RESEARCH MEMORANDUM

METHODS FOR CALCULATING THRUST AUGMENTATION AND LIQUID CONSUMPTION FOR VARIOUS TURBOJET-AFTERBURNER FUELS

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NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

WASHINGTON

October 10, 1956

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METHODS FOR CALCULATING THRUST AUGMENTATION AND LIQUID CONSUMPTION FOR VARIOUS TURBOJET-AFTERBURNER FUELS

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SUMMARY

Methods are presented for calculating net thrust using air specific-impulse data for various fuels. Nomographic solutions are given to adapt the methods to turbojet-afterburner calculations. These nomographs can be used to compute net thrusts obtained by expanding exhaust gases to either a Mach number of 1.0 or the ambient static pressure at the nozzle exit. Thermodynamic data for several fuels are also presented.

INTRODUCTION

Turbojet-propelled aircraft often require thrust augmentation for takeoff, maneuverability, and supersonic flight. Afterburning with high-energy fuels rather than hydrocarbons may produce greater augmented thrust with lower total fuel flows. In order to predict and compare performances of turbojet-afterburner fuels, calculations must be made with theoretical and experimental data.

This report gives methods for computing net thrusts and total liquid flows for fuels burned in jet engines having various component efficiencies and operating at various flight conditions. Air specific-impulse data, which are available for many new fuels, are used in these methods to account for the energy, mass, and nature of combustion products. Nomographic solutions are presented for calculating net thrusts for expansion of exhaust gases to either Mach number 1.0 or ambient static pressure at the nozzle exit. The ranges of variables for the nomographs were selected for calculations of turbojet-afterburner performance.

The calculation methods for expansion of combustion products to a Mach number of 1.0 are practical ones for many present and future turbojet afterburners having variable-area convergent exhaust nozzles. However, turbojet engines that produce high pressure ratios will require variable-area convergent-divergent nozzles to yield best performances. In these cases, the maximum net thrust would be obtained if combustion

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products were completely expanded, but for many engines the nozzle weight, drag, and complexity would make complete expansion impractical.

Then, the nomographic methods can be used to predict or bracket net thrust for all turbojet afterburners. Two examples are given to show calculation procedures and the differences in net thrusts computed for expansion of exhaust gases to unit Mach number and to ambient static pressure. Thermodynamic data for ideal combustion of several fuels are presented in graphical form for convenient use with these calculation methods.

ANALYTICAL METHODS

Air specific impulse is used as a variable in the calculation methods of this report. For ideal, adiabatic combustion of a fuel, air specific impulse is defined in terms of the stream thrust obtained by expanding the combustion products adiabatically to a Mach number of 1.0. However, air specific impulse can also be expressed as a function of the following variables: (1) fuel type, (2) equivalence ratio, (3) total temperature, and (4) total pressure, all at the point considered, and (5) inlet-air temperature. Then, for frozen-composition, adiabatic expansion, air specific impulse has a constant value at all points in the stream, regardless of Mach number. Equation (B1) confirms this.

Therefore, air specific impulse is used in the equations of the calculation methods to represent the stream thrust, energy, mass, and nature of combustion products. The equations are general, but application to turbojet-afterburner calculations is stressed. All symbols and complete derivations for the equations in this section are given in appendixes A and B, respectively.

Net Thrust

Air specific impulse was used to convert the general expression for net thrust,

$$F_n = m_{10}V_{10} - m_0V_0 + A_{10}(p_{10} - p_0) \qquad (1)$$

to the following expression:

$$\frac{F_n}{w_a} = S_{a,10} f(M_{10}, p_{10}, p_0, \gamma_{10}) - \frac{V_0}{g} = \frac{S_{a,10}}{M_{10}\sqrt{2(1+\gamma_{10})\left(1 + \frac{\gamma_{10}-1}{2} M_{10}^2\right)}}$$

$$\left[1 + \gamma_{10} M_{10}^2 - \left(1 + \frac{\gamma_{10}-1}{2} M_{10}^2\right)^{\frac{\gamma_{10}}{\gamma_{10}-1}} \left(\frac{p_0}{p_{10}}\right)\right] - \frac{V_0}{g} \tag{2}$$

If the nozzle-exit Mach number is 1.0, equation (2) reduces to

$$\frac{F_n}{w_a} = S_{a,10} f(p_{10}, p_0, \gamma_{10}) - \frac{V_0}{g} = S_{a,10}\left[1 - f(\gamma_{10})\left(\frac{p_0}{p_{10}}\right)\right] - \frac{V_0}{g}$$

$$= S_{a,10}\left[1 - \frac{(1+\gamma_{10})^{\frac{1}{\gamma_{10}-1}}}{(2)^{\frac{\gamma_{10}}{\gamma_{10}-1}}}\left(\frac{p_0}{p_{10}}\right)\right] - \frac{V_0}{g} \tag{3}$$

This is the basic equation used in the approximate (figs. 1 and 2) and exact (figs. 3 to 6) nomographic solutions for turbojet-afterburner net thrust produced by expansion of exhaust products through a choked convergent nozzle.

The function $f(\gamma_{10})$ is practically constant, varying from 0.793 at $\gamma_{10} = 1.345$ to 0.803 at $\gamma_{10} = 1.225$. If $f(\gamma_{10}) = 0.8$ is used and afterburner losses are neglected, a simplified form of equation (3) results. This approximate expression and its limitations are discussed in appendix C.

For complete expansion of combustion products,

$$p_{10} = p_0$$

and the equation for net thrust becomes

$$\frac{F_n}{w_a} = S_{a,10}f(P_{10},p_0,\gamma_{10}) - \frac{V_0}{g} = S_{a,10}\sqrt{\frac{\gamma_{10}^2}{\gamma_{10}^2 - 1}\left[1 - \left(\frac{p_0}{P_{10}}\right)^{\frac{\gamma_{10}-1}{\gamma_{10}}}\right]} - \frac{V_0}{g} \tag{4}$$

Figure 7 is the nomographic solution for equation (4); figures 3, 4, 5, and 7 comprise the exact nomographic method for computing the turbojet-afterburner net thrust obtained when combustion products are completely expanded.

Equations (2) to (4) can be used to compute net thrust for various afterburner fuels if the ratio of ambient static pressure to nozzle-exit total pressure p_0/P_{10} is known. For turbojet-afterburner problems, the ratio of afterburner-inlet total pressure to ambient static pressure P_5/p_0 is generally known. Then, if the total-pressure ratios across the flameholder $(P_6/P_5)_F$, the combustion zone $(P_9/P_6)_M$, and the nozzle $(P_{10}/P_9)_N$ are computed, p_0/P_{10} can be found from the following identity:

$$\frac{p_0}{P_{10}} = \frac{p_0}{P_5}\left(\frac{P_5}{P_6}\right)_F\left(\frac{P_6}{P_9}\right)_M\left(\frac{P_9}{P_{10}}\right)_N \tag{5}$$

Flameholder Total-Pressure Ratio

The flameholder total-pressure ratio $(P_6/P_5)_F$ and the combustion-zone-inlet Mach number M_6 can be calculated from the following equation:

$$\left(\frac{P_6}{P_5}\right)_F = \frac{M_5}{M_6}\left(\frac{1 + \frac{\gamma_5 - 1}{2}M_6^2}{1 + \frac{\gamma_5 - 1}{2}M_5^2}\right)^{\frac{\gamma_5+1}{2(\gamma_5-1)}} = 1 - C_D\left[\frac{\gamma_5 M_5^2}{2\left(1 + \frac{\gamma_5 - 1}{2}M_5^2\right)^{\frac{\gamma_5}{\gamma_5-1}}}\right] \tag{6}$$

In order to use equation (6), the afterburner-inlet Mach number M_5, specific-heats ratio γ_5, and flameholder drag coefficient $(C_D = \Delta P/q)$ must be known, and the duct area, stream energy, mass, and composition must be constant across the flameholder.

An approximate solution for equation (6) is given by line A of figure 1. Figure 3 is an exact nomographic solution for $(P_6/P_5)_F$ and M_6.

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Combustion-Zone Total-Pressure Ratio

The combustion-zone total-pressure ratio $(P_9/P_6)_M$ is given as a function of values, upstream and downstream of the combustion zone, of air specific impulse ($S_{a,6}$ and $S_{a,9}$), Mach number (M_6 and M_9), and ratio of specific heats (γ_6 and γ_9). The following equations are valid for a constant-area duct:

$$\frac{M_9\sqrt{1 + \frac{\gamma_9 - 1}{2} M_9^2}}{1 + \gamma_9 M_9^2} = \frac{M_6\sqrt{1 + \frac{\gamma_6 - 1}{2} M_6^2}}{1 + \gamma_6 M_6^2} \sqrt{\frac{1 + \gamma_6}{1 + \gamma_9}} \left(\frac{S_{a,9}}{S_{a,6}}\right) \qquad (7)$$

$$\left(\frac{P_9}{P_6}\right)_M = \frac{(1 + \gamma_6 M_6^2)\left(1 + \frac{\gamma_9 - 1}{2} M_9^2\right)^{\frac{\gamma_9}{\gamma_9 - 1}}}{(1 + \gamma_9 M_9^2)\left(1 + \frac{\gamma_6 - 1}{2} M_6^2\right)^{\frac{\gamma_6}{\gamma_6 - 1}}} \qquad (8)$$

The use of air specific impulse in equation (7) eliminates separate treatments of energy and mass addition and trial-and-error methods across the combustion zone. Approximate (fig. 1 and lines A to C of fig. 2) and exact (fig. 4 and lines A to C of fig. 5) nomographic solutions for equations (7) and (8) are presented.

Nozzle Total-Pressure Ratio

The nozzle total-pressure ratio $(P_{10}/P_9)_N$, the velocity coefficient, and a kinetic-energy coefficient are accepted conventions that express exhaust-nozzle losses. For convenience, the total-pressure-ratio method was selected as a means of treating nozzle losses in the nomographic solutions. The ratio $(P_{10}/P_9)_N$ is assigned in the approximate method. In the exact nomographic solutions, $(P_{10}/P_9)_N$ can be entered on line F of figure 5. This figure gives the solution for equation (5).

Assumptions for Nomographic Methods

The three nomographic solutions for net thrust depend on the following assumptions: 3991

(1) The mass, energy, and nature of combustion products are represented at any point in the stream by the equivalent air specific-impulse value.

(2) Values of air specific impulse and ratio of specific heats are constant across the flameholder and also from the end of the combustion zone to the exit of the exhaust nozzle.

(3) The afterburner cross-sectional area is constant from the inlet to the exhaust-nozzle inlet.

(4) All energy and mass additions occur with negligible friction downstream of the flameholder and upstream of the exhaust nozzle. The complete afterburner friction loss is represented by flameholder and exhaust-nozzle total-pressure ratios.

The exact nomographic method of figures 3 to 6 computes the net thrust produced by expanding exhaust gases to a Mach number of 1.0 at the exit of a convergent nozzle. The net thrust for expansion of combustion products to the ambient static pressure at the exhaust-nozzle exit can be calculated with the exact nomographic method of figures 3, 4, 5, and 7.

Figures 1 and 2 are an approximate nomographic method for computing the net thrust obtained by expanding exhaust products to unit Mach number at the nozzle exit. This method can be used for quick, approximate comparisons of performances of afterburner fuels. The approximate nomographic solution depends on the assumptions for the exact methods, and it is also restricted to the following assumptions:

(1) The average value for the afterburner-inlet ratio of specific heats is 1.325.

(2) The average afterburner-exit specific-heats ratio equals 1.275.

(3) The total-pressure ratio across the flameholder is 0.95.

(4) The product of flameholder and exhaust-nozzle total-pressure ratios equals 0.92.

Turbojet-Afterburner Fuel Performance

The net thrust of an entire engine can be calculated with these nomographs by using ideal values of air specific impulse and specific-heats ratio corresponding to combustion at the over-all equivalence ratio
3991 of a blend of the primary and afterburner fuels. The over-all combustion process is assumed to occur at the afterburner combustion pressure with air at the turbojet-engine-inlet temperature.

Afterburner performance can be compared using the following conventions:

Augmented net thrust ratio:

$$\frac{F_{n,eab}}{F_{n,e}} = \frac{\left[S_{a,10}\, f(P_{10}, p_0, \gamma_{10}) - \frac{V_0}{g}\right]_{eab}}{\left[S_{a,10}\, f(P_{10}, p_0, \gamma_{10}) - \frac{V_0}{g}\right]_{e}} \tag{9}$$

Augmented liquid ratio:

$$\frac{w_{f,eab}}{w_{f,e}} = \frac{\left[\varphi\left(\frac{w_f}{w_a}\right)_s\right]_{eab}}{\left[\varphi\left(\frac{w_f}{w_a}\right)_s\right]_{e}} \tag{10}$$

Specific fuel consumption:

$$sfc = \frac{3600}{S_f} = \frac{3600 w_f}{F_n} = \frac{3600\left[\varphi\left(\frac{w_f}{w_a}\right)_s\right]_{eab}}{\left[S_{a,10}\, f(P_{10}, p_0, \gamma_{10}) - \frac{V_0}{g}\right]_{eab}} \tag{11}$$

Effects of combustion efficiency can be introduced in basic engine and afterburner calculations with the following expression:

$$\eta_B = \frac{\varphi_{id}}{\varphi_{ac}} \tag{12}$$

where φ_{id} and φ_{ac} are ideal and actual equivalence ratios for a given value of air specific impulse. This is an approximate method, which is good for high combustion efficiencies.

DISCUSSION

Several methods are presented for computing jet-engine net thrust. In these methods air specific-impulse data for various fuels are used to account for the energy, mass, and nature of combustion products. Application of the methods to turbojet-afterburner calculations is stressed.

Equation (2) is a general expression for net thrust. An approximate equation for net thrust of exhaust gases expanding through a choked convergent nozzle, neglecting afterburner losses and variation of specific-heats ratio, is presented in appendix C. An approximate nomographic solution for expansion of combustion products to Mach number 1.0 at the exhaust-nozzle exit is shown in figures 1 and 2. This method depends on assumed values of flameholder and exhaust-nozzle total-pressure ratios and ratios of specific heats.

An exact nomographic method for calculating net thrust obtained by expanding exhaust gases to a Mach number of 1.0 at the nozzle exit is given in figures 3 to 6. This solution includes effects of combustion products for various fuels and of afterburner component efficiencies.

Figures 3, 4, 5, and 7 are an exact nomographic method for computing net thrust for expansion of exhaust products to ambient static pressure at the nozzle exit. These two exact nomographic solutions (figs. 3 to 6 and figs. 3, 4, 5, and 7) are identical with the exception of the function $f(P_{10},p_0,\gamma_{10})$ in the net-thrust equation (compare eqs. (3) and (4)). This difference occurs between lines C and E of figures 6 and 7.

Examples

Two examples are given in detail in appendix C to show procedures and differences in results obtained with the three nomographic methods. For a turbojet engine operating at conditions indicated by the following: (1) an altitude of 30,000 feet, (2) a flight Mach number of 0.81, and (3) a ratio of afterburner-inlet total pressure to ambient static pressure of 3.98, the following results were computed for JP-4 fuel used in the primary engine and afterburner:

	Calculation method					
	Approximate (figs. 1 and 2): expansion to $M_{10} = 1.0$		Exact (figs. 3 to 6): expansion to $M_{10} = 1.0$		Exact (figs. 3, 4, 5, and 7): expansion to $p_{10} = p_0$	
	Afterburner equivalence ratio, φ_{ab}					
	0	1.0	0	1.0	0	1.0
$\frac{F_n}{w_a}$, $\frac{\text{lb thrust}}{\text{lb air/sec}}$	52.7	99.2	52.8	98.4	54.3	100.8
sfc, $\frac{\text{lb fuel/hr}}{\text{lb thrust}}$	1.14	2.46	1.14	2.48	1.11	2.42
$\frac{F_{n,eab}}{F_{n,e}}$		1.88		1.86		1.86
$\frac{w_{f,eab}}{w_{f,e}}$		4.05		4.05		4.05

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The results give good agreement for this particular example. However, when the approximate method for expansion to Mach number 1.0 is used for specific afterburner problems, the assumptions of the solution should be checked carefully against the actual component efficiencies. The agreement of results computed for complete expansion with those for a nozzle-exit Mach number of 1.0 stems from the low ratio of afterburner-inlet total pressure to ambient static pressure.

The second example was selected to show the differences in net thrusts calculated with the two exact nomographic methods for a turbojet engine operating with a high pressure ratio. Values of all of the variables are given in appendix C; the following values indicate the operating conditions of the turbojet engine and afterburner using JP-4 fuel: (1) a flight altitude of 50,000 feet, (2) a flight Mach number of 2.5, and (3) a ratio of afterburner-inlet total pressure to ambient static pressure of 19.04. The following results were computed:

	Calculation method			
	Exact (figs. 3 to 6): expansion to $M_{10} = 1.0$		Exact (figs. 3, 4, 5, and 7): expansion to $p_{10} = p_0$	
	Afterburner equivalence ratio, φ_{ab}			
	0	1.0	0	1.0
$\frac{F_n}{w_a}$, $\frac{\text{lb thrust}}{\text{lb air/sec}}$	29.6	89.4	43.5	112.8
sfc, $\frac{\text{lb fuel/hr}}{\text{lb thrust}}$	1.96	2.73	1.33	2.16
$\frac{F_{n,eab}}{F_{n,e}}$		3.02		2.59
$\frac{w_{f,eab}}{w_{f,e}}$		4.20		4.20

The expansion of exhaust gases to ambient static pressure rather than to Mach number 1.0 produced 47- and 26-percent increases in net thrust for afterburner equivalence ratios of 0 and 1.0, respectively. The corresponding decreases in specific fuel consumption are 32 and 21 percent. However, the advantages would be attended by increases in nozzle weight, drag, and complexity. The best performance would be obtained using a compromise based on these factors and net thrust.

Figures

The meanings and uses of the figures are discussed briefly in the following outline.

Figure 1. - Figures 1 and 2 comprise the approximate nomographic method for determining turbojet-afterburner net thrust for expansion of combustion products to a nozzle-exit Mach number of 1.0. Simplifying assumptions in this method are $\gamma_5 = \gamma_6 = 1.325$; $\gamma_9 = \gamma_{10} = 1.275$; $(P_6/P_5)_F = 0.95$; and $(P_6/P_5)_F\ (P_{10}/P_9)_N = 0.92$. Figure 1 computes the functions of specific-heats ratio and Mach number, upstream $f(\gamma_6,M_6)_M$ and downstream $f(\gamma_9,M_9)_M$ of the afterburner combustion zone, required

to calculate the combustion total-pressure ratio $(P_9/P_6)_M$. The value of afterburner combustion-zone-exit (exhaust-nozzle-inlet) Mach number M_9 is computed while determining $f(\gamma_9,M_9)_M$.

Values of afterburner-inlet Mach number M_5 and of air specific impulse upstream $(S_{a,6} = S_{a,5})$ and downstream $(S_{a,9} = S_{a,10})$ of the afterburner combustion zone are located on lines A, B, and D, respectively, of figure 1. The straight lines are drawn in the order indicated by the arrowheads and number sequence. Then the values of $f(\gamma_6,M_6)_M$, $f(\gamma_9,M_9)_M$, and M_9 can be read from lines A and E.

<u>Figure 2</u>. - Figure 2 calculates the value of net thrust divided by the air-flow rate F_n/w_a for expansion of gases to Mach number 1.0. In figure 2, $f(\gamma_6,M_6)_M$ and $f(\gamma_9,M_9)_M$ from figure 1, ratio of afterburner-inlet total pressure to ambient static pressure P_5/p_0, and air specific impulse at the exhaust-nozzle throat $(S_{a,10} = S_{a,9}$ for frozen-composition expansion) are located on lines A, B, D, and F, respectively. For the nonafterburning case (but with afterburner in place) $f(\gamma_6,M_6)_M = f(\gamma_9,M_9)_M$, $(P_9/P_6)_M = 1.0$, and $S_{a,5} = S_{a,6} = S_{a,9} = S_{a,10}$.

With the straight lines drawn as shown, the value of $S_{a,10}\left[1 - f(\gamma_{10})\dfrac{p_0}{P_{10}}\right]$ is fixed on line G. The flight altitude and Mach number M_0 values (which remain unchanged for the afterburning and nonafterburning cases when augmented thrust ratios are computed) are placed on lines H and K, respectively. Then the appropriate straight lines are drawn beginning with the altitude on H, passing through M_0 on K, and intersecting line L; then connecting the intersection on L with $S_{a,10}\left[1 - f(\gamma_{10})\dfrac{p_0}{P_{10}}\right]$ on G. Finally, the value of F_n/w_a can be read from line I (if the high-range scales were used on lines F and G) or line J (if low-range scales were used).

<u>Figure 3</u>. - Figures 3 to 6 are the exact nomographic method for determining turbojet-afterburner net thrust produced by expanding exhaust products to a Mach number of 1.0 at the nozzle exit. Figures 3, 4, 5, and 7 are the exact nomographic solution for turbojet-afterburner net thrust obtained with complete expansion of combustion products. Figure 3 yields values of total-pressure ratio across the flameholder $(P_6/P_5)_F$ and Mach number downstream of the flameholder M_6 (combustion-zone-inlet Mach number).

The ratio of specific heats at the afterburner inlet γ_5 and the flameholder drag coefficient ($C_D = \Delta P/q$) are located on lines A and D. The value of afterburner-inlet Mach number M_5 is placed on lines B and F. With the straight lines drawn as indicated, the value of $(P_6/P_5)_F$ is given on line E; then M_6 can be read from line G.

