

FILE COPY  
NO. *21*

# CASE FILE COPY

TECHNICAL MEMORANDUMS  
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

THIS DOCUMENT ON LOAN FROM THE FILES OF  
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS  
LANGLEY MEMORIAL AERONAUTICAL LABORATORY  
LANGLEY FIELD, HAMPTON, VIRGINIA

RETURN TO THE ABOVE ADDRESS.

REQUESTS FOR PUBLICATIONS SHOULD BE ADDRESSED  
AS FOLLOWS:

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS  
1724 STREET, N.W.,  
WASHINGTON 25, D.C.

*JM*  
No. 334

## THEORY OF FLAPPING FLIGHT

By Alexander Lippisch

From "Flugsport," June 17, 1925.

Washington,  
October, 1925.

Reproduced by  
**NATIONAL TECHNICAL  
INFORMATION SERVICE**  
Springfield, Va. 22151

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL MEMORANDUM NO. 334.

THEORY OF FLAPPING FLIGHT.\*

By Alexander Lippisch.

The announcement of the 1925 Rhoen soaring-flight contest contains, among other things, the effort of a prize for flights in muscle-propelled aircraft. Obviously the aviator must first construct his flying machine and then try his luck with it.

Before attempting to construct such a flying machine, the aviator will first try to post himself theoretically on the possible method of operating the flapping wings. It is not possible to include all the aspects of this difficult problem in the present article. We will, however, give, on the basis of a simpler case, a graphic and mathematical method, which renders it possible to determine the power required, so far as it can be done on the basis of the wing dimensions.

We will first consider the form of the flight path through the air. The simplest form is probably the curve of ordinary wave motion (Fig. 3, at the top). This curve would be produced, in flight at uniform speed, by alternately moving the wings up and down (Fig. 1). No account is taken of the fact that a longer stroke (if the wing of a bird is taken as a pattern) is made by the outer than by the inner portion of

---

\* From "Flugsport," June 17, 1925, pp. 246-253.

a wing. The wing is here assumed to move as a whole, without the twisting of any portion with relation to the rest. The case is therefore as simple mechanically as can be conceived. On the other hand, the method of operation of a wing working like the wing of a bird can be derived from the results obtained in the simpler case. Moreover, this case assumes still greater importance when it represents the converse of dynamic soaring flight in air moving in the form of waves.

After finding the flight curve, we must next determine the change in the angle of attack while passing through the different phases of the wave. Since the angle of attack, however, is directly proportional to the lift, we will, for simplicity, determine the lift distribution along the flight path. We will refer the air forces themselves to the horizontal axis of the flight path and resolve them into a horizontal and a vertical component (Fig. 2). We thus obtain

$$y = c_y S \frac{\gamma}{2g} v^2 \quad (1)$$

$$x = c_x S \frac{\gamma}{2g} v^2 \quad (2)$$

For the nondimensional coefficients  $c_y$  and  $c_x$ , we obtain the values

$$c_y = c_a + c_w \tan\varphi \quad (3)$$

$$c_x = c_a \tan\varphi - c_w \quad (4)$$

$c_y$  and  $c_x$  are generally represented, as in the case of  $c_a$  and  $c_w$

by the 100-fold values  $c_y$  and  $c_x$ . The angle  $\varphi$  is the momentary difference in direction between the horizontal and the tangent to the flight path and is found by the differentiation of the equation of the flight curve. The general formula is:

$$y = m \sin x \quad (5)$$

in the scale of the actual motion of the wing:

$$y = \left[ m \sin \left( \frac{2 \pi x}{\tau_0 v} \right) \right] \frac{\tau_0 v}{2 \pi} \quad (6)$$

Herein:  $v$  = flight speed (m/sec.),

$\tau_0$  = time taken for a complete stroke (sec.),  
with return to original position;

$x$  = distance flown (m), measured on the  
horizontal line.

Then 
$$\tan \varphi = m \cos \left( \frac{2 \pi x}{\tau_0 v} \right) \quad (7)$$

With the help of these formulas, we can calculate the momentary values of  $x$  and  $y$ , or the corresponding values of  $c_x$  and  $c_y$ , and plot them along the flight path as running functions of  $x$ .

