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ON SLENDER-BODY THEORY AT TRANSONIC SPEEDS

By KEITH C. HARDER and E. B. KLUNKER



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Langley Field, Va.

National Advisory Committee for Aeronautics

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REPORT 1315

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SUMMARY

The basic ideas of the slender-body approximation have been applied to the nonlinear transonic-flow equation for the velocity potential in order to obtain some of the essential features of slender-body theory at transonic speeds. The results of the investigation are presented from a unified point of view which demonstrates the similarity of slender-body solutions in the various Mach number ranges. The primary difference between the results in the different flow regimes is represented by a certain function which is dependent upon the body area distribution and the stream Mach number. The transonic area rule and some conditions concerning its validity follow from the analysis.

INTRODUCTION

Slender-body theory originated with Munk's report (ref. 1) in 1924 in which the forces on slender airships were calculated for low-speed flight. In 1938 Tsien (ref. 2) pointed out that Munk's airship theory also applied to the flow past inclined pointed bodies at supersonic speeds. The subject gained new importance in 1946 with the appearance of Jones's report (ref. 3) in which it was shown that the basic ideas of the slender-body approximation could be used to calculate the forces on slender lifting wings at both subsonic and supersonic speeds provided that proper account was taken of trailing-vortex sheets. Since Jones's report, the subject has received wide treatment. In an important paper in 1949, Ward (ref. 4) developed a general unifying theory for the flow past smooth slender pointed bodies at supersonic speeds. This theory contains as special cases the lifting planar wings of Jones and the slender nonlifting bodies treated by Von Kármán (ref. 5). The corresponding problem at subsonic speeds has been examined by Adams and Sears (ref. 6) who also extended the slender-body concepts to shapes which are "not so slender." Lighthill (ref. 7) has given a method for calculating the flow past bodies with discontinuities in slope. Keune (ref. 8) has developed solutions for slender wings with thickness, and various lifting configurations have been treated by Heaslet, Spreiter, Lomax, Ribner, and others (refs. 9 to 13).

The slender-body theory presented in references 2 to 13 has been based upon the linearized equation for the velocity potential. In the present report, the basic ideas of the slender-body approximation are applied to the nonlinear transonic equation for the velocity potential in order to gain some insight into the essential features of slender-body theory at transonic speeds. The attempt has been made to present the results from a unified point of view which demonstrates the similarity of the slender-body solutions in the various Mach number ranges.

The authors wish to acknowledge the valuable suggestions of Dr. Adolf Busemann of the Langley Laboratory during the preparation of this report.

SLENDER-BODY APPROXIMATION

Slender-body theory deals with that class of shapes whose length is large compared with any lateral dimension. For such shapes at both subsonic and supersonic speeds, the flow in planes normal to the stream direction can be approximated by solutions of Laplace's equation. The justification is that for very slender wings or bodies the variation of the geometrical properties in the stream direction is small and, consequently, the rate of change of the longitudinal component of the velocity in the stream direction is also small. The various slender-body solutions have all been developed on the basis of the linearized potential equation. However, a similar development can be made on the basis of the nonlinear transonic equation.

The simplest differential equation for the disturbance potential Φ which is generally valid at transonic speeds (ref. 14, for example) is

$$[1-M^2-(\gamma+1)M^2\Phi_x]\Phi_{xx}+\Phi_{yy}+\Phi_{zz}=0 \qquad (1)$$

where x, y, and z are rectangular coordinates, M is the stream Mach number, and γ is the ratio of specific heats at constant pressure and constant volume. With l the characteristic length and b the characteristic width (such as the largest lateral dimension of the configuration), the nondimensional coordinates x_1, y_1, and z_1 defined by $x=lx_1$, $y=by_1$, and $z=bz_1$ and the nondimensional potential Φ_1 defined by $\Phi=\frac{b^2}{l}\Phi_1$ (x_1, y_1, z_1) are all of the order of 1 in the vicinity of the configuration. In this coordinate system, equation (1) becomes

$$\left(\frac{b}{l}\right)^2\left[1-M^2-(\gamma+1)M^2\left(\frac{b}{l}\right)^2\Phi_{1_{x_1}}\right]\Phi_{1_{x_1x_1}}+\Phi_{1_{y_1y_1}}+\Phi_{1_{z_1z_1}}=0 \qquad (2)$$

[1] Supersedes NACA Technical Note 3815 by Keith C. Harder and E. B. Klunker, 1956.

