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RESEARCH MEMORANDUM

ANALYSIS OF TWO-STAGE-TURBINE EFFICIENCY CHARACTERISTICS IN TERMS OF WORK AND SPEED REQUIREMENTS

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RESEARCH MEMORANDUM

ANALYSIS OF TWO-STAGE-TURBINE EFFICIENCY CHARACTERISTICS IN TERMS OF WORK AND SPEED REQUIREMENTS

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SUMMARY

The effects of variations in mean-section velocity diagrams on two-stage-turbine efficiency characteristics are analyzed. The fundamental correlating parameter used is a work-speed parameter λ (ratio of square of mean-section blade speed to required specific work output), used previously in single-stage-turbine efficiency studies, which is the same as the familiar Parson's characteristic number. The range of the parameter considered is that of interest for turbojet accessory- and turbopump-drive turbines. The assumptions and limitations are the same as those used in single-stage analyses. The principal limit, that of impulse conditions across any blade row, results in a range of stage-work split and velocity diagram for a given λ. The points yielding the maximum value of efficiency for a given λ are used in describing the results. The efficiencies considered are total or aerodynamic, rating, and static.

For zero exit whirl a maximum total efficiency occurs at λ of 0.50, the efficiency decreasing somewhat as λ approaches its lower limit of 0.125. As λ is reduced from 0.50, the first stage develops progressively more work until, at the lower limit, a 75-25 work split exists. The principal effect of imposing negative exit whirl is to reduce the lower limit of λ. However, as λ is reduced from 0.125 the efficiency decreases markedly, and the work split tends to return to an equal split. For λ below 0.25 the maximum efficiencies change only slightly over the range of turbine exit whirls considered.

Comparison of the results for two-stage and single-stage turbines shows considerable improvement in efficiency for the two-stage turbine over the λ range studied. Thus, multistaging is often desirable to obtain high efficiency. Variations in total efficiencies of one- and two-stage turbines, compared on the basis of an average stage λ, are approximately the same.

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INTRODUCTION

As part of the general program concerned with the study of turbine performance characteristics, the NACA Lewis laboratory is currently analyzing the effect of work and speed requirements on turbine efficiency. The fundamental parameter λ used in this analysis is the same as Parson's characteristic number described in reference 1, which is defined as the ratio of the square of the mean-section blade speed to the specific work output at design conditions. In reference 2 this parameter is used in the study of the efficiency characteristics of single-stage 4353
turbines. This parameter is also used in studying the effect of including downstream stator blades on the efficiency of single-stage turbines in reference 3, which indicates that over a certain range of λ a significant improvement in efficiency is attainable by using such stators.

In this report two-stage-turbine efficiency characteristics are analyzed as a function of work and speed requirements in terms of the effect of changing the required mean-section velocity diagram. The fundamental assumptions and limits used in references 2 and 3 are also used herein. Since the limit of impulse conditions across any blade row (described in ref. 2) applies to three blade rows in this study, the resulting range of operation becomes very important and is considered in detail in describing the consequent limits on the parameters affected.

The three types of efficiency considered herein are the same as those studied in references 2 and 3:

(1) Total or aerodynamic efficiency, which includes all aerodynamic losses

(2) Rating efficiency, which, in addition to the aerodynamic losses, considers the turbine exit whirl a loss (used in jet-engine analysis)

(3) Static efficiency, which, in addition to the aerodynamic losses, considers the turbine exit total velocity head a loss (used in turbopump and accessory-drive analyses)

A complete description of the analytical results for zero exit whirl is presented to indicate the type of results obtained when the pertinent diagram parameters vary as prescribed. This case also covers most of the λ range considered and is of most interest in the turbojet-engine field. With respect to turbopumps and auxiliary drives, very high specific work outputs are desired, requiring λ values below that obtainable for zero exit whirl within the specified impulse limit. As a result, negative turbine exit whirl is required to permit reductions in λ to the range of these turbines. These cases wherein exit whirl is used are also described, but in less detail than that for zero exit whirl, with primary emphasis on the maximum-efficiency characteristics. The two-stage total-efficiency characteristics are then compared with the single-stage results of reference 2 using average stage-work outputs and speeds as a basis to indicate the validity of extending the results to turbines of many stages. Finally, the significance of assumptions and constants used in the analysis is discussed in terms of their effects on the turbine-efficiency levels presented, particularly at low values of λ.

METHOD OF ANALYSIS

As discussed in the INTRODUCTION, the fundamental parameter to be used in relating the two-stage-turbine efficiency characteristics to the work and speed requirements is the work-speed parameter λ defined as

$$\lambda = \frac{U^2}{gJ\Delta h'} \tag{1}$$

where U is the mean-section blade speed and $\Delta h'$ is the required specific work output. (All symbols are defined in appendix A.) The equations relating the efficiencies to this parameter and others defining the velocity-diagram characteristics are presented in this section.

Efficiency Equations

The three types of efficiency to be considered herein are total, rating, and static. These efficiencies are used in references 2 and 3 for the single-stage turbine, and their significance is pointed out in the INTRODUCTION. For the present analysis the total efficiencies of both stages are first considered, since they must be known before the over-all efficiency is obtained.

Stage total efficiency. - The total efficiency of a stage, which includes the effects of all the aerodynamic losses, is defined as

$$\eta_{ST} = \frac{\Delta h'_{ST}}{\Delta h'_{id,ST}} \tag{2}$$

where $\Delta h'_{ST}$ is the actual specific work output and $\Delta h'_{id,ST}$ is the ideal specific work output corresponding to the total-pressure ratio across the turbine stage. The equation relating this efficiency to the work-speed and velocity-diagram parameters is derived in reference 2 as

$$\eta_{ST} = \frac{\lambda_{ST}}{\lambda_{ST} + BC} \tag{3}$$

where

$$B = K \frac{A}{w} \tag{3a}$$

and

$$C = 4\lambda_{ST} \frac{(V_x^2)_{av}}{gJ\Delta h'_{ST}} + \left(\frac{V_{u,1}}{\Delta V_{u,ST}}\right)^2 + \left(\frac{V_{u,1}}{\Delta V_{u,ST}} - \lambda_{ST}\right)^2 + \left(\frac{V_{u,1}}{\Delta V_{u,ST}} - \lambda_{ST} - 1\right)^2 \tag{3b}$$

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In equation (3b) the subscript 1 represents that station between the stage stator and rotor. This equation is derived in reference 2 by use of a number of assumptions. Although this derivation is not presented herein, the assumptions involved are briefly reviewed:

(1) The stage specific energy loss $\Delta h'_{id,ST} - \Delta h'_{ST}$ is assumed equal to the sum of the stator and rotor specific losses $L_S + L_R$. These blade losses, expressed in units of Btu per pound, are defined as the difference between the ideal and actual specific kinetic energy obtained through expansion to the blade exit static pressure. The validity of this assumption is investigated in appendix C of reference 2, where it is shown to be correct except for the small effect of absolute enthalpy variations through the stage.

(2) The stage specific loss $L_R + L_S$ is assumed proportional to the surface area per unit weight flow and the specific kinetic-energy level of the flow; that is,

$$L_S + L_R \propto \frac{A}{w} E$$

This assumption can be shown to be valid from boundary-layer considerations.

(3) The specific kinetic-energy level E is assumed to be representable by the average of the specific kinetic energies entering and leaving the blade rows where the velocities involved are relative to the blade rows.

Throughout the analysis, the constant of proportionality K and ratio of surface area to weight flow A/w are assumed constant, yielding a constant value of B. The same value of B = 0.030 used for the single-stage analyses is used herein. Then, after the analysis is presented, the significance of these parameters (K, A/w, and B) is discussed in terms of their effect on the efficiency levels presented.

One other parameter requiring consideration is $(V_x^2)_{av}/gJ\Delta h'_{ST}$ appearing in the term C of equation (3). During most of the single-stage analysis, this parameter was assumed constant at 0.49, a value corresponding approximately to that encountered in a number of turbojet-engine turbines investigated at the Lewis laboratory. For the subject analysis it is assumed that $(V_x^2)_{av}/gJ\Delta h' = 0.245$, where $\Delta h'$ represents the total specific work output of the two stages. The significance of this parameter is also discussed later. The specification of the parameter $(V_x^2)_{av}/gJ\Delta h'$ requires an alteration in the first term of equation (3b) as follows:

$$4\lambda_{ST} \frac{(V_x^2)_{av}}{gJ\Delta h'_{ST}} = 4\lambda_{ST} \frac{(V_x^2)_{av}}{gJ\Delta h'} \frac{gJ\Delta h'}{gJ\Delta h'_{ST}}$$

$$= 4\lambda_{ST} \frac{(V_x^2)_{av}}{gJ\Delta h'} \frac{\dfrac{U^2}{gJ\Delta h'_{ST}}}{\dfrac{U^2}{gJ\Delta h'}}$$

Introducing

$$\lambda_{ST} = \frac{U^2}{gJ\Delta h'_{ST}}$$

and

$$\lambda = \frac{U^2}{gJ\Delta h'}$$

then

$$4\lambda_{ST} \frac{(V_x^2)_{av}}{gJ\Delta h'_{ST}} = 4 \frac{\lambda_{ST}^2}{\lambda} \frac{(V_x^2)_{av}}{gJ\Delta h'} \tag{4}$$

Equations (3) and (4) can now be used in obtaining the equation for the total efficiency of the two stages. Figure 1 presents the velocity diagram and station nomenclature for two-stage turbines. Using this nomenclature and equation (4), equation (3) can be altered to yield the following equation for the first-stage total efficiency:

$$\eta_a = \frac{\lambda_a}{\lambda_a + BC_a} \tag{5}$$

where

$$B = K\,\frac{A}{w} \tag{5a}$$

and

$$C_a = 4\,\frac{\lambda_a^2}{\lambda}\,\frac{(V_x^2)_{av}}{gJ\Delta h'} + \left(\frac{V_{u,1}}{\Delta V_{u,a}}\right)^2 + \left(\frac{V_{u,1}}{\Delta V_{u,a}} - \lambda_a\right)^2 + \left(\frac{V_{u,1}}{\Delta V_{u,a}} - \lambda_a - 1\right)^2 \tag{5b}$$

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A similar set of equations for the second-stage total efficiency can also be written as

$$\eta_b = \frac{\lambda_b}{\lambda_b + BC_b} \tag{6}$$

where

$$B = K\,\frac{A}{w} \tag{6a}$$

and

$$C_b = 4\,\frac{\lambda_b^2}{\lambda}\,\frac{(V_x^2)_{av}}{gJ\Delta h'} + \left(\frac{V_{u,3}}{\Delta V_{u,b}}\right)^2 + \left(\frac{V_{u,3}}{\Delta V_{u,b}} - \lambda_b\right)^2 + \left(\frac{V_{u,3}}{\Delta V_{u,b}} - \lambda_b - 1\right)^2 + \left[\frac{\lambda_b}{\lambda_a}\left(\frac{V_{u,1}}{\Delta V_{u,a}} - 1\right)\right]^2 \tag{6b}$$

The last expression occurs as a result of stator inlet whirl.

Over-all total efficiency. - The general equation for the over-all total efficiency of the two-stage turbine is

$$\bar{\eta} = \frac{\Delta h'}{\Delta h'_{id}} \tag{7}$$

where $\Delta h'$ is the total specific work output across the turbine, and $\Delta h'_{id}$ is the ideal specific work output corresponding to the total-pressure ratio across the turbine. If the effect of reheat is not considered,

$$\bar{\eta} = \frac{\Delta h'_a + \Delta h'_b}{\Delta h'_{id,a} + \Delta h'_{id,b}} \tag{8}$$

If the numerator and denominator of equation (8) are multiplied through by

$$\frac{\Delta h'_a}{\Delta h'_{id,a}} \frac{\Delta h'_b}{\Delta h'_{id,b}} \frac{U^2}{gJ\Delta h'_a \Delta h'_b}$$

the following equation is obtained:

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$$\eta = \frac{\frac{\Delta h'_a}{\Delta h'_{id,a}} \frac{\Delta h'_b}{\Delta h'_{id,b}} \left(\frac{U^2}{gJ\Delta h'_a} + \frac{U^2}{gJ\Delta h'_b}\right)}{\frac{\Delta h'_a}{\Delta h'_{id,a}} \frac{U^2}{gJ\Delta h'_a} + \frac{\Delta h'_b}{\Delta h'_{id,b}} \frac{U^2}{gJ\Delta h'_b}}$$

Finally, using the definitions of η_a, η_b, λ_a, and λ_b,

$$\bar{\eta} = \frac{\eta_a \eta_b (\lambda_a + \lambda_b)}{\eta_a \lambda_a + \eta_b \lambda_b} \tag{9}$$

Throughout the major portion of the report, the effect of reheat is not considered, as this would require specification of an enthalpy level. At the conclusion of the report, however, the significance of this reheat effect is discussed in terms of its effect on the efficiency levels presented. The derivation of an efficiency equation similar to equation (9) but including this reheat effect is presented in appendix B.

