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RESEARCH MEMORANDUM

DESIGN PROCEDURE FOR TRANSPIRATION-COOLED STRUT-SUPPORTED TURBINE ROTOR BLADES

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SUMMARY

The procedure currently employed by the NACA Lewis laboratory in the design of transpiration-cooled strut-supported turbine rotor blades is discussed. The strut is the internal blade supporting member and also serves to partition the blade into separate cooling-air passages. Orifices in the blade base, which meter the cooling-air to each internal passage, are used in conjunction with a constant chordwise permeability. A compromise variable spanwise permeability is currently employed.

Details on the design of both the strut and the porous blade shell are included. Limitations in the present design procedure are also discussed.

INTRODUCTION

The NACA Lewis laboratory made exploratory tests on a transpiration-cooled turbine rotor blade in a modified production turbojet engine in order to evaluate blade design and fabrication methods (ref. 1). Since the completion of these tests, an additional theoretical study which permits the design of improved blades was made (ref. 2). Methods for determining the local coolant flow required to maintain a constant prescribed wall temperature for a transpiration-cooled gas turbine blade are presented in reference 3. In this reference, only chordwise variations were determined; effects of gas-to-wall temperature ratio are included. A similar investigation is reported in reference 4, in which temperature-ratio effects were accounted for by the use of a correction factor. The current design practice is to employ, in principle, the chordwise methods of reference 3 in conjunction with the spanwise variations described in reference 2.

Porous materials currently available for use in transpiration-cooled turbine blades may not possess sufficient strength to withstand the high stresses imposed by the high rotative speeds of the turbine.

As a result, such blades may require some internal nonporous load-carrying member, commonly called a strut, to which a porous shell is attached. In addition to serving as the blade supporting member, the strut also serves as a device for dividing the blade into compartments. The amount of coolant flow that passes through the porous wall is dependent on the difference in the squares of the absolute pressure levels on opposite sides of the wall. A considerable pressure variation exists around the exterior of the blade. The division of the blade interior into separate passages and the use of different-sized orifices in the blade base to meter the cooling air to the various passages minimize both the local blade overcooling and the amount of cooling air required (ref. 5). When porous materials that can withstand the high stresses imposed on rotating turbine blades are developed and a blade supporting member is no longer required, some kind of sheet-metal divider may be used to partition the blade interior. 3905

The present report outlines the methods now used by the NACA Lewis laboratory in the design of strut-supported transpiration-cooled turbine rotor blades. A brief discussion of the strut design is presented first. Following this, for a prescribed shell temperature, the methods of determining the ideal-coolant-flow requirements and blade-shell permeability are discussed. However, the fabrication of porous materials has not yet reached the state where such ideal specifications can always be met, and the blade designer may be restricted to a shell permeability different from the ideal one. Finally, methods currently used to determine the coolant flow and orifice sizes for a shell of prescribed permeability are discussed.

DESIGN PROCEDURE

The blade-design procedure is divided into two parts: (1) the design of the strut, and (2) the design of the porous blade shell.

Design of Blade Strut

Strut and shell materials can be chosen after the blade application and shape are specified. The allowable stress level is then determined and the strut design established. A number of fins are integral with the strut to permit attachment of the porous shell and to provide a number of internal passages for the cooling air (see fig. 1).

Strut fins and cooling-air passages. - The selection of the number of strut fins, or of cooling-air passages, depends upon several factors. The distance between fins must be such that the blade shell is sufficiently stiff to minimize shell vibrations. Current design employs an average external pressure for each cooling-air passage at each spanwise position; keeping the strut fins close together minimizes the external pressure variation across each passage and hence results in a more efficient design. The width of the fins must be large enough to permit the attachment of the shell and small enough so that a minimum of the porous surface is blocked.

The cross sections of a strut and blade shell are shown in figure 1. The fins are usually located opposite each other on the two sides of the strut; this positioning is made so that pressure can be applied to one blade surface when shell attachment is made on the other without damaging the shell. The cooling-passage areas are usually set up so that the area variation from root to tip is linear with the tip areas being limited by the tip profile. The overhang of the blade shell at the leading and trailing sections must be designed to minimize vibration. Furthermore, the shell radii at the leading and trailing edges must be of sufficient size that the porous shell will not crack.

Strut stresses. - In general, a rotating turbine blade is subjected to centrifugal, gas-bending (steady state and vibratory), and thermal stresses. The various spanwise sections of the strut are alined so that the centers of gravity of these sections fall on a nearly radial line. In this way, bending stresses due to centrifugal forces are minimized. In order to partially compensate for gas-bending stresses, the blade may be tilted toward the suction side. In this way, total bending stresses are minimized. Despite considerable effort already applied to the study of vibratory and thermal stresses in turbine blades, the mechanisms involved are so complex that little information on these stresses is available. In air-cooled turbine blades, blade shells are usually thin and structures are often brazed; both thin walls and brazing have the effect of changing the stress-rupture properties. Moreover, after blade fabrication, the stress-rupture properties of the blade materials are not known. In order to account for the unknown vibratory and thermal stresses, the effects of braze attack, and other fabrication flaws, it is current practice to employ a safety factor known as the stress-ratio factor (see ref. 6) in cooled-blade designs. For a given average blade temperature, the stress-ratio factor at any spanwise blade section is simply the ratio of the allowable stress for bar stock to the centrifugal stress at the section.

The allowable stress for the strut metal is determined from a stress-rupture curve at a specified temperature and for a specified strut-material life. Typical examples of stress-rupture curves for bar stock of several high-temperature alloys are shown in figure 2, where the stress-rupture for a 100-hour life is plotted against metal temperature. The critical blade section is that section where the stress-ratio factor is a minimum. A method for determining the centrifugal stress along the span of a solid turbine blade is given in reference 7. Modification of equation (4) in reference 7 is employed herein.

The centrifugal stress at any spanwise location X, with the blade shell considered as dead weight, may be found from

$$\sigma_X = \frac{\Gamma_{st}\omega^2}{A_{X,st}g}\int_X^L A_{x,st}(r_r + x)dx + \frac{\Gamma_{sh}\omega^2}{A_{X,st}g}\int_X^L A_{x,sh}(r_r + x)dx \qquad (1)$$

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The symbols are defined in the appendix. The variation in the strut centrifugal stress along the blade span can be obtained by use of equation (1). A stress-ratio factor at the blade critical section of approximately 1.5 to 2 is currently used to allow for the unknown vibratory and thermal stresses, braze attack, and fabrication flaws.

The original strut design may lead to erratic pressure and flow distributions inside the various cooling-air passages. Alterations of the strut design (changes in the passage cross-sectional area distributions) may then be required. If such alterations are necessary, the altered strut design will have to be stress-checked again.

