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# RESEARCH MEMORANDUM

## RAPID ESTIMATION OF BENDING FREQUENCIES OF ROTATING BEAMS

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## SUMMARY

A procedure is presented in the form of charts which permits the rapid estimation of the natural bending frequencies of helicopter rotor blades, both rotating and nonrotating. Since the approach is based on Southwell's equation, an evaluation of the method with regard to such things as higher modes, blade offset, and variable mass and stiffness distributions is also given. The evaluation shows that, when nonrotating beam bending modes are used, Southwell's equation yields reasonably accurate bending frequencies for rotating helicopter blades. Example comparisons of frequencies estimated using the charts with values given by the manufacturer for several actual blades show that the simplified procedure yields good practical results.

## INTRODUCTION

The purpose of this paper is to present results in chart form which permit the rapid estimation of bending frequencies of rotor blades. The proposed method of frequency determination makes use of the familiar Southwell form; thus, it is also the purpose of this paper to show that this approach works quite well when such things as higher modes, blade offset, and variable mass and stiffness distributions are considered. The paper is divided into three parts as follows: In the first part a review and an evaluation of the Southwell approach is given, in the second part frequency charts are presented, and in the third part the results of applying these charts to some actual helicopter blades are given to indicate the order of accuracy obtainable in practical cases with this procedure.

## REVIEW AND EVALUATION OF SOUTHWELL APPROACH

The basic form of the Southwell equation which defines the bending frequencies of a rotating beam can be obtained by energy considerations and is given in reference 1 as

$$\omega_R^2 = \left[\frac{\int_0^L EI(y'')^2\,dx}{\int_0^L my^2\,dx}\right] + \left[\frac{\int_0^L T_1(y')^2\,dx}{\int_0^L my^2\,dx}\right]\Omega^2 \tag{1}$$

where

| | |
|---|---|
| $\omega_R$ | bending frequency of rotating beam |
| $\Omega$ | rotational speed of beam |
| m | mass distribution for beam |
| EI | stiffness distribution for beam |
| $T_1\Omega^2$ | tension force in beam |
| L | length of beam |
| x | spanwise coordinate along beam |
| $y, y', y''$ | beam mode shape and derivatives with respect to $x$ |

Equation (1) yields exact values for the bending frequencies of a rotating beam if the mode shapes of the rotating beam are known. Since these exact shapes are usually not known, however, it is necessary to assume mode shapes from which approximate frequencies can then be estimated. If the nonrotating mode shape is substituted into the first term of equation (1) and the coefficient of $\Omega^2$ is replaced by $K$, the Southwell constant, the Southwell equation takes the following form:

$$\omega_R^2 = \omega_{NR}^2 + K\Omega^2 \tag{2}$$

where

| | |
|---|---|
| $\omega_{NR}$ | nonrotating bending frequency of beam |
| K | Southwell constant |

If the hinge or point of fixity of the rotating beam is offset from the axis of rotation, it may easily be shown that the Southwell constant can be written in the form:

$$K = K_0 + K_1 \epsilon \qquad (3)$$

where

$K_0$ zero-offset Southwell constant

$K_1$ offset correction coefficient for Southwell constant

$\epsilon$ distance hinge is offset from axis of rotation, percent beam length

If equation (3) is substituted into equation (2),

$$\omega_R^2 = \omega_{NR}^2 + \left(K_0 + K_1 \epsilon\right) \Omega^2 \qquad (4)$$

If certain constants for the beam are introduced into equation (4), it can be written as

$$\omega_R^2 = a_n^2 \frac{EI_0}{m_0 L^4} + \left(K_0 + K_1 \epsilon\right) \Omega^2 \qquad (5)$$

where $I_0$ and $m_0$ are measured at the root of the beam and

$L$ length of beam outboard of hinge or point of fixity

$a_n$ nonrotating frequency coefficient for beam vibrating in nth mode

In order to provide a basis for estimating the accuracy, usefulness, and possible limitations of the Southwell approach, a systematic series of blades was selected and frequencies were calculated by the Southwell approach and a more exact process. Figure 1 shows the cases studied using both methods. These cases include the uniform cantilever with 0, 5, and 10 percent offset and the uniform hinged beam with the same offsets. The "linear" type beams are beams in which the mass and stiffness both vary linearly from the root value to zero at the tip. Zero- and 10-percent offset were treated for this type of beam with both the cantilever and hinged root.

The Southwell frequencies were obtained with the use of the mode shapes for the nonrotating beam whereas the true or reference frequencies were obtained by a Rayleigh-Ritz energy procedure involving expansion of the rotating beam modes in terms of the nonrotating modes of a uniform beam. For the cantilever beams, five nonrotating uniform modes were used in the expansion. For the hinged beams, a pendulum mode was used in addition to the five nonrotating modes. Some of the results of this investigation are shown in figures 2 to 5.

The variation of bending frequency with rotational speed for a uniform hinged beam is shown in figure 2. The range of $\left[\frac{\Omega}{\left(\omega_{NR}\right)_{1st}}\right]^2$ in this figure is roughly 25 percent above that encountered in current helicopters. From the results shown it is evident that the Southwell results are quite accurate, the maximum error being about 3 percent in frequency squared or only about $1\frac{1}{2}$ percent in frequency. This maximum error is about the same for all three modes.

In order to avoid confusion, results for the 5-percent-offset case are not shown in this figure. They fall roughly midway between the 0- and 10-percent-offset curves and show the same type of agreement between exact and Southwell results.

Frequency results for the "linear" type hinged beam are shown in figure 3. From this figure it is apparent that the Southwell results are extremely accurate, even for the highest rotational speeds shown.

A comparison of frequency results for the uniform and the "linear" type hinged beam is given in figure 4. The most important thing to be noted from this comparison is the difference in slope between the dashed and solid line for each mode. The slope of each of these lines is directly proportional to the Southwell constant. The large difference in slope, particularly evident for the first mode, indicates that a single value of the Southwell constant for each mode could not adequately predict the variations of frequency with rotational speed which are shown in the figure.