Since the angle  $\varphi$  is generally very small, it is not necessary to compute  $c_y$ , because the member  $c_w \tan \varphi$  is smaller than the possible error in  $c_a$ . We therefore put  $c_y = c_a$  (approximately) and thus determine the lift distribution directly.  $c_y$  and  $c_x$  are positive, when there is a thrust or negative drag. In order to determine the best kind of lift distribution

we will consider three cases:

Case 1 (Fig. 3,a), when the lift is constant. No change occurs in the angle of attack with relation to the flight curve.

Case 2 (Fig. 3,b), when the lift fluctuates between its maximum value on the down-stroke and its minimum value on the up-stroke. The angle of attack with relation to the flight curve is greater during the down-stroke and smaller during the up-stroke.

Case 3 (Fig. 3,c), when the lift fluctuates still more. The angle of attack with relation to the horizontal remains nearly constant ( $\alpha_m = - 3.3^\circ$ ).

Further data may be obtained from the diagrams. We learn from the diagrams that there is no forward wing thrust in case 1, while there is in case 2 and still more in case 3.

The diagrams were drawn for a wing with the Göttingen profile No. 433 and an aspect ratio of 1 : 10, the maximum value of the angle  $\varphi$  (in passing through the middle position) being  $\varphi_{\max} = \pm 8^\circ$  and  $\tan \varphi_{\max} = 0.14 = m$ .

The equation for the flight curve is accordingly

$$y = \left[ 0.140 \cdot \sin \left( \frac{2 \pi x}{\tau_0 v} \right) \right] \frac{\tau_0 v}{2 \pi}$$

Any desired values may be given  $\tau_0$  and  $v$ , so long as it is simply a matter of the computation or graphic representation of the coefficients  $c_x$  and  $c_y$ .

For the computation of the required power and of the flight speed, we must determine the mean values of  $c_x$  and  $c_y$  by the integration of the hatched areas. In order to do this, we represent the polar curves by the following equation:

$$c_w = \left( \frac{S}{b^2 \pi} + k \right) c_a^2 - p c_a + q \quad (8)$$

This form of equation for a parabola agrees best with the form of ordinary polars because it takes account of the fact that the minimum resistance or drag occurs at small positive values of  $c_a$ . When computed (by way of example) for profile 433, it gives the following values:

$$c_w = \left( \frac{S}{b^2 \pi} + 0.0138 \right) c_a^2 - 0.0093 c_a + 0.0151$$

The agreement fully suffices for our investigation and within the limits  $c_a = 1.2$  to  $c_a = -0.2$ , as shown by Fig. 4.

The mean values obtained from the integration are then

$$c_{x_{\text{mean}}} = \frac{1}{2} c_a \left[ m - \Delta c_a \left( \frac{S}{b^2 \pi} + k \right) \right] - \left[ \left( \frac{S}{b^2 \pi} + k \right) c_{a_m}^2 - p c_{a_m} + q \right]$$

$$c_{y_{\text{mean}}} \cong c_{a_{\text{mean}}} \quad (9)$$

In the expression for  $c_{x_{\text{mean}}}$ ,  $\Delta c_a$  denotes the fluctuation of the different values of  $c_a$  about its mean value. Hence, if (as in Fig. 3,c), the value of  $c_a$  fluctuates between  $c_{a_{\text{max}}} = 1.2$  and  $c_{a_{\text{min}}} = -0.2$ , then the mean value of  $c_a$  is

$$c_{a_m} = \frac{1.20 + (-0.20)}{2} = 0.50 \text{ and } \Delta c_a = \pm 0.70$$

We might, however, represent the distribution of  $c_a$  by the following formula:

$$c_{ax} = \Delta c_a \cos \left( \frac{2 \pi x}{\tau_0 v} \right) + c_{a_{\text{mean}}} \quad (10)$$

The expression

$$\left[ \left( \frac{S}{b^2 \pi} + k \right) c_{am}^2 - p c_{am} + q \right] = c_{w_{\text{mean}}}$$

i.e., it is the value of  $c_w$  corresponding to the mean value of  $c_a$ .

