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CHOICE OF PROFILE FOR THE WINGS OF AN AIRPLANE

By A. Toussaint and E. Carafoli

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Simplified General Method for Drawing Airplane Profiles

The above method was useful for obtaining a better understanding of the problem. It leads, nevertheless, to a somewhat laborious drawing which becomes quite complicated when we take a transformation function having terms of a high degree.

Besides, it is extremely difficult to determine by that method the parameters which govern the profile form, with the object of obtaining "families of profiles," that is, profiles the form of which evolves in a continuous manner by modification of one of the characteristics, such as C_{m_0}, the maximum relative thickness; e/l, the suitable distribution of the relative thickness along the chord, etc.

*From L'Aeronautique, January, 1928. For Part I see N.A.C.A. Technical Memorandum No. 468.

The method given below overcomes these difficulties. It depends on the following considerations (See Comptes Rendus de l'Academie des Sciences, "Notes" de M. Carafoli, October 24, 1927, and November 14 and 28, 1927.

We have seen that, by changing the axes, the transformation function can always be put into the form

$$z = \zeta + \frac{c^2}{\zeta} + \sum_{n=2}^{n} \frac{x_n}{\zeta^n} \qquad (12')$$

in which c^2 is a real quantity.

Let Ox and Oy be a system of axes, M the center of the generating circle passing through B_1, and B' the point of zero velocity which should correspond to the profile tip (Fig. 11). It can be shown that the auxiliary circle M_1, corresponding to the partial transformation

$$z_1 = \frac{c^2}{\zeta} \quad \text{or} \quad z_1 \zeta = c^2$$

passes through the point C_1 so that $OB_1 \times OC_1 = c^2$, and the center M_1 is located at the intersection of the line OM_1 (symmetrical to OM with respect to Oy) with the line $C_1 M_1$ parallel to B_1M (Fig. 8).

The position of B' on the generating circle is defined by the value of the moment coefficient C_{m_0}, according to the formula

$$C_{m_0} = 8\pi \frac{c}{l^2} \tau \qquad (10')$$

in which τ, the angle of B'M with Ox, corresponds to

$(\beta - \gamma)$ of the preceding notations (if $MB'O = \beta$ and $B'OB_1 = \gamma$) and l is the profile chord, the value of which is approximately $2\,(OB_1 + OC_1)$.

It follows from equation (10'), τ being expressed in degrees, that

$$\tau^\circ \cong \frac{C_{m_0}}{8\pi}\,\frac{180}{\pi}\,\frac{4(OB_1+OC_1)^2}{OB_1 \times OC_1} = C_{m_0}\ 36.4\ \frac{\left(1 - \frac{1}{2}\,\frac{B_1C_1}{OB_1}\right)^2}{1 - \frac{B_1C_1}{O\,B_1}}$$

$$\cong 36.4\ C_{m_0}$$

For the point B' $(\zeta = -\,OB'e^{-i\gamma})$, which must correspond to the trailing edge of the profile, we should have

$$\left(\frac{dz}{d\zeta}\right)_{B'} = \frac{\zeta - \frac{c^2}{\zeta} - \sum\limits_{n=2}^{n} \frac{n\,x_n}{\zeta}}{\zeta^n} = 0 \qquad (35)$$

If the point C' corresponding to B' for the transformation $z_1 = \frac{c^2}{\zeta}$, be on the auxiliary circle M_1, then $\left(\zeta - \frac{c^2}{\zeta}\right)$ represents $C'B'e^{i\sigma}$ (Fig. 11), from which we derive

$$\left\{\begin{array}{l} C'B'e^{i\sigma} - \left(\sum\limits_{n=2}^{n} \dfrac{n\,x_n}{\zeta^n}\right) = 0 \\ \text{or} \\ \sum\limits_{n=2}^{n} \dfrac{n\,x_n}{(-1)^n\,(O\,B')^n\ e^{-in\gamma}} = C'B'e^{i\sigma} \end{array}\right. \qquad (36)$$

If, for example, the transformation function be

$$z = \zeta = \frac{c^2}{\zeta} + \left(\frac{\bar{x}_p}{\zeta^p} + \frac{\bar{x}_q}{\zeta^q} + \frac{\bar{x}_r}{\zeta^r}\right),$$

the above formula can then be written

$$\frac{p\ x_p}{(-1)^p\ (OB')^p\ e^{-ip\gamma}} + \frac{q\ x_q}{(-1)^q\ (OB')^q\ e^{-iq\gamma}} + \frac{r\ x_r}{(-1)^r\ (OB')^r\ e^{ir\gamma}} = C'B'e^{i\sigma} \qquad (36')$$

so that the three terms of the first member constitute a polygon C'IHB' the geometrical resultant of which is B'C' (Fig. 12).

Conversely, each polygon C'IHB' would correspond to three terms p, q, r of the transformation function, which are determined by means of the formula (36')

$$\frac{p\ x_p\ e^{ip\gamma}}{(-1)^p\ (OB')^p} = C'Ie^{i\sigma p}\ \ldots,$$

and so on.

In particular, the polygon could be reduced to any number of sides aligned on C'B' (as, for example, C'I'H'B').
In practice, it even suffices to consider a single supplementary term of any degree n, in which case we have

$$\frac{n\ x_n\ e^{in\gamma}}{(-1)^n\ (OB')^n} = C'B'e^{i\sigma}\ *$$

*We have already pointed out the advantage of the case in which the only complementary term is of the degree n = 3, or

$$z = \zeta + \frac{c^2}{\zeta} + \left(\frac{x_3}{\zeta^3}\right)$$

We should thus add to the two terms, which give us the point P, the segment $P\,P_n$ of the modulus

$$\left|\frac{x_n}{\zeta^n}\right| = (-1)^n \frac{C'B'}{n} \frac{OB'}{OP'}^n$$

and of amplitude

$$\varphi_n = \sigma - n\gamma - n\theta$$

In order to compute the velocity W_p at the point P_n of the profile, it suffices to construct the vector

$$Q'R' = \left|\frac{n\ x_n}{\zeta^n}\right| = (-1)^n\ C'B'\left(\frac{OB'}{OP'}\right)^n$$

starting from the corresponding point Q' with direction φ_n and magnitude $P'R'$, and we get, as before,

$$W_p = 2V\,\frac{P'O'}{MB} \times \frac{OP'}{P'R'} \tag{25'}$$

Utilizing these principles, the profile design would involve the following operations, which are entirely geometrical and require no special knowledge of the problem.

