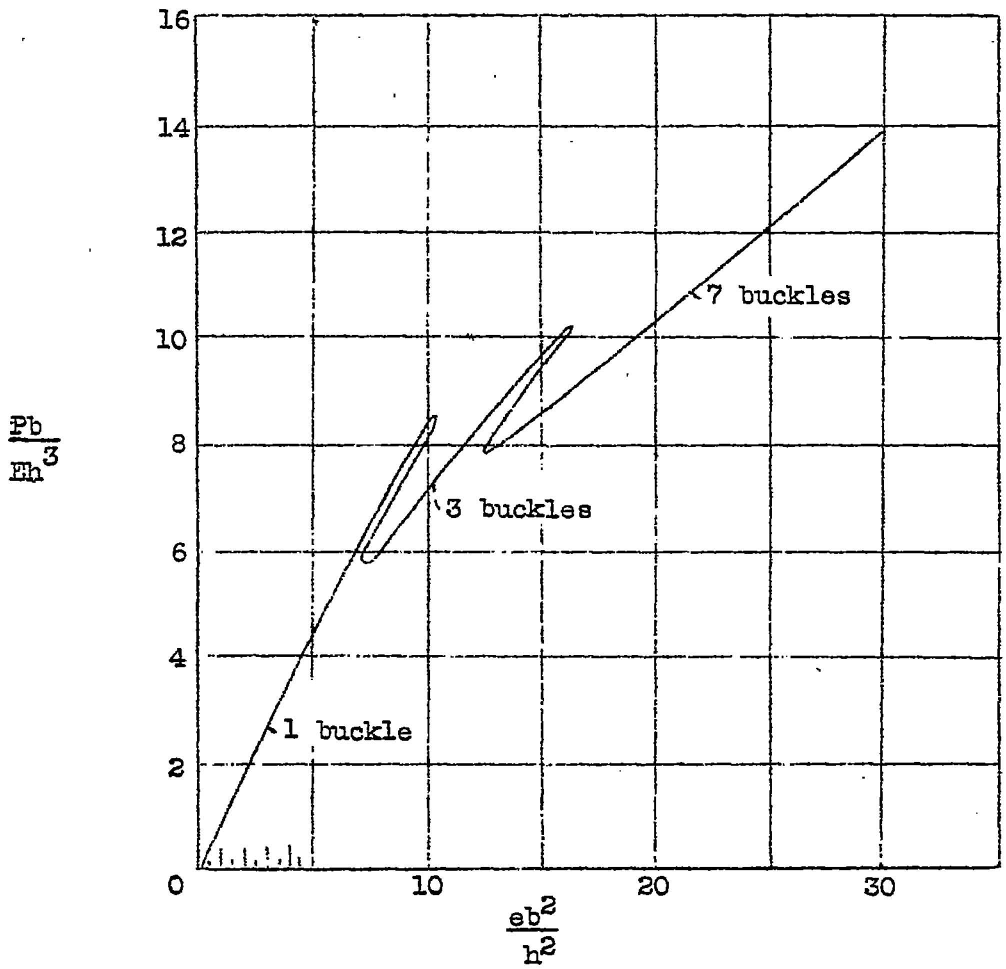


Figure 8.- Axial load P as a function of average edge strain e. Normal pressure, $p = 12.02Eh^4/b^4$, (b = plate width, h = plate thickness).