For sufficiently small values of the width parameter b/l, the terms involving derivatives in the stream direction can be neglected to obtain the result that the flow approximately satisfies Laplace's equation

$$\Phi_{yy}+\Phi_{zz}=0 \tag{3}$$

in the crossflow plane. Equation (3) represents the slender-body approximation to equation (1).

The surface boundary condition is

$$\frac{\partial\Phi}{\partial n}=\frac{dn}{dx}(1+\Phi_x)\approx\frac{dn}{dx}$$

where n is the outward normal to the configuration in the crossflow plane. For slender configurations the surface boundary condition can be integrated (ref. 4, for example) to give

$$\int_\nu \frac{\partial\Phi}{\partial n}\,d\nu=S'(x) \tag{4}$$

where ν is any contour enclosing the shape, $S(x)$ is the cross-sectional area distribution of the shape, and the prime denotes differentiation with respect to the indicated argument.

In the slender-body approximation, the potential satisfying equation (1) and the surface boundary condition is represented in the neighborhood of the configuration as a solution of equation (3) plus a function of integration $G(x)$. Thus for $r=\sqrt{y^2+z^2}<\rho$, say, where $\rho>b$,

$$\Phi(x,y,z)=\phi(y,z;x)+G(x) \qquad (r\leqq\rho) \tag{5}$$

where ϕ is a solution of the Laplace equation in the crossflow plane with x appearing as a parameter introduced by the shape of the cross section at x. The function ϕ, being independent of the stream Mach number, can be evaluated for an incompressible flow past the shape under consideration. The function $G(x)$ is determined from considerations involving the complete equation for transonic flow (eq. (1)) and, consequently, is dependent upon the stream Mach number and upon the shape of the configuration. Although the analytic expression for $G(x)$ at transonic speeds is not known, it will be shown that the only geometrical property of the configuration which influences this function is the cross-sectional area distribution—just as at subsonic and supersonic speeds. This property of $G(x)$ is established by comparing the slender-body solution with the solution for the flow past a body of revolution. As a preliminary to these considerations it is necessary to examine the expression for the velocity potential in more detail.

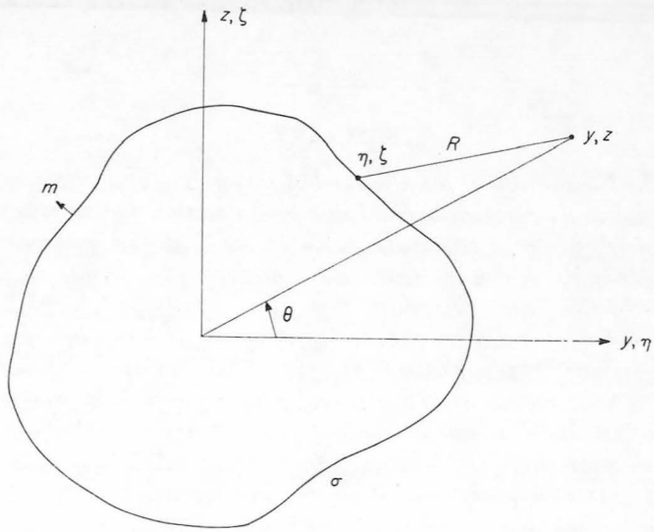
The flow past a slender configuration is given by the solution of equation (3) satisfying the boundary conditions of the problem and can be expressed in nondimensional terms by

$$\Phi(x,y,z)=\frac{b^2}{l}\left[\varphi\left(\frac{y}{l},\frac{z}{l};\frac{x}{l}\right)+g\left(\frac{x}{l}\right)\right]$$
$$=\frac{b^2}{l}\left[\frac{1}{2\pi}\int_\sigma\left(\frac{\partial\varphi}{\partial m}-\varphi\frac{\partial}{\partial m}\right)\left(\log\frac{R}{r}+\log\frac{r}{l}\right)d\sigma+g\left(\frac{x}{l}\right)\right] \tag{6}$$

where σ is the contour bounding the cross-sectional area of the configuration and/or the trailing-vortex system in the y,z-plane, m is the unit outward normal,

$$\phi(y,z;x)=\frac{b^2}{l}\varphi\left(\frac{y}{l},\frac{z}{l};\frac{x}{l}\right),\quad G(x)=\frac{b^2}{l}g\left(\frac{x}{l}\right),\quad R=\sqrt{(y-\eta)^2+(z-\zeta)^2},$$

and $r=\sqrt{y^2+z^2}$ as shown in the following sketch:



Since r is independent of the surface normal, equation (6) can be written as

$$\Phi(x,y,z)=\frac{b^2}{l}\left[s'\left(\frac{x}{l}\right)\log\frac{r}{l}+\int_\sigma\left(\frac{\partial\varphi}{\partial m}-\varphi\frac{\partial}{\partial m}\right)\log\frac{R}{r}\,d\sigma+g\left(\frac{x}{l}\right)\right] \qquad (r\leqq\rho) \tag{7}$$

where use has been made of equation (4) and where $S(x)=b^2s\left(\frac{x}{l}\right)$. The variation of Φ with the azimuth angle θ is contained entirely in the line integral. Two of the basic assumptions used in the derivation of equation (7) are that both the perturbation velocities and the perturbation-velocity gradients in the stream direction are small. In order to satisfy these assumptions $s''\left(\frac{x}{l}\right)$ and $s'''\left(\frac{x}{l}\right)$ must be bounded. These conditions imply that equation (7) applies only to shapes that are smooth and free from discontinuities. Moreover, an additional restriction on the asymmetry of the shape is sometimes required (ref. 4); namely, the radius of curvature of the configuration in the crossflow plane must be of the order of b where the shape is convex outward.

For a body of revolution at zero incidence the contour integral in equation (7) vanishes and

$$\Phi_o(x,y,z)=\frac{b^2}{l}\left[s_o'\left(\frac{x}{l}\right)\log\frac{r}{l}+g_o\left(\frac{x}{l}\right)\right] \qquad (r\leqq\rho_o) \tag{8}$$

where the subscript o is used to denote values for a body of revolution. Since a body of revolution is completely defined in terms of the cross-sectional area distribution, this is the

only geometric parameter which enters into $g_o\left(\frac{x}{l}\right)$. Thus, $g_o\left(\frac{x}{l}\right)$ is of the form $g_o\left(\frac{x}{l};s_o\right)$ where the dependence upon the body shape is contained in s_o. Further consideration of the region of validity of the slender-body solution is necessary in order to show the corresponding dependence for $g\left(\frac{x}{l}\right)$.

Examination of equation (7) shows that the variation of the potential with the azimuth angle becomes vanishingly small for $r \geqq r_1$, since the logarithm in the contour integral is of the order b/r for $\frac{b}{r} \ll 1$. The magnitude of the terms neglected in equation (1) are now compared with those retained, in order to show that r_1 lies within the region where the slender-body solution is a valid approximation. The ratio of the neglected terms $[1-M^2-(\gamma+1)M^2\Phi_x]\Phi_{xx}$ to any of the remaining terms for $r>r_1$ is of the order

$$\left(\frac{r}{b}\right)^2\left(\frac{b}{l}\right)^2\left\{(1-M^2)\left[0\left(\log\frac{r}{l}\right)+0(1)\right]+\left(\frac{b}{l}\right)^2\left[0\left(\log^2\frac{r}{l}\right)+0\left(\log\frac{r}{l}\right)+0(1)\right]\right\}=\epsilon$$

where $0(\ )$ denotes order of, $0(1)$ denotes nonsingular terms, and the functions $g'\left(\frac{x}{l}\right)$ and $g''\left(\frac{x}{l}\right)$ are considered to be regular. From this ratio it can be seen that, for a given Mach number and degree of approximation ϵ, the region of validity of the slender-body solution ($r \leqq \rho$), measured in terms of body widths ρ/b, can be made as large as desired by suitably restricting b/l. Consequently, for $\frac{b}{l} \ll 1$, $r_1 < \rho$ and the flow field external to r_1 are nearly axisymmetric so that

$$\Phi(x,y,z)=\frac{b^2}{l}\left[s'\left(\frac{x}{l}\right)\log\frac{r}{l}+g\left(\frac{x}{l}\right)\right] \qquad (r_1<r<\rho) \qquad (9)$$

In addition, for a given degree of approximation, larger values of the width parameter b/l are permitted at transonic speeds than in the other speed ranges since the quantity $1-M^2$ is much larger at subsonic and supersonic speeds than at transonic speeds.