Design of Blade Shell

The steps in the design calculation of a transpiration-cooled blade shell with prescribed chordwise and spanwise variations in shell temperature are: (1) the determination of the gas velocity relative to the blade and the pressure distribution around the blade; (2) the calculation of the local required cooling-air flow ρv necessary to maintain the prescribed wall temperature; (3) the calculation of the spanwise variation of the static pressure of the cooling air in each internal cooling-air passage; and (4) the determination, for a selected type of porous-shell material, of the required spanwise variation in shell permeability for each cooling-air passage. These calculations result in so-called ideal values and, in general, specify both chordwise and spanwise permeability variations. Each of the above mentioned steps is discussed in detail, subsequently.

Gas velocity and pressure distributions around blade. - The chordwise and spanwise variations in the velocity and pressure of the engine gases around the turbine blade must be determined either from theory or experiment. These distributions depend upon the engine flight and operating conditions and the turbine geometry. Graphical two-dimensional stream-filament theories for predicting these distributions for impermeable blades in the cascade row are given in references 8 and 9. Lack of experimental data on the effects of flow through a porous wall on the main-stream-flow conditions makes it necessary to use these theories in the current design of permeable blades; experiments on these effects are required. Typical examples of these chordwise distributions, calculated by the method of reference 8, are shown in figure 3.

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Local ideal cooling-air flow. - The spanwise variation in local ideal cooling-air flow ρv for each cooling-air passage depends upon the local values of effective gas temperature, cooling-air temperature, gas pressure around the blade, gas velocity relative to the blade, and the prescribed shell temperature; it is independent of cooling-air supply pressure and shell permeability. The determination of the local values of ρv is discussed in detail in reference 3, and only a brief outline will be given herein.

The transition points from laminar to turbulent flow in the external-gas stream are taken as the points of minimum pressure on the blade pressure and suction surfaces. Over the portion of the blade surface having laminar flow, the exact solutions of the laminar-boundary-layer equations for wedge-type flow with transpiration cooling and a constant wall temperature (refs. 10 to 12) are used to determine the local coolant flows. For the turbulent-flow region, the Rannie-Friedman theory (refs. 13 and 14) for flow over a permeable flat plate is used. These theories will be sketched briefly, and the adequacy of each will be discussed in the section DESIGN LIMITATIONS.

The spanwise variation in effective gas temperature T_g for each cooling-air passage must be found either from temperature measurements of impermeable blades or by estimating the value of the recovery factor (ref. 3). The rate of heat addition to the coolant on its way through the flow passages is usually not well known and has to be estimated (ref. 2). An assumed spanwise variation in the coolant temperature is therefore necessary.

Laminar flow: Since the wall temperature is prescribed, the value of the temperature parameter $1 - \varphi = \frac{T_w - T_c}{T_g - T_c}$ may be determined. The Euler number $Eu = \frac{y}{W}\frac{dW}{dy}$ is found from the chordwise gas-velocity distributions previously determined, y being the peripheral chordwise distance from the blade leading edge to the point under consideration on either blade surface (see fig. 4). In figure 5, results of the wedge-type laminar boundary-layer solutions are plotted as $(1 - \varphi)$ against the coolant-flow parameter $-f_w$; with Euler number Eu and the temperature ratio T_g/T_w known, the value of $-f_w$ may be found from the figure. The parameter $-f_w$ is given in turn as (see ref. 3)

$$-f_w = \frac{2}{Eu + 1}\frac{v}{W}\sqrt{Re} \tag{2}$$

where v is the cooling-air ejection velocity at the porous wall based on porous-surface area (not porous flow area). The gas Reynolds number $Re = \rho W y/\mu$, density $\rho = p_e/RT_w$, and viscosity μ are based on the prescribed wall temperature T_w. Hence the local cooling-air flow is

$$\rho v = \rho \frac{-f_w(Eu + 1)}{2} \frac{W}{\sqrt{Re}} \tag{3}$$

It may be mentioned here that the use of the wall temperature rather than the effective gas temperature in the evaluation of the cooling-air density and viscosity is based on judgment rather than experience, in the absence of conclusive experimental data.

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The preceding procedure may also be used for the cooling-air passage supplying the blade leading edge (stagnation point), which is considered to be in the laminar region, with the following modifications. At the stagnation point, the Euler number Eu is unity, and for a circular cylinder of diameter D the velocity near the stagnation point is given approximately by (ref. 15)

$$W = 3.63\ U \frac{y}{D} \tag{4}$$

where U is the upstream-approach gas velocity. As an approximation, equation (4) may be used for the leading edge of the turbine blade if twice the leading edge radius is substituted for D. Substituting equation (4) and $Eu = 1$ into equation (3) gives

$$\rho v = 1.905\rho \frac{(-f_w)\ U}{\sqrt{Re_D}} \tag{5}$$

as the required cooling-air flow for the leading-edge passage. Here the Reynolds number Re_D is given by $Re_D = \rho UD/\mu$, with density ρ and viscosity μ based on the prescribed wall temperature as before.

Turbulent flow: The cooling-air flow ρv necessary to maintain a prescribed wall temperature for a flat plate in turbulent-flow transpiration cooling depends on the local values of $1 - \varphi$, gas Reynolds number Re, Prandtl number Pr, and viscosity μ. The expression for ρv taken from reference 16, with the modification that gas properties are based on wall temperature rather than gas temperature, is

$$\rho v = \frac{Re^{0.9} \ln\left[1 - \frac{2.11}{Re^{0.1}} + \frac{2.11}{Re^{0.1}(1 - \varphi)}\right]}{71.3\ Pr^{2/3} \frac{y}{\mu}} \tag{6}$$

Equation (6) is used to calculate the spanwise distribution of cooling-air flow ρv for those coolant passages lying in the turbulent-flow region of the blade.

Spanwise variation in cooling-air pressure in internal blade passages. - The spanwise variation in the cooling-air pressure in each of the blade coolant passages is calculated, with the use of the theory given in reference 2. In order to apply this theory, the coolant-passage geometry, the spanwise variation in cooling-air flow ρv, and 3905 the spanwise variation of cooling-air static temperature T_c must be known for each passage. With an assumed variation in cooling-air temperature, it is necessary to solve simultaneously the following pair of equations:

$$\frac{dw}{dx} = -b(\rho v) \tag{7}$$

and

$$\left[1 - \left(\frac{w}{A_c}\right)^2 \frac{RT_c}{gp_i^2}\right] \frac{dp_i}{dx} = \frac{p_i}{gRT_c} r\omega^2 \cos\beta - \frac{2RT_c}{gp_i}\left(\frac{w}{A_c}\right) \frac{d\left(\frac{w}{A_c}\right)}{dx} - \left(\frac{w}{A_c}\right)^2 \left(\frac{R}{gp_i}\right)\left(\frac{dT_c}{dx} + \frac{T_c}{A_c}\frac{dA_c}{dx} + \frac{fT_c}{2D_h}\right) \tag{8}$$

The need for an assumption of a spanwise variation in cooling-air temperature could be eliminated and the actual variation calculated if the appropriate energy equation were solved simultaneously with equations (7) and (8). This would make the calculation considerably more tedious. In view of this and of the various assumptions required for the application of other theoretical analyses to the blade design, the simpler method, as presented in reference 2, was employed.