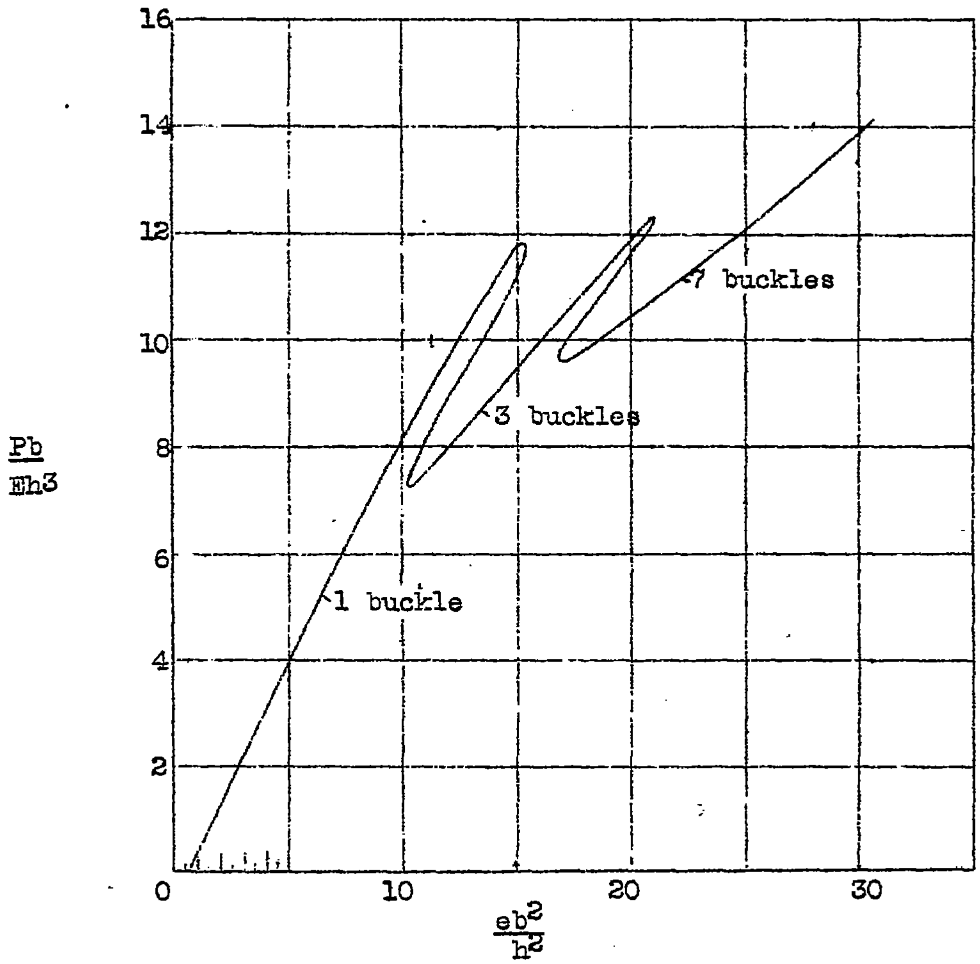


Figure 9.- Axial load P as a function of average edge strain e. Normal pressure, $p = 24.03Eh^4/b^4$, (b = plate width, h = plate thickness).