Over-all rating efficiency. - As indicated in the INTRODUCTION, when a turbine is used in a jet engine, the kinetic energy involved in the turbine exit whirl is considered a loss, because this whirl does not contribute to engine thrust. The equation for this efficiency is derived in reference 2 for single-stage turbines. For the two-stage turbine, it is desired to use the turbine exit-whirl parameter $V_{u,4}/\Delta V_{u,b}$ in the rating-efficiency equation, as follows. Neglecting enthalpy levels as was done in reference 2, equation (19) of that reference can be rewritten here for two-stage turbines with appropriate nomenclature as

$$\bar{\eta}_x = \frac{\bar{\eta}}{1 + \frac{V_{u,4}^2}{2gJ\Delta h'_{id}}} \tag{10}$$

$$= \frac{\bar{\eta}}{1 + \frac{1}{2} \frac{V_{u,4}^2}{\Delta V_{u,b}^2} \frac{\Delta V_{u,b}^2}{U^2} \frac{U^2}{gJ\Delta h'} \frac{\Delta h'}{\Delta h'_{id}}}$$

Then, using the definitions of λ_b, λ, and $\bar{\eta}$,

$$\bar{\eta}_x = \frac{\bar{\eta}}{1 + \frac{\bar{\eta}}{2}\frac{\lambda}{\lambda_b^2}\left(\frac{V_{u,4}}{\Delta V_{u,b}}\right)^2} \tag{11}$$

Over-all static efficiency. - The INTRODUCTION also points out that in many applications the turbine efficiency is based on the ratio of total to static pressure; that is, the entire exit kinetic energy is considered a loss. An equation for this efficiency can be written in a form similar to equation (10) as

$$\bar{\eta}_s = \frac{\bar{\eta}}{1 + \frac{V_{x,4}^2 + V_{u,4}^2}{2gJ\Delta h_{id}'}}$$

$$= \frac{\bar{\eta}}{1 + \frac{1}{2}\frac{\Delta h'}{\Delta h_{id}'}\left(\frac{V_{x,4}^2}{gJ\Delta h'} + \frac{V_{u,4}^2}{\Delta V_{u,b}^2}\frac{\Delta V_{u,b}^2}{U^2}\frac{U^2}{gJ\Delta h'}\right)}$$

If it is assumed that $V_{x,4}^2 = (V_x^2)_{av}$ (as in ref. 2), the equation for $\bar{\eta}_s$ can finally be written as

$$\bar{\eta}_s = \frac{\bar{\eta}}{1 + \frac{\bar{\eta}}{2}\left[\frac{(V_x^2)_{av}}{gJ\Delta h'} + \frac{\lambda}{\lambda_b^2}\left(\frac{V_{u,4}}{\Delta V_{u,b}}\right)^2\right]} \tag{12}$$

As stated in deriving the stage total efficiency, in this analysis $(V_x^2)_{av}/gJ\Delta h'$ is to be specified a constant and equal to 0.245.

Work Considerations

The over-all work-speed parameter can be related to those of the two stages through consideration of the work and speed characteristics. The total turbine work output is, of course, equal to the sum of that being developed in each stage; that is,

$$\Delta h' = \Delta h_a' + \Delta h_b'$$

Dividing through by U^2 and multiplying through by gJ yields

$$\frac{gJ\Delta h'}{U^2} = \frac{gJ\Delta h'_a}{U^2} + \frac{gJ\Delta h'_b}{U^2}$$

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Finally, using the definitions of λ, λ_a, and λ_b,

$$\frac{1}{\lambda_a} = \frac{1}{\lambda} - \frac{1}{\lambda_b} \qquad (13)$$

Exit-Whirl Considerations

As will be described later, the turbine exit-whirl parameter $V_{u,4}/\Delta V_{u,b}$ is to be considered one of the major variables. It is here desired to obtain an equation for $V_{u,3}/\Delta V_{u,b}$ in terms of this parameter for later use. By definition,

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$$V_{u,3} - V_{u,4} = \Delta V_{u,b}$$

Therefore, dividing through by $\Delta V_{u,b}$ and solving for $V_{u,3}/\Delta V_{u,b}$ yield

$$\frac{V_{u,3}}{\Delta V_{u,b}} = 1 + \frac{V_{u,4}}{\Delta V_{u,b}} \qquad (14)$$

Calculation Procedure

The equations involved in the two-stage-turbine efficiency calculations for a given set of conditions have all been presented. These equations are (5), (6), (9), (11), (12), (13), and (14). These equations are presented in functional form in table I together with an outline of the procedure used. Table I(b) shows that four of the parameters are required for solution of the equations and that a range of values must be specified. The limits imposed on these ranges are discussed in the following section. Once particular values of these four parameters are selected, all the equations can then be solved, as indicated in table I(b).

Limits

The ranges of the four parameters, $V_{u,4}/\Delta V_{u,b}$, λ, λ_b, and $V_{u,1}/\Delta V_{u,a}$, required to calculate turbine efficiency characteristics are obtained from considerations of certain aerodynamic limitations, some absolute and others arbitrarily selected to encompass a range of interest. These limits will now be described in detail.

Turbine exit-whirl parameter $V_{u,4}/\Delta V_{u,b}$. - In selecting the range of the turbine exit-whirl parameter $V_{u,4}/\Delta V_{u,b}$, the use of positive values was not considered, because (1) maximum total efficiencies do not occur in this range and (2) the positive whirls contribute to reductions in the rating and static efficiencies. Therefore, zero exit whirl was selected as the upper limit. As will be shown, negative whirls permit a reduction in λ from that permitted for zero exit whirl and are, there- 4353
fore, of considerable interest. A lower limit on this parameter was selected to be -0.4, since this value was considered a lower limit on the range of feasibility and interest. Thus,

$$-0.4 \leq \frac{V_{u,4}}{\Delta V_{u,b}} \leq 0 \qquad (15)$$

represents the range of this parameter considered.

Over-all work-speed parameter λ. - The lower limit imposed upon the range of λ was selected with regard to the reaction characteristics of the two rotors and interstage stator. The first-stage stator was not considered, since it always has high reaction. In general, experience has shown that when negative reaction (i.e., a static-pressure rise across a blade row) is encountered, large increases in the blade losses occur. In view of this experience, a limit of impulse across any blade row is specified for this analysis (as in the single-stage-turbine analysis of ref. 2). Since constant axial component of velocity is assumed, impulse conditions occur when the relative whirl velocity leaving the blade row is equal to but opposite in sign from the relative entering whirl velocity. The impulse limits for the three blade rows may then be expressed as follows:

$$\text{First-stage rotor:} \quad W_{u,1} = -W_{u,2} \qquad (16a)$$

$$\text{Second-stage stator:} \quad V_{u,2} = -V_{u,3} \qquad (16b)$$

$$\text{Second-stage rotor:} \quad W_{u,3} = -W_{u,4} \qquad (16c)$$

The first-stage-rotor impulse limit can be considered in a manner similar to that for the single-stage turbine of reference 2. Since

$$\Delta V_{u,a} = W_{u,1} - W_{u,2}$$

the equation

$$W_{u,1} = \frac{\Delta V_{u,a}}{2}$$

can be obtained with equation (16a). Then, since

$$V_{u,1} = W_{u,1} + U$$

the equation

$$\frac{V_{u,1}}{\Delta V_{u,a}} = \lambda_a + \frac{1}{2} \tag{17}$$

is easily obtained.

With respect to the second-stage-stator impulse limit, let equation (16b) be modified to

$$\frac{V_{u,2}}{\Delta V_{u,a}} = - \frac{V_{u,3}}{\Delta V_{u,b}} \frac{\Delta V_{u,b}}{U} \frac{U}{\Delta V_{u,a}} \tag{18}$$

Then, using the definitions of λ_a and λ_b, equation (14), and

$$\frac{V_{u,1}}{\Delta V_{u,a}} - \frac{V_{u,2}}{\Delta V_{u,a}} = 1 \tag{19}$$

equation (18) is modified to yield

$$\frac{V_{u,1}}{\Delta V_{u,a}} = 1 - \frac{\lambda_a}{\lambda_b}\left(1 + \frac{V_{u,4}}{\Delta V_{u,b}}\right) \tag{20}$$

The impulse limit on the second-stage rotor (eq. (16c)) can be treated similarly to that of the first-stage rotor, to yield

$$\frac{V_{u,3}}{\Delta V_{u,b}} = \lambda_b + \frac{1}{2}$$

Then, using equation (14),

$$\frac{V_{u,4}}{\Delta V_{u,b}} = \lambda_b - \frac{1}{2} \tag{21}$$

The lower limit of λ occurs when all blade rows except the first-stage stator reach impulse conditions simultaneously. When this limit is specified, equations (17), (20), and (21) can be used with equation (13) to solve for λ in terms of $V_{u,4}/\Delta V_{u,b}$ to yield

$$\lambda = \frac{\frac{1}{2} + \frac{V_{u,4}}{\Delta V_{u,b}}}{4\left(1 + \frac{V_{u,4}}{\Delta V_{u,b}}\right)} \qquad (22)$$

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This lower limit of λ is presented in figure 2 as a function of $V_{u,4}/\Delta V_{u,b}$. As negative exit whirl is imposed, the permissible value of λ is reduced, going to zero at $V_{u,4}/\Delta V_{u,b} = -0.5$. Of course, a practical lower limit of λ exists. Reference 3 indicates a practical single-stage-turbine lower limit of 0.10. Using this limit as a stage limit for the two-stage turbine, a practical lower limit on the order of 0.05 results.

The upper limit imposed upon λ was somewhat arbitrary but intended to cover the range considered of interest. For zero exit whirl, the upper limit of λ was selected as 0.50, since this could be obtained with $\lambda_a = \lambda_b = 1$, a most conservative value that yields maximum stage and total efficiency (see ref. 2). The upper limit of λ, which was arbitrarily chosen as the lowest λ for a given exit whirl at which there would be an equal work split ($\lambda_a = \lambda_b$), is also shown in figure 2. This also corresponds to impulse conditions across the second-stage rotor. A larger λ for the same exit whirl was not considered, since it would be more advantageous to go to less exit whirl.

Second-stage work-speed parameter λ_b. - For specified values of $V_{u,4}/\Delta V_{u,b}$ and λ, a range of λ_b must be selected. For a given value of λ, variations in λ_b result in variations in the work split between the two stages (eq. (13)). As λ_b is increased, the first stage produces progressively more work output. The upper limit on this parameter will then be at a point where impulse conditions exist across both the first-stage rotor and the second-stage stator. Therefore, using equations (13), (17), and (20) gives the following equation for this upper limit of λ_b:

$$\lambda_b = \frac{\frac{3}{2} + \frac{V_{u,4}}{\Delta V_{u,b}}}{\frac{1}{2\lambda} - 1} \qquad (23)$$

As λ_b is reduced, the work split of the turbine is altered, the second stage producing progressively more work output. The lower limit on λ_b will then occur at impulse conditions across the second-stage rotor. Equation (21) can therefore be rearranged as follows to obtain this lower limit for a given turbine exit whirl:

$$\lambda_b = \frac{1}{2} + \frac{V_{u,4}}{\Delta V_{u,b}} \tag{24}$$

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<u>First-stage-stator exit-whirl parameter $V_{u,1}/\Delta V_{u,a}$.</u> - For specified values of $V_{u,4}/\Delta V_{u,b}$, λ, and λ_b, a range of $V_{u,1}/\Delta V_{u,a}$ must be selected. Variations in this parameter result in variation in the split of whirl between the entrance and exit of the first-stage rotor by the equation

$$\frac{V_{u,1}}{\Delta V_{u,a}} - \frac{V_{u,2}}{\Delta V_{u,a}} = 1 \tag{25}$$

The upper limit on $V_{u,1}/\Delta V_{u,a}$ occurs when impulse conditions exist across the first-stage rotor. Therefore, equation (17) as written defines the upper limit on this parameter, since λ_a is known, given λ and λ_b (eq (13)).

The lower limit of $V_{u,1}/\Delta V_{u,a}$ occurs when impulse conditions exist across the second-stage stator. Equation (20) can be used directly to obtain the lower limit on this parameter (λ_a can again be obtained using eq. (13)).

Once the limits on the four parameters were determined, a sufficient number of points were arbitrarily selected between these limits to establish with sufficient accuracy the trends of the results. The calculations of the efficiency characteristics then proceeded.

RESULTS OF ANALYSIS

In presenting the results of the two-stage-turbine efficiency analysis, attention is first given to the condition of zero exit whirl, since this condition exists over most of the λ range studied and is of considerable interest in the turbojet-engine application. The effects of exit whirl, which are of principal interest with respect to turbopump- and auxiliary-drive turbines, are then discussed. Next, the efficiency characteristics of the two-stage turbine are compared with those obtained in the single-stage analyses of references 2 and 3.

Finally, the significance of assumptions and constants used in the analysis are discussed in terms of their effect on the turbine-efficiency levels presented, particularly at low λ values.

Zero Exit Whirl

General efficiency characteristics. - The two-stage-turbine overall efficiency characteristics for zero exit whirl $(V_{u,4}/\Delta V_{u,b} = 0)$ are presented in figures 3 and 4. Figure 3 shows the total-efficiency characteristics, which for this case are the same as the rating, and figure 4 presents the static-efficiency characteristics. In order to permit a distinction between the various groups of points as well as to discern easily those conditions which yield the peak efficiency, efficiency is presented as a function of the first-stage-stator exit-whirl parameter $V_{u,1}/\Delta V_{u,a}$, with λ and λ_b as parameters. 4353

The reaction limits are shown in figures 3 and 4 in the following manner:

(1) The circles of $\lambda_b = 0.50$ represent second-stage rotor impulse conditions (eq. (21)).

(2) The dashed lines represent second-stage-stator impulse conditions (eq. (20)).

(3) The dot-dashed lines represent first-stage-rotor impulse conditions (eq. (17)).