Frequency results for uniform cantilever beams are presented in figure 5. The range of $\left[\frac{\Omega}{\omega_{NR_{1st}}}\right]^2$ corresponds roughly to that covered for the hinged beams; the difference in scale is due to the fact that the first bending frequency of a uniform hinged beam is about four times greater than the first bending frequency of a uniform cantilever beam. For each mode the lower dashed and solid curve are for zero offset and the upper pair are for 10-percent offset.

From the figure it is apparent that the Southwell results are very accurate for the second and third modes. For the first mode, however, the error in frequency is somewhat larger, about 5 percent. Similar results obtained for cantilever beams with mass and stiffness distribution varying linearly from the root value to zero at the tip show about the same type of agreement between the Southwell and the more exact results as are given in figure 5. Since the error in Southwell results for the first mode is roughly the same for both these cases it is reasonable to assume that applying a correction factor determined by the error shown in figure 5 will result in more accurate prediction of the first mode bending frequency of other cantilever beams as well.

From the foregoing evaluation of the Southwell approach it was concluded that Southwell constants based on the nonrotating beam mode shapes lead to reasonably accurate bending frequencies of rotating helicopter blades. The evaluation also showed that the Southwell constants vary appreciably with beam mass and stiffness distribution.

## CHARTS FOR FREQUENCY DETERMINATION

In order to provide a means for rapidly estimating rotor-blade bending frequencies, the nonrotating frequency coefficients, zero-offset Southwell constants, and offset correction coefficients for Southwell constants have been computed for a series of beams with linear mass and stiffness distributions. The range of mass and stiffness distributions was selected to encompass variations found in currently manufactured blades with some latitude for new design. All the constants are based on the mode shapes of the nonrotating beam which were obtained by standard numerical iteration procedures. The cantilever modes were obtained using 10 stations while the hinged modes were obtained using 15 stations.

The variation of the nonrotating frequency coefficient $a_n$, with beam mass and stiffness distribution is shown in figure 6. The abscissa is the ratio of tip mass to root mass, 1.0 represents a constant mass beam, and 0 the case where the mass varies linearly to zero at the tip.

The solid curves are for beams with constant stiffness along the length, the dashed curves for beams where the stiffness drops to half the root value at the tip, and the long- and short-dashed curves for beams where the stiffness is zero at the tip.

Figure 7 permits selecting an accompanying value for the Southwell constant for beams with zero hinge offset. The variation with stiffness distribution appears quite small, particularly for the first mode. Actually, however, there is a maximum difference of about 5 percent. The variation with mass distribution is obviously somewhat more pronounced for all the modes shown. The zero-offset Southwell constant is not shown for the zero or pendulum mode since it is always unity independent of the mass and stiffness distribution of the beam.

In order to account for cases where the hinge is offset from the axis of rotation, offset correction coefficients $K_1$ have also been computed for this family of beams. These coefficients, when multiplied by the offset given as a fraction of the free beam length, yield the correction to be added to the zero-offset Southwell constant.

The variation of this offset coefficient is shown in figure 8. It is evident that this variation is quite similar to that shown in figure 7 except for the zero mode. An expression for the zero-mode offset coefficients presented here is given in reference 2.

Results similar to those presented in figures 6 to 8 have also been obtained for cantilever beams and are shown in figures 9 to 11. The effect of root fixity on the Southwell constant for the various modes can be deduced by comparing the curves in figures 8 and 9 with those in figures 10 and 11. It should be mentioned that a more accurate estimation for the first bending frequency of a rotating cantilever beam should be obtainable by reducing the frequency obtained using the charts and Southwell's equation by a small percentage which may be quickly estimated from figure 5.

## EXAMPLE RESULTS

In order to illustrate the type of accuracy which can be expected in using the frequency charts of figures 6 to 11, bending frequencies have been estimated for the first three modes of four existing helicopter blades, all of which are hinged. The following procedure was used in this estimation:

(1) Straight lines were faired through the $m$ and $EI$ distributions for the blade, large values near the root being ignored.

(2) From these fairings, the effective root values of $m$ and $EI$ and the necessary tip-to-root ratios were obtained.

(3) By using these ratios, values of $a_n$, $K_0$, and $K_1$ were obtained from the charts.

(4) Substitution of these constants into the Southwell equation yielded the bending frequencies at zero and at rated rotor speed.

The results are shown in table I. The $m$ and $EI$ distributions for the blades are shown on the left-hand side of the table. The actual distribution is given by the solid lines. The dashed line is the linear approximation selected to represent this variation. It should be emphasized that these linear approximations used in estimating the frequencies were the initial ones selected and were not juggled to obtain the best agreement. The frequencies shown as exact in this figure are values furnished by the manufacturer.

If the exact and estimated results for the blades are compared, it is evident that the results are quite accurate when the crudeness of the linear approximations used is considered. It is interesting to note that in all cases the estimated frequency of the rotating beam is more accurate than that for the nonrotating beam; this indicates that the linear approximations yield more accurate values for the Southwell constants than for the nonrotating frequency coefficients.

Although no comparisons have been made for fixed end blades, it is believed that even more accurate results should be obtainable for this end condition since large values of root stiffness can be more accurately accounted for by using the offset correction factors.

All the results presented in this paper are for bending vibration in a plane normal to the plane of rotation. Frequencies for vibration in any other plane can be easily determined from these results, however, by means of a simple formula proposed by Lo and Renbarger (ref. 3).

## CONCLUDING REMARKS

To summarize, it has been shown that the Southwell approach, based on nonrotating mode shapes, provides a reasonably accurate means of predicting frequency changes due to rotational speed for most current helicopter blades. It has been found, however, that a single Southwell constant for each mode cannot yield accurate results for all cases, particularly if the mass distribution of the blades is quite different. In order to aid the designer, charts have been presented which permit the rapid estimation of nonrotating frequencies, of Southwell constants, and of the offset correction coefficients for Southwell constants, from which reasonably accurate frequencies can be readily obtained for rotating blades. In example applications, the method gave good estimations of the bending frequencies of actual rotor blades.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 18, 1954.

