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STRENGTH CALCULATIONS ON AIRPLANES

By A. Baumann

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Every strength calculation, including those on airplanes, must be preceded by a determination of the forces to be taken into account. In the following discussion, it will be assumed that the magnitudes of these forces are known and that it is only a question of how, on the basis of these known forces, to meet the prescribed conditions on the one hand and the practical requirements on the other.

Allowable stresses in ordinary airplanes.- These were originally determined only by experiment and it was the service of Bach to bring order and harmony out of the great mass of disconnected data and establish the ratio of 3 : 2 : 1 for the allowable stresses of constant, increasing and alternating loads. There it became customary to associate the allowable stress with the breaking strength and to adopt, for a static load, a safety factor of 4 to 5. It is now customary to use the data thus obtained, where no special restrictions are placed on the constructor, either with respect to weight, change of shape, price, etc. Bach's preface to the first edition of his book on the mechanical principles plainly indicates, how-

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ever, that he never thought of establishing definitely fixed coefficients for all cases, but preferred instead to leave a free hand to the judgment of the constructor.

According to the above ratio, a safety factor of 4 to 5 for a static load indicates the need of a 6 to 7.5 safety factor for an increasing load and a 12 to 15 safety factor for a fluctuating load. As already mentioned, these large safety factors were originally based on experimental results. In order to judge of their necessity, it would be important to know, in each instance, on what basis they were calculated. It would also be necessary to know how uniform was the strength of the building material which served as their basis, since it is obvious that any building material, the properties of which fluctuate greatly in different samples, calls for greater safety factors than a building material of great uniformity. The accuracy of production and assembling and the precautions against eventual wear and possible excessive static or dynamic stressing also have much to do with the magnitude of the safety factor to be chosen.

As regards metal building materials, it may be assumed that both their quality and accuracy of production have, in general, improved. Moreover, the methods of calculation have also been refined and increasing attention has been accorded the dynamic processes. Originally every strength calculation was purely static and only in the course of time, probably beginning with the inertia forces of the crank mechanism, did dynamic action receive any attention. The necessary result was that the actual forces acting on any structural member were found to be greater than those obtained by static calculation. A correspondingly high safety factor had to be employed to offset the difference between the basis of the computation and actuality. This holds especially true for increasing and fluctuating stresses, since both cases of loading are hardly conceivable without accompanying effect of inertia.

The question arises as to how far the safety factor can be reduced, on the assumption of an absolutely uniform building material and the fulfillment of all other requirements regarding accurate calculation and production. With such a basis, we could then make additions according to the particular case and thus increase the safety factor as required.

This may be illustrated by an example. The allowable static tensile stress for wrought iron is calculated at 900 kg/cm² (12800 lb./sq.in.). We can assume a mean tensile strength of about 3300 kg/cm² (46937 lb./sq.in.) for this wrought iron, with 5% variation above and below, so that the minimum strength is accordingly 3150 kg/cm² (44803 lb./sq.in.). If we assume that the yield point for this wrought iron is located at 55% of the breaking strength, then the yield point will be reached at 1750 kg/cm². In general, the working stress must be kept considerably below the yield point, if the structural part is to perform its task, because, in only a very few cases, is any appreciable elongation of the structural member possible without producing disturbances of some kind. The 900 kg/cm² (12800 lb./sq.in.) allowable stress denotes, with respect to the 1750 kg/cm² (24890 lb./sq.in.) limit, a safety factor of 1.9. We would hardly wish to go below a safety factor of 1.5, so that a stress of 900 kg/cm² (12800 lb./sq.in.) is not so much below the lowest allowable stress as people generally think.