At λ values equal to or below 0.25 these three limits box in an area of permissible operation, this area rapidly decreasing as λ is reduced. At the lower limit of $\lambda = 0.125$ (see fig. 2) the area is zero, yielding a one-point operation. The limits are not shown for λ greater than 0.25, because at these values the turbine is quite conservative, the average λ_{ST} being 0.50 or greater. The stages therefore move away from impulse conditions at λ values greater than 0.25 unless they are badly mismatched with large differences between λ_a and λ_b.

Inspection of figure 3 shows that, for a given λ, a certain combination of λ_b and $V_{u,1}/\Delta V_{u,a}$ yields maximum total efficiency. Comparison of these points with corresponding points in figure 4 shows that they also yield the peak static efficiency. Consideration of these points is important, since this maximum-efficiency condition is usually desired, as discussed in greater detail in the next section. However, for a given λ, the peak-efficiency point lies on a fairly flat curve, so that a wide spread of λ_b and $V_{u,1}/\Delta V_{u,a}$ at a given λ results in only a small penalty in efficiency.

Maximum-efficiency characteristics. - Figure 5 presents the maximum total- and static-efficiency points from figures 3 and 4 as a function of the over-all work-speed parameter λ. In the range of λ from 0.25 to 0.50 the efficiencies do not vary to any great extent (0.89 and 0.81 at $\lambda = 0.50$ to 0.88 and 0.79 at $\lambda = 0.25$). As λ is reduced below 0.25, the efficiencies decrease markedly, reaching values of 0.82 and 0.74 at the lower limit of $\lambda = 0.125$. The static efficiency is, of course, less than the total efficiency, because the exit velocity head is considered a loss in the former.

The work split of the two stages corresponding to these maximum-efficiency conditions can be obtained from figure 6, which presents the ratio of second-stage work output to total work output $\Delta h_a'/\Delta h'$ as a function of the over-all work-speed parameter λ. This ratio is easily obtained from the equation

$$\frac{\Delta h_a'}{\Delta h'} = \frac{\dfrac{U^2}{gJ\Delta h'}}{\dfrac{U^2}{gJ\Delta h_a'}} = \frac{\lambda}{\lambda_a} \tag{26}$$

The figure shows that, at $\lambda = 0.50$, the first stage develops half the total work output; that is, the work split of the two stages is 50-50. As λ is reduced, the first stage performs progressively more work, until a work split of 60-40 is obtained at $\lambda = 0.20$. Below $\lambda = 0.20$ the work split changes much more severely in the same direction, until at the lower limit of $\lambda = 0.125$ a split of 75-25 is obtained. This trend occurs because the exit-whirl limitation forces the first stage to do a progressively larger percentage of the work.

Effect of Turbine Exit Whirl on Maximum-Efficiency Characteristics

The discussion of the limits imposed on λ pointed out that, as negative turbine exit whirl is imposed, the two-stage turbine is permitted to increase its work capabilities for a given blade speed - that is, to operate at λ values below 0.125. The efficiency characteristics of the two-stage turbine were therefore computed over a range of exit-whirl parameter $V_{u,4}/\Delta V_{u,b}$ from zero (already described) to -0.4 in increments of 0.1. For each value of this parameter, a complete set of calculations were made similar to those presented for zero exit whirl in figures 3 and 4. An example of the results obtained for these whirl cases is shown in figure 7 for $V_{u,4}/\Delta V_{u,b}$ of -0.2. [Since the use of this exit whirl yields rating efficiencies different from total (see eq. (11)), an additional graph is required in describing the efficiency characteristics.]

Inspection of figures 7(a) and (b) shows not only that the total and rating efficiencies are different but also that, for a given λ, the peak efficiencies can occur at different values of λ_b. Therefore, in describing the effects of exit whirl on the maximum-efficiency characteristics of the turbine, these differences must be considered. Table II presents the values of maximum total, rating, and static efficiency as a function of λ for all the whirl cases considered, along with the corresponding values of λ_b and $V_{u,1}/\Delta V_{u,a}$. For values of λ of 0.25 and less, these maximum efficiencies occur at $V_{u,1}/\Delta V_{u,a}$ of approximately 2/3 regardless of whirl. 4353

The maximum efficiencies in table II are presented in figure 8. The points for zero exit whirl are, of course, the same as those presented in figure 5. Figure 8 shows that imposing progressively more negative exit whirl permits a reduction in the permissible λ, a fact already described in the discussion of the limits imposed on λ. The figure also shows that, for a given λ where a range of whirls can be considered ($\lambda = 0.125$, e.g.), imposing negative exit whirl does not reduce the efficiency. In fact, the total efficiency improves somewhat as the whirl is imposed. This is true because the second-stage velocity diagram approaches that corresponding to the maximum stage total efficiency described in reference 2. The improvement in the total efficiency offsets the reductions in rating and static efficiencies that would occur because of the added exit-whirl loss, so that these efficiencies vary only slightly with whirl. As a result of these effects, curves can be drawn through the points of maximum efficiency to describe the efficiency trends very closely. These curves are used later in comparing the results with the single-stage results of references 2 and 3.

Figure 8 shows that the efficiencies decrease markedly at low values of λ. For example, $\overline{\eta}$ decreases from approximately 0.83 at $\lambda = 0.125$ to approximately 0.74 at $\lambda = 0.05$. This rapid decrease in total efficiency at low λ values, which is discussed at length in reference 2, is attributed to the increased viscous losses incurred when extracting the turbine work through use of increased flow velocities. The efficiency, of course, decreases to zero as $\lambda \to 0$.

The decrease in $\overline{\eta}_x$ is even more marked than that of $\overline{\eta}$. For example, $\overline{\eta}_x$ declines from 0.82 at $\lambda = 0.125$ to approximately 0.61 at $\lambda = 0.05$. This difference is due to the increased whirl necessarily imposed at the low λ values, which is considered as a loss in computing $\overline{\eta}_x$. The trends in $\overline{\eta}_s$ shown in figure 8 are the same as those of $\overline{\eta}_x$. The only difference is that the level is lower because of the additional loss of the axial component of dynamic pressure.

From the results presented in figure 8, it can be concluded that, in the low ranges of λ desired for turbopump and accessory-drive applications, relatively low efficiencies must be expected compared with those obtained at the high λ values used in turbojet-engine applications.

Effect of Turbine Exit Whirl on Stage-Work Split

Table II includes a compilation of the ratio of first-stage work to over-all work for the maximum rating and static efficiencies used in figure 8. These ratios are shown in figure 9 as a function of λ. The zero-exit-whirl points are the same as those used in figure 6. For a given value of exit whirl the lower limit of λ is the value corresponding to impulse conditions in the two rotors and the second-stage stator, as previously discussed. Since this condition specifies one-point operation, the efficiency obtained is the peak efficiency and the point represents a lower limit on λ. The lower-limit points in figure 9 result in a straight line (dot-dashed line) passing through an equal work split $\Delta h_a'/\Delta h' = 1/2$ at $\lambda = 0$. Equations (22), (24), and (13) can be used to solve for the equation of this limiting curve to give

$$\frac{\lambda}{\lambda_a} = 2\lambda + \frac{1}{2} \qquad (27)$$

In figure 9 this impulse-limit curve combines with that for zero exit whirl (solid line) to envelop the other points considered. These two curves intersect at the maximum work split of 75-25. Figure 9 shows some scatter in the points, since the efficiencies have a fairly flat peak and the calculated points were not necessarily at the absolute peak. It can be concluded, nevertheless, that the general effect of imposing the negative exit whirl is to equalize the work of the two stages.

Comparison with Single-Stage Results

As pointed out in the section METHOD OF ANALYSIS, the same fundamental assumptions and limits used for the single-stage analyses of references 2 and 3 are used herein. Consequently, a direct comparison of these results can be made to indicate the effect of staging on the level of turbine efficiency. This comparison is made in figure 10, where the over-all total, rating, and static efficiencies are presented as a function of λ. The curves representing the maximum-efficiency characteristics of the two-stage turbine are the faired curves of figure 8. The curves representing the efficiencies obtained in the single-stage-turbine analysis corresponding to maximum rating and static efficiencies are from figures 9 to 11 of reference 2. For the range of λ considered, these curves are for impulse conditions across the rotor. Finally, the curves representing the efficiency characteristics of single-stage turbines utilizing downstream stators $\left(\text{designated a } 1\frac{1}{2}\text{-stage turbine}\right)$ are taken from figures 5 to 7 of reference 3. The range of λ shown in figure 10 is that considered of interest in the two-stage-turbine analysis (0 to 0.50).

The two-stage turbine has the highest total efficiency over the range of λ considered. This is true because, for the same λ, each stage of the two-stage turbine can operate at a much higher λ_{ST} and 4353
consequently at higher stage efficiencies. The total efficiency of the $1\frac{1}{2}$-stage turbine is lower than that of the single-stage turbine because the upstream stator and rotor are no more efficient than the single-stage turbine, while the downstream stators add to the viscous loss and then reduce the total efficiency from that of the single stage.

The two-stage turbine also has the highest rating efficiency, the difference between these results and those for the single-stage turbine becoming more pronounced. The greater difference is a result of the large amounts of negative exit whirl required in the single-stage turbine, especially in the low λ range, to maintain the rotor at impulse conditions. The curve for the $1\frac{1}{2}$-stage turbine, which lies, in general, somewhere between those for the single- and two-stage turbines, indicates the range of λ wherein the downstream stators offer an efficiency advantage in the operation of single-stage turbines. However, even with these stators, the single-stage turbine does not yield rating efficiencies that approach those of the two-stage turbine.

The trends of the static-efficiency characteristics are the same as those of the rating efficiency, except that the difference in efficiency levels for the single- and two-stage turbines is much more pronounced. This increased difference is attributed to the assumed values of the parameter $(V_x^2)_{av}/gJ\Delta h'$ used in the analyses. For the single-stage analysis $(V_x^2)_{av}/gJ\Delta h'$ was 0.49, whereas for the two-stage analysis it is 0.245. As a result, the axial component of turbine exit dynamic head is a much larger percentage of the single-stage-turbine work output and, consequently, reduces the static efficiency considerably.

Thus, figure 10 shows that, for the three efficiencies considered, the two-stage turbine yields the greatest efficiency over the range of λ considered. These results indicate the desirability of multistaging in many turbines in which high efficiency is important at low λ values. Furthermore, for a given efficiency level, multistaging becomes desirable in order to achieve minimum values of λ. For turbopump applications, where the blade speed is fixed because of stress considerations, the

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increase in specific work output obtained with a multistage turbine results in a corresponding reduction in the turbine weight flow.

Efficiency Comparison Using an Average Stage Work-Speed Parameter

In many applications in which extremely low work-speed parameters are incurred, it becomes necessary to use several turbine stages in order to operate at such a value of stage work-speed parameter as to obtain reasonably high efficiency. One method of extending the results of this analysis to the cases for which the additional stages are required is to study the total-efficiency characteristics using an average stage specific work $\Delta h'_{ST,av}$. This average stage specific work can be used to obtain an average λ_{ST}, defined as $\lambda_{ST,av} = U^2/gJ\Delta h'_{ST,av}$. A relation between $\lambda_{ST,av}$ and λ can then be obtained as

$$\lambda_{ST,av} = \lambda n \tag{28}$$

where n is the number of stages involved. The mean-section blade speed U^2 that would be used in the equation for λ and $\lambda_{ST,av}$ would represent the average of the blade speeds for each stage.

The single- and two-stage total efficiencies are compared on this basis in figure 11. The range of $\lambda_{ST,av}$ considered is from 0 to 1.00, unity being that value yielding the maximum stage total efficiency. For the two-stage turbine, the maximum total-efficiency points from table II are used; for the single-stage turbine, the zero-exit-whirl condition described in reference 2 is used for $\lambda_{ST,av}$ from 0.50 to 1.00, and below 0.50 impulse conditions are used.

As shown in figure 11, the total-efficiency results for the single- and two-stage turbines correlate fairly well on an average stage-work basis. Thus, it appears that the trends of this figure could be used in the study of multistage turbines. For example, if a total efficiency of 0.80 is desired and an over-all work-speed parameter λ of 0.02 is imposed, from figure 11 a value of $\lambda_{ST,av}$ of approximately 0.20 is required. Thus,

$$n = \frac{\lambda_{ST,av}}{\lambda} = \frac{0.20}{0.02} = 10 \text{ stages}$$

This example does not account for the effect of reheat between stages, which in a multistage turbine could possibly reduce the number of stages considerably. This analysis is intended merely to serve as a rough guide for estimating the number of stages of a turbine with high specific work output. In extending the results of this analysis to the multistage turbine, the rating and static efficiencies are not considered, because these efficiencies are a function of various components of the turbine exit dynamic pressure. Since this pressure, in general, would not vary in proportion to the number at stages, a correlation similar to that for the total efficiency could not be expected.

Significance of Assumptions and Constants Used in Analysis

Four of the assumptions and constants used in the analysis will now 4353
be discussed in terms of their effect on the efficiency levels presented, particularly in the low range of λ where some of the assumptions are questionable. These effects are studied for λ of 0.20, 0.11, and 0.042, with corresponding exit whirls of 0, -0.2, and -0.4. These are three of the maximum-efficiency points presented in table II. The calculated results obtained in the study of these four effects are presented in figure 12 and are described in detail in the following paragraphs.