Figure 4. - Figure 4 computes values of $f(\gamma_6,M_6)_M$, $f(\gamma_9,M_9)_M$, and M_9 in a manner similar to that of figure 1, but without the assumptions of values of flameholder total-pressure ratio and inlet and exit ratios of specific heats that were made to simplify figure 1. In figure 4 the values of $\gamma_6 = \gamma_5$, M_6, $S_{a,6} = S_{a,5}$, $S_{a,9} = S_{a,10}$, and $\gamma_9 = \gamma_{10}$ are located on lines A, B, E, G, and K, respectively. The straight lines are drawn as shown, where all lines are constructed without requiring the points for the values of $f(\gamma_6,M_6)_M$ and $f(\gamma_9,M_9)_M$ of lines D and H, respectively. These functions are obtained by extending the first and last straight lines of figure 4 to intersect lines D and H, respectively. 3991

If it is desired, the afterburner combustion-zone-exit Mach number M_9 can be read from line J. Values of $f(\gamma_6,M_6)_M$ and $f(\gamma_9,M_9)_M$, which are used in figure 5 to compute $(P_9/P_6)_M$, can be obtained from lines D and H.

Figure 5. - Figure 5 yields either the value of exhaust-nozzle-throat total pressure P_{10} or the ratio of exhaust-nozzle-throat total pressure to ambient static pressure P_{10}/p_0. The functions $f(\gamma_6,M_6)_M$ and $f(\gamma_9,M_9)_M$ from figure 4, $(P_6/P_5)_F$ from figure 3, the assumed nozzle total-pressure ratio $(P_{10}/P_9)_N$, and either the afterburner-inlet total pressure P_5 or the ratio of afterburner-inlet total pressure to ambient static pressure P_5/p_0 are located on lines A, B, D, F, and H, respectively, of figure 5. Again, for the nonafterburning case, $f(\gamma_6,M_6)_M = f(\gamma_9,M_9)_M$, and $(P_9/P_6)_M = 1.0$.

With the straight lines constructed as shown, either P_{10} or P_{10}/p_0 can be read from line I.

Figures 6. - Figures 6 are the same nomograph showing calculations for the two examples. This nomograph yields the value of F_n/w_a for expansion of exhaust gases to a nozzle-exit Mach number of 1.0 as computed by the exact nomographic method. Figures 6 are identical with figure 2 for lines E to L; therefore, this part of figures 6 is not described here.

If P_{10} was computed with figure 5, its value is located on line B of figures 6. Then either the altitude or the ambient pressure is placed on line A. A straight line drawn through these two points gives the value of p_0/P_{10} on line C. If P_{10}/p_0 was calculated with figure 5, its value is entered on line C, and lines A and B are not used. Then, $\gamma_{10} = \gamma_9$ is located on line D; the short length of line D, representing a range of γ_{10} from 1.21 to 1.35, indicates the small effect of γ_{10} variation on the value of $f(\gamma_{10})$ in equation (3). A straight line drawn through p_0/P_{10} on C and γ_{10} on D yields the value of $\left[1 - f(\gamma_{10}) \frac{p_0}{P_{10}}\right]$ at the intersection with line E.

From this point on the procedures are identical with those for figure 2. The value for F_n/w_a can be read from either line I or line J, depending on whether high or low scales were used on lines F and G.

Figures 7. - Figures 7 show the exact nomographic calculations, for the two examples, of F_n/w_a produced by complete expansion of exhaust products. These two nomographs are identical and differ from figures 6 only between lines C and E. The points of similarity are not repeated.

After p_0/P_{10} is determined on line C of figures 7, its value is located on line C', which crosses line E. Then the value of γ_{10} is placed on line D, and a straight line is drawn through the points on lines D and C' to intersect line E at the value of $f(P_{10}, p_0, \gamma_{10})$. From this point, procedures are identical with those of figures 2 and 6.

The augmented net thrust ratio can be obtained by dividing F_n/w_a for the turbojet engine with afterburning by F_n/w_a for the turbojet engine without afterburning (eq. (9)). Thermodynamic data for combustion of various fuels must be used with the nomographs and equations (9) to (11) to calculate augmented net thrust ratio, augmented liquid ratio, and specific fuel consumption. Some of these data were obtained from references 1 to 3.

However, to simplify turbojet-afterburner calculations, these and new thermodynamic data were collected and are presented in graphical forms convenient for afterburner analyses.

Figure 8. - In figure 8 afterburner equivalence ratio $\varphi_{ac,ab}$ is given as a function of ideal primary-combustor equivalence ratio $\varphi_{id,e}$ $(= \eta_{B,e}\, \varphi_{ac,e})$ and over-all equivalence ratio $\varphi_{ac,eab}$. The

assumption for this figure is that all unburned fuel entering the afterburner is charged to the afterburner fuel quantity.

Figure 9. - Variations of over-all stoichiometric fuel-air ratio $(w_f/w_a)_{s,eab}$ with primary-combustor equivalence ratio using JP-4 primary fuel $(\varphi_{ac,e})$ and with over-all equivalence ratio are given in figure 9. These values are given for a 60 percent magnesium slurry in JP-4, pentaborane, and JP-4 afterburner fuels. The value of $(w_f/w_a)_{s,eab}$ is required to solve equations (10) and (11). 3991

Figure 10. - Figure 10 shows the variation of weight fraction of non-JP-4 fuel in the over-all fuel mixture with actual primary-combustor equivalence ratio using JP-4 primary fuel and with over-all equivalence ratio. The afterburner fuels for which these data are given are a 60 percent magnesium slurry in JP-4 fuel and pentaborane.

Figures 8, 9, and 10 give relations to obtain values of equivalence ratio, stoichiometric fuel-air ratio, and weight fraction of non-JP-4 fuel for the over-all fuel mixture from similar variables for the separate fuels in the afterburner and in the primary combustors (JP-4 fuel). This is done because the ideal thermodynamic properties of the exhaust gases leaving the afterburner can be treated as those that would result from burning a blend of the primary and afterburner fuels with air at the engine-inlet total temperature (ref. 4). The thermodynamic properties of the afterburner exhaust gases also depend on the static pressure at that point.

Figure 11. - Figure 11 presents variations of combustion temperature with equivalence ratio and inlet-air temperature for JP-4 fuel combustion at a pressure of 2 atmospheres. This figure is used to convert given turbine-outlet temperature data to afterburner-inlet equivalence ratio and air specific-impulse values required for the nomographic solutions. The data for a pressure of 2 atmospheres can be used without correction, because the pressure effect is negligible at low values of temperature (or equivalence ratio) such as those at the turbine outlet (ref. 3).

Figure 12. - Variations of air specific impulse for slurries with varying concentrations of magnesium in JP-4 fuel are given as functions of equivalence ratio and inlet air temperatures for combustion at 2 atmospheres in figure 12. Corrections for pressures other than 2 atmospheres are given in a later figure. The data for JP-4 fuel alone (zero percent magnesium) can be used for afterburner-inlet $(S_{a,5} = S_{a,6})$ and -outlet $(S_{a,9} = S_{a,10})$ air specific-impulse values, when JP-4 fuel is being used in the afterburner. When a 60 percent magnesium slurry in JP-4 fuel is used as afterburner fuel, exit air specific-impulse values $(S_{a,9} = S_{a,10})$ can be obtained from figure 12 by interpolation. This interpolation requires over-all values of equivalence ratio and weight fraction of magnesium taken from figures 8 and 10, respectively.

Figure 13. - Figure 13 shows variations of air specific impulse with equivalence ratio and inlet-air temperature for combustion at 2 atmospheres of blends of several concentrations of pentaborane in JP-4 fuel. The figure can be used, in a manner similar to that of figure 12, when pentaborane afterburner fuel is analyzed.

Ideal values of air specific impulse and temperature for combustion at 2 atmospheres are obtained from figures 11 to 13 using over-all values of equivalence ratio and fraction of non-JP-4 fuel from figures 8 and 10,
3991 respectively.

Figure 14. - Air specific impulse corrected for combustion pressure is given in figure 14 as a function of air specific impulse for a combustion pressure of 2 atmospheres. In general, no pressure corrections are required for the relatively low air specific-impulse values corresponding to afterburner-inlet conditions (ref. 3).

The data given in figures 8 to 14 are sufficient for calculations using the approximate nomographic method (figs. 1 and 2). However, if the exact nomographic methods are used, afterburner-inlet and -exit values of specific-heats ratio ($\gamma_5 = \gamma_6$ and $\gamma_9 = \gamma_{10}$, respectively) are required. Because the major variable of the calculation method is air specific impulse, it is convenient to define the ratio of specific heats as a function of air specific impulse.

Figure 15. - Figure 15 gives variations of specific-heats ratio for ideal combustion products of JP-4 fuel and of slurries of 25 and 50 percent magnesium in JP-4 fuel. For any one of these compositions the value of specific-heats ratio is defined by the intersection of any two of the lines for constant air specific impulse, equivalence ratio, and temperature. Ratios of specific heats for over-all magnesium weight fractions below 50 percent can be obtained by interpolation between values for 0 and 25 percent or 25 and 50 percent magnesium.

Figure 16. - Variations of specific-heats ratio with air specific impulse and pentaborane concentration (for ideal exhaust products at the ideal combustion temperatures) for blends of pentaborane and JP-4 fuel are given in figure 16. The data in figure 16 are for combustion with a 100° F inlet-air temperature at a combustion pressure of 2 atmospheres.

CONCLUDING REMARKS

Methods were derived to compute jet-engine net thrust using air specific-impulse data for various fuels. Solutions for the methods treating expansion of combustion products to a Mach number of 1.0 and to the ambient static pressure at the exhaust-nozzle exit are presented in nomographic form. The ranges of variables for the nomographs were selected to apply to turbojet-afterburner calculations.

Two examples are given to show procedures and differences in net thrusts computed using the nomographic methods. For a turbojet engine operating at (1) an altitude of 50,000 feet, (2) a flight Mach number of 2.5, (3) a ratio of afterburner-inlet total pressure to ambient static pressure of 19.04, and (4) with JP-4 fuel burned in primary engine and afterburner, expansion of exhaust gases to ambient static pressure, rather than to a Mach number of 1.0 at the nozzle exit, gave the following computed changes:

Afterburner equivalence ratio, φ_{ab}	Percent increase in net thrust	Percent decrease in specific fuel consumption
0	47	32
1.0	26	21

These values do not indicate effects of nozzle weight, drag, and complexity.

Thermodynamic data for ideal combustion of several fuels were collected or calculated and are presented in graphical form for convenient use in turbojet-afterburner calculations.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, January 27, 1956

APPENDIX A

SYMBOLS

A	area, sq ft
a	sonic velocity, ft/sec
C_D	drag coefficient, $\frac{\Delta P}{q} = \frac{\Delta P}{\frac{1}{2g}\rho V^2}$
F_n	net thrust, lb
f	function
g	gravitational constant, 32.17 ft/sec^2
M	Mach number
m	mass rate, slugs/sec
n	point number
P	total pressure, lb/sq ft abs
p	static pressure, lb/sq ft abs
q	dynamic pressure, lb/sq ft
R	gas constant, ft-lb/(lb gas)($^\circ$R)
S_a	air specific impulse, lb stream thrust/(lb air/sec) at $M = 1$
S_f	fuel specific impulse, lb thrust/(lb fuel/sec)
sfc	specific fuel consumption, (lb fuel/hr)/lb thrust
T	total temperature, $^\circ$R
t	static temperature, $^\circ$R
V	velocity, ft/sec
w	fluid weight rate, lb/sec

x weight fraction of condensed phase

γ ratio of specific heats

η_B combustion efficiency

ρ density, lb/cu ft

φ equivalence ratio 3991

Subscripts:

a air

ab afterburner

ac actual value

e engine without afterburning (using afterburner as tailpipe)

eab engine with afterburning

F friction (or flameholder)

f fuel

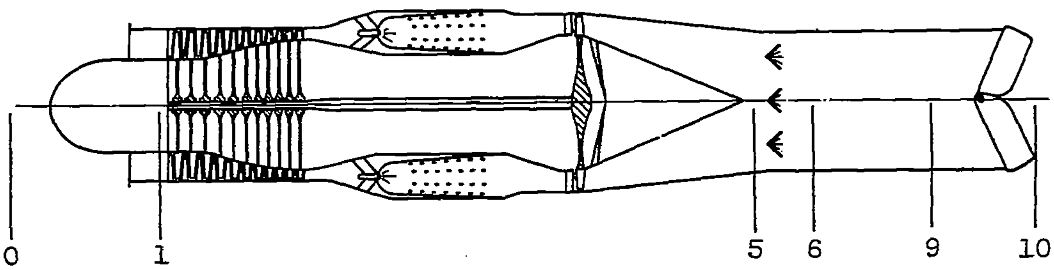
id ideal value

M momentum (heat addition)

N nozzle

s stoichiometric

t total



0 ambient location

1 at compressor inlet

5 at afterburner inlet

6 downstream of afterburner flameholder (afterburner combustion-zone inlet)

9 at nozzle inlet (afterburner combustion-zone exit)

10 at nozzle exit

APPENDIX B

DERIVATIONS

The following are derivations of the final expressions (eqs. (2) to (8)) used to solve the afterburner performance problem:

Air Specific Impulse

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By definition,

$$S_a = \frac{mV + pA}{w_a}$$

where $V = a$ or $M = 1.0$. Then

$$S_a = \frac{pA}{w_a}(1 + \gamma) = \frac{pA}{w_t}\left(1 + \frac{w_f}{w_a}\right)(1 + \gamma)$$

But

$$\frac{pA}{w_t} = \frac{1}{M}\sqrt{\frac{(1 - x)Rt}{\gamma g}} = \frac{1}{M}\sqrt{\frac{(1 - x)RT}{\gamma g\left(1 + \frac{\gamma - 1}{2}M^2\right)}} = \left(\sqrt{\frac{2(1 - x)RT}{\gamma(1 + \gamma)g}}\right)_{M=1.0}$$

where R is equal to the universal gas constant divided by the average molecular weight of the gases alone. This is equivalent to considering solid or liquid phases to possess zero volume or infinite molecular weight. Therefore,

$$S_a = \left(1 + \frac{w_f}{w_a}\right)(1 + \gamma)\sqrt{\frac{2(1 - x)RT}{\gamma(1 + \gamma)g}} = \left(1 + \frac{w_f}{w_a}\right)\sqrt{\frac{2(1 + \gamma)(1 - x)RT}{\gamma g}} \tag{B1}$$

Net Thrust

By definition,

$$F_n = m_{10}V_{10} - m_0V_0 + A_{10}(p_{10} - p_0) \tag{1}$$

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$$\frac{F_n}{w_a} = \frac{m_{10}V_{10} + p_{10}A_{10}}{w_a} - \frac{p_0A_{10}}{w_a} - \frac{m_0V_0}{w_a} = \frac{p_{10}A_{10}}{w_a}\left(1 + \gamma_{10}M_{10}^2 - \frac{p_0}{p_{10}}\right) - \frac{V_0}{g}$$

$$= \left(1 + \frac{w_{f,10}}{w_{a,10}}\right)\frac{1}{M_{10}}\sqrt{\frac{(1-x_{10})R_{10}T_{10}}{\gamma_{10}g\left(1 + \frac{\gamma_{10}-1}{2}M_{10}^2\right)}}\left[1 + \gamma_{10}M_{10}^2 - \left(1 + \frac{\gamma_{10}-1}{2}M_{10}^2\right)^{\frac{\gamma_{10}}{\gamma_{10}-1}}\left(\frac{p_0}{p_{10}}\right)\right] - \frac{V_0}{g}$$

Then, the general expression for net thrust at any nozzle-exit Mach number is

$$\frac{F_n}{w_a} = \frac{S_{a,10}}{M_{10}\sqrt{2(1+\gamma_{10})\left(1 + \frac{\gamma_{10}-1}{2}M_{10}^2\right)}}\left[1+\gamma_{10}M_{10}^2 - \left(1 + \frac{\gamma_{10}-1}{2}M_{10}^2\right)^{\frac{\gamma_{10}}{\gamma_{10}-1}}\frac{p_0}{p_{10}}\right] - \frac{V_0}{g} \tag{2}$$

Assuming $M_{10} = 1.0$,

$$\frac{F_n}{w_a} = S_{a,10} - \frac{p_0A_{10}}{w_a} - \frac{V_0}{g}$$

$$= S_{a,10} - \frac{p_0w_{10}}{p_{10}w_a}\sqrt{\frac{(1 - x_{10})R_{10}t_{10}}{\gamma_{10}g}} - \frac{V_0}{g}$$

$$= S_{a,10} - \frac{p_0}{p_{10}}\left(\frac{1 + \gamma_{10}}{2}\right)^{\frac{\gamma_{10}}{\gamma_{10}-1}}\left(1 + \frac{w_f}{w_a}\right)\sqrt{\frac{2(1 - x_{10})R_{10}T_{10}}{\gamma_{10}(1 + \gamma_{10})\ g}} - \frac{V_0}{g}$$

Therefore,

$$\frac{F_n}{w_a} = S_{a,10}\left[1 - \frac{(1 + \gamma_{10})^{\frac{1}{\gamma_{10}-1}}}{(2)^{\frac{\gamma_{10}}{\gamma_{10}-1}}}\frac{p_0}{p_{10}}\right] - \frac{V_0}{g} \tag{3}$$

The general expression for net thrust reduces to the following equation:

$$\frac{F_n}{w_a} = \left(1 + \frac{w_f}{w_a}\right)\frac{V_{10}}{g} - \frac{V_0}{g} = \left(1 + \frac{w_f}{w_a}\right)\frac{a_{10}M_{10}}{g} - \frac{V_0}{g}$$

for complete expansion of exhaust gases, where

$$p_{10} = p_0$$

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The nozzle-exit Mach number is given by

$$M_{10} = \sqrt{\left[\left(\frac{P_{10}}{p_{10}}\right)^{\frac{\gamma_{10}-1}{\gamma_{10}}} - 1\right]\frac{2}{\gamma_{10} - 1}} = \sqrt{\left[\left(\frac{P_{10}}{p_0}\right)^{\frac{\gamma_{10}-1}{\gamma_{10}}} - 1\right]\frac{2}{\gamma_{10} - 1}}$$

while the corresponding sonic velocity is

$$a_{10} = \sqrt{\gamma_{10}g(1-x_{10})R_{10}t_{10}} = \sqrt{\frac{\gamma_{10}g(1-x_{10})R_{10}T_{10}}{1 + \frac{\gamma_{10} - 1}{2} M_{10}^2}} = \sqrt{\frac{\gamma_{10}g(1 - x_{10})R_{10}T_{10}}{\left(\frac{P_{10}}{p_{10}}\right)^{\frac{\gamma_{10}-1}{\gamma_{10}}}}}$$

Then,

$$\frac{F_n}{w_a} = \frac{\left(1 + \frac{w_f}{w_a}\right)}{g}\sqrt{\left[\left(\frac{P_{10}}{p_0}\right)^{\frac{\gamma_{10}-1}{\gamma_{10}}} - 1\right]\frac{2}{\gamma_{10} - 1}}\sqrt{\frac{\gamma_{10}g(1 - x_{10})R_{10}T_{10}}{\left(\frac{P_{10}}{p_0}\right)^{\frac{\gamma_{10}-1}{\gamma_{10}}}}} - \frac{V_0}{g}$$

$$= \left(1 + \frac{w_f}{w_a}\right)\sqrt{\frac{2(1+\gamma_{10})(1-x_{10})R_{10}T_{10}}{\gamma_{10}g}}\sqrt{\frac{\gamma_{10}^2}{\gamma_{10}^2-1}\left[\left(\frac{P_{10}}{p_0}\right)^{\frac{\gamma_{10}-1}{\gamma_{10}}} - 1\right]\left(\frac{p_0}{P_{10}}\right)^{\frac{\gamma_{10}-1}{\gamma_{10}}}} - \frac{V_0}{g}$$

$$\frac{F_n}{w_a} = S_{a,10}\sqrt{\frac{\gamma_{10}^2}{\gamma_{10}^2 - 1}\left[1 - \left(\frac{p_0}{P_{10}}\right)^{\frac{\gamma_{10}-1}{\gamma_{10}}}\right]} - \frac{V_0}{g} \tag{4}$$

[redacted]

Flameholder Total-Pressure Ratio

Assuming an adiabatic flow process with equal areas before and after the flameholder and the definition of flameholder drag coefficient,

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$$C_D = \frac{\Delta P}{q} = \frac{P_5 - P_6}{\frac{\rho_5 V_5^2}{2g}} = \frac{2\gamma_5 g R_5 t_5 (P_5 - P_6)}{\gamma_5 p_5 V_5^2}$$

$$= \left(1 - \frac{P_6}{P_5}\right)\left[\frac{2\left(1 + \frac{\gamma_5 - 1}{2} M_5^2\right)^{\frac{\gamma_5}{\gamma_5 - 1}}}{\gamma_5 M_5^2}\right]$$