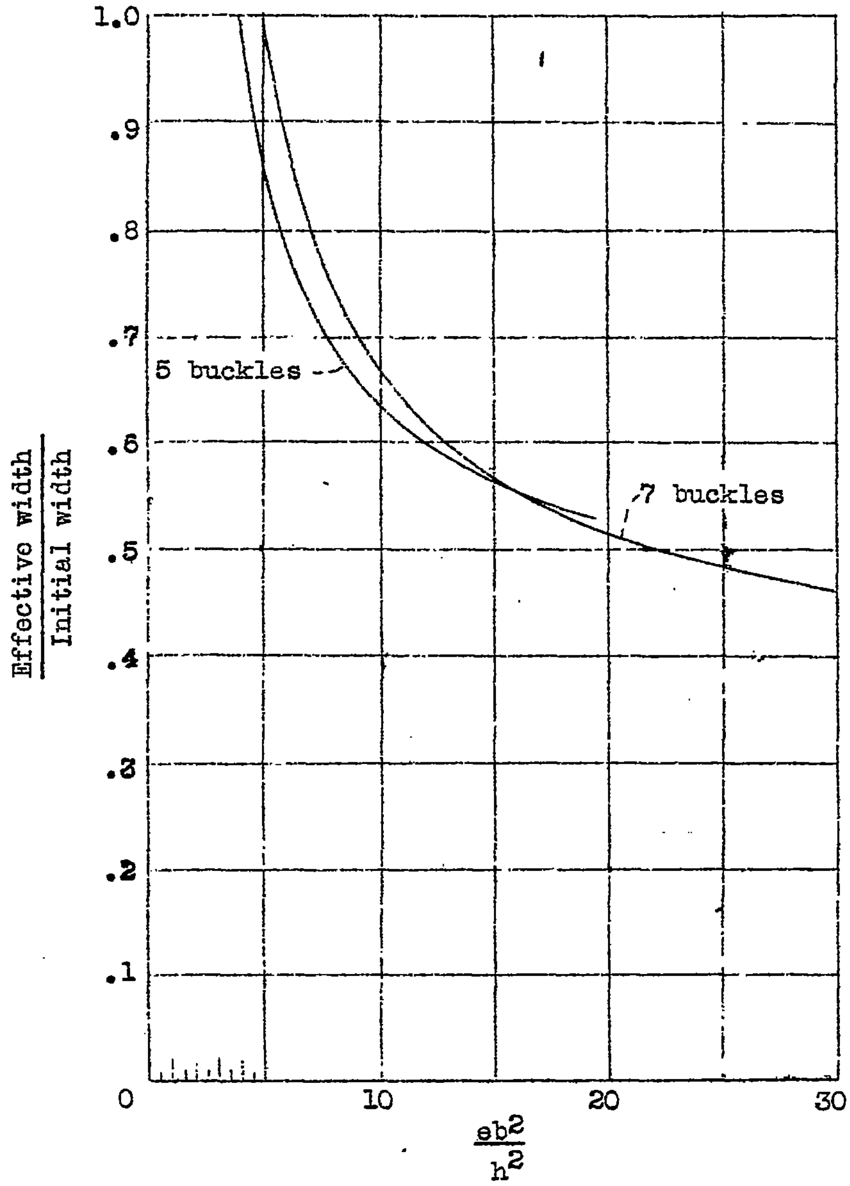


Figure 10.- Variation of ratio of effective width to initial width with edge strain e. Normal pressure, $p = 0$, (b = plate width, h = plate thickness).