Thus we had, by way of example, in case 3 (Fig. 3,c) the following assumptions:

$$\Delta c_a = \pm 0.70 \quad m = \tan \varphi_{\text{max}} = 0.140 \quad \left( \frac{S}{b^2 \pi} + k \right) = 0.0446$$

$$\left( \frac{S}{b^2} = \frac{1}{10} \right) \quad c_{am} = 0.50 \quad c_{wm} = 0.0216$$

from which we then derive

$$c_{x_{\text{mean}}} = \frac{0.70}{2} [0.140 - 0.70 - 0.0446] = 0.0216$$

$$c_{x_{\text{mean}}} = 0.0165 \quad (c_x = 1.65)$$

The maximum forward motion occurs for

$$\Delta c_a = \frac{m}{8 \left( \frac{S}{b^2 \pi} + k \right)}$$

whereby  $c_x$  becomes

$$c_{x_{\text{max}}} = \frac{m^2}{8 \left( \frac{S}{b^2 \pi} + k \right)} - c_{w_{\text{mean}}} \quad (11)$$

This maximum is, however, of only conditional importance, because  $\Delta c_a$ , even for small values of  $m$ , exceeds the burble point or the upper and lower interruptions of the polar curve.

For our case, it would be

$$\Delta c_a = \frac{0.140}{0.0892} = 1.13$$

For the mean values of  $c_a = 0.5$ , the limits would be  $c_{a_{\max}} = 2.23$  and  $c_{a_{\min}} = -1.23$ , which far exceed the attainable limits of the polar.

The wing thrust would then be

$$c_{x_{\max}} = \frac{0.0196}{0.3563} - 0.0216 = 0.0334 \quad (c_x = 3.34)$$

hence more than doubled.

The ideal form of a flapping wing must therefore be sought in a wing whose polar curve covers a very wide range, as the result of suitable devices (adjustable profile, slots or rotor on the leading edge).

The resistance generated by the lift in the up and down motion of the wings and which must be overcome by the driving force is the total power consumption in the vertical thrust of the wings.

The course of the work diagram for case 3 is shown in Fig. 5.

By adding the work of the down-stroke and that of the up-stroke, we obtain the hatched area in the form of an ellipse,



one of whose axes is  $\Delta c_a$ .

Consequently, the mean air resistance to be overcome by the driving mechanism, is

$$P_m = 0.78 \Delta c_a S \frac{\gamma}{2g} v^2 \quad (12)$$

The vertical component of the flight path is

$$S_0 = 2 m \frac{v}{\pi} \quad (13)$$

Hence the force expressed in HP. is

$$N = 0.000415 m \Delta c_a S v^3 \quad (14)$$

whereby  $\frac{\gamma}{2g} = \frac{1}{16}$ , i.e., for normal atmospheric conditions.

Now, however,

$$v = 4 \frac{\sqrt{\frac{g}{S}}}{\sqrt{c_{a_{\text{mean}}}}}$$

and hence

$$N = 0.021 m g \frac{\sqrt{\frac{g}{S}}}{\sqrt{\frac{c_{a_{\text{m}}}^3}{\Delta c_a^2}}} \quad (15)$$

If, in the example of case 3 (Fig. 3,c), we put  $g = 122 \text{ kg}$ , we have

$$\sqrt{\frac{g}{S}} = 3.18 S = 12 \text{ m}^2 \quad \text{and then}$$

$$v = 4 \frac{3.18}{0.707} = 18 \text{ m/sec.}$$

The required power is

$$N = 0.000415 \times 0.140 \times 0.70 \times 12 \times 5832$$

or

$$N = 0.021 \times 0.140 \times 122 \frac{3.18}{\sqrt{\frac{0.50^3}{0.70^2}}}$$

$$N = 2.8 \text{ HP.}$$

Thereby it is tacitly assumed that the coefficient of the mean drag  $c_{w_m}$ , increased by the coefficient of the structural drag  $c_s$ , is equal to the forward thrust. The flight is therefore uniform only when the following condition is satisfied:

$$m = \Delta c_a \left( \frac{S}{b^2 \pi} + k \right) + \frac{3 c_g}{\Delta c_a} \quad (17)$$

In Fig. 6,  $N$  and  $c_g$  are plotted against each other for different values of  $c_g$ . This diagram shows that the required power is proportional to the increase in the drag.