But

$$\frac{P_6}{P_5} = \frac{p_6\left(1 + \frac{\gamma_6 - 1}{2} M_6^2\right)^{\frac{\gamma_6}{\gamma_6 - 1}}}{p_5\left(1 + \frac{\gamma_5 - 1}{2} M_5^2\right)^{\frac{\gamma_5}{\gamma_5 - 1}}} = \frac{w_6 M_5 A_5}{w_5 M_6 A_5}\sqrt{\frac{(1 - x_6) R_6 t_6 \gamma_5 g}{(1 - x_5) R_5 t_5 \gamma_6 g}}\left[\frac{\left(1 + \frac{\gamma_6 - 1}{2} M_6^2\right)^{\frac{\gamma_6}{\gamma_6 - 1}}}{\left(1 + \frac{\gamma_5 - 1}{2} M_5^2\right)^{\frac{\gamma_5}{\gamma_5 - 1}}}\right]$$

$$= \frac{M_5}{M_6}\sqrt{\frac{(1 - x_6) R_6 \gamma_5}{(1 - x_5) R_5 \gamma_6}}\left[\frac{\left(1 + \frac{\gamma_6 - 1}{2} M_6^2\right)^{\frac{\gamma_6 + 1}{2(\gamma_6 - 1)}}}{\left(1 + \frac{\gamma_5 - 1}{2} M_5^2\right)^{\frac{\gamma_5 + 1}{2(\gamma_5 - 1)}}}\right]$$

assuming $T_5 = T_6$. And, since over the range of conditions to be used, $R_5 = R_6$, $x_5 = x_6$, and $\gamma_5 = \gamma_6$ can be assumed,

$$\frac{P_6}{P_5} = \frac{M_5}{M_6}\left(\frac{1 + \frac{\gamma_5 - 1}{2} M_6^2}{1 + \frac{\gamma_5 - 1}{2} M_5^2}\right)^{\frac{\gamma_5+1}{2(\gamma_5-1)}}$$

Substituting in the equation for C_D yields the drag coefficient as a function of M_5 and M_6: 3991

$$C_D = \frac{\Delta P}{q} = \left[1 - \frac{M_5}{M_6}\left(\frac{1 + \frac{\gamma_5 - 1}{2} M_6^2}{1 + \frac{\gamma_5 - 1}{2} M_5^2}\right)^{\frac{\gamma_5+1}{2(\gamma_5-1)}}\right] \frac{2\left(1 + \frac{\gamma_5 - 1}{2} M_5^2\right)^{\frac{\gamma_5}{\gamma_5-1}}}{\gamma_5 M_5^2}$$

And, rearranging the equation for C_D,

$$\left(\frac{P_6}{P_5}\right)_F = \frac{M_5}{M_6}\left(\frac{1 + \frac{\gamma_5 - 1}{2} M_6^2}{1 + \frac{\gamma_5 - 1}{2} M_5^2}\right)^{\frac{\gamma_5+1}{2(\gamma_5-1)}} = 1 - C_D\left[\frac{\gamma_5 M_5^2}{2\left(1 + \frac{\gamma_5 - 1}{2} M_5^2\right)^{\frac{\gamma_5}{\gamma_5-1}}}\right] \tag{6}$$

Combustion-Zone-Exit (Nozzle-Inlet) Mach Number

Assuming constant-area frictionless flow in the combustion zone,

$$p_6(1 + \gamma_6 M_6^2) = p_9(1 + \gamma_9 M_9^2)$$

$$\frac{w_6}{M_6}\sqrt{\frac{(1 - x_6)R_6 t_6}{\gamma_6 g}}\,(1 + \gamma_6 M_6^2) = \frac{w_9}{M_9}\sqrt{\frac{(1 - x_9)R_9 t_9}{\gamma_9 g}}\,(1 + \gamma_9 M_9^2)$$

[redacted]

$$\left(1 + \frac{w_f}{w_a}\right)_6 \sqrt{\frac{(1 - x_6)R_6T_6}{\gamma_6\left(1 + \frac{\gamma_6 - 1}{2} M_6^2\right)g}} \left(\frac{1 + \gamma_6 M_6^2}{M_6}\right)$$

$$= \left(1 + \frac{w_f}{w_a}\right)_9 \sqrt{\frac{(1 - x_9)R_9T_9}{\gamma_9\left(1 + \frac{\gamma_9 - 1}{2} M_9^2\right)g}} \left(\frac{1 + \gamma_9 M_9^2}{M_9}\right)$$

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Then,

$$\frac{M_9\sqrt{1 + \frac{\gamma_9 - 1}{2} M_9^2}}{1 + \gamma_9 M_9^2} = \frac{M_6\sqrt{1 + \frac{\gamma_6 - 1}{2} M_6^2}}{1 + \gamma_6 M_6^2} \sqrt{\frac{1 + \gamma_6}{1 + \gamma_9}} \times$$

$$\left[\frac{1 + \left(\frac{w_f}{w_a}\right)_9}{1 + \left(\frac{w_f}{w_a}\right)_6}\right] \sqrt{\frac{2(1 + \gamma_9)(1 - x_9)R_9T_9}{\gamma_9 g} \, \frac{\gamma_6 g}{2(1 + \gamma_6)(1 - x_6)R_6T_6}}$$

And, substituting S_a for its identity,

$$\frac{M_9\sqrt{1 + \frac{\gamma_9 - 1}{2} M_9^2}}{1 + \gamma_9 M_9^2} = \frac{M_6\sqrt{1 + \frac{\gamma_6 - 1}{2} M_6^2}}{1 + \gamma_6 M_6^2} \sqrt{\frac{1 + \gamma_6}{1 + \gamma_9}} \left(\frac{S_{a,9}}{S_{a,6}}\right) \tag{7}$$

Combustion-Zone Total-Pressure Ratio

Assuming constant-area frictionless flow in the combustion zone,

$$p_9(1 + \gamma_9 M_9^2) = p_6(1 + \gamma_6 M_6^2)$$

and

$$\left(\frac{P_9}{P_6}\right)_M = \frac{(1 + \gamma_6 M_6^2)\left(1 + \frac{\gamma_9 - 1}{2} M_9^2\right)^{\frac{\gamma_9}{\gamma_9 - 1}}}{(1 + \gamma_9 M_9^2)\left(1 + \frac{\gamma_6 - 1}{2} M_6^2\right)^{\frac{\gamma_6}{\gamma_6 - 1}}} \tag{8}$$

APPENDIX C

USE OF NOMOGRAPHIC SOLUTIONS

Before giving solutions for two afterburner problems by the three nomographic methods (figs. 1 to 7), two approximate forms of equation (3) should be mentioned. As was indicated in the equation for net thrust produced by expansion of exhaust products to a nozzle-exit Mach number of 1.0, 3991

$$\frac{F_{n,10}}{w_a} = S_{a,10}\left[1 - f(\gamma_{10})\frac{p_0}{P_{10}}\right] - \frac{V_0}{g} \qquad (3)$$

the function of the specific-heats ratio at the exhaust-nozzle throat

$$f(\gamma_{10}) = \frac{(1 + \gamma_{10})^{\frac{1}{\gamma_{10}-1}}}{(2)^{\frac{\gamma_{10}}{\gamma_{10}-1}}}$$

varies from 0.793 at $\gamma_{10} = 1.345$ to 0.803 at $\gamma_{10} = 1.225$.

Therefore, an excellent approximation of the net thrust expression is given by the following equation:

$$\frac{F_{n,10}}{w_a} = S_{a,10}\left(1 - 0.8\frac{p_0}{P_{10}}\right) - \frac{V_0}{g}$$

This is the basic equation used in the approximate nomographic method (figs. 1 and 2).

The effects of neglecting afterburner total-pressure losses can be indicated by considering the following approximate equation:

$$\frac{F_{n,10}}{w_a} = S_{a,10}\left(1 - 0.8\frac{p_0}{P_5}\right) - \frac{V_0}{g}$$

and examining the error

$$\frac{0.8\left(\frac{p_0}{P_{10}} - \frac{p_0}{P_5}\right)}{1 - 0.8\frac{p_0}{P_{10}}}$$

This would be the error in $F_{n,10}$ for a choked convergent exhaust nozzle when $V_0 = 0$.

If the following afterburner-component total-pressure ratios existed but were neglected:

$$(P_6/P_5)_F = 0.94$$

$$(P_9/P_6)_M = 1.0 \text{ (no afterburning) to } 0.93 \text{ (at unit equivalence ratio)}$$

$$(P_{10}/P_9)_N = 0.97$$

the following ranges of errors would occur at the specified ratios of turbine-outlet total pressure to ambient pressure P_5/p_0:

$\frac{P_5}{p_0}$	$100 \times \frac{0.8\left(\frac{p_0}{P_{10}} - \frac{p_0}{P_5}\right)}{1 - 0.8\frac{p_0}{P_{10}}}$, percent	
	No afterburning	Unit equivalence ratio
2	8.1	14.1
5	1.95	3.7
10	.88	1.66
20	.47	.78

If V_0 did not equal zero, the errors in $F_{n,10}$ would be larger than those shown.

Most current turbojet engines fall in the P_5/p_0 range from 2 to 5, where errors caused by neglecting afterburner losses are large. Therefore, some method is required to yield afterburner-component total-pressure ratios. The nomographic solutions that are described herein simplify and combine the calculations for afterburner losses and, in turn, yield solutions to the basic net thrust equations.

Because the details of procedures and operations of the nomographic solutions complicate an introduction to the methods, they are given in a later section. The solutions of two afterburner problems are presented first to reveal the general application of the nomographic methods. The following examples illustrate the use of figures 1 to 7:

Values of F_n/w_a and sfc (with and without afterburning) and augmented ratios of net thrust and liquid weight for afterburning at unit equivalence ratio are to be predicted for a turbojet engine. The following data were obtained in simulated flight tests:

Altitude, ft	30,000
Flight Mach number, M_0	0.81
Compressor-inlet total temperature, T_1, $^{\circ}R$	460
Afterburner-inlet total temperature, T_5, $^{\circ}R$	1660
Afterburner-inlet total pressure, P_5, lb/sq ft abs	2500
Afterburner-inlet Mach number, M_5	0.22
Engine combustion efficiency, $\eta_{B,e}$	0.98
Fuel	JP-4 (octene-1)

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The predicted afterburner characteristics are as follows:

Flameholder drag coefficient, $\Delta P/q$	2.0
Afterburner equivalence ratio, φ_{ab}	1.0
Afterburner combustion efficiency, $\eta_{B,ab}$	0.85
Nozzle total-pressure ratio, $(P_{10}/P_9)_N$	0.97
Fuel	JP-4

With the preceding values and the thermodynamic data relating S_a, γ, and φ from figures 11, 12, and 15 (or combinations of figs. 13, 14, and 16), the following values are obtained:

$$\left(\frac{w_f}{w_a}\right)_s = 0.0678 \qquad S_{a,5} = 100$$

$$\varphi_{id,e} = 0.242 \qquad \gamma_5 = 1.33$$

Then, using equation (12),

$$\varphi_{ac,e} = \frac{0.242}{0.98} = 0.247$$

$$\varphi_{id,eab} = 0.242 + 0.85(1 - 0.242) = 0.886$$

$$S_{a,10} = 163$$

$$\gamma_{10} = 1.256$$

$$\text{Augmented liquid ratio} = \frac{w_{f,eab}}{w_{f,e}} = \frac{1.0}{0.247} = 4.05.$$

Solution by Approximate Nomographic Method (Figs. 1 and 2)

The following are the assumptions for the approximate method:

(1) Mass and energy contents of stream at any point are represented by the equivalent air specific-impulse value.

(2) Air specific impulse and ratio of specific heats are constant both before and after the combustion zone.

(3) The afterburner cross-sectional area is constant from the inlet to the exhaust-nozzle inlet.

(4) All energy and mass additions occur with negligible friction downstream of the flameholder and upstream of the nozzle.

(5) A convergent exhaust nozzle with unit throat Mach number is used.

(6) The value of the afterburner-inlet ratio of specific heats is 1.325.

(7) The afterburner-exit specific-heats ratio is 1.275.

(8) The total-pressure ratio across the flameholder is 0.95.

(9) The product of the flameholder and exhaust-nozzle total-pressure ratios (representing all assumed friction losses) is 0.92.

<u>Determination of net thrust with afterburning</u>. -

(A) Solution for total-pressure ratio across combustion zone $(P_9/P_6)_M$:

(1) Locate the value (1_{eab}) for M_5 (0.22) on the left scale of line A of figure 1.

(2) Place a point (2_{eab}) on line B of figure 1 at 100, the value of $S_{a,5} = S_{a,6}$.

(3) Draw a straight line through the two points (1_{eab} and 2_{eab}) to intersect line C at point 3_{eab}.

(4) Locate $S_{a,9} = S_{a,10} = 163$ on line D of figure 1 (point 4_{eab}).

(5) Construct a straight line through points 3_{eab} and 4_{eab} crossing line E of figure 1 at point 5_{eab}.

(6) Read the value of $f(\gamma_6, M_6)_M$ from the right scale of line A of figure 1 and locate the corresponding point $[f(\gamma_6, M_6)_M = 1.034]$ on line A of figure 2 (point 6_{eab}).

(7) Place a point (7_{eab}) on line B of figure 2 at $f(\gamma_9, M_9)_M = 1.104$, which is the value given by the left scale of line E of figure 1.

(8) Draw a straight line through points 6_{eab} and 7_{eab} to intersect line C at point 8_{eab}, which gives the total-pressure ratio across the combustion zone $(P_9/P_6)_M$ as 0.937. 3991

(B) Determination of $f(P_{10}, p_0, \gamma_{10}) = [1 - 0.8\ (p_0/P_{10})]$:

(9) At 30,000 feet $p_0 = 629$ pounds per square foot absolute and $P_5/p_0 = 3.98$; locate this value on line D of figure 2 (point 9_{eab}).

(10) Construct a straight line through points 8_{eab} and 9_{eab} crossing line E at point 10_{eab}, which indicates that $\left[1 - f(\gamma_{10})\ \frac{p_0}{P_{10}}\right] = 0.766$.

(C) Solution for value of $f(S_{a,10}, P_{10}, p_0, \gamma_{10}) = S_{a,10}\left(1 - 0.8\ \frac{p_0}{P_{10}}\right)$:

(11) Place a point (11_{eab}) on line F of figure 2 at $S_{a,10} = 163$ (left scale).

(12) Draw a straight line through points 10_{eab} and 11_{eab} to intersect line G at point 12_{eab}, where $S_{a,10}\left[1 - f(\gamma_{10})\ \frac{p_0}{P_{10}}\right] = 124.2$.

(D) Determination of V_0/g:

(13) Locate the altitude of 30,000 feet on the left scale of line H (point 13_{eab}), which gives the ambient sonic velocity a_0 (995).

(14) Place a point (14_{eab}) on line K of figure 2 at the value of the flight Mach number ($M_0 = 0.81$).

(15) Construct a straight line through points 13_{eab} and 14_{eab} intersecting line L of figure 2 at the value $V_0/g = 25.0$ (point 15_{eab}).

(E) Solution for value of net thrust = $f(S_{a,10}, P_{10}, p_0, \gamma_{10}, V_0)$:

(16) Finally, draw a straight line between points 12_{eab} and 15_{eab} crossing line I at point 16_{eab}, which yields the answer,

$$\frac{F_n}{w_a} = S_{a,10}\left[1 - f(\gamma_{10})\frac{p_0}{P_{10}}\right] - \frac{V_0}{g} = 99.2$$

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Determination of net thrust without afterburning. -

(A) Solution for value of $f(P_{10}, p_0, \gamma_{10}) = \left(1 - 0.8\,\frac{p_0}{P_{10}}\right)$:

(1) Locate point 1_e on line C of figure 2 where with no afterburning $(P_9/P_6)_M = 1.0$.

(2) Point 2_e on line D is identical with 9_{eab} $(P_5/p_0 = 3.98)$.

(3) Draw a straight line through points 1_e and 2_e intersecting line E, $f(P_{10}, p_0, \gamma_{10})$, at point 3_e.

(B) Determination of value of $f(S_{a,10}, P_{10}, p_0, \gamma_{10}) = S_{a,10}\left(1 - 0.8\,\frac{p_0}{P_{10}}\right)$:

(4) Locate $S_{a,10} = S_{a,9} = S_{a,6} = S_{a,5} = 100$ on line F of figure 2 using the right scale (point 4_e).

(5) Construct a straight line through 3_e and 4_e to point 5_e on line G, $f(S_{a,10}, P_{10}, p_0, \gamma_{10}) = 77.7$ (on right scale).

(C) Solution for value of V_0/g:

(6) Points 6_e, 7_e, and 8_e on figure 2 are identical with points 13_{eab}, 14_{eab}, and 15_{eab}.

(D) Determination of net thrust = $f(S_{a,10}, P_{10}, p_0, \gamma_{10}, V_0)$:

(7) Draw a straight line connecting points 5_e and 8_e and crossing line J at the answer (point 9_e),

$$\frac{F_n}{w_a} = S_{a,10}\left[1 - f(\gamma_{10})\frac{p_0}{P_{10}}\right] - \frac{V_0}{g} = 52.7$$

<u>Results</u>. - The results for the approximate nomographic solution for $M_{10} = 1.0$ are as follows: 3991

$$\left(\frac{F_n}{w_a}\right)_e = 52.7 \text{ lb thrust/(lb air/sec)}$$

$$\left(\frac{F_n}{w_a}\right)_{eab} = 99.2 \text{ lb thrust/(lb air/sec)}$$

$$\left.\begin{aligned} sfc_e &= \frac{(0.247)(0.0678)(3600)}{52.7} = 1.14 \text{ (lb fuel/hr)/lb thrust} \\ sfc_{eab} &= \frac{(0.0678)(3600)}{99.2} = 2.46 \text{ (lb fuel/hr)/lb thrust} \end{aligned}\right\} \quad \text{(eq. (11))}$$

$$\frac{F_{n,eab}}{F_{n,e}} = \frac{99.2}{52.7} = 1.88 \text{ at } \frac{w_{f,eab}}{w_{f,e}} = 4.05 \qquad \text{(eqs. (9) and (10))}$$

Solution by Exact Nomographic Method for $M_{10} = 1.0$ (Figs. 3 to 6)

The following are the assumptions for the exact nomographic method:

(1) The mass and energy contents of the stream at any point are represented by the equivalent air specific-impulse value.

(2) Air specific impulse and ratio of specific heats are constant both before and after the combustion zone.

(3) The afterburner cross-sectional area is constant from the inlet (upstream of the flameholder) to the exhaust-nozzle inlet (combustion-zone exit).

(4) All energy and mass additions occur with negligible friction downstream of the flameholder and upstream of the nozzle.

(5) A convergent exhaust nozzle with unit throat Mach number is used.

<u>Determination of net thrust with afterburning</u>. -

(A) Solution for value of $(P_6/P_5)_F$:

(1) On line A of figure 3 locate point 1_{eab} at $\gamma_5 = 1.33$.

(2) Place the value $M_5 = 0.22$ on line B (point 2_{eab}).

(3) Draw a straight line through points 1_{eab} and 2_{eab} to 3_{eab}, the intersection with line C of figure 3.

(4) Locate the drag coefficient $\Delta P/q = 2.0$ on line D (point 4_{eab}).

(5) Construct a straight line through points 3_{eab} and 4_{eab} to intersect line E of figure 3 at the total-pressure ratio across the flameholder $(P_6/P_5)_F = 0.937$ (point 5_{eab}).

(B) Solution for value of M_6:

(6) Mark the point $M_5 = 0.22$ on line F of figure 3 at 6_{eab}.

(7) Draw a straight line through points 5_{eab} and 6_{eab} crossing line G of figure 3 at $M_6 = 0.236$, which is point 7_{eab}.

(C) Determination of values of M_9, $f(\gamma_6,M_6)$, and $f(\gamma_9,M_9)$ to give $(P_9/P_6)_M$:

(8) Locate point 8_{eab} at $\gamma_6 = \gamma_5 = 1.33$ on line A of figure 4.

(9) Place the value $M_6 = 0.236$ (from line G of fig. 3) on line B of figure 4 at point 9_{eab}.

(10) Draw a straight line through points 8_{eab} and 9_{eab} to intersect 10_{eab} on line C.

(11) Extend this straight line to cross line D of figure 4 at point 11_{eab}, where $f(\gamma_6,M_6)_M = 1.035$.

(12) Locate the value of $S_{a,6} = S_{a,5} = 100$ at point 12_{eab} on line E.

(13) Construct a straight line passing through points 10_{eab} and 12_{eab} and crossing line F at point 13_{eab}.

(14) Place point 14_{eab} on line G at $S_{a,9} = 163$.

(15) Draw a straight line through points 13_{eab} and 14_{eab} to intersect line I at point 15_{eab}.

(16) Mark the value $\gamma_9 = 1.256$ at point 16_{eab} on line K of figure 4.

(17) Construct a straight line through points 15_{eab} and 16_{eab} to 3991
cross line J at point 17_{eab}, which gives $M_9 = 0.455$, if the value of that variable is desired.

(18) Extend this straight line to intersect line H at point 18_{eab}, where $f(\gamma_9,M_9)_M = 1.1075$.

(D) Solution for value of $(P_9/P_6)_M$:

(19) Locate the value $f(\gamma_6,M_6) = 1.035$ (from line D on fig. 4) on line A of figure 5 as point 19_{eab}.

(20) Place $f(\gamma_9,M_9) = 1.1075$ (from line H of fig. 4) on line B of figure 5 at point 20_{eab}.

(21) Draw a straight line through points 19_{eab} and 20_{eab} to intersect line C at point 21_{eab}, giving the value of the total-pressure ratio across the combustion zone $(P_9/P_6)_M = 0.9347$.

(E) Solution for the value of $(P_{10}/P_5)_{F,M,N}$:

(22) Locate $(P_6/P_5)_F = 0.937$ (from line E on fig. 3) on line D of figure 5 as point 22_{eab}.