In the region $r>r_1$, the flow about a slender configuration is nearly axisymmetric and Φ must be identical to some Φ_o in this region. If Φ_o is the potential of the associated axisymmetric flow which gives rise to the same velocities as Φ for $r>r_1$, then, from equations (8) and (9), $s_o=s$ and $g_o=g$. Thus, g is determined as the function g_o. Since the only geometrical property affecting g_o is s_o, and since $s=s_o$, the only geometrical property influencing g is s. Thus, $g\left(\frac{x}{l}\right)$ is of the form $g\left(\frac{x}{l};s\right)$ where the dependence upon the shape of the configuration is contained entirely within $s\left(\frac{x}{l}\right)$.

In the preceding discussion the region of validity of the slender-body approximation to Φ_o was tacitly assumed to be at least as large as that for Φ. This condition is certainly true since the singular terms in the two solutions are the same.

A complete discussion of the validity of the slender-body approximation at transonic speeds would require the analytic expression for $g\left(\frac{x}{l}\right)$. In the absence of this information, such considerations are admittedly somewhat speculative. Even so, it is of interest to explore the nature of the approximation since some elementary considerations suggest that the slender-body solution will provide a reasonable approximation in regions where it might be expected to be poor—in the neighborhood of weak shock waves. Because of the nature of the slender-body solution, the flow is represented only in a small neighborhood of the configuration, and the shocks are represented as surfaces of discontinuity normal to the stream direction. Moreover, for slender configurations at transonic speeds, only near normal shock waves are to be expected.

In the slender-body approximation the term $[1-M^2-(\gamma+1)M^2\Phi_x]\Phi_{xx}$ is required to be small compared with any of the other terms in the transonic differential equation for all values of r less than ρ. If this condition is to be satisfied in the neighborhood of weak shock waves, the quantities

$$g''\left(\frac{x}{l}\right)\left[1-M^2-(\gamma+1)M^2\Phi_{o_x}\right] \qquad (10a)$$

and

$$g''\left(\frac{x}{l}\right)\frac{\partial}{\partial x}\int_\sigma\left(\frac{\partial\varphi}{\partial m}-\varphi\frac{\partial}{\partial m}\right)\log\frac{R}{r}\,d\sigma \qquad (10b)$$

must be bounded there. Since the disturbance velocities are bounded for shapes which satisfy the assumptions of slender-body theory, the quantities in expressions (10) will be bounded at shock waves if $g''\left(\frac{x}{l}\right)$ is bounded. The transonic differential equation admits of solutions having velocity discontinuities which are compatible with the transonic approximation to the shock-wave relations (see appendix). Since the development in the appendix does not require that ϕ_{xx} be singular, it seems reasonable to suppose that ϕ_{xx} and, hence, $g''\left(\frac{x}{l}\right)$ are bounded in the vicinity of shock waves. In addition, the coefficient of $g''\left(\frac{x}{l}\right)$ in expression (10a) has a mean value of 0 for the admissible normal shock waves and the contour integral in expression (10b) vanishes at values of x/l for which the configuration is axisymmetric.

The slender-body solutions in the various Mach number ranges are similar in that they are all represented by equation (7) although the function $g\left(\frac{x}{l}\right)$ differs for the various Mach number ranges. Ward (ref. 4) has determined the function $g\left(\frac{x}{l}\right)$ for supersonic flows and Adams and Sears (ref. 6)

have obtained a corresponding expression for subsonic flows. Although an analytic expression for this function at transonic speeds is not known, it has been established that the only geometric property of the body influencing $g\left(\frac{x}{l}\right)$ is the area distribution. Moreover, the transonic similarity rule for bodies of revolution (ref. 14 or 15) shows that $g\left(\frac{x}{l}\right)$ can be expressed in the form