Effect of variation in weight flow. - The effect of variations in weight flow on single-stage-turbine efficiency characteristics is described in considerable detail in reference 2. In this study it was assumed that variations in weight flow affect two of the parameters used in the analysis. These parameters are

$$B = K\frac{A}{w} \propto \frac{1}{w} \tag{29}$$

and

$$\frac{(V_x^2)_{av}}{gJ\Delta h'} \propto w^2 \tag{30}$$

The values of $B = 0.030$ and $(V_x^2)_{av}/gJ\Delta h' = 0.245$ assumed in the analysis were obtained from reference 2 and are based on turbojet-engine turbines where high equivalent weight flows per unit frontal area are required. In turbopump and auxiliary-drive applications, however, very low equivalent weight flows per unit frontal area are encountered. Thus, it is important to know the effect of this characteristic on the efficiency levels in the low λ range where these drive-turbines operate.

The effect of varying B and $(V_x^2)_{av}/gJ\Delta h'$ in the manner prescribed by equations (29) and (30) is presented for the three examples in figure 12(a) in terms of a weight-flow ratio w/w_r, where w_r is that weight flow corresponding to $B = 0.030$ and $(V_x^2)_{av}/gJ\Delta h' = 0.245$. The general

effect of reducing the weight flow is to decrease the total efficiency, this effect becoming more pronounced as λ is reduced. Similar effects can be observed for rating efficiency. The trends of the static efficiency are somewhat different because of the reduction in exit axial kinetic energy as the weight flow is reduced. At $\lambda = 0.20$, $\overline{\eta}_s$ first increases and then decreases as the weight flow is reduced; while at
4353 $\lambda = 0.042$, $\overline{\eta}_s$ decreases steadily as the weight flow is reduced.

It may be concluded that, in general, the effect of reduction in weight flow is to reduce the efficiency levels from those presented in the analysis. Thus, with respect to weight flow alone, the efficiencies of turbines for turbopump and auxiliary-drive applications will probably be below those presented in figure 8.

Effect of increases in K. - The constant of proportionality K appearing in the term B relates the kinetic-energy loss per unit surface area to the average free-stream kinetic-energy level of the gas. Since no absolute values of K are used, increases in K must be studied in terms of increases in B in the total-efficiency equation (3) as

$$\frac{K}{K_r} = \frac{B}{B_r} \tag{31}$$

The effect of increasing K in this manner on the efficiency characteristics for the three examples is shown in figure 12(b). For all levels considered, increases in K result in considerable reduction in efficiency, this reduction being much more pronounced at very low λ values.

Increases in K from the level used in the analysis will probably occur in turbopump and auxiliary-drive turbines as a result of a number of considerations, including the following:

(1) Reduction in Reynolds number

(2) Increased required trailing-edge blockage as a result of practical limitations with small blades

(3) General dimensional and tolerance problems that occur with small blading

(4) Increased effects of tip clearange or seal leakage

(5) Increase in effective K when partial admission is used, due to eddying and unsteady effects in the unused portion of the rotor

(6) Use of supersonic flow Mach numbers that could introduce high shock losses

From consideration of these and other effects it is clear that large reductions in efficiency level from that presented in figure 8 would occur for these drive-turbines as a result of significant increases in K alone.

Effect of increases in exit axial kinetic energy. - In the calculation of the static efficiency (eq. (12)), the axial component of kinetic energy at the turbine exit was assumed equal to the average across the turbine. In general, however, the axial component of kinetic energy increases from turbine inlet to exit and, as a result, the exit axial kinetic-energy level is somewhat greater than the average. The significance of this effect is shown in figure 12(c), where efficiency is shown as a function of the ratio of exit axial kinetic energy to average axial kinetic energy, that is 4353

$$\frac{(V_x^2)_4}{(V_x^2)_{av}} = \frac{\dfrac{(V_x^2)_4}{gJ\Delta h'}}{\dfrac{(V_x^2)_{av}}{gJ\Delta h'}}$$

Of course, the total and rating efficiencies are unaffected, since this ratio does not enter into the calculations. The static efficiency is considerably affected, however, $\overline{\eta}_s$ decreasing as the ratio is increased above 1. The effect is most pronounced at increased λ because of the higher $\overline{\eta}$ (see eq. (12)). Thus, it is evident that the levels of static efficiency presented in the analysis are somewhat greater than those expected experimentally.

Effect of reheat. - In the derivation of the equation for over-all total efficiency (eq. (9)), the reheat effect was not considered; $\Delta h'_{id,a}$ was merely added to $\Delta h'_{id,b}$ in the denominator of equation (8). In order to determine the significance of the reheat effect, an equation for $\overline{\eta}$ is derived in appendix B similar to equation (9) but including these effects. The parameter chosen to describe this effect is the ratio of specific enthalpy drop across the turbine to inlet enthalpy level $\Delta h'/h'_0$. This equation is

$$\overline{\eta} = \frac{\eta_a \eta_b (\lambda_a + \lambda_b)\left(1 - \dfrac{\lambda}{\lambda_a}\dfrac{\Delta h'}{h'_0}\right)}{\eta_a \lambda_a + \eta_b \lambda_b - \lambda \dfrac{\Delta h'}{h'_0}\left(1 + \dfrac{\lambda_b}{\lambda_a}\eta_b\right)} \qquad \text{(B12)}$$

Neglecting the effect of reheat merely assumes that $\Delta h'/h_0' \rightarrow 0$ or that the change in enthalpy is much less than the enthalpy level.

The efficiencies were computed for the three examples over a range
4353 of $\Delta h'/h_0'$ with $\bar{\eta}$ computed from equation (B6). Figure 12(d) shows that the general effect is to increase the efficiency level to some degree. For example, if $\Delta h'/h_0' = 0.30$, the reheat effect increases the efficiency level 1 to 2 points. Thus, this reheat effect offsets to a small extent the reductions in efficiency caused by the factors previously described.

SUMMARY OF RESULTS

An analysis of the effect of work and speed requirements on the efficiency characteristics of two-stage turbines has been presented. Assumptions and limitations were the same as those used in single-stage studies previously reported. The principal limit, that of impulse across any blade row, was studied in detail and resulted in a range of stage-work split and velocity-diagram parameters for a given work-speed parameter λ. The combination of these parameters yielding the maximum efficiency for a given λ and exit-whirl condition was used in describing the results of the analysis. These results are as follows:

1. The turbine with zero exit whirl attained a maximum total efficiency of approximately 0.89 at $\lambda = 0.50$. The efficiency then decreased as λ was reduced until at the lower limit of $\lambda = 0.125$ the total efficiency was approximately 0.82. The static-efficiency trends were similar but at lower efficiency values. As λ was reduced from 0.50 the stage-work split was altered from 50-50, the first stage developing progressively more of the work until at the lower limit of $\lambda = 0.125$ the work split was 75-25.

2. The primary effect of imposing negative exit whirl was to reduce the lower limit of λ at which the impulse limit was encountered. As λ was reduced below 0.125 in this manner, the efficiencies decreased quite markedly. The work split also tended to return to an equal split, theoretically returning to 50-50 as λ approached zero.

3. For a given λ where a range of turbine exit whirl was considered (0.25 and below), the maximum total efficiency increased slightly as negative exit whirl was imposed. The maximum rating and static efficiencies were approximately constant over the range of whirl considered for a given λ. These maximum efficiencies occurred in all cases at λ values below 0.25 at a first-stage-stator exit-whirl parameter of approximately 2/3.

4. Comparison of results for the two-stage analysis with those previously obtained for single-stage turbines revealed that, over the entire range of λ considered, the two-stage turbine yielded significantly higher efficiencies. This result indicates that, in applications where very low values of λ are encountered, multistage turbines become necessary when high efficiencies are desired.

5. When an average stage work-speed parameter was used, the total-efficiency characteristics of both the single- and two-stage turbines 4355
correlated well. Thus, it appears that the efficiency trends obtained in this manner can be used in the study of multistage turbines.

In conclusion, it is to be emphasized that the purpose of the analysis presented in this report is to indicate the effect of variations in work and speed requirements on the efficiency level. Constants used over the range of λ considered were obtained from well-designed high-specific-weight-flow turbojet turbines. A study of these constants and assumptions indicated that altering them in the direction of the range encountered in turbopump and accessory-drive turbines tends to reduce the efficiency considerably. Therefore, the efficiency levels presented in the analysis are probably greater than those expected for these turbines with low work-speed parameters.

Lewis Flight Propulsion Laboratory
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Cleveland, Ohio, June 13, 1957

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APPENDIX A

SYMBOLS

A	turbine blade surface area, sq ft
B	parameter equal to $K \frac{A}{w}$
C	parameter describing velocity-diagram type for stage total-efficiency equation
E	specific kinetic-energy level, Btu/lb
g	acceleration due to gravity, 32.17 ft/sec^2
h	specific enthalpy, Btu/lb
$\Delta h'$	specific work output, Btu/lb
J	mechanical equivalent of heat, 778.2 ft-lb/Btu
K	constant of proportionality
L	loss in kinetic energy, Btu/lb
n	number of stages
p	pressure, lb/sq ft
U	mean-section blade speed, ft/sec
V	absolute gas velocity, ft/sec
W	relative gas velocity, ft/sec
w	weight-flow rate, lb/sec
η	total efficiency, based on total-pressure ratio across turbine
η_s	static efficiency, based on ratio of total to static pressure across turbine
η_x	rating efficiency, based on total pressure upstream of turbine and pressure downstream of turbine equal to sum of static pressure and axial component of velocity head
λ	work-speed parameter, $U^2/gJ\Delta h'$

Subscripts:

a	first stage
av	average
b	second stage
id	ideal 4353
R	rotor
r	reference
S	stator
ST	stage
u	tangential component
x	axial component
0	upstream of turbine
1	station between first-stage stator and rotor
2	station between stages
3	station between second-stage stator and rotor
4	downstream of turbine

Superscripts:

'	absolute total state
—	over-all

APPENDIX B

DEVELOPMENT OF EQUATION FOR EFFECT OF REHEAT ON OVER-ALL TOTAL EFFICIENCY

The over-all total-efficiency equation with reheat is developed in terms of enthalpy ratios from the enthalpy-entropy diagram of figure 13. The general equation for over-all total efficiency (eq. (7)) can be written as

$$\bar{\eta} = \frac{\Delta h'}{h_0' - \bar{h}_{4,id}'} \tag{B1}$$

Dividing equation (B1) by h_0' gives

$$\bar{\eta} = \frac{\dfrac{\Delta h'}{h_0'}}{1 - \dfrac{\bar{h}_{4,id}'}{h_0'}} \tag{B2}$$

Figure 13 shows that the following enthalpy ratios are equal, since they represent isentropic drops in enthalpy as a result of the same pressure ratio:

$$\frac{h_2'}{h_{4,id}'} = \frac{h_{2,id}'}{\bar{h}_{4,id}'} \tag{B3}$$

Solving equation (B3) for $\bar{h}_{4,id}'$ and substituting into equation (B2) yield

$$\bar{\eta} = \frac{\dfrac{\Delta h'}{h_0'}}{1 - \dfrac{h_{2,id}' h_{4,id}'}{h_0' h_2'}} \tag{B4}$$

The stage total efficiencies expressed in terms of enthalpy are

$$\eta_a = \frac{\Delta h_a'}{\Delta h_{a,id}'} = \frac{\Delta h_a'}{h_0' - h_{2,id}'} \tag{B5a}$$

$$\eta_b = \frac{\Delta h_b'}{\Delta h_{b,id}'} = \frac{\Delta h_b'}{h_2' - h_{4,id}'} \tag{B5b}$$

Equations (B5a) and (B5b) can be solved for $h'_{2,id}/h'_0$ and $h'_{4,id}/h'_2$, respectively, and substituted into (B4) to give

$$\bar{\eta} = \frac{\frac{\Delta h'}{h'_0}}{1 - \left(1 - \frac{\Delta h'_a}{\eta_a h'_0}\right)\left(1 - \frac{\Delta h'_b}{\eta_b h'_2}\right)} \tag{B6}$$

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Expanding terms in the denominator yields

$$\bar{\eta} = \frac{\frac{\Delta h'}{h'_0}}{\frac{\Delta h'_a}{\eta_a h'_0} + \frac{\Delta h'_b}{\eta_b h'_2} - \frac{\Delta h'_a \Delta h'_b}{\eta_a \eta_b h'_0 h'_2}} \tag{B7}$$

Now

$$\frac{\Delta h'_a}{h'_0} = \frac{\Delta h'_a}{\Delta h'} \frac{\Delta h'}{h'_0} = \frac{\lambda}{\lambda_a} \frac{\Delta h'}{h'_0} \tag{B8}$$

Also,

$$\left.\begin{aligned}
\frac{\Delta h'_b}{h'_2} &= \frac{\Delta h'_b}{\Delta h'} \frac{\Delta h'}{h'_0} \frac{h'_0}{h'_2} \\
&= \frac{\lambda}{\lambda_b} \frac{\Delta h'}{h'_0} \frac{h'_0}{h'_2} \\
&= \frac{\lambda}{\lambda_b} \frac{\Delta h'}{h'_0} \left(1 - \frac{\Delta h'_a}{h'_0}\right)^{-1} \\
&= \frac{\lambda}{\lambda_b} \frac{\Delta h'}{h'_0} \left(1 - \frac{\lambda}{\lambda_a} \frac{\Delta h'}{h'_0}\right)^{-1}
\end{aligned}\right\} \tag{B9}$$

