
REPORT No. 12.

**EXPERIMENTAL RESEARCHES ON THE
RESISTANCE OF AIR.**

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CHAPTER I.

CLASSIFICATION OF EXPERIMENTAL METHODS.

1. REACTIONS EXERTED BY THE AIR ON A BODY IN RELATIVE MOVEMENT WITH IT.

When a body is in movement relative to the air with which it is surrounded, it is subject to a system of forces to which is given the name of "reactions exerted on the body by the air." These reactions are variable, especially as regards (1) the form of the body, (2) the position which it occupies in relation to the surrounding medium, (3) the various circumstances of its movement (time elapsed from origin of movement to present moment—velocity relative to the air), and, finally, (4) the mass of the fluid which surrounds the body in movement.

We shall not develop in detail the difficulties presented by each of these problems, of which certain have received only very imperfect solutions.

We shall, in what follows, consider only the case of a body surrounded completely by a great mass of air, relative to which it has a movement, established a long time previously, and of which the velocity and direction are constant and readily determined.

The reactions exerted by the air on the body in movement relative to it are reduced to a force and a couple. We shall assume that the body under experiment possesses, at the least, a plane of symmetry, thus eliminating the couple from the reactions of the air and reducing them to a single force, to which we shall give the name of "resistance of the air on the body in movement relative to it."

When we consider the movement of the body relative to the air which surrounds it, we have not only in view a movement of translation, but also a movement of simple rotation and likewise a movement of rotation combined with a movement of translation. In other words, we shall study here the problem of the propeller as well as that of the wings of an airplane.

2. MANNER OF PRODUCING THE MOVEMENT OF A BODY RELATIVE TO THE AIR WHICH SURROUNDS IT—BODY MOVABLE.

Various experimental methods may be utilized in order to produce the movement of a body in reference to free air.

In an indefinite mass of air, at rest as a whole, the following types of movement may be given to the body:

- (a) A movement of rectilinear translation;
- (b) A movement of rotation about the axis of a mechanism;
- (c) An oscillating movement, as in the case of a pendulum.

The methods by means of some form of mechanism or by means of a pendulum have been but little employed in France and we shall omit special reference to them.

The method employing the motion of translation may be applied in two forms:

- (1) The body is allowed to fall freely in air, as calm as possible.
- (2) The body is carried on some form of car which is moved in calm air.

In France the method of free fall has given rise to important investigations made by MM. Cailletet and Colardeau and especially by M. G. Eiffel.

The method by means of a car is now utilized by the Aerodynamic Institute of Saint-Cyr, at the laboratory of military aerostation of Chalais-Meudon, and also by M. the Duke of Guiche. At Saint-Cyr and at Chalais-Meudon, the car is composed of a carriage moving on rails. M. de Guiche employs an automobile as a carrier.

A variant of the method of the car has been installed at the laboratory of military aviation at Vincennes. On a stretched cable a little hanging car rolls, carrying, attached below it, the objects under test with the necessary instruments.

The dimensions of the bodies on which the experiments are carried out may be of the order of those which are utilized in aviation itself. In other words, it is possible to operate upon equipment as used in actual aviation, or at least presenting dimensions differing but little from those used in practice.

From this point of view the method by displacement through the air opens up a field of investigation more extended than the method in which an artificial current of air is employed.

3. MANNER OF PRODUCING THE MOVEMENT OF A BODY RELATIVE TO THE AIR WHICH SURROUNDS IT—ARTIFICIAL CURRENT OF AIR.

It is possible, in fact, to realize in an entirely different manner the relative movement of a body through the air.

Instead of moving the body under test, a fixed position is given to such body placed in an artificial current of air.

The body may then be disposed in the free air in front of the orifice through which the air enters under regulation by means of suitable devices. This method has been employed by M. Rateau.

The body under investigation may also be placed in an inclosure or integral part of the apparatus for the regulation of the current of air. It is placed, for example, in a part of a large cylindrical pipe which receives a current of air produced by a fan and of which the velocity, at a certain distance from the walls, has been rendered sensibly parallel to the pipe.

This method, furthermore, may be subject to certain variations:

(a) The body under investigation alone is placed in the inclosure in the interior of which the artificial current of air is produced. The apparatus for measuring the reactions of the air are on the exterior of this inclosure, their connection with the interior being made through the solid wall which limits the conduit.

This method is known under the name of the "tunnel method." It has not been largely employed in France. There exists at the present time at the Aerodynamic Institute of Saint-Cyr a tunnel of which the practical use has been interrupted by the present war.

(b) The apparatus employed for determining the circulation of the air is enlarged into a chamber of suitable size, traversed between two of its parallel walls by a cylinder of moving air. On the outside of the latter and within the chamber are located the experimenters with the necessary measuring apparatus.

We propose to call this the "Eiffel method."

In France this method has given very complete results. It is for us the characteristic method in connection with the use of an artificial current of air.

From the point of view of the convenience of carrying on the experiments, especially in large numbers, the last method is superior to the method by displacement in free air. The latter demands, in fact, that the external air shall be as calm as possible. This condition can only be realized on certain days and then only for certain hours of a given day. If along the right-line path of the body under investigation the wind should have everywhere the same intensity and the same direction, due allowance might be made for its existence.

But many investigations, notably those of M. Maurain at the Aerotechnic Institute of Saint-Cyr, show that at any given point in the air the wind is frequently subject to continued changes in direction and intensity.

But even if it allows the experimenter to regulate the conditions of any one investigation, the method by the use of the artificial current of air can only be applied to models reduced in size in comparison with actual practice in aviation. We shall see later the reason for this limitation.

One question immediately presents itself: How may the results obtained in the study of models be transformed in order to furnish information applicable to apparatus of full size? What is the law of similitude which makes possible the transformation of an investigation on a small scale to corresponding phenomena on a large scale. This is the matter which we shall especially develop at a later point.

A further question presents itself: Do the methods mentioned above, namely, the displacement of the body under investigation and the method by the artificial current of air, lead to the same results? M. Eiffel maintains the affirmative, relying upon the fundamental principle of relative movement. M. de Guiche maintains the negative, arguing that the tunnel method does not realize fully the conditions which permit the application of such a principle.

We shall return to this question at a later point, in connection with the comparison of the results obtained by these two experimenters.

4. STUDIES OF AIRPLANES IN FREE FLIGHT.

The methods which we have just considered require that the body under investigation be connected in a fixed manner with a support. The latter has, under good conditions, its dimensions reduced as much as possible. It is also removed as far as possible from the body under investigation, so that its presence will produce the minimum of disturbance. It is none the less true, however, that the airplane, thus studied, is not in the precise condition of free evolution in the open air.

For this reason investigations have been undertaken on airplanes during their free flight in the air. Unfortunately, the field of such investigation is limited. It can not be carried through at the will of the experimenter; that is to say, of the pilot, who must first of all guard against danger of fall. Such experiments give complex results often difficult of analysis. Nevertheless it can not be denied that such results may have a very considerable practical value.

Experiments of this character were inaugurated in 1910 by MM. Gaudart and Legrand with a Voisin biplane. These experiments were, however, neither sufficiently systematic nor numerous to lead to significant results.

Quite otherwise are the researches made by Commander Dorand, at Villacoublay, on a biplane of his own construction piloted by M. Labouchère. At the Institute of Saint-Cyr, MM. Toussaint and Lepère, Toussaint and the Lieutenant of Aviation Gouin, have made important experiments on a Maurice Farman biplane and on a Blériot monoplane. Ingenious apparatus capable of registering the movement of the pilot was employed to furnish important indications regarding the operation of such actual aviation equipment.

5. THE TOTAL RESISTANCE OF THE AIR AND THE DETERMINATION OF THE PRESSURES AT EACH POINT OF THE SURFACE OF THE BODY UNDER INVESTIGATION.

Let us return to the methods which, in a laboratory, may be employed in determining the resistance of the air upon a body in movement relative to it.

With regard to the method of measuring this resistance two types may be characterized:

(1) Determination, by means of a balance, of the total resistance on the entire body under investigation.

(2) Determination, at each point of the body, of the reaction exerted by the air at this point; a study, in some manner topographical in character, regarding the pressures resulting from the relative movement of the body and the air.

This investigation immediately leads, through a geometrical composition of the individual forces thus determined, to a knowledge of the complete resistance of the air.

The method by means of the balance has given wonderful results in the laboratory of M. Eiffel and at the Institute of Saint-Cyr. M. de Guiche has applied this method solely to the analysis of the distributed pressures.

Such is the general classification of the experimental methods at present in use in France for the study of the problems of aerodynamics. We proceed to give in detail the fundamental principles of these investigations in a further study of the French aerodynamic laboratories.

CHAPTER II.

THE AERODYNAMIC LABORATORIES OF FRANCE.

1. THE EIFFEL LABORATORIES—EXPERIMENTS MADE AT THE EIFFEL TOWER.

The Eiffel Tower was the first laboratory utilized by the celebrated engineer in his researches in aerodynamics, carried on during the past 10 years. Bodies thrown from one of the platforms of the tower have permitted a study of free fall in calm air.

The study of this movement admits, furthermore, of being made by two different methods.

The first of these methods consists in determining the velocity of uniform movement which succeeds the varying movement. To this velocity corresponds a resistance of the air equal to the weight P of the body. By augmenting the weight of the body without changing the surface, as by the addition of suitable ballast, it is possible to increase, at the same time, the limiting uniform velocity V of the movement. The comparison of the different values of P with the corresponding values of V provides a means for developing a law of variation of resistance as a function of velocity. Such is the principle of the method applied in 1892 by Cailletet and Colardeau from the second story of the Eiffel Tower (120 meters = 394 feet above the ground).

Instead of limiting himself to the study of that part of the free fall that corresponds to uniform movement, M. Eiffel registers the values of the velocity and of the resistance of the air at each instant of the fall. The principle involved in this investigation is the following:

The surface under investigation, a plane for example, falls freely, remaining horizontal. It is supported by a spring, of which the displacements are inscribed on a cylinder revolving with a velocity directly proportional to that of the fall of the system under investigation. The compression of the spring, as a result of the resistance of the air, gives rise to a force which, by a suitable calibration, may be determined as a function of the displacements of this spring. This force produces equilibrium with the following system of forces

- (1) The weight of the system.
- (2) The forces of inertia which act upon it.
- (3) The resistance of the air.

It is then possible to calculate this last force when the acceleration of the system is known. To this end it is sufficient to inscribe, by means of a tuning fork, the time of fall on the same cylinder whereon are recorded the compressions of the spring.

In a certain experiment, when the combined weight of the plate with its spring and support was 4.494 kilograms (9.887 pounds), the

following determinations were made, in one case at the end of 60 meters of fall (196.8 feet) and in the other case at the end of 95 meters (311.6 feet):

| | |
|----------------------------|-----------|
| At the end of 60 meters: | |
| Force of inertia..... | 3.76 kg. |
| (Absolute value.) | |
| Tension of spring..... | 4.15 kg. |
| Resistance of the air..... | 4.90 kg. |
| | <hr/> |
| Difference..... | =0.75 kg. |
| At the end of 95 meters: | |
| Force of inertia..... | 3.36 kg. |
| (Absolute value.) | |
| Tension of spring..... | 6.15 kg. |
| Resistance of the air..... | 7.30 kg. |
| | <hr/> |
| Difference..... | =1.15 kg. |

These numbers show that under the existing conditions (total weight of plates, of spring, and of support rather high) the difference between the tension of the spring and the resistance of the air is clearly measurable.

By this method, M. Eiffel has studied the resistance of the air on planes of which the surfaces varied from 1/16 square meter (0.67 square foot) to 1 square meter (10.77 square feet) and of which the velocities of fall ranged between 18 and 40 meters per second (59 to 131.2 feet). These high velocities have made it possible to operate in the open air with high precision in calm weather and as long as wind velocities did not exceed 2 to 3 meters per second (6.56 to 9.84 feet per second). The results obtained by these experiments are excellent for planes falling horizontally. They are less worthy of confidence for planes inclined to the vertical.

2. THE EIFFEL LABORATORY—METHOD BY THE USE OF AN ARTIFICIAL CURRENT OF AIR.

This method, which consists in placing a model in the cylinder of air flow created by a fan, should be applied with the following precautions:

(1) It is necessary that the model should be placed in a mass of air theoretically indefinite, practically very great, and having a velocity constant in magnitude and in direction.

The section of the cylinder of air should be sufficiently large, in order that at the periphery the velocity of the air may be sensibly the same in magnitude and in direction as that of the air which has not yet approached the obstacle. In this method it is necessary to realize, first of all, a cylindrical current of air, and then to introduce into this current a body of dimensions so small by comparison that its presence shall not produce any sensible disturbances at the periphery of the current. Experience has shown that the ratio of the greatest dimension of the model to the diameter of the cylindrical current should not exceed 45 per cent.

(2) It is very necessary that the model under investigation shall be practically isolated in the current of air; that is to say, that the support of the model shall play only a negligible rôle and shall introduce no perturbations of importance.

(3) It is necessary that the model adopted shall not be too small in size if it is desired to extend in a more or less significant manner the results obtained with such model to full-sized apparatus.

In fact, when a study is made of the distribution of pressure over the various points of a plate, for example, either on the face directly exposed to the action of the current of air, or on the reverse face, it is found that this distribution becomes regular only at a certain distance from the border. There exists, both in front and behind, a central zone in which a regular regimen is established, which is manifest by isobars parallel to the forward edge. In order that this central zone may be studied, it is necessary that the dimensions of the plate under investigation be sufficiently large. In fact, the width of the marginal zone in which the pressures are irregularly distributed does not vary proportionally with the dimensions of the plate. The experiments of M. de Guiche show that this width varies but little with the dimensions of the plate. In operating on thin rectangular planes with the attacking edge perpendicular to the direction of movement, M. de Guiche has found that the marginal bands of irregular condition have a sensibly constant width, equal to 20 centimeters (7.88 inches) in front and to 40 or 50 centimeters (15.76 to 19.7 inches) at the rear. He concludes that it is well not to operate with planes having a spread less than 1 meter (3.28 feet). In the study of curved surfaces, M. de Guiche found marginal bands of disturbance of which the sensibly uniform width scarcely exceeded 20 centimeters (7.88 inches) on the two faces. It is well, therefore, to use only surfaces whose spreads are superior to 40 centimeters (15.76 inches). When lesser spreads are employed the results obtained by the use of small models do not permit of deducing, in a sufficiently precise manner, the results which would be given by the wings of an airplane of normal size. In the case of very small models the mode of distribution of pressure has but a very remote relation to that which would be found on wings of normal dimensions.

This condition of only using, for experimental purposes, models of sufficient dimensions leads, in the method by the use of an artificial current of air, to the employment of very large sections for the cylinder of air. For example, a study of planes having a spread of 1 meter (3.28 feet) can only be made in a cylinder of air of which the diameter is greater than $100/.45 = 220$ centimeters (86.7 inches), approximately. As to curved surfaces, it would be sufficient to provide a cylinder of a diameter greater than $40/.45 = 89$ centimeters (35 inches), approximately.

In France, M. Rateau has utilized the method by the use of an artificial current of air. His apparatus comprises the following items:

- (1) A helicoidal fan, 1.2 meters diameter (47.3 inches).
- (2) A wooden chamber, 1.5 meters on the side (59.1 inches). The purpose of this chamber is to suppress, by means of suitable partitions, the turbulence produced by the fan and to create a current of air with the velocities of movement equal and parallel throughout.
- (3) An outlet orifice of 0.7 meter (27.6 inches) diameter from whence issues a cylindrical current of the same diameter.
- (4) A weighing balance located in the outside air at a little distance from the orifice through which the current of air issues.

Such an apparatus would permit of realizing velocities of the air reaching 35 meters (114.8 feet) per second. The diameter of the cylinder of air, however, is too small. M. Rateau was not justified in introducing into this current plates 30 by 50 centimeters (11.8 by 19.7 inches) or biplanes 15 by 50 centimeters (5.9 by 19.7 inches) (separation of planes 20 to 30 centimeters=7.9 to 11.8 inches). The supports of the objects under investigation and of the measuring equipment were too large, causing very considerable perturbations. The experiments of M. Rateau should be noted as of historic interest, but they can be scarcely considered as having a definitive value.

Much more complete and certain are the results obtained by the installation of M. Eiffel. The Eiffel apparatus comprises the following items:

- (1) An orifice from which issues a cylindrical current of air.
- (2) An experimental chamber where the air is at a pressure less than normal and where are located the experimenters and the measuring apparatus.
- (3) A diffuser.
- (4) A fan placed at the end of the diffuser.

The current of air which enters through the orifice in the wall of the experimental chamber and which leaves through the opening of the diffuser placed in the wall opposite and parallel to the first, is a current of air produced by aspiration and not by pressure, as in the installation of M. Rateau. The aspiration removes the influence due to the turbulence produced by the fan, and a regulating box for the current of air in front of the orifice is not necessary. However, certain grillages placed in the openings for entrance to and issue from the experimental chamber play the rôle of regulators for the current of air.

In the laboratory of the Champ de Mars the fan was placed near the opening through which the air leaves the chamber. A large conduit of wood received the air issuing from the fan and, gradually reducing in size, conducted it through a passage ending in the shed, whence the air was drawn into the orifice through which it entered the experimental chamber.

The cylinder of air had a diameter of 1.5 meters (59.1 inches).

The maximum velocity of this air was equal to 18 meters (59 feet) per second, or 65 kilometers (40.4 miles) per hour. The fan employed, of the centrifugal type, delivered 31 meters³ (1,095 cubic feet) per second, corresponding to a velocity of 18 meters per second and to the circular section of 1.5 meters diameter, requiring 60 horsepower.

The laboratory installed at Auteuil by M. Eiffel is much more powerful. The cylinder of air of the large equipment has a diameter of 2 meters (6.56 feet). Velocities of the air may be realized from 2 to 30 meters (6.56 to 98.4 feet) per second. A second smaller cylinder of air, having a diameter of 1 meter (3.28 feet), parallel to the first, provides velocities from 2 to 40 meters (6.56 to 131.2 feet) per second.

The 60 horsepower is, however, not exceeded in this new installation, in which the delivery of 90 cubic meters of air (3,179 cubic feet) per second may be realized, corresponding to the circular section of 2 meters diameter and a velocity of 100 kilometers (65.14 miles) per hour.

