TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 400

ON THE MUTUAL REACTION OF WINGS AND BODY

By J. Lennertz

From "Zeitschrift für Flugtechnik und Motorluftschiffahrt"
January 14, 1927

Washington
February, 1927

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ON THE MUTUAL REACTION OF WINGS AND BODY.*

By J. Lennertz.

At the suggestion of Professor Von Karman, I have made a few theoretical investigations of the mutual reaction of the wings and body of an airplane, the results of which are given here. The treatment of the problem is very complicated for the usual body shapes. Therefore, there was assumed as the basis of my calculations, a strongly idealized body shape, namely, a cylinder extended to infinity at both ends and having its axis parallel to the direction of motion of the airplane. Even with this cylindrical body, the calculation is very instructive. The results are to be regarded as rough approximations for a long airplane body and also for an airship which is provided with fins. In my calculations, I have considered only a monoplane in which the axis of the wing is rectilinear.

If a wing of infinite span, having a constant distribution of the circulation, is attached to a cylindrical body so that their axes intersect, then there is generated the same lift as if the body were replaced by a section of wing having the same span as the diameter of the body. In this case, the body has no effect on the wing. (The same holds true for a sphere with infinitely wide wings.)

*"Beitrag zur theoretischen Behandlung des gegenseitigen Einflusses von Tragfläche und Rumpf." From "Zeitschrift für Flugtechnik und Motorluftschiffahrt," January 14, 1927, pp. 11-13.

In the case of wings of <u>finite</u> span, the total lift and its distribution over the span are obtained from a consideration of the flow at an infinite distance from the wing and with the help of the theory of momentum. If the distribution of the circulation is constant over the span, then, according to the Prandtl wing theory, the wing and free vortices can be replaced by a roughly horseshoe-shaped vortex (See Fig. 7). If the free parallel branches of this "horseshoe" vortex are reflected on the surface of the body and if they are joined at the head, there is produced, inside the space formerly occupied by the body, a "horseshoe" vortex of the same intensity, but in the opposite direction. At an infinite distance from the wing, both "horseshoe" vortices give perfect flow. The law of momentum then gives us the total lift of the airplane

$$A = \rho V \Gamma s \left(1 - \frac{d^2}{s^2 + 4 e^2}\right),$$

in which ρ is the density of the air; V, the velocity of the basic flow; Γ, the circulation about the wing; s, the span of the airplane; d, the diameter of the body; and e, the shortest distance of the wing axis from the cylinder axis (The displacement of the wing axis from the ℄ of the body, taken as positive in the direction of the lift).

If we replace the body by sections of wing projecting into the space originally occupied by the body and which will produce the same lift as the body, then the true or effective span is obtained by reducing the original span by the distance between the inside ends of the added sections of wings (Fig. 8). Assuming a "horseshoe" vortex as a substitute for the wings, the presence of the body necessitates a change in the angle of attack. This change is the result of the downwash velocity, which the parallel vortex arms inside the body induce on the wings. The angle, by which the effective angle of attack of the wing is diminished by the body, is

$$\epsilon = \frac{\Gamma}{4\,\pi\,V}\,\sigma^2 \left[\frac{\sigma^2 y - \frac{s}{2}}{\left(\sigma^2\, y - \frac{s}{2}\right)^2 + (\sigma^2 - 1)^2 e^2} - \frac{\sigma^2\, y + \frac{s}{2}}{\left(\sigma^2\, y + \frac{s}{2}\right)^2 + (\sigma^2 - 1)^2 e^2} \right],$$

in which y is the coördinate in the direction of the span, measured from the axis of the body and $\sigma^2 = \frac{s^2 + 4\,e^2}{d^2}$. Simultaneously with the effective angle of attack, the induced drag of the wing is also changed by the body. In Fig. 1, the effect of the body on the wing drag for $e = 0$ is plotted against the ratio $b = \frac{s}{d}$. In Fig. 2, the effect of the body on the wing drag for a fixed width ratio is plotted against the displacement of the wing. According to this figure, the body causes a slight reduction in the wing drag, when the wing is located above or below the body. It is also worth noting that the lift and the change in the drag are direct functions of e

W_o

and that therefore the raising or lowering of the wings by the same distance from the middle position affect the lift and drag in the same manner.

Furthermore, the problem of the minimum induced drag for a given lift is considered for the case when the wing axis and the body axis intersect. For a wing without body, we hereby obtain the condition that a constant downward velocity is produced in infinity on the vortex ribbon. The minimum condition, which is produced in this case, likewise with the aid of the law of momentum and energy, corresponds, at an infinite distance behind the wing, to a flow, which can be produced as follows (Fig. 3).