The method of reference 2 includes the effect of fluid friction forces, centrifugal forces, and change of momentum on the flow of cooling air in each passage. The theory is applicable one-dimensionally in a spanwise direction. Hence the values of the various quantities involved are taken each as an average over the chordwise width of each passage at every spanwise location. The friction factor f for turbulent flow through impermeable tubes is used in the absence of friction data for flow through permeable tubes.

With the variations of the quantities b and ρv known in the spanwise direction, equation (7) may be integrated directly in order to find the variation of coolant weight flow w with distance x:

$$w = w_t + \int_x^L b(\rho v)\, dx \tag{9}$$

where w_t is a prescribed coolant weight flow leaving the passage at the tip. For passages ending blindly, w_t is zero. Since the variation of w with x is known, the variation of the gradient $\frac{d(w/A_c)}{dx}$ with x may be found from the relation

$$\frac{d\left(\frac{w}{A_c}\right)}{dx} = \frac{-b(\rho v) - \frac{w}{A_c}\frac{dA_c}{dx}}{A_c} \tag{10}$$

since the variation of the passage cross-sectional area A_c with x is assumed to be known. For a strut design with a cooling-air passage having a large negative spanwise gradient in flow area dA_c/dx, there exists the possibility of the gradient $\frac{d(w/A_c)}{dx}$ changing sign along the blade span (eq. (10)). This would result in an erratic curve of w/A_c against x, which would mean eccentric velocity and pressure distributions in the coolant passage. This is apparent if the coolant-flow Mach number is expressed in terms of w/A_c and the pressure p_i as

$$M = \frac{\frac{w}{A_c}}{p_i}\sqrt{\frac{RT_c}{\gamma g}} \tag{11}$$

To eliminate such undesirable effects, it might be necessary to modify the strut design in order to reduce the large spanwise gradients in passage area.

All quantities in equation (8) are then known with the exception of the cooling-air static pressure p_i; so a numerical solution of equation (8) may be applied to find p_i as a function of x, when a boundary value $p_{i,r}$ of p_i at the passage entrance $(x = 0)$ is prescribed. This boundary value $p_{i,r}$ must be chosen sufficiently large so that the resulting calculated internal pressure p_i is greater than the external gas pressure p_e at each spanwise position in order to ensure ejection of cooling air through the porous wall. For radial passages, $\beta = 0$ and $r = r_r + x$ in equation (8).

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Required spanwise variation in shell permeability. - With the spanwise variations of cooling-air flow ρv, gas pressure p_e, and cooling-air pressure p_i known for each cooling-air passage, the permeability of the porous wall of each passage must vary spanwise in such a manner that the required ρv is obtained with the given pressures p_i and p_e.

For several types of porous shell materials, the correlation between cooling-air flow ρv and the pressures p_i and p_e is well approximated by an equation of the form

$$\rho v = C_K(p_i^2 - p_e^2)^n \tag{12}$$

over a fairly wide range of flow (ref. 2). The value of the exponent n is determined empirically and is a function of the type of porous material; the value of C_K is a function of material permeability and thickness, and of cooling-air properties based on the porous-wall temperature. Since the spanwise distributions of ρv, p_i, and p_e are known, equation (12) may be used to calculate the required spanwise variation in the value of C_K for each passage. Then the required spanwise variation in the ratio of shell permeability to thickness K/τ may be determined from the values of C_K since the porous wall temperature is known. For a typical sintered material, for which $n = 2/3$,

$$C_K = \frac{9.66(10^3)\ \frac{K}{\tau}\left(\frac{\mu_{sl}^2 T_{sl}}{\mu^2 T_w}\right)^{2/3}}{\frac{\mu_{sl}}{\mu}}$$

For a typical wire-cloth material, for which $n = 5/8$,

$$C_K = 3.050\left(\frac{K}{\tau}\,\frac{1}{\mu T_w}\right)^{5/8}$$

Necessary compromises in specification of shell permeability. - The preceding methods yield the ideal values necessary to maintain a prescribed blade-shell temperature. Although variations in the shell temperature may be prescribed, it is current practice to prescribe a uniform blade-shell temperature equal to the maximum allowable for the particular shell material. In this way, the amount of coolant can be held to a minimum. Calculations made according to the preceding methods for such a prescribed constant wall temperature result in both chordwise and spanwise variations in required shell permeability. Since ideal specifications are difficult to achieve in fabrication, compromises on shell permeability may be required.

The use of orifices for metering cooling air to the blade passages will partially compensate for the use of a constant chordwise permeability, so a constant chordwise permeability has been accepted in current design. Attention is therefore centered on the type of spanwise variation obtainable. The calculation methods employed when a spanwise permeability is prescribed follow.

Calculation of wall temperature and cooling-air flow for prescribed spanwise permeability. - With the spanwise variation of shell permeability specified, the spanwise variation in wall temperature and the coolant weight flow w_r to each passage may be calculated as follows: 3905

By noting that

$$\frac{dw}{dx} = A_c \frac{d\left(\frac{w}{A_c}\right)}{dx} + \frac{w}{A_c}\frac{dA_c}{dx} \tag{13}$$

and by using equation (12), equation (7) may be rewritten as

$$A_c \frac{d\left(\frac{w}{A_c}\right)}{dx} + \frac{w}{A_c}\frac{dA_c}{dx} = -bC_K\left(p_i^2 - p_e^2\right)^n \tag{14}$$

Equation (14) must be solved simultaneously with equation (8) to find the variation of p_i and w/A_c with spanwise distance x. The boundary conditions are $p_i = p_{i,r}$ at $x = 0$ and $w/A_c = w_t/A_{c,t}$ at $x = L$, where $p_{i,r}$, w_t, and $A_{c,t}$ are known or are prescribed values. The variations with x of all quantities in equations (8) and (14) are known except w/A_c, p_i, and C_K. The first two of these must be found, whereas C_K depends on the shell permeability and shell temperature. The variation of shell permeability with x is known, but the shell temperature must be calculated.