(23) Construct a straight line passing through points 21_{eab} and 22_{eab} and crossing line E at point 23_{eab}, where $(P_9/P_5)_{F,M} = 0.876$.

(24) Mark the nozzle total-pressure ratio $(P_{10}/P_9)_N = 0.97$ at point 24_{eab} on line F of figure 5.

(25) Draw a straight line through points 23_{eab} and 24_{eab} to intersect line G at point 25_{eab}, where the total-pressure ratio across the afterburner is given as $(P_{10}/P_5)_{F,M,N} = 0.851$.

(F) Determination of values of p_0/P_{10}:

(26) If the afterburner-inlet total pressure (P_5 = 2500 lb/sq ft abs) is known, locate that value at point 26_{eab} on line H of figure 5 (using the left scale). If the pressure ratio $P_5/p_0 = 3.98$ is known, mark it as point $26'_{eab}$ on line H (using the right scale).

(27) Construct a straight line through points 25_{eab} and 26_{eab} or 25_{eab} and $26'_{eab}$ to intersect line I of figure 5 at either point 27_{eab} where P_{10} = 2130 pounds per square foot absolute (right scale) or point $27'_{eab}$ where $(P_{10}/p_0) = 3.38$ (left scale), respectively.

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(28) If P_{10} was determined at point 27_{eab}, locate the altitude of 30,000 feet (or the ambient pressure, if known) on line A of figure 6(a) at point 28_{eab}.

(29) Place a point 29_{eab} on line B at P_{10} = 2130 pounds per square foot absolute (from line I, point 27_{eab}, of fig. 5).

(30) Draw a straight line through points 28_{eab} and 29_{eab} to cross line C of figure 6(a) at point 30_{eab}, giving $p_0/P_{10} = 0.297$. If P_{10}/p_0 was determined as point $27'_{eab}$ on line I of figure 5, enter it on line C of figure 6(a) using the left scale.

(G) Solution for the value of $\left[1 - f(\gamma_{10}) \frac{p_0}{P_{10}}\right]$:

(31) Locate $\gamma_{10} = \gamma_9 = 1.256$ on line D of figure 6 at point 31_{eab}. The length of this line shows the small effect of γ_{10} on the value of $f(\gamma_{10})$ in equation (3).

(32) Construct a straight line passing through points 30_{eab} and 31_{eab} and crossing line E at point 32_{eab}, where $\left[1 - f(\gamma_{10}) \frac{p_0}{P_{10}}\right] = 0.76$.

(H) Determination of the value of $S_{a,10}\left[1 - f(\gamma_{10}) \frac{p_0}{P_{10}}\right]$:

(33) Place point 33_{eab} at $S_{a,10} = S_{a,9} = 163$ on line F of figure 6(a).

(34) Draw a straight line through points 32_{eab} and 33_{eab} to intersect line G at point 34_{eab}, giving the value $S_{a,10}\left[1 - f(\gamma_{10})\ \frac{p_0}{P_{10}}\right] = 123.4.$

(I) Solution for value of V_0/g:

(35) Locate the altitude (30,000 ft) on line H of figure 6(a) at point 35_{eab}, which gives the sonic velocity (right scale). 3991

(36) Enter the value $M_0 = 0.81$ as point 36_{eab} on line K of figure 6(a).

(37) Construct a straight line through points 35_{eab} and 36_{eab} crossing line L at point 37_{eab}, where $V_0/g = 25.0$.

(J) Determination of net thrust:

(38) Draw a straight line between points 34_{eab} and 37_{eab} intersecting line I of figure 6(a) at point 38_{eab}, where the final result is given as $\frac{F_n}{W_a} = S_{a,10}\left[1 - f(\gamma_{10})\ \frac{p_0}{P_{10}}\right] - \frac{V_0}{g} = 98.4.$

Determination of net thrust without afterburning. -

(A) Solution for value of (P_{10}/P_5):

(1) The following points for the engine with and without afterburning are identical:

Figure 3: 1_e and 1_{eab}, 2_e and 2_{eab}, 3_e and 3_{eab}, 4_e and 4_{eab}, 5_e and 5_{eab}, 6_e and 6_{eab}, and 7_e and 7_{eab}.

Figure 5: 9_e and 24_{eab}, and 11_e and 26_{eab} or $11_e'$ and $26_{eab}'$.

Figure 6(a): 13_e and 28_{eab}, 20_e and 35_{eab}, 21_e and 36_{eab}, and 22_e and 37_{eab}.

Therefore, no explanation will be given where these points are encountered.

(2) Locate $(P_6/P_5)_F = (P_9/P_5)_{F,M} = 0.937$ on line E of figure 5 at point 8_e (taken from point 5_e on line E of fig. 3).

(3) Draw a straight line through points 8_e and 9_e to intersect line G of figure 5 at point 10_e, giving the afterburner total-pressure ratio $(P_{10}/P_5)_{F,M,N} = 0.91$.

3991 (B) Determination of value of p_0/P_{10}:

(4) Construct a straight line through points 10_e and 11_e (if P_5 is used) or 10_e and $11_e'$ (if P_5/p_0 is used) to intersect line I of figure 5 at point 12_e (P_{10} = 2280 lb/sq ft abs) or point $12_e'$ ($P_{10}/p_0 = 3.60$), respectively.

(5) Place a point (14_e) on line B of figure 6(a) at the value $P_{10} = 2280$.

(6) Draw a straight line through points 13_e and 14_e to intersect line C at 15_e, where $p_0/P_{10} = 0.279$. If P_{10}/p_0 was found on line I of figure 5 (point $12_e'$), enter that value on line C of figure 6(a) using the left scale.

(C) Solution for the value of $\left[1 - f(\gamma_{10}) \frac{p_0}{P_{10}}\right]$:

(7) Locate $\gamma_{10} = \gamma_5 = 1.33$ at point 16_e on line D of figure 6(a).

(8) Construct a straight line through points 15_e and 16_e intersecting line E at point 17_e, giving the value $\left[1 - f(\gamma_{10}) \frac{p_0}{P_{10}}\right] \doteq 0.778$.

(D) Determination of the value of $S_{a,10}\left[1 - f(\gamma_{10}) \frac{p_0}{P_{10}}\right]$:

(9) Enter the value $S_{a,10} = S_{a,5} = 100$ on line F at point 18_e.

(10) Draw a straight line through points 17_e and 18_e to cross line G of figure 6(a) at point 19_e, where $S_{a,10}\left[1 - f(\gamma_{10}) \frac{p_0}{P_{10}}\right] = 77.8$.

(E) Solution for net thrust:

(11) Construct a straight line between points 19_e and 22_e intersecting line J of figure 6(a) at point 23_e, which gives the final answer,

$$\frac{F_n}{w_a} = S_{a,10}\left[1 - f(\gamma_{10})\frac{p_0}{p_{10}}\right] - \frac{V_0}{g} = 52.8.$$

Results. - The results for the exact nomographic method for $M_{10} = 1.0$ are the following: 3991

$$\left(\frac{F_n}{w_a}\right)_e = 52.8 \text{ lb thrust/(lb air/sec)}$$

$$\left(\frac{F_n}{w_a}\right)_{eab} = 98.4 \text{ lb thrust/(lb air/sec)}$$

$$\left.\begin{aligned} sfc_e &= \frac{(0.247)(0.0678)(3600)}{52.8} = 1.14 \text{ (lb fuel/hr)/lb thrust} \\ sfc_{eab} &= \frac{(0.0678)(3600)}{98.4} = 2.48 \text{ (lb fuel/hr)/lb thrust} \end{aligned}\right\} \text{(eq. (11))}$$

$$\frac{F_{n,eab}}{F_{n,e}} = \frac{98.4}{52.8} = 1.86 \quad \text{at} \quad \frac{w_{f,eab}}{w_{f,e}} = 4.05 \qquad \text{(eqs. (9) and (10))}$$

Solution by Exact Nomographic Method for $p_{10} = p_0$

(Figs. 3, 4, 5, and 7)

The assumptions are identical with those for the exact nomographic method for $M_{10} = 1.0$ (figs. 3 to 6) with the exception of the nozzle-exit condition. In this exact nomographic method (figs. 3, 4, 5, and 7), it is assumed that the combustion products expand to the ambient static pressure at the exhaust-nozzle exit.

Determination of net thrust with and without afterburning. - The procedures for the exact calculation method for $p_{10} = p_0$ are identical with those of the exact nomographic solution for $M_{10} = 1.0$ with the exception of the steps for determining the function $f(P_{10},p_0,\gamma_{10})$ in the net thrust equation (compare eqs. (3) and (4)). This difference occurs between lines C and E of figures 6 and 7. The points of similarity will not be discussed.

Procedure for using figure 7. - The procedure for determining $f(P_{10},p_0,\gamma_{10})_{p_{10}=p_0}$ using lines C', D, and E of figures 7 is as follows:

(1) Locate the value of the ratio of ambient static pressure to nozzle-exit total pressure (p_0/P_{10}, taken from line C) on line C' (point $30'_{eab}$ for the turbojet engine with afterburning or point $15'_e$ for the nonafterburning case).

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(2) Place the value of nozzle-exit specific-heats ratio γ_{10} on line D (point 31_{eab} or 16_e).

(3) Draw a straight line through these points (31_{eab} and $30'_{eab}$ or 16_e and $15'_e$) to intersect line E at the value of $f(P_{10},p_0,\gamma_{10})_{p_{10}=p_0}$ (point 32_{eab} or 17_e).

Results. - The results for the exact nomographic method for $p_{10} = p_0$ (fig. 7(a)) are the following:

$$\left(\frac{F_n}{w_a}\right)_e = 54.3 \text{ lb thrust/(lb air/sec)}$$

$$\left(\frac{F_n}{w_a}\right)_{eab} = 100.8 \text{ lb thrust/(lb air/sec)}$$

$$\left.\begin{aligned} sfc_e &= \frac{(0.247)(0.0678)(3600)}{54.3} = 1.11 \text{ (lb fuel/hr)/lb thrust} \\ sfc_{eab} &= \frac{(0.0678)(3600)}{100.8} = 2.42 \text{ (lb fuel/hr)/lb thrust} \end{aligned}\right\} \text{(eq. (11))}$$

$$\frac{F_{n,eab}}{F_{n,e}} = \frac{100.8}{54.3} = 1.86 \text{ at } \frac{w_{f,eab}}{w_{f,e}} = 4.05 \qquad \text{(eqs. (9) and (10))}$$

Comparison of Results

Comparison of the results by the three methods reveals good agreement in all categories. However, when the approximate nomographic method for $M_{10} = 1.0$ is used for solutions to specific afterburner problems, the assumptions of the method should be checked carefully against the actual component characteristics. The agreement of results calculated for $p_{10} = p_0$ with those for $M_{10} = 1.0$ occurred because the turbojet engine of this example has a low ratio of afterburner-inlet total pressure to ambient static pressure.

The following example shows net thrusts and total fuel flows computed for a turbojet engine operating with a high pressure ratio, with and without afterburning. The following variables are assigned:

Altitude, ft	50,000
Flight Mach number, M_0	2.5
Compressor-inlet total temperature, T_1, °R	884
Afterburner-inlet total temperature, T_5, °R	2001
Ratio of afterburner-inlet total pressure to ambient static pressure, P_5/p_0	19.04
Afterburner-inlet Mach number, M_5	0.246
Engine combustion efficiency, $\eta_{B,e}$	0.99
Fuel (primary engine)	JP-4 (octene-1)
Afterburner flameholder drag coefficient, $\Delta P/q$	1.6
Afterburner equivalence ratio, φ_{ab}	1.0
Afterburner combustion efficiency, $\eta_{B,ab}$	1.0
Fuel (afterburner)	JP-4
Nozzle total-pressure ratio, $(P_{10}/P_9)_N$	0.97

With the preceding values and figures 9, 11, 12, 14, and 15, the following values result:

$$\left(\frac{w_f}{w_a}\right)_s = 0.0678 \qquad S_{a,5} = 110$$

$$\varphi_{id,e} = 0.236 \qquad \varphi_{ac,e} = \frac{0.236}{0.99} = 0.238$$

$$\gamma_5 = 1.316 \qquad \gamma_{10} = 1.253$$

$$S_{a,10} = 173.4$$

The ratios of nozzle-exit total pressure to ambient static pressure were computed from figures 3 to 5 (calculations not shown):

$$\left(\frac{P_{10}}{p_0}\right)_e = 17.36 \qquad \left(\frac{P_{10}}{p_0}\right)_{eab} = 16.11$$

Figure 6(b) gives the calculations for net thrusts produced by expansion of combustion products to a Mach number of 1.0 at the exhaust-nozzle exit. The nomographic calculations for net thrusts resulting from complete expansion of exhaust gases are shown in figure 7(b). The results obtained with both methods are given in the following table:

	Nozzle-exit condition			
	$M_{10} = 1.0$		$p_{10} = p_0$	
	Afterburner equivalence ratio, φ_{ab}			
	0	1.0	0	1.0
$\frac{F_n}{w_a}$, $\frac{\text{lb thrust}}{\text{lb air/sec}}$	29.6	89.4	43.5	112.8
sfc, $\frac{\text{lb fuel/hr}}{\text{lb thrust}}$	1.96	2.73	1.33	2.16
$\frac{F_{n,eab}}{F_{n,e}}$		3.02		2.59
$\frac{w_{n,eab}}{w_{f,e}}$		4.20		4.20

Procedures and Operations

The following are the detailed procedures and operations used when solving afterburner problems in general with the nomographs for the approximate and exact methods. The construction lines and numbering system used for the previous example are followed (1_{eab}, 2_{eab}, 3_{eab} . . ., or 1_e, 2_e, 3_e . . ., for computing sequences with points for the engine with afterburner or the engine alone, respectively, and arrowheads representing construction direction of straight lines).

Approximate nomographic method for $M_{10} = 1.0$. - Figures 1 and 2 are based on the assumptions given in the ANALYTICAL METHODS section and the previous example for the approximate nomographic method for expansion of combustion products to a Mach number of 1.0 at the exhaust-nozzle exit.

Figure 1:

Step (1)

Point 1_{eab}. - Locate M_5 on the left scale of line A of figure 1. 3991

Operation 1_{eab}. - M_5 and the assumptions ($\gamma_5 = \gamma_6 = 1.325$ and $(P_6/P_5)_F = 0.95$) determine M_6 (eq. (6)); M_6 and γ_6 yield the following functions: $\dfrac{M_6\sqrt{\left(1 + \dfrac{\gamma_6 - 1}{2} M_6^2\right)(1 + \gamma_6)}}{1 + \gamma_6 M_6^2}$ required to compute M_9 and represented by the location on line A (scale not shown), and $f(\gamma_6,M_6)_M = \dfrac{1 + \gamma_6 M_6^2}{\left(1 + \dfrac{\gamma_6 - 1}{2} M_6^2\right)^{\frac{\gamma_6}{\gamma_6 - 1}}}$ required to compute $(P_9/P_6)_M$. Read the value of $f(\gamma_6,M_6)_M$ from the right scale of line A to be used as point 6_{eab} on line A of figure 2.

Point 2_{eab}. - Locate $S_{a,6}$ on line B.

Operation 2_{eab}. - The point represents division by $S_{a,6}$.

Point 3_{eab}. - Draw a straight line through points 1_{eab} and 2_{eab} to intersect line C at point 3_{eab}.

Operation 3_{eab}. - The point represents the value of

$$\frac{M_6\sqrt{\left(1 + \frac{\gamma_6 - 1}{2} M_6^2\right)(1 + \gamma_6)}}{(1 + \gamma_6 M_6^2)\, S_{a,6}}.$$

Step (2)

Point 4_{eab}. - Locate $S_{a,9}$ on line D.

Operation 4_{eab}. - The point represents multiplication by $S_{a,9}$.

Point 5_{eab}. - Draw a straight line through points 3_{eab} and 4_{eab} to intersect line E at point 5_{eab}.

Operation 5_{eab}. - The point represents the value of

$$\frac{M_9\sqrt{\left(1+\frac{\gamma_9-1}{2}M_9^2\right)(1+\gamma_9)}}{1+\gamma_9 M_9^2}=\frac{M_6\sqrt{\left(1+\frac{\gamma_6-1}{2}M_6^2\right)(1+\gamma_6)}}{1+\gamma_6 M_6^2}\left(\frac{S_{a,9}}{S_{a,6}}\right)$$

(eq. (7)), which with the assumption ($\gamma_9 = \gamma_{10} = 1.275$) yields the value of M_9 (right scale of line E); M_9 and γ_9 in turn give the function

$$\frac{1+\gamma_9 M_9^2}{\left(1+\frac{\gamma_9-1}{2}M_9^2\right)^{\frac{\gamma_9}{\gamma_9-1}}}=f(\gamma_9,M_9)_M$$

required to compute the value of $(P_9/P_6)_M$. Read $f(\gamma_9,M_9)_M$ from the left scale of line E to be used as point 7_{eab} on line B of figure 2.

Figure 2:

Step (1)

Point 6_{eab}. - Locate $f(\gamma_6,M_6)_M$ (value from point 1_{eab} of fig. 1) on line A of figure 2.

Operation 6_{eab}. - The point represents the value of $f(\gamma_6,M_6)_M$.

Point 7_{eab}. - Locate $f(\gamma_9,M_9)_M$ (value from point 5_{eab} of fig. 1) on line B.

Operation 7_{eab}. - The point represents division by $f(\gamma_9,M_9)_M$.

Point 8_{eab}. - Draw a straight line through points 6_{eab} and 7_{eab} to intersect line C at point 8_{eab}.

Operation 8_{eab}. - The point represents the value of the total-pressure ratio $(P_9/P_6)_M$ across the combustion zone:

$$\left(\frac{P_9}{P_6}\right)_M = \frac{(1 + \gamma_6 M_6^2)\left(1 + \frac{\gamma_9 - 1}{2} M_9^2\right)^{\frac{\gamma_9}{\gamma_9 - 1}}}{(1 + \gamma_9 M_9^2)\left(1 + \frac{\gamma_6 - 1}{2} M_6^2\right)^{\frac{\gamma_6}{\gamma_6 - 1}}} \tag{8}$$

Point 1_e. - Locate at 1.00 on line C. 3991

Operation 1_e. - The point represents engine operation without afterburning. Calculations for this condition are similar to those with afterburning from here, and points for the engine alone are noted particularly parenthetically ($n_e = n_{eab} - 7$).

Step (2)

Point 9_{eab} (or 2_e). - Locate (P_5/p_0) on line D.

Operation 9_{eab} (or 2_e). - The point represents multiplication of reciprocal of $(P_9/P_6)_M$ by $f(\gamma_{10}) \frac{p_0}{P_5} \left(\frac{P_5}{P_6} \frac{P_9}{P_{10}}\right)$ where assumed values give $\left(\frac{P_6}{P_5} \frac{P_{10}}{P_9}\right) = 0.92$ and $f(\gamma_{10}) = \frac{(1 + \gamma_{10})^{\frac{1}{\gamma_{10} - 1}}}{(2)^{\frac{\gamma_{10}}{\gamma_{10} - 1}}} = 0.8$.

Point 10_{eab} (or 3_e). - Draw a straight line through points 8_{eab} (or 1_e) and 9_{eab} (or 2_e) to intersect line E at point 10_{eab} (or 3_e).

Operation 10_{eab} (or 3_e). - The point represents the value of

$$f(P_{10}, p_0, \gamma_{10}) = 1 - f(\gamma_{10}) \frac{p_0}{P_{10}} = 1 - \frac{(\gamma_{10} + 1)^{\frac{1}{\gamma_{10} - 1}}}{(2)^{\frac{\gamma_{10}}{\gamma_{10} - 1}}} \frac{p_0}{P_5} \frac{P_5}{P_6} \frac{P_6}{P_9} \frac{P_9}{P_{10}}$$

Step (3)

Point 11_{eab} (or 4_e). - Locate $S_{a,10}$ on line F. ($S_{a,10,eab} = S_{a,9,eab}$; $S_{a,5,e} = S_{a,6,e} = S_{a,9,e} = S_{a,10,e}$.)

Operation 11_{eab} (or 4_e). - The point represents multiplication by $S_{a,10}$. NOTE: Lines F and G have high (left) and low (right) range scales. If the high-range scale is used on line F ($S_{a,10}$), the $f(S_{a,10},P_{10},p_0,\gamma_{10})$ value will be that given by the high-range scale of line G, and the final answer F_n/w_a must be read on line I. If the low-range scales for $S_{a,10}$ and $f(S_{a,10},P_{10},p_0,\gamma_{10})$ are used, the final result F_n/w_a must be read on line J.

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Point 12_{eab} (or 5_e). - Draw a straight line through points 10_{eab} (or 3_e) and 11_{eab} (or 4_e) to intersect line G at point 12_{eab} (or 5_e).

Operation 12_{eab}. - The point represents the value of

$$f(S_{a,10},P_{10},p_0,\gamma_{10}) = S_{a,10}\left[1 - f(\gamma_{10})\,\frac{p_0}{P_{10}}\right] = S_{a,10}\left[1 - \frac{(1+\gamma_{10})^{\frac{1}{\gamma_{10}-1}}}{(2)^{\frac{\gamma_{10}}{\gamma_{10}-1}}}\left(\frac{p_0}{P_{10}}\right)\right]$$

Mark this point to be used in step (5).

Step (4)

Point 13_{eab} (or 6_e). - Locate the flight altitude on line H (left scale).