Now the greatest allowable stress for the same material is 600 kg/cm² (8534 lb./sq.in.) with increasing load and 300 kg/cm² (4267 lb./sq.in.) with fluctuating load. These give, with reference to the yield point, respective safety factors of 2.5 and 5. These figures are certainly ample and allow for additional dynamic stresses. It is often said that such a safety factor is necessary, in consideration of the fatigue of the material, especially with a fluctuating load, but we may refer in this connection, to the customary stressing of springs, which are subjected to just such fluctuations and for which, with respect to the static load, we are accustomed to reckon, for railway cars, on 4500-6500 kg/cm² (64000-92450 lb./sq.in.) for a breaking strength of 15000 kg/cm² (213350 lb./sq.in.). These figures correspond, with reference to the breaking strength, to safety factors of 2.3 - 3.3. In the case of the springs, we can refrain from any reference to the yield point, since tempered spring steel does not have such a point. It is difficult to say just how great the working stresses are, but they may amount to 1.2 - 1.5 times the static load, according to conditions. This means that the fluctuating stresses would be 5400-6800 (instead of 4500·(64000)) kg/cm² (76806-96718 lb./sq.in.) and 7800-9800 (instead of 6500·(92450)) kg/cm² (110942-139388 lb./sq.in.). The safety factor, with respect to the working stress, would fall to 1.8 - 1.5 or 1.9 - 1.5 (on an average 1.6), while the usual stipulation would require 6 - 7.5 and possibly 13-15. Thereby such springs meet the endurance requirements. Though springs do occasionally break, it would be difficult to say whether the breaks, in the majority of cases, are due to over-stressing or to secondary stresses arising from torsion. For the springs of motor cars we calculate, according to the conditions, with stresses of the springs, with respect to the static load, of 3000 to 5000 kg/cm² (42700-71117 lb./sq.in.), with a deflection of 50-150 mm (1.97 to 5.91 in.) for the static load. It is customary to make a further allowance of 50-150 mm (1.97 to 5.91 in.) for dynamic stresses, which would be equivalent to 6000-10000 kg/cm², (85340-142233 lb./sq.in.), if the same ratio should hold between deflection and load. The safety factors accordingly correspond throughout to the same ratios as for railway cars.

From the above we may conclude

1. That very large safety factors are employed in ordinary

mechanical construction, but that the force of circumstances sometimes compels the use of smaller safety factors, which may, in individual instances, entail no disadvantage.

2. That manifestly a distinction must be made between materials which have a yield point or undergo very great permanent elongations, increasing with the load, and materials with which such is not the case.

3. That for materials with pronounced yield points, the latter are in most cases important and decisive for the choice of the safety factor and that, when the customary safety factor is applied to the yield point, it cannot be regarded as excessive.

4. That the uniformity of the building material is of decisive importance.

Allowable stresses in airplanes.- In airplane construction, where all unnecessary weight must be avoided, we are always compelled to employ small safety factors. The building specifications, which soon became customary, relieved the constructor of a large share of responsibility. There is a question as to the expediency of this method, but anyway it is very convenient for the constructor. We will not discuss these building specifications, but only state what follows from them and what considerations devolve upon the constructor as regards the practical requirements, choice of building materials and safety factors.

The building specifications stipulate definite breaking strengths of the airplane framework for certain cases of loading. The simplest way to meet these requirements, as regards the mental labor, is to make a breaking test, probably preceded by a more or less rough estimate. This experimental method, however, can hardly give the best results, unless carried out intelligently and greatly extended, since it lies in the nature of the case that this method only determines what portions of the structure are too weak and not what are too strong. When a premature break occurs, it shows only what part must be strengthened, thereby increasing the weight of the structure. In many instances, moreover, it cannot be told where the break originates because it often happens that other members give way in extremely rapid succession, following the perhaps unnoted failure of one member. It is therefore advisable, under all circumstances, not to be satisfied with the breaking test alone, but to precede it with the most thorough calculation possible. In this case the question of the safety factors again becomes important.

In connection with the above-mentioned general principles, the nature of the building material must also be considered. For the supporting parts of airplanes the chief materials are: wood, in its natural condition and as plywood; hard steel (piano wire), steel tubing; mild steel, in the form of plates and welded pieces; light metals, such as duralumin and aluminum in combination with steel; in the future perhaps "elektron," "silumin," etc.