At each step of the numerical simultaneous solution of equations (8) and (14), starting at $x = 0$ where $p_i = p_{i,r}$, the wall temperature T_w is calculated by an iteration scheme as follows: For an assumed value of T_w, a value of ρv is found from equation (12). Because the coolant velocity v may be determined as $v = \frac{\rho v}{\rho} = \frac{(\rho v)RT_w}{p_e}$ for the laminar region of the blade surface the value of $-f_w$ may be found from equation (2), with the gas Reynolds number Re and the density ρ based on the assumed value of T_w. Then, by evaluating the ratio T_g/T_w, using the assumed value of T_w, a value of $1 - \varphi$ is read from figure 5. Since $1 - \varphi = \frac{T_w - T_c}{T_g - T_c}$, and T_c and T_g are known, a value of T_w may be determined. If this value does not agree with the value originally assumed in the determination of ρv from equation (12), iteration is required until agreement is reached. For the leading-edge passage, the only modification to this procedure is that the Euler number Eu is set equal to unity and $-f_w$ is calculated from the equation

$$-f_w = 0.525 \frac{v}{U} \sqrt{Re_D} \tag{15}$$

where the Reynolds number Re_D is based on the upstream-gas velocity U and the leading-edge diameter D.

In the turbulent-flow region, equation (6) may be solved for $1 - \varphi$ to give

$$1 - \varphi = \frac{\frac{2.11}{Re^{0.1}}}{\exp\left[\frac{71.3(\rho v)Pr^{2/3}}{Re^{0.9}} \frac{y}{\mu}\right] + \frac{2.11}{Re^{0.1}} - 1} \tag{16}$$

With an assumed value of T_w, a value of ρv is found from equation (12), and a value of $1 - \varphi$ is then calculated from equation (16), again basing ρ and μ on the assumed value of T_w. From this value of $1 - \varphi$, a value of T_w may be found, and again iteration is required until this value agrees with the value assumed originally.

Since the boundary conditions are imposed at opposite ends of the cooling-air passage, a trial-and-error process is generally necessary to find the simultaneous solution of equations (8) and (14) which satisfies the prescribed boundary conditions (ref. 2). This trial-and-error procedure combined with the iteration process necessary at each step for the determination of the wall temperature makes the numerical solution of equations (8) and (14) cumbersome. If only the calculation of the consumption of cooling air is desired, and if an estimate of the spanwise variation in wall temperature T_w can be made in advance, the variation of C_K with x is known and equations (8) and (14) may be solved without iteration for T_w. An addition simplification in this case is possible if only a qualitative estimate of the cooling-air consumption is desired. A short-form solution of equation (8) for the internal cooling-air pressure p_i may be obtained by omitting all terms except those involving centrifugal force due to rotation (ref. 2). In the case of a stator blade, this is equivalent to the assumption of a constant spanwise pressure p_i. For a radial passage ($\beta = 0$) and an assumed linear spanwise variation in cooling-air temperature ($T_c = T_{c,r} + ax$), equation (8) may be integrated in closed form to yield

$$\ln\left(\frac{p_i}{p_{i,r}}\right) = \frac{\omega^2 T_{c,r}}{gRa^2}\left(\frac{ar_r}{T_{c,r}} - 1\right) \ln\left(1 + \frac{a}{T_{c,r}} x\right) + \frac{\omega^2}{gRa} x \qquad (17)$$

The variation of p_i with x may then be calculated from equation (17); and since the variation of C_K and p_e with x are known, the spanwise distribution of ρv may be found from equation (12). Then the cooling-air flow w_r to the passage is found by setting $x = 0$ in equation (9):

$$w_r = w_t + \int_0^L b(\rho v)dx \qquad (18)$$

Although equation (17) is generally used as an approximate solution of equation (8) for M_r less than 0.2 (ref. 2), it is still useful for higher M_r since only a qualitative estimate of the cooling-air flow is desired.

The total cooling-air flow for a cooled turbine is given by

$$w_{to} = N\Sigma w_r \qquad (19)$$

if there are N cooled blades. The ratio of the total cooling-air flow w_{to} to the engine gas weight flow is an important quantity for evaluating the effectiveness of a method of turbine cooling. A comparison of this ratio for the ideal and the prescribed permeability cases gives a quantitative picture of the potential of transpiration cooling for a given application and how much this potential is compromised by fabrication limitations.

Required orifice size. - The use of small orifices at the blade base for metering the cooling-air flow to each blade passage is discussed in reference 5. Although this reference included only chordwise variations in cooling-air flow ρv, cooling-air pressure p_i, external-gas pressure p_e, and shell permeability, the method presented therein is used for approximating results when there are significant spanwise variations in these quantities.

The purpose of orifices at design engine operation is to reduce the constant chordwise coolant supply pressure p_a at the blade base

to the required pressure $p_{i,r}$ at each cooling-air passage entrance, while passing the required coolant weight flow w_r for the passage. Since the value of $p_{i,r}$ and w_r for the passages may vary widely, a different orifice size will probably be required for each passage. In reference 16, it is shown with experimental data that the flow through small orifices obeys the laws governing flow through nozzles. If the equations for subsonic flow and critical flow through flow nozzles are solved for the nozzle flow area A_n, the results are, in the symbols
3905 of the present report,

$$A_n = \sqrt{\frac{R}{2g}}\,\frac{w_r}{B}\sqrt{\frac{T_{c,r}}{p_{i,r}(p_a - p_{i,r})}} \tag{20}$$

and

$$A_n = 2.067\sqrt{\frac{R}{2g}}\,\frac{w_r}{B}\,\frac{\sqrt{T_{c,r}}}{p_a} \tag{20a}$$

Equation (20) is applicable for subcritical pressure drops across an orifice $\left(\frac{p_{i,r}}{p_a} > 0.528\right)$, whereas equation (20a) is for supercritical pressure drops $\left(\frac{p_{i,r}}{p_a} \leq 0.528\right)$. The values of $T_{c,r}$ and p_a are prescribed, and the values of w_r and $p_{i,r}$ are known from the calculations of the pressure distribution in the interior of each blade passage. The quantity B is a nozzle coefficient whose value may be estimated from experimental flow correlations (ref. 16). The required orifice area A_n for each cooling-air passage may then be calculated from one of these equations. It is, of course, necessary that the available coolant supply pressure p_a be greater than the largest of the required pressures $p_{i,r}$ for the coolant passages.

DESIGN LIMITATIONS

Much development work remains to be done in the fabrication of porous materials. Improvement and checking of theoretical analyses must also be completed before improved designs are to be obtained.

For example, the methods of references 8 and 9 for determining the chordwise variations in gas velocity and pressure were developed for use with impermeable blades in a cascade. Lack of experimental data on the effects of fluid injection through a porous wall on main-stream flow conditions necessitated the use of the methods of reference 8 or 9 in designing transpiration-cooled blades. Adequate experimental data to justify the use of this method for transpiration cooling are needed.