## REFERENCES

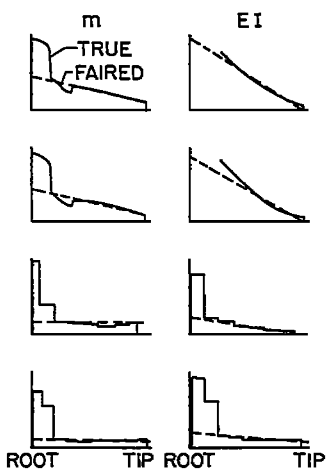
1. Liebers, F.: Contribution to the Theory of Propeller Vibrations. NACA TM 568, 1930.

2. Prewitt, R. H., and Wagner, R. A.: Frequency and Vibration Problems of Rotors. Jour. Aero. Sci., vol. 7, no. 10, Aug. 1940, pp. 444-450.

3. Lo, Hsu, and Renbarger, Jack L.: Bending Vibrations of a Rotating Beam. Proc. First U. S. Nat. Cong. Appl. Mech. (Chicago, Ill., 1951), A.S.M.E., 1952, pp. 75-79.

TABLE I

EXACT AND ESTIMATED FREQUENCIES FOR SEVERAL MANUFACTURED BLADES




| MODE | $\omega_{NR}$, RADIANS/SEC | | $\omega_{R}$, RADIANS/SEC | |
|---|---|---|---|---|
| | EXACT | EST. | EXACT | EST. |
| 1st | 17.3 | 17.4 | 49.2 | 47.7 |
| 2nd | 48.5 | 50.0 | 86.8 | 85.7 |
| 3rd | 95.5 | 101.0 | 137.0 | 137.8 |
| 1st | 21.6 | 21.1 | 50.6 | 49.2 |
| 2nd | 58.9 | 60.5 | 92.4 | 92.2 |
| 3rd | 112.1 | 122.0 | 148.0 | 154.0 |
| 1st | 21.9 | 21.1 | 74.0 | 78.3 |
| 2nd | 63.7 | 59.5 | 132.0 | 134.4 |
| 3rd | 126.0 | 125.5 | 200.0 | 207.5 |
| 1st | 13.4 | 14.6 | 37.9 | 37.8 |
| 2nd | 43.7 | 41.6 | 71.0 | 70.3 |
| 3rd | 94.9 | 94.5 | 125.0 | 124.0 |

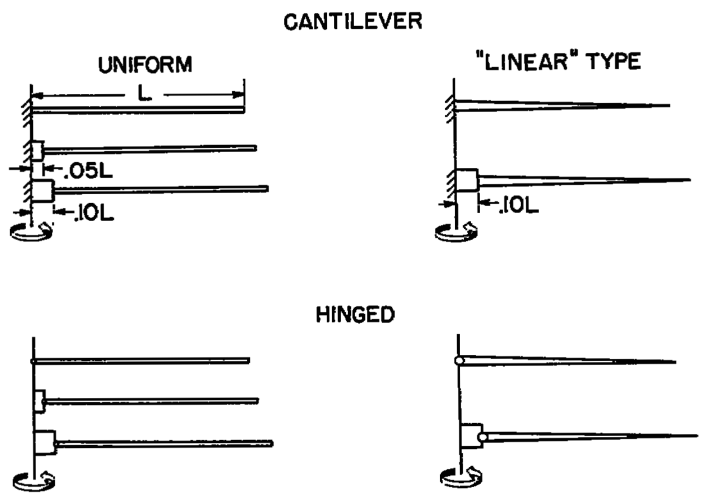



Figure 1.- Beams treated by "exact" and Southwell methods.

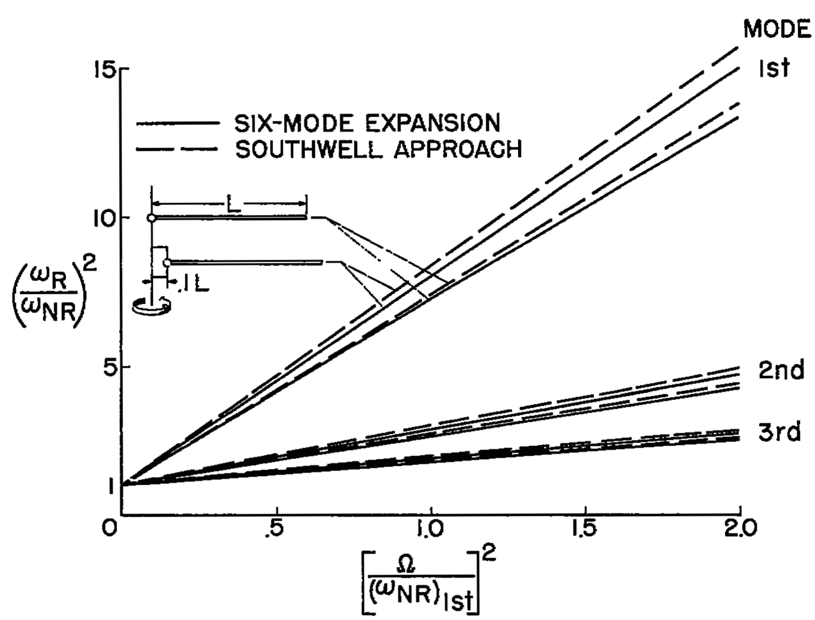



Figure 2.- Bending frequencies of uniform hinged beam.

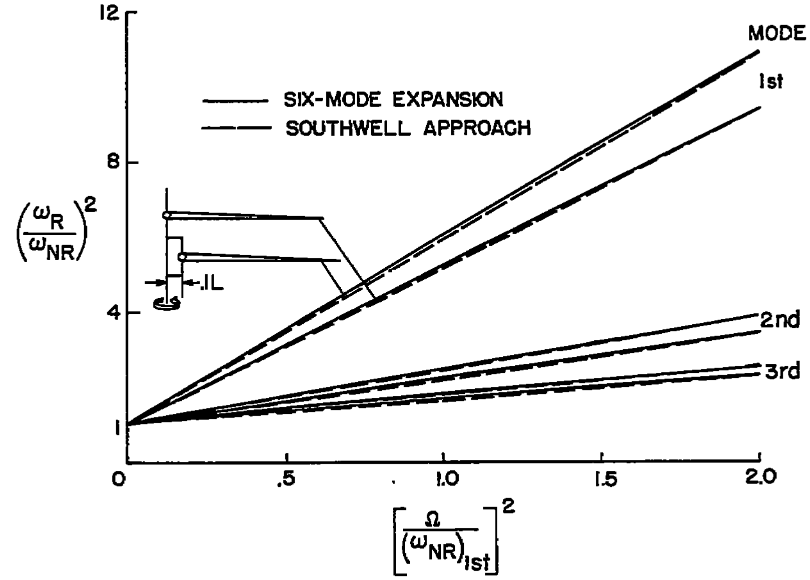



Figure 3.- Bending frequencies of "linear" type hinged beam.