For wood and plywood, there is no elastic limit to be reckoned with, but for wood in its natural condition there is a considerable lack of uniformity in strength, which is mostly eliminated in plywood. The lack of any yield point renders it possible, in the case of wood, aside from the allowance for its lack of uniformity, to reckon with very small safety factors. The same is true for hard steel, whereby care must be taken to insert turnbuckles or other tightening devices in cables and wires for restoring the original length in case of stretching. The parts made from soft steel usually elongate so little, that the permanent set when the yield point is exceeded, although percentilely greater, has no important effect on the shape of the whole structure. Moreover, in such cases, readjustments are usually possible by means of tightening devices. A higher safety factor is often adopted for such parts, especially as an extra precaution in the case of welds. Nevertheless, it has been found in many instances that the stressing of such individual parts in use has exceeded the yield point, which, of course, should be avoided.

In flattening out the course of an airplane after diving or while banking in curving flight, three times the static load is often exceeded. Thereby, in accordance with the building specifications, a breaking strength, corresponding to five times the static load, is employed as the basis of the calculation. Inasmuch as the yield point lies at 60% of the breaking strength, the former is therefore reached at a threefold load and exceeded at any load beyond that.

It is, however, not dangerous in itself if, as the result of exceeding the yield point, an appreciable elongation of a structural member occurs, as long as it does not affect the shape of the whole supporting structure in such a way as to interfere with its proper functioning. The situation is such that as a result of stretching the member, the yield point of the material at this location is raised, so that, if an equal stress occurs a second time, no further permanent elongation results.

As regards metal airplanes, the problems are more complicated and difficult. This is so because in contrast with wood, there is a far greater range in the choice of building materials. If, for instance, duralumin is selected, there is the choice between several different kinds, differing from one another in breaking strength, yield point and elongation. The relations are such that the elongation at rupture generally decreases with increasing breaking strength.

The constructor cares most for a sufficient breaking strength and elongation. For duralumin we have:

	Breaking strength	Coefficient of elongation %	Yield point
681 B 1/3	3800- 4100 kg/cm² 54049-58316 lb./sq.in	18-21	About 2650 kg/cm² 37692 lb./sq.in.
681 B	3800- 4200 kg/cm² 54049-59738 lb./sq.in.	18-20	2750 kg/cm² 39114 lb./sq.in.
681 B 1/2	4100- 4400 kg/cm² 58316-62583 lb./sq.in.	13-16	3150 kg/cm² 44803 lb./sq.in.
681 B strongly stretched	- 6000 kg/cm² 85340 lb./sq.in.	3	About 5000- 6000 kg/cm² 71117-85340 lb./sq.in.

From the standpoint of strength, the harder kinds, with a smaller elongation, would be preferable and, with corresponding homogeneity, a considerable saving in weight and material would be possible. An equal or greater advantage could, however, probably be obtained by taking the yield point as the basis instead of the breaking strength. In the first case, the breaking strengths are in the ratio of 2 : 3. In the second case, the yield points are in the ratio of 1 : 2. If we assume, on the basis of the breaking strength and the maximum working load (taking into consideration the presumable dynamic load of three times the static load), a safety factor of 1.6, which would correspond to a breaking load of about 5, we obtain:

	Stress	Yield point
681 B 1/3	2500 kg/cm² 35558 lb./sq.in.	2650 kg/cm² 37692 lb./sq.in.
681 B	2500 kg/cm² 35558 lb./sq.in.	2750 kg/cm² 39114 lb./sq.in.
681 B 1/2	2650 kg/cm² 37692 lb./sq.in.	3150 kg/cm² 44803 lb./sq.in.
681 B strongly stretched	3750 kg/cm² 53337 lb./sq.in.	About 5000 kg/cm² 71117 lb./sq.in.