The dashed line was plotted with reference to the horsepower of a propeller-driven airplane having the same flight speed. The efficiency, in this case, was assumed to be very good (up to 0.8), since the efficiency of the wing-flapping mechanism had hitherto been disregarded. Moreover, energy is consumed only as the result of the resistance or drag due to inertia. This drag is, at most, very small for the right mutual adjustment of the flapping parts.

In Figs. 7 and 8, the aspect ratio and the weight are plotted against the horsepower. Fig. 7 shows that the effect of the aspect ratio is very small above  $\frac{b^2}{S} = 10$  and any

further gain would be more than offset by the necessary increase in weight. The weight increase plays a much more important part (the same as for a kite).

The horsepowers thus obtained are naturally minimum values. On the other hand, we must bear in mind that it doubtless gives still more effective wing motions.

We have also disregarded the fact that accelerations and retardations are produced (both horizontally and vertically) by the fluctuations in the engine power and the forward thrust which affect either the shape of the flight curve or, if the latter is maintained, the flight path with respect to the aircraft.

The accelerations naturally depend on the mass of the aircraft, as well as on the rapidity of the strokes. The greater the mass, the less rapid the strokes can be.

If we take nature as our pattern, it appears that, with increasing weight, the time for a complete stroke in an aircraft like our example, may be much greater without any unfavorable result. Any mathematical computation would carry us too far.

Another question appears to be of more practical importance, namely, what would be the effect of other shapes of the flight path? We will discuss this question and the problem of flapping flight after the manner of birds in our next article.

We will content ourselves today with the knowledge ob-

tained from our simple example, which (notwithstanding the apparent technical difficulties of flapping flight, as compared with the advantages of propeller-driven flight) seems to justify serious attempts to solve this problem.

Translation by Dwight M. Miner,  
National Advisory Committee  
for Aeronautics.