This increase in efficiency is realized by interposing between the experiment chamber and the fan a divergent orifice forming a diffuser. This diffuser has a diameter equal to 2 meters at the outlet from the chamber; it connects with the ring of a helicoidal fan of 3.8 meters diameter (12.46 feet), providing for the flow of the air a useful section of 9 square meters (96.9 square feet). The reduction of velocity which is produced by passing through the diffuser, as a result of the progressive increase in diameter, raises the pressure of the air by a certain quantity and diminishes correspondingly the power which must be furnished to the fan in order to bring the air to atmospheric pressure.

The measure of the total resistance of the air is made in the experiment room by means of a balance, into the detail of which we can not here enter.

The pressures exerted on each point of the body subjected to the action of the current of air are determined by means of orifices of very small diameter fixed normally to the surface at various points of the body and connected with manometers.

This installation likewise provides for determining, with reduced models of screw propellers, the thrust of the propeller and the power required on the shaft. By means of a small electric motor the propeller is turned in the cylinder of air produced by the fan. It is assumed that by this means the same conditions are realized as though the propeller itself advanced through the air.

The propeller models have a diameter which does not exceed 1 meter (3.28 feet). In a cylinder of air of 2 meters (6.56 feet) diameter the propeller model is thus surrounded by a mass of air sufficiently thick to represent action in an indefinite medium.

3. THE AEROTECHNIC INSTITUTE OF SAINT-CYR.

The Aerotechnic Institute of Saint-Cyr was created by M. Henri Deutsch de la Meurthe, who gave it as a gift to the University of Paris. Its purpose is to follow lines of research, both theoretical and practical, tending to the improvement of the means of aerial locomotion in all its forms, these researches being carried out under conditions as nearly as possible similar to those actually in practice.

In order to realize this program, there was constructed on the level a railway 1,350 meters in length (4,428 feet), on which are operated special cars.

The car for the tests of surfaces and of airplanes is an electric tractor with normal gauge.

The principal characteristics are as follows:

Weight, not equipped, 5 tons.
 Length, 6 meters (19.7 feet).
 Width, 2 meters (6.56 feet).
 Height, 1.10 meters (3.61 feet).

The current is taken by lateral shoes sliding on conductors placed on either side of the line.

The motor is of 130 horsepower capacity, separately excited. It is geared to two intermediate shafts, upon which are placed pinions which transmit the movement to the forward and rear axles by Renold chains. All the shafts of the motor and axles are provided with roller bearings.

The brake is applied electrically, with a safety provision, by means of shoes at the rear, engaging at the end of the line on a sliding way.

The control is exercised from an operating cab by means of a controller and an adjustable automatic accelerator.

The maximum velocity of the car is about 20 meters (65.6 feet) per second.

The car is furnished with a special mounting providing for registering the following items:

(1) The vertical component force or lift.
 (2) The horizontal component force or drift. (These together define the resistance of the air in magnitude.)

(3) A rotating couple, from which, with the preceding, may be derived the location of the resistance considered as a single force.

The relative velocity of the body under trial with relation to the air is determined by measuring the absolute velocity of the car with reference to the ground and adding or subtracting the velocity of the wind according to the direction of the line on which the car runs.

The absolute velocity of the car with reference to the ground is measured by means of a registering speed instrument giving directly the revolutions of an axle. Furthermore, there is installed along the line a system of electric contacts inscribing points of reference on the cylinder of a precision chronograph. This cylinder permits a measurement (to 1/200 second or to 1/10 second) of the time required by the car for traversing a known distance—95.9 meters exactly (314.55 feet).

In order to have the correction due to the wind, measurement is made, at a fixed point on the line where the tests are made, of the magnitude and direction of the wind. The velocity of the wind is measured by an anemocinematographe, which is extremely sensitive (20 millimeters (0.788 inch) for 1 meter (3.28 feet) per second velocity). The direction of the wind is measured by a self-registering wind vane. Furthermore, the correction for the wind can not be made with satisfactory rigor unless the average velocity of the wind is at least equal to 2 meters (6.56 feet) per second.

A car of the same type as the preceding, but more robust, serves for the study of an entire airplane. These experiments, still in an introductory stage, have been carried out on a two-passenger machine of the Blériot type. The characteristics of this monoplane are as follows:

Tail plane (pigeon tail).
 Span, 11.10 meters (36 feet).
 Length, 9 meters (29.5 feet).
 Surface of the wings, 25.35 square meters (272.8 square feet).
 Angle of the chord of the wings with the tail plane, 6°.

The ensemble of this apparatus was studied for three positions of the depth rudder:

- (a) Rudder in the prolongation of the tail plane.
- (b) Rudder turned 18° downward with reference to the tail plane.
- (c) Rudder turned 51° upward with reference to the tail plane.

Experimental speeds 15 to 18 meters (49.2 to 59 feet) per second (54 to 65 kilometers (33.5 to 40.4 miles) per hour).

A special car serves for the study of propellers.

The propellers are full sized. They are mounted on the special car which they serve to propel. This car carries a framework which

makes it possible to carry the propeller at a distance from the car itself and to operate the propeller as though it were placed in an indefinite medium. On the car is an 80-horsepower motor which operates the propeller shaft by means of a transmission similar to that which is found on dirigible balloons.

The axles of the car are free and the speed of the car is due solely to the pull of the propeller. A lever brake is provided which is applied by the operator who rides on the car. This brake is, furthermore, similar to that which is used on the car for testing surfaces. The control of the car is carried out from an operating cab similar to that for the car previously described.

The return of the propeller car is obtained by running the propeller backward at reduced speed in order to avoid undue strain. The pull of the propeller is measured by means of a dynamometer inserted between a movable bar articulated at its base and the fixed framework of the car.

The power absorbed by the propeller is measured in two different ways:—

(a) By means of a wattmeter registering the electric power required by the motor.

(b) By means of a transmission dynamometer registering the couple required to drive the propeller.

These two pieces of apparatus are standardized by means of a Renard brake, which is attached to the shaft in place of the propeller.

The velocity of the car is determined by the same two methods as above noted for investigating surfaces.

The number of revolutions of the propeller is measured by means of a registering counter.

The relative velocity is obtained by making a correction for the wind, the same as in the case for surfaces.

The velocities realized have not exceeded 20 meters (65.6 feet) per second.

The Aerotechnic Institute has undertaken to make a series of tests on airplanes in free flight.

M. Toussaint, assistant director of the institute, has carried out the following investigations:

First, on a Maurice Farman biplane, piloted by Capt. Etévé.

Second, on a Blériot monoplane, piloted by Lieut. Gouin.

The following items of equipment were installed:

(1) A recorder of relative velocity giving the speed of the airplane relative to the air.

(2) A registering wind vane giving the inclination of the wind to the chord of the planes.

(3) A registering clinometer giving the longitudinal inclination of the airplane relative to the horizontal.

(4) A registering barograph, very sensitive up to 500 meters.

(5) A registering revolution counter giving the speed of the engine.

(6) A register of the movement of the depth rudder.

(7) A register of the warping of the planes.

All these registering instruments, except the revolution register, are provided with cylinders of such proportions as to give a length of diagram of 292 millimeters (11.5 inches) in 26 minutes—that is, 11.2 millimeters (0.442 inch) per minute. With this velocity of movement of the record one may judge the periods of steady condi-

tions exceeding 10 to 20 seconds duration. Quantitative measures can not be properly drawn from periods of steady condition having less than this duration.

In order to assure a perfect agreement in time between the different diagrams, or, in other words, in order to be perfectly sure that the points taken as corresponding do indeed relate exactly to the same instant of flight, each register is provided with a supplementary pen moved by an electromagnet. Throughout the course of the flight the pilot by closing the circuit at sufficiently close intervals causes these pens to register simultaneously on all the diagrams.

The reference points thus traced are all in synchronism, and thus provide an assurance of perfect accord for measurements made in the vicinity of these points.

In order to protect the instruments from vibrations and shocks, they are provided with elastic suspension in their cases.

We shall not enter into the details of the description of these various instruments. We note simply that the register for velocity relative to the air, as well as the direction vane, should be placed in such manner as to avoid the turbulence produced by the propeller, the planes, and the body.

The register for the movements of the depth rudder and for the warping of the planes was installed on the Blériot. This gives on one cylinder the movements of the handwheel barrel. It comprises simply a system of axes with tracing points, parallel between themselves and perpendicular to the axis of the registering cylinder. Each one of these axes carries, on the one hand, a lever connected to the barrel by a wire, and, on the other hand, a stylus or marking pen. A return spring fixed on the lever serves constantly to maintain the wire under tension. This wire is attached to the barrel at the same point as the cable which operates the corresponding control; it passes over little pulleys and is thus carried to the register. The movements of the stylus for the position of the barrel corresponding to horizontal flight and to various inclinations of the depth rudder are marked on the register, the axis of the propeller in repose being horizontal. Similarly reference marks are determined in repose for the movements of the stylus registering the warping movements of the planes. In the apparatus of the institute the needle rises when the barrel is carried to the left; it descends when it is carried to the right.

The Aerotechnic Institute possesses also an installation for the study of small models by means of a fan. The fan absorbs 120 horsepower, providing for a flow in an experimental section of 2 meters (6.56 feet) diameter of a current of air of 40 meters (131.2 feet) per second or 144 kilometers (89.4 miles) per hour. The total delivery amounts to 125 cubic meters (4,412 cubic feet) per second. This apparatus is under trial.

There is also at the Institute of Saint-Cyr a covered installation for the study of small models. The diameter of the rotunda is equal to 38 meters (125 feet). The turning arm is about 16 meters (52.5 feet) in length. The velocity at the extremity is 90 to 95 kilometers (56 to 59 miles) per hour.

4. THE LABORATORY OF M. DE GUICHE.

M. de Guiche has instituted a systematic series of experiments by the method of displacing the body under investigation in quiet air.

The body (use so far has been made of plates in various forms) is carried by an automobile. Two vertical pillars fixed to the machine carry at their upper extremities a horizontal axis on which are placed the various plates under investigation. These plates are provided at their lateral extremities with two graduated half circles, which, by means of a needle carried by each pillar, serve to measure their inclination relative to the line of movement. The pillars are of sufficient height and so placed that the turbulence produced by the automobile itself is not felt in their neighborhood. In particular, M. de Guiche has found it advantageous to carry the mounting on the rear of the automobile rather than on the front. The wheels and the body throughout are, furthermore, suitably shielded in order to diminish turbulence.

M. de Guiche thus carries out a sort of topographical measure of the pressures exerted on the different points of the plates both on the front and rear sides.

As we have seen previously, M. Eiffel likewise employs this method, but he determines separately the pressures at the different holes and measures each time the velocity of the current of air by means of a Pitot tube. As this velocity is somewhat variable, the pressures are reduced to what they would be if the velocity were constantly equal to 10 meters (3.28 feet) per second.

But in operating this by successive observations there is danger of finding between the determinations marked discordance.

Therefore, M. de Guiche determines at the same time, and by a single experiment, the pressures at selected points in as large a number as possible. To this end the plates which are to serve for the experiment are pierced, each with holes forming two series of lines cutting each other at right angles (for planes, lines at the maximum slope and horizontal lines). In a single experiment, and at the same instant, the pressures in all these points of the several lines are measured.

To this end, 20 orifices ending at these holes on the various lines are provided with 20 rubber tubes connecting them with 20 little manometers. These manometers are mounted side by side in a frame before a glass provided with transverse divisions in half millimeters.

The indications of all these manometers are inscribed at any given instant by placing them in a photographic chamber where the atmospheric pressure acts upon their open ends. By lighting the interior of the chamber with electric lamps, there is readily produced on a photographic plate the levels of the liquid in the different manometers.

It may be asked if the pressure of the atmosphere actually prevails in the interior of the photographic chamber. The latter can not, in effect, be perfectly tight, if it is not desired that it should operate as an air thermometer. It is then possible that during the movement of the automobile, currents of air passing through small apertures of the photographic chamber may tend to produce a variation of pressure within. M. de Guiche has assured himself by many experi-

ments that this effect is entirely absent and that atmospheric pressure prevails throughout the interior of the photographic chamber.

At the Institute of Saint-Cyr, M. Maurain, who has employed the manometric method, undertook to establish in spite of the perturbations caused by the movement of the vehicle, the atmospheric pressure on the open part of the manometers which he uses for measuring pressure. To this end the open part is connected to a space, completely tight, which communicates with the outside air by means of a tube placed at the extremity of an antenna. The latter is placed quite far from the surface in order to escape variations of pressure which are produced in its vicinity (2.3 meters, 7.54 feet) in front of the surface under investigation and 3.35 meters (10.9 feet) above the body of the car. At the extremity of this antenna is a tube of which the useful part is horizontal and cylindrical and is terminated by a pointed closed cone pointing in the direction of movement. In the cylindrical and lateral part of the tube are formed small circular openings of 1 millimeter (0.04 inch) in diameter which place it in communication with the atmosphere. M. Maurain has assured himself that this tube placed in a current of air of 20 meters (65.6 feet) per second approximately parallel to the current assumes indeed the pressure of the atmosphere.

The pressures thus determined directly are reduced to the values which they would have at a velocity of 10 meters (32.8 feet) per second. Curves of equal pressure may then be traced, thus indicating the condition of the surface of the body and showing the distribution of pressure over the same.

Furthermore, the surface of the body is divided into strips of a certain width. By a calculation of averages, the mean pressure is determined for each strip. Multiplying this by the surface of each strip, we have the resistance exerted by the air on the given strip. As the surface of the body under investigation is perfectly smooth, the friction of the air on such surface may be neglected. The force determined is then normal to the surface of the body. By combining these forces by the well-known methods of graphical statics, the resistance of the air over the entire plate may be obtained in magnitude and location.

Finally, in the experiments of M. de Guiche the automobile should move through a mass of air motionless as a whole. To this end the experiments are made at certain hours on favorable days over a road traversing a forest. The road is about 30 meters (98.4 feet) wide and is bordered with small brush. The straight part in which the measures are taken is determined at its two ends by easy turns. There is thus avoided the establishment of a regular current of air along the axis of the road.

The velocity of the automobile relative to the ground is measured, correcting it, if necessary, for the component of the wind along the direction of motion.

5. THE LABORATORY OF CHALAIS-MEUDON AND THE EXPERIMENTS OF M. LE COMMANDANT DORAND.

The aerotechnic laboratory of Chalais-Meudon is celebrated throughout the entire world by reason of the work of Col. Charles Renard. It is there that were made the studies which have brought the solution of the problem of the dirigibility of balloons. It is there that

have been established by precision experiments some of the fundamental laws of air resistance. It is there, finally, that were made certain experiments by Capt. Ferber, experiments which should soon lead to the French solution of the problem of aviation.

M. le Commandant Dorand has brilliantly continued the labors of these eminent predecessors. The experiments made here on air propellers have carried a long step forward this complex question.

They were made on propellers of normal size by means of the car method, which was later applied at the Institute of Saint-Cyr.

The propeller to be tested is mounted on a car moving on a railroad. This car carries a dynamo shunt excited, by means of which the propeller is turned at a constant speed. On the same car are placed registering apparatus for thrust, speed of car, speed of rotation, and power absorbed.

The railroad, of 1 meter gauge (3.28 feet), on which moves the dynamometer car, has for half of its length a uniform grade. It is prolonged by a level run and then by an ascent, destined to diminish the velocity of the car before application of the automatic brakes. There is utilized in this manner the weight of the equipment, the effect of which added to the propulsive effort of the propeller gives rapidly to the vehicle a high velocity, and brings it back to the point of departure after the experiment. The current intended for the electric motor is brought by two insulated rails, on which move sliding shoes of bronze similar to those which are used for electric railroads.

In order to obtain a constant electric resistance in the circuit, no matter where the car may be on the track, the current is brought to the rails at two opposite extremities. By this means the instruments for the measuring of electric quantities may be located at a fixed point and not on the vehicle.

The energy is transmitted to the propeller shaft by means of a chain.

In order to measure the tractive pull of the propeller there is carried at the end of the propeller shaft (turning in roller bearings) a roller thrust bearing which transmits the tractive effort to a manometric cell. A registering dynamometer thus gives at each instant the thrust of the propeller.

The power absorbed by the propeller is obtained by means of indications furnished by an ammeter and a voltmeter of the recording type, previously standardized for the different speeds of the motor by the aid of a Renard brake.

Chronographs indicating the origin of time are mounted on each of the registering equipments and are put in movement automatically at the same moment.

The part of the apparatus provided for the measuring of the propeller thrust is movable relative to the car. It results that the tractive effort directly measured represents the algebraic sum of the following forces:

- (a) Tractive effort on the level;
- (b) $+ph$, p being the weight of the moving system and h the change of level per meter run;
- (c) Inertia $= -\frac{P}{g} \frac{dv}{dt}$, g being the acceleration due to gravity, and

$\frac{dv}{dt}$ the acceleration of the motion of the car.

The tractive effort on the level, which interests us here, is then equal to the tractive effort measured directly, diminished by ph and augmented by $\frac{p}{g} \frac{dv}{dt}$. For an acceleration which remains near 0.8 meter second (2.6 feet per sec.), this last correction is

$$p \times \frac{0.8}{9.81} = 0.0815 p.$$

As p is always less than 60 kilograms (132 pounds), this correction is about 5 kilograms (11 pounds).

The maximum speed of the dynamometer car has been, in these experiments, equal to 14.6 meters (47.9 feet) per second.

M. le Commandant Dorand made, at the military aerodrome of Villacoublay, numerous trials with a biplane of his construction, and piloted by Labouchère.

This flying laboratory is a biplane with planes stepped toward the front; it is provided with an engine of 60 horsepower, with tractor propeller.

During a horizontal flight the following measures were made:

- (a) Thrust of the propeller.
- (b) Speed of rotation of the propeller shaft or of the engine.
- (c) Speed of the airplane relative to the air.
- (d) Angle of incidence of the planes.

The measures are instantaneous. At the desired moment in horizontal flight the pilot presses on a button and thus determines the registration of all the measures to be made.

The frame of the engine is mounted in a dynamometer balance on a shaft carried on roller bearings. The moment of the thrust of the propeller relative to this axis is balanced by that of two hydraulic cells. Recording manometers give the pressure at the cells and thus furnish, through the lever relation, the thrust of the screw itself. The recording drums are set in motion at the instant of start of the airplane; chronograph markers moved electrically indicate on each sheet the precise place where the reading is to be made.