Operation 13_{eab} (or 6_e). - The point represents the sonic velocity a_0 (ft/sec) at the flight altitude; a_0 can be read from the right scale.

Point 14_{eab} (or 7_e). - Locate the flight Mach number M_0 on line K.

Operation 14_{eab} (or 7_e). - The point represents multiplication by M_0.

Point 15_{eab} (or 8_e). - Draw a straight line through points 13_{eab} (or 6_e) and 14_{eab} (or 7_e) to intersect line L at point 15_{eab} (or 8_e).

Operation 15_{eab} (or 8_e). - The point represents the value of $V_0/g = (a_0 M_0)/g$.

Step (5)

Point 16_{eab} (or 9_e). - Draw a straight line through points 12_{eab} (or 5_e) and 15_{eab} (or 8_e) to intersect line I (or line J) at point 16_{eab} (or 9_e).

Operation 16_{eab} (or 9_e). - The point represents the value of

$$\frac{F_{n,10}}{w_a} = S_{a,10}\left[1 - f(\gamma_{10})\,\frac{p_0}{p_{10}}\right] - \frac{V_0}{g} \qquad (3)$$

<u>Exact nomographic method for $M_{10} = 1.0$.</u> - Figures 3 to 6 are based on the assumptions given in the ANALYTICAL METHODS section and the examples for the exact nomographic method for expansion of exhaust products to Mach number 1.0 at the nozzle exit.

Figure 3:

Step (1)

Point 1_{eab} (or 1_e). - Locate γ_5 on line A of figure 3.

Point 2_{eab} (or 2_e). - Locate M_5 on line B.

Point 3_{eab} (or 3_e). - Draw a straight line through points 1_{eab} (or 1_e) and 2_{eab} (or 2_e) to intersect line C at point 3_{eab} (or 3_e).

Operations 1_{eab} (or 1_e), 2_{eab} (or 2_e), and 3_{eab} (or 3_e). - The function

$$\frac{\gamma_5 M_5^2}{2\left(1 + \frac{\gamma_5 - 1}{2} M_5^2\right)^{\frac{\gamma_5}{\gamma_5 - 1}}}$$

is computed; point 3_{eab} (or
3991 3_e) represents the value of this function.

Step (2)

Point 4_{eab} (or 4_e). - Locate the value of $C_D = \Delta P/q$ on line D.

Operation 4_{eab} (or 4_e). - The point represents multiplication by $\Delta P/q$.

Point 5_{eab} (or 5_e). - Draw a straight line through points 3_{eab} (or 3_e) and 4_{eab} (or 4_e) to intersect line E at point 5_{eab} (or 5_e).

Operation 5_{eab} (or 5_e). - The point represents the value of

$$\left(\frac{P_6}{P_5}\right)_F = 1 - C_D\left[\frac{\gamma_5 M_5^2}{2\left(1 + \frac{\gamma_5 - 1}{2} M_5^2\right)^{\frac{\gamma_5}{\gamma_5 - 1}}}\right] \qquad (6)$$

Step (3)

Point 6_{eab} (or 6_e). - Locate M_5 on line F.

Point 7_{eab} (or 7_e). - Draw a straight line through points 5_{eab} (or 5_e) and 6_{eab} (or 6_e) to intersect line G at point 7_{eab} (or 7_e).

Operations 6_{eab} (or 6_e) and 7_{eab} (or 7_e). - $(P_6/P_5)_F$ and M_5 (also γ_5) determine M_6 (eq. (6)). Point 7_{eab} (or 7_e) represents the value of M_6. Read M_6 to be used for point 9_{eab} on line B of figure 4. The variation of γ_5 in equation (6) also affects M_6 to a very small extent over the range of Mach numbers used. In the case yielding the maximum difference in M_6 due to γ_5 variation, the effect was less than 0.1 percent as γ_5 changed from 1.29 to 1.35. For this reason the influence of γ_5 variation is not included in step (3).

Figure 4:

Step (1) 3991

Point 8_{eab}. - Locate $\gamma_6 = \gamma_5$ on line A of figure 4.

Point 9_{eab}. - Locate M_6 on line B (value of M_6 from point 7_{eab}, fig. 3).

Point 10_{eab}. - Draw a straight line through points 8_{eab} and 9_{eab} to intersect line C at point 10_{eab}.

Operations 8_{eab}, 9_{eab}, 10_{eab}. - γ_6 and M_6 determine a function necessary to compute M_9. Point 10_{eab} represents the value of

$$\frac{M_6\sqrt{\left(1 + \frac{\gamma_6 - 1}{2} M_6^2\right)(1 + \gamma_6)}}{1 + \gamma_6 M_6^2}.$$

Point 11_{eab}. - Extend the straight line drawn to locate point 10_{eab} to intersect line D at point 11_{eab}.

Operations 8_{eab}, 9_{eab}, and 11_{eab}. - γ_6 and M_6 fix the value of a function

$$f(\gamma_6,M_6)_M = \frac{1 + \gamma_6 M_6^2}{\left(1 + \frac{\gamma_6 - 1}{2} M_6^2\right)^{\frac{\gamma_6}{\gamma_6 - 1}}}$$

required to compute $(P_9/P_6)_M$. Read the value of $f(\gamma_6,M_6)_M$ for point 11_{eab} to be used to locate point 19_{eab}, line A, figure 5.

Step (2)

Point 12_{eab}. - Locate $S_{a,6} = S_{a,5}$ on line E.

Operation 12_{eab}. - The point represents division by $S_{a,6}$.

Point 13_{eab}. - Draw a straight line through points 10_{eab} and 12_{eab} to intersect line F at point 13_{eab}.

Operation 13_{eab}. - The point represents the value of the function $\frac{M_6\sqrt{\left(1 + \frac{\gamma_6 - 1}{2} M_6^2\right)(1 + \gamma_6)}}{(1 + \gamma_6 M_6^2)S_{a,6}}$.

Step (3)

Point 14_{eab}. - Locate the value of $S_{a,9}$ on line G.

Operation 14_{eab}. - The point represents multiplication by $S_{a,9}$.

Point 15_{eab}. - Draw a straight line through points 13_{eab} and 14_{eab} to intersect line I at point 15_{eab}.

Operation 15_{eab}. - The point represents the value of

$$\frac{M_9\sqrt{\left(1 + \frac{\gamma_9 - 1}{2} M_9^2\right)(1 + \gamma_9)}}{1 + \gamma_9 M_9^2} = \frac{M_6\sqrt{\left(1 + \frac{\gamma_6 - 1}{2} M_6^2\right)(1 + \gamma_6)}}{1 + \gamma_6 M_6^2}\left(\frac{S_{a,9}}{S_{a,6}}\right) \quad (7)$$

Step (4)

Point 16_{eab}. - Locate $\gamma_9 = \gamma_{10}$ on line K.

Point 17_{eab}. - Draw a straight line through points 16_{eab} and 15_{eab} to intersect line J at point 17_{eab}.

Operation 17_{eab}. - The function determined as point 15_{eab} and γ_9 (16_{eab}) fix the value of M_9 that can be read from point 17_{eab}.

Point 18_{eab}. - Extend the straight line drawn to locate point 17_{eab} to intersect line H at point 18_{eab}.

Operation 18_{eab}. - M_9 (17_{eab}) and γ_9 (16_{eab}) yield the function $f(\gamma_9, M_9)_M = \dfrac{1 + \gamma_9 M_9^2}{\left(1 + \dfrac{\gamma_9 - 1}{2} M_9^2\right)^{\frac{\gamma_9}{\gamma_9 - 1}}}$ required to compute $(P_9/P_6)_M$. Read the value for $f(\gamma_9, M_9)_M$ at point 18_{eab} to be used as point 20_{eab}, line B, figure 5. 3991

Figure 5:

Step (1)

Point 19_{eab}. - Locate the value of $f(\gamma_6, M_6)_M$ (point 11_{eab}) on line A of figure 5.

Point 20_{eab}. - Locate $f(\gamma_9, M_9)_M$ (from point 18_{eab}) on line B.

Operation 20_{eab}. - The point represents division by $f(\gamma_9, M_9)_M$.

Point 21_{eab}. - Draw a straight line through points 19_{eab} and 20_{eab} to intersect line C at point 21_{eab}.

Operation 21_{eab}. - The point represents the value of equation (8):

$$\left(\frac{P_9}{P_6}\right)_M = \frac{f(\gamma_6, M_6)_M}{f(\gamma_9, M_9)_M} = \frac{(1 + \gamma_6 M_6^2)\left(1 + \dfrac{\gamma_9 - 1}{2} M_9^2\right)^{\frac{\gamma_9}{\gamma_9 - 1}}}{(1 + \gamma_9 M_9^2)\left(1 + \dfrac{\gamma_6 - 1}{2} M_6^2\right)^{\frac{\gamma_6}{\gamma_6 - 1}}}$$

Step (2)

Point 22_{eab}. - Locate $(P_6/P_5)_F$ (from point 5_{eab}) on line D.

Operation 22_{eab}. - The point represents multiplication by $(P_6/P_5)_F$.

Point 23_{eab}. - Draw a straight line through points 21_{eab} and 22_{eab} to intersect line E at point 23_{eab}.

Operation 23_{eab}. - The point represents the value of $\left(\frac{P_9}{P_5}\right)_{F,M} = \left(\frac{P_6}{P_5}\right)_F \left(\frac{P_9}{P_6}\right)_M$.

Point 8_e. - Locate the value of $(P_6/P_5)_F$ (from 5_e) on line E.

Operation 8_e. - The point represents engine operation without afterburning $(P_9/P_6 = 1.00)$. Calculations for this condition are similar to those with afterburning from this point on. Therefore, points for the engine operating without afterburning are noted parenthetically $(n_e = n_{eab} - 15)$.

Step (3)

Point 24_{eab} (or 9_e). - Locate the selected nozzle total-pressure ratio $(P_{10}/P_9)_N$ on line F.

Operation 24_{eab} (or 9_e). - The point represents multiplication by $(P_{10}/P_9)_N$.

Point 25_{eab} (or 10_e). - Draw a straight line through points 23_{eab} (or 8_e) and 24_{eab} (or 9_e) to intersect line G at point 25_{eab} (or 10_e).

Operation 25_{eab} (or 10_e). - The point represents the product of afterburner-component total-pressure ratios:

$$\left(\frac{P_{10}}{P_5}\right)_{F,M,N} = \left(\frac{P_6}{P_5}\right)_F \left(\frac{P_9}{P_6}\right)_M \left(\frac{P_{10}}{P_9}\right)_N$$

Step (4)

Point 26_{eab} (or 11_e). - Locate the value of P_5 on line H. [Point $26'_{eab}$ (or $11'_e$). - If the engine compression ratio is known, locate P_5/p_0 on line H.]

Operation 26_{eab} (or 11_e). - The point represents multiplication by P_5. [Operation $26'_{eab}$ (or $11'_e$). - The point represents multiplication by P_5/p_0.]

Point 27_{eab} (or 12_e). - Draw a straight line through points 25_{eab} (or 10_e) and 26_{eab} (or 11_e) to intersect line I at point 27_{eab} (or 12_e). [Point $27'_{eab}$ (or $12'_e$). - Draw a straight line through points 25_{eab} (or 10_e) and $26'_{eab}$ (or $11'_e$) to intersect line I at point $27'_{eab}$ (or $12'_e$).] 3991

Operation 27_{eab} (or 12_e). - The point represents the value of the nozzle-exit total pressure $P_{10} = P_5 \left(\frac{P_{10}}{P_5}\right)_{F,M,N}$. Read this value on the right scale of line I to be used as point 29_{eab} (or 14_e), line B, figure 6. [Operation $27'_{eab}$ (or $12'_e$). - The point represents the over-all engine compression ratio P_{10}/p_0. Read the value for P_{10}/p_0 at point $27'_{eab}$ (or $12'_e$) to be used as point 30_{eab} (or 15_e), line C, fig. 6.] NOTE: The two scales (P_5 and P_5/p_0; P_{10} and P_{10}/p_0) on each of lines H and I are not related by positions on the lines. Therefore, if one scale is used initially (e.g., a scale marked with '), the corresponding scale must be used for the next line (').

Figure 6:

Step (1)

Point 28_{eab} (or 13_e). - Locate the altitude on line A of figure 6.

Operation 28_{eab} (or 13_e). - The point represents the values of ambient pressure p_0 (given on the right scale of line A) at the flight altitude.

Point 29_{eab} (or 14_e). - Locate P_{10} (from point 27_{eab} (or 12_e)) on line B.

Operation 29_{eab} (or 14_e). - The point represents division by P_{10}.

Point 30_{eab} (or 15_e). - Draw a straight line through points 28_{eab} (or 13_e) and 29_{eab} (or 14_e) to intersect line C at point 30_{eab} (or 15_e). If (P_{10}/p_0) was determined for point $27'_{eab}$ (or $12'_e$), locate the value on the left scale of line C.

Operation 30_{eab} (or 15_e). - The point represents the value of equation (5):

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$$\frac{p_0}{P_{10}} = p_0 \left[\frac{1}{P_5}\left(\frac{P_5}{P_6}\right)_F \left(\frac{P_6}{P_9}\right)_M \left(\frac{P_9}{P_{10}}\right)_N\right]$$

The left scale (P_{10}/p_0) is the reciprocal of the right scale.

Step (2)

Point 31_{eab} (or 16_e). - Locate $\gamma_{10} = \gamma_9$ on line D.

Operation 31_{eab} (or 16_e). - The point represents multiplication by

$$f(\gamma_{10}) = \frac{(1 + \gamma_{10})^{\frac{1}{\gamma_{10}-1}}}{(2)^{\frac{\gamma_{10}}{\gamma_{10}-1}}}$$

Point 32_{eab} (or 17_e). - Draw a straight line through points 30_{eab} (or 15_e) and 31_{eab} (or 16_e) to intersect line E at point 32_{eab} (or 17_e).

Operation 32_{eab} (or 17_e). - The point represents the value of $f(P_{10},p_0,\gamma_{10}) = 1 - f(\gamma_{10})\,\frac{p_0}{P_{10}}$.

Steps (3) to (5) for figure 6 correspond to steps (3) to (5) for figure 2, with 32_{eab} (or 17_e) being a value comparable to 10_{eab} (or 3_e).

Exact nomographic method for $p_{10} = p_0$. - Figures 3, 4, 5, and 7 comprise the exact nomographic method for complete expansion of exhaust products. This calculation method is identical with the exact nomographic method for $M_{10} = 1.0$ with the exception of the function $f(P_{10}, p_0, \gamma_{10})$ in the net thrust equation (compare eqs. (3) and (4)). This difference occurs between lines C and E of figures 6 and 7. With these regions excepted, all equations, assumptions, nomographs, procedures, and operations for figures 3 to 6 apply to the exact nomographic method for $p_{10} = p_0$. The points of similarity will not be repeated. 3991

The following are procedures and operations for determining $f(P_{10}, p_0, \gamma_{10})_{p_{10}=p_0}$ using lines C', D, and E of figure 7:

Point $30'_{eab}$ (or $15'_e$). - Locate the value of p_0/P_{10} (from point 30_{eab} or 15_e on line C) on line C'.

Point 31_{eab} (or 16_e). - Place the value of γ_{10} on line D.

Point 32_{eab} (or 17_e). - Draw a straight line through points 31_{eab} (or 16_e) and $30'_{eab}$ (or $15'_e$) to intersect line E at point 32_{eab} (or 17_e).

Operations $30'_{eab}$ (or $15'_e$), 31_{eab} (or 16_e), and 32_{eab} (or 17_e). - The function $f(P_{10}, p_0, \gamma_{10})_{p_{10}=p_0} = \sqrt{\frac{\gamma_{10}^2}{\gamma_{10}^2 - 1}\left[1 - \left(\frac{p_0}{P_{10}}\right)^{\frac{\gamma_{10}-1}{\gamma_{10}}}\right]}$ is computed; point 32_{eab} (or 17_e) represents the value of this function.

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2. Tower, Leonard K., and Gammon, Benson E.: Analytical Evaluation of Effect of Equivalence Ratio, Inlet-Air Temperature, and Combustion Pressure on Performance of Several Possible Ram-Jet Fuels. NACA RM E53G14, 1953.

3. Tower, Leonard K.: Analytic Evaluation of Effect of Inlet-Air Temperature and Combustion Pressure on Combustion Performance of Boron Slurries and Blends of Pentaborane in Octene-1. NACA RM E55A31, 1955.

4. Turner, L. Richard, and Bogart, Donald: Constant-Pressure Combustion Charts Including Effects of Diluent Addition. NACA Rep. 937, 1949. (Supersedes NACA TN's 1086 and 1655.)

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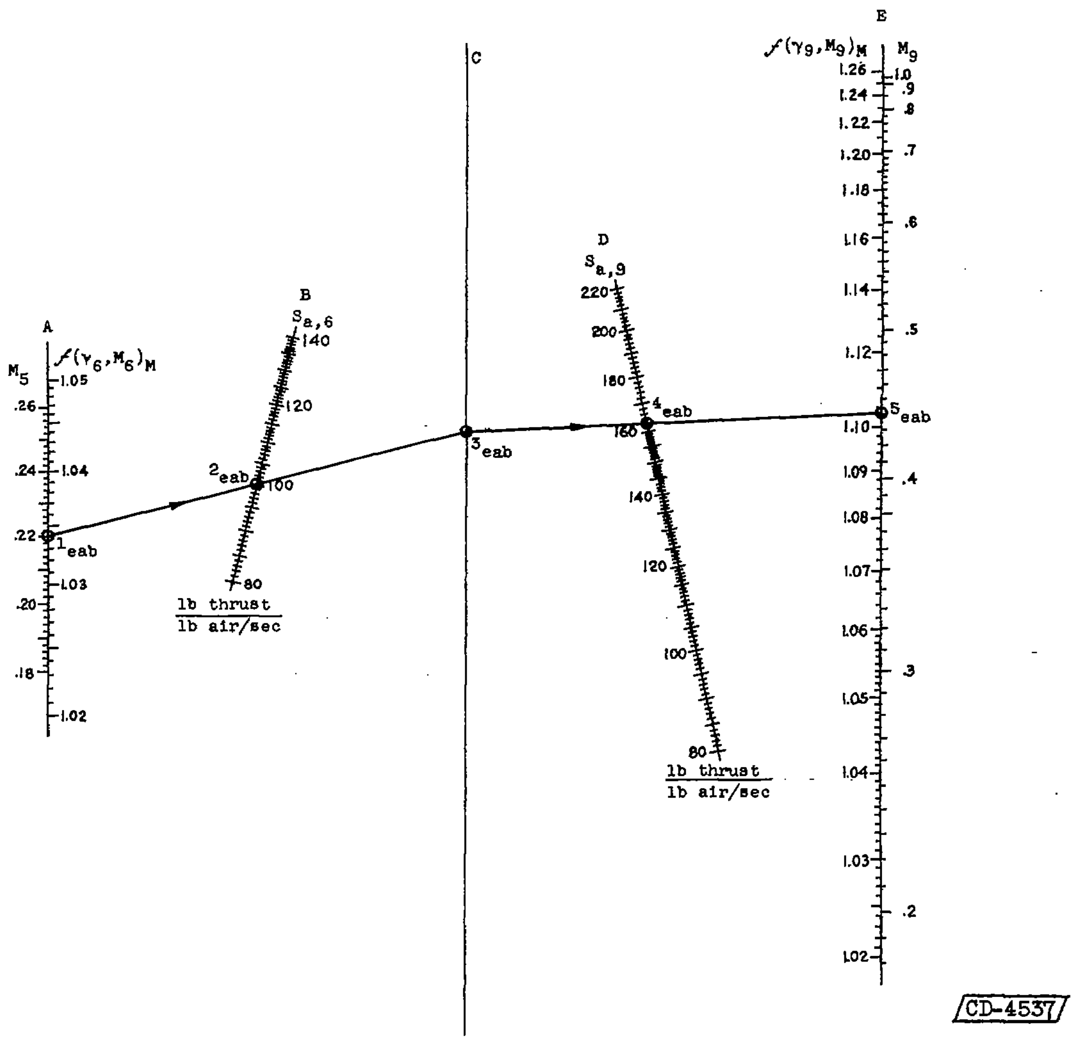


Figure 1. - Nomograph for approximate determination of Mach number downstream of combustion zone and functions used to compute total-pressure ratio across combustion zone. (A large working copy of this figure may be obtained by using the request card bound in the back of the report.)

