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A METHOD FOR THE DIRECT DETERMINATION OF WING-SECTION DRAG

By A Betz

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A METHOD FOR THE DIRECT DETERMINATION OF WING-SECTION DRAG.*

By A. Betz.

The wing theory teaches us that a large part of the drag
of a wing is due to the losses at the tips, constituting the
so-called induced drag. It is almost independent of the shape
of the wing section (or profile) and can be determined mathe-
matically with a fair degree of accuracy. The rest of the
drag depends on the shape of the wing section and is there-
fore called the wing-section (or profile) drag. It is ordi-
narily found by subtracting the calculated theoretical induced
drag from the total wing drag measured in a wind tunnel. This
method, however, gives rise to various inaccuracies. The wing-
section drag is often only a small fraction of the total wing
drag, so that a slight error in the determination of the lat-
ter makes considerable percentile difference in the former.
Moreover, the theoretical calculation of the induced drag is
only approximately correct and the points therein disregarded
principally affect the wing-section drag, on account of the
smallness of the latter. The problem therefore becomes impor-
tant when we wish to test in a wind tunnel, the effect of the
index value. For this purpose we must use wings with very

* From "Zeitschrift für Flugtechnik und Motorluftschiffahrt,"
February 14, 1925, pp. 42-44.
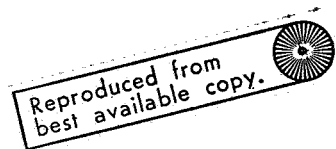
long chords, though we cannot correspondingly increase the
span, on account of the limited dimensions of the wind tunnel.
Hitherto, in such cases, we have placed barriers at the ends
of the wings, in order to produce a uniform current by pre-
venting the air from flowing around the wing tips.* Even
this method, however, has sources of error. The flow around
the wing is disturbed in the vicinity of the walls by the
boundary layer of air on the walls. This disturbance can
cause a separation of the flow at the wing tips. The result-
ing decrease in lift is, however, again accompanied by induced
drag. Although this is smaller than the induced drag without
walls, its magnitude is still so uncertain as to render any
accurate determination of the wing-section drag impossible.
We can, indeed, by special precautionary measures, diminish
the disturbing effect of the walls, but all such measures are
of the nature of makeshifts and the increase in accuracy is
not very great.

On account of these difficulties, Mr. Ackeret undertook
to work out a method for calculating the wing-section drag di-
rectly from the energy loss of the air. If we measure the
so-called total pressure $p + \frac{\rho}{2} v^2$ (p = static pressure of
the air and $\frac{\rho}{2}$ = dynamic pressure), then this quantity is con-
stant in a constant or potential flow. If losses occur during

* Betz, "Untersuchung einer Schukowskyschen Tragfläche,"
"Zeitschrift für Flugtechnik und Motorluftschiffahrt," 1915,
p. 173. Wieselsberger, "Der Einfluss des Kennwertes auf die
Luftkräfte von Tragflügeln" in "Ergebnisse der Aerodynamischen
Versuchsanstalt zu Göttingen," Report I, p. 54.

the course of the flow, they are manifested by a decrease in the total pressure. When, therefore, we measure the total pressures behind the wing in a line perpendicular to its span, we must be enabled to determine the wing-section drag of the corresponding front portion of the wing from the difference between the total pressures behind and in front of the wing.

Aside from the fact that by such a method, we can obtain the wing-section drag directly, (i.e., not simply as the difference between the total and induced drags), this method has the further advantage of enabling us to determine the wing-section drag for each individual section. This may be important, inasmuch as the wing-section drag varies throughout the span. This method has still another great advantage, in that the requisite determination can also be made on full-sized airplanes. This affords a means for comparing the results obtained on the model and on a full-sized airplane at just the point where the effect of the index value is greatest. Before employing this method, it is necessary to understand the theoretical principles on which it is based. This will be explained in what follows.

In order that the method may be more easily understood, we will first consider the simpler case when there is no lift, but only drag, and when the streamlines at the measuring point behind the obstacle are nearly parallel. Moreover, the flow is assumed not to deviate much from the two-dimensional flow.

The velocity in front of the body is denoted by v_0 and the static pressure by p_0. These two quantities vary somewhat from place to place, due to the disturbance caused by the body, but the total pressure $g_0 = p_0 + \frac{\rho}{2} v_0^2$ is constant (Fig. 1). At the measuring point behind the body, the pressure p and the velocity v likewise vary from place to place and, if no losses occur, the total pressure is here also $g = p + \frac{\rho}{2} v^2 = g_0$. Only in a small region behind the body is g smaller than g_0 (Fig. 2).