The use of wedge solutions of boundary-layer equations in predicting heat transfer (or in determining the ideal required coolant flow) over the laminar-flow portion of the blade has not been verified experimentally. Wedge solutions have been used to predict the heat transfer to impermeable bodies of arbitrary cross section within required engineering accuracy. The extension of this method to transpiration cooling appears reasonable, but experimental verification is desirable. For the turbulent part of the blade, the Rannie-Friedman theory (refs. 13 and 14) for a permeable flat plate is employed. It has long been known that the effect of a pressure gradient on heat transfer in the turbulent boundary layer is much less than in the laminar region; hence, use of the flat-plate theory also appears reasonable. A recent experimental investigation (ref. 17) indicates that one of the assumptions made in the Rannie-Friedman theory may be inaccurate. The experimental results indicate that flow through a porous wall affects velocity distributions in both the laminar sublayer and the turbulent core of the boundary layer. The theory used in the blade design assumed an effect on velocity profile only in the laminar sublayer. Other theoretical analyses of transpiration cooling in the turbulent-flow region have been made (refs. 18 and 19), but each awaits experimental verification. As a consequence, the Rannie-Friedman theory (refs. 13 and 14) has been maintained in the design procedure. The theoretical method for determining the pressure variation within each cooling-air passage has not, as yet, been checked experimentally. The flow correlations for porous materials have been verified experimentally. References 20 and 22 present experimentally determined flow correlations for wire cloth, and references 16 and 23, for sintered materials.

CONCLUDING REMARKS

With respect to coolant-flow, the ideal shell for a transpiration-cooled strut-supported turbine rotor blade should possess both a variable chordwise and a variable spanwise permeability. The use of orifices for metering cooling air to the blade passages will partially compensate for the use of a constant chordwise permeability. A constant chordwise permeability and the use of different-sized orifices in the blade base are used in current-design methods; by these means, the variations of the gas pressure around the blade periphery may be accounted for, and the blade-shell fabrication problem is considerably simplified.

The ideal spanwise variation in permeability for each cooling-air passage may be determined from the methods of this report. Since these variations differ for the different passages and these ideal permeability variations are difficult to fabricate, it may be necessary to determine some compromise spanwise variation. The choice of this compromise variation in spanwise permeability is left to the discretion of the designer. The compromise is primarily dictated by the manufacturer's ability to meet the specifications.

In order to compensate for unknown vibratory and thermal stresses, fabrication flaws, and the effects of braze attack, use is made of a safety factor (stress-ratio factor) in the design of the load-carrying strut.

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Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, October 26, 1955

APPENDIX - SYMBOLS

The following symbols are used in this report:

A cross-sectional area, sq ft

a coefficient in linear spanwise variation of cooling-air temperature, $T_c = T_{c,r} + ax$

B orifice flow coefficient

b chordwise peripheral width of coolant passage, ft

C_K quantity containing permeability coefficient, eq. (12), $lb(ft^4)^n/(sec)(sq\ ft)(lb^2)^n$

D leading-edge diameter, ft

D_h coolant-passage hydraulic diameter, ft

Eu Euler number of gas flow over blade, $\frac{y}{W} \frac{dW}{dy}$

f friction factor

$-f_w$ coolant-flow parameter for laminar-flow region, eq. (2)

g acceleration due to gravity, ft/sec^2

K permeability coefficient, sq ft

L coolant-passage length (see fig. 4), ft

M cooling-air-flow Mach number

N number of cooled turbine blades

n exponent, eq. (12)

Pr Prandtl number

p static pressure, lb/sq ft

R gas constant, ft-lb/(lb)($^{\circ}$F)

Re gas Reynolds number, $\rho Wy/\mu$

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Re_D gas Reynolds number at blade leading edge, $\rho UD/\mu$

r radius, ft

T temperature, °R

U gas approach velocity upstream of blade leading edge, ft/sec

v cooling-air velocity through porous wall (based on total porous-surface area), ft/sec

W gas velocity relative to blade, ft/sec

w weight flow of cooling air, lb/sec

X specified spanwise position on blade, ft

x spanwise distance from blade base (see fig. 4), ft

y peripheral distance from blade leading edge (see fig. 4), ft

β angle between velocity vector and direction of increasing radius

Γ metal density, lb/cu ft

γ ratio of specific heats

μ absolute viscosity (based on porous-wall temperature), lb/(ft)(sec)

ρ density (based on porous-wall temperature), lb/cu ft

σ centrifugal stress, lb/sq ft

τ porous blade-shell thickness, ft

φ temperature-difference ratio, $T_g - T_w/T_g - T_c$

ω angular velocity, sec^{-1}

Subscripts:

a cooling-air supply at blade base

c cooling air or cooling-air passage

cr critical

e external gas flow

g effective gas temperature

i internal coolant flow

le leading edge

n orifice

r coolant-passage entrance (blade root)

sh shell

sl NACA sea-level standard conditions

st strut

t coolant-passage tip

to total

X specified spanwise position on blade, ft

w porous wall

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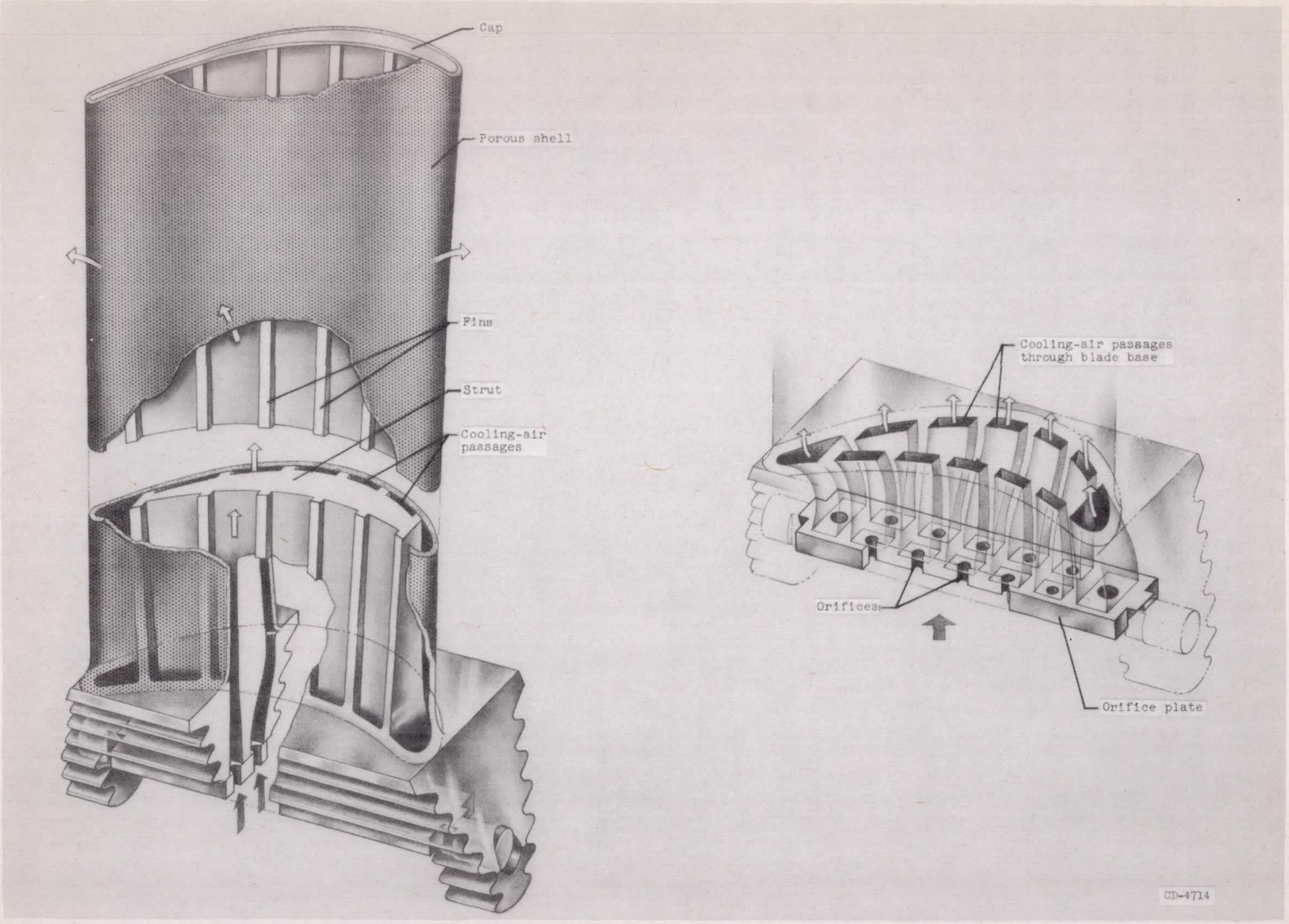