Substituting equations (B8) and (B9) into (B7) results in

$$\bar{\eta} = \frac{\dfrac{\Delta h'}{h_0'}}{\dfrac{\lambda}{\lambda_a}\dfrac{\Delta h'}{\eta_a h_0'} + \dfrac{\lambda}{\lambda_b}\dfrac{\Delta h'}{\eta_b h_0'}\left(1 - \dfrac{\lambda}{\lambda_a}\dfrac{\Delta h'}{h_0'}\right)^{-1} - \dfrac{\lambda^2}{\lambda_a\lambda_b}\left(\dfrac{\Delta h'}{h_0'}\right)^2 \dfrac{\left(1 - \dfrac{\lambda}{\lambda_a}\dfrac{\Delta h'}{h_0'}\right)^{-1}}{\eta_a\eta_b}} \tag{B10}$$

Cancelling out $\Delta h'/h_0'$, putting all three terms in the denominator in terms of a common denominator, and inverting yield

$$\bar{\eta} = \frac{\eta_a\eta_b\lambda_a\lambda_b\left(1 - \dfrac{\lambda}{\lambda_a}\dfrac{\Delta h'}{h_0'}\right)}{\lambda\lambda_b\eta_b\left(1 - \dfrac{\lambda}{\lambda_a}\dfrac{\Delta h'}{h_0'}\right) + \lambda\lambda_a\eta_a - \lambda^2\dfrac{\Delta h'}{h_0'}} \tag{B11}$$

Since equation (13) can be altered to

$$\frac{\lambda_a\lambda_b}{\lambda} = \lambda_a + \lambda_b$$

equation (B11) can be modified to

$$\bar{\eta} = \frac{\eta_a\eta_b(\lambda_a + \lambda_b)\left(1 - \dfrac{\lambda}{\lambda_a}\dfrac{\Delta h'}{h_0'}\right)}{\eta_a\lambda_a + \eta_b\lambda_b - \lambda\dfrac{\Delta h'}{h_0'}\left(1 + \dfrac{\lambda_b}{\lambda_a}\eta_b\right)} \tag{B12}$$

Equation (B12) becomes the same as (9) when $\Delta h'/h_0' \to 0$; that is, when no reheat effect is considered. Equation (B12) was used to obtain the curves in figure 12(d), where this reheat effect is presented for the three examples.

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TABLE I. - FUNCTIONAL RELATIONS AND CALCULATION PROCEDURE USED IN OBTAINING TWO-STAGE-TURBINE EFFICIENCY CHARACTERISTICS

(a) Functional relations

Equation	Functional relation
(5)	$\eta_a = f(\lambda, \lambda_a, V_{u,1}/\Delta V_{u,a})$
(6)	$\eta_b = f(\lambda, \lambda_b, V_{u,3}/\Delta V_{u,b})$
(9)	$\bar{\eta} = f(\eta_a, \eta_b, \lambda_a, \lambda_b)$
(11)	$\bar{\eta}_x = f(\bar{\eta}, \lambda, \lambda_b, V_{u,4}/\Delta V_{u,b})$
(12)	$\bar{\eta}_s = f(\bar{\eta}, \lambda, \lambda_b, V_{u,4}/\Delta V_{u,b})$
(13)	$\lambda_a = f(\lambda, \lambda_b)$
(14)	$V_{u,3}/\Delta V_{u,b} = f(V_{u,4}/\Delta V_{u,b})$

(b) Calculation procedure

Quantity	Obtained from	Quantities needed						
$V_{u,4}/\Delta V_{u,b}$	Range specified		●				●	●
λ	Range specified	●		●	●		●	●
λ_b	Range specified	●			●	●	●	●
$V_{u,1}/\Delta V_{u,a}$	Range specified			●				
λ_a	Eq. (13)			●		●		
$V_{u,3}/\Delta V_{u,b}$	Eq. (14)				●			
η_a	Eq. (5)					●		
η_b	Eq. (6)					●		
$\bar{\eta}$	Eq. (9)						●	●
$\bar{\eta}_x$	Eq. (11)							
$\bar{\eta}_s$	Eq. (12)							

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TABLE II. - RESULTS OF MAXIMUM-EFFICIENCY CALCULATIONS

Turbine exit-whirl parameter, $\frac{V_{u,4}}{\Delta V_{u,b}}$	Over-all work-speed parameter, λ	Maximum over-all total efficiency, $\overline{\eta}$			Maximum over-all rating and static efficiencies, $\overline{\eta}_x$ and $\overline{\eta}_s$				
		λ_b	$\frac{V_{u,1}}{\Delta V_{u,a}}$	$\overline{\eta}$	λ_b	$\frac{V_{u,1}}{\Delta V_{u,a}}$	$\overline{\eta}_x$	$\overline{\eta}_s$	$\frac{\Delta h_a'}{\Delta h'}$
0	0.500	1.00	1.00	0.894	1.00	1.00	0.894	0.806	0.50
	.375	.80	.90	.891	.80	.90	.891	.803	.53
	.250	.60	.67	.878	.60	.67	.878	.793	.58
	.200	.50	.66	.866	.50	.66	.866	.783	.60
	.150	.50	.61	.842	.50	.61	.842	.763	.70
	.125	.50	.67	.817	.50	.67	.817	.743	.75
-0.1	0.200	0.50	0.69	0.869	0.50	0.69	0.866	0.783	0.60
	.196	.50	.70	.867	.50	.70	.864	.782	.61
	.170	.40	.65	.859	.40	.65	.856	.774	.58
	.140	.40	.58	.842	.40	.58	.839	.761	.65
	.111	.40	.65	.810	.40	.65	.808	.735	.72
-0.2	0.150	0.30	0.60	0.854	0.40	0.61	0.838	0.760	0.62
	.130	.30	.62	.843	.40	.62	.825	.749	.68
	.110	.30	.58	.824	.30	.58	.808	.735	.63
	.094	.30	.64	.799	.30	.64	.786	.717	.69
-0.3	0.100	0.20	0.55	0.825	0.30	0.65	0.780	0.712	0.67
	.090	.20	.60	.813	.26	.64	.766	.700	.66
	.080	.20	.60	.797	.23	.62	.748	.685	.65
	.071	.20	.61	.775	.20	.61	.730	.670	.64
-0.4	0.050	0.10	0.53	0.746	0.12	0.58	0.615	0.572	0.59
	.047	.10	.55	.735	.11	.58	.601	.560	.59
	.044	.10	.53	.721	.11	.58	.585	.546	.58
	.042	.10	.57	.707	.10	.57	.572	.535	.57

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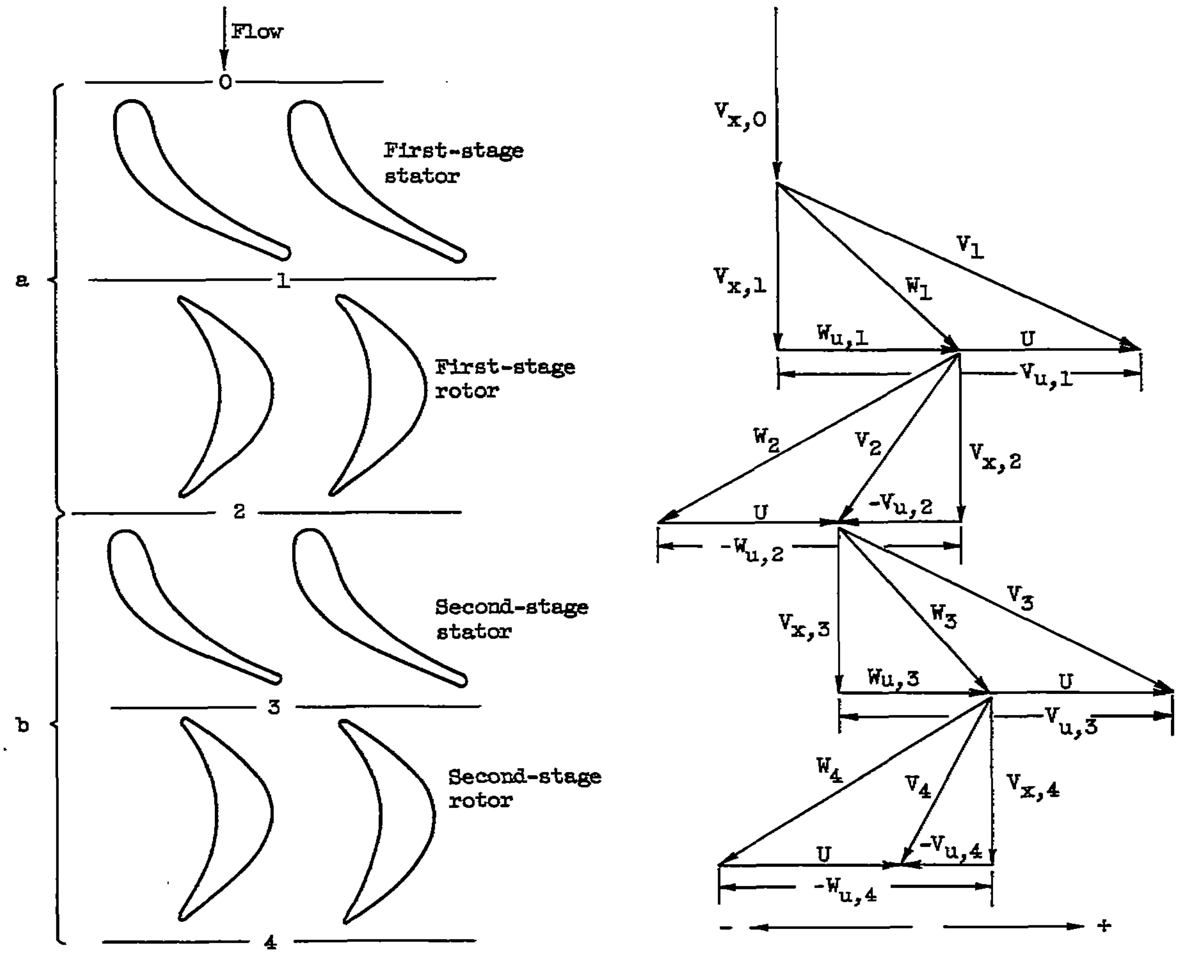

Figure 1. - Velocity diagrams and nomenclature.

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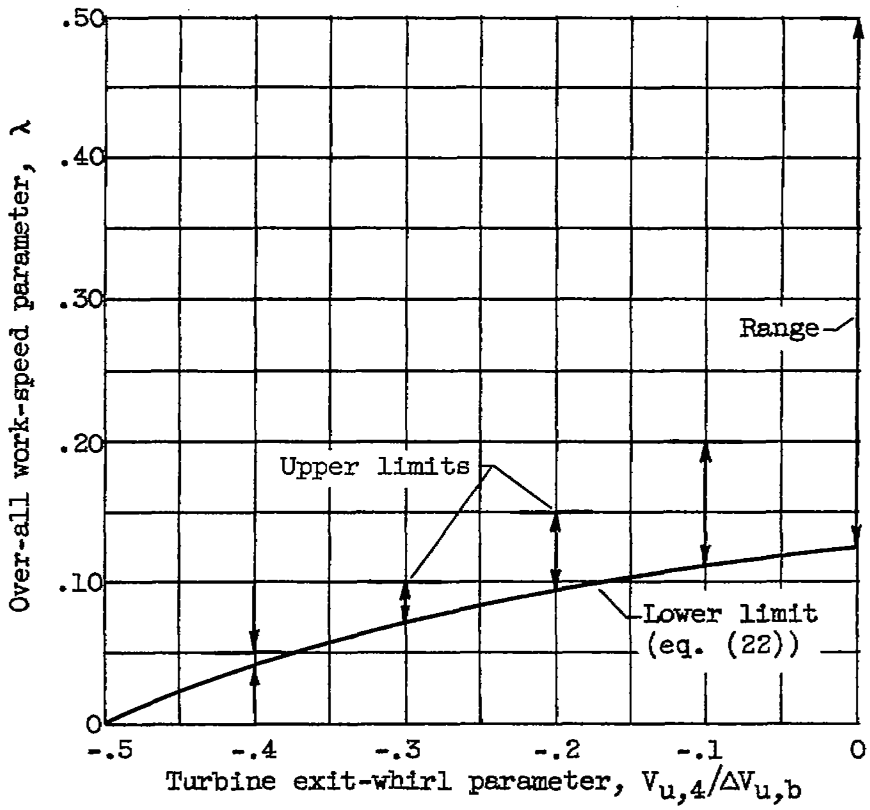

Figure 2. - Graphical description of limits imposed on over-all work-speed parameter.