We can determine the drag of the body only with the help of the impulse law We imagine, before and behind the body, a control surface which extends to infinity both above and below and is perpendicular to the plane l of the diagram. The force exerted on a section of the body of the length l is then found as the difference of the pressures and of the impulses on both control surfaces

$$W = l \int_{-\infty}^{+\infty} [(p_0 - p) + \rho (v_0^2 - v^2)] d\,y$$

If we put

$$p_0 + \frac{\rho}{2} v_0^2 = g_0 \text{ and } p + \frac{\rho}{2} v^2 = g$$

we obtain

$$W = l \int (g_0 - g)\, d\,y + \frac{\rho}{2} l \int_{-\infty}^{+\infty} (v_0^2 - v^2) d\,y$$

The first of the two integrals contains the difference of the total pressures and is relatively simple to determine. The integral does not need to be extended to infinity, since g dif-

fers from g_O only on a limited region, the vortical region behind the body (Fig. 2), so that, outside of this region, the integrand $g_O - g$ everywhere vanishes. The second integral is less favorable in this respect. We can, however, transform it in such a manner that only one integration is required in the vortical region.

We will imagine a potential flow which, outside of the vortical region, is identical with the flow producing the drag and let v' denote its velocity. Then, outside the vortical region, $v' = v$ behind the body and $v_O' = v_O$ in front of the body. In the vortical region, however, v' is greater than v (Fig. 2). In order to bring this into harmony with the continuity condition, we must assume sources for the potential flow, whose total yield for the section of the body concerned is $E = l \int (v' - v)d\,y$. A simple potential flow is known to produce no drag. If, however, sources are connected with the body, as in the present case, a negative drag is produced, $W' = - \rho\, v_\infty\, E$, wherein v_∞ denotes the velocity in infinity.* If we perform the same calculation for this potential flow, as above in the determination of W, since every where

$$= p + \frac{\rho}{2} v'^{2} = g_O$$

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* See Lagally, "Berechnung der Kräfte und Momente, die strömende Flüssigkeiten auf ihre Begrenzung ausüben in "Zeitschrift für angewandte Mathematik und Mechanik," 1922, p. 409. The law is entirely analogous to the known law of Kutta and Schukowsky, namely, that $A = \rho\, v\, \Gamma\, l$ (A = lift and Γ = circulation) and can also be demonstrated in a very similar manner.

we obtain

$$W' = -\rho \, v_\infty \, E = \frac{\rho}{2} \, l \int_{-\infty}^{+\infty} (v_0{}^2 - v'{}^2) d \, y.$$

By subtracting this equation from the above equation for W, we obtain

$$W + \rho \, v_\infty \, E = l \int (g_0 - g) d \, y + \frac{\rho}{2} l \int (v'{}^2 - v^2) d \, y$$

If we introduce $E = l \int (v' - v) d \, y$, we then obtain, after a simple calculation,

$$W = l \int (g_0 - g) d \, y - \frac{\rho}{2} \, l \int (v' - v)(2v_\infty - v' - v) d \, y.$$

Moreover, the second integral is to be extended likewise only over the vortical region, since it disappears outside of the same, where $v' - v$ is everywhere zero.

The difference $(g_0 - g)$ of the total pressures can be very easily determined by placing before and behind the obstacle, a bent tube (or even an ordinary Pitot tube) with its mouth toward the air stream, and connecting it with both arms of a micromanometer (Fig. 1). The determination of the quantities contained in the second integral is somewhat more troublesome. For this purpose, moreover, it is also necessary to measure the static pressures. A little deliberation shows, however, that with a properly selected measuring point, the share of the second integral is so small that a rough estimate of it is usually sufficient. The course of the pressures is plotted in Fig. 2. The farther from the experimental body we get, the smaller are the differences between g_0 and g and

between p and p. Then v' approximates v_∞ and $\dfrac{v_\infty - v}{v_\infty}$

approximates $\dfrac{g_0 - g}{2\frac{\rho}{2}v_\infty^2}$. Thereby, however, the integrand of the

second integral $\dfrac{(g_0 - g)}{4\frac{\rho}{2}v^2_\infty}$, (hence in the ratio $\dfrac{g_0 - g}{4\frac{\rho}{2}v^2_\infty}$

is smaller than in the first. If, therefore, the maximum value

of $g_0 - g$ is smaller than $\dfrac{1}{5}\dfrac{\rho}{2}v^2_\infty$, which can usually be at-

tained, then the share of the second integral is less than $\dfrac{1}{20}$

of the first.