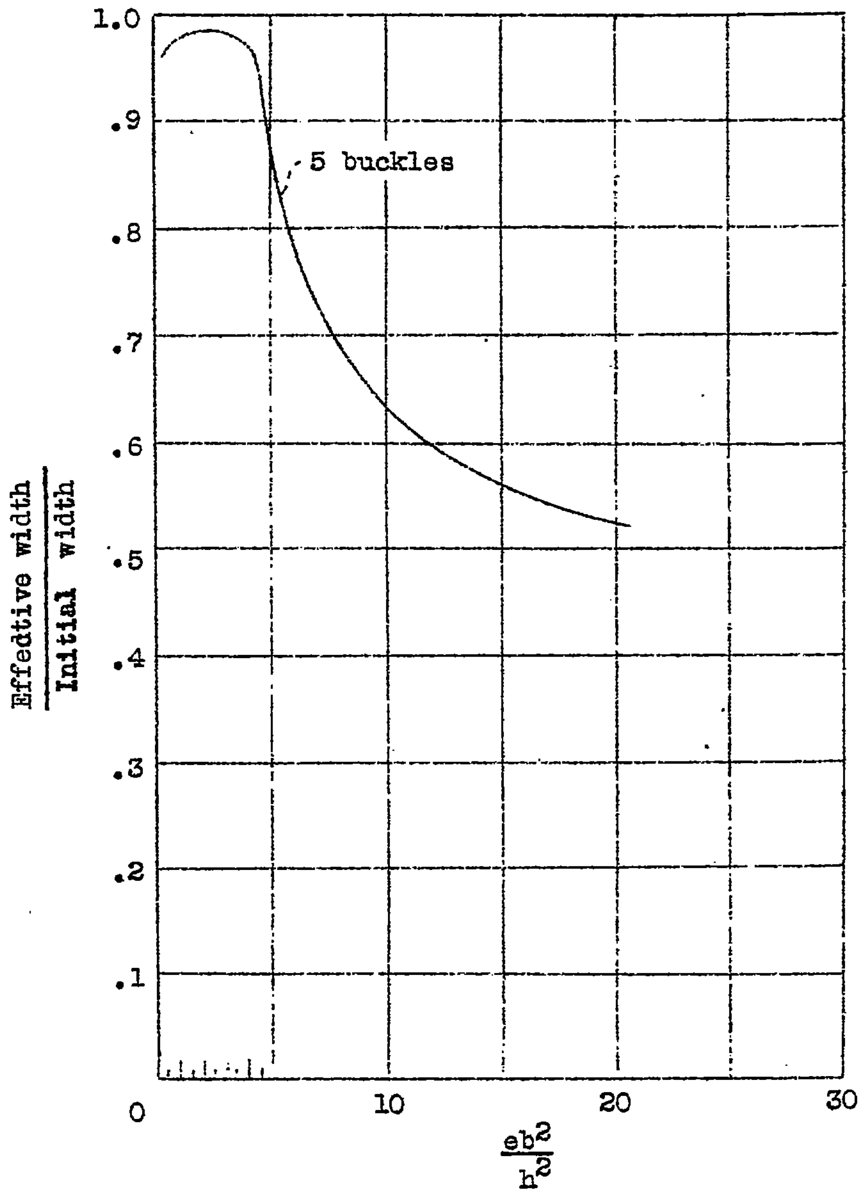


Figure 11.- Variation of ratio of effective width to initial width with edge strain e. Normal pressure, $p = 2.40Eh^4/b^4$, (b = plate width, h = plate thickness).