Figure 1. - Transpiration-cooled strut-supported turbine rotor blade. (Arrows indicate direction of cooling-air flow.)

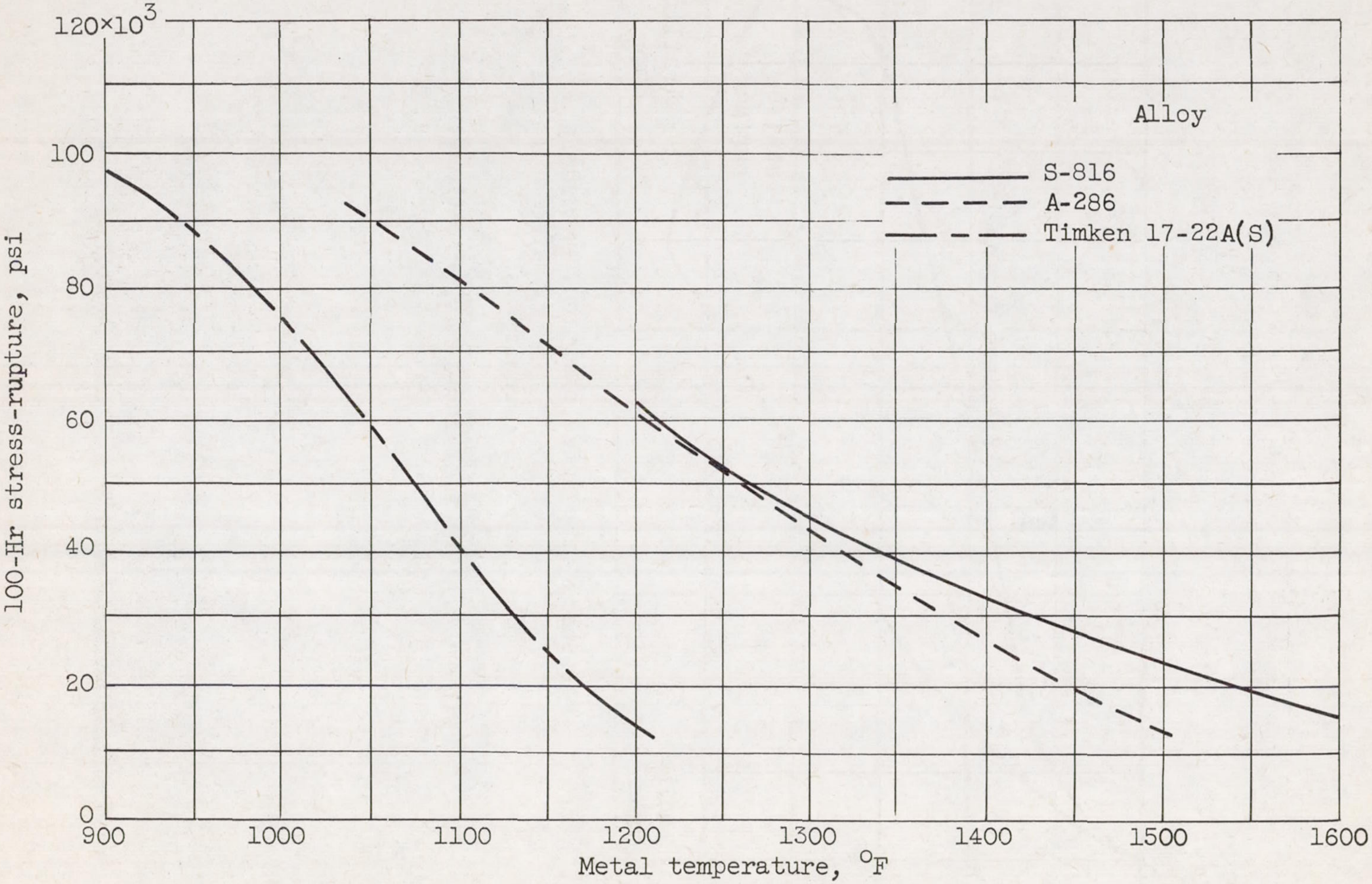

Figure 2. - Stress-rupture properties of several high-temperature alloys (bar stock).