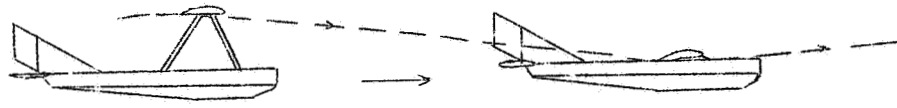


Fig.1

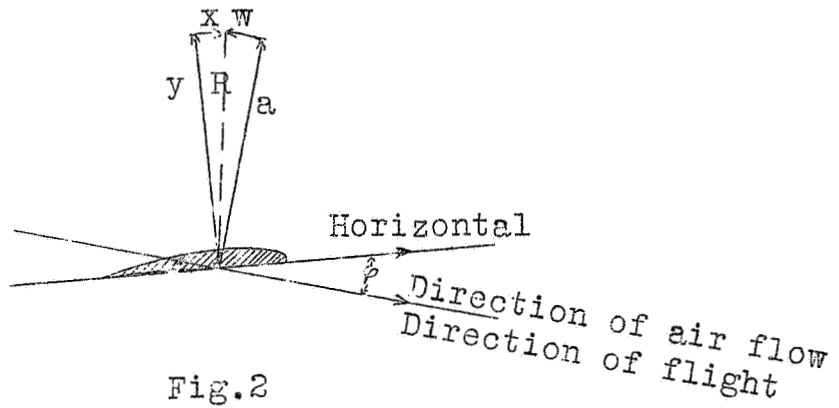


Fig.2

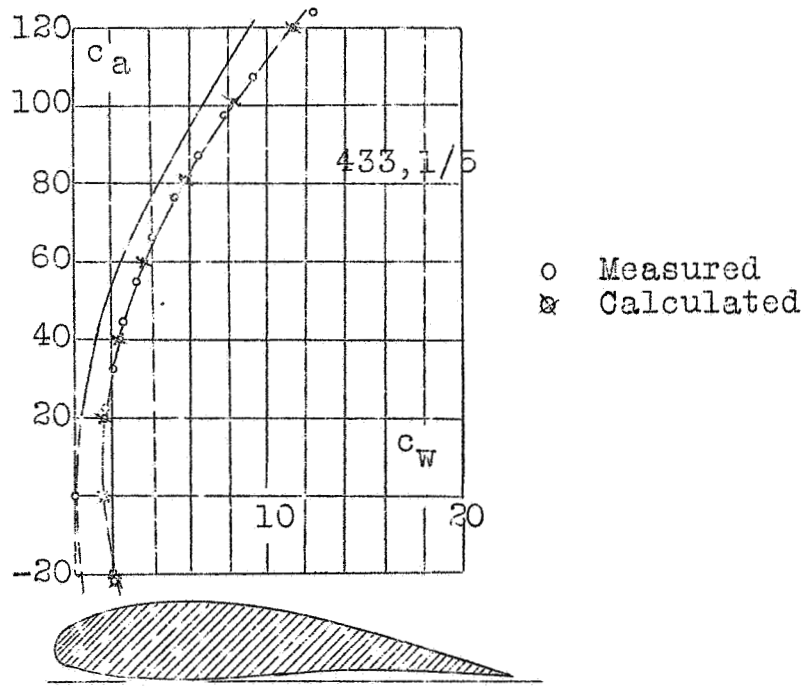
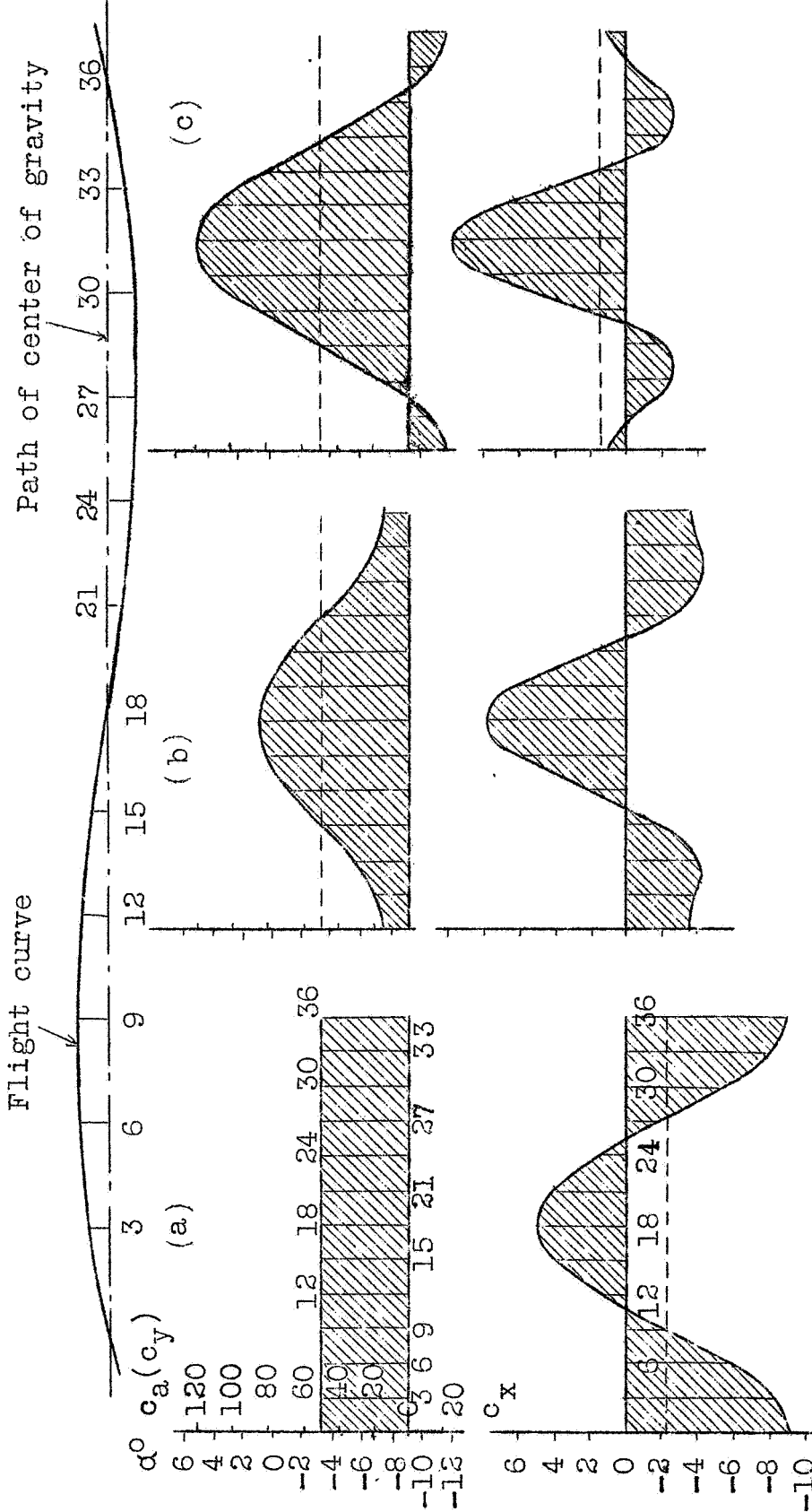