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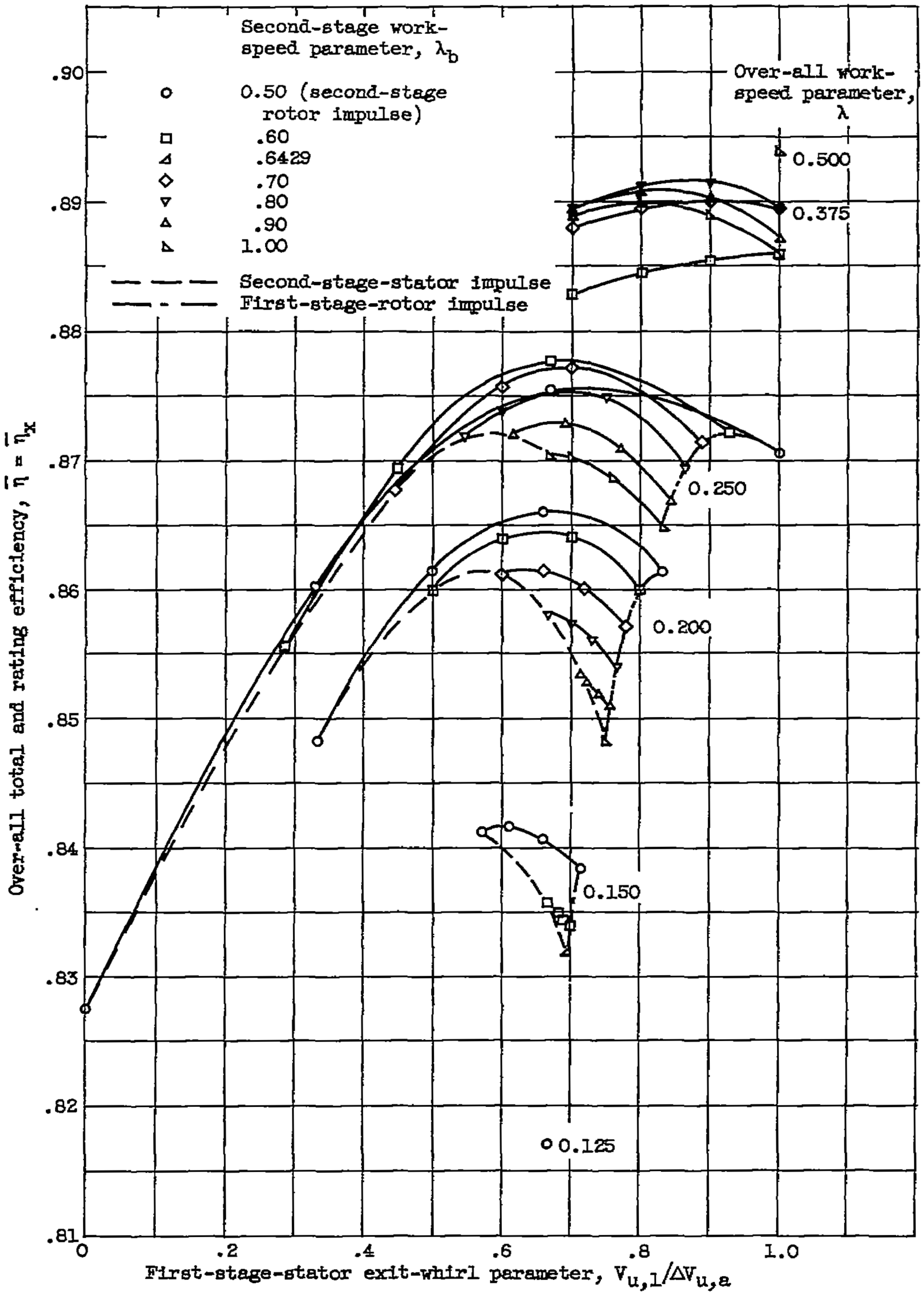


Figure 3. - Over-all total-efficiency characteristics; turbine exit-whirl parameter, $V_{u,4}/\Delta V_{u,b}$, 0.

[redacted]

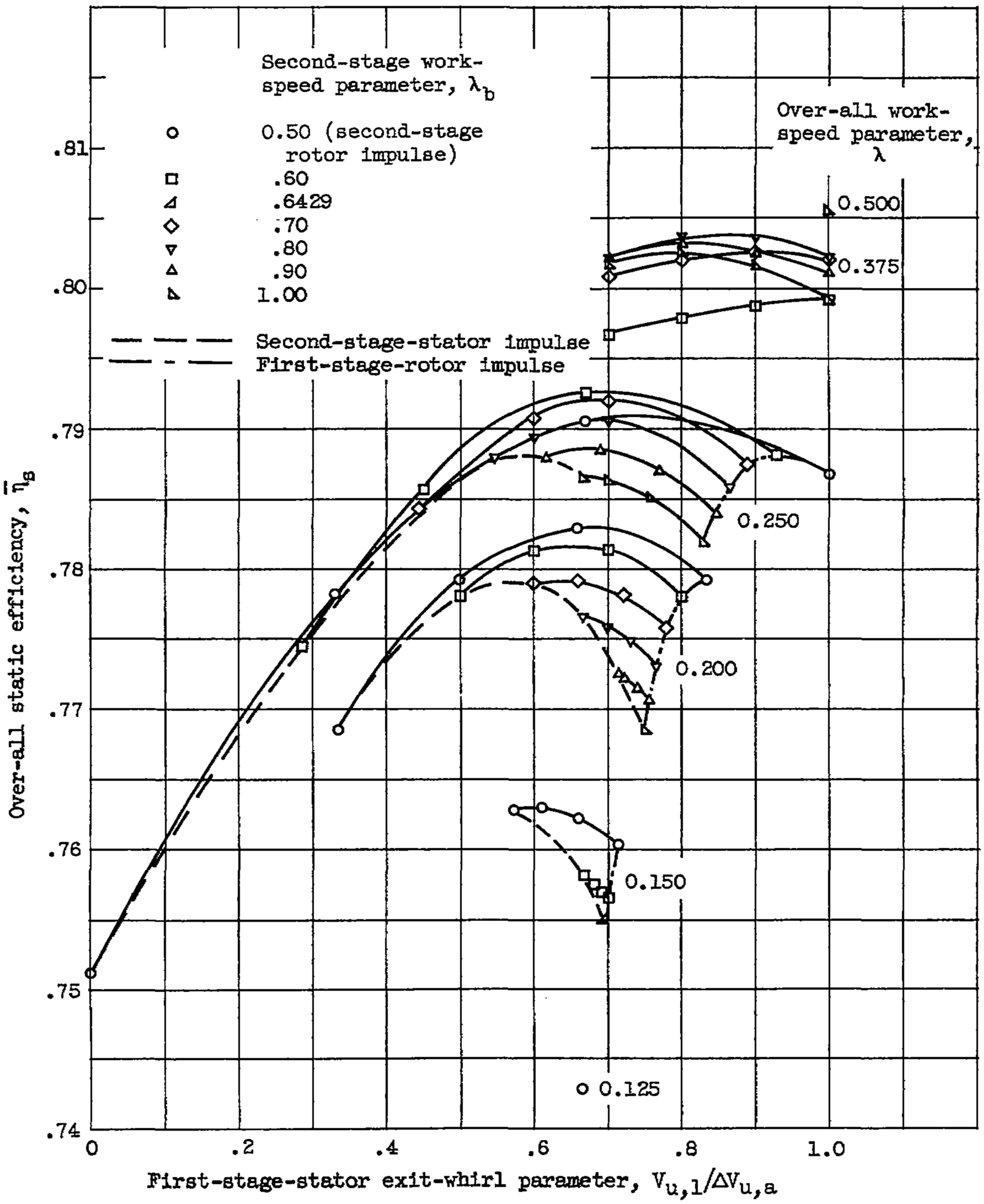


Figure 4. - Over-all static-efficiency characteristics; turbine exit-whirl parameter, $V_{u,4}/\Delta V_{u,b}$, 0.

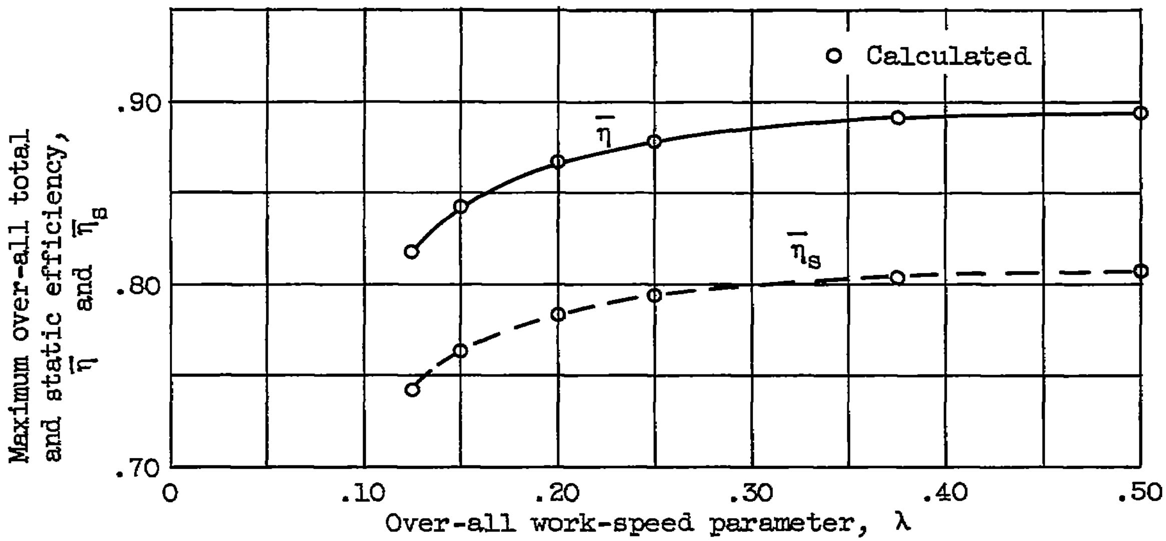


Figure 5. - Two-stage-turbine maximum-efficiency characteristics; turbine exit-whirl parameter, $V_{u,4}/\Delta V_{u,b}$, 0.

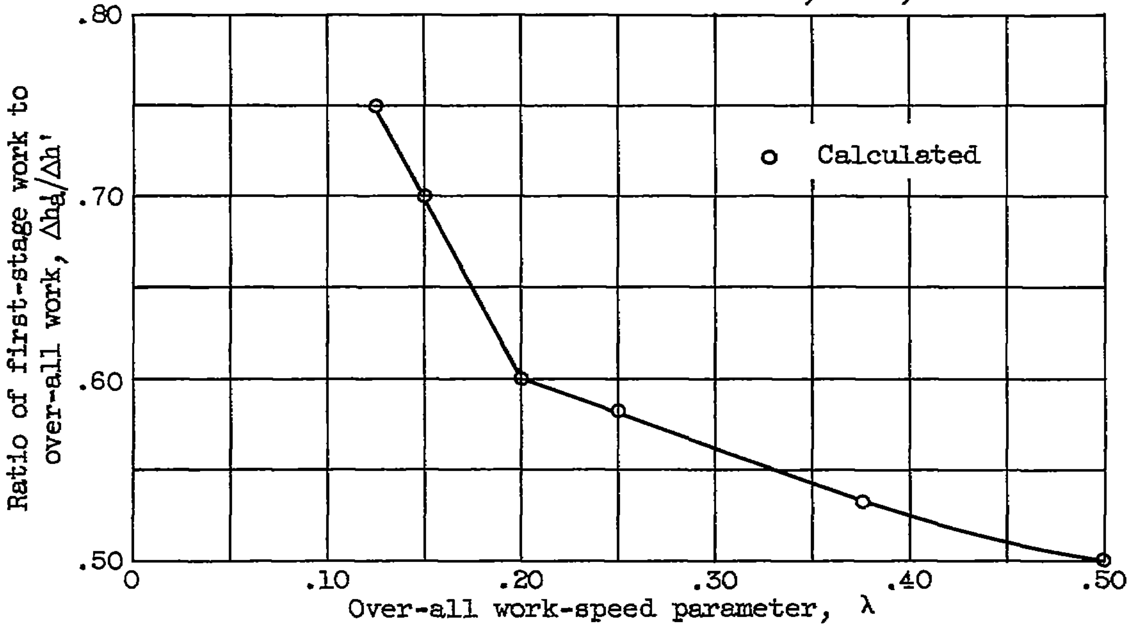
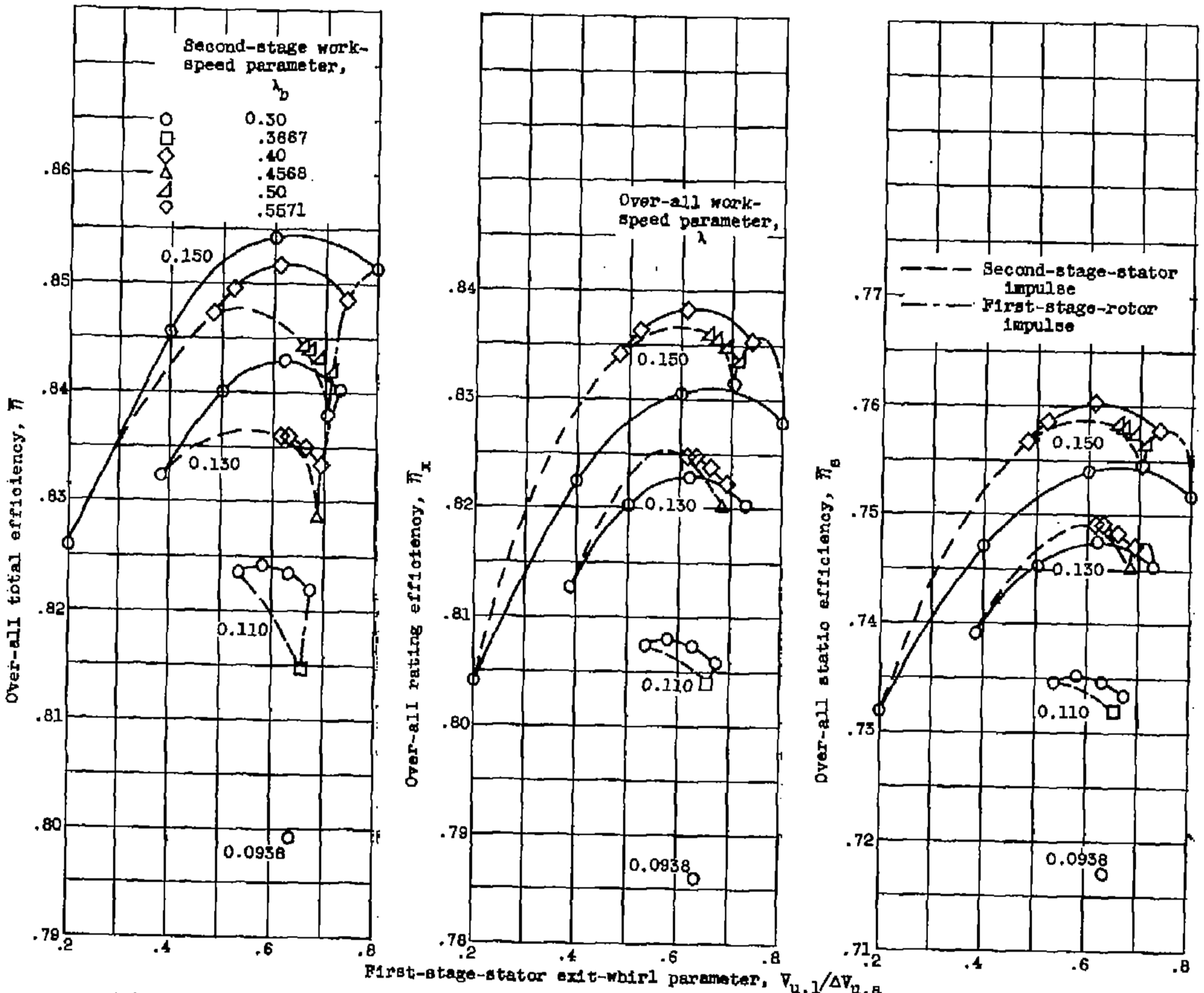