The action of the air on that part of the balance comprising the engine and its supports gives rise to a force which must be added to that which is measured, if it is desired to know the true thrust of the propeller. In order to make this correction, the entire system without the propeller is placed in a current of air. The velocity of this is measured as well as the resistance which it develops.

The measure of the thrust of the propeller gives the resistance of the airplane at any given instant of its horizontal flight.

In order to measure the speed of rotation of the engine, there is mounted on the shaft a cylinder of ebonite covered over half its circumference with a sheet of copper. Two brushes installed in the circuit of an electric chronograph rest on this cylinder. At each revolution of the engine there is an interruption of the current and hence a jog on the chronograph sheet.

The measure of the relative velocity through the air is made by means of a Venturi tube. We may note here that the operation of this apparatus has been made at the Aerotechnic Institute of Saint-Cyr, the subject of a very careful and systematic investigation.

The angle of incidence is obtained by means of a clinometer formed by a pendulum dampened in glycerine. A pointer which moves over a scale indicates at each instant the angle of the chord of the planes with the horizon.

Experiments in gliding flight have also been made with this equipment. They require the knowledge of the gradient of the path during the glide, or the angle of the relative air movement with the horizon. This item is furnished by means of a vane with horizontal axis.

6. THE AVIATION LABORATORY OF VINCENNES AND THE EXPERIMENTS OF CAPT. OLIVE.

The ministry of war has instituted, under the charge of the artillery school at Vincennes, a laboratory which is specially concerned with researches in aviation.

Among the investigations carried out in this laboratory, we should note here the measurements made by Capt. Olive on airplanes of normal dimensions. The principle of the method was as follows:

Let us consider a body rigidly suspended from a trolley which rides on a rectilinear inclined cable. Let us assume that the body possesses a plane of symmetry which also contains the cable. The body under investigation, descending along the cable in such manner that its plane of symmetry is displaced parallel to itself, is subjected, at each instant, to the following forces:

- (1) The action of gravity applied at the center of gravity.
- (2) The resistance of the air applied at a point which we may designate by ρ .
- (3) The force of inertia applied at the center of gravity.

These forces may be decomposed parallel and perpendicular to the cable. The components of the weight are opposed to those of the resistance of the air; those of the force of inertia are parallel to them.

Graphic record is made of the magnitude of the components parallel and perpendicular to the cable. When the system is in a state of rest, the weight acts alone. The mode of recording thus provides for the elimination of the effects due to gravity.

The body under investigation is allowed to descend along the cable, with the trolley, to which it is rigidly attached. At a given moment the acceleration, α , of the system is measured and record is made of the components of the resistance of the air and of the force of inertia.

The acceleration α is measured in the following manner: When a pendulum is mounted on a support which, itself, is given a movement of translation with an accelerated velocity, it tends to place itself at each instant in a position such that the angular deviation from the vertical is connected with the acceleration α by the equation

$$\beta = \frac{\alpha}{g}$$

The angle β is, in general, quite small. This principle has been realized in the following manner:

A car moves on an aerial monorail formed by a cable stretched between two posts. The airplane under investigation is suspended from this car by means of a bar to which it is attached through a

network of wires forming triangles, thus giving the equivalent of a rigid connection in every direction. In order that these wires may remain under tension during the experiment, it is necessary that the weight of the airplane shall be greater than the upward thrust received from the air during the movement.

The bar connecting with the car is attached thereto by means which transmit separately to dynamometer springs, on the one hand, the forces normal to the cable, and, on the other, the forces parallel to the cable. The movements of the springs are recorded on a moving cylinder.

The cable on which the car rolls shows a general inclination in such manner that the movement of the apparatus may be produced by the action of gravity.

The experiment should be carried out in a part of the cable where the inclination is practically constant in order to avoid the need of taking account of the varying components of gravity and of the force of inertia produced by the curvature of the cable.

At Vincennes the cable, which is 155 meters (508 feet) long, is carried by two pillars which are installed, one on the summit of a hill of about 20 meters (65.6 feet) height, the other on the crest of an embankment. The maximum deflection of the cable varies between 1.4 meters (4.6 feet) without load to 5 meters (16.4 feet) for a load of 1,000 kilograms (2,204 pounds) placed at the center, the tension remaining constant. The general slope is 12 per cent. In the experiments the weight carried was 700 kilograms (1,543 pounds) and the maximum velocity realized was 12 meters per second (39.4 feet per second).

7. THE EXPERIMENTS OF COMMANDANT LAFAY AT THE PHYSICAL LABORATORY OF THE POLYTECHNIC SCHOOL.

Commandant Lafay has installed in the physical laboratory of the Polytechnic School the apparatus previously used by M. Rateau. He has utilized this apparatus in order to solve, often in an elegant manner, a series of interesting problems in aerodynamics. For example, he has proposed to make visible the paths of air stream lines which surround solid bodies of different forms placed within a cylinder of air issuing from the discharge orifice of a fan.

The methods which give the total resistance of the air enable us generally to determine separately the lift and the drift. From these we may then find the ratio of support, which is nothing but the ratio of the lift to the drift. As a matter of fact, in a horizontal flight, the smaller this ratio, the greater the weight carried by a given effort of propulsion. If we call ϕ the angle of the lift with the total resistance, the relation of the drift to the lift is equal to $\text{tang. } \phi$. By reason of the importance of this angle, which Commandant Raibaud of the aviation laboratory at Vincennes proposed to call the angle of support, M. Lafay has constructed an apparatus by which it can be determined directly without going through the determinations of the lift and drift.

He has operated on different stuffed birds and has tried to determine the value of this ratio of support for these birds.

He has been brought to the conclusion that a stuffed bird behaves, on the whole, as a mediocre flyer, certainly inferior to one of our good monoplanes.

These experiments which have dealt with stuffed birds can not, with certainty, be extended to living birds. However, this result leads us to admit only with reserve the statements of those who claim that birds are perfect flyers, presenting for the angles of attack which they utilize ratios of support far superior to those of our apparatus.

M. Lafay has set himself another problem which is very important in regard to the practice of aviation.

The experiments made with models of wings or of airplanes are, as a general rule, exclusively static. The wind of the blowing apparatus whose action is utilized is maintained in a constant state during the period of each observation. Now, when an airplane moves through the air it is subject on the part of the air to actions which vary quite rapidly not only in intensity, but also in direction. Although the inertia of an airplane prevents it from obeying all these instantaneous forces, it may be questioned if the static experiments of the laboratory can be applied without restriction to machines in practice. M. Lafay has striven to give a few indications regarding this question. He has tried a few dynamic experiments, i. e., he has tried to study the variable forces produced by a wind which changes rapidly its direction or its intensity.

The experimental study of this problem presents great difficulties, due chiefly to the disturbing actions of inertia. In order to avoid them, he was led to build models as light as possible, attached to elastic carriers, and to make use of the deformations of these carriers in order to measure the forces. But if these deformations are to be very slight, in order that the energy acquired by the system be negligible, they must, however, be sufficiently great and sufficiently regular to permit of arriving, by means of an appropriate optical amplification, at a correct evaluation of the forces which produce them. The results of this amplification must be such that they can be registered photographically on account of the rapidity of evolution in the phenomena under investigation. Finally, the model and its elastic support would not fail to take on a vibratory movement under the action of the inevitable irregularities of the blowing apparatus, except for the precaution of adding just enough dampening to make the apparatus practically aperiodic, without, however, retarding too much its dynamic indications.

M. Lafay has produced an aerodynamometer capable of satisfying these contradictory conditions.

These experiments have been interrupted by the war. However, from those which have been made it seems to be proven that for changes of speed and direction having the degree of rapidity of those which may normally take place in aviation, the resistance of the air at a given moment has a value little different (10 per cent at the most) from that which one would obtain in permanent régime, keeping invariable the conditions which characterize, at the instant under consideration, the movement of the avion relative to the air.

Consequently, we can deduce from static experiments, properly directed, the elements which are necessary for the calculation of the forces sustained by a machine in given circumstances, as, for example, those which accompany its rapid righting after a diving flight, or its entrance into an ascending or descending current of air.

8. AERODYNAMIC STUDIES PERFORMED IN OTHER LABORATORIES.

A certain number of aerodynamic experiments have been made in other laboratories.

Mention may be made of the experiments on propellers at a fixed point performed by M. Auclair in the experimental laboratory of the Conservatoire des Arts et Métiers. This young savant was the first to give precise results on the influence of the back of the blades, an influence which had been noted as early as 1900 by M. Rateau.

L'Institute Marey, in the Parc des Princes, is continuing the fine studies of Marey on the flight of birds.

M. Houssaye, in his laboratory of L'Ecole Normale Supérieure, at Paris, is studying the resistance of water on the forms of fishes.

M. Magnan, in the laboratory of l'Ecole des Hautes Etudes at the Sorbonne, is trying to deduce from the study of the dimensions of birds some coefficients which will be useful for the construction of airplanes.

CHAPTER III.

DIAGRAMS REPRESENTING THE RESULTS OF EXPERIMENTS.

I. PROPOSED NOTATIONS.

Let us consider a reduced size model of the body under investigation. Let λ be the ratio of the homologous linear dimensions taken in the body and in the model.

Let us suppose that the model is tried at a relative velocity V and that the results of the experiments are reduced to what they would be for a velocity V_1 . If r_{v_1} and r_v are the actions of the air on the model at speeds V_1 and V , we have the relation

$$\frac{r_{v_1}}{r_v} = \left(\frac{V_1}{V}\right)^2 \dots \dots \dots (1)$$

On the other hand, let us take the body under investigation. Let V be its velocity relative to the air, and suppose that the actions of the air are reduced to what they would be if the velocity had the value V_2 . Denote by R_v and R_{v_2} these values of the resistance of the air. We have the relation

$$\frac{R_{v_2}}{R_v} = \left(\frac{V_2}{V}\right)^2 \dots \dots \dots (2)$$

The relations (1) and (2) give:

$$\frac{R_{v_2}}{R_v} \times \frac{r_v}{r_{v_1}} = \left(\frac{V_2}{V_1}\right)^2 \dots \dots \dots (3)$$

But since r_v and R_v are relative to the model and to the body under investigation at the same speed V , we have

$$\frac{r_v}{R_v} = \frac{1}{\lambda^2} \dots \dots \dots (4)$$

Carrying this value into (3) we have

$$\frac{R_{v_2}}{r_{v_1}} = \lambda^2 \left(\frac{V_2}{V_1}\right)^2 \dots \dots \dots (5)$$

In experiments in aerodynamics the values usually taken are $V_1 = V_2 = 10$ meters per second (32.8 feet per second). Measure is then taken of r_v on the model or R_v on a body of normal size. Equations (1) and (2) then give

$$r_{10} = r_v \left(\frac{10}{V}\right)^2 \dots \dots \dots (6)$$

$$R_{10} = R_v \left(\frac{10}{V}\right)^2 \dots \dots \dots (7)$$

The methods for the measurement of $r\nu$ give at the same time:

(a) The component of $r\nu$ along the direction of relative air movement.

(b) The component of $r\nu$ normal to the direction of relative air movement.

With M. Eiffel, let us call r_x and r_y , F_x and F_y the components of r_{10} and R_{10} along and normal to the direction of relative air movement, respectively.

We have then the relations:

$$r_x = \text{component of } r\nu \text{ along direction of relative air movement} \times \left(\frac{10}{V}\right)^2 \dots \dots \dots (8)$$

$$r_y = \text{component of } r\nu \text{ along normal to direction of relative air movement} \times \left(\frac{10}{V}\right)^2 \dots \dots \dots (9)$$

$$F_x = \text{component of } R\nu \text{ along direction of relative air movement} \times \left(\frac{10}{V}\right)^2 \dots \dots \dots (10)$$

$$F_y = \text{component of } R\nu \text{ along normal to direction of relative air movement} \times \left(\frac{1}{V}\right)^2 \dots \dots \dots (11)$$

We may note that $r_x V^2$, $r_y V^2$, $F_x V^2$ and $F_y V^2$ are quantities of the order of force.

M. Eiffel, in his experiments on models of airplanes, takes $V_2 = 1$ met./sec. = 3.28 ft./sec. and $V_1 = 10$ met./sec. = 32.8 ft./sec.

Equation (5) then gives

$$\frac{R_x}{r_{10}} = \left(\frac{\lambda}{10}\right)^2 \dots \dots \dots (12)$$

M. Eiffel calls R_x and R_y the components of R_1 along and normal to the direction of relative air movement.

We have then

$$\left. \begin{aligned} R_x &= r_x \left(\frac{\lambda}{10}\right)^2 \\ R_y &= r_y \left(\frac{\lambda}{10}\right)^2 \end{aligned} \right\} \dots \dots \dots (13)$$

R_x and R_y are quantities of the same order as r_x and r_y .

If the model of the airplane is on a scale $1/10$, $\lambda = 10$ and we have

$$\left. \begin{aligned} R_x &= r_x \\ R_y &= r_y \end{aligned} \right\} \dots \dots \dots (14)$$

The numerical values calculated for the model apply directly to the airplane of normal size.

When the problem involves the wings of airplanes, M. Eiffel places

$$\left. \begin{aligned}
 K_x &= \frac{\text{component of } r_v \text{ along direction of relative air movement.}}{S V^2} = \frac{r_x}{S \times 10^2} \\
 K_y &= \frac{\text{component of } r_v \text{ along normal to direction of relative air movement.}}{S V^2} = \frac{r_y}{S \times 10^2}
 \end{aligned} \right\} (15)$$

$$K_i = \sqrt{K_x^2 + K_y^2} \dots \dots \dots (16)$$

In this equation *i* is the angle between the direction of relative air movement and a reference line attached to the wing, generally the chord of the profile of the wing in its plane of symmetry.

K_x, *K_y*, *K_i* are quantities of the order of density.

S should be a mean between the surface of the wing exposed directly to the action of the air and the surface on the back. Builders of airplanes usually consider *S* equal to the greatest projection of the wing on a horizontal plane.

2. STUDY OF THE WINGS OF AN AIRPLANE—POLAR DIAGRAMS OF M. EIFFEL.

M. Eiffel represents the properties of the wings of an airplane by means of what he calls simple polar diagrams.

On two rectangular axes, he plots as abscissæ the values of *K_x* and as ordinates the values of *K_y*, the same scale being used for both. The curve thus traced in the *K_x* *K_y* plane is called the "first simple polar." A point of the curve corresponds to a determinate value of the angle *i*. The radius vector from the origin to this point represents the quantity *K_i*. The angle of this radius vector with the axis of *K_y* is the angle between the resistance of the air and the normal to the direction of relative air movement. If this angle is denoted by *θ* we have

$$\text{tang. } \theta = \frac{K_x}{K_y} \dots \dots \dots (17)$$

The tangent drawn from the origin to the polar gives the value of *θ*, *θ_m*, for which the ratio $\frac{K_x}{K_y}$ is a minimum.

To each point of the curve corresponds a value of the angle *i* and a value of the angle *θ*. If *θ* = *i* the resistance of the air is normal to the chord of the profile; if *θ* < *i*, the resistance of the air is forward of the normal to the chord. For *θ* > *i* it falls behind the chord.

This mode of representation (*K_x* and *K_y* represented to the same scale) is not suitable for the values of the angle *i* corresponding to the small values employed in aviation. In fact, for these values of the angle *i* the polar diagram approaches very close to a straight line slightly inclined to the axis of *K_y*. The comparison of one

wing with another by simple superposition of diagrams is a delicate operation. In particular it is almost impossible to compare the wings regarding the minimum value of $\frac{K_x}{K_y}$.

Accordingly, M. Eiffel constructs what he calls the "second simple polar." He takes for K_x a scale five times larger than for K_y . In this mode of representation, a vector joining the origin with a point on the curve is no longer equal to K_i , and the angle of this vector with the axis of K_y is no longer the angle θ . However, the same as for the small values of i , less than 10° , K_i is very little different from K_y , and for the values K_i the ordinates of the new curve may be taken. It is convenient to add to this curve a scale representing values of $\frac{K_x}{K_y}$. On a parallel to the axis of K_x , passing through a point of K_y , values are plotted of $\frac{K_x}{K_y}$ corresponding to one of the intersections with the new curve of the radius vector starting from the origin and ending at this point. Let us call this line the axis of $\frac{K_x}{K_y}$.—In order that $\frac{K_x}{K_y}$ may correspond to an angle i , it is necessary that the vector just named should cut the second polar curve. The minimum value of $\frac{K_x}{K_y}$ is then given by the point where the tangent from the origin to the polar curve meets the axis of $\frac{K_x}{K_y}$.

3. STUDY OF THE HORIZONTAL MOVEMENT OF AN AIRPLANE—THE LOGARITHMIC POLAR CURVE.

In order to study the horizontal movement of an airplane, M. Eiffel has pointed out a very ingenious representation, to which he has given the name of logarithmic polar.

Let us consider a model of an airplane and let i be the angle made between the direction of relative air movement and a straight reference line intimately connected with the apparatus, for example, a straight line doubly tangent to the lower part of the principal planes, near the fuselage. To this value of the angle i , the experiment on the model will give corresponding values of the resistance of the air, of which the projections parallel and perpendicular to the air movement are r_x and r_y . To these, equations (13) serve to give the corresponding values of R_x and R_y relative to an airplane of full size.

Furthermore, let

Q = the weight of the actual airplane.

P = the power required to maintain horizontal flight with a relative velocity V .

The equations

$$\left. \begin{aligned} P &= R_x V^2 \\ Q &= R_y V^2 \end{aligned} \right\} \dots \dots \dots (18)$$

define the correlative values of P , Q , V , R_x and R_y , and hence of the angle i which corresponds to the horizontal flight of an airplane of determinate form (especially of an airplane in which the depth rudder

occupies a determinate position) when the axis of the propeller is parallel to the path of flight.

Let us consider such an airplane.