$$g\left(\frac{x}{l}\right)=\frac{s'\left(\frac{x}{l}\right)}{4\pi}\log\left[(\gamma+1)M^2\left(\frac{b}{l}\right)^2\right]+f\left(\frac{x}{l};K\right)$$

where the similarity parameter is

$$K=\frac{1-M^2}{(\gamma+1)M^2\left(\frac{b}{l}\right)^2}$$

AERODYNAMIC FORCES

Since the slender-body solutions are all represented by equation (7), formal expressions for the aerodynamic forces can be determined which are valid throughout the Mach number range. Consequently, many of the essential features of slender-body theory at transonic speeds can be obtained without resorting to detailed calculations.

LIFT

The most significant difference between the slender-body solutions at subsonic, transonic, and supersonic speeds is that the function $g\left(\frac{x}{l}\right)$ differs in these various speed ranges. However, the term in the pressure arising from the function $g\left(\frac{x}{l}\right)$ makes only a uniform contribution to the pressure at any value of x and, therefore, cannot affect the lift distribution or the lift. Thus, within the slender-body approximation, the lift distribution depends only upon the function φ and, consequently, is independent of the stream Mach number. Several investigators (for example, Heaslet, Lomax, and Spreiter (ref. 9)) have previously noted that the linearized slender-body theory gave consistent results, even at a Mach number of 1, for planar systems.

According to slender-body theory, the lift distribution can be obtained completely from solutions of Laplace's equation in the crossflow plane. Since this equation is linear, the lift is proportional to the angle of attack even at transonic speeds. Ward has obtained an especially simple form for the drag due to lift in which

$$D_L=\frac{1}{2}\alpha L$$

where α is the angle of attack measured from zero lift and L is the lift.

DRAG

By computing the momentum change of the fluid passing through a cylinder enclosing the body, the drag D is determined as

$$\frac{D}{q}=\frac{D_b}{q}-\left(\frac{b^2}{l}\right)^2\left[2\int_0^1 s'(\xi)g'(\xi)d\xi+\int_{\sigma'}\varphi\frac{\partial\varphi}{\partial n}d\sigma'\right] \qquad (11)$$

where the body extends from $\xi=0$ to $\xi=1$, σ' denotes the contour of the body at the stern which in the case of wings or wing-body combinations includes the trailing-vortex sheet, q is the stream dynamic pressure, and D_b is the base drag. Equation (11) is valid throughout the Mach number range provided the appropriate forms of the function $g\left(\frac{x}{l}\right)$ are employed. The line integral is zero for nonlifting configurations if the body is closed or if the body ends in a cylindrical section whose elements are parallel to the stream. The effect of Mach number (excluding the variation of base drag with Mach number) is contained in the term involving $g\left(\frac{x}{l}\right)$.

When the subsonic form of $g\left(\frac{x}{l}\right)$ is used in equation (11), the correct result is obtained that the drag of nonlifting configurations is zero. By using the supersonic form of $g\left(\frac{x}{l}\right)$, the drag varies with Mach number as $[s'(1)]^2\log(M^2-1)$. For pointed bodies, or for bodies which end in a cylindrical section, the supersonic slender-body theory indicates that the drag is independent of Mach number. For bodies which do not satisfy these conditions, the supersonic result indicates that the drag approaches infinity as the Mach number approaches 1. These results from linear theory cannot be considered satisfactory at transonic speeds since they give a discontinuity in the drag as the Mach number is increased through 1; whereas experimental data show that the drag starts to increase rapidly at a subsonic Mach number and varies smoothly through 1. However, the few known solutions of the nonlinear transonic-flow equation are in good agreement with experiment in this regard. It would be expected, therefore, that the drag rise of slender shapes would be correctly approximated by equation (11) once the transonic form of $g\left(\frac{x}{l}\right)$ is known.