The simplified method of drawing a profile with a pointed trailing edge includes the following operations:

1. Draw the two axes Ox and Oy and lay out the length OB_1 along the negative direction of Ox. OB_1 is generally taken as one-fourth the chord of the profile to be obtained (Fig. 11).

2. Choose OC_1 or B_1C_1 determining the point C_1, the

position of which relative to B_1 characterizes the thickness distribution along the profile chord. In the case represented by the figure, we have B_1C_1 positive or negative according to whether C_1 is on the right or on the left of B_1. When B_1C_1 is positive, the profile is less sharp at the rear and conversely. In practice, we take

$$\frac{B_1C_1}{O\ B_1} \leqq 0.05.$$

3. Choose OM_O which characterizes the apparent mean camber of the profile (by analogy with the case of Joukowski profiles). For profiles with small C_{m_O} it is convenient to take

$$\frac{OM_O}{OB_1} \leqq 0.10.$$

4. Choose the center M of the generating circle on the straight line B_1M_O in such a manner that M_OM would satisfy the following formula in terms of the maximum relative thickness e/l we wish to obtain

$$\frac{M_OM \pm 0.4\ B_1C_1}{OB_1} = \frac{0.77\ \frac{e}{l}}{1 - 0.77\ \frac{e}{l}}\ .$$

The minus sign is taken when B_1C_1 is positive and vice versa.

5. Draw the generating circle with the center M and the radius MB_1.

6. Draw the auxiliary circle corresponding to the partial

transformation

$$z_1 = \frac{c^2}{\zeta} \quad \text{or} \quad z\zeta = c^2 = OB_1 \times OC_1 .$$

This circle will then pass through C_1. Its center M_1 will be at the intersection of the line OM_1 (symmetrical to OM with respect to Oy) with the line $C_1 M_1$ parallel to $B_1 M$.

7. Carry out the operations corresponding to the first two terms of the function, by geometric summation of the vectors OP' and OQ' of equal amplitudes and opposite signs (Trefftz, geometrical construction).

When C_1 is inside the generating circle, the points P define a profile with rounded trailing edge, but, when C_1 is outside of the circle, the contour of the points P forms a figure 8 with a small bulge toward the rear.

8. Define, on the generating circle, the point of zero velocity B', with the object of obtaining a C_{m_0} fixed _a priori_. For the latter, we draw B'M making an angle τ with Ox, such that $\tau^0 = 36.4\ C_{m_0}$. The point C', on the auxiliary circle M_1, corresponds to the point B', so that

$$B'OB_1 = B_1OC' = \gamma$$

9. Construct the representative segments of the other terms of the transformation function.

Since, in practice, it is important to take a single complementary term of the degree $n = 3$, the segment to be constructed, starting from each of the points P, will have the magnitude

$$PP_8 = \left|\frac{x_3}{\xi^3}\right| = -\frac{C'B'}{3}\left(\frac{OB'}{OP'}\right)^3$$

and the direction

$$\varphi = \sigma - 3\gamma - 3\theta$$

Remarks.- The above method defines, without ambiguity, the parameters which govern the evolution of the profile form. These parameters are

$$C_{m_0};\quad \frac{OM_0}{OB_1};\quad \frac{M_0M \mp 0.4\ B_1C_1}{OB_1};\quad \frac{B_1C_1}{OB_1};$$

and the degree n of the complementary term.

The knowledge of these parameters permits the definition of "profile groups" of evolutive forms and characteristics. For example:

(a) Groups with variable C_{m_0}, with constant apparent camber and relative thickness and otherwise general form characterized by the same B_1C_1/OB_1 and the same degree n.

(b) Groups with variable apparent camber, with constant C_{m_0} and relative thickness, as well as same B_1C_1/OB_1 and n.

(c) Groups with variable thickness distribution (by variation of B_1C_1/OB_1), all other characteristics remaining constant.

(d) Groups with variable relative thickness, with all other parameters constant.

(e) Groups with variable n, and all other characteristics constant. In practice, there seems to be little interest, in the present state of the problem, in varying n and $n = 3$ is therefore adopted.

Remarks.- The preceding general method also permits the drawing of profiles with rounded trailing edges (Comptes Rendus, Vol. 185, p.842, Oct. 24, 1927). It can be adapted to the particular case of complementary terms leading to corrective terms of constant modulus (Comptes Rendus, Vol. 185, p.1014, November, 1927).

By extension we can pass from a pointed profile to a profile with a reflexed trailing edge.(Comptes Rendus, Vol. 185, p.1189, Nov. 28, 1927.

Aerodynamic Characteristics of Theoretical Profiles

The aerodynamic characteristics of theoretical profiles result from the application of the same conformal transformation to the sustaining flow around the generating circle, according to Joukowski's hypothesis.

It is known that this hypothesis consists in determining the circulation Γ such that the point of the profile in the z plane corresponds to a point of zero velocity on the generating circle in the ζ plane.

Under these conditions, the aerodynamic characteristics of the theoretical profile, corresponding to the case of a wing of infinite span, will be given by the following formulas:

Lift coefficient

$$C_z \approx 8\pi \frac{a}{l} (\alpha + \beta) \tag{17}$$

Moment coefficient at the angle of zero lift

$$C_{m_{(\Gamma)}} = C_{m_0} = 8\pi \frac{c^2}{l^2} (\beta - \gamma)$$

in which

α is the angle of incidence with respect to OB';

β is the amplitude of the first axis with respect to OB';

γ is the amplitude of the second axis with respect to OB';

c^2 is the modulus of the parameter x_1;

a is the radius of the generating circle;

l is the profile chord, measured from the drawing, after it is completed. (It is found, moreover, that the value thus obtained differs but little from the approached value.)

In order to get the moment coefficient with reference to the leading edge of the profile, it is sufficient to determine the position of the profile focus F. The latter is found on a line MF making the angle $2(\gamma - \beta)$ with the first axis (MB').