Equations (18) give immediately

$$\left. \begin{aligned} \log. R_x &= \log. P - 3 \log. V \\ \log. R_y &= \log. Q - 2 \log. V \end{aligned} \right\} \dots \dots \dots (19)$$

or

$$\left. \begin{aligned} \log. R_x &= \log. P - \frac{3}{\sqrt{13}} \times \sqrt{13} \log. V \\ \log. R_y &= \log. Q - \frac{2}{\sqrt{13}} \times \sqrt{13} \log. V \end{aligned} \right\} \dots \dots \dots (20)$$

The experiments on a model permit, for various values of i , the determination of corresponding values of R_x and R_y .

On two rectangular axes let us plot to the same scale, on the axis of abscissæ, distances proportional to the various values of $\log. R_x$; on the axis of ordinates, distances proportional to the various values of $\log. R_y$. We shall thus obtain in the plane of the axes a curve to which M. Eiffel has given the name of logarithmic polar. Each point on this curve corresponds to a determinate value of the angle i which is inscribed on the curve.

Let us consider a vector OM_i running from the origin O to a point M_i on the curve. This vector has for projections on the axes of coordinates the values $\log. R_x$ and $\log. R_y$. But equations (20) show that this vector is the resultant of a broken line of which the vectors are

- $\log. P$ directed along the axis of $\log. R_x$.
- $\log. Q$ directed along the axis of $\log. R_y$.

$\sqrt{13} \times \log. V$ directed in the third angle of the coordinate planes ($-\log. R_x, -\log. R_y$), and making with the axis of $\log. R_x$ an angle of which the cosine is equal to

$$-\frac{3}{\sqrt{13}} \text{ (See fig. 1.)}$$

If the two extremities, O and M_i , of the broken line are preserved, the segments may be run through in any order whatever. Thus, for example, we may have any one of the following orders:

- $\log. P, \sqrt{13} \times \log. V, \log. Q;$
- $\log. Q, \log. P, \sqrt{13} \times \log. V;$
- $\sqrt{13} \times \log. V, \log. Q, \log. P.$

It is well known that starting from the point O one should, following the broken line, end at a point M_i of the logarithmic polar. The directions of the vectors are, furthermore, well known. If we take two of the vectors of the broken line, the trace of this line permits immediately the determination of the third.

We may thus solve graphically by means of the logarithmic polar a series of problems relating to the horizontal flight of an airplane

when the axis of the propeller is parallel to the path. We might, for example, desire to know what weight should be given to the apparatus in order to obtain a given velocity with a given power.

In this problem the vectors $\log. P$ and $\sqrt{13} \times \log. V$ are known in magnitude and direction; it is easy to trace them. From the extremity of the vector $\sqrt{13} \times \log. V$ there is drawn a straight line parallel to the axis of $\log. R_v$, which is continued to its point of intersection with the logarithmic polar. The vector $\log. Q$ is thus constructed; it gives the weight Q which is sought. At the same time, the point of intersection of this vector with the polar curve determines the angle i of the flight.

Let us now consider a velocity V_o which is, for example, the normal actual velocity of the airplanes (100 kilometers (62.1 miles) per hour). Then equations (18) may be written

$$\left. \begin{aligned} \frac{P}{V_o^3} &= R_x \left(\frac{V}{V_o} \right)^3 \\ \frac{Q}{V_o^2} &= R_v \left(\frac{V}{V_o} \right)^2 \end{aligned} \right\} \dots \dots \dots (21)$$

From these we derive

$$\left. \begin{aligned} \log. R_x &= \log. \left(\frac{P}{V_o^3} \right) - \frac{3}{\sqrt{13}} \sqrt{13} \times \log. \frac{V}{V_o} \\ \log. R_v &= \log. \left(\frac{Q}{V_o^2} \right) - \frac{2}{\sqrt{13}} \sqrt{13} \times \log. \frac{V}{V_o} \end{aligned} \right\} \dots \dots \dots (22)$$

On the axis $\log. V$ let us take a point V_o such that

$$OV_o = \sqrt{13} \times \log. V_o \quad (\text{fig. 1.})$$

The vector $V_o V$ then represents

$$\sqrt{13} \times \log. \frac{V}{V_o}$$

Let us then carry this vector over to $C_o B$ on the vector AB , and then project C_o to A_o on the axis $\log. R_x$. Finally, lead the vector $A_o B_o$ to the ordinate parallel to the axis $\log. R_v$. To the contour $OABM_t$, in which

$$OA = \log. P, AB = \sqrt{13} \times \log. V, BM_t = \log. Q,$$

we thus substitute the contour $OA_o B_o M_t$, which is its equivalent since it has the same resultant, and which is such that

$$OA_o = \log. \left(\frac{P}{V_o^3} \right), A_o B_o = \sqrt{13} \times \log. \frac{V}{V_o}, B_o M_t = \log. \left(\frac{Q}{V_o^2} \right)$$

We have as a result:

$$\text{Vector } OA_o = \text{vector } OA + \text{vector } AA_o$$

$$\text{Vector } AA_o = -3 \log. V_o$$

$$\text{Vector } OA_o = \log. P - 3 \log. V_o = \log. \left(\frac{P}{V_o^3} \right)$$

As we shall have constantly to consider a vector V_oV or A_oB_o or $\sqrt{13} \times \log. \frac{V}{V_o}$, it is natural to carry the point V_o to the origin.

When the velocity of the airplane is equal to V_o , the vector A_oB_o disappears; the points A_o and B_o become coincident with the point M_x (fig. 2). It is, in fact, easy to see that we have

$$M_xA_o = 3 \log. \frac{V}{V_o}, B_oM_x = 2 \log. \frac{V}{V_o}$$

The coordinates $\log. R_x$ and $\log. R_y$ of the point have then for values

$$\left. \begin{aligned} \log. R_x &= \log. \left(\frac{P_o}{V_o^3} \right) \\ \log. R_y &= \log. \left(\frac{Q_o}{V_o^2} \right) \end{aligned} \right\} \dots \dots \dots (23)$$

From these equations we derive

$$\left. \begin{aligned} R_x &= \frac{P_o}{V_o^3} \\ R_y &= \frac{Q_o}{V_o^2} \end{aligned} \right\} \dots \dots \dots (24)$$

We are therefore able to develop a correspondence between a point M_x on the axis of abscissæ (fig. 1) and a value R_x , such that

$$OM_x = \log. R_x,$$

and a value P_o of the useful power such that

$$OM_x = \log. \left(\frac{P_o}{V_o^3} \right)$$

In other words, the axis of abscissæ may be graduated in terms of useful power.

In the same way we may graduate the axis of ordinates in terms of weight.

Let us now suppose that, in a problem, we have given the useful power P and the speed V . The axis of abscissæ, which is the scale for P , gives immediately the point A_o , such that

$$V_oA_o = \log. \left(\frac{P}{V_o^3} \right) \quad (\text{See fig. 2.})$$

The vector V_oV is such that

$$V_oV = \sqrt{13} \times \log. \frac{V}{V_o}.$$

The contour $V_oA_oB_o$ may be traced. By carrying B_oM_i parallel to $\log. R_y$ and extending to the point of intersection with the polar curve, there is found the vector

$$B_oM_i = \log. \left(\frac{Q}{V_o} \right).$$

If this vector is led down from V_o on the scale of R_y , which is at the same time the scale of weight, the extremity of the segment gives immediately the weight Q which is sought.

In the system of units (meter, kilogram, second) generally used, P is expressed in kilometer-seconds, V in meter-seconds, Q in kilograms (weight). It is more convenient, for practical application, to graduate the scales for P and V in horsepower and in kilometers per hour. To this end it is sufficient to divide the indications of the first scale by 75 for horsepower and to multiply by 3.6 the numbers relating to velocity.

If the velocity W is less than V_o , the segment is directed opposite to the segment V_oV . The contour to consider is $V_o A_o' B_o' M_i$. (See fig. 2.) We have, in fact, in this case

$$\left. \begin{aligned} \log. R_x &= \log. \left(\frac{P}{V_o^3} \right) + 3 \log. \frac{V_o}{W} \\ \log. R_y &= \log. \left(\frac{Q}{V_o^2} \right) + 2 \log. \frac{V_o}{W} \end{aligned} \right\}$$

It is easily seen that these equations represent the projections on the two axes of the contour $V_o A_o' B_o' M_i$.

We are thus led to the following rule:

If we follow a broken line starting from the origin and ending on the polar curve, the direction in which each vector is traversed is the direction in which such vector should be placed, starting from the origin, on the corresponding scale in order to give its value.

It results immediately that if we have the contour $V_o ABCD$ (fig. 2), the vector BC corresponds to a speed greater than the vector BD , these two speeds being, furthermore, inferior to V_o .

Let us suppose, now, that the results found with the model do not correspond to the conditions which had been fixed *a priori* for the airplane. The question may then arise of changing proportionately the dimensions of the apparatus.

Let N be the ratio of the lineal dimensions of the second apparatus to those of the first; N , for example, might be 1.10 for an increase of 10 per cent in the dimensions.

The fundamental equations of horizontal flight for this new apparatus will be

$$\left. \begin{aligned} \frac{P}{V_o^3} &= R_x N^2 \left(\frac{V}{V_o} \right)^3 \\ \frac{Q}{V_o^2} &= R_y N^2 \left(\frac{V}{V_o} \right)^2 \end{aligned} \right\} \dots \dots \dots (25)$$

It is not necessary to construct special polar curves corresponding to various values of N in order to determine the value suited to this number. We find, in fact, from equations (25)

$$\left. \begin{aligned} \log. R_x &= \log. \left(\frac{P}{V_o^3} \right) - 3 \log. \frac{V}{V_o} - 2 \log. N \\ \log. R_y &= \log. \left(\frac{Q}{V_o^2} \right) - 2 \log. \frac{V}{V_o} - 2 \log. N \end{aligned} \right\} \dots \dots (26)$$

To the vectors,

$$\log. \left(\frac{P}{V_0^3} \right), \log. \left(\frac{Q}{V_0^2} \right), \sqrt{13} \times \log. \frac{V}{V_0},$$

it is convenient to add a fourth vector,

$$\sqrt{8} \times \log. N.$$

This is directed along the line making the angle $5 \frac{\pi}{4}$ with the axis abscissæ (Axis $V_0 N$, fig. 2).

If N is greater than unity, the values of $\log. N$ are laid off along this axis; if N is less than unity, they are laid off along the line $\frac{\pi}{4}$ with the axis of the abscissæ.

If then the values fixed in advance are P , Q , V , it is sufficient, in order to have the value of N which will permit of realizing these values, to draw the fourth segment in a suitable direction until it meets the polar curve. The fourth segment indicates, furthermore, by its intersection with the polar curve, a suitable angle of flight.

Instead of terminating the polygonal contour running from the origin to a point of the curve by the vector $\sqrt{8} \times \log. N$, we may trace this segment first. In other words, instead of starting from the origin of coordinates as the origin of contour, we may start from a point situated on the axis of N . We then see immediately by the figure what becomes of the properties of an airplane, for which the dimensions have been multiplied by the number determined by the point on the axis of N which was taken for the point of departure. Every broken line drawn between this point and the polar curve gives the system of values P , Q , V , which corresponds to the modified apparatus.

We can not here indicate the solution of all the problems for which the consideration of the logarithmic polar provides. To this end, reference should be made to the work of M. Eiffel noted in the bibliography attached to this paper. However, we may note, in résumé, the results to which this study of the logarithmic polar leads.

For all the forms of apparatus studied by M. Eiffel, the logarithmic polar curves always present the same general characteristic, that of figure 3. Beginning with small values of the angles of incidence, we find the angles of horizontal flight for which the properties are the following:

- (1) Angle i_1 for which R_x is minimum (fig. 3).

This angle is given by the point of contact of the tangent to the polar, parallel to the axis of ordinates.

Horizontal flight under this angle corresponds to the maximum speed for a given power, or to the minimum power for a given speed.

- (2) Angle i_2 for which $\frac{R_x}{R_y}$ is a minimum (fig. 3).

This angle is given by the point of contact of the tangent to the polar curve, drawn parallel to the axis of N .

Horizontal flight under this angle corresponds to the minimum tractive force required for a given weight, or to the maximum weight for a given tractive force.

- (3) Angle i_3 for which $\frac{R_x^2}{R_y^3}$ is a minimum (fig. 3).

This angle is given by the point of contact of the tangent to the polar curve, drawn parallel to the axis V_0V .

Horizontal flight under this angle corresponds to the minimum power for a given weight, or to the maximum weight for a given power.

(4) Angle i_4 for which R_y is maximum (fig. 3).

This angle is given by the point of contact of the tangent to the polar curve, drawn parallel to the axis of abscissæ.

Horizontal flight under this angle corresponds to the maximum weight for a given speed, or to the minimum speed for a given weight.

It is seen that to each one of the angles of incidence, i_1, i_2, i_3, i_4 , we may relate two magnitudes, of which one is maximum or minimum when the other is given. Each one of these angles is the most favorable angle regarding the two corresponding magnitudes.

As these polar curves, in their useful part, do not have a point of inflection, it follows that the nearer the angle of horizontal flight lies to one of the angles i_1, i_2, i_3, i_4 , the better are the conditions with regard to the group of magnitudes which correspond to these angles.

Let us take an example. The weight carried by an airplane is judged to be too small. It is desired to gain weight at the expense of speed, but at the same time preserving the same expenditure of power. It is sufficient to approach the point for which the weight will be maximum for a given power. It is well to give to the apparatus a construction such that horizontal flight (with the axis of the propeller in the direction of the path) is made under an angle as near as possible to the values indicated for i_3 .

We have constructed the logarithmic polar curve for a given position of the depth rudder. We have, by means of this curve, studied the properties of horizontal flight for an apparatus under different angles of flight. We have then supposed that for all these angles the air resistance passed sensibly through the point of intersection of the axis of the propeller and of the vertical through the center of gravity. For each apparatus this is sensibly true for an average position for the depth rudder.

But we may approach still more closely to reality. Experiments made on a model with various positions of the depth rudder give the resultants of the air resistance which pass exactly through the point γ of intersection of the axis of the propeller and of the vertical through the center of gravity. We then have sufficient data for the following tables:

| Position of rudder. | Characteristics of the resistance of the air, passing through γ . |
|---------------------|--|
| A. ----- | R_{xA}, R_{yA}, i_A |
| B. ----- | R_{xB}, R_{yB}, i_B |
| C. ----- | R_{xC}, R_{yC}, i_C |

With these data we may construct the curves

$$R_y = f_1(R_x) \quad \text{Ordinary polar curve.}$$

$$R_y = f_2(i)$$

$$\log. R_y = f_3(\log. R_x) \quad \text{Logarithmic polar curve.}$$

A point of one of these curves gives, not only the values of R_x, R_y, i , but indicates at the same time the corresponding position of the depth rudder.

4. THE CHARACTERISTIC COEFFICIENTS OF PROPELLERS ACCORDING TO G. EIFFEL.

The experimental study of the propeller includes the following quantities:

- (1) The thrust, Π .
- (2) The effective power, P_e , delivered to the propeller shaft.
- (3) The number n of revolutions per minute of time.
- (4) The velocity of flight, V .

These experimental data permit the determination of the following:

- (a) The useful power expended in propulsion

$$P_u = w \times V \dots \dots \dots (27)$$

- (b) The angular velocity, $2\pi n$. The speed of rotation of the extremity of the blade of the propeller πnD (D equals the diameter of the propeller)

- (c) The moment of the couple of rotation

$$C = \frac{P_e}{2\pi n} \dots \dots \dots (28)$$

- (d) The efficiency of the propeller

$$\rho = \frac{P_u}{P_e} = \frac{\Pi}{C} \times \frac{V}{2\pi n} \dots \dots \dots (29)$$

What are then the coefficients which should be considered in the study of a propeller?

- (1) The direction of the relative velocity at the extremity of a blade; it is characterized by the ratio

$$\frac{V}{\pi nD}$$

- (2) Magnitudes of the nature of a density,

$$\frac{\Pi}{n^2 D^4}, \frac{C}{n^2 D^5}, \frac{P_e}{n^3 D^6}, \frac{P_u}{n^3 D^6} \dots \dots \dots (G_1)$$

- (3) The abstract number ρ .

Let us assume, as a first approximation, that the magnitudes here mentioned are functions of a single ratio $\frac{V}{nD}$.

It is naturally the same with the magnitudes.

$$\frac{\Pi}{n^2 D^4} \left(\frac{V}{nD}\right)^m; \frac{C}{n^2 D^5} \left(\frac{V}{nD}\right)^p; \frac{P_e}{n^3 D^6} \left(\frac{V}{nD}\right)^q; \frac{P_u}{n^3 D^6} \left(\frac{V}{nD}\right)^{q'};$$

whatever may be the exponents m, p, q, q' .

In the group (G_1) the velocity V does not enter. We may then dispose the preceding exponents in such manner as to define two other groups of coefficients which do not contain D or n .

By taking $m = -4, p = q = q', = -5$, we have the group (G_2)

$$\frac{\Pi n^2}{V^4}, \frac{C n^5}{V^5}, \frac{P_e n^2}{V^5}, \frac{P_u n^2}{V^5} \dots \dots \dots (G_2)$$

By taking $m = p = -2, q = q' = -3$, we derive the coefficients

$$\frac{\Pi}{D^2 V^2}, \frac{C}{D^2 V^2}, \frac{P_e}{D^2 V^3}, \frac{P_u}{D^2 V^3} \dots \dots \dots (G_3)$$

With the efficiency ρ , the preceding groups define 13 coefficients which may serve to characterize a problem.

Among these coefficients it is sufficient to note more especially the following:

$$\frac{P_e n^2}{V^5}, \frac{P_e}{D^2 V^3}, \rho \dots \dots \dots (G_4)$$

Two problems, in fact, often arise, as under (1) and (2) following.

(1) Suppose an airplane for which we must choose a propeller directly connected with the motor.

We have given, the speed of translation, V , of the airplane, the power, P_e , and the number of revolutions, n , of the engine. These data enable us to calculate the coefficient $\frac{P_e n^2}{V^5}$.

What is the diameter which should be given the propeller?

What is the efficiency of this propeller under the preceding conditions of operation?

Let us take a given type of propeller. Suppose a study of a model of this propeller has permitted us to construct curves giving as a function of $\frac{V}{nD}$ the various values of $\frac{P_e n^2}{V^5}$ and of ρ .