It is obvious that, in using 681 B and 681 B 1/3, the stress to be anticipated in operation is very near the yield point, while in using 681 B 1/2, this stress is considerably below the yield point, although this stress is greater than in the first case. If the maximum dynamic load were 3.5 (instead of 3) times the static load, which is entirely possible for swift airplanes, then we would have, with a five-fold breaking load, a safety factor of only 1.4, instead of 1.6, with the following results:

	Stress	Yield point
681 B 1/3	2850 kg/cm² 40536 lb./sq.in.	2650 kg/cm² 37692 lb./sq.in.
681 B	2850 kg/cm² 40536 lb./sq.in.	2750 kg/cm² 39114 lb./sq.in.
681 B 1/2	3000 kg/cm² 42670 lb./sq.in.	3150 kg/cm² 44803 lb./sq.in.
681 B strongly stretched	4300 kg/cm² 61160 lb./sq.in.	About 5000 kg/cm² 71117 lb./sq.in.

Here the yield point in the first two cases would be exceeded, although the breaking strength would meet the building specifications and no objection could be made from the standpoint of danger of breaking, to the safety factor of 1.4, the homogeneity of the material being assumed. The harder materials naturally have an advantage over the softer. One may therefore be tempted to recommend the harder kinds, even though they have very small elongation.

Value of the elongation. Why then does the constructor prefer the materials having a greater coefficient of elongation and renounce the advantages of greater strength?

In the first place, because, corresponding to what has already been accomplished, there is, up to a certain point, greater safety in the smaller strength with a greater coefficient of elongation, since the yield point itself goes higher after it has been exceeded. The exceeding of the yield point, however, is possible only with a corresponding loss of energy (in the work done during elongation and in changing the shape). If, therefore, excessive stressing occurs as the result of the effects of inertia, there is the possibility that the energy of inertia, causing the excessive stressing, will be consumed by the work of changing the shape, so that the stress will not increase beyond the breaking point, as might be the case, should no great elongation occur. Such an effect can probably be produced only by unusual landing shocks, but not in actual flight. This reasoning, which can be justified in many instances in general mechanical construction, hardly holds good for an airplane wing.

Another reason is that the workability of the material decreases with the elongation. A structural material of only 3% elongation cannot be changed much by bending, crimping or stamping, without rupturing. Up to a certain point, this can be offset by suitable shaping and by methods not always easily determined, as, for example, drilling holes instead of punching them. It nevertheless remains true that the employment of stronger material with a smaller elongation is not always possible.

A third reason, especially in the fabrication of slender parts involving the use of thin metal plates, may be seen in the fact that the sensitiveness of a material to local accidental stresses is inversely proportional to the elongation.

A further reason, which, however, is not important in airplane construction, is that the greater the elongation of the material the less its sensitiveness to temperature stresses.

All in all, we can therefore conclude that, in so far as workability considerations are not decisive, the greater strength due to high yield points is more important for a material than the elongation.

In this connection, it must also be remembered, when material with a greater elongation is preferred on account of its workability that, when the working is accomplished by exhausting the elongation, the finished piece then consists of a material having a smaller elongation. There is then all the less reason for constructing other parts, to which like considerations do not apply, from materials having greater elongation.

As regards the tendency to fatigue, the conditions are generally such that hard materials possess it to a less degree than soft materials, because there is less danger of exceeding the yield point of the former than of the latter.

In this connection, especial attention is called to the following. In most cases, not all the structural members are stressed the same, nor have the same yield point. In other words, when an airplane has to support a certain load, the yield point is not reached at the same instant in all the members. This may produce unpleasant results, according to the degree of interdependence of the individual members. If the yield point is exceeded in one member, so that it is permanently distorted and tends to maintain its position, then the adjoining member, which has not been permanently deformed, tends by its elasticity to restore the original shape. As to whether and how far this tendency is successful depends on the conditions. If it succeeds, then very troublesome conditions are produced, since the overstressed member is again distorted in the opposite direction and a disastrous fatigue of the material is soon produced. This also holds true, in a somewhat modified form, for building materials which have no yield point in the narrower sense, but undergo, in some other way, increasing permanent elongation with increasing load.

In these cases, the fact that the exceeding of the yield point raises the latter for succeeding loads, probably does not improve the situation. At any rate, I know of no experiments on such cases. The structural member is brought back to its original position, which is possible only by again disturbing the structure of the material and it is a question whether, in the repetition of the process after remedying the distortion, the yield point will correspond to the load first attained. Under all circumstances, the occurrence of such loads and deformations should be avoided.