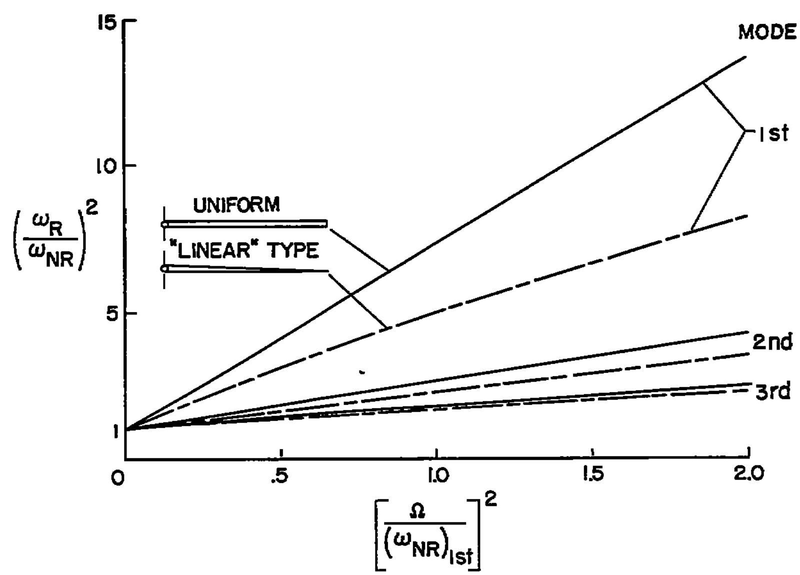



Figure 4.- Comparison of uniform and "linear" type hinged beams.

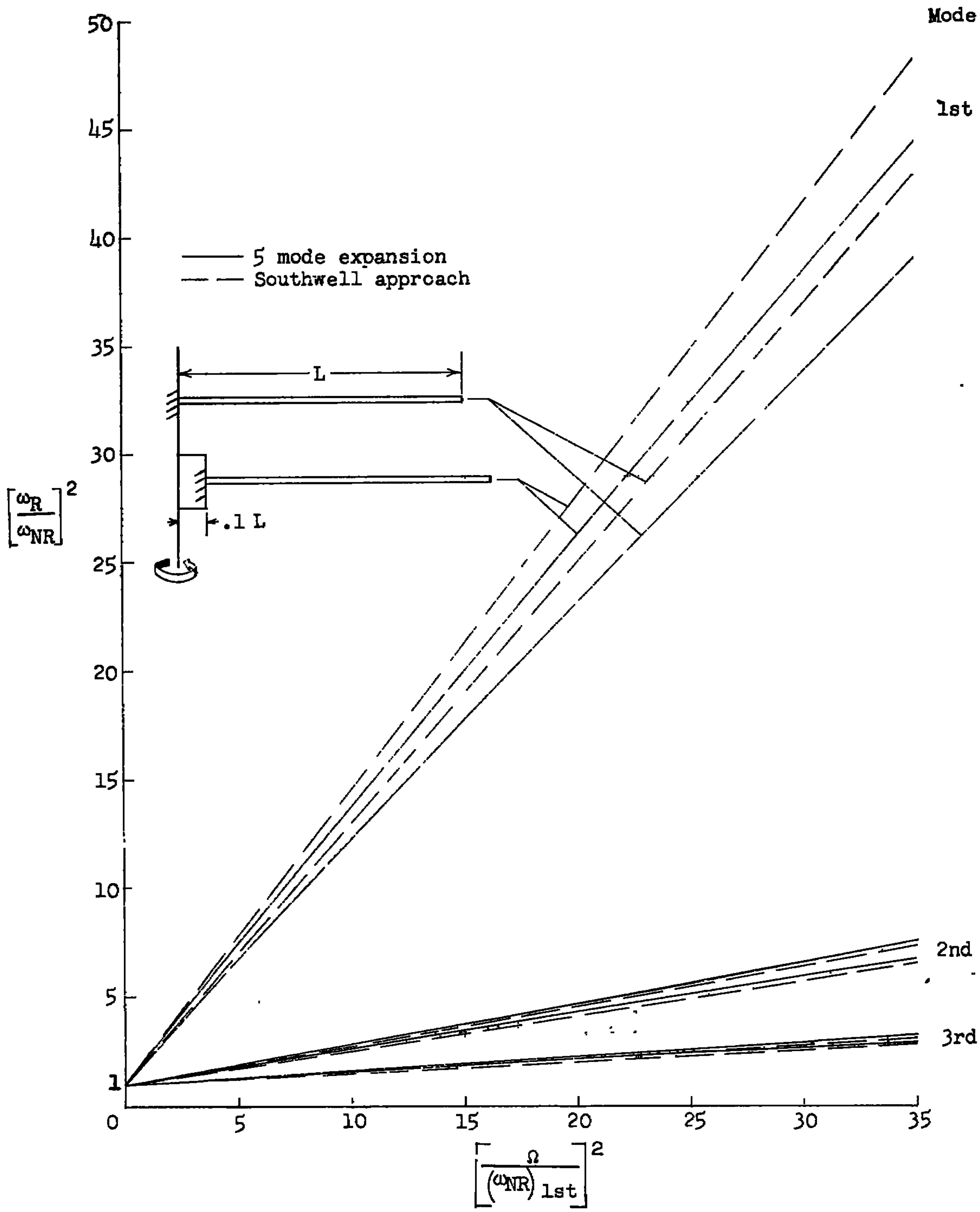



Figure 5.- Effect of rotational speed on the bending frequencies of a uniform cantilever beam.