It is then easy to take from one of these curves the value of $\frac{V}{nD}$ which corresponds to the particular value of $\frac{P_e n^2}{V^5}$ suited to the propeller. This value of $\frac{V}{nD}$ gives immediately the diameter of the propeller as desired.

At the same time, on the other curve corresponding to the value of $\frac{V}{nD}$ thus determined, we may read the value of the efficiency ρ .

(2) Suppose an airplane with chain-connected propeller or a dirigible.

We have given the power, P_e , of the engine, the speed of translation, V , the diameter of the propeller, D , and, in consequence, the coefficient $\frac{P_e}{D^2 V^3}$.

It is required to determine the number of revolutions of the propeller and its efficiency.

Let us suppose that for a given type of propeller we have, by means of model experiments, constructed curves giving as a function of $\frac{V}{nD}$ values of $\frac{P_e}{D^2 V^3}$ and of the efficiency ρ .

On the first curve the particular value calculated for $\frac{P_e}{D^2 V^3}$ gives the corresponding value of $\frac{V}{nD}$, and, in consequence, the number of revolutions, n , of the propeller. To this value of $\frac{V}{nD}$ there corresponds a value of the efficiency ρ which is read on the second curve of efficiencies.

What has been said shows that the construction of two curves relating to two coefficients of the groups (G_1) , (G_2) , (G_3) (the efficiency ρ being joined to each one of these groups) suffices for the determination for the operation of a propeller under determinate conditions. We may represent as a function of $\frac{V}{nD}$ the following:

$$\frac{\Pi}{n^2 D^4} \text{ and } \frac{C}{n^2 D^5} \text{ of the group } (G_1),$$

$$\frac{\Pi n^2}{V^4} \text{ and } \frac{C n^2}{V^5} \text{ of the group } (G_2),$$

$$\frac{P_e}{n^2 D^6} \text{ and } \frac{P_u}{n^2 D^5} \text{ of the group } (G_3),$$

etc.

We may indeed construct but a single curve, that which corresponds to the power P_e , for example, on the condition of noting on such curve the various values of the efficiency.

We have assumed, as a first approximation, that any coefficient Γ of one of the preceding groups is represented by a single curve in the plane $(\Gamma, \frac{V}{nD})$.

In reality, to each value of $\frac{V}{nD}$ there correspond in the plane $(\Gamma, \frac{V}{nD})$ various values of Γ . These latter correspond to varying values of nD or of πnD (velocity of rotation at the extremity of the blade). Instead of having for Γ a single representative curve in the plane $(\Gamma, \frac{V}{nD})$, there are several curves, of which each one corresponds to a particular value of nD .

However, for values of nD varying by 10 units (D expressed in meters, n in revolutions per second) in the field of values of nD comprised between 25 and 50 (values actually met with in practice), the curves corresponding to the various values of nD differ but little from an average curve, which is the one here considered.

5. STUDY OF THE PROPERTIES OF PROPELLERS—THE LOGARITHMIC DIAGRAM OF M. EIFFEL.

We have now to consider the representation as a function of $\frac{V}{nD}$ of the coefficients $\frac{P_e}{n^2 D^5}$ and $\frac{P_u}{n^2 D^5}$.

We develop this representation by taking for abscissæ values of $\log. \frac{V}{nD}$ and for ordinates values for $\log. \frac{P_e}{n^2 D^5}$ and $\log. \frac{P_u}{n^2 D^5}$ (fig. 4).

We thus obtain what M. Eiffel calls the logarithmic diagram for propellers.

When these diagrams are constructed we may read directly, by means of a single scale and from axes suitably chosen, the values of the 13 coefficients, ρ and the groups (G_1) , (G_2) , (G_3) .

These same diagrams give us also directly the various values of the magnitudes

$$V, n, D, \Pi, C, P_e, P_u, \rho.$$

Let us now show how, by means of these diagrams, the values of the 13 characteristic coefficients of a propeller may be read. We have the following relations:

$$\left. \begin{aligned} \log. \frac{\Pi}{n^2 D^4} &= \log. \left[\frac{\Pi V}{n^2 D^5} \times \frac{nD}{V} \right] = \log. \frac{P_u}{n^2 D^5} - \log. \frac{V}{nD} \\ \log. \frac{C}{n^2 D^5} &= \log. \left[\frac{2\pi n C}{n^2 D^5} \times \frac{1}{2\pi} \right] = \log. \frac{P_e}{n^2 D^5} - \log. 2\pi \\ \log. \rho &= \log. \frac{P_u}{P_e} = \log. \frac{P_u}{n^2 D^5} - \log. \frac{P_e}{n^2 D^5} \end{aligned} \right\} \dots \text{Group (G}_1\text{)}$$

$$\left. \begin{aligned} \log. \frac{\Pi n^2}{V^4} &= \log. \left[\frac{\Pi V}{n^2 D^5} \times \frac{n^5 D^5}{V^4} \right] = \log. \frac{P_u}{n^2 D^5} - 5 \log. \frac{nD}{V} \\ \log. \frac{C n^3}{V^5} &= \log. \left[\frac{2\pi n C}{n^2 D^5} \times \frac{n^5 D^5}{V^4} \times \frac{1}{2\pi} \right] = \log. \frac{P_e}{n^2 D^5} \\ &\quad - 5 \log. \frac{V}{nD} - \log. 2\pi \end{aligned} \right\} \dots \text{Group (G}_2\text{)}$$

$$\left. \begin{aligned} \log. \frac{P_e n^2}{V^5} &= \log. \left[\frac{P_e}{n^2 D^5} \times \frac{n^5 D^5}{V^5} \right] = \log. \frac{P_e}{n^2 D^5} - 5 \log. \frac{V}{nD} \\ \log. \frac{P_u n^2}{V^5} &= \log. \left[\frac{P_u}{n^2 D^5} \times \frac{n^5 D^5}{V^5} \right] = \log. \frac{P_u}{n^2 D^5} - 5 \log. \frac{V}{nD} \\ \log. \frac{\Pi}{D^2 V^3} &= \log. \left[\frac{\Pi V}{n^2 D^5} \times \frac{n^3 D^3}{V^3} \right] = \log. \frac{P_u}{n^2 D^5} - 3 \log. \frac{V}{nD} \\ \log. \frac{C}{D^2 V^3} &= \log. \left[\frac{2\pi n C}{n^2 D^5} \times \frac{n^3 D^3}{V^3} \times \frac{1}{2\pi} \right] = \log. \frac{P_e}{n^2 D^5} \\ &\quad - 2 \log. \frac{V}{nD} - \log. 2\pi \end{aligned} \right\} \dots \text{Group (G}_3\text{)}$$

$$\left. \begin{aligned} \log. \frac{P_e}{D^2 V^3} &= \log. \left[\frac{P_e}{n^2 D^5} \times \frac{n^3 D^3}{V^3} \right] = \log. \frac{P_e}{n^2 D^5} - 3 \log. \frac{V}{nD} \\ \log. \frac{P_u}{D^2 V^3} &= \log. \left[\frac{P_u}{n^2 D^5} \times \frac{n^3 D^3}{V^3} \right] = \log. \frac{P_u}{n^2 D^5} - 3 \log. \frac{V}{nD} \end{aligned} \right\}$$

On the axis of abscissæ $\left(\log. \frac{V}{nD} \right)$ let us take the point which corresponds to $\frac{V}{nD} = 1$. According to the mode of graduation of the scale of abscissæ, the vector having for origin this point and for extremity a point on the axis of abscissæ, is, in absolute value, equal to $1 - \log. \alpha$, $\frac{\alpha}{10}$ (α whole number), being the value of $\frac{V}{nD}$ which corresponds to a point at the extremity of the vector. The values of α inferior to 10 correspond to the points on the axis of abscissæ situated to the left of the point $\left(\frac{V}{nD} = 1 \right)$; the values of α superior to 10 cor-

respond to the points on the axis of abscissæ situated on the right of the point $\left(\frac{V}{nD}=1\right)$. The vectors issuing from the point $\left(\frac{V}{nD}=1\right)$ measure then, with their sign, the values of $\log. \frac{V}{nD}$.

This being understood, let us draw through the point $\left(\frac{V}{nD}=1\right)$ right lines having for angular coefficients, 1, 3, 5. Let us take these lines as origins for vectors parallel to the axis of ordinates and terminating, either on the polar curve, $\log. \frac{P_e}{n^3 D^5}$ or $\log. \frac{P_u}{n^3 D^5}$. The vectors thus defined measure

$$\begin{aligned} & \log. \frac{\Pi}{n^2 D^4}, \text{ (right line for angular coefficient equal to 1);} \\ & \log. \frac{\Pi}{D^2 V^2}, \log. \frac{P_e}{D^2 V^2}, \log. \frac{P_u}{D^2 V^2}, \text{ (right line for angular coefficient} \\ & \text{equal to 3);} \\ & \log. \frac{\Pi n^2}{V^4}, \log. \frac{P_e n^2}{V^4}, \log. \frac{P_u n^2}{V^4}, \text{ (right line for angular coefficient} \\ & \text{equal to 5).} \end{aligned}$$

In tracing the right line ordinate ($\log. 2\pi$) and taking this line as the origin of vectors parallel to the axis of ordinates, and terminating at one or the other of the logarithmic polars, these vectors represent

$$\log. \frac{C}{n^2 D^5}, \log. \frac{C n^2}{V^4}, \log. \frac{C}{D^2 V^2}.$$

For these two last it is necessary, furthermore, to trace through the point $\left(\frac{V}{nD}=1, \text{ord.}=\log. 2\pi\right)$ the right line for angular coefficient 5 and the right line for angular coefficient 2.

Finally the vector $\log. \rho$ is represented by the difference of the ordinates $\log. \frac{P_u}{n^3 D^5}$ and $\log. \frac{P_e}{n^3 D^5}$ of the two logarithmic polars corresponding to the same value of $\frac{V}{nD}$.

In practice it is important to know as a function of V, n, D , the following:

(1) The useful power, P_u (from the viewpoint of the operation of the airplane).

(2) The effective power, P_e (from the viewpoint of the motor to install on the airplane).

(3) The efficiency ρ .

Let us consider the logarithmic polar

$$\left[\log. \frac{P_e}{n^3 D^5}, \log. \frac{V}{nD} \right].$$

We have

$$\left. \begin{aligned} \log. \frac{V}{nD} &= \log. V - \frac{1}{\sqrt{10}} \times \sqrt{10} \log. n - \frac{1}{\sqrt{26}} \sqrt{26} \log. D \\ \log. \frac{P_e}{n^3 D^5} &= \log. P_e - \frac{3}{\sqrt{10}} \sqrt{10} \log. n - \frac{5}{\sqrt{26}} \sqrt{26} \log. D \end{aligned} \right\} \dots (30)$$

Let us trace the directions ON and OD (fig. 4) making with the positive direction of the axis of abscissæ, the first the angle $(\pi + a_1)$, and the second the angle $(\pi + a_2)$, the angles a_1 and a_2 being given by the relations:

$$\text{tang. } a_1 = 3, \text{ tang. } a_2 = 5 \dots \dots \dots (31)$$

The two equations (30) express that, for the contour OAB $\left[OA = \log. \frac{V}{nD}, AB = \log. \frac{P_e}{n^3 D^5} \right]$ we may substitute the contour $OA_1B_1C_1B$, which has the same resultant. This new contour is such that:

OA_1 is parallel to the axis of $\frac{V}{nD}$ and has for magnitude $\log. V$; A_1B_1 is parallel to OD and has for magnitude $\sqrt{26} \times \log. D$; B_1C_1 is parallel to ON and has for magnitude $\sqrt{10} \times \log. n$; C_1B is parallel to the axis of $\frac{P_e}{n^3 D^5}$ and has for magnitude $\log. P_e$.

If the proper graduations have been made on the various axes parallel to the sides of this contour (graduation in $\log. V$ on the axis of $\frac{V}{nD}$; graduation in $\log. P_e$ on the axis of $\frac{P_e}{n^3 D^5}$; graduation in $\sqrt{10} \log. n$ on the axis of n ; graduation in $\sqrt{26} \log. D$ on the axis of D), it is easy, by the construction of the contour in question, to determine any one of the vectors, knowing the magnitudes of the others. It is sufficient to remark that the contour, starting from the point O , must always end on a point of the logarithmic polar.

But this construction may be transformed in the following manner:

In present practice with airplanes, normal conditions of operation lead to the employment of propellers of a diameter of about 3 meters (9.84 feet) turning at about 800 revolutions per minute. If, for these conditions near the normal, the vectors A_1B_1 and B_1C_1 are zero, the contour $OA_1B_1C_1B$ is reduced to the contour OAB . The construction relative to normal operation is very much simplified, since it is reduced to the tracing of two lines instead of four. Now the vectors A_1B_1 and B_1C_1 are zero, if the normal values (800 revolutions per minute, 3 meters) coincide with the origin O . We are therefore led, for n and D , to a change of origin, which may be made in the following manner:

Let us consider a particular number of revolutions n_0 and a diameter D_0 for the propeller. We have then:

$$\left. \begin{aligned} \frac{V}{nD} &= \frac{V}{n_0 D_0} \times \frac{n_0}{n} \times \frac{D_0}{D} \\ \frac{P_e}{n^3 D^5} &= \frac{P_e}{n_0^3 D_0^5} \times \left(\frac{n_0}{n}\right)^3 \times \left(\frac{D_0}{D}\right)^5 \end{aligned} \right\} \dots \dots \dots (32)$$

$$\left. \begin{aligned} \log. \frac{V}{nD} &= \log. \frac{V}{n_0 D_0} - \frac{1}{\sqrt{10}} \sqrt{10} \log. \frac{n}{n_0} - \frac{1}{\sqrt{26}} \sqrt{26} \log. \frac{D}{D_0} \\ \log. \frac{P_e}{n^3 D^5} &= \log. \frac{P_e}{n_0^3 D_0^5} - \frac{3}{\sqrt{10}} \sqrt{10} \log. \frac{n}{n_0} - \frac{5}{\sqrt{26}} \sqrt{26} \log. \frac{D}{D_0} \end{aligned} \right\} \dots \dots \dots (33)$$

These equations mean that for the contour OAB , and, in consequence, for the contour $OA_1B_1C_1B$, we have substituted the equivalent contour $OA_1'B_1'C_1'B$ (fig. 4), in which:

$$OA_1' = \log. \frac{V}{n_o D_o} \left(\text{directed along the axis of } \log. \frac{V}{nD} \right)$$

$$A_1'B_1' = \sqrt{26} \log. \frac{D}{D_o} \left(\text{directed along } OD \right)$$

$$B_1'C_1' = \sqrt{10} \log. \frac{n}{n_o} \left(\text{directed along } ON \right)$$

$$C_1'B = \log. \frac{P_e}{n_o^3 D_o^5} \left(\text{directed along the axis of } \log. \frac{P_e}{n^3 D^5} \right)$$

For $n=n_o$ and $D=D_o$ the vectors $A_1'B_1'$ and $B_1'C_1'$ are zero. The points $n=n_o$, $D=D_o$ are the origins of the vectors $\sqrt{10} \log. \frac{n}{n_o}$, $\sqrt{26} \log. \frac{D}{D_o}$.

The angles α_1 and α_2 defined by the relations (31) are too large and lead to ill-proportioned diagrams. We shall substitute for them the angles α_1' and α_2' such that:

$$\text{tang. } \alpha_1' = \frac{3}{2}, \text{ tang. } \alpha_2' = \frac{5}{2} \dots \dots \dots (34)$$

The angular coefficients of the axes ON and OD are one-half less than the preceding, defined by the relations (31).

To this end it is sufficient to plot the ordinates on a scale one-half that of the abscissae. The ordinate of a point on the axis of n , instead of being equal to 3 times the abscissa of this point is only 3/2 times.

In the place of equations (33) we shall substitute the following:

$$\left. \begin{aligned} \log. \frac{V}{nD} &= \log. \frac{V}{n_o D_o} - \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{2} \log. \frac{n}{n_o} - \frac{2}{\sqrt{29}} \cdot \frac{\sqrt{29}}{2} \log. \frac{D}{D_o} \\ \log. \frac{P_e}{n^3 D^5} &= \log. \frac{P_e}{n_o^3 D_o^5} - \frac{3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{2} \log. \frac{n}{n_o} - \frac{3}{\sqrt{29}} \cdot \frac{\sqrt{29}}{2} \log. \frac{D}{D_o} \end{aligned} \right\} (35)$$

$\log. \frac{V}{nD}$ and $\log. \frac{V}{n_o D_o}$ are represented according to the same scale as before, OA_1' for example, in the two cases. $\log. \frac{P_e}{n^3 D^5}$ and $\log. \frac{P_e}{n_o^3 D_o^5}$ are represented according to a scale one-half less (for example, $C_1''B' = \frac{C_1'B}{2}$). As to the axes of n and of D , they have angular coefficients which are one-half of the preceding. They have

the directions designated by ON' and OD' in figure 4. The vectors which are laid out along these axes have for magnitude:

$$\frac{\sqrt{13}}{2} \log \frac{n}{n_0} \text{ and } \frac{\sqrt{29}}{2} \log \frac{D}{D_0}$$

The new contour is $OA_1' B_1'' C_1'' B'$, which is equivalent to the contour OAB' , the point B' being a point of the logarithmic polar, of which the ordinates are laid off to a scale one-half that of the abscissæ.

In practice, we have given directly the speed V and the power P_e . It is then necessary to inscribe on the various points on the axis of $\frac{V}{nD}$ and of $\frac{P_e}{n^2 D^3}$ the corresponding values of V and of P_e .

Let us take an example. Suppose that, on the axis of $\frac{V}{nD}$ ($\log \frac{V}{nD}$), we have marked at a given point $\frac{V}{nD} = 0.7$. The vector comprised between the origin and this point represents $\log (0.7)$. In a proposed problem, a certain velocity of translation is given, such that, in laying out, on the axis of $\frac{V}{nD}$, a suitable vector, we should find the point 0.7. If this is so, the speed V should have the value derived from the equation

$$\frac{V}{n_0 D_0} = 0.7$$

in which

$$n_0 = 800 \text{ revolutions per minute} = 13.33 \text{ revolutions per second.}$$

$$D_0 = 3 \text{ meters.}$$

From this we find

$$V = 0.7 \times 3 \times 13.33 = 28 \text{ meters per second} = 100.8 \text{ kilometers per hour.}$$

On the axis of $\frac{V}{nD}$, adjacent to the division 0.7 we write the number 100.8. If then at any time we have a speed of 100.8 kilometers per hour, we know that the vector $\log \frac{V}{n_0 D_0}$ which must be laid off on the axis of abscissæ, will be such that its origin is at the point O , while its extremity is at the point marked 0.7 or 100.8.