The distance MF can then be calculated by the formula

$$MF = \frac{c^2}{a} \tag{19}$$

Knowing thus the position of the focus F, we then measure the distance FA as far as the leading edge and obtain

$$C_{m_{(A)}} = C_{m_{(F)}} + C_z \frac{AF}{l}$$

or

$$C_{m_{(A)}} = C_{m_o} + C_z \frac{AF}{l} \tag{20}$$

In general $\frac{AF}{l}$ is approximately 0.25.

Diagrams of Aerodynamic Pressures on a Theoretical Profile

The velocity at a given point on the profile being W_p, the pressure coefficient can be defined by the expression

$$C_p = 1 - \left(\frac{W_p}{V}\right)^2, \tag{21}$$

V being the aerodynamic velocity.

The velocity W_p is calculated by the general formula

$$W_p = W_c \left|\frac{d\zeta}{dz}\right| = \frac{W_c}{\left|\frac{dz}{d\zeta}\right|} \tag{22}$$

in which W_c is the velocity corresponding to the point on the generating circle.

For all points P' of the generating circle (Fig. 13), we have

$$W_c = 2V \frac{\overline{P'D'}}{\overline{MP'}} = 2V \frac{\overline{P'D'}}{a}, \tag{23}$$

$P'D'$ being the length of the perpendicular dropped from P' to the line V which passes through B'.

Moreover, we have

$$\frac{1}{\left|\frac{dz}{d\zeta}\right|} = \frac{\zeta'}{\left|\zeta' - \frac{c^2}{\zeta'} - \frac{2x_2\ e^{-3i\gamma}}{\zeta'^2} - \cdots - \frac{nx_n\ e^{-(n+1)i\gamma}}{\zeta'^n}\right|} \tag{24}$$

or

$$\zeta' = \overline{OP'}.$$

Considering the point Q' conjugate of P' on the auxiliary circle M_1 (Fig. 14), we have

$$\overline{P'Q'} = \left|\zeta' - \frac{c^2}{\zeta'}\right|$$

If we introduce

$$Q'RI_1 = \left|\frac{2x_2\ e^{-3i\gamma}}{\zeta'^2}\right|$$

of the amplitude φ_2 previously calculated, we get

$$P'R'_1 = P'Q' - Q'R'_1 = \left|\zeta' - \frac{c^2}{\zeta'} - \frac{2x_2\ e^{-3i\gamma}}{\zeta'^2}\right|$$

Similarly

$$R'_1\ R'_2 = \left|\frac{3x_3\ e^{-4i\gamma}}{\zeta'^3}\right|$$

with the amplitude φ_3 or $(\varphi_3 + \gamma)$ and we get

$$P'R'_2 = \left|\zeta' - \frac{c^2}{\zeta'} - \frac{2x_2\ e^{-3i\gamma}}{\zeta'^2} - \frac{3x_3\ e^{-4i\gamma}}{\zeta'^3}\right|$$

and so on, as far as the point $R'_{(n-1)}$ corresponding to the term in x_n.

It must be noted that $Q'R'_1 = 2$ times the positional correction previously calculated for the term in x_2. In like manner $R'_{(n-1)}R'_{(n-2)} = n$ times the correction calculated for the term in x_n.

The construction of the segment $P'R'_{(n-1)}$ utilizes there-

fore the elements of the profile design.

Finally, for each point P, consequent on P', we have

$$W_p = 2V \frac{\overline{P'D'}}{a} \frac{\overline{OP'}}{\overline{P'R'}_{(n-1)}}, \tag{25}$$

from which C_p is derived.

The factors $\frac{OP'}{P'R'_{(n-1)}}$ are calculable, once for all, for all points of the profile.

The factors PD' vary with the angle of attack α for each of the points P'.

Aerodynamic Characteristics of a Wing of Finite Span, with a Theoretical Profile

1. Case of rectangular wings

The aerodynamic characteristics of a wing having a finite span are deduced from the aerodynamic characteristics of the profile of a wing of infinite span by allowing for the modifications brought about in the theoretical flow by limiting the span.

Prandtl's theory teaches us that these modifications are interpreted in practice by the fact that: "The effective angle of attack at all sections of the finite wing is equal to the real angle of attack diminished by the induced angle of the section in question." This induced angle depends essentially on the law of lift distribution along the span.

For rectangular monoplane wings the simplifying hypothesis

of an elliptical distribution of the lift is frequently used. It can be considered as a sufficiently practical approximation when we wish to calculate the mean aerodynamic characteristics for the whole wing. If we wish to analyze what takes place in a particular wing section, it is convenient to assume a lift distribution along the wing span. In this connection, we have proposed to adopt the simplified approximate solution of Fuchs*, because the theoretical solution of Betz necessitates very laborious calculations. The former consists in assuming that the law of distribution of the circulation or of the lift on a rectangular wing of span $\Lambda = \frac{L}{l}$ is represented by the formula

$$\Gamma_{(x)} = \Gamma_0 \sqrt{1 - \left(\frac{2x}{L}\right)^2} \left[1 + \frac{\Lambda}{2\Lambda + 3\pi} \left(\frac{2x}{L}\right)^2 .\right]$$

Figure 12 represents the comparison of the lift distributions for a rectangular wing of aspect ratio 5 and for the same mean lift of 1.

The curve EEE corresponds to an elliptical distribution. The curve BBB corresponds to the evolutive distribution as expressed by formula (26).

The straight line RR corresponds to a rectangular distribution.

Lastly, the dotted line represents the distribution experimentally determined for a mean angle of attack.

A sufficient agreement is noted between the theoretical distribution obtained by formula (26) and the experimental dis-

*A. Toussaint and E. Carafoli, "Verifications experimentales de la Theorie des ailes sustentatrices," L'Aerophile, April 1-15, 1927.

tribution, except at the wing tips, where the latter exhibits abnormal deviations due to disturbances of the lateral edges.

In adopting this distribution, approximate but at the same time developed with Λ, one easily finds that the mean lift coefficient C_z on the wing is given by the formula

$$C_{z_{(\Lambda)}} = C_{z_{(\infty)}} \frac{\pi}{4} \frac{1 + \frac{a_1}{4}}{1 + 2\pi \frac{a}{L} \left(1 - \frac{a_1}{2}\right)} \tag{27}$$

in which

$$C_{z_{(\infty)}} = 8\pi \frac{a}{l} (\alpha + \beta) \tag{17}$$

and

$$a_1 = \frac{\Lambda}{2\Lambda + 3\pi} .$$

As before, a is the radius of the generating circle. It is sometimes expressed by an approximate formula in terms of the profile chord l. Since we are dealing with theoretical profiles, it is not necessary to resort to this approximation.