Conclusions.- After all, it appears opportune, at least for metal airplanes, to take into account not simply the breaking strength of the structure, in the sense of the building specifications, but also the distortion and the effect of the yield point. At the present time, there is lacking in this respect, every detail regarding both the safety factor with relation to the maximum working stress and with relation to the yield point. It seems at least necessary to require, in building specifications, in addition to the breaking strength, a statement of the maximum permanent distortion under the influence of a stipulated load. It might also be advisable to precede the breaking test with a repeated loading and unloading with a stipulated multiple load (of about 3.5-fold). The proparation of the suggested specifications might well be left to the committee on constructive problems in our association.

In recent times the number of light metals has been greatly increased. It would be opportune to investigate their adaptability to airplane construction in connection with the above suggestions. The published data on their characteristics are incomplete. It would, in any case, be desirable for our association to undertake their investigation by having a committee work out some plan for having these investigations made in one or more laboratories, with special regard to the requirements of airplane construction. The means for these investigations could be provided from the emergency funds of the German scientific societies.

For the strength calculations of an airplane, we could start with the anticipated stresses with respect to speed, banking ability and pressure distribution on the wings. This load multiple would lie at 2.5 for slower airplanes and 3.5 for faster ones, without exceeding the maximum limit by increasing the flight characteristics beyond what is now customary.

Attention should then be given to the selection of the building materials, with special regard to their homogeneity and the determination of their yield points. Their allowable stressing should then be so chosen, with regard to the degree of uniformity that, at a 2.5 to 3.5-fold load, the stress would still lie below the yield point. It should be borne in mind, however, that the increases in stress are not proportional to the loads in all cases, but sometimes at a higher rate. The elongation, to be assumed as desirable, depends on the preliminary processes of riveting, bending, crimping, etc. It is doubtful, however, whether a minimum coefficient of elongation should be prescribed, as in other building specifications (e.g., boiler construction). There seem to be more reasons against than for such a course.

Not all building materials have a yield point. The elastic limit would then have to be substituted in its place. This is always an arbitrarily fixed quantity, concerning which an agreement is necessary. The question is at what percentage of permanent set it shall be placed. The stresses should remain below the limit of elasticity, as previously below the yield point.

It would not be desirable to make, on the basis of such considerations, strict building and calculating stipulations, which might lull to sleep the sense of responsibility and obstruct progress. Only guiding principles should be stated, in accordance with which the strength characteristics of new building materials should be investigated with regard to their suitability for airplane construction.

It would be necessary to investigate another property of building materials, which is important for all light construction and especially for airplane building. This is the strength of the materials in obstructed contraction and elongation. Such an obstruction may be due to various causes. The best known cause is that of notching, where the material adjoining the notch exercises a stiffening effect and obstructs the contraction of the cross-section, which must go hand in hand with the elongation. Such an obstruction can also occur when, at any point, forces act in different directions, as is conceivable at least at junction points.

Time is required for the development of every contraction and elongation. If the effect of the force follows so quickly as to allow no time for the elongation (landing shock, jolts, etc.), a condition must arise which is similar to that produced by notching.

It depends entirely on the properties of the building material: on the one hand, its cohesion; and on the other hand, the prevention, connected with the flow, of the breaking along crystal surfaces, whether an obstructed contraction increases or diminishes the strength. Hence, under certain conditions, we find little notch tenacity with great strength and good elongation.

The strength characteristics of the new building materials should be determined under such conditions. Such investigations are all the more desirable, since the notch tenacity, which, as already mentioned, depends on similar processes, has so far as known, extremely low values for all light metals. These values, however, differ from one another, between 0.48 (269) and 1.5 (840) while they run as high as 12 m-kg/cm² (6700 in.-lb./sq.in.) for soft steel.

Inasmuch as we do not have reliable data, we must adopt higher safety factors in cases where we have to do with stresses like those described above.