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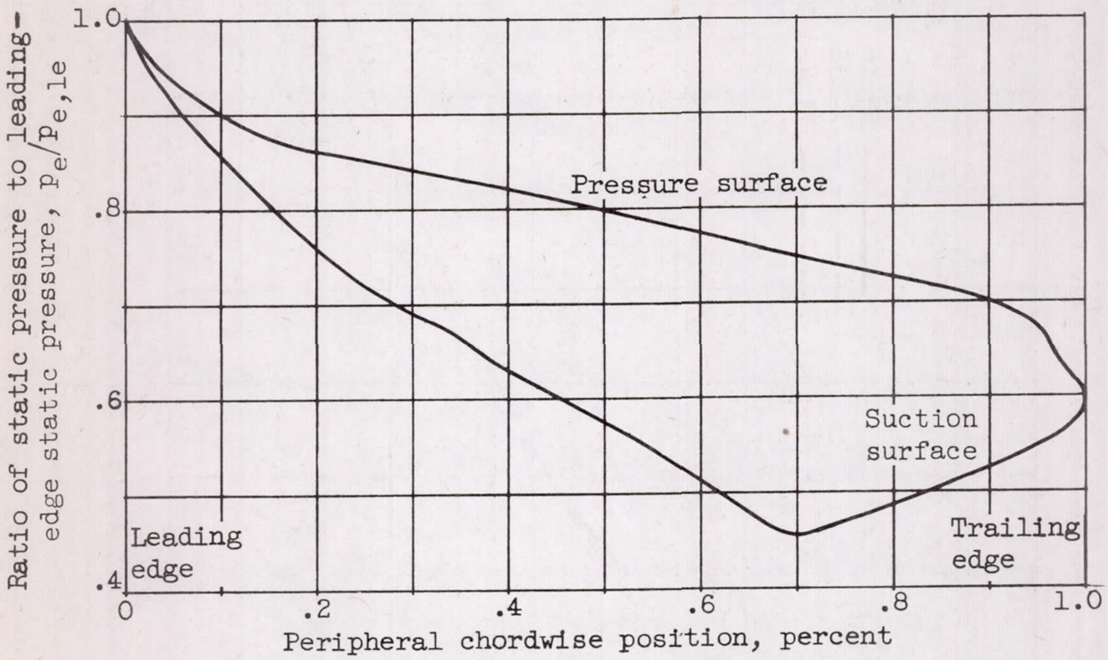
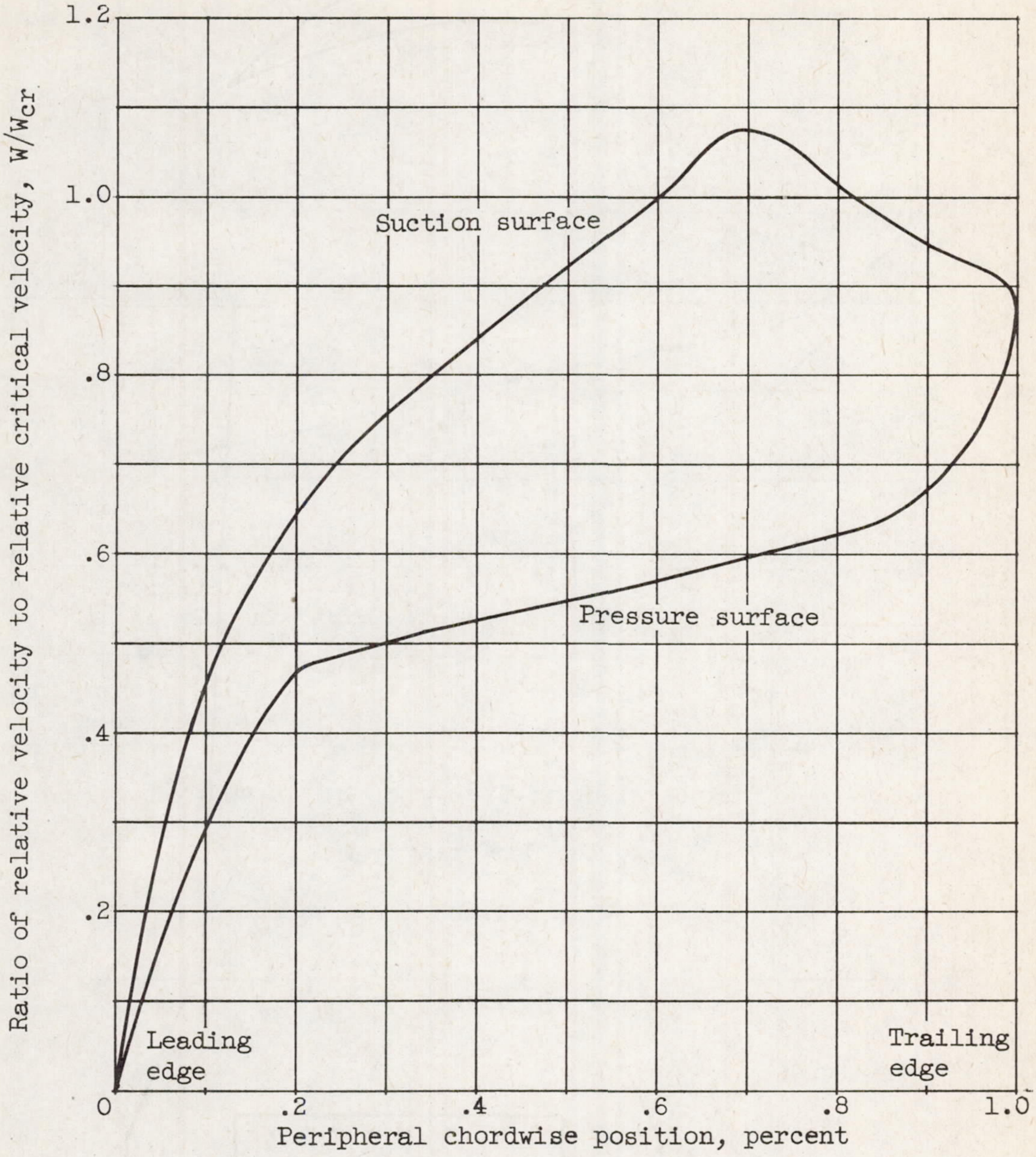


(a) Gas pressure.

Figure 3. - Typical chordwise variations.



(b) Gas velocity relative to blade.

Figure 3. - Concluded. Typical chordwise variations.