The expression for $C_{z_{(\Lambda)}}$ may also be written

$$\left\{\begin{array}{l} C_{z_{(\Lambda)}} = A_{(\Lambda)} (\alpha + \beta) \\ \text{with} \\ A_{(\Lambda)} = 8\pi \dfrac{a}{l} \dfrac{\pi}{4} \dfrac{1 + \frac{a_1}{4}}{1 + 2\pi \frac{a}{L} \left(1 - \frac{a_1}{2}\right)} \end{array}\right. \tag{27'}$$

If the angle of attack $(\alpha + \beta)$, referred to the axis of zero lift, is expressed in degrees, we have

$$C_{z(\infty)} = 8\pi \frac{a}{l} \frac{(\alpha + \beta)^\circ}{57.3} \tag{17'}$$

and thus

$$\left\{\begin{array}{ll} C_{z(\Lambda)} & = A^\circ_{(\Lambda)} \; (\alpha + \beta)^\circ \\ \text{with} & \\ A^\circ_{(\Lambda)} & = 8\pi \dfrac{a}{l} \dfrac{1}{57.3} \dfrac{\pi}{4} \dfrac{1 + \dfrac{a_1}{4}}{1 + 2\pi \dfrac{a}{L} \left(1 - \dfrac{a_1}{2}\right)} \end{array}\right. \tag{27''}$$

The expression for the moment coefficient $C_{m(A)}$ remains unchanged and we have

$$C_{m(A)} = 8\pi \frac{c^2}{l^2} \frac{(\beta - \gamma)^\circ}{57.3} + \frac{AF}{l} C_{z(\Lambda)}. \tag{20'}$$

With the simplifying hypothesis of an elliptical lift distribution, which assumes $a_1 = 0$, we would have

$$C_{z(\Lambda)E} = C_{z(\infty)} \frac{\pi}{4} - \frac{1}{1 + 2\pi \frac{a}{L}} \tag{27'''}$$

<u>Remark I</u>.- The formulas relating to the expression of the mean lift coefficient $C_{z(\Lambda)}$ for a wing of finite span depend essentially on the accepted law of lift distribution along the span.

In using the approximate distribution represented by formula (26), we desired principally to attract the attention of technical people to the necessity of considering, for rectangular wings, a distribution derivable from the aspect ratio. Consequently, we can employ the theoretical solution established by Betz.

The formulas relating to the expressions for the moment coefficients $C_{m(A)}$ and C_{m_0}, as well as the determination of the angle of zero lift, do not depend on the above law of distribution, nor on the theory of finite span. These aerodynamic characteristics are inherent in the profile form, according to the theory of infinite span.

Remark II.- The experiments conducted in the Aerodynamic Laboratory show that the mean lift coefficients experimentally determined are generally a little smaller than the theoretical values computed by the preceding formulas. This slight discrepancy results: first, from the approximation made in regard to the lift distribution; secondly, from the divergence between the real and the theoretical air flow; thirdly, from the approximations inherent in the theory of finite span.

Likewise, the experimental values of C_{m_0} and of β are a little smaller than their theoretical values. These differences seem to be due almost exclusively to divergencies between the real and theoretical flows.

The comparison of the experimental and theoretical values of C_z, C_{m_0}, and β will permit figuring the discrepancies between the experimental and theoretical aerodynamic characteristics for different groups of profiles.

The induced polar is computed with a sufficient practical approximation by the formulas of Prandtl's theory.

2. Case of wings of elliptical or similar plan form. In this case, we still have $a_1 = 0$, but the expression for the mean lift coefficient becomes

$$C_{z(\Lambda)E} = C_{z(\infty)} \frac{1}{1 + 2\pi \frac{a_o}{L}}$$

or again

$$\left\{ \begin{array}{l} C_{z(\Lambda)E} = A^o_{(\Lambda)E} \ (\alpha + \beta)^o \\ \text{with} \\ A^o_{(\Lambda)E} = 8\pi \frac{a_o}{l_o} \ \frac{1}{57.3} \ \frac{1}{1 + 2\pi \frac{a_o}{L}} \end{array} \right. \tag{28'}$$

a_o and l_o relating to the middle section.

The expression for the moment coefficient $C_{m(A)}$ can be calculated according to the plan form

Remarks.- The reduction factor to account for the discrepancy between theoretical and experimental lifts is a little smaller than the one relating to rectangular wings.

3. Case of wing cells.- This problem is more complex but, for a first approximation, we can use the preceding formulas by giving to Λ the value $\frac{K^2L^2}{S}$, which is employed in calculations of induced drag.

Diagrams of Aerodynamic Pressures on Wings of Finite Span

The pressure coefficient at a point on any section of a finite wing is computed as already mentioned, taking care, however, of the induced angle in the section under consideration. In other words, the direction of the aerodynamic velocity, passing through B', makes with Ox an effective angle $(\alpha - \varphi_y)$, φ_y being the induced angle relative to the section located at a distance y from the middle section.

For rectangular wings, having a distribution derived in terms of Λ, we have

1. At the middle section $(y = 0)$

$$\varphi_o = \frac{2\pi \frac{a}{L} \left(1 - \frac{a_1}{2}\right)}{1 + 2\pi \frac{a}{L} \left(1 - \frac{a_1}{2}\right)} i \tag{29}$$

2. In any section located at the distance y from the middle section

$$\varphi_{(y)} = 2\pi \frac{a}{L} \left[1 - \frac{a_1}{2} + 3a_1 \left(\frac{2y}{L}\right)^2\right] (i - \varphi_o) \tag{30}$$

or again

$$\varphi_{(y)} = \frac{2\pi \frac{a}{L} \left[1 - \frac{a_1}{2} + 3a_1 \left(\frac{2y}{L}\right)^2\right]}{1 + 2\pi \frac{a}{L} \left(1 - \frac{a_1}{2}\right)} i,$$

in which $i = \alpha + \beta$ is the actual angle of attack with reference to the axis of zero lift.