In conclusion, I would again recommend, in the cases arising in airplane construction, where the calculations are made with very high stresses and small safety factors, that the safety factors be based on the breaking strengths as determined by tensile tests, and also that the committee on constructive problems make a thorough investigation of the indicated questions.

Discussion

Prof. Reissner: "Mr. Baumann has called attention to all the difficulties and the possibilities of their solution now confronting the calculation of the safety factors of airplanes, even after the determination of the distribution of the stresses in all the structural members. The committee on constructive problems of our association has already considered the very problems posed by Mr. Baumann, but has thus far been unable to agree entirely on their formulation. I hope that Mr. Baumann will cooperate more than hitherto with our committee, so that we may ultimately solve these important problems."

Engineer Weyl: "I wish to speak briefly on a point which seems to me to be of very great importance for the further development of our aviation.

"This is the question of the strength of our airplanes. What strength requirements shall the airplane builder take as the basis of his designs? This problem is yet far from being solved. We believe, however, that the German building specifications ("Bau- und Liefervorschriften," 1918) do not guarantee the requisite degree of strength under all flight conditions for high-powered airplanes of the most modern types. These modern airplanes are much swifter and have a much higher speed limit in diving than the 1918 types. Moreover, a transition to airplane types with cantilever wings of good aspect ratio is gradually taking place.

"This question is likewise not settled in other countries. A compilation of the strength requirements in the different countries, by Iperide Leveratto ("Rendiconti Tecnici," April 15, 1924, p. 18) gives the following data:

"The Spad VII single-seat pursuit plane with a 140 HP. Hispano-Suiza engine, a well-known French combat plane from the year 1917, has for the case of "flattening out" (German "A" case) an actual strength of 7.9 times the load. According to the German "Bau- und Liefervorschriften," this airplane (a single-strutter with intermediate struts) had to have, in the breaking test, a load multiple of only 6.5; according to the United States specifications, 8.5; according to the French specifications, 6.7; according to the Italian specifications, 10.3.

"You see how widely the strength requirements differ in the different countries for this relatively weak and not very efficient airplane. It is still worse in the case of an efficient modern single-seat pursuit plane of 300 HP. For instance, the Italian Piaggio single-seat pursuit plane, a low-wing cantilever airplane which I will discuss more minutely in my lecture, should have, at the breaking point, according to the 1918 B.L.V., a load multiple of 6.5; according to the United States specifications, 8.5; according to the French specifications, not less than 14.9; according to the Italian specifications, 10.5. As a matter of fact, the breaking test with this airplane showed a strength of 18 times the normal load. In any case, therefore, this airplane is much too strong, i.e., much too heavy." (This airplane has recently been reconstructed on this account.)

"The case is different for very large airplanes. For instance, the Caproni three-engine airplane, type 3, has an actual wing strength of 6.12 times the load, while the French specifications require only 3.2 for an airplane of this kind, i.e., less than the 1918 B.L.V.

"The French doubtless have the greatest strength requirements for small airplanes. They take, as the basis of the load factor, a relationship, according to which the strength is proportional to the inverse value of the lift and to the third power of the speed. This seems logical. At any rate, we can make no further progress by classifying the airplane strength according to the German 1918 B.L.V. We will also be obliged to take flight-mechanical relationships and drag coefficients as the basis of the load multiples. As to how high we have to go with the strength factors of modern airplanes is now the subject of a thorough theoretical investigation in the German Experimental Institute for Aviation."

Prof. Schlink likewise expressed his appreciation of Prof. Baumann's lecture and asserted that it was high time to make more thorough investigations concerning the safety factors of materials employed in airplane building. Anyway, they could not be as small as for other technical structures, since there are static indeterminates in every airplane. Moreover, he called attention to the need of special care, in strength problems, to give the different parts of an airplane the same safety factor. Too little attention is now being given this point and much research work must be done in this connection. With reference to wood as a building material, he remarked that it varied greatly in strength and that in this year's soaring-flight contest in the Rhoen Mountains, it was found that powerful stresses were produced in wood by the action of moisture, which might cause breaks under favorable conditions.

Translation by Dwight M. Miner,
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