Figure 6. - Work split corresponding to turbine maximum total efficiency; turbine exit-whirl parameter, $V_{u,4}/\Delta V_{u,b}$, 0.

[redacted]



(a) Total efficiency.

(b) Rating efficiency.

(c) Static efficiency.

Figure 7. - Over-all efficiency characteristics; turbine exit-whirl parameter, $V_{u,4}/\Delta V_{u,b}$, -0.2.

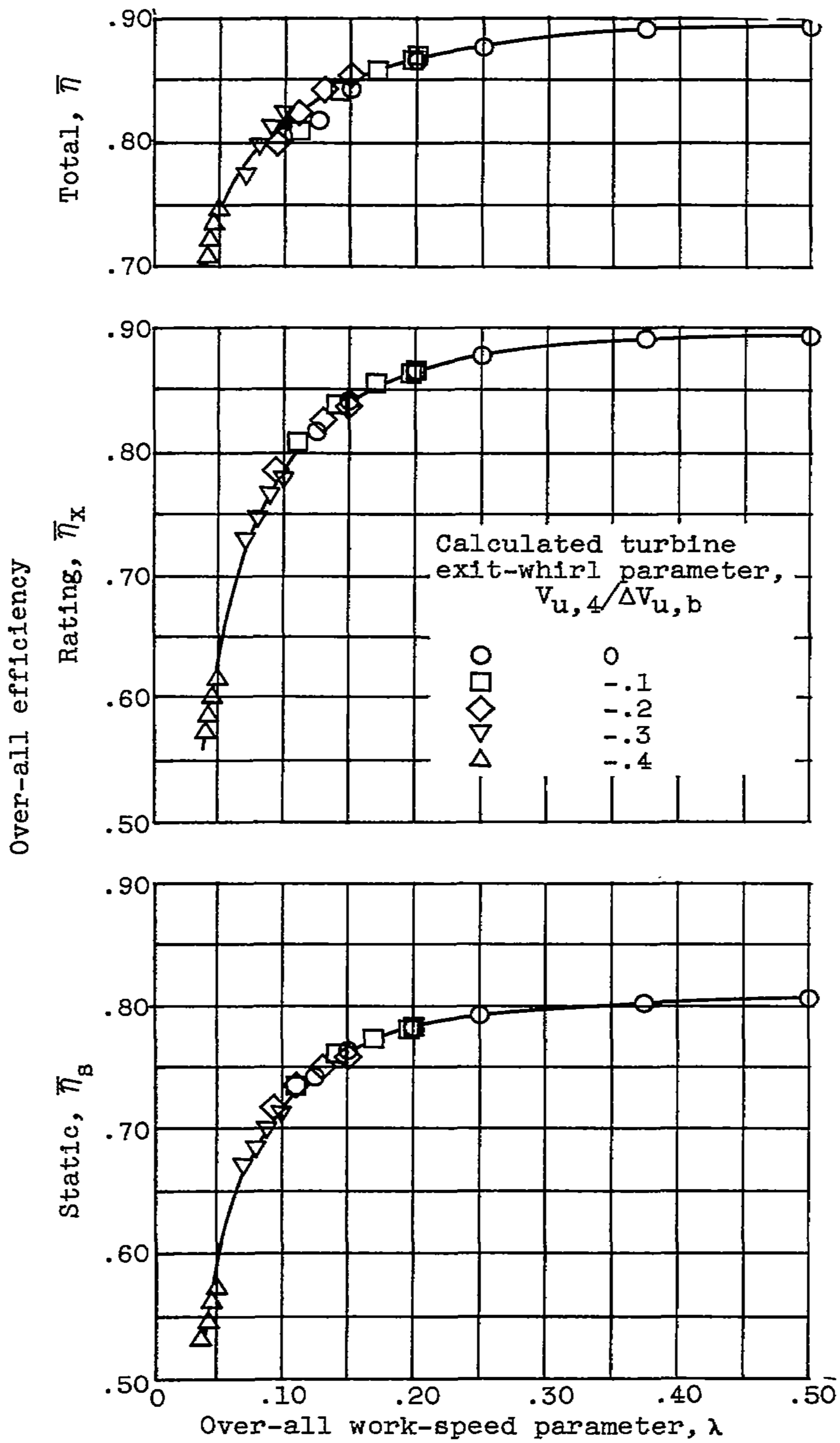


Figure 8. - Over-all maximum-efficiency characteristics with exit whirl.

[redacted]

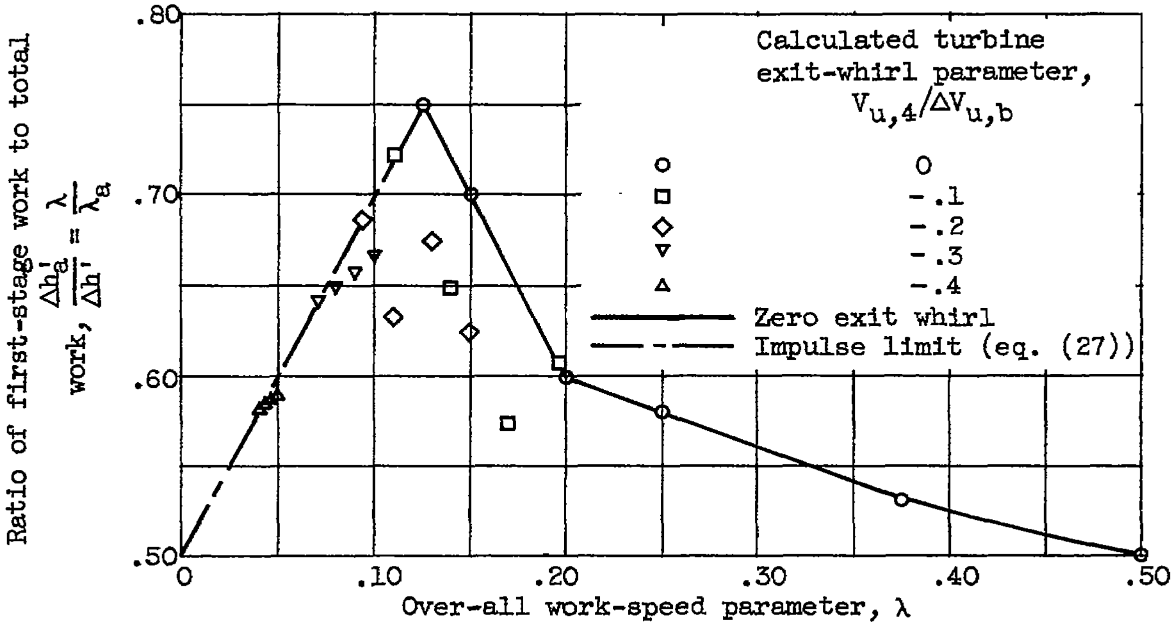


Figure 9. - Work-split characteristics with exit whirl.

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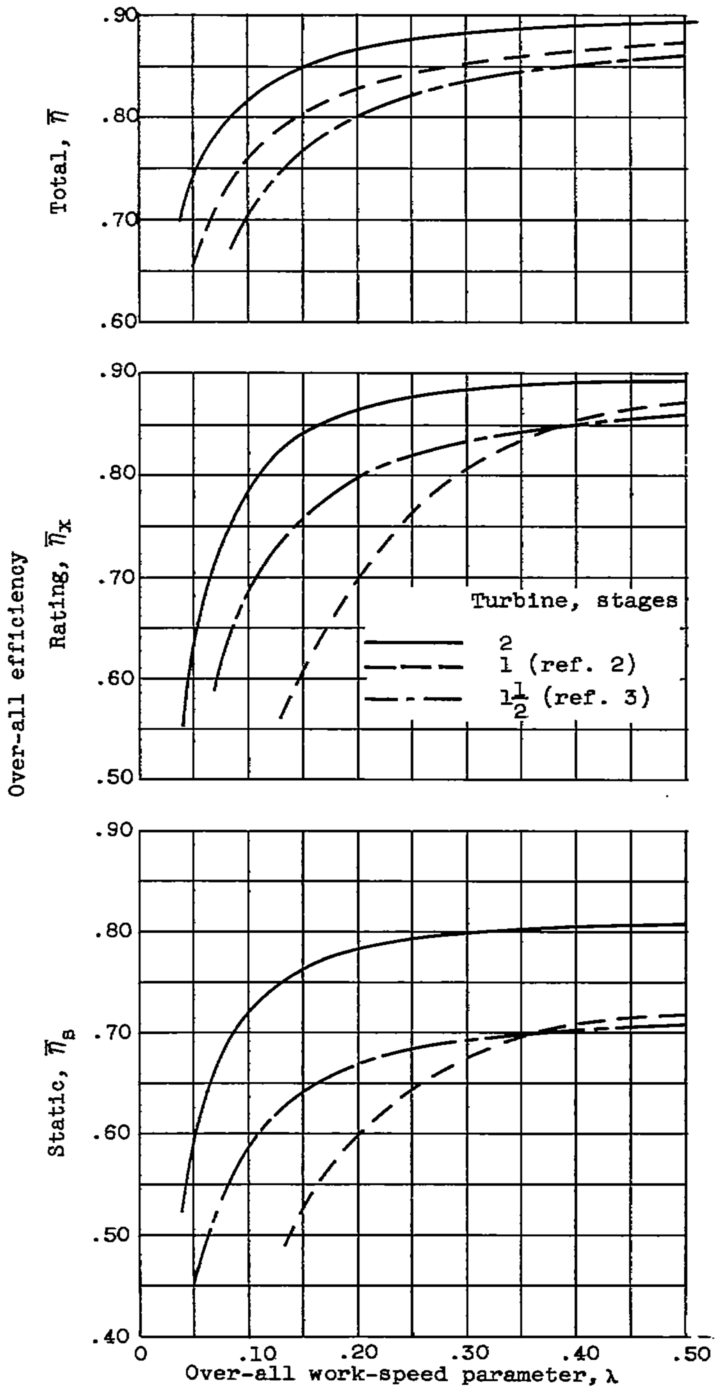

Figure 10. - Comparison of 1-, $1\frac{1}{2}$-, and 2-stage-turbine efficiency characteristics over range of work-speed parameter from 0 to 0.50.