Following the same principle, the axis of ordinates is graduated in horsepower. Let it be desired to find the point on this axis to which corresponds a power $P_e = 100$ horsepower. In the construction of the broken line we have to trace the vector $\frac{P_e}{n_0^2 D_0^3}$, in which $P_e = 100 \times 75 = 7,500$ kilogram-meters.

$$n_0 = 13.33 \text{ revolutions per second; } D_0 = 3$$

We have then

$$\frac{7,500}{13.33^2 \times 3^3} = 0.013$$

Adjacent to the point already marked 0.013 on the axis of ordinates, we write the number 100.

6. STUDY OF THE PROPERTIES OF SCREW PROPELLERS—THE DIAGRAM OF THE AEROTECHNIC INSTITUTE OF SAINT-CYR.

At the Aerotechnic Institute of Saint-Cyr, for each propeller, there are first made certain observations with the propeller held at a fixed point (not propelling the car) and the following curve is then constructed:

Abscissæ N = revolutions per minute.
 Ordinates Π_o = tractive pull in kilograms.
 $P_e^{(o)}$ = horsepower on shaft.

Following this, with the propeller used to propel the car, similar measures are taken.

For a speed = V and revolutions per minute = N or revolutions per second = n , let the traction or thrust = Π and power on the shaft = P_e . We then compute, for the same rotative speed of the propeller, the ratios

$$\frac{\Pi}{\Pi_o}, \frac{P_e}{P_e^{(o)}}, \rho = \frac{\Pi V}{75 P_e}$$

It is assumed, as a sufficient first approximation in practice, that these ratios are simple functions of $\frac{V}{nD}$. Curves are next plotted, for which the values of $\frac{V}{nD}$ are abscissæ and the values of the preceding ordinates.

Let us consider, for a given type of propeller, the ratios:

$$\alpha_o = \frac{\Pi_o}{n^2 D^2}, \beta_o = \frac{P_e^{(o)}}{n^3 D^5}$$

According to Col. Charles Renard, who was the first to propose the use of these expressions in the study of the screw propeller at a fixed point, these ratios, for each type of propeller, are constant; they are the same for all similar propellers deduced from the type. If this is so, the ratios $\frac{\Pi}{\Pi_o}, \frac{P_e}{P_e^{(o)}}$ are proportional to the magnitudes $\frac{\Pi}{n^2 D^2}, \frac{P_e}{n^3 D^5}$, considered by M. Eiffel.

But if the coefficients α_o, β_o of Col. Renard, $\frac{\Pi}{n^2 D^2}, \frac{P_e}{n^3 D^5}$ of M. Eiffel, vary with n , it may be assumed that this variation will remain nearly the same at all speeds of translation. The curve which represents $\frac{\Pi}{\Pi_o}$ as a function of $\frac{V}{nD}$ may be the same, whatever the value of n , so long as the curve representing $\frac{\Pi}{n^2 D^2}$ as a function of $\frac{V}{nD}$ varies with n . This is the reason for the mode of representation adopted at the Aerotechnic Institute of Saint-Cyr.

CHAPTER IV.

THE RESULTS OF EXPERIMENT.

1. WITHIN WHAT LIMITS MAY THE RESULTS OF EXPERIMENTS MADE ON MODELS BE APPLIED TO FULL-SIZED MACHINES?

We have seen, Chapter II, that experimenters have studied the problem of the resistance of the air either on reduced size models, or on models nearly full size or on actual full-size models. The most complete and important results are those obtained by M. Eiffel on models.

One question presents itself immediately: Are all these experimental results comparable among themselves? To what extent, for actual airplanes, may we use the results of experiments carried out on a reduced scale?

Consider a body surrounded by the air and having a velocity of translation V relative to it. If S is a suitably chosen surface, distinctive or characteristic of the form or design, the resistance of the air may be expressed by the equation:

$$R = KS V^2 \dots \dots \dots (1)$$

K being a coefficient of the nature of a density.

We may first note a law sufficiently exact in a great number of cases, and which was formulated in the seventeenth century by Huyghens, Mariotte, and Pardies as follows:

If the density of the air remains the same (experiments carried out in air at sensibly the same temperature and pressure), the coefficient K depends solely on the form of the body studied.

For similar bodies, the coefficient K is constant, whatever may be the value of the velocity V . The realization of an experiment on a reduced scale similar to an experiment full size is easy. We may choose at will the scale of the model and the velocity for the experiment.

We have assumed that the body is given a movement of translation relative to the air; such a restriction is not essential. The movement of the body in the air may be more complex, accompanied, for example, by rotation; such is the case of a screw propeller. In any such case, however, the peripheral velocities for the model and for the full-sized machine should be in the same ratio as the velocities of translation or advance.

For sustaining propellers, the speeds of advance are zero, and this condition is fulfilled. If D is the diameter of the propeller and n the number of revolutions in unit time, if S is the area of the circle swept by the blades, we draw readily from (1) the laws announced in 1903 by Col. Charles Renard; viz, for similar propellers we have:

$$\frac{\Pi_o}{n^2 D^4} = \text{constant}; \quad \frac{P_u^{(o)}}{n^3 D^5} = \text{constant}$$

where Π = traction or thrust of propeller and P_u = useful power expended.

It follows immediately, from what has been said above, that these formulæ may be extended, with different coefficients, to *propulsive* propellers when the combination of speed of advance and of rotation gives, for homologous points, speeds equally inclined to the axis.

In other words, provided the values of $\frac{V}{nD}$ are the same for two propellers geometrically similar, the results of experiments made on one of them are applicable to the other. We may apply to traction, screw propellers geometrically similar to equation (1) by considering that the coefficient K is a function of $\frac{V}{nD}$.

For the wings of an airplane and for airplanes complete, M. Eiffel assumes that K is a simple function of the form of the body under investigation. It is on this assumption that the formulæ of paragraph 1 of Chapter III have been established.

M. Eiffel bases his conclusions in this regard on a comparison of results of tests made, on the one hand, by Commandant Dorand on a biplane in free flight, and, on the other hand, by himself by means of a fan on a model of this airplane built to a scale of 1/14.5.

Let us denote by $R_x V^2$ the drift (equal to the thrust of the propeller) measured on the airplane full size. Let us call μ_x the coefficient by which it is necessary to multiply the results of the experiments on the full-sized machine in order to pass to the model. The drift of the model at a scale of 1/14.5 will be, at the velocity V , equal to

$$\frac{1}{\mu_x} \times \frac{R_x V^2}{(14.5)^2}$$

This drift, brought to a speed of 10 meters (32.8 feet) per second, has for value,

$$r_x' = \frac{1}{\mu_x} \times \frac{R_x V^2}{(14.5)^2} \times \left(\frac{10}{V}\right)^2 \dots \dots \dots (2)$$

Similarly, the thrust on the model, calculated from the thrust $R_y V^2$ measured on the airplane and brought to the speed of 10 meters per second, has for value,

$$r_y' = \frac{1}{\mu_y} \times \frac{R_y V^2}{(14.5)^2} \times \left(\frac{10}{V}\right)^2 \dots \dots \dots (3)$$

These values r_x' and r_y' may be compared with the values r_x and r_y measured directly on the model placed before the fan.

From this the following values were found:

$$\begin{array}{ll} \mu_x = 0.99 & r_x' = \frac{1}{0.99} r_x \\ \mu_y = 1.01 & r_y' = \frac{1}{1.01} r_y \end{array}$$

M. Eiffel believed that he was justified in concluding from these results that the law as stated above (law of similitude, supposing K constant) was verified within 1 per cent. We do not consider the conclusion justified. The measurements of M. Eiffel and those of

Commandant Dorand were not made under the same conditions. In those of M. Eiffel the model of the propeller was fixed; in those of Commandant Dorand the propeller revolved before sustaining planes. Now the wind from the propeller has on the wings a certain action; it therefore seems to result that the agreement within 1 per cent between the results of M. Eiffel and those of Commandant Dorand shows that if the experiments had been carried out under conditions really comparable, the results obtained would have shown a definite divergence.

Comparative experiments on surfaces and their models (ratio of 1/10) have, furthermore, been made at the same relative velocity at the laboratory of Saint-Cyr (method by car) and at the Eiffel laboratory. For varying values of the angle of incidence i , the values of K_x and K_y were plotted for a certain number of surfaces. The following results were obtained:

(a) The curves for K_y have the same general form; those determined by the car are, in general, above those determined from the model; they are, moreover, readily distinguished from them.

(b) The curves for K_x likewise have the same general form; those determined by the car are sometimes above and sometimes below the others. On the whole, it is very difficult to draw any conclusions from these divergencies. On the one hand the forces to be measured are very small; on the other, with the surfaces parallel to the current of air, the nature of the surfaces (duck for the airplane at Saint-Cyr and well polished wood for the model), with their roughness or slight deformations, may assume the highest importance.

(c) As regards the centers of thrust (intersection of the resultant of the air resistance with the chord of the profile of the surface in the plane of symmetry), the agreement is in general satisfactory. However, for certain surfaces, M. Eiffel finds this point a little farther from the attacking edge than the Institute of Saint-Cyr; for other surfaces, it is the inverse.

According to this result, it does not seem that M. Soreau is correct in assuming that the law of similitude is less exact in regard to the location of the resultant of the resistance than in regard to its magnitude. However, we shall find later certain other experimental results which seem to justify this conclusion.

(d) As to the distribution of the pressure (both above and below normal) over the two faces of the surfaces, the general agreement is quite satisfactory. The systematic divergence is marked by this fact that the values of the drop of the pressures below normal (negative pressure or suction) on the upper face are greater for the car than for the fan, which agrees with the comparison of the values of K_y . For certain angles, near the attacking edge, the experiments with the car indicated a relatively greater value of the drop below normal pressure.

In the comparison of the results obtained with the surfaces tested full size on the car and with models 1/10 tested by the fan (same relative velocity), there was found the same general form of curve for the results, with the vertical force appearing systematically a little greater for the car than for the fan.

M. de Guiche deduces from his experiments on curved aerofoils the following conclusions:

Two aerofoils of the same spread, but with different profiles geometrically similar, are not comparable.

He has studied the two following forms:

(a) Thick aerofoil (maximum thickness, 101 millimeters (4 inches)) with circular curvature.

Chord=120 centimeters (47.3 inches).

Maximum height of mean line segment+length of chord=1/10 (mean line segment=intersection of plane of symmetry with the surface equidistant between the two faces, upper and lower).

Maximum height of upper line segment+length of chord=1/7 (upper line segment=intersection of plane of symmetry with the upper face of aerofoil).

Spread=170 centimeters (68 inches).

Radius of curvature of the upper face=130 centimeters (51 inches).

Radius of curvature of the lower face=252 centimeters (99.3 inches).

The two surfaces are joined by half circles of 14 millimeters (56 inches) radius.

(b) Aerofoil of which the section is a proportional reduction of the preceding, in the ratio of one-half.

Maximum thickness=50 millimeters (2 inches).

Spread=170 centimeters (68 inches).

Chord=60 centimeters (23.7 inches).

Radius of curvature of upper face=65 centimeters (25.5 inches).

Radius of curvature of lower face=126 centimeters (49.7 inches).

Radius of connecting circles=7 millimeters (0.28 inch).

These similar profiles are subject to reductions below the normal pressure very different at corresponding points. For the same angle of incidence there is no relation between the distribution of pressures above and below normal on the two faces. The variations of the thrust and the center of the thrust are very different in the two cases. On form (a) the point of application of the resultant advances constantly toward the attacking edge as the angle of incidence increases up to 25° (limit of the experiment). On form (b), on the contrary, the point of application of the resultant advances toward the attacking edge until the angle i =about 12° , then for values of i greater than 12° it returns toward the center of the wing.

The curve of the values of K_p is sensibly the same for the two forms, but the curve of the values of K_v takes very different forms in the two cases. For form (a) the values of K_v increase constantly up to $i=25^\circ$. For form (b) K_v passes through a maximum for $i=12^\circ$. This appears to be the critical angle for this form; it marks at the same time the maximum of K_v and the most advanced position of the force.

We have insisted on these results of experiment which show clearly that, for wings and for airplanes, the law of similitude based on an assumed constant value of K is not exact.

Nevertheless, we believe that this law is sufficient for the guidance of constructors regarding the aerodynamic qualities of aerofoils, or of complete airplanes for which the design is in hand.

Regarding screw propellers, this law is not of itself sufficient. It is not enough that the experiments on a propeller full size and on its model should be made in such manner that $\frac{V}{nD}$ shall have the same value. It is necessary, further, that the speed of advance V and, in consequence, the peripheral speed πnD shall be the same. Equation (1) is applicable for propellers moving forward through the air on condition of considering K as a function not only of $\frac{V}{nD}$ but of V .

It is thus necessary, in order to keep the revolutions of the propeller within an upper limit of 3,600, to hold to a scale not exceeding one-third for the models of screw propellers.

Theoretical considerations developed by M. Jouguet, and which would require too much space for extended notice here, show that this departure from the law of Huyghens, Mariotte, and Pardies is due to the compressibility of the air, which becomes of importance at the high relative velocities of the blades and the air which are realized in the case of screw propellers.

These same theoretical considerations show that, for the speeds of advance used in aviation (25 to 30 meters (82 to 98.4 feet) per second), a more exact law of similitude may be realized by corrections for the perturbation due to the viscosity of the air. To this end it would serve to test the model at a speed equal to that of the airplane multiplied by the linear ratio between airplane and model.

If V is the speed of advance of the airplane and v that of the model at the scale ratio of $\frac{1}{\lambda}$, we have for the resistances of the air

$$\frac{R_V}{r_v} = \frac{K_V}{K_v} \lambda^2 \left(\frac{V}{v} \right)^2$$

If then we take $\frac{V}{v} = \frac{1}{\lambda}$ and $K_V = K_v$, we shall have $R_V = r_v$.

2. THE DISPLACEMENT OF A BODY UNDER TEST THROUGH THE AIR AND THE MOVEMENT OF THE AIR WITH RESPECT TO A FIXED BODY.

M. de Guiche has found for certain surfaces, notably for planes, distributions of pressure differing from those found by M. Eiffel. The latter has found, on the following face, only zones of pressure below the normal. M. de Guiche, on the contrary, for angles of incidence less than 20° , has found zones of pressure above the normal. Now the experiments in the two cases are very numerous and have been made with the greatest care. The differing results are, then, due to phenomena which appear in one of the modes of experimentation and not in the other.

The principle of relative motion is, in fact, not in question. What is in doubt is, whether, by the method of the tunnel, conditions can be developed which permit the assumption of equal relative motions in the two cases.

Let us consider a solid which moves with a velocity V in an indefinite medium originally at rest. At a given moment, far from the origin of movement, we may distinguish two regions in the space surrounding the body under investigation. One, formed of regions far removed from the solid, is not disturbed by the passage of the latter. At each of these points there is a velocity relative to the moving body of $-V$, equal, parallel and in the opposite direction to the velocity of translation V . The other region, situated near the body, is disturbed by the movement of the latter. Relative to the moving body, each point in this region has a velocity which is the resultant of

(a) A velocity $-V$ equal, parallel, and opposite in direction to the velocity of the body.

(b) A velocity W due to the motion of the solid body.

This last velocity is a function of the disturbance brought into the medium by the passage of the body. It depends, furthermore, not

only on the form of the body, on its dimensions, on the degree of polish of its surface, but also on the properties of the medium, its viscosity, and the initial conditions of the movement.

Let us now assume the solid fixed and the surrounding medium to be displaced relative to it. This mode of experimentation should realize the conditions which require the application of the principle of relative movement. For this it is necessary that, from the moment when permanent conditions are established, the velocity at each point in the medium disturbed by the presence of the solid should be equivalent to the resultant of the two velocities — V and W .

It is assumed in the method of the tunnel that such a resultant is obtained by giving to the fluid, in the undisturbed parts, a velocity — V . In particular, in the experiments in the midst of a moving column of air, it is assumed that it suffices to draw the air into one of the extremities of the tunnel sufficiently removed from the body placed in the interior. Now this fact does not imply that the velocity, in the disturbed part, is equivalent to the resultant of the two velocities — V and W .

The viscosity of the fluid may intervene in such manner as to cause the state of the disturbed region to depend on the mode of producing the relative motion. Experiment alone can decide.

Now, as we shall see at a later point, comparison of the experiments of M. de Guiche and of M. Eiffel shows that the phenomena observed in the method with the tunnel are not always identical with those which result from the movement of a solid body in free air.

We should note, however, that the study of pressures below normal made at the Institute of Saint-Cyr (method by use of car) do not show, in comparison with the experiments of M. Eiffel, the differences which M. de Guiche has noted. The question involved here does not seem to be completely elucidated. In particular it seems desirable that M. de Guiche should reproduce certain of his experiments with his apparatus at Saint-Cyr in order to establish the atmospheric pressure in the free branches of the manometers used for measuring the pressures above and below the normal.

In any case, the solution of such a question can be of interest only from the viewpoint of the science of aerodynamics. It does not present the same interest from the viewpoint of practical aviation. From the latter viewpoint, we consider as equivalent the two methods of producing the relative movement between the air and the body under investigation.

3. THE AERODYNAMICS OF THE PLANE—PLANES ORTHOGONAL.

When the body is a thin orthogonal plane we may, in practice, take for the coefficient K of equation (1) the value $K_{90} = 0.080$ (0.0015 pound second-feet). M. Eiffel has found that this coefficient varies

(a) With the extent of the surface;

(b) For a surface of given extent, with the form of the contour.

M. Soreau has proposed, in order to represent the various values found by M. Eiffel, the empirical formula:

$$K_{90} = \frac{0.0888 S^{0.06}}{1 + 0.116 S^{0.06}}$$

The surfaces on which M. Eiffel has operated are, in general, too small; the perturbations caused by the influence of the bound-

aries are preponderant. It would be necessary to operate on surfaces of which one of the sides has a length greater than 1 meter (3.28 feet). With such surfaces, steady conditions, characterized by isobars, parallel to the sides of the plate, would be established and would assume the preponderance. We might thus determine for K_{90} a value characteristic of this regular regimen, and which would be independent of the area of the plate and of the form of its contour.