Fig.4



Case 1.  
 Mean lift  $c_{a,m} = 50.0$   
 Mean negative drag  $c_{x,m} = -2.16$   
 Lift variation  $\Delta c_a = \pm 0.0$   
 Variation in ang. of attack  $\Delta \alpha = \pm 0^\circ$  (referred to flight curve)

Case 2.  
 Mean lift  $c_{a,m} = 50.0$   
 Mean negative drag  $c_{x,m} = -0.02$   
 Lift variation  $\Delta c_a = \pm 35.0$   
 Variation in ang. of attack  $\Delta \alpha = \pm 4.1^\circ$  (referred to flight curve)

Case 3.  
 Mean lift  $c_{a,m} = 50.0$   
 Mean negative drag  $c_{x,m} = +1.65$   
 Lift variation  $\Delta c_a = \pm 70.0$   
 Variation in ang. of attack  $\Delta \alpha = \pm 8.3^\circ$  (referred to flight curve)

$\alpha_{max} = 5.0^\circ$   
 $\alpha_{min} = -11.6^\circ$

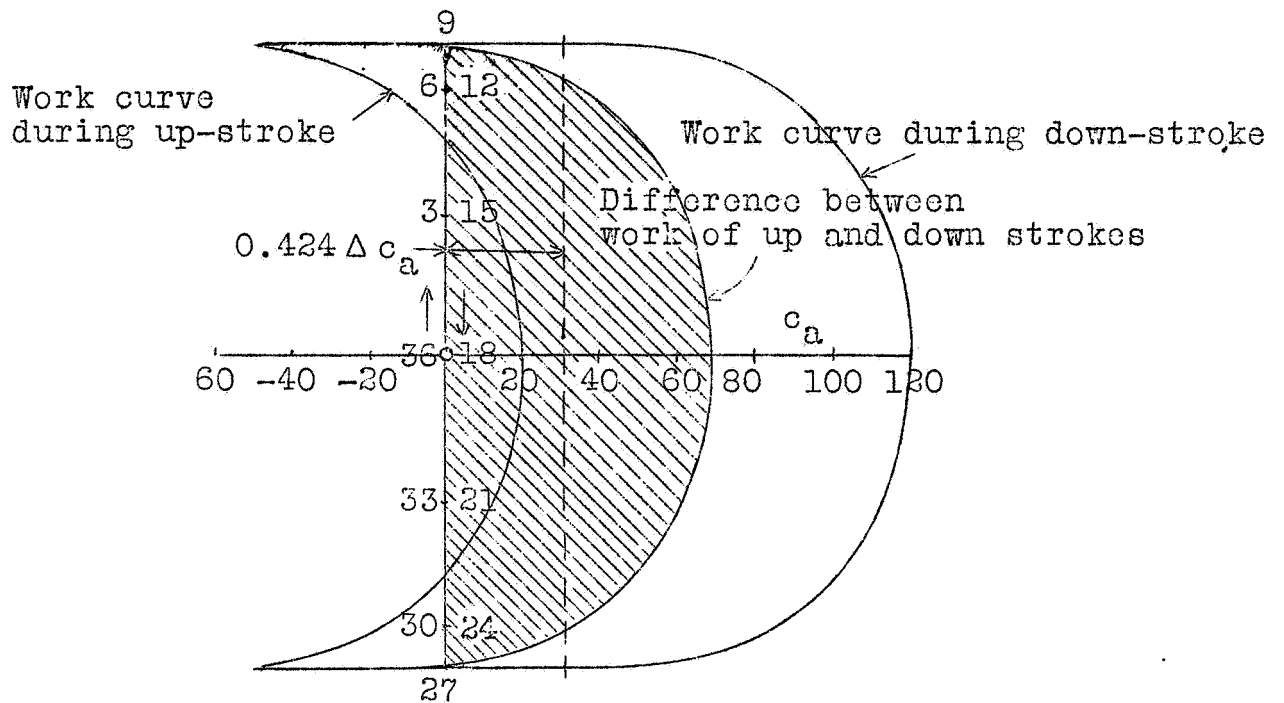


Fig.5

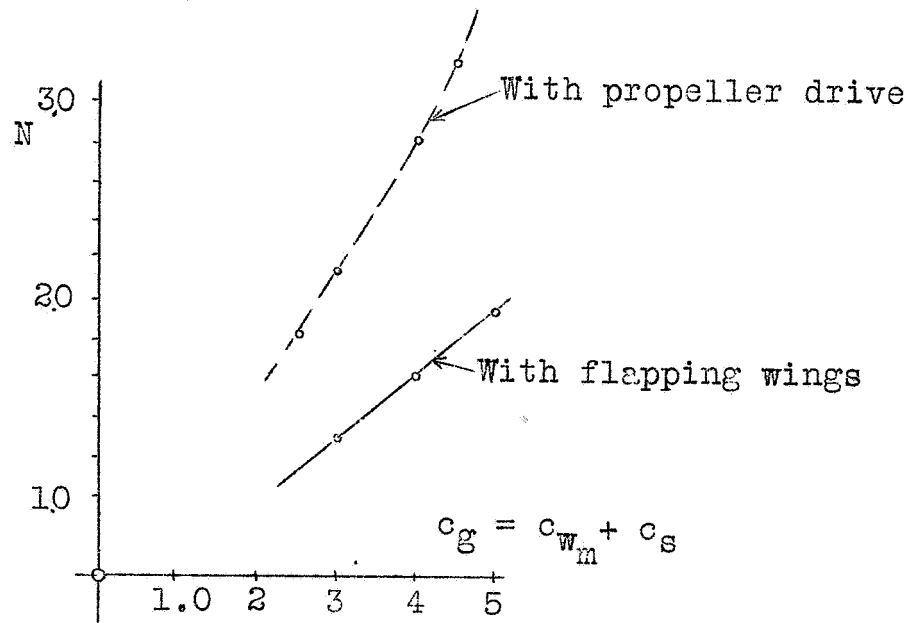


Fig.6

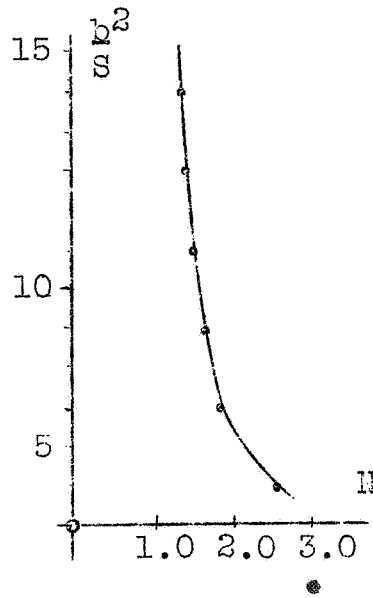


Fig.7

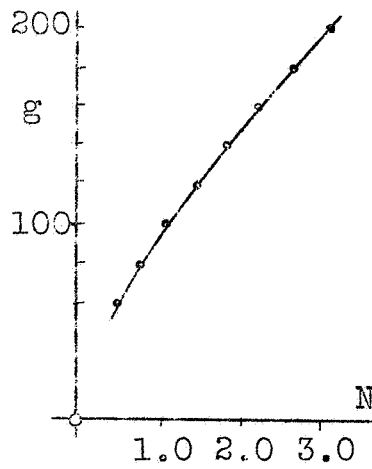


Fig.8