For elliptical or similar wings, with elliptical lift distribution, we have

$$\varphi_0 = \varphi(y) = \frac{2\pi \frac{a_0}{L}}{1 + 2\pi \frac{a_0}{L}} i \qquad (31)$$

Aerodynamic Characteristics of Profile (Fig. 8)

The chord length is laid out on the drawing, namely,

$$l = 23.8 \text{ cm},$$

while the approximate computation gives

$$l = 2\ (\lambda + OC') = 23.54 \text{ cm}$$

The focus F falls on MF, making an angle $2\ (\gamma - \beta) = -4^\circ$ with the first axis; and we have

$$\overline{MF} = \frac{c^2}{a} = \frac{34.6}{6.45} = 5.38 \text{ cm}$$

We then measure

$$\overline{AF} = 6.15 \text{ cm},$$

from which

$$\frac{\overline{AF}}{l} = \frac{6.15}{23.8} = 0.258.$$

The aerodynamic characteristics of the profile will then be

$$C_{z(\infty)} = 8\pi \frac{a}{l} (\alpha + \beta) = 6.8\ (\alpha + \beta)$$

or, for $(\alpha + \beta)$ expressed in degrees,

$$C_{z(\infty)} = \frac{6.8}{57.3} (\alpha + \beta)^\circ = 0.119 (\alpha + \beta)^\circ,$$

$$C_{m_0} = C_{m(F)} = 8\pi \frac{c^2}{l^2} \frac{(\beta - \gamma)^\circ}{57.3} = 0.0536$$

and

$$C_{m(A)} = 0.0536 + 0.258\, C_{z(\infty)} \,.$$

Lastly, for the experimental comparisons, we take the angle $\alpha_c = -1.3^\circ$ between the tangent chord line and the axis Ox. From this it follows that the theoretical angle of zero lift, with respect to this chord, has the value

$$5.3^\circ - 1.3^\circ = 4^\circ \,.$$

Experimental Verification of the Aerodynamic Characteristics

With the preceding profile drawing, we have dealt with rectangular wings with spans corresponding to the aspect ratios 2, 3, 4, 5, and 6, with the same chord length

$$l = 23.8 \text{ cm.}$$

The graph (Fig. 16) represents the experimental results of tests with these wings.

We see that the experimental angle of zero lift is the same for all the aspect ratios and that its value is 3.6°, while the theoretical value was found to be 4°.

The experimental C_{m_0}, which is also the same for all aspect ratios, is 0.045, while the theoretical value is 0.0536.

The mean slope of the curve (C_z, C_m) is 0.255 in place of the theoretical value of 0.258.

Figure 17 represents the lift coefficients for the various aspect ratios converted to the infinite aspect ratio by the application of formula (27). The representative points are grouped along a mean line near the straight line representing the theoretical lift of the profile.

$$C_{z_{(\infty)}} = 0.119\ (\alpha + \beta).$$

The reduction coefficient k for the experimental and theoretical lifts and angles β is thus found to be $k = 0.94$.

Lastly, Figure 18 represents the experimental points of the various polars converted to the single aspect ratio $\Lambda = 5$. It is seen that the points thus corrected are grouped in a single polar blending with the experimental polar for $\Lambda = 5$.

Experimental Verification of the Pressures on the Middle Section

The pressure diagrams for the middle section were calculated as already mentioned. The comparison of these theoretical pressures with the experimental pressures shows a very good agreement for the angles of attack at which the flow does not develop important separations.

This comparison between the theoretical and experimental aerodynamic characteristics confers an undeniable scientific character upon the investigation of wings with theoretical profiles. Under these conditions, the experimental data, which could be established by systematic tests of different profile groups, would be susceptible of a general interpretation.

For example, in determining the law of evolution for the reduction coefficient between the theoretical and experimental characteristics for a group of theoretical profiles, we could, by a legitimate interpolation, directly apply the suitable coefficient to all profiles of the group, without having to resort to a new experimental investigation.

These remarks also apply to the aerodynamic characteristics which the present theories do not enable us to calculate. In particular, we could establish, for each group of theoretical profiles, the law of evolution of the maximum C_z and of the minimum C_x in terms of one or several characteristics of the drawing.

Thus, the choice of the profile to give an airplane wing will be found to be completely determined by satisfying the required conditions for the aerodynamic characteristics and structural considerations.

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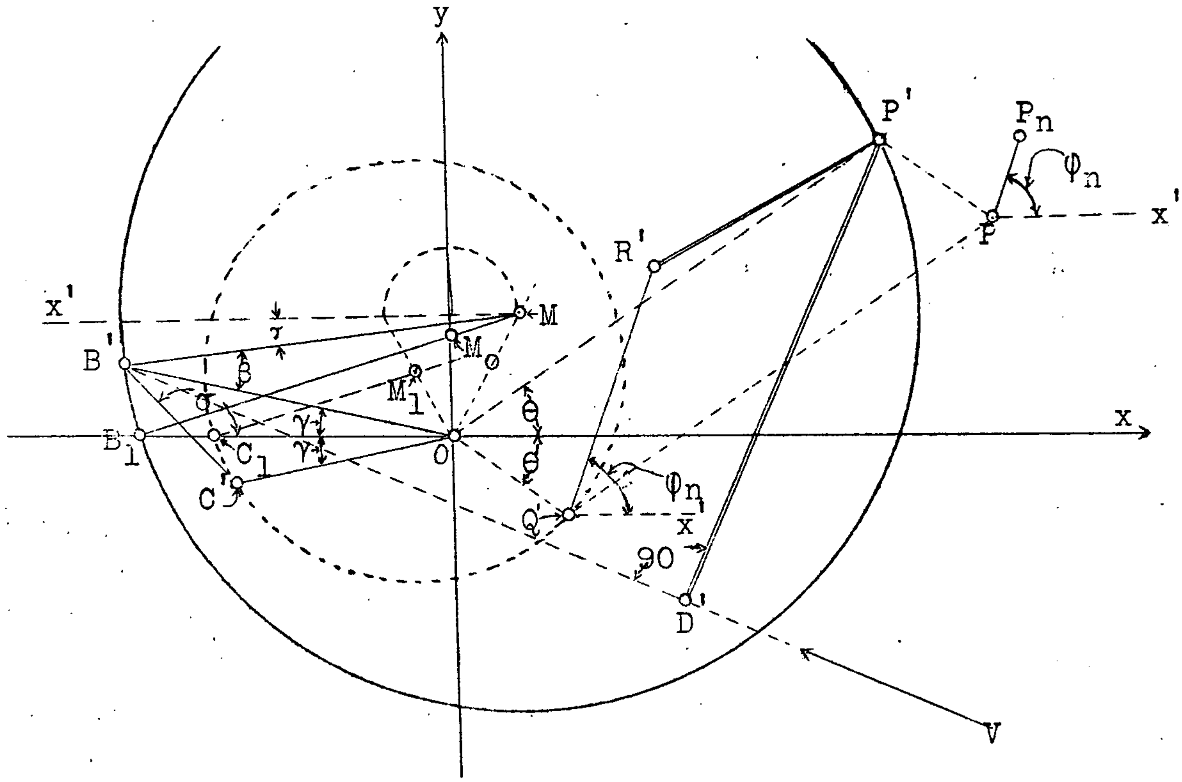
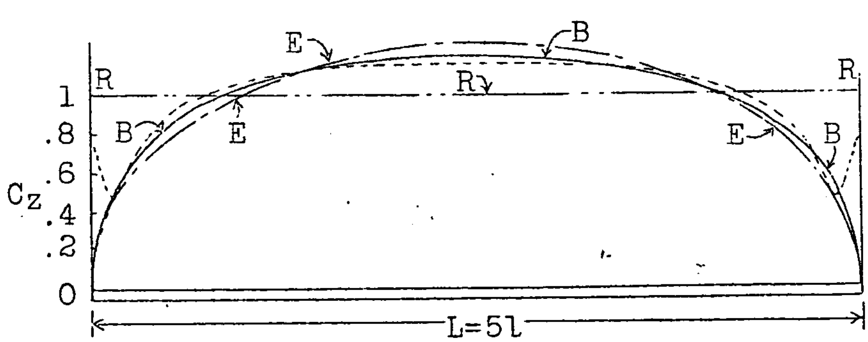