Experiments over so wide a field have not been made. However, M. de Guiche, studying a rectangular plate 100 by 60 centimeters (39.4 by 23.6 inches), found $K_{90}=0.083$.

In order that the thickness of the plate normal to the direction of relative motion may have any influence, it is necessary that this thickness be of the order of the transverse dimensions of the plate. From the point where the thickness of the plate is equal to one of the transverse dimensions, the coefficient K_{90} varies, first decreasing and then increasing as the thickness increases.

M. Eiffel has studied, by means of the fan, the combination formed by two thin orthogonal planes separated by a variable distance. He has been led to the following conclusions, applicable, for example, to disks 30 centimeters (11.8 inches) diameter:

(1) The force on the combination diminishes, first, in proportion as the separation increases. It passes through a minimum for a certain value of the separation, and then increases again.

(2) The force on the combination does not begin to exceed the force on an isolated plate until the plates are very considerably separated.

(3) The force on the combination has always been less than the sum of the forces on each of the plates taken alone. The effect of shielding due to the forward plate makes itself felt, even when the plates are very far apart.

(4) The pressure on the plate exposed directly to the wind is approximately independent of the separation, but it always moderately exceeds the pressure on an isolated plate.

(5) The shielded plate is first drawn toward the forward plate. This attraction varies, first increasing in absolute value with the separation of the plates, passing through a maximum corresponding to the minimum of the total force, and then decreases to zero. This attraction is then changed into repulsion, of which the absolute value increases with increase in the separation of the plates.

(6) On the plate directly exposed to the wind, the mean pressures on the leading face (above normal) and on the following face (below normal) are sensibly independent of the separation of the plates.

(7) The mean pressures on the leading and following faces of the shielded plate are below normal, so long as the plates are not very far apart.

The mean drop in pressure on the leading face is, in absolute value, at first greater than that on the following face.

The total mean force on the plate is such as to tend to bring together the two plates (attraction for the shielded plate).

For a suitable separation of the two plates, not only the mean forces but also the forces at each point on the shielded plate are sensibly zero, and the resultant mean force is zero.

Finally, when the separation of the plates increases beyond a certain limit, the mean total pressure on the shielded plate becomes such as to tend to produce further separation (repulsion of the shielded plate).

4. AERODYNAMICS OF THE PLANE—PLANE INCLINED TO THE DIRECTION OF RELATIVE MOVEMENT—PLANE ISOLATED.

(a) DISTRIBUTION OF PRESSURES, ABOVE AND BELOW NORMAL.

(1) As soon as the plane exceeds certain dimensions there is found, on both the leading and following faces, a central zone where a regular regimen becomes established. The existence of this zone is shown by isobars parallel to the attacking edge. In this entire zone the phenomenon is defined by the distribution of the pressures along the line of steepest gradient. The study of this serves, in many cases, to characterize a surface.

M. Eiffel has studied only this mode of distribution of pressures above and below normal. The dimensions of the sides of the planes which are normal to the wind were, in some cases, too small and the experimenter was able to observe only the pressures in the disturbed zone near the boundaries of the plane.

(2) The boundary zones of disturbance have a width sensibly constant, which is, for the leading edge, some 20 centimeters (7.9 inches), and for the following edge, from 40 to 50 centimeters (15.8 to 19.7 inches). In consequence, in order that the regular regimen may be in evidence the plane should have a spread at least twice the width of the zones of disturbance.

(3) Special study of the leading face of plane:

Angles of incidence less than 20° .

Bourlet has proposed the following formula deduced from theoretical considerations:

Total pressure on leading face:

$$P = \frac{4}{3} \frac{C}{100} (\sin i)^{0.4} S l^{-1} V^2$$

System of units, kilogram-meter-second.

S = area.

l = depth in direction of wind.

V = speed.

C = coefficient which depends on the form of the plane.

C is of the form.

$$C = A - B \frac{L}{S}$$

where L = perimeter of plane.

M. de Guiche has found by experiment:

$$A = 3.8 \quad B = 0.11$$

With these values the formula becomes:

$$P = \left(0.050 - 0.0014 \frac{L}{S} \right) (\sin i)^{0.4} S l^{-1} V^2$$

For surfaces sufficiently large

$$P = 0.050 (\sin i)^{0.4} S l^{-1} V^2$$

This formula is not applicable for angles of incidence, i , such that the pressures measured along a line of greatest slope become less than normal. The pressures, thus measured, decrease in fact continuously from the forward edge. They may even become negative in the vicinity of the following edge. The existence of pressures below the normal comes into evidence for smaller angles of incidence as the ratio of the fore and aft to the transverse dimensions is smaller. In the experiments of M. de Guiche it was determined that such pressure below normal began to appear as follows:

For surface 180 by 120 centimeters (70.87 by 47.24 inches) from $i = 20^\circ$.
 For surface 180 by 80 centimeters (70.87 by 31.50 inches) from $i = 10^\circ$.
 For surface 180 by 40 centimeters (70.87 by 15.75 inches) from $i = 8^\circ$.

The lateral turbulent zones show pressures less than those found at an equal distance from the attacking edge.

(4) Special study of leading face of plane:

Angles of incidence exceeding 20° .

The formula of Bourlet is not applicable. The maximum pressure is no longer at the leading edge. This becomes, when the angle increases, a zone of lesser pressure like the three other sides.

(5) Special study of following face of plane:

Angles of incidence less than 20° .

In the regular zone (isobars parallel to the attacking edge), for depths of plane sufficient to render negligible the influence of the boundaries, the pressure at a given point and for a given angle of incidence appears to be a function solely of its distance from the attacking edge.

M. Eiffel has not observed counter pressures on this back face of the plane. On the other hand, M. de Guiche has very clearly observed them. This is one of the differences between the results obtained by one or the other of the methods of experimentation.

(6) Special study of following face of plane:

Angles of incidence exceeding 20° .

There are no longer any counter pressures on the following face. The distribution of the pressures below normal becomes quite uniform.

(b) TOTAL FORCE.

(1) According to M. Eiffel, the total force passes through a maximum for an angle of incidence of about 37° .

According to M. de Guiche the total force increases up to angles of 45° ; beyond that it remains sensibly constant.

For angles of inclination comprised between 0 and 10° , rectangular plates for which the transverse dimensions are greater than the fore and aft are subject to the greatest total pressures.

For angles less than 10° , the following formula may be taken:

$$\frac{K_i}{K_{90}} = \left(3.2 + \frac{n}{2}\right) \frac{i}{100} \quad (i \text{ in degrees})$$

(2) The center of pressure, that is to say, the point where the line of the resultant force intersects the thin plane, is at the center of the plate when the plane is perpendicular to the line of relative move-

ment. In proportion as the inclination decreases, the center of pressure advances toward the leading edge, even to the smallest values of the angle.

(3) The resultant of the air pressures is, for inclinations exceeding about 10° , sensibly perpendicular to the plane. For smaller angles of inclination the angle of the resultant with the perpendicular to the line of relative motion is greater than the inclination of the plane; the resultant is inclined to the normal to the plane behind this normal.

(4) Thick plates.

If the thickness of a plane plate is increased, leaving the ends plane and at right angles with the two faces, there is introduced but slight change as compared with the phenomena described for thin planes, except that the head resistance is increased as the thickness is made greater.

(5) Thick plane plate with leading or following edge provided with cutwater.

With a cutwater on the following edge, the ratio drift/lift is notably less than without. It is more advantageous to find the following than the leading edge.

It is preferable to have the forward cutwater edge toward the lower rather than the upper side.

Finally, the law of variation of the center of force is entirely different from that indicated above for planes.

5. AERODYNAMICS OF THE PLANE—PLANES IN TANDEM.

M. de Guiche has experimented with three elements of aluminum of 1 meter (39.4 inches) spread, 20 centimeters (7.9 inches) fore and aft width, and 8 millimeters (0.32 inches) thickness. He has studied the distribution of the pressures along the median line of maximum gradient, forward and aft, in the three following cases:

- (a) Elements in contact (interval zero).
- (b) Elements separated by interval of 5 centimeters (1.97 inches).
- (c) Elements separated by interval of 10 centimeters (3.94 inches).

(1) Study of forward face:

The character of the variation of the diminution of the pressures from the leading to the following edge is the same for the separated elements as for those in contact. The pressure diminishes continuously from the leading to the following edge.

Second element: If we consider the second or middle element, the maximum pressure in the vicinity of the leading edge does not become sensibly equal to that for the leading edge of the first element unless the separation of the two elements is equal to 10 centimeters (3.94 inches). For the distance of 5 centimeters (1.97 inches) the maximum pressure on the second element is inferior to the maximum pressure on the first element. As regards the forward face, then, the second element does not behave as if it were alone unless the distance of the two elements is equal to 10 centimeters (3.94 inches).

Third element: The maximum pressure near the leading edge is always clearly less than that for the second element.

Total pressure on forward face: For angles of incidence less than 15° , it is less for the separated elements than when they form a continuous surface.

For angles between 15° and 25° , it is greater for the separated than for the continuous elements.

For angles greater than 25° , it is sensibly the same for the separated as for the continuous elements.

(2) Study of rear face:

The first element shows pressures below normal, similar to those for the isolated plane (no pressures above normal).

On the second element, similarly, only pressures below normal are observed. For small angles of incidence (4° to 6°) the drop below normal pressure near the forward edge is much more pronounced than for the first element.

This relation is less marked in proportion as the angle of incidence is increased; it is even reversed for angles exceeding 30° .

On the third element, positive pressures are observed for very small angles of incidence and then pressures below normal for greater angles.

As to the total drop in pressure on the rear face, it is in general less for the separated than for the continuous elements.

(3) Total force.

This is, in general, less for the separated than for the continuous elements.

The center of pressure of the combination, defined by its distance from the forward edge of the first element, changes less (with variable angle of incidence) for the elements with separation than when continuous.

6. AERODYNAMICS OF PLANE AEROFOILS, ARRANGED STEPWISE.

M. de Guiche has studied the disposition of planes stepwise.

The planes are parallel; the leading edge of each element is on the normal passing through the following edge of the preceding element.

The steps are arranged in two ways: Direct, if in going in the direction of the wind one descends the steps; reverse, if in going in the direction of the wind one mounts the steps.

The planes of brass, which M. de Guiche has used, have a spread of 100 centimeters (39.4 inches), a depth of 12 centimeters (4.7 inches), a thickness of 4.5 millimeters (0.18 inches); the separation of the planes was either 2 centimeters (0.79 inches) or 4 centimeters (1.58 inches). The number of planes was three.

(1) Planes direct: Forward or lower face.

First element: Diminution of pressure from the leading to the following edge.

Second element: Inversely as compared with the case of planes in tandem, the leading edge is subject to a notable negative pressure and the maximum positive pressure is produced to the rear of the leading edge.

Third element: The maximum positive pressure is produced at a point lying behind the leading edge, but, in general, the leading edge is not subject to a negative pressure.

These phenomena (relative to the second and third elements) become more pronounced for the separation of 4 centimeters than for that of 2 centimeters.

(2) Planes direct: Following or upper face.

First element: This is subject to a negative pressure. It behaves nearly as though it were alone.

Second and third elements: The negative pressures are less marked than for the first element. There are even positive pressures produced near the following edge.

(3) Planes reverse: Forward or lower face.

This system has been studied at angles of incidence of 6° , 8° , and 15° , and for a separation of the elements equal to 4 centimeters (1.58 inches).

First element: If for an incidence of 6° we consider the displacement cylinder of the first element, it is seen that the second and the third elements project into this cylinder. There results, for this element, a diminution of pressure on the leading edge. The latter is no longer subject to the maximum pressure, which is carried aft. This phenomenon disappears at incidences of 8° and 15° . The decrease in the pressure then follows continuously from the leading edge aft.

Second and third elements: For angles of incidence of 6° and 8° , the phenomena are the same as for the steps in direct form.

At 15° , the second element is almost shut out by the first; this element is subject entirely to a negative pressure. The third element is subject in part to positive pressure; there is, however, negative pressure near the leading edge.

(4) Planes reverse: Rear or upper face.

At the incidence of 8° , the negative pressure on the second and third elements is marked near the leading edge; it is from 1.5 to 2.5 times as great as on the first element.

There is here a phenomenon similar to that which has been observed with Venturi tubes disposed in series.

(5) Total force.

The total force on the system is less than for a continuous plane of the same surface.

The drift is more considerable on account of the more numerous edges.

This type of combination is not to be recommended.

M. Eiffel has determined the total force and the drift for planes parallel to each other and without stepwise interval. We shall return later to the consideration of curved aerofoils, for which the results are similar.

7. AERODYNAMICS OF CURVED AEROFOILS—ISOLATED.

There has been made in France a very considerable number of experiments on curved aerofoils, isolated.

M. Eiffel has operated on models; a very considerable number of them have a spread of 90 centimeters (35.5 inches) and a depth along the chord of 15 centimeters (5.9 inches) (aspect ratio = 6). He has not used aerofoils of less than 45 centimeters (17.8 inches) spread with the same depth as above. M. Eiffel considers the mean surface; that is, the surface equidistant from the two actual surfaces of the

aerofoil. The nominal height of the curved contour is the ratio of the height of the arc for this mean surface to the chord.

M. de Guiche has studied aerofoils for which the spread is 170 centimeters (67 inches) and the depth along the chord 120 centimeters (47.3 inches) (aspect ratio = 1.4). However, in certain cases the depth was reduced to 60 centimeters (23.6 inches).

At the Institute of Saint-Cyr there have been studied aerofoils for which the spread varies from 5 to 10 meters (16.4 to 32.8 feet), the depth varying from 2 to 2.5 meters (6.56 to 8.40 feet).

(a) DETERMINATION OF THE PRESSURES POSITIVE AND NEGATIVE.

(1) Beyond a certain value of the spread there develops a regular regimen, involving the entire surface with the exception of two lateral turbulent bands. This regimen is indicated by isobars parallel to the leading edge.

It appears that the fairly uniform width of the bands of turbulence does not exceed 20 centimeters (7.9 inches). It is desirable, therefore, to use no aerofoils with a spread less than 40 centimeters (15.8 inches).

(2) Each one of the faces of the aerofoils joins in the support, but not equally. The pressures supported by the lower face, at ordinary incidences of flight, assume a smaller share of the total force than the negative pressures on the back.

(3) The lower face is subject to the influence of the upper face, but is itself without influence on the latter.

(4) The curvature of the back determines the distribution of the negative pressures; it deviates upward the lines of air flow.

An exaggerated height of the arc for the curvature on the back produces a harmful counter pressure on the following edge of the wing.

The displacement of the maximum height of arc toward the leading edge carries a corresponding displacement of the maximum value of the negative pressure, and in consequence a reduction in the drift value.

The ideal would be to suppress in an aerofoil harmful pressures, positive or negative (that is to say, to have on the upper face only negative pressures and on the lower face only positive pressures), and to find the pressure of the atmosphere only at the following edge, where the lines of air flow join together without shock. The key of the problem seems to involve the maximum height of the arc and position between the leading and following edges.

(5) The negative pressures are not modified by the form of the leading edge; this has only a local influence. It has been said that this edge should be rounded, under a penalty of a reduction of the negative pressures on the back of the aerofoil. There is nothing to this. It is better to make it sharp in order to facilitate its penetration.

A French engineer, M. Constantin, has proposed a concave form for the leading edge. M. de Guiche found that this form did not show the advantages which its inventor had anticipated. However, it is only fair to say that by means of the fan, M. Eiffel arrived at an opposite conclusion.

(6) In general for angles involved in aviation, the modes of variation of pressures, positive and negative, are similar in all the results obtained by different experimenters.

The modes of variation of the negative pressures on the back fall into two principal types.

(a) The negative pressure starts from a certain value, often small, near the leading edge; it then increases continuously passing aft, passes through a maximum, then decreases regularly to the following edge.

This mode of distribution is found with thick aerofoils (monoplane type) of 80 to 100 millimeters (3.15 to 3.79 inches) maximum thickness.

(b) The negative pressure is very pronounced at the leading edge. Passing aft, the value decreases, passes through a minimum, then increases, passes through a second maximum, in general less than the first, and finally decreases regularly to the neighborhood of the following edge.

M. Eiffel has found such modes of variation with thin planes (biplane type) of 20 to 35 millimeters (0.79 to 1.38 inches) thickness.

The combination of the two modes (a) and (b) is found very marked in the case of aerofoils presenting steps on the back. The lower face has a regular curvature, but the upper face is formed stepwise. The combination thus constituted gives the impression of being formed of two or three aerofoils joined one behind another.

With one projecting ridge (up to incidences of 5°) there is found, in going from the leading to the following edges, a maximum of negative pressure, a minimum, and finally a maximum.

With two projecting ridges (up to incidences of 5°) there is found a maximum of negative pressure, a minimum, a maximum, a minimum, and finally a maximum.

For an incidence of 10° , the distribution is the same as for mode (b), but with several maxima and minima in the depth of the surface (one projecting ridge, two minima and a maximum; two projecting ridges, two minima and two maxima).

(7) We have now examined the mode of distribution of the negative pressures on the back of the aerofoil. If next we consider the lower face, there are found, in general, no negative pressures except near the following edge. On the remainder of the face the pressures are positive. In general, in passing from the leading edge aft, the pressure increases first a little, passes a maximum, and then decreases to the following edge.

However, certain aerofoils (with projecting ridges on the back) show, for angles of incidence near 0° , negative pressures near the leading edge.

(8) Two aerofoils of the same spread and with similar but not equal sections are not comparable.

(9) However, as pointed out by M. de Guiche, it is the aerofoil with the largest value of the aspect ratio which presents the most marked advantages. It is desirable that extended investigations should be made on aerofoils of different depths with the same spread—that is to say, on aerofoils with varying values of the aspect ratio and geometrically similar in section—in order to determine if for a given section, there is a best value of this ratio, and what is this value.

It may be noted that M. Eiffel considers 6 as this best value of the aspect ratio.

(6) Total resultant and point of application.

(1) The total resultant force continuously increases with the angle of incidence, at least within the range of interest in aviation.