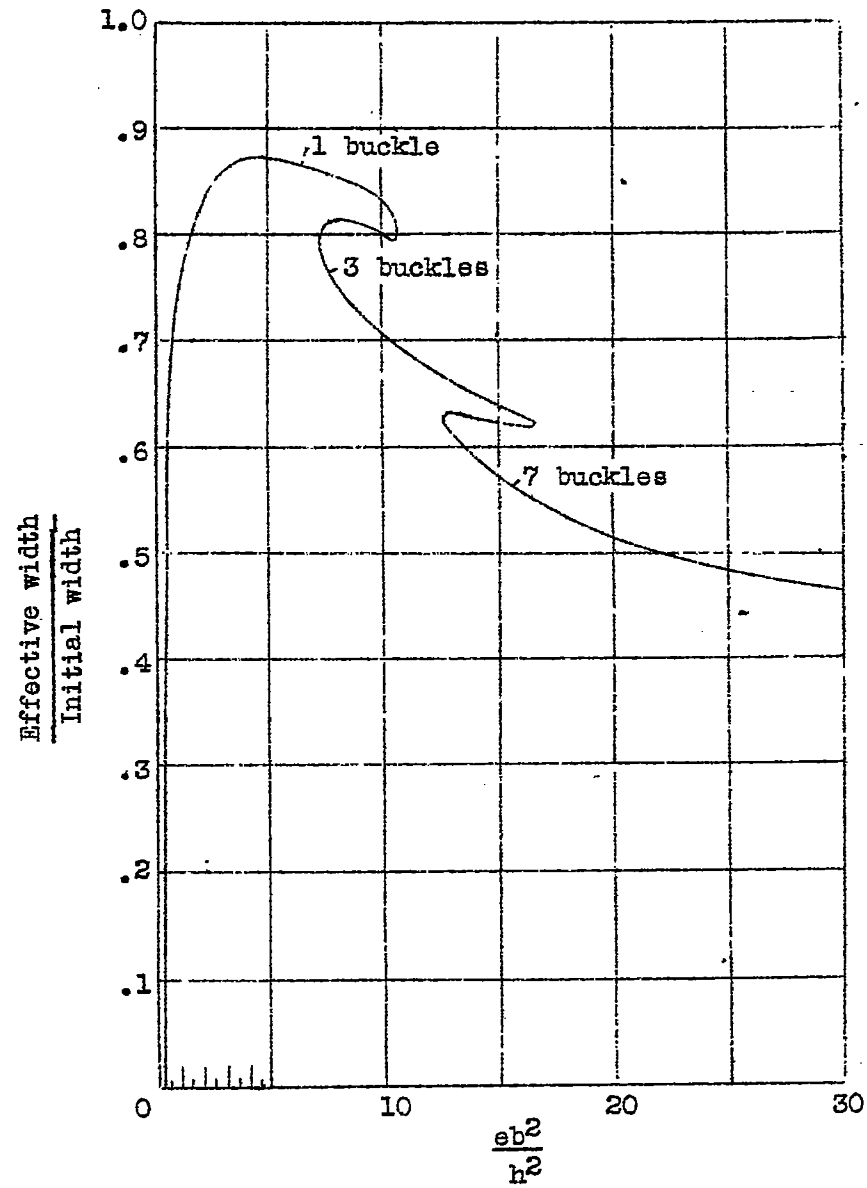


Figure 12.- Variation of ratio of effective width to initial width with edge strain e. Normal pressure, $p = 12.02Eh^4/b^4$, (b = plate width, h = plate thickness).

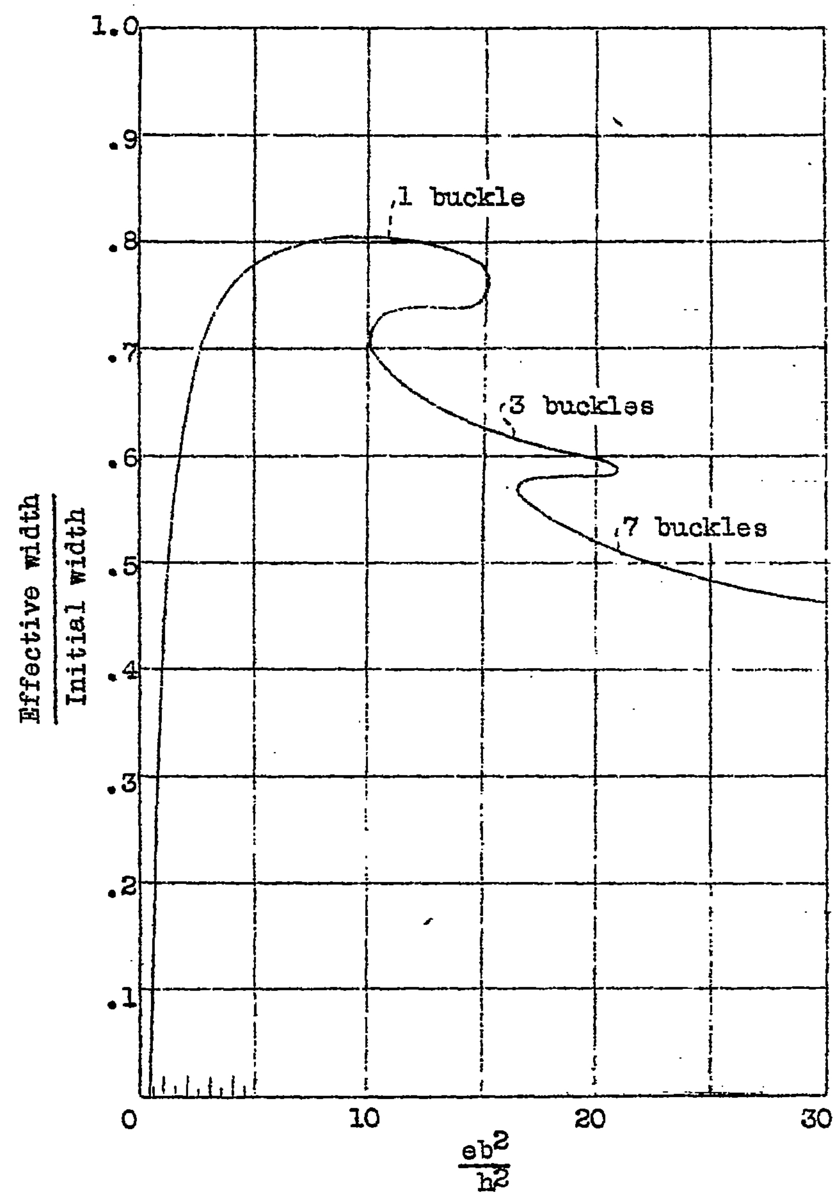


Figure 13.- Variation of ratio of effective width to initial width with edge st;ain e. Normal pressure, $p = 24.03Eh^4/b^4$ (b = plate width, h = p'ate thickness).