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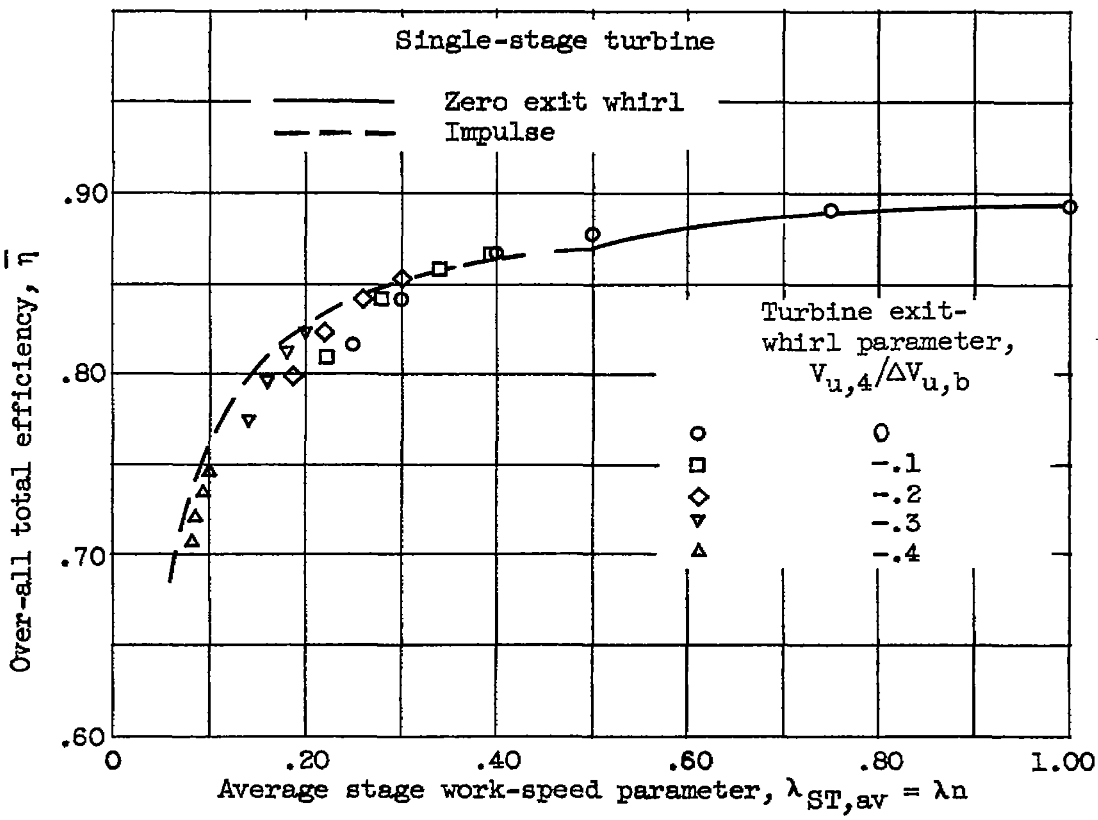
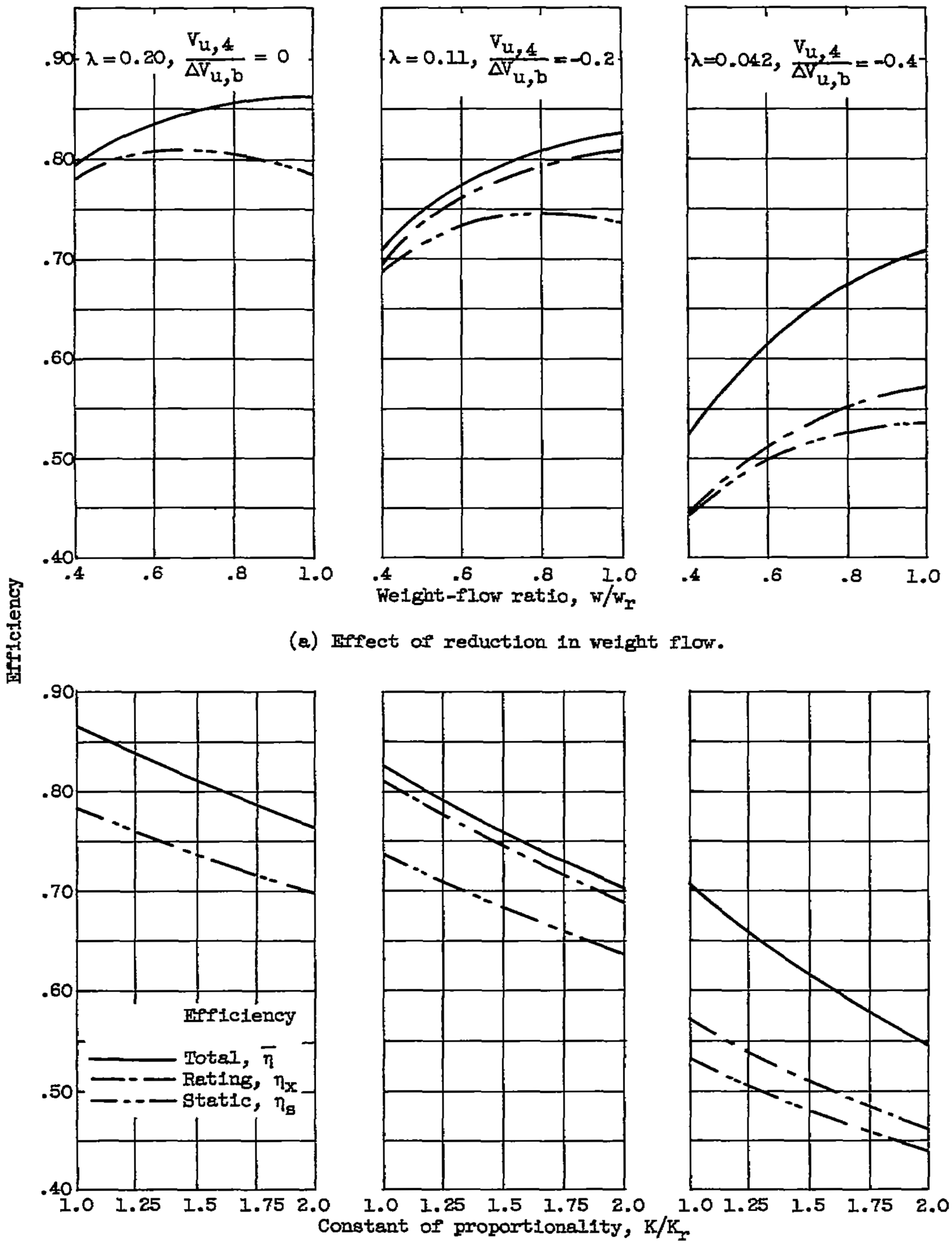


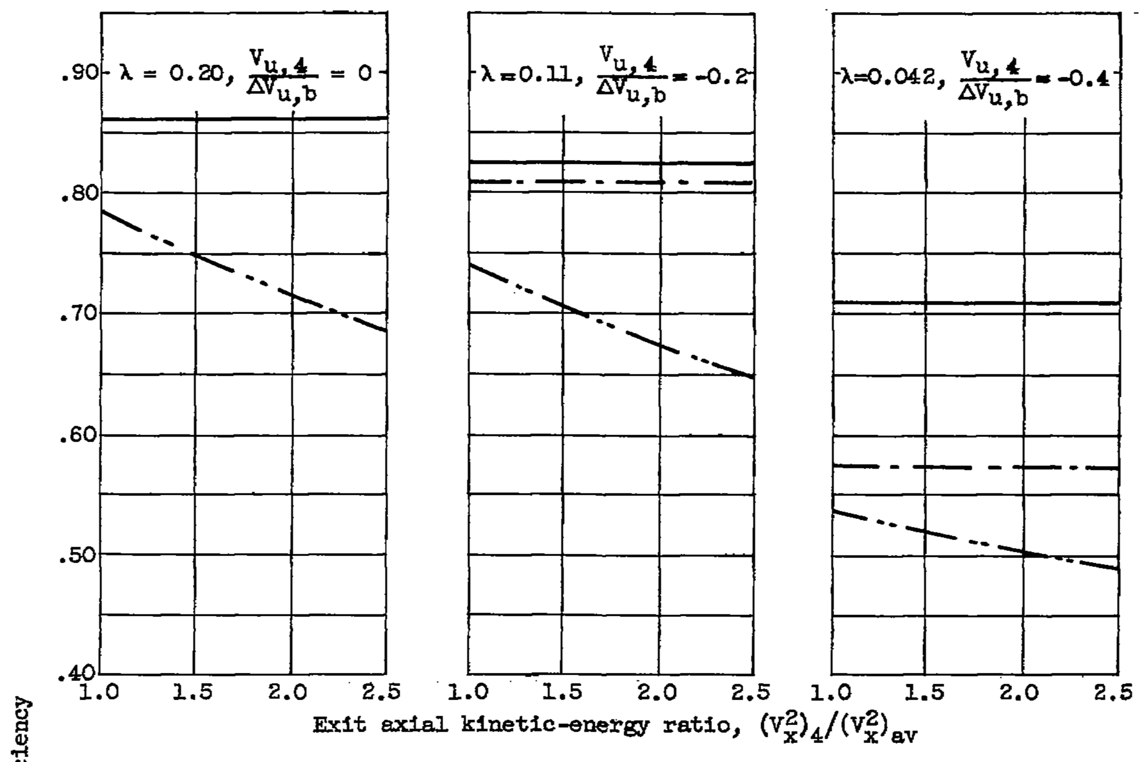
Figure 11. - Comparison of two-stage and single-stage total efficiencies on a per stage basis.



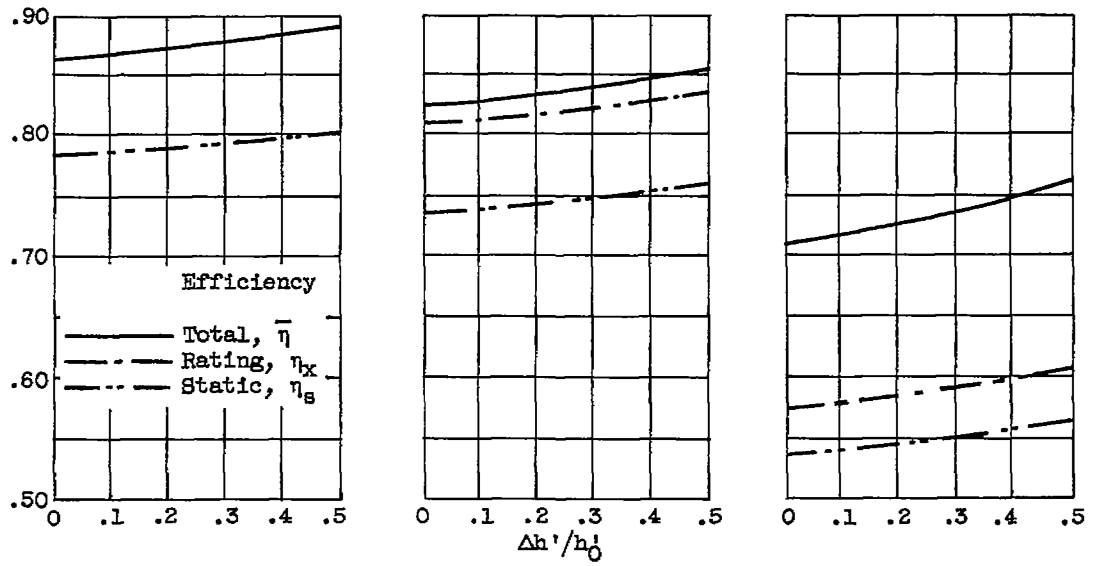
(a) Effect of reduction in weight flow.

(b) Effect of increased kinetic-energy losses.

Figure 12. - Effect of assumptions and constants on over-all turbine performance.



(c) Effect of increased exit axial kinetic energy.



(d) Effect of reheat.

Figure 12. - Concluded. Effect of assumptions and constants on over-all turbine performance.

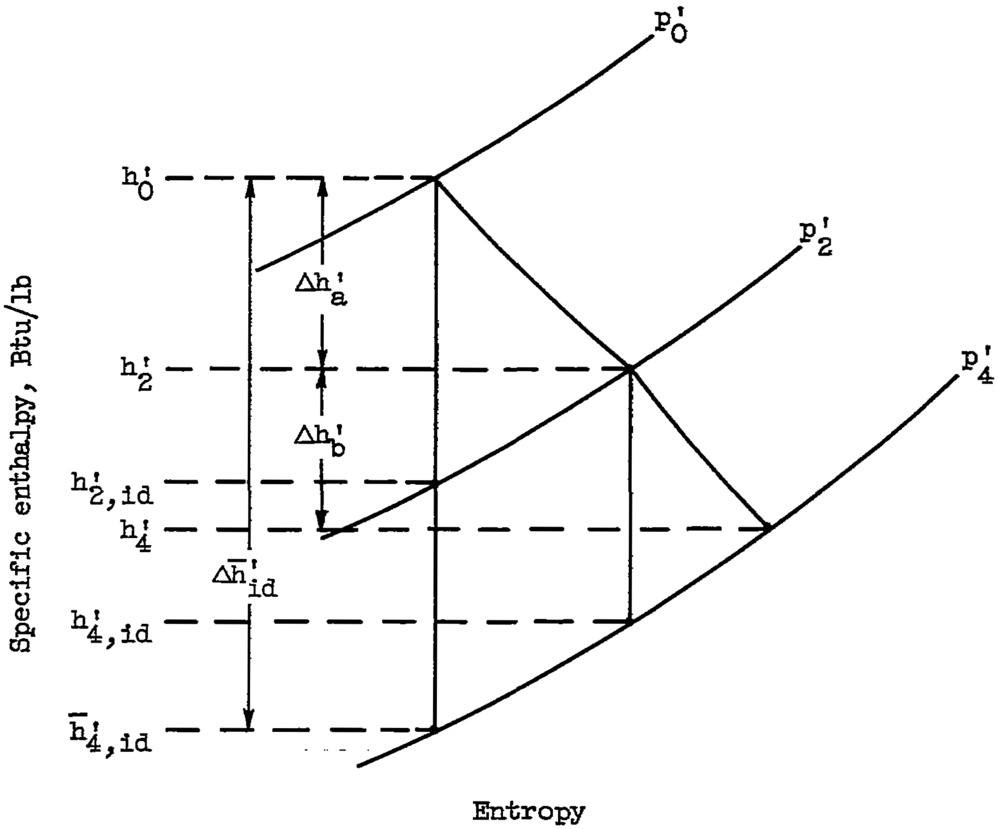


Figure 13. - Two-stage-turbine characteristics on enthalpy-entropy diagram.

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