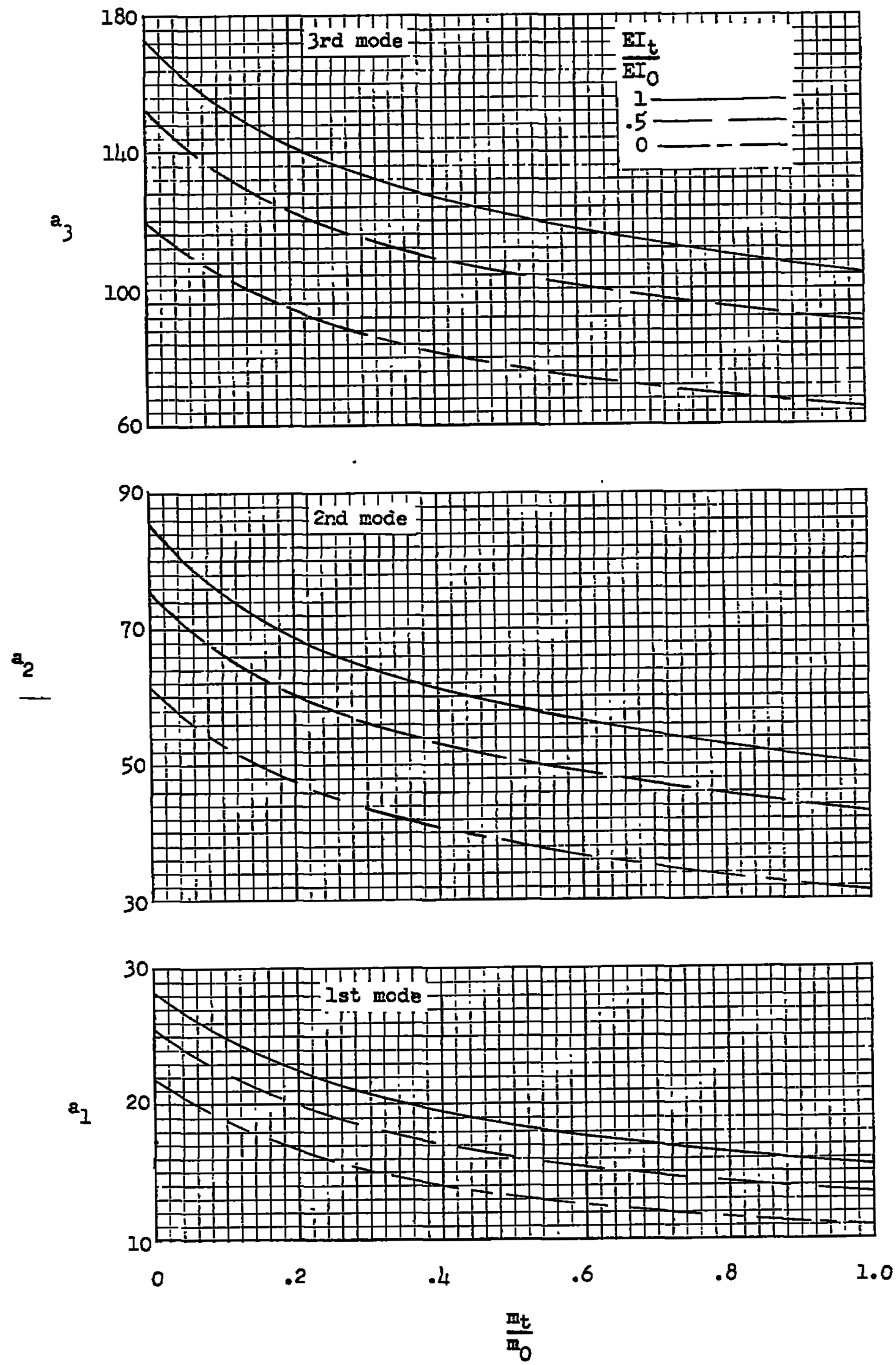



Figure 6.- Bending frequency coefficients $a_n$ for hinged beams with linear mass and stiffness distributions.