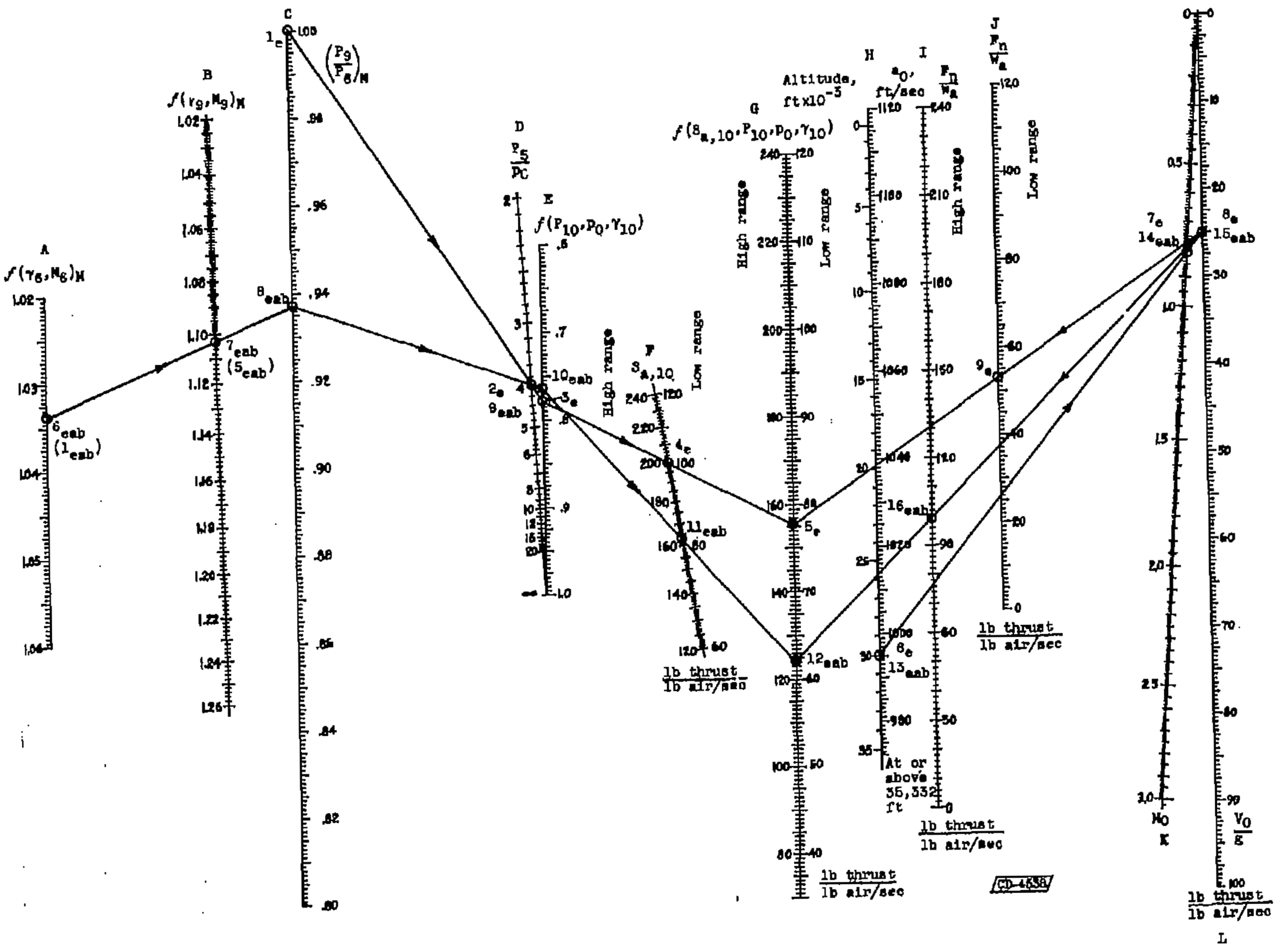


Figure 2. - Nomograph for approximate determination of net thrust for expansion of exhaust products to nozzle-exit Mach number of 1.0. (A large working copy of this figure may be obtained by using the request card bound in the back of the report.)

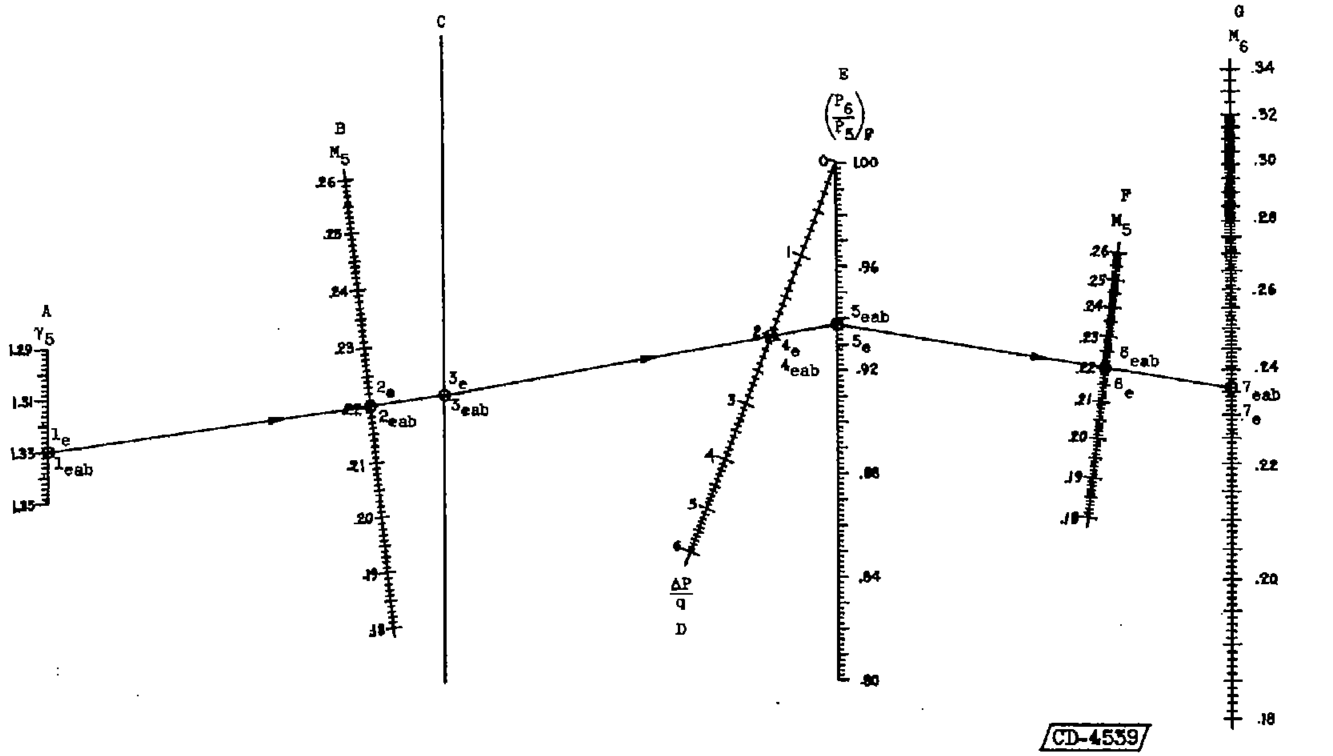


Figure 3. - Nomograph for determination of total-pressure ratio across flameholder and Mach number downstream of flameholder. (A large working copy of this figure may be obtained by using the request card bound in the back of the report.)

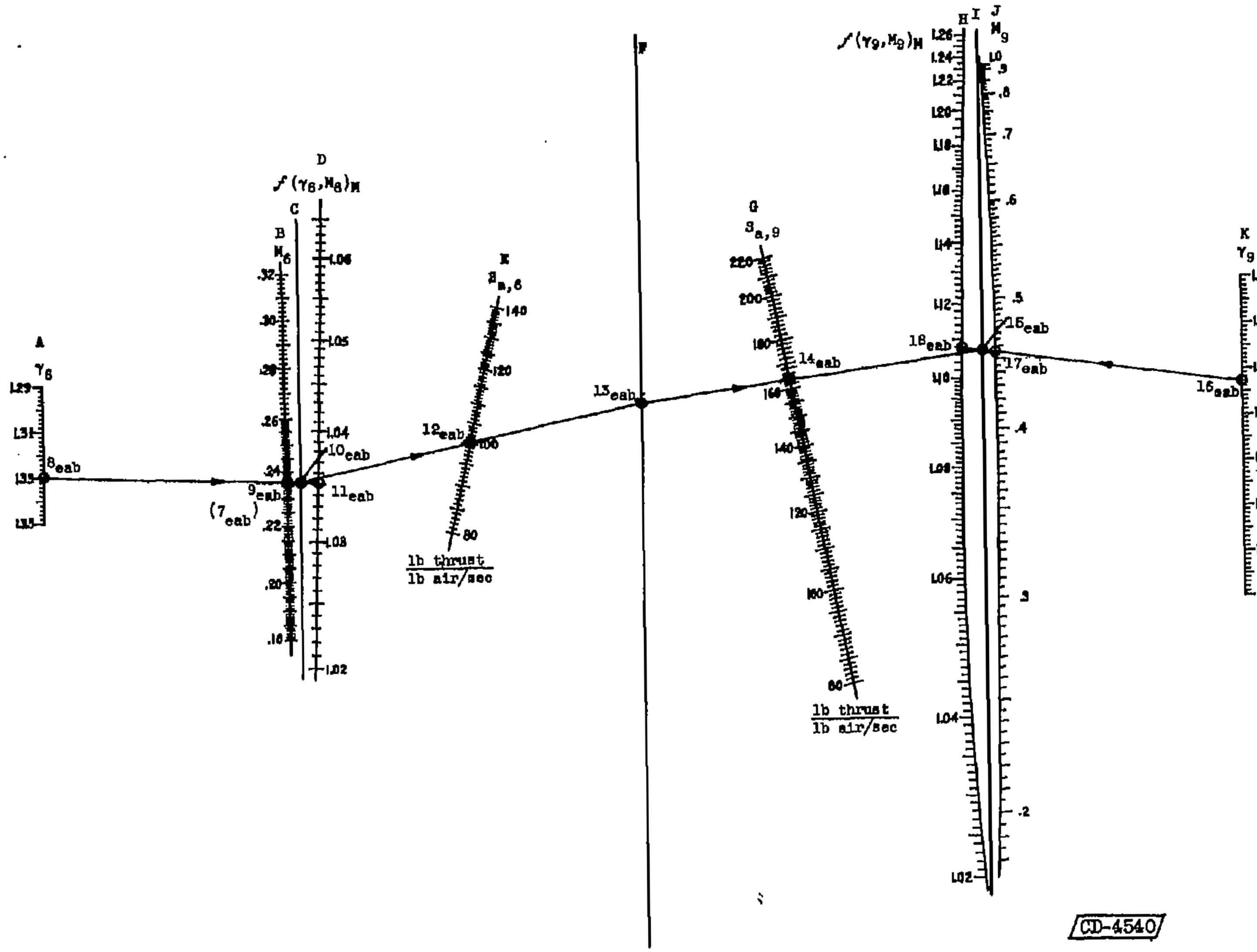


Figure 4. - Nomograph for determination of Mach number downstream of combustion zone and functions used to compute total-pressure ratio across combustion zone. (A large working copy of this figure may be obtained by using the request card bound in the back of the report.)

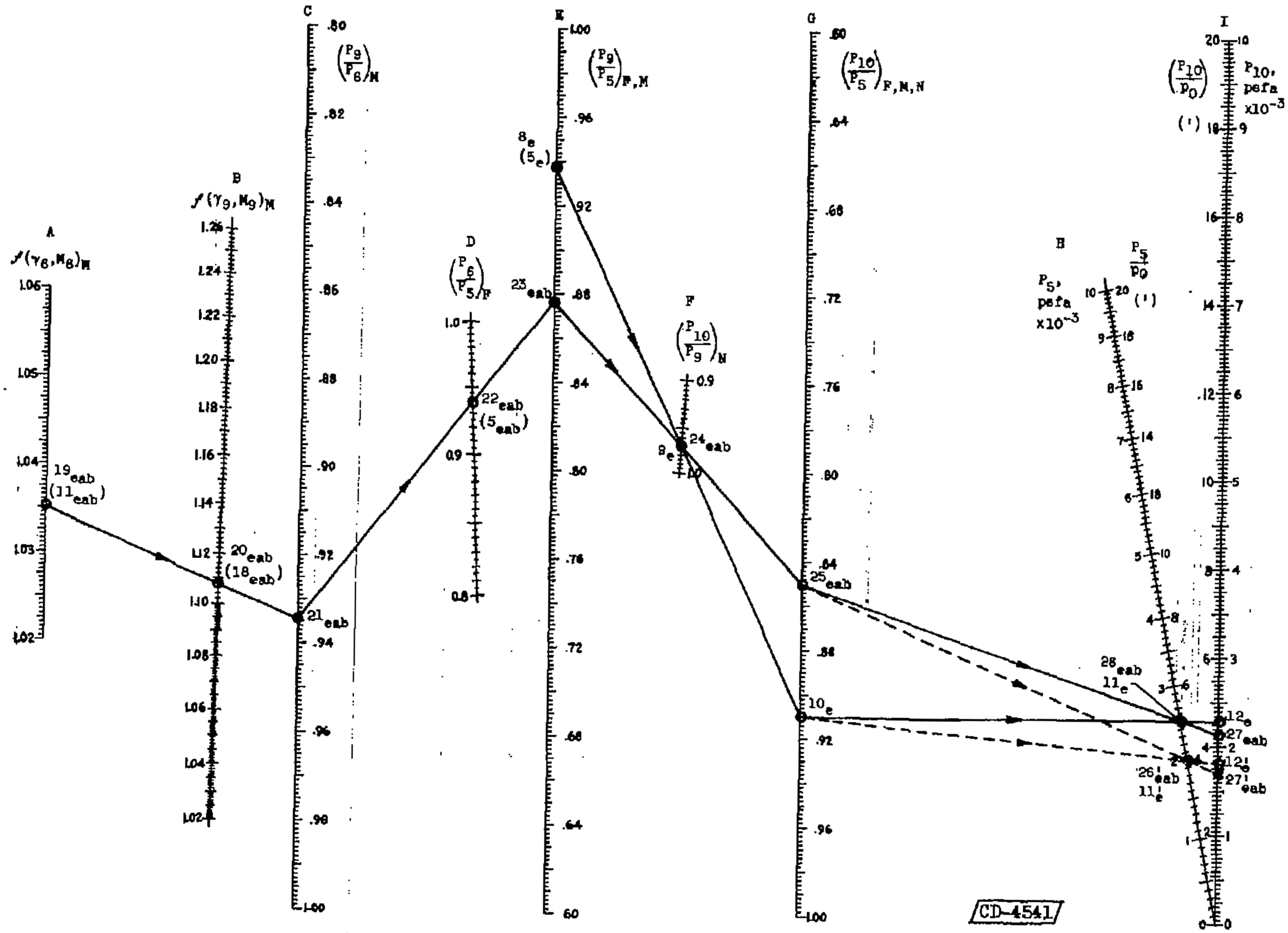
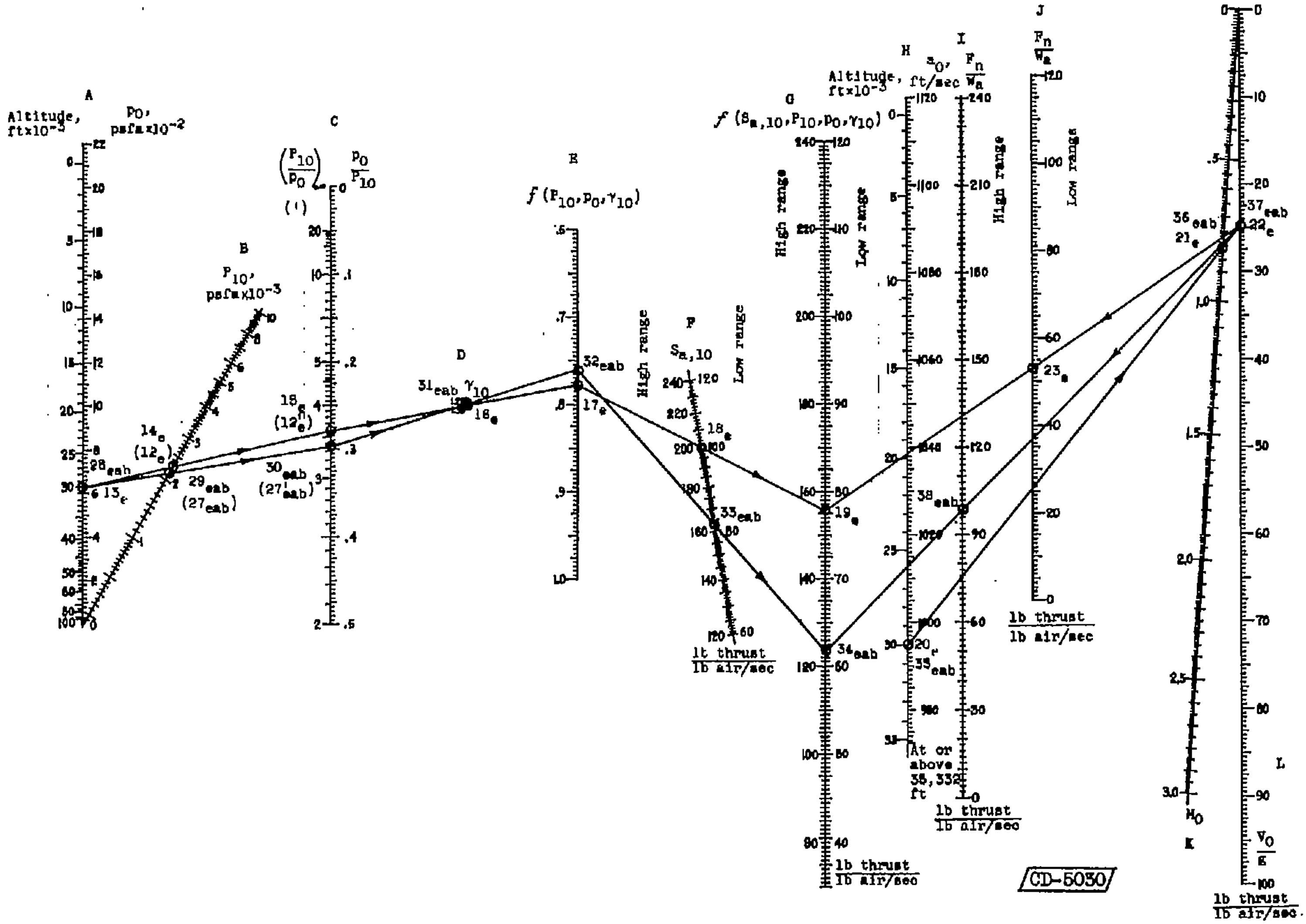
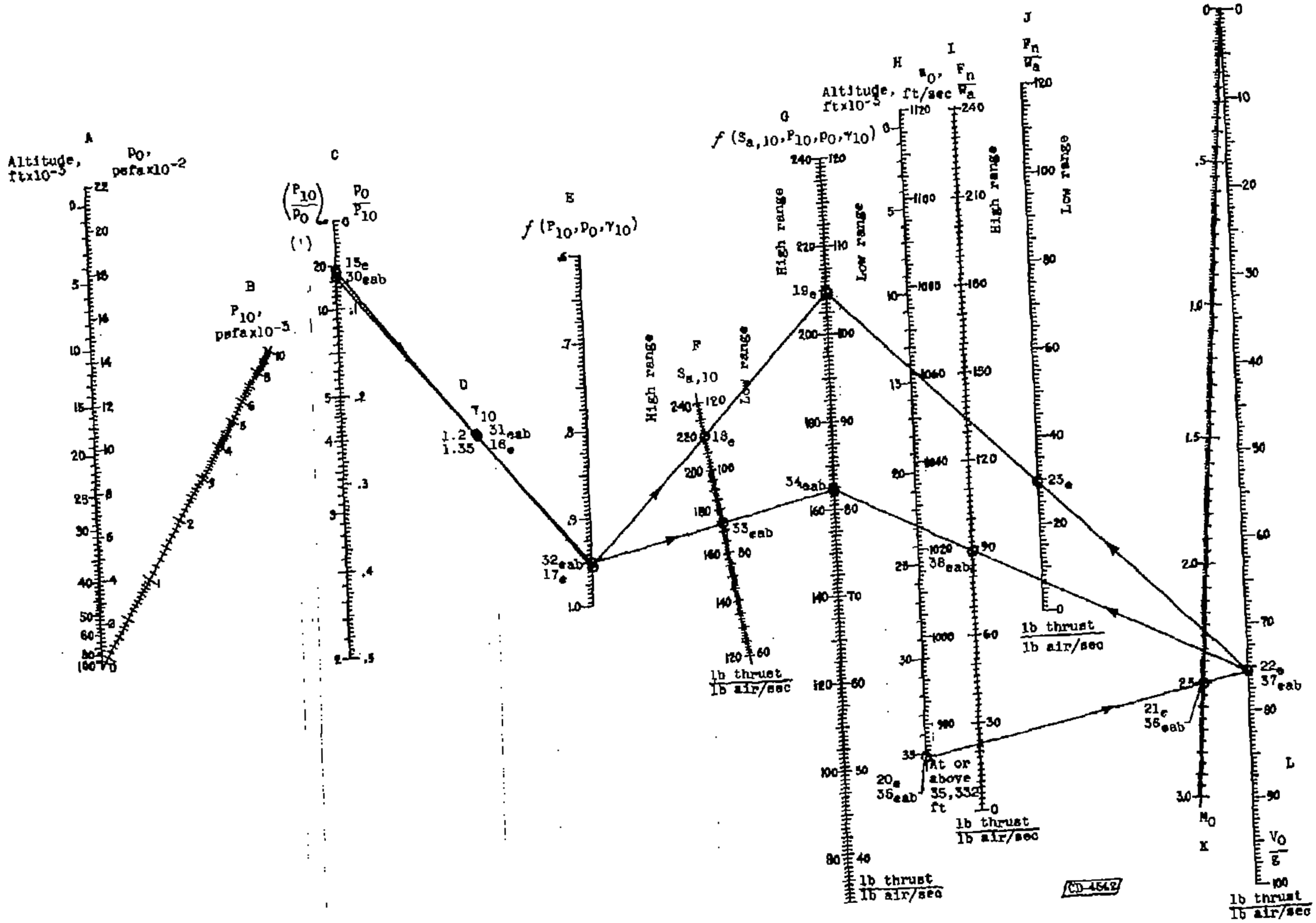


Figure 5. - Nomograph for determination of total-pressure ratio across combustion zone and total pressure at nozzle exit. (A large working copy of this figure may be obtained by using the request card bound in the back of the report.)