In our previous deliberations, we have assumed, for the

sake of simplicity, that there was no lift and, consequently,

no drag. We will now turn, however, to the case of a lift-

producing wing, which, indeed, interests us most. Here the

air behind the wing has a vertical component w (Fig. 3), in

addition to the horizontal velocity component v. The total

pressure is $p + \dfrac{\rho}{2}(v^2 - w^2)$. If we utilize the impulse law

in the same manner as in the first calculation, we obtain

$$W = l\int_{-\infty}^{+\infty}[(p_0 - p) + (v_0^2 - v^2)]\,d\,y = l\int(g_0 - g)\,d\,y +$$

$$+ \frac{\rho}{2}\,l\int_{-\infty}^{+\infty}(v_0^2 - v^2 + w^2)\,d\,y$$

If we likewise imagine here a potential flow with corresponding

sources of the output $E = l\int(v' - v\,d\,y)$* then such a poten-

tial flow causes an induced drag W_i, since the downward ve-

locity w is due essentially to the vortices passing off the

wing tips. Since we have not changed the value of w, the

lift also retains its value. The induced drag is therefore

* More accurately $l\int(\sqrt{v'^2 + w^2} - \sqrt{v^2 + w^2})\,d\,y$, but the
difference is negligible.

the same in both cases   Taking into consideration the effect of the sources, we obtain, for the potential flow, the drag

$$W' = W_i - \rho\, v_\infty\, E = \frac{\rho}{2}\, l \int_{-\infty}^{+\infty} (v_0^2 - v'^2 + w^2)\, d\,y$$

By subtracting this equation from the preceding one for W and also introducing the wing-section drag for $W - W_i = W_p$, we obtain

$$W_p = -\rho\, v_\infty\, l \int (v' - v)\, dy + l \int (g_0 - g)\, dy + \frac{\rho}{2}\, l \int (v'^2 - v^2)\, dy$$

$$= l \int (g_0 - g)\, dy - \frac{\rho}{2}\, l \int (v' - v)\,(2\,v_\infty - v' - v)\, dy.$$

which is exactly the same result as for the body without lift. The same considerations regarding the order of magnitude of the second member also hold good for the case with lift.

We can therefore determine the wing-section drag by this method without any difficulties in principle. For this purpose it is necessary, however, to plot a whole curve instead of making only a single measurement. If the method proves to be good otherwise, we hope that the process of application can be made simple enough to be practically utilizable in the majority of the cases.

Translation by Dwight M. Miner,
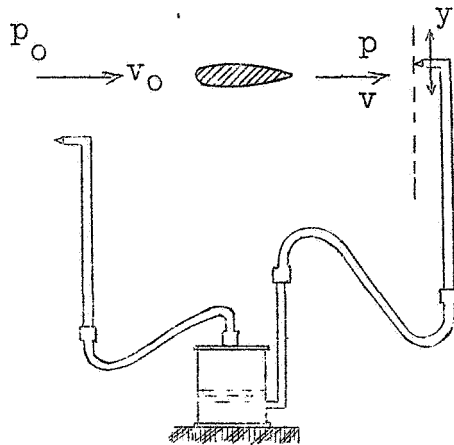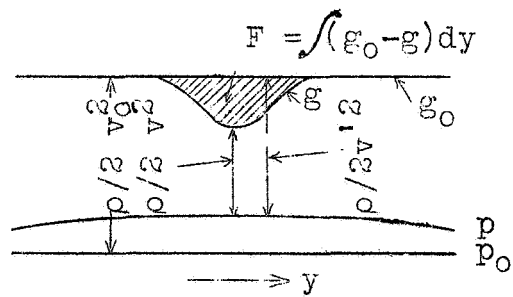National Advisory Committee
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$p_O \longrightarrow v_O$ $\longrightarrow p$ y

v

Fig.1

$$F = \int (g_O - g)\,dy$$

g g_O

p
p_O

$\longrightarrow y$

Fig.2

v_O

p_O

p

v

w

Fig.3