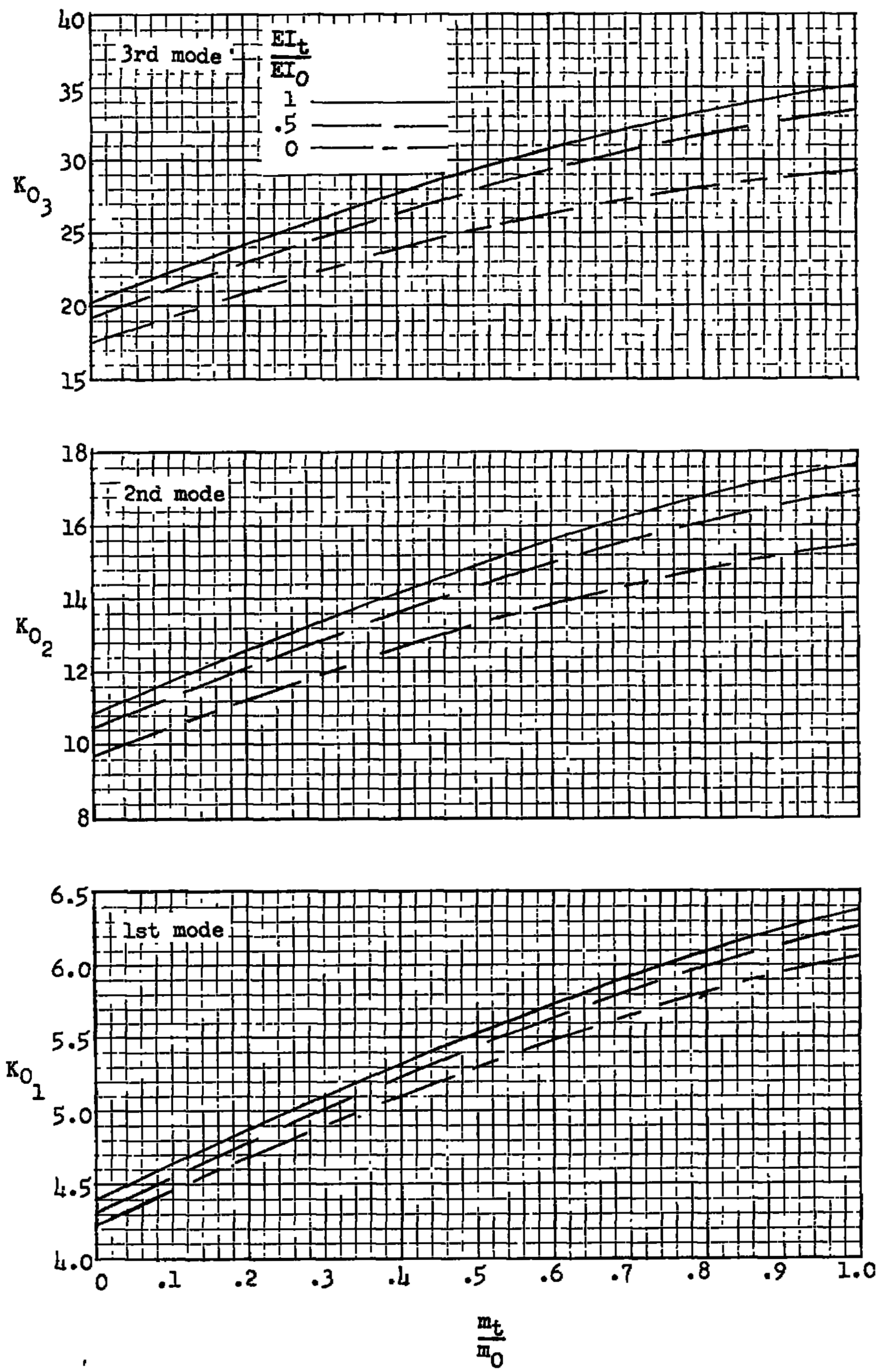



Figure 7.- Zero-offset Southwell constants $K_{0_n}$ for hinged beams with linear mass and stiffness distributions.

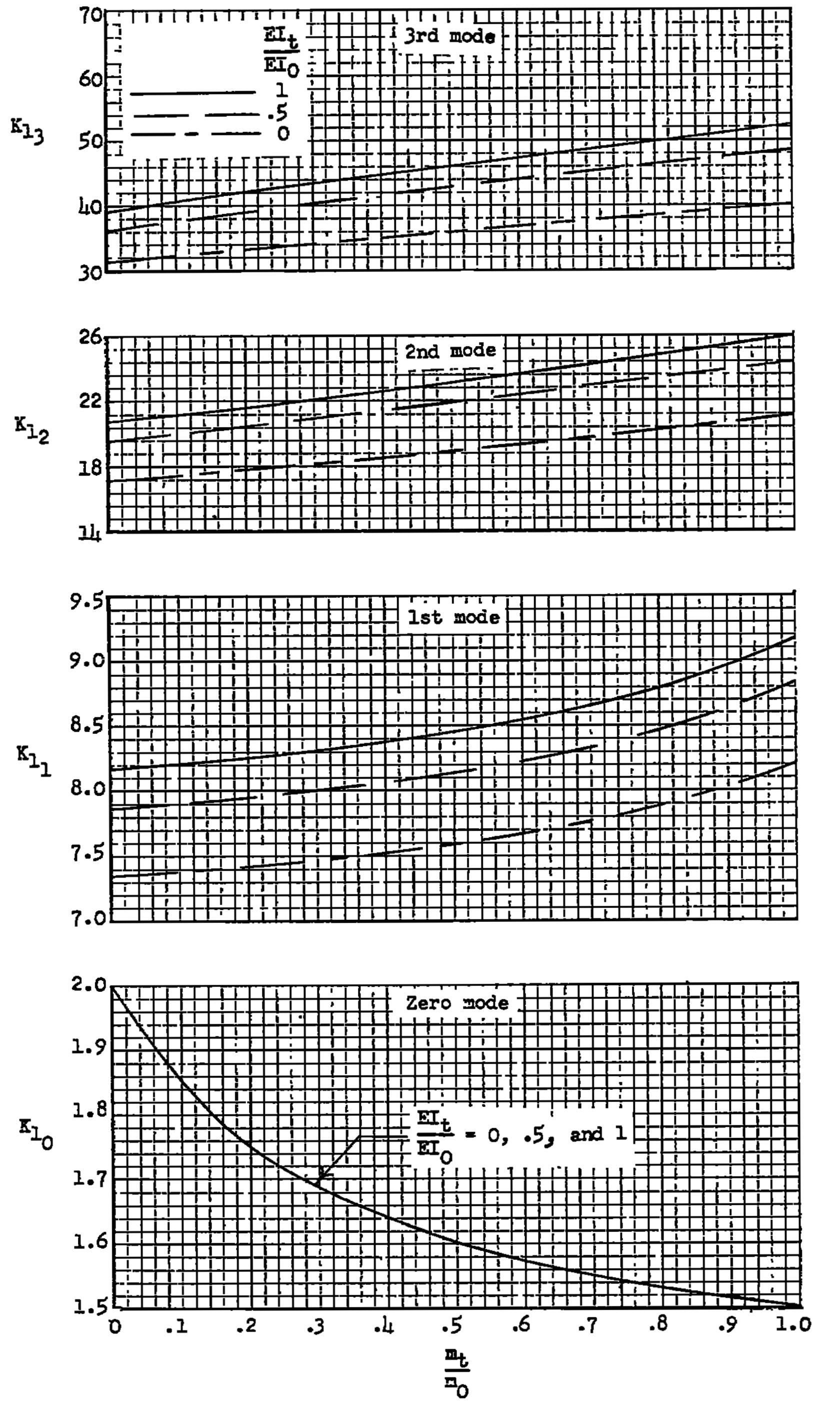



Figure 8.- Offset correction coefficients for Southwell constants $K_{1_n}$ for hinged beams with linear mass and stiffness distributions.