A fluid motion is first selected which flows over the vortex ribbon and the body and, at infinity, has a constant upward velocity. This motion is superposed on a flow about the body which, at infinity, has a constant downward velocity of the same magnitude as the first upward velocity. The lift distribution, which follows from this condition and corresponds to the elliptical distribution on a wing, is shown in Fig. 4 for various span ratios. The induced drag, with the previous designations in terms of the lift, becomes

$$W = \frac{A^2}{\pi \rho V^2} \; \frac{8\,d\,s\,(s^2 - d^2)^2}{(s^4 + d^4)^2} \; \frac{1}{\pi} - \frac{2}{\pi^2} \text{ arc sin } \frac{b^2 - 1}{b^2 + 1} \; .$$

In Fig. 5 the lift is plotted against the induced drag both with and without reference to the body. The ratio of the share of the lift due to the body to the total lift is

$$\frac{A_R}{A} = \frac{4}{\pi}\left[b\,\frac{b^2 - 1}{b^4 + 1} + \frac{(b^2 + 1)^2}{2\,(b^4 + 1)}\,\text{arc sin}\,\frac{2\,b}{b^2 + 1} - \frac{b^2}{b^4 + 1}\,\pi\right].$$

A few values for this ratio are given in the following table:

b=	10	8	6	4	2
$\frac{A_R}{A}$ =	0.216	0.258	0.317	0.399	0.378

In addition to the total air forces and their distribution over the span, the lift distribution in the direction of the length of the body is also of special interest. This is determined for wings of infinite span and for the ratio $b = 10$ with constant circulation distribution over the wing span, as also for $b = 2$ with the circulation distribution which follows from the condition of minimum drag for a given lift. The free vortices are again mirrored on the body surface for the case of wings of finite width. Thereby the condition of smooth flow about the body is fulfilled at a long distance before and behind the wing. In the vicinity of the wing, this condition must be satisfied by the superposition of a supplementary flow. The accurate numerical solution of the resulting marginal-value problem can not be obtained. If we now develop all the velocity components in Fourier series according to the angle of circulation about the cylindrical body, we find that only occasionally the first harmonic term furnishes a contribution to the lift distribution along the body. Hence the correction in the vicinity of the wing can be made by the assumption of a double-source thread along the body axis, whereby the edge condition will be fulfilled simply for the first Fourier term of the velocity component normal to the surface of the body. For the determination of the intensity of the double-source thread we obtain from the boundary condition an integral equation which produces, by transition to finite intervals, a system of twice infinitely many linear equations with just as many unknowns. The unknowns were found according to a method indicated by Professor O. Toeplitz. The lift distribution was then determined with the aid of the Bernoulli theorem. Fig. 6 represents the lift distribution along the body for the calculated cases, the difference being very small between $b = \infty$ and $b = 10$. The more exact derivation of these results will be given in my Aachen dissertation, which will soon be published.

Translation by Dwight M. Miner,
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for Aeronautics.

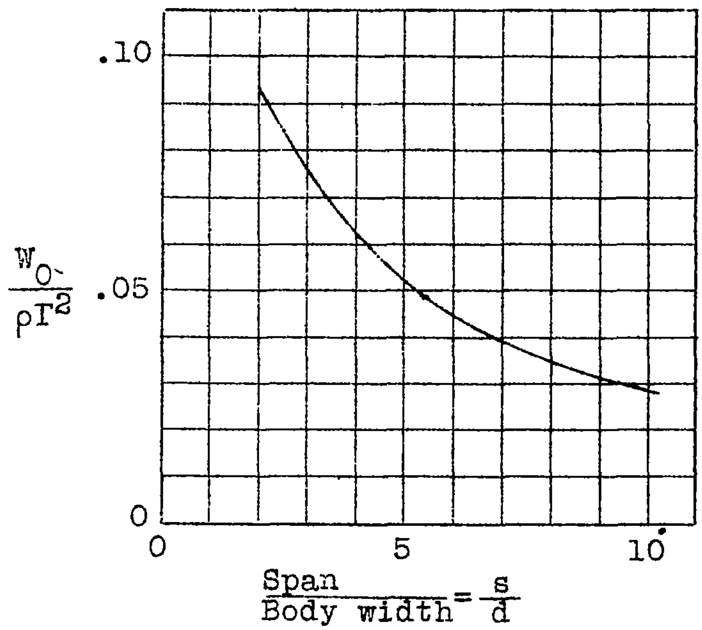

Fig.1

W_0 = change in drag (?)