(a) Example 1.

Figure 6. - Nomograph for determination of net thrust for expansion of exhaust products to nozzle-exit Mach number of 1.0. (A large working copy of this figure may be obtained by using the request card bound in the back of the report.)



(b) Example 2.

Figure 6. - Concluded. Nomograph for determination of net thrust for expansion of exhaust products to nozzle-exit Mach number of 1.0. (A large working copy of this figure may be obtained by using the request card bound in the back of the report.)

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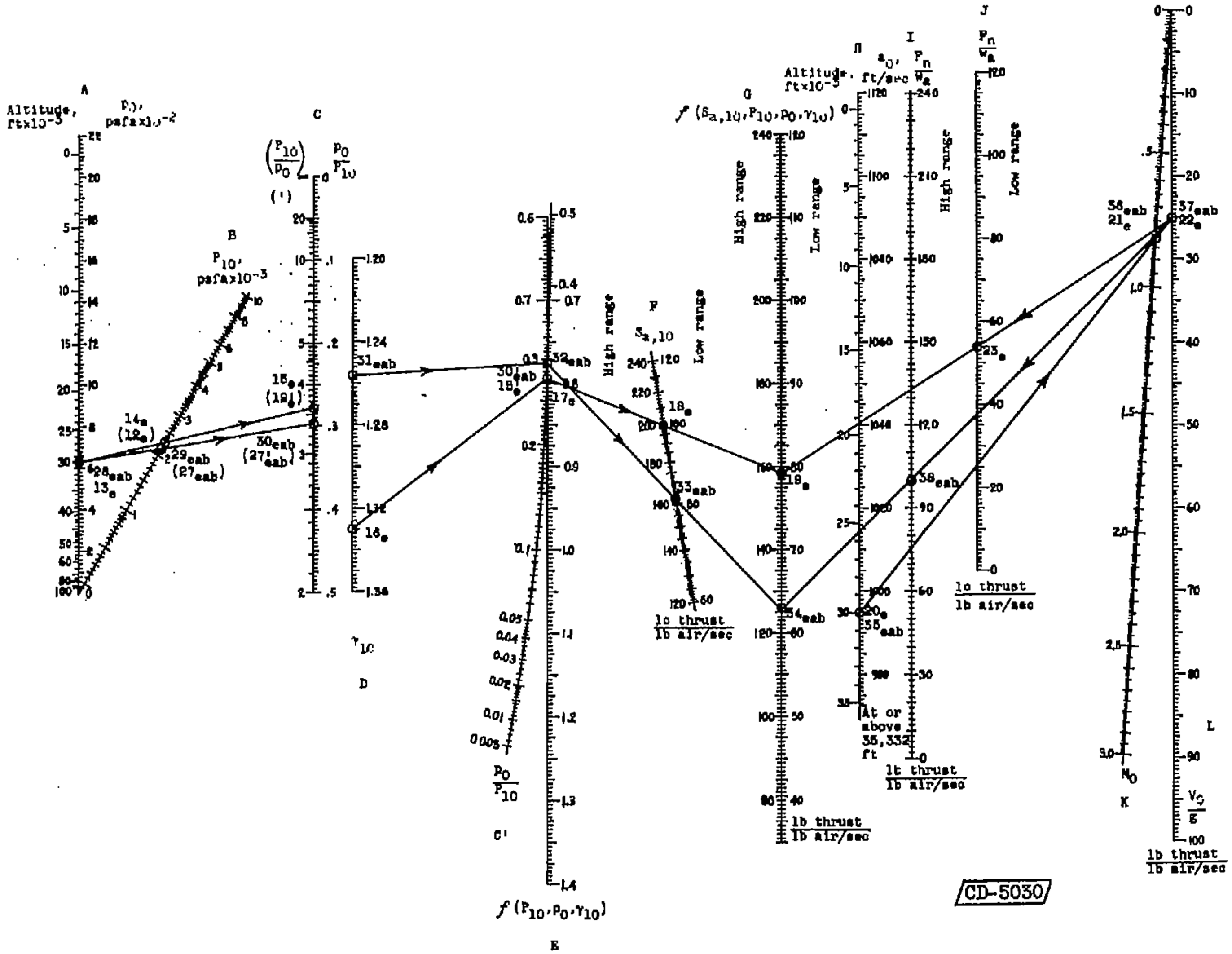
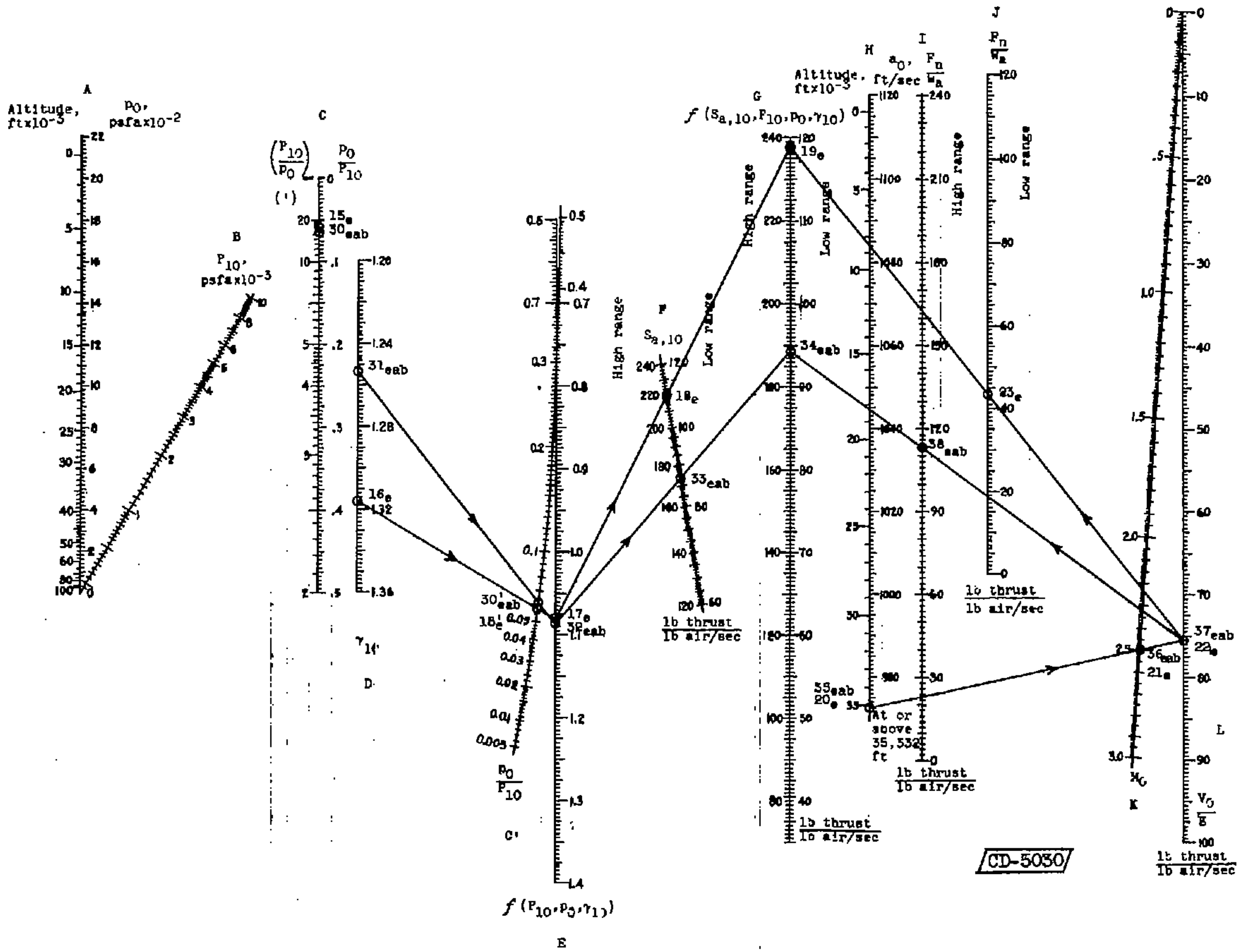


(a) Example 1.

Figure 7. - Nomograph for determination of net thrust for complete expansion of exhaust products. (A large working copy of this figure may be obtained by using the request card bound in the back of the report.)



(b) Example 2.

Figure 7. - Concluded. Nomograph for determination of net thrust for complete expansion of exhaust products. (A large working copy of this figure may be obtained by using the request card bound in the back of the report.)

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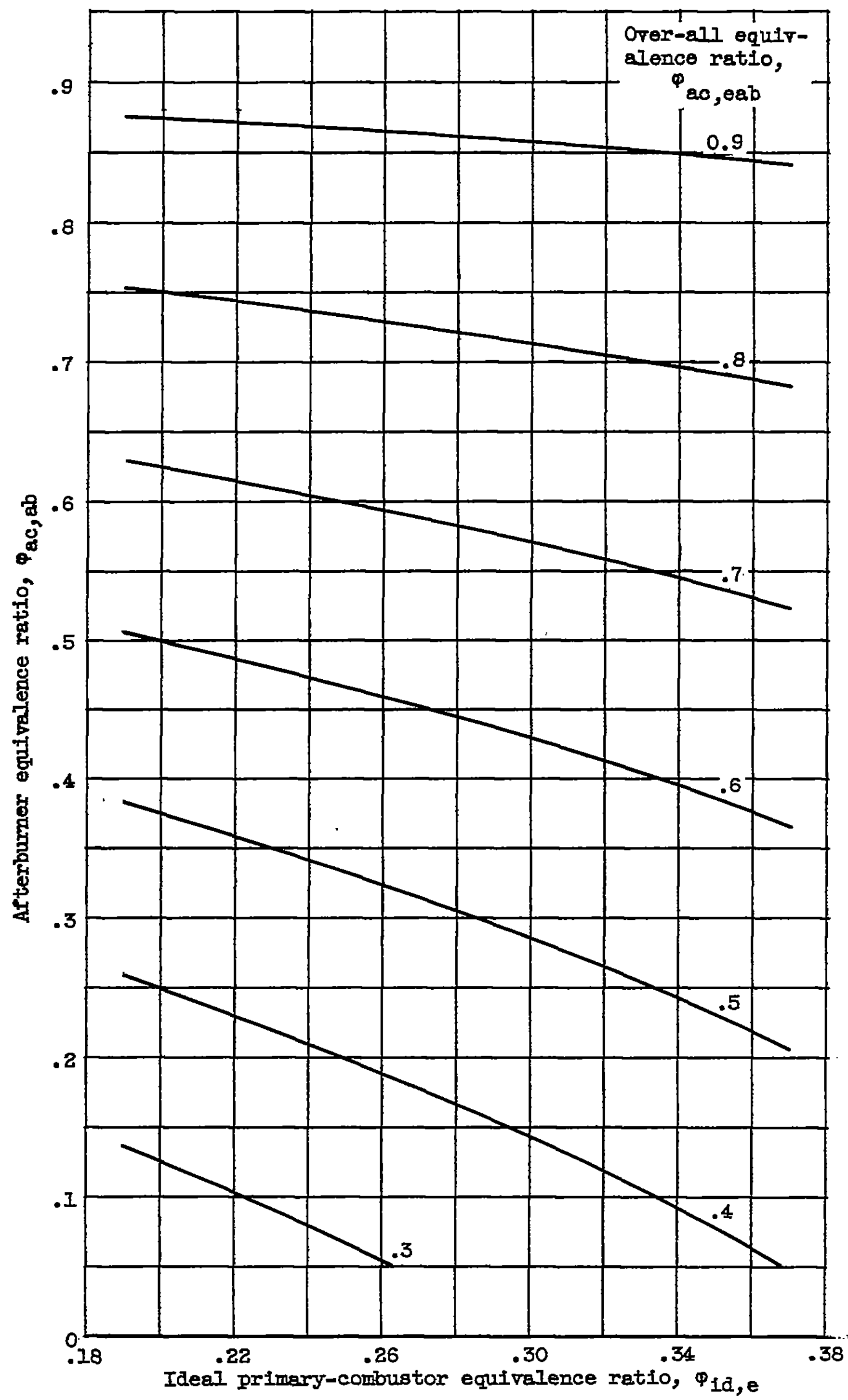

Figure 8. - Variation of afterburner equivalence ratio with ideal primary-combustor equivalence ratio and over-all equivalence ratio.

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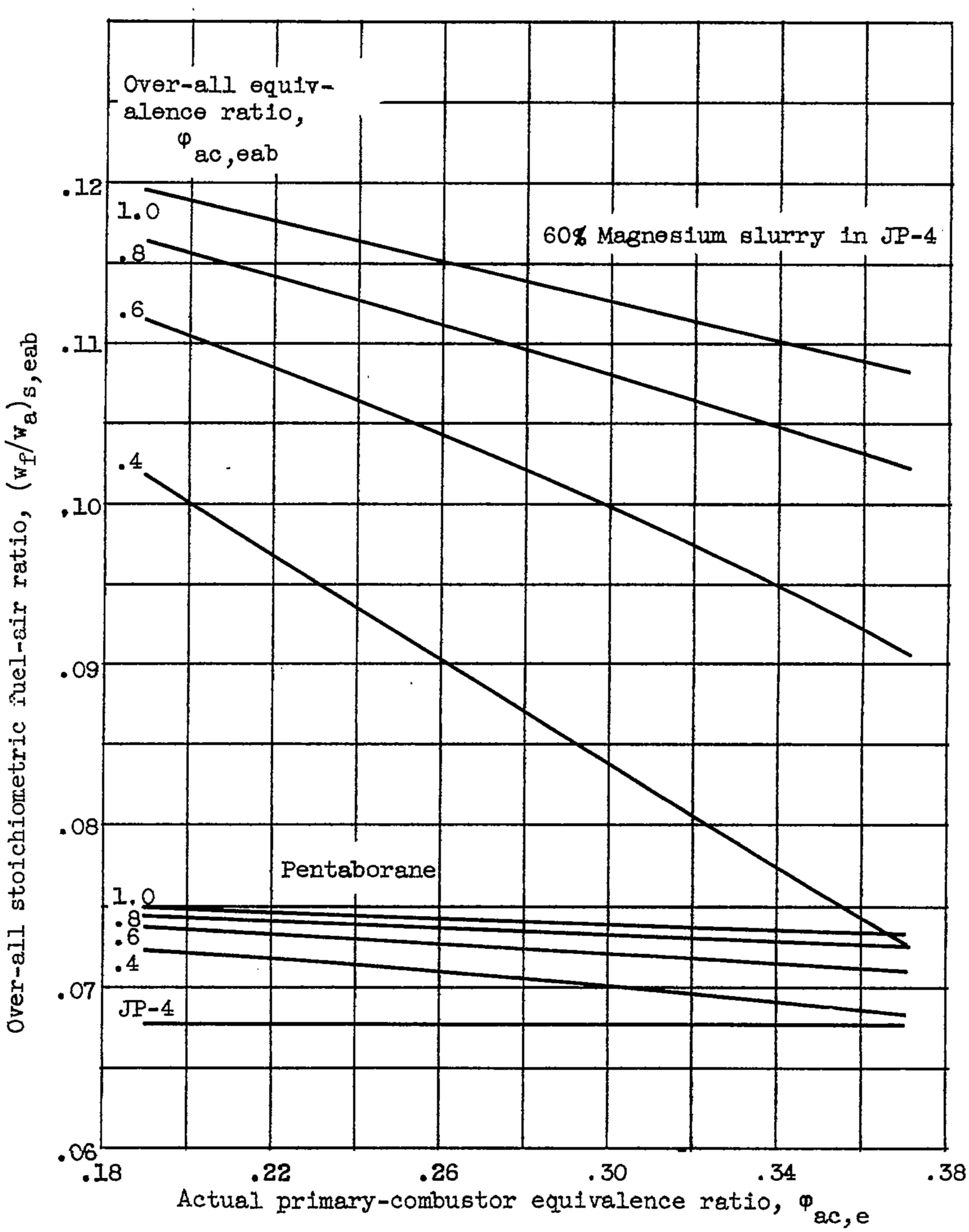

Figure 9. - Over-all stoichiometric fuel-air ratio for three afterburner fuels used with JP-4 fuel in primary combustors.

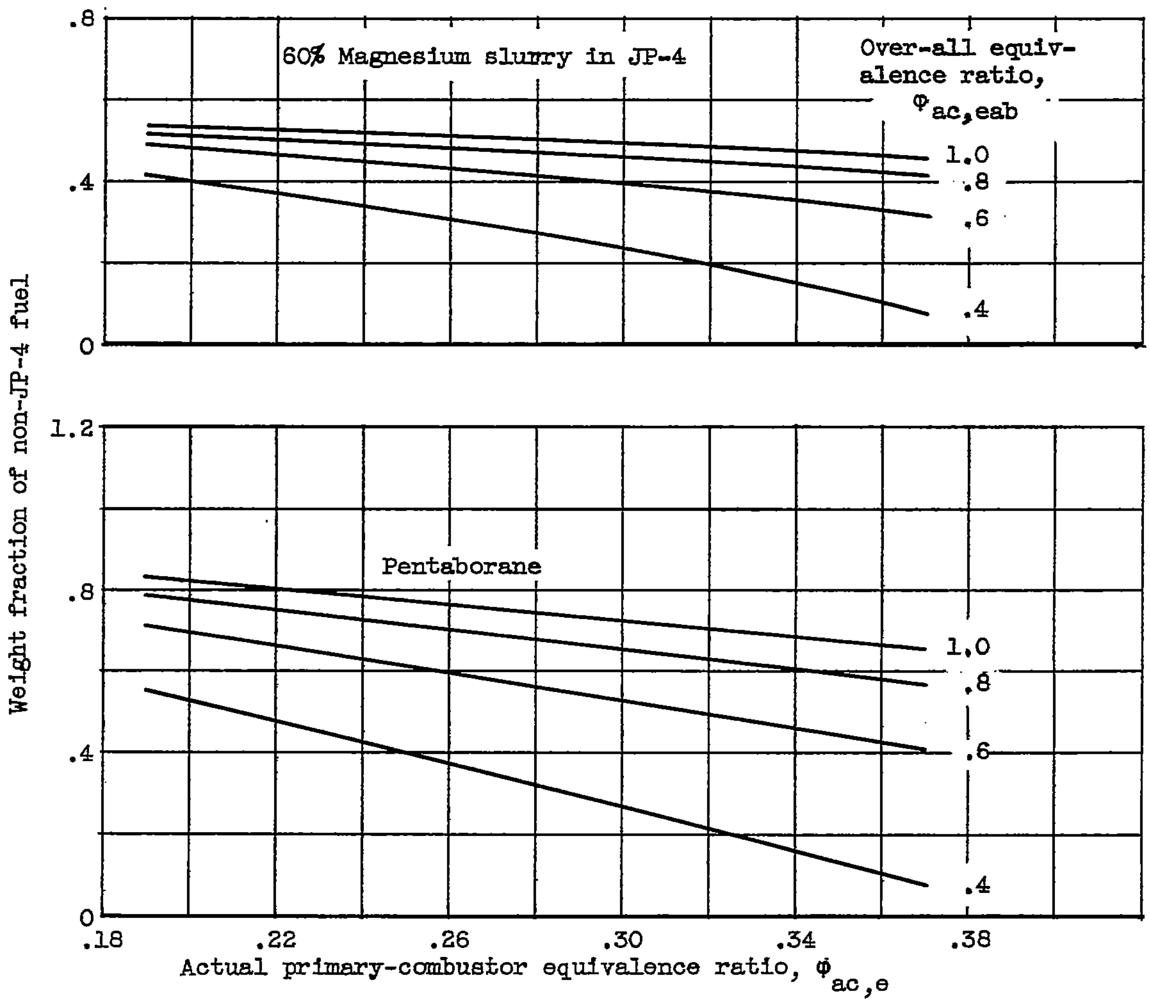

Figure 10. - Weight fraction of non-JP-4 fuel in over-all fuel mixture for two afterburner fuels used with JP-4 fuel in primary combustors.