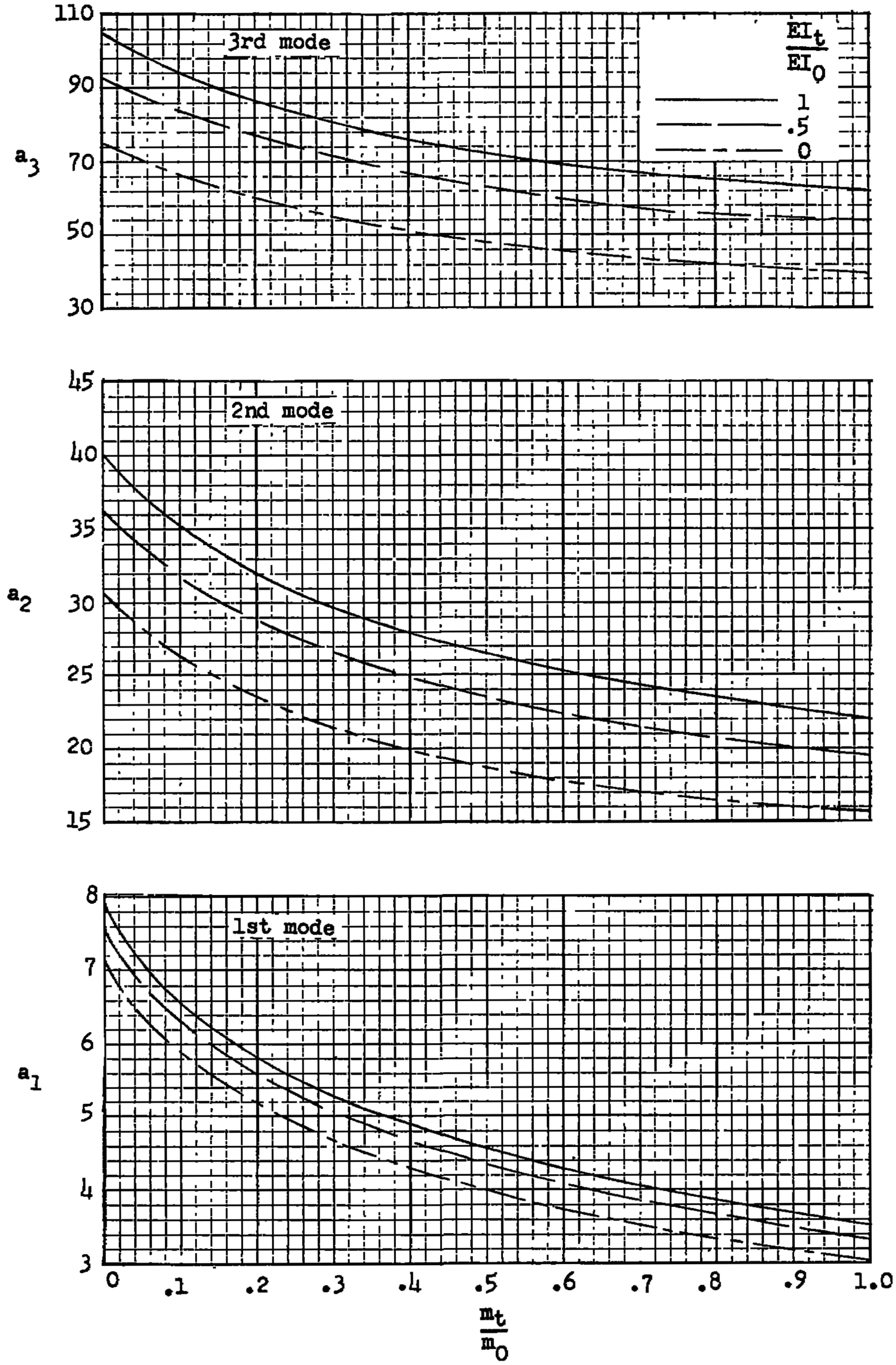



Figure 9.- Bending frequency coefficients $a_n$ for cantilever beams with linear mass and stiffness distributions.