TRANSONIC AREA RULE

The body shape enters into the function $g\left(\frac{x}{l}\right)$ only as a function of the cross-sectional area distribution throughout the Mach number range. This property of the slender-body solutions leads to an important result even though the analytic expression for $g\left(\frac{x}{l}\right)$ is not known at transonic speeds. Examination of equation (11) shows that the body cross-sectional shape enters into the slender-body drag expression only through the contour integral evaluated at the stern of the configuration. For a fixed base contour, then, the drag of nonlifting configurations depends only on the axial distribution of the body cross-sectional area and is

independent of the cross-sectional shape. Thus, within the slender-body approximation, the drag of a nonlifting configuration is the same as that of the associated body of revolution having the same streamwise distribution of cross-sectional area provided the base contour is fixed. It is in this sense that an equivalent body of revolution is associated with a wing-body combination. This result, often referred to as the area rule, is especially significant at transonic speeds where larger values of the width parameter b/l are permitted than in other speed ranges.

The property of the dependence of the drag upon the distribution of cross-sectional area has previously been obtained by Ward (ref. 4) and Graham (ref. 16) for supersonic flow and has been observed experimentally by Whitcomb (ref. 17, for example) at transonic speeds. The importance of this result was first noted by Whitcomb who demonstrated that the area rule could be used as a basis for the design of low-drag wing-body combinations at transonic speeds. From the preceding development, the transonic area rule is subject to the restrictions of slender-body theory with the additional condition that the base contour be fixed.

LANGLEY AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *January 18, 1954.*

APPENDIX

ON SOLUTIONS OF THE TRANSONIC DIFFERENTIAL EQUATION HAVING VELOCITY DISCONTINUITIES

The transonic differential equation for the disturbance velocity potential (eq. (1)) can be written as

$$\frac{-1}{2(\gamma+1)M^2}\frac{\partial}{\partial x}[1-M^2-(\gamma+1)M^2\Phi_x]^2+\Phi_{yy}+\Phi_{zz}=0 \quad \text{(A1)}$$

From the conservation laws, the tangential velocities across a shock wave are continuous and the normal velocity is discontinuous. Consider first the possibility that Φ_x is discontinuous across a surface normal to the x-coordinate. In order for the differential equation to admit such solutions, the values of Φ_x on each side of the discontinuity must give rise to the same value for the first term in equation (A1). With the subscripts 1 and 2 denoting quantities immediately upstream and downstream, respectively, of the surface of discontinuity, this condition is satisfied by

$$[1-M^2-(\gamma+1)M^2\Phi_{1_x}]=-[1-M^2-(\gamma+1)M^2\Phi_{2_x}]$$

or

$$1-M^2-\frac{(\gamma+1)M^2}{2}(\Phi_{1_x}-\Phi_{2_x})=0$$

which is the first-order approximation to the normal-shock relations.

By considering discontinuities in all three velocity components (i. e., oblique shock waves), the resulting expression relating the disturbance velocities on each side of the discontinuity is identical to the first-order approximation for the entire shock polar. Thus, the transonic differential equation admits of solutions having velocity discontinuities which are consistent with the first-order approximation to the entire shock polar. Stated another way, the transonic approximation to the differential equation and shock relations are consistent.

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Copies obtainable from NACA, Washington

1. Flow, Subsonic (1.1.2.1)
2. Flow, Mixed (1.1.2.2)
3. Flow, Supersonic (1.1.2.3)
4. Wing-Fuselage Combinations - Airplanes (1.7.1.1.1)

I. Harder, Keith C.
II. Klunker, E. Bernard
III. NACA Rept. 1315
IV. NACA TN 3815

NACA Rept. 1315
National Advisory Committee for Aeronautics.
ON SLENDER-BODY THEORY AT TRANSONIC SPEEDS. Keith C. Harder and E. B. Klunker. 1957. ii, 6p. (NACA Rept. 1315. Supersedes TN 3815)

The basic ideas of the slender-body approximation have been applied to the nonlinear transonic-flow equation for the velocity potential in order to obtain some of the essential features of slender-body theory at transonic speeds. The results of the investigation are presented from a unified point of view which demonstrates the similarity of slender-body solutions in the various Mach number ranges. The transonic area rule and some conditions concerning its validity follow from the analysis.

Copies obtainable from NACA, Washington

1. Flow, Subsonic (1.1.2.1)
2. Flow, Mixed (1.1.2.2)
3. Flow, Supersonic (1.1.2.3)
4. Wing-Fuselage Combinations - Airplanes (1.7.1.1.1)

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