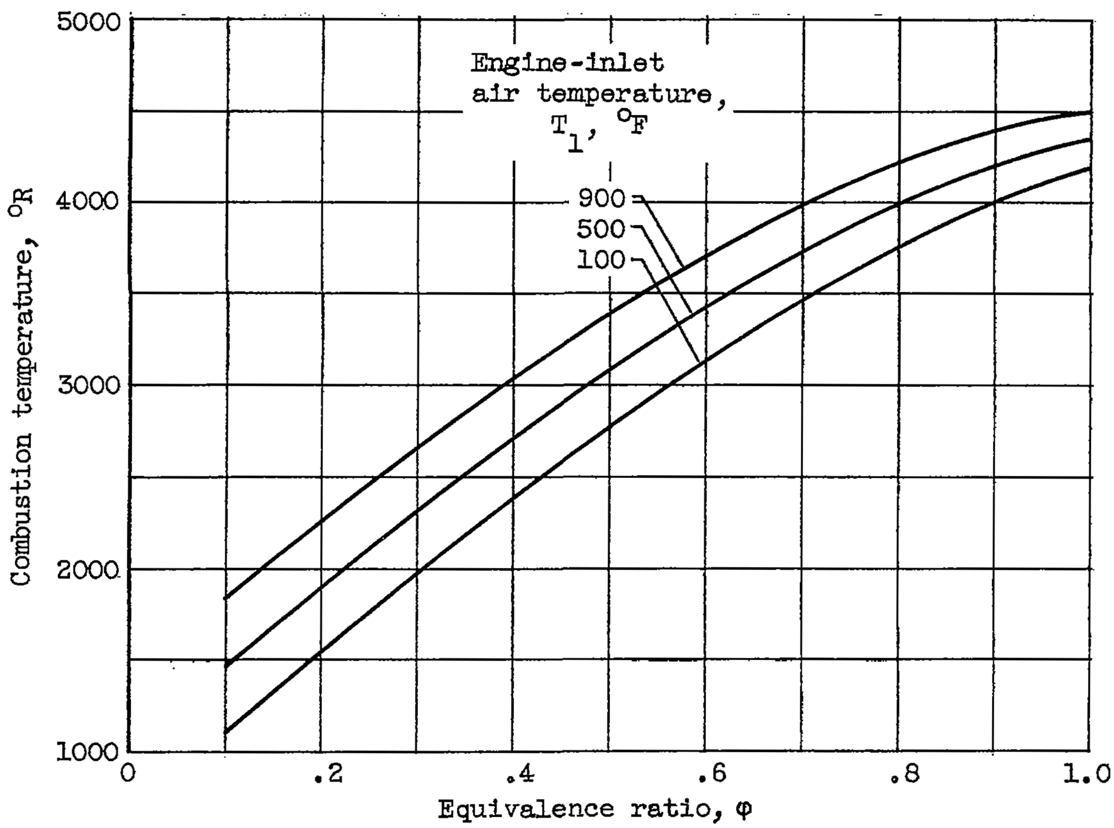

Figure 11. - Variation of combustion temperature with equivalence ratio and inlet air temperature for JP-4 fuel at combustion pressure of 2 atmospheres.

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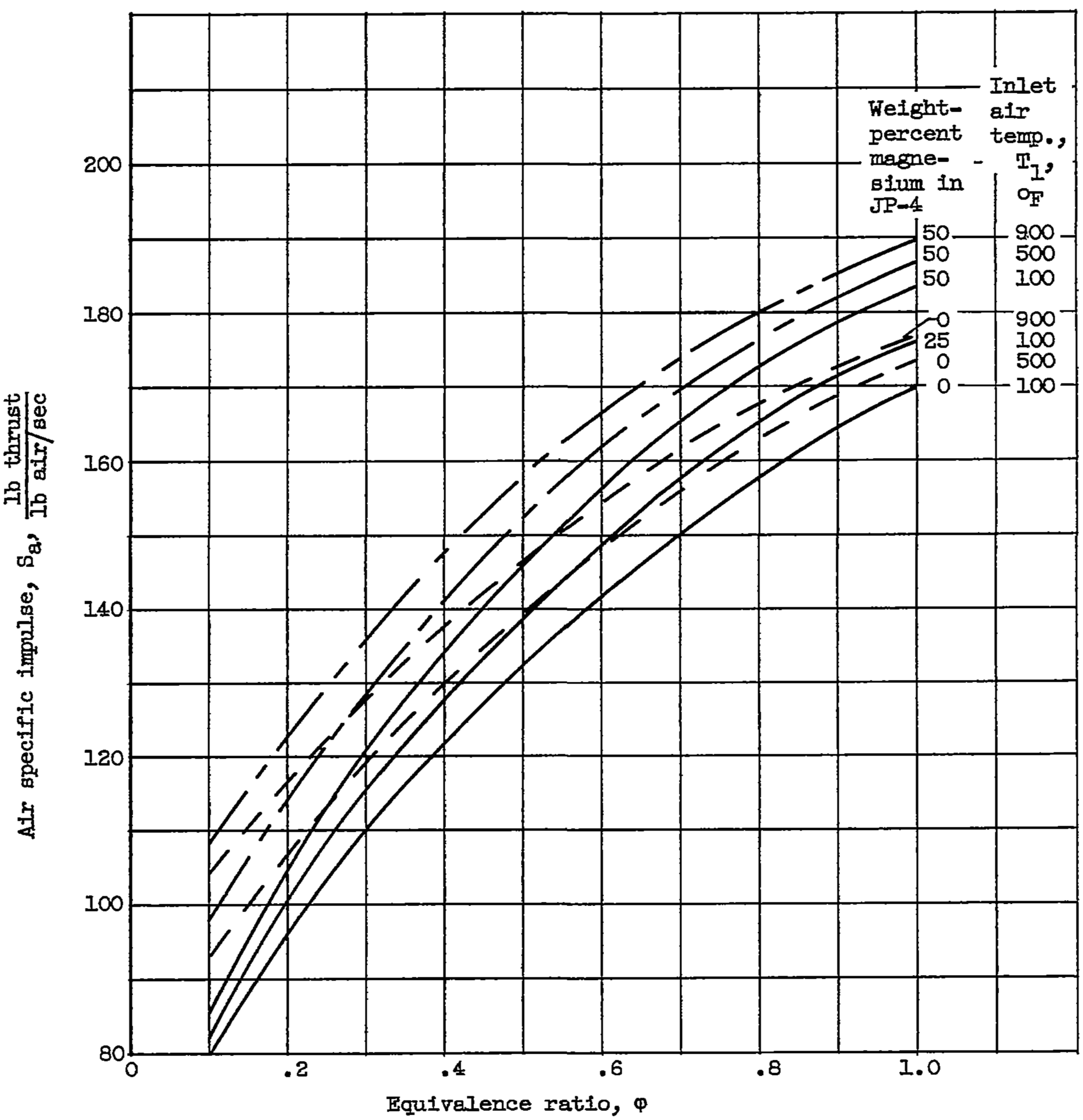

Figure 12. - Variation of air specific impulse with equivalence ratio, magnesium concentration, and inlet air temperature for slurries of magnesium in JP-4 fuel at combustion pressure of 2 atmospheres.

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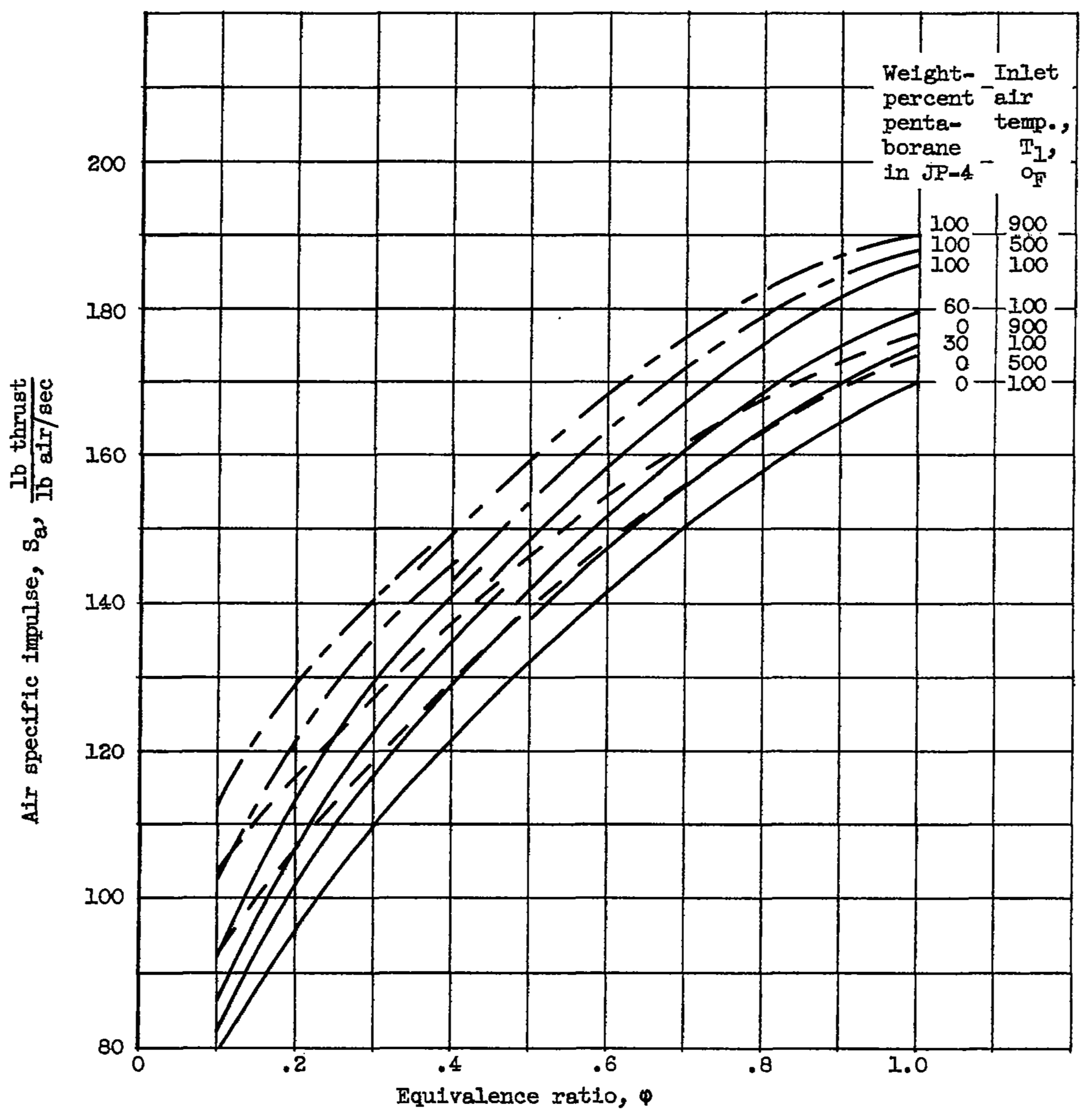

Figure 13. - Variation of air specific impulse with equivalence ratio, pentaborane concentration, and inlet air temperature for blends of pentaborane and JP-4 fuels at combustion pressure of 2 atmospheres.

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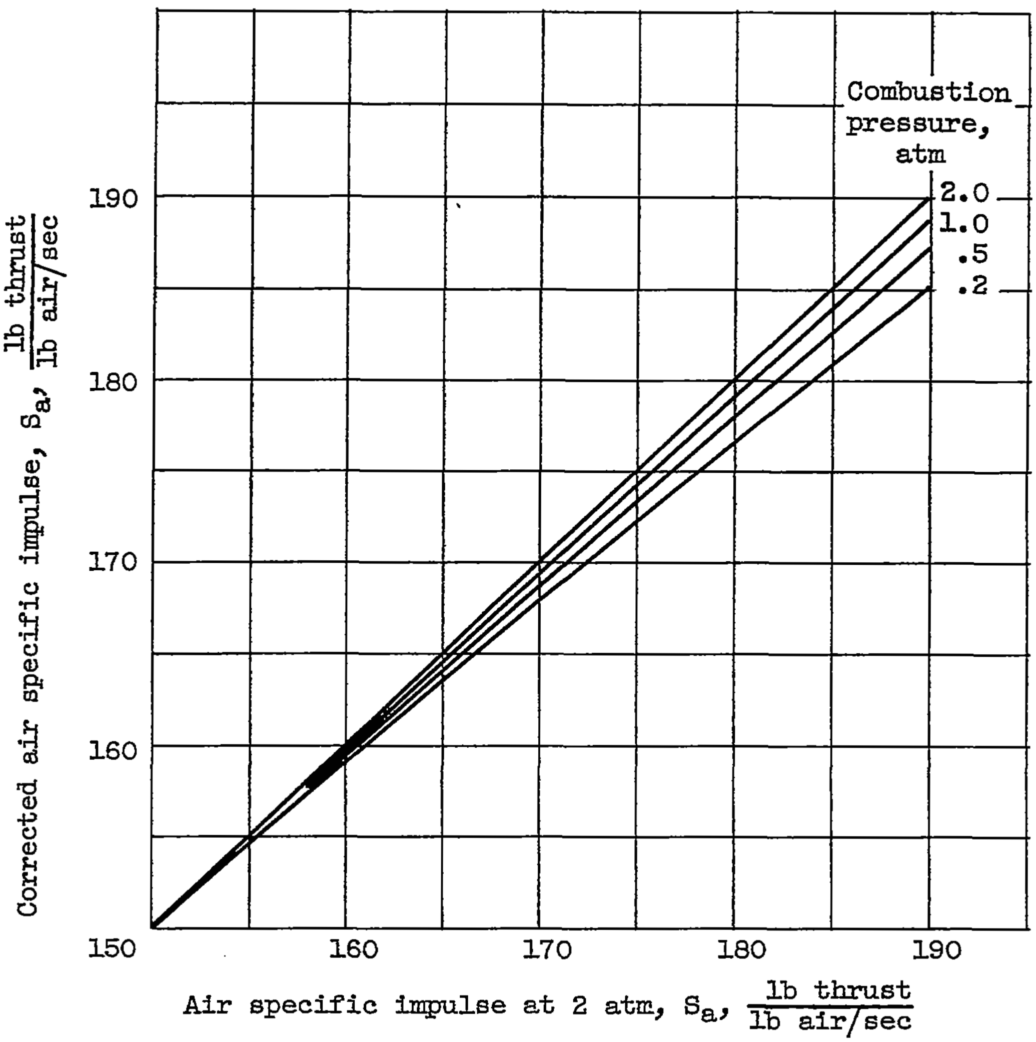

Figure 14. - Variation of air specific impulse with combustion pressure and air specific impulse at combustion pressure of 2 atmospheres for inlet air temperature of 100° F.

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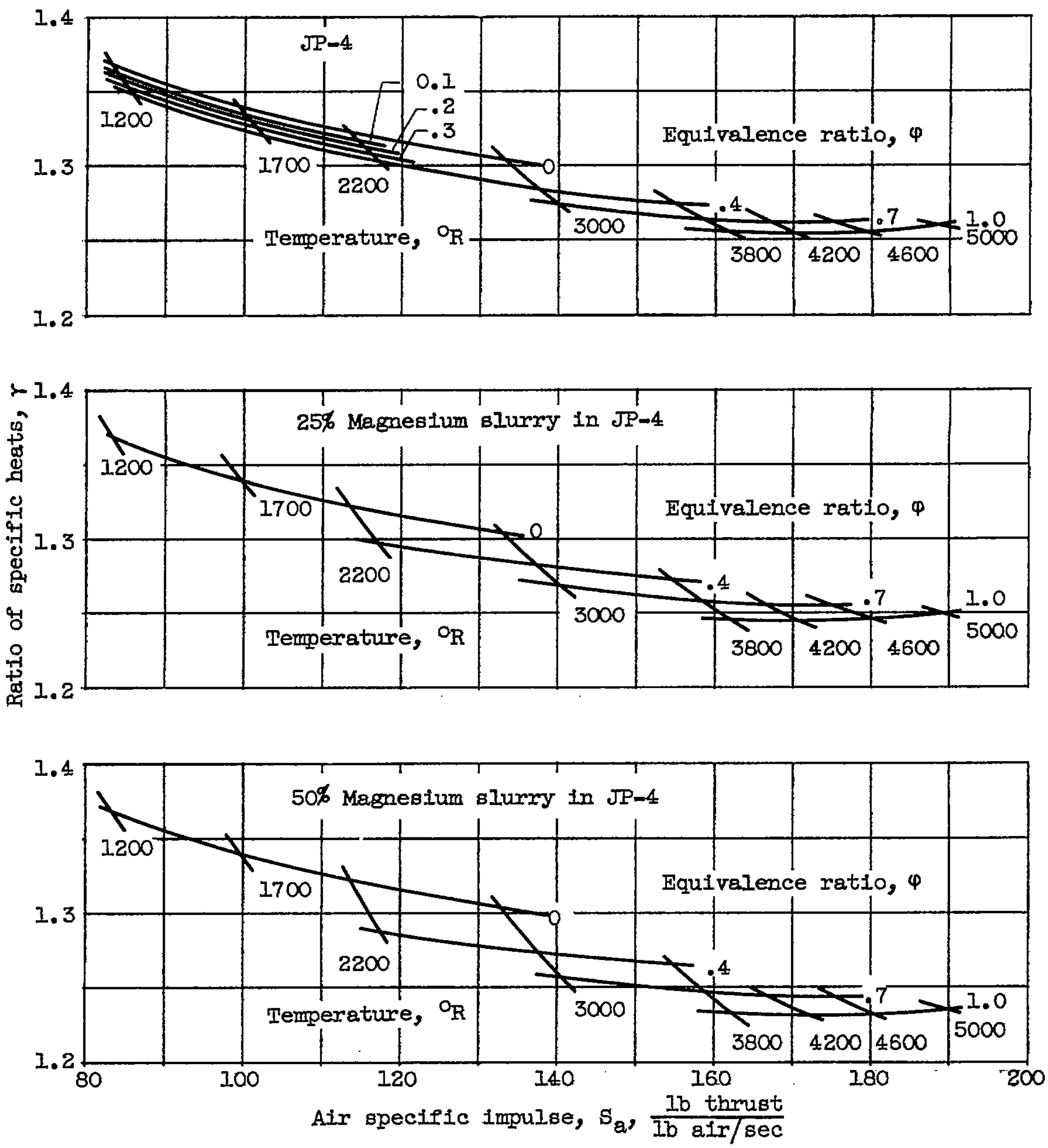

Figure 15. - Variation of specific-heats ratio with air specific impulse, equivalence ratio, and magnesium concentration for combustion of magnesium slurries in JP-4 fuel at combustion pressure of 2 atmospheres.

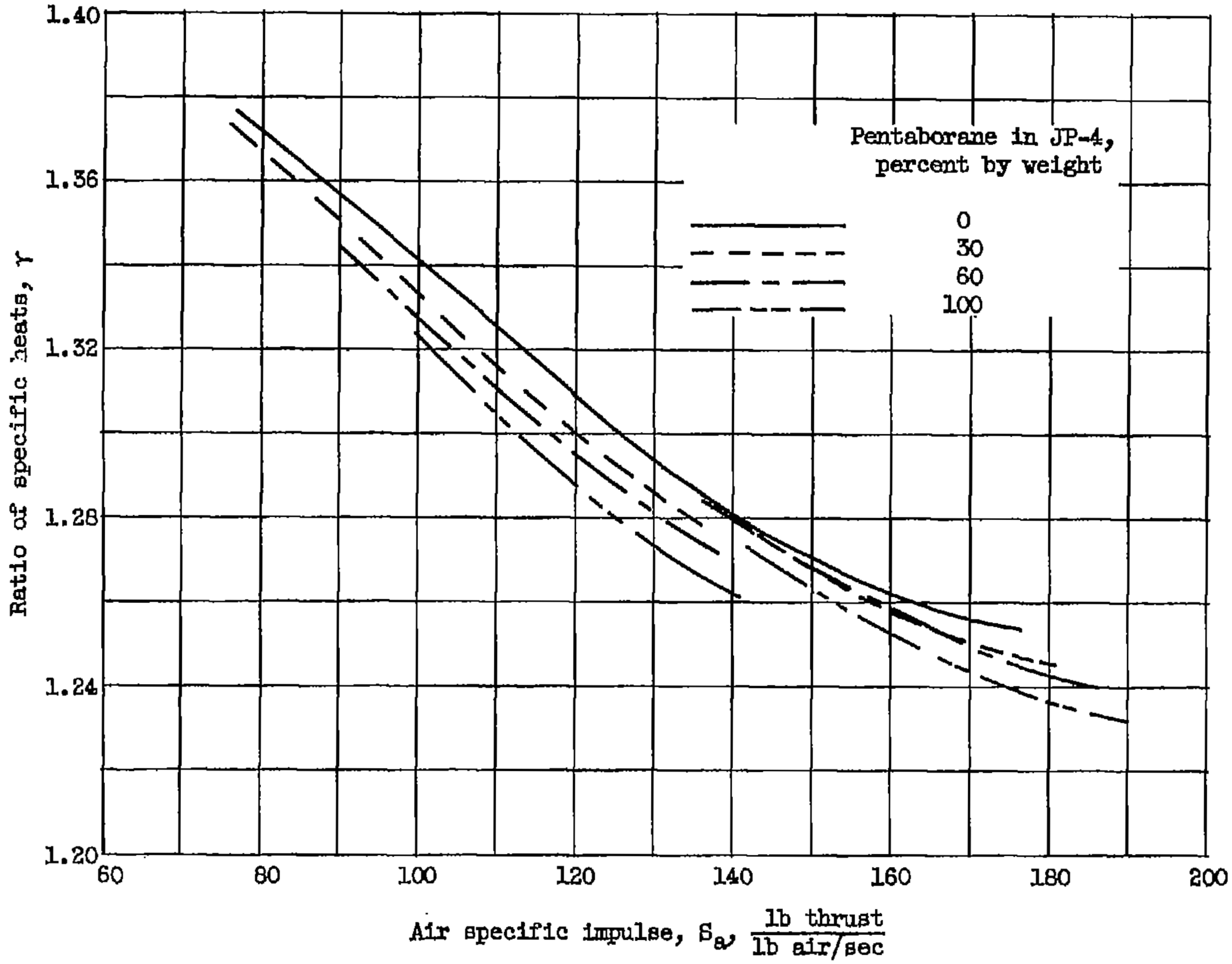

Figure 16. - Variation of specific-heats ratio with air specific impulse and pentaborane concentration for combustion of blends of pentaborane and JP-4 fuels at combustion pressure of 2 atmospheres for inlet air temperature of 100° F.

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