y
x
x'
x'
P'
P_n
φ_n
P
R'
M
M_o
M_1
B'
β
σ
γ
γ
B_1
C_1
C'
O
θ
θ
φ_n
Q
x'
90
D'
V

Fig.11



E
B
R
R
R
1
.8
.6
.4
.2
0
C_z
B
E
E
B
L=5l

Fig.15

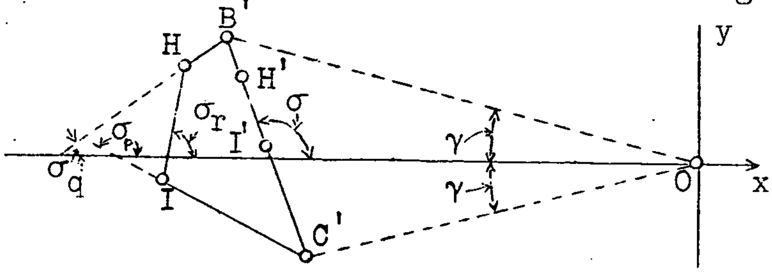


Fig.12

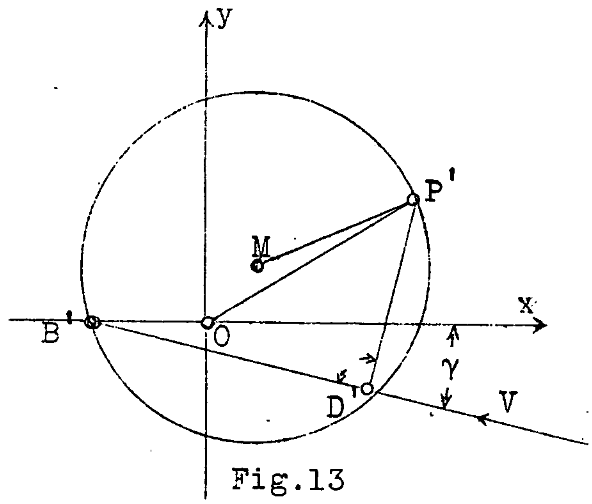


Fig.13

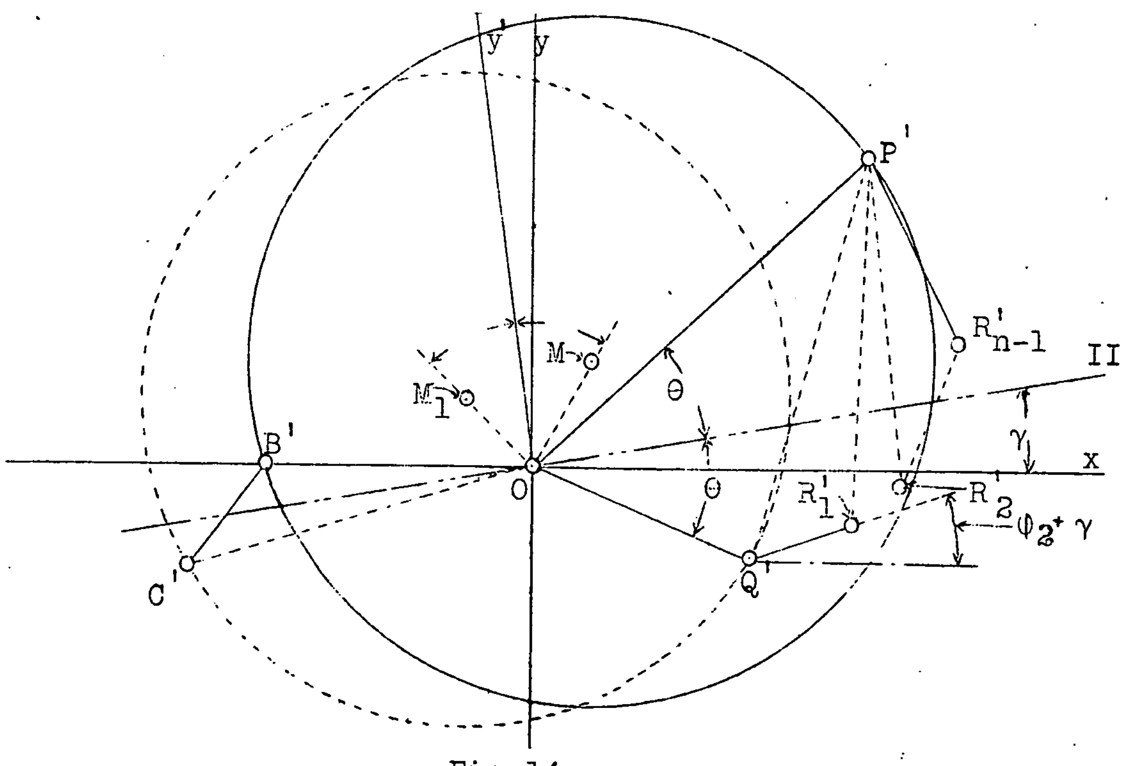


Fig.14

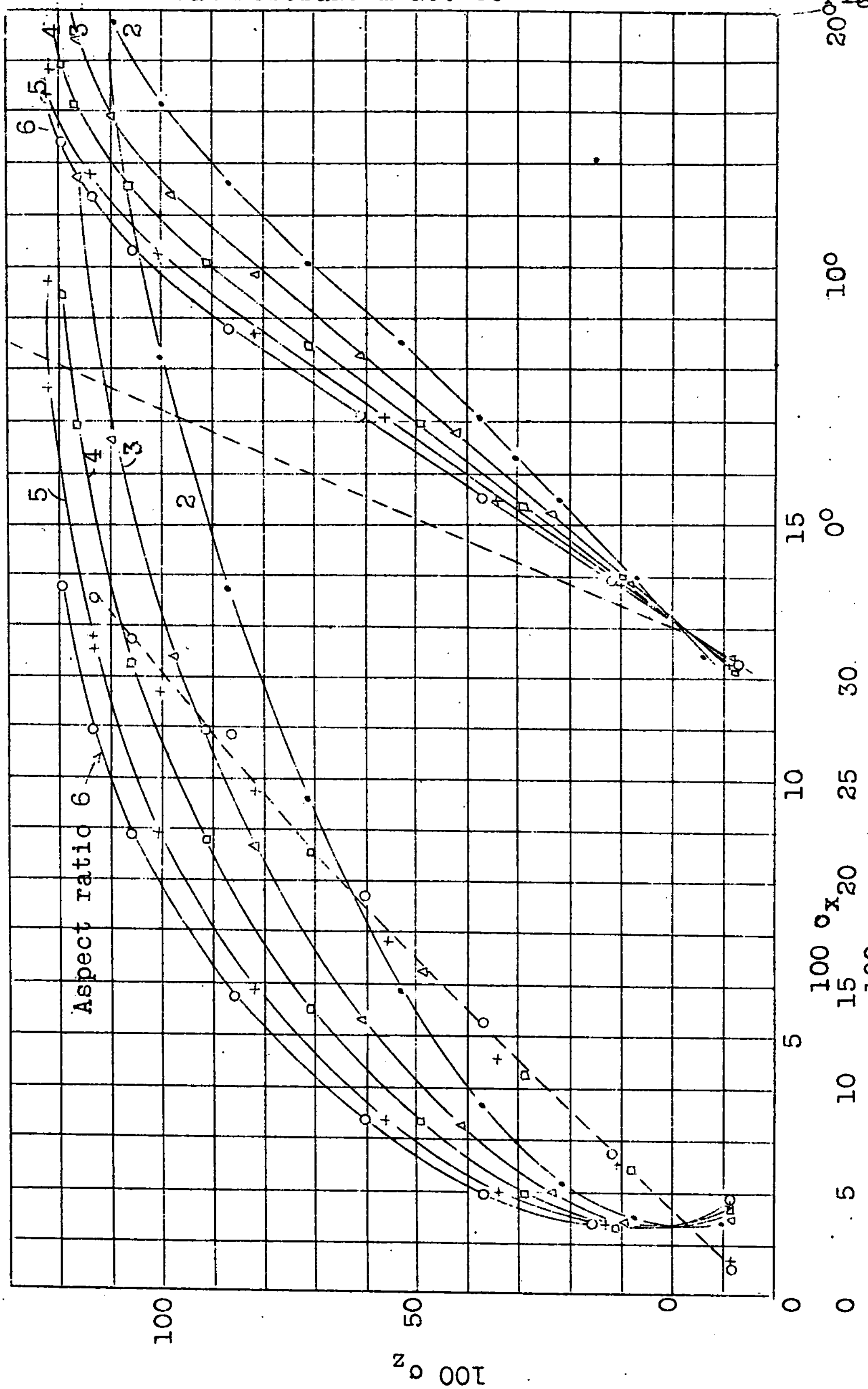


Fig.16 S.C.281 (Von Mises) airfoil. Aspect ratios 6,5,4,3 & 2.