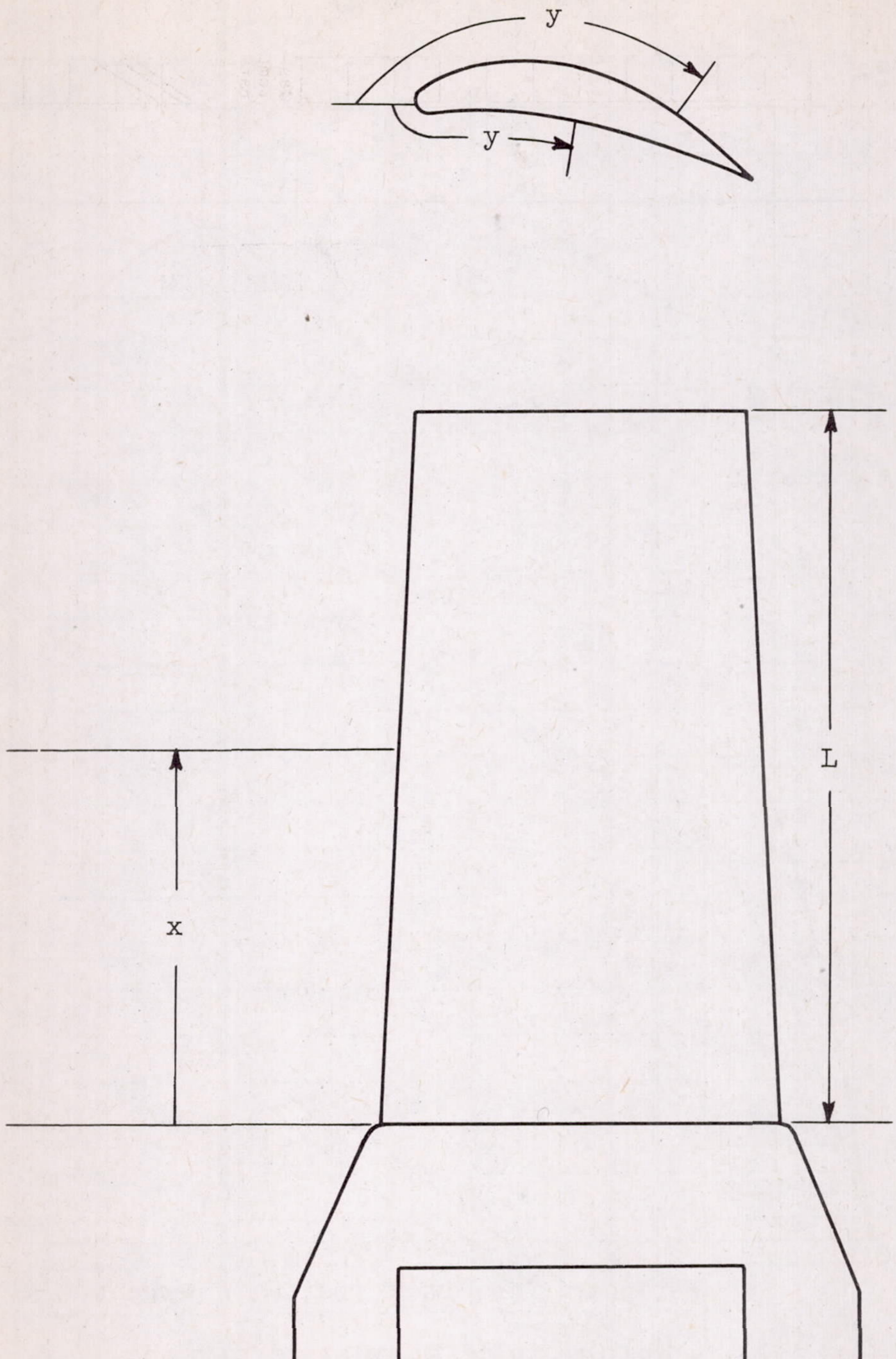
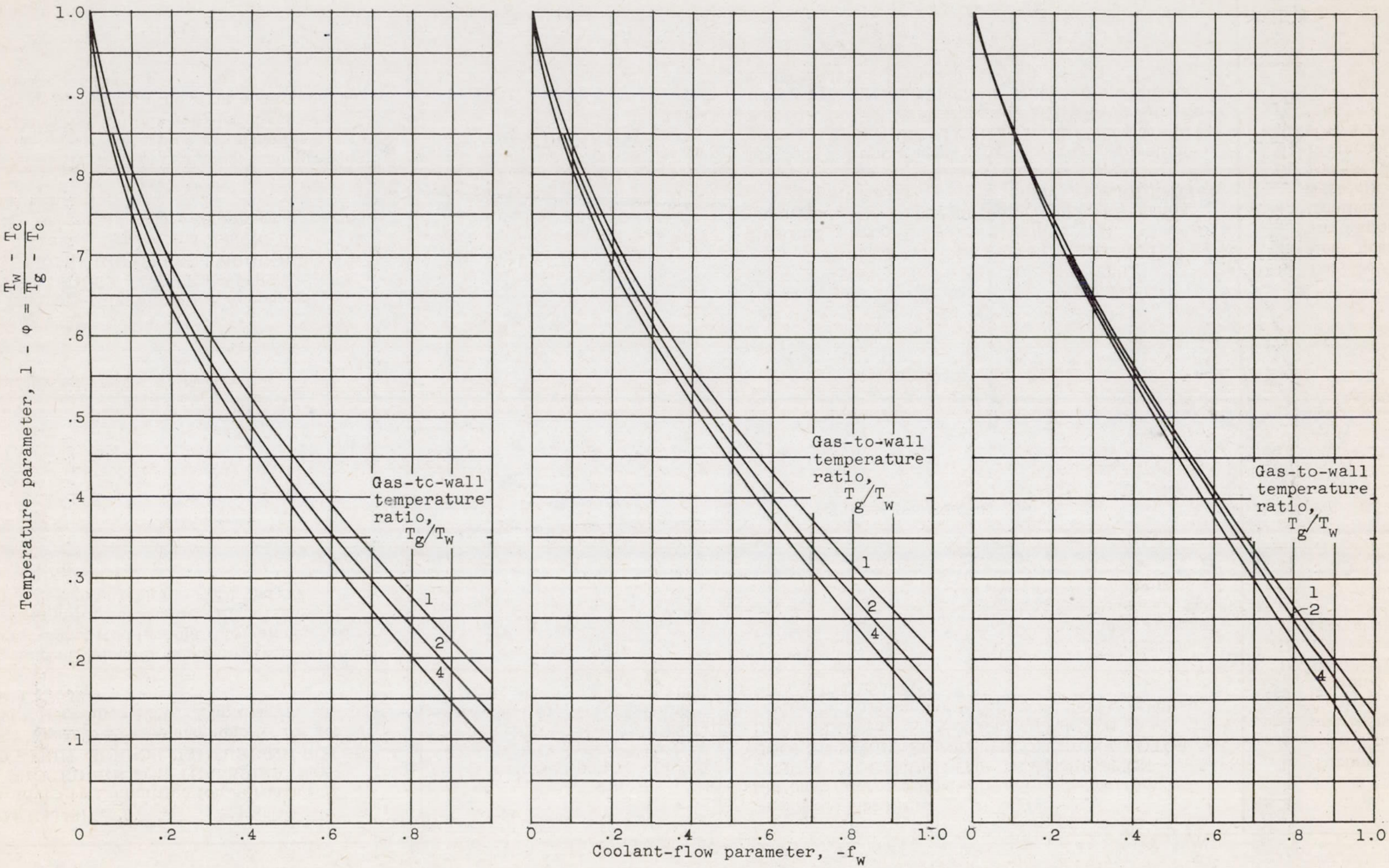


Figure 4. - Coordinate system used in transpiration-cooled blade-design procedure.



(a) Euler number, 1.0.

(b) Euler number, 0.5.

(c) Euler number, 0.

Figure 5. - Coolant-flow and temperature parameters of transpiration-cooled wall in laminar gas-flow region. Prandtl number for air, 0.7.