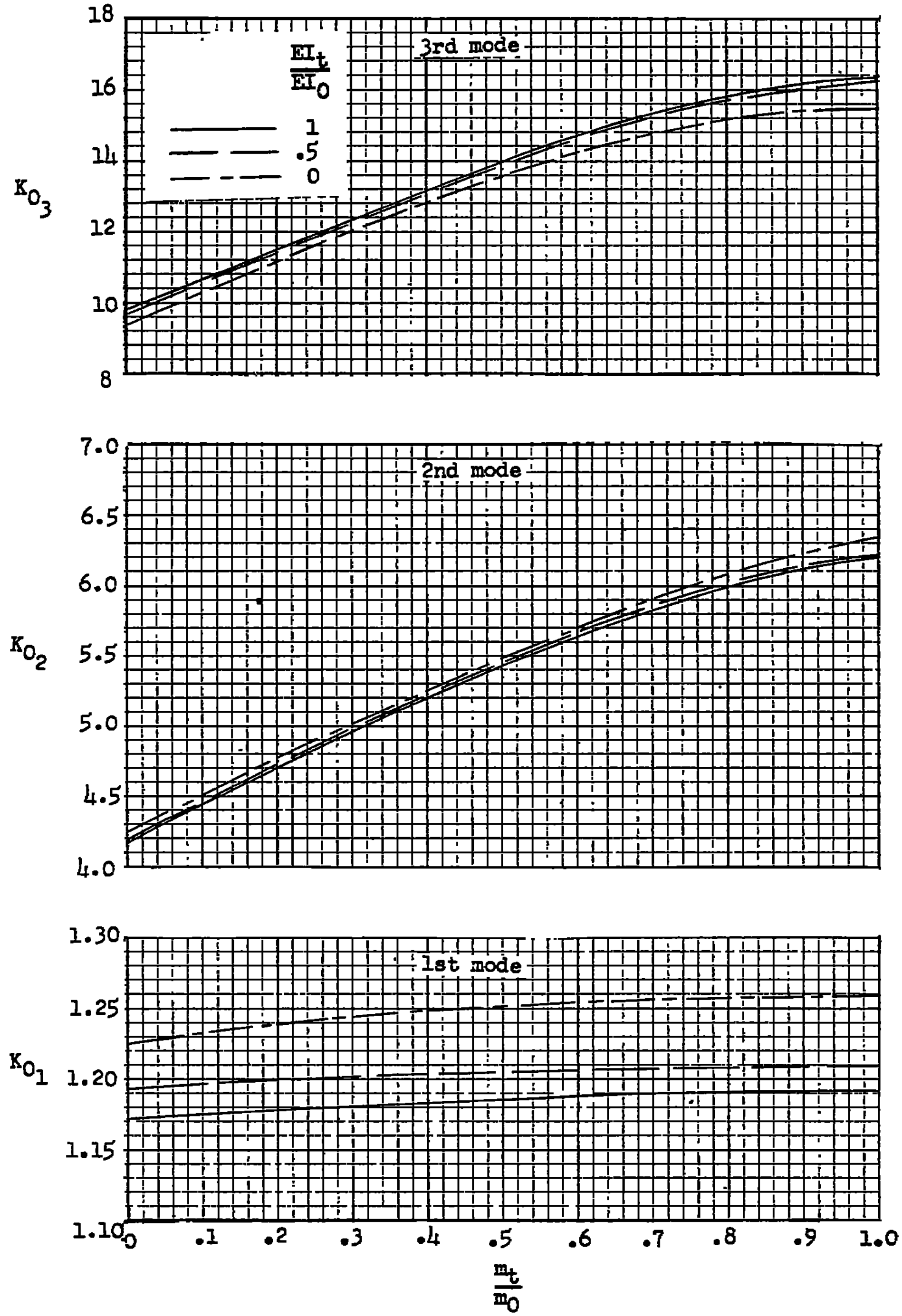



Figure 10.- Zero-offset Southwell constants $K_{0_n}$ for cantilever beams with linear mass and stiffness distributions.

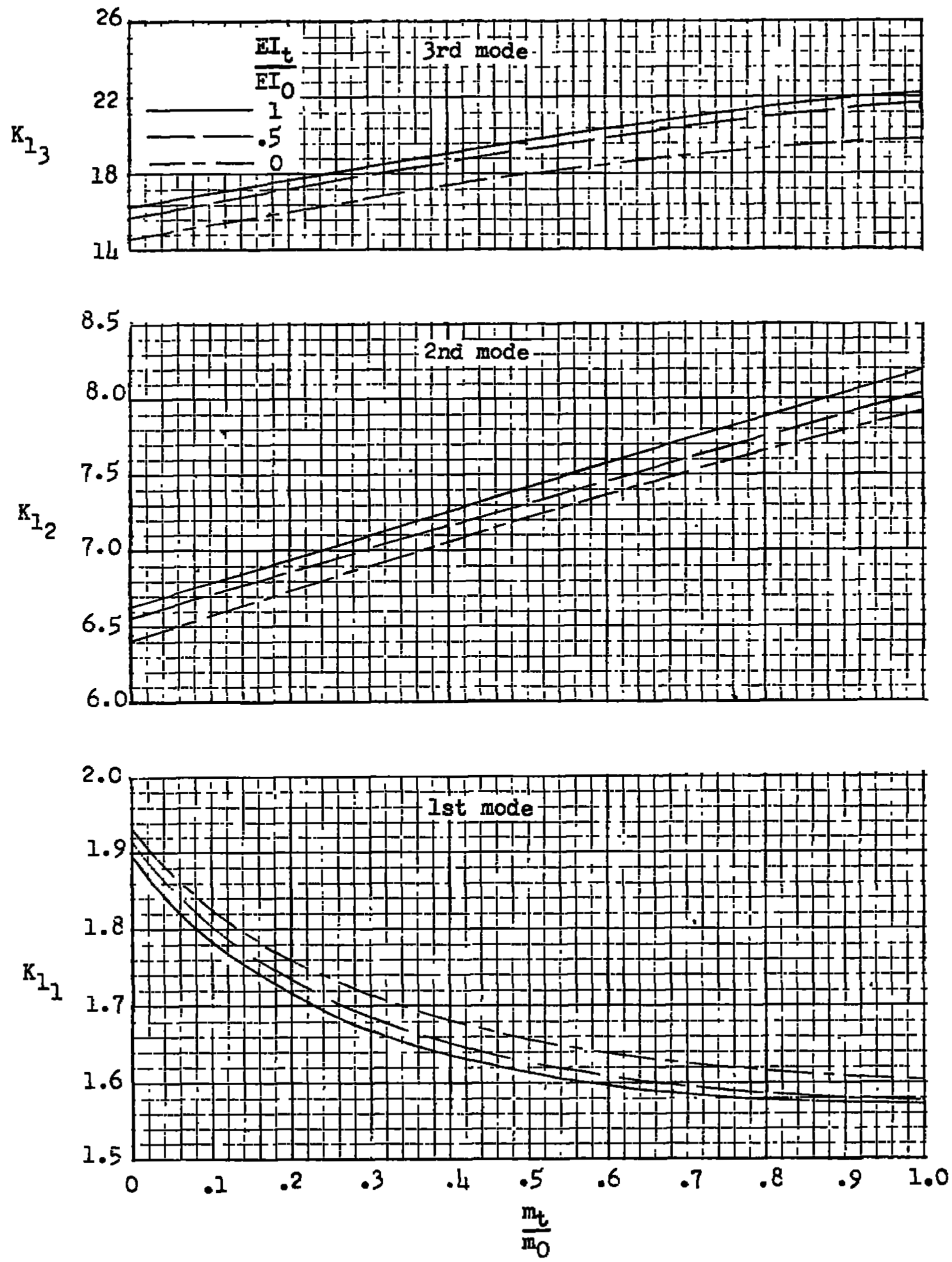



Figure 11.- Offset correction coefficients for Southwell constants $K_{1_n}$ for cantilever beams with linear mass and stiffness distributions.