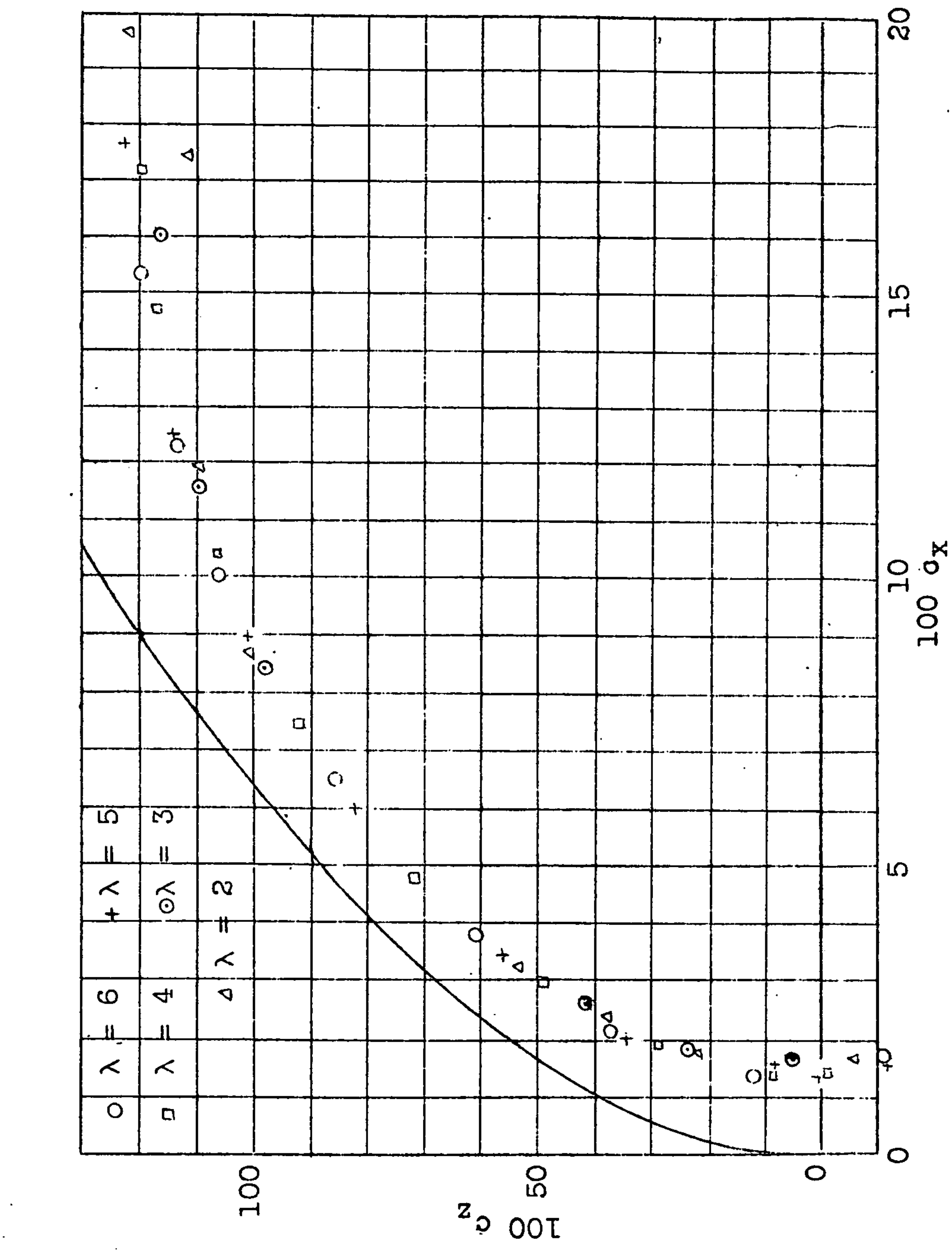


Fig.17 Polars converted to λ = 5. S.C. 281 (Von Mises) airfoil. Aspect ratios 6,5,4,3 & 2.

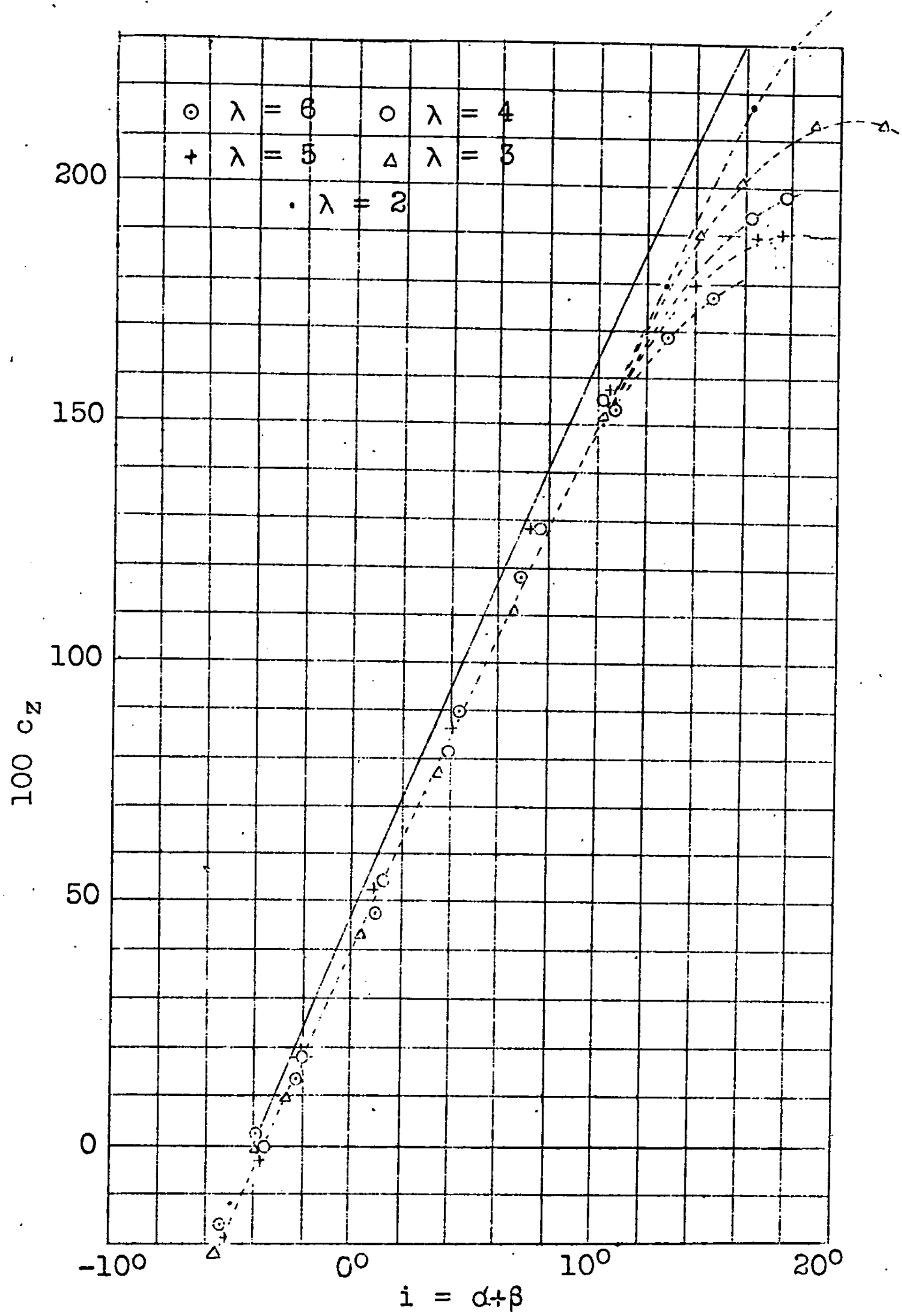


Fig.18 Lift coefficient converted to infinite aspect ratio. S.C. 281 (Von Mises) airfoil. Aspect ratios 6,5,4,3&2.