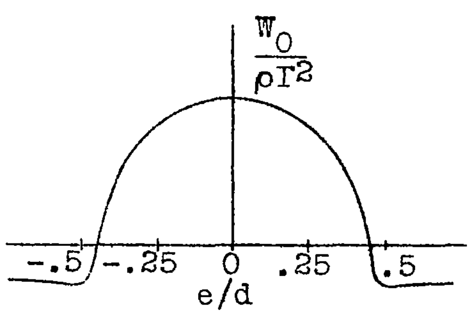

Fig.2

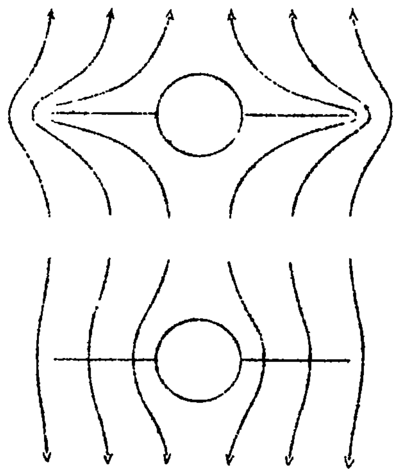

Fig.3

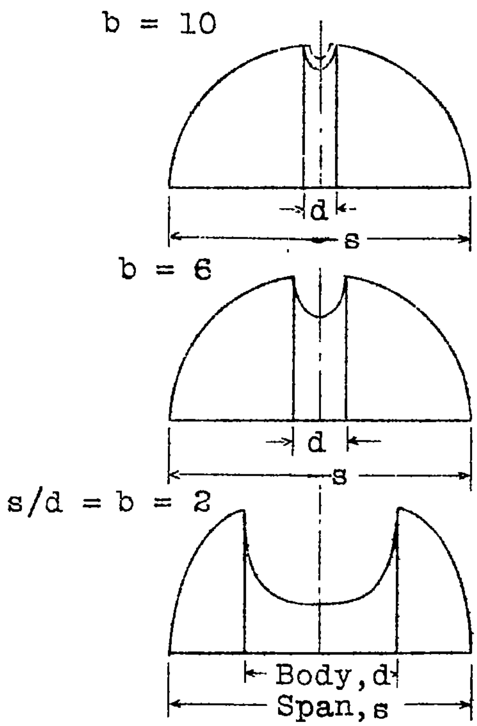

Fig.4

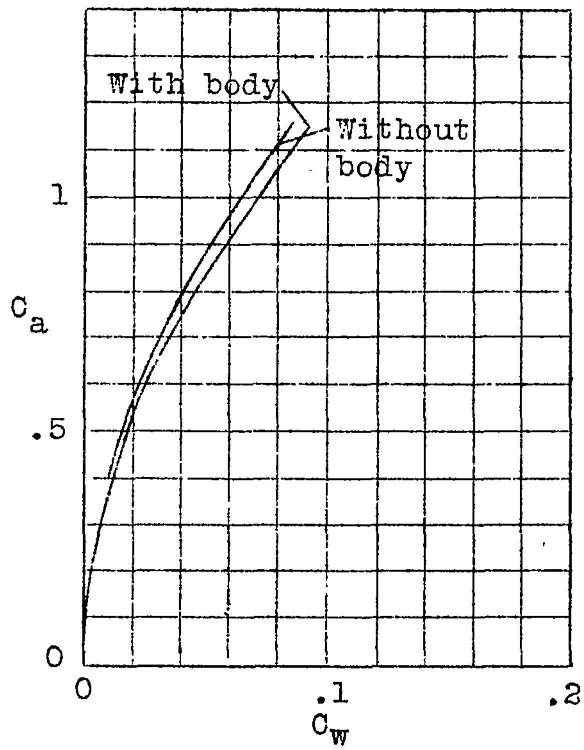

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Fig.5

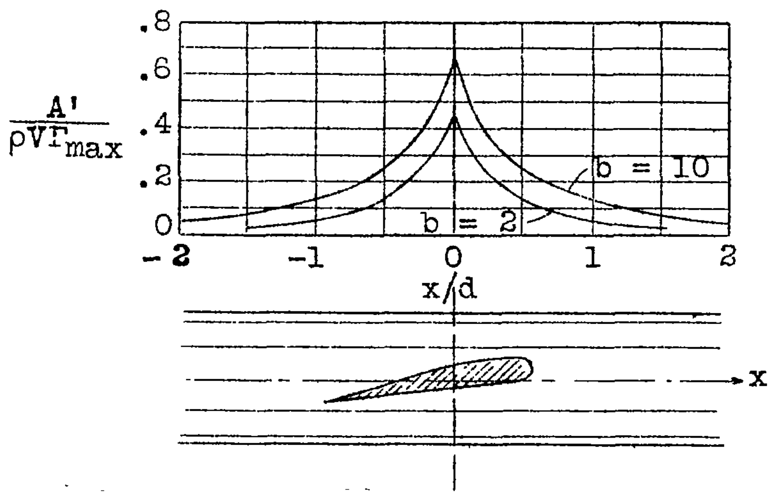

Fig.6

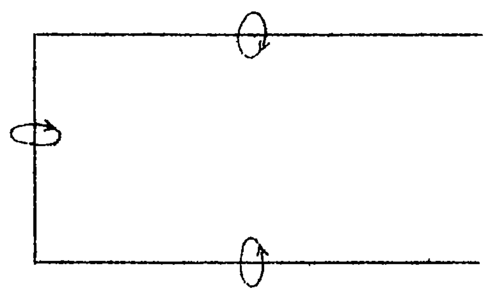

Fig.7

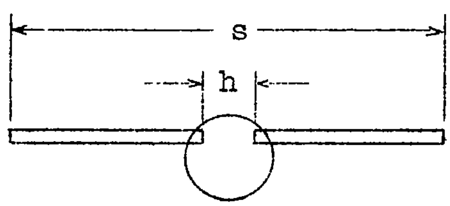

Fig.8 Effective span s-h