As to the force center (intersection of the total resultant with the chord of the section) it approaches nearer and nearer to the leading edge as the angle of incidence increases from zero, at least within the range involved in aviation. This is the inverse of what takes place with a plane.

If we pass beyond the angles involved in aviation (angles less than 10°), the force center again recedes from the leading edge as the incidence increases.

(2) If aerofoils of varying thickness have the same surface of mean curvature, they are the more advantageous as the thickness is less.

M. Eiffel has shown, in effect, that under these conditions the ratio $\frac{K_x}{K_y}$ continuously increases with the thickness of the aerofoil.

It follows that if it is desired to compare the qualities of two aerofoils, it is necessary to use only forms with the same maximum thickness and with the same aspect ratio.

(3) A distinction may be drawn between thick and thin sections for aerofoils.

Thick aerofoils are suited more especially to monoplanes, because they must contain solid structural members. Such aerofoils, in general, have a thickness of about 90 millimeters (3.55 inches) at one-third and 50 millimeters (1.97 inches) at two-thirds of the depth from the leading edge.

Thin aerofoils are used for biplanes. Their maximum thickness varies between 30 and 90 millimeters (1.18 and 3.55 inches).

For good, thick aerofoils and for an angle of incidence $i=5.6^\circ$ we have $\frac{K_x}{K_y}=0.079$, with $K_x=0.0043$ and $K_y=0.055$.

These are values suited to a monoplane.

With $i=2.1^\circ$ we have $\frac{K_x}{K_y}=0.069$, with $K_x=0.0019$ and $K_y=0.027$.

But this angle is too small for a normal angle of incidence for a plane.

For a thin aerofoil of thickness equal to 63 millimeters (2.48 inches) and an angle of incidence $i=5.3^\circ$ we have $\frac{K_x}{K_y}=0.058$, with $K_x=0.0023$ and $K_y=0.040$.

For a thin aerofoil of thickness equal to 45 millimeters (1.77 inches) and an angle of incidence $i=8^\circ$ we have $\frac{K_x}{K_y}=0.091$, with $K_x=0.0055$ and $K_y=0.060$.

These values are suited to the wings of biplanes.

(4) The lateral edges of the wings exert a feeble influence on their quality. However, the trapezoidal form with the larger base behind seems more effective than the rectangular form.

(5) With certain forms of wing (wing provided along the leading edge with a concave edge forming a sort of crest, wing with thick,

rounded leading edge), M. Eiffel has found discontinuities in the curves of K_x and K_y in relation with different regimens of the flow of the air. But such phenomena are exceptional.

8. AERODYNAMICS OF CURVED AEROFOILS—COMBINATIONS OF CURVED AEROFOILS—ARRANGEMENT IN BIPLANE.

M. Eiffel has determined the varying values of R_x, R_y, F_x, F_y , for the various parts of a Dorand biplane, these parts being subject to experimental investigation assembled in complete form, or separate.

This biplane consists of a principal cell formed of two identical planes 14.5 by 2.25 meters (47.56 by 7.38 feet). These planes are separated in height by a distance of 1.95 meters (6.4 feet), that is, by a height sensibly equal to the depth of the plane. They are stepped a distance of 85 centimeters (2.79 feet), upper plane leading. They are joined by two series of 10 oblique struts. A forward equilibrator and a tail-plane element formed by two parallel planes are mounted on a cross-braced fuselage. These two elements are conjugate, constituting thus a secondary control, completely mobile. The tail-plane element forms with the principal planes a ∇ angle, plainly marked.

(1) With an apparatus thus formed, the sustentation for the cell alone is notably greater than for the biplane entire. The tail-plane element, far from aiding in the support, receives on the back the air deviated by the principal planes. It thus reduces the carrying power.

(2) The influence of the two principal parallel planes, separated by a distance sensibly equal to their depth, is evidenced by a loss of carrying power of about 20 per cent of that for the complete but isolated cell. If we denote the total carrying surface of the planes by S , the effective supporting surface of the biplane is $\frac{S}{1.2}$.

(3) The upper plane, in the presence of the lower plane, behaves as though it were isolated. The lower plane, under the influence of the upper plane, loses about one-third of the carrying power of the isolated plane.

In a biplane, the lower plane then operates poorly with regard to carrying power. It may, without inconvenience, be reduced, if such reduction brings other advantages.

(4) For the ordinary angles of incidence, the head resistance of all the parts aside from the principal cell is about 7 per cent of the sustentation. The head resistance of all parts aside from the planes is about 10 per cent of the sustentation.

These values are applicable to biplanes.

(5) The stepwise arrangement of the planes does not give any appreciable advantage with regard to sustentation and head resistance, but renders the construction more difficult.

Such a stepwise arrangement is not, in general, to be recommended.

9. THE AERODYNAMICS OF CURVED AEROFOILS—COMBINATIONS OF CURVED AEROFOILS—AEROFOILS IN TANDEM.

Let us consider two aerofoils of different spread, situated one behind the other (planes in tandem). If the aerofoil with the smaller spread is leading, it is said that the two elements form a "duck"

type. If the aerofoil with the lesser spread follows, it is considered equivalent to the "ordinary monoplane" type.

The angle made by the chords of the two aerofoils in the plane of symmetry is the angle of *décalage*, or simply the *décalage*, of one of these planes with relation to the other.

If, in the plane of symmetry, the leading edge of the following plane is in the prolongation of the chord of the leading profile, it is said that the vertical displacement of the two planes is zero.

M. Eiffel has made a series of experiments with two planes of the same type, of which one has an aspect ratio of 6 (90 by 15 centimeters (35.5 by 5.9 inches)) and the other of 3 (45 by 15 centimeters (17.8 by 5.9 inches)).

The "duck" type has been studied (vertical displacement zero) with values of the angle of *décalage* varying from 2° to 6° and separations of the planes equal to $\frac{4}{3}$ and $\frac{8}{3}$ their width.

The "ordinary monoplane" type has been studied with a *décalage* of 4° and separations of the elements identical with the preceding.

Finally, M. Eiffel has studied the tandem type with equal elements.

The angle of incidence of this combination of elements is the angle relative to the chord of the leading element.

(1) "Duck" type.

From the viewpoint of sustentation and of head resistance, it is advantageous to increase the separation of the elements, and not to exceed a certain angle of *décalage*.

Suppose that, for various angles of incidence, there have been determined the total resultants of the air forces on the combination of elements. We shall denote the aggregate of these by the term "bundle of resultants."

In the study of this bundle, the following results are obtained:

(a) The bundle is always located toward the middle of the interval which separates the two elements. It is then in this region that the center of gravity of an airplane of this type should be found.

(b) For a given distance between the elements, the bundle is so much the more extended as the *décalage* is greater.

(c) For a given *décalage*, the bundle is so much the more extended as the distance between the elements is greater.

This longitudinal change in the bundle of resultants has relation with the longitudinal stability of an airplane of this type.

(d) For a given distance between the elements, the bundle of resultants is displaced toward the forward element in proportion as the *décalage* is increased.

In an airplane of the "duck" type, if the *décalage* is increased it is necessary to move the center of gravity forward.

Let us suppose that in such an airplane the center of gravity is on the propeller shaft. For equilibrium under a certain angle of incidence, it is necessary that the resultant corresponding to this angle pass through the center of gravity. From this, let us drop normals on the other resultants of the bundle, resultants which correspond to varying angles of incidence. It is then easy to calculate the moments of these resultants with reference to the center of gravity. These are the stabilizing moments. They are considered positive when they tend to turn their lever arm in direction inverse to the movement of the hands of a watch. They are negative in opposite case.

Let us represent these stabilizing moments by setting off as abscissæ the angles of incidence, and as ordinates the stabilizing moments.

The airplane is longitudinally stable when, for increasing values of the angle, the curve of the moments descends continuously from left to right, cutting the axis of abscissæ at the point of equilibrium. It is unstable when, for increasing angles, the curve of moments rises from left to right.

When study is made of such curves for the "duck" type, it is seen that, from the viewpoint of stability, it is not well to realize too great an angle of décalage for the two elements.

The manageability of the airplane requires also that the décalage shall not be too great, and that the distance between the elements shall also not be too considerable. It is desirable that the stabilizing moments should not exceed 50 kilogram meters (361 pound-feet).

From this same viewpoint it is desirable that the center of gravity of the airplane should not be too low.

M. Biffel has studied a vertical displacement of the elements approximately equal to one-quarter of the depth of an element. The effect of such a displacement is so little sensible that it may be taken as negligible.

(2) "Ordinary monoplane" type.

The influence of the elements, one on the other, is evidenced by a reduction of the sustentation and by an increase in the head resistance in relation to the sustentation and to the head resistance of the elements without mutual action.

The relative diminution of the sustentation is independent of the fore and aft separation of the elements. The resultants are grouped on the forward element. The center of gravity of an airplane provided with such planes must be located in this region.

The bundle of the resultants is opened out considerably when the distance between the elements is doubled.

(3) Tandem with equal elements.

From the viewpoint of sustentation and of head resistance, this type is clearly inferior to the "ordinary monoplane" type.

In this last type it is therefore not advantageous to increase, beyond a certain limit, the spread of the tail-plane element.

The biplane arrangement is also preferable to the tandem type with equal elements.

It may be said that this last arrangement is not to be recommended in the construction of apparatus for aviation.

(4) In a tandem, the following element is influenced by the forward element. M. Biffel has studied the conditions of operation of such an element.

(a) If we designate by i_c the angle of incidence of the influenced element to the path of the combination of the elements, the force on this element is equal to the force which would be exerted on the element isolated, for which the angle of incidence would be

$$i_r = i_c - \beta$$

The angle β depends on all the factors which fix the relative positions of the elements, that is to say, on the distance between the elements, on the vertical displacement, on the angular décalage, and on the relative spread of the two elements.

(b) Whatever may be the characteristics of the combination of the elements, the drift of the influenced element is practically equal to that for the same element isolated.

From the viewpoint of the drift, there is no need of distinguishing between the real angle of incidence i_r and the apparent angle i_a .

(c) Whatever may be the characteristics of the combination of elements, the leading element of a tandem behaves like an isolated element.

(d) Case of tail planes.

When the law of variation of the real angles of incidence i_r in relation to the apparent angles i_a is known, and also the values of K_v as a function of the angles of incidence for the isolated tail plane, it is possible to determine the force acting on a tail plane placed behind an ordinary monoplane.

Let us take an example. Consider an ordinary monoplane of which the principal plane has dimensions of 10 by 2 meters (32.8 by 6.56 feet) and the tail element is formed by a plane 3 by 1 meters (9.84 by 3.28 feet) placed 5 meters (16.4 feet) behind the principal plane, with vertical displacement zero. Let us suppose that the angular décalage of the tail element (form V) with relation to the principal plane is 6° . If the normal angle for horizontal flight is 6° (angle of the principal plane with the horizontal trajectory) the apparent angle of the tail plane making a V with the principal plane is zero.

Let us assume that the law of variation of the real angles of incidence as a function of the apparent angles gives -5.4° for the real angle of incidence of the tail plane. The study of the plane gives then $K_v = -0.02$. The force on the tail plane, for a speed of 30 meters (98.4 feet) per second, is $-0.02 \times 3 \times 900 = -54$ kilograms (119 pounds).

Now if the tail plane were isolated and making with the trajectory an angle of zero, the force would be zero. Such a negative force of 54 kilograms (119 pounds) is of the greatest importance with regard to equilibrium.

10. THE APPARATUS OF AVIATION.

M. Eiffel has made, by the fan method, a great number of tests on models of certain forms of apparatus. From these tests we may deduce a certain number of rules, which we shall state at a later point; rules which may serve to establish the preliminary design of an airplane.

The interesting experiments at the Institute of Saint-Cyr on an airplane entire (by means of the car) or on an airplane in free flight are not yet sufficiently numerous to give ground for rules of construction for airplanes. However, these results merit statement.

(1) M. Eiffel has shown fully the use which may be made of a study of the logarithmic diagram for the conditions of operation of an airplane in horizontal movement. It is thus that he has studied the régime of maximum speed for a given power and also the economical régime.

The maximum speed for horizontal flight depends more especially on the engine installed on board the avion (see fig. 3, point i_1).

The economical régime, or régime of minimum power for given weight (see fig. 3, point i_3), is of great interest. In fact, when an avion rises with the maximum vertical speed, it is placed in condi-

tions such that the useful power developed shall be minimum, the excess of power being utilized for raising the airplane to the greatest possible height.

The limiting speeds of an airplane for planing are:

- (a) The maximum speed of normal horizontal flight.
- (b) The speed corresponding to the minimum slope.

This minimum is defined by the minimum value of $\frac{R_x}{E_y}$. The angle which corresponds to this minimum is the best angle of planing of Col. Charles Renard.

The motive quality or sustaining quality of an airplane introduced by the Constructor Louis Bréguet has for value

$$g = \frac{\rho \sqrt{\frac{Q}{S}}}{\rho \frac{P_M}{Q} - V_m};$$

in which

ρ = efficiency of propeller.

$\frac{Q}{S}$ = weight in kilograms carried per square meter of surface.

$\rho \frac{P_M}{Q}$ = useful work (kilogram-meter-second) of the motor propeller combination per kilogram of weight carried. This power corresponds to the efficiency ρ of the propeller and to the full power P_M of the motor.

V_m = maximum vertical speed in meters per second.

(2) Ordinary monoplanes.

The following coefficients result from the experiments of M. Eiffel.

(a) The loads sustained in relation to the sustaining surface vary between 25 and 35 kilograms per square meter (5.12 to 7.17 pounds per square foot).

(b) The maximum speeds of horizontal flight are comprised between 26.4 and 33.3 meters per second (86.6 and 109.3 feet per second) or 95 and 120 kilometers per hour (59 and 74.6 miles per hour).

The speeds for the economical régime vary between 19.44 and 25 meters per second (63.8 and 82 feet per second) or 70 and 90 kilometers per hour (43.5 and 55.9 miles per hour).

Let us give the name "portance" to the ratio:

$$\frac{Q}{S} \times \frac{1}{V^2}$$

The portance for maximum speed of horizontal flight varies between 0.025 and 0.040. The portance for economical speeds varies between 0.040 and 0.070. The values utilized vary, therefore, between 0.025 and 0.070.

(d) The maximum useful power (maximum horizontal flight) per 100 kilograms (220 pounds) of weight carried varies between 8 and 11 horsepower.

The minimum useful power (economical régime) per 100 kilograms (220 pounds) of weight carried varies between 5 and 6 horsepower.

The useful power expended in raising 100 kilograms (220 pounds) with the maximum vertical speed varies between 1.5 and 6 horsepower.

(e) The maximum vertical speeds vary between 2.3 and 4.25 meters per second (7.55 and 13.94 feet per second).

(f) Let us assume 6 horsepower per 100 kilograms (220 pounds) for the economical régime.

Let there be an expenditure of 2 horsepower per 100 kilograms (220 pounds) for climbing. This permits of raising 100 kilograms (220 pounds) a distance of 450 meters (147.6 feet) in five minutes.

In a preliminary design we may assume a useful power of 8 horsepower per 100 kilograms of weight carried.

If the propeller has a mean efficiency of 0.70, the power developed on the shaft is $8/0.7 = 11.5$ horsepower per 100 kilograms of weight carried.

In a preliminary design for a monoplane, it is necessary to count on 11 to 12 horsepower per 100 kilograms (220 pounds) of total weight carried, say 120 horsepower for an airplane of which the total weight in flying condition is equal to 1,000 kilograms (2,204 pounds). The consumption per horsepower will be 0.32 to 0.52 kilograms (0.71 to 1.15 pounds) of gasoline and oil, and the weight per horsepower of the engine-propeller equipment, 2 to 3.2 kilograms (4.41 to 7.06 pounds).

(g) The minimum values of $\frac{R_x}{R_y}$ are comprised between 0.16 and 0.20.

The best planing angles are comprised between 9° and 11.3° (mean angle = 10°).

The ratios of the limiting speeds for planing are comprised between 1.27 and 1.48.

(h) The values of the motive quality are comprised between 0.83 and 1.05.

(3) Biplanes.

(a) The loads carried in relation to the carrying surface vary between 15 and 30 kilograms per square meter (3.07 and 6.15 pounds per square foot).

(b) The maximum speeds for normal horizontal flight are comprised between 19.44 and 27.8 meters per second (63.8 and 91.2 feet per second), or 70 and 100 kilometers per hour (43.5 and 62.1 miles per hour). The economical speeds vary between 13.9 and 22.2 meters per second (45.6 and 72.8 feet per second), or 50 to 80 kilometers per hour (31 and 49.7 miles per hour).

(c) The values of the portance for maximum speeds are comprised between 0.035 and 0.045 and the values for economical speeds between 0.060 and 0.065.

The values utilized lie between 0.035 and 0.065—that is to say, within narrower limits than for monoplanes.

(d) The maximum useful power per 100 kilograms (220 pounds) of weight carried varies between 5 and 7 horsepower.

The minimum useful power per 100 kilograms (220 pounds) of weight carried varies between 4 and 5 horsepower.

The useful power expended in lifting 100 kilograms (220 pounds) with the maximum vertical speed varies from 0.5 to 2.5 horsepower.

(e) The maximum vertical speeds vary between 0.5 and 1.6 meters per second (1.64 and 5.25 feet per second).

(f) Let us assume 5 horsepower per 100 kilograms (220 pounds) minimum useful power and 2 horsepower per 100 kilograms (220 pounds) for useful power required for climbing; it is seen that, for 100 kilograms (220 pounds) of weight carried, there will be required a useful power of 7 horsepower, or a power of 10 horsepower absorbed by the shaft, assuming 0.70 for the mean efficiency of the propeller. This indicates a power of 100 horsepower for an airplane of 1,000 kilograms (2,204 pounds).

For a biplane as compared with a monoplane, there is therefore required less power for the same weight carried.

(g) The minimum values of $\frac{R_x}{R_v}$ are contained between 0.142 and 0.228. The best planing angles range between 8° and 11° . The ratios of the limiting speeds of planing are comprised between 1.08 and 1.22.

(h) The values of the motive quality are comprised between 0.75 and 1.17.

(4) Hydravions.

(a) The loads in relation to the carrying surface vary between 30 and 40 kilograms per square meter (6.15 and 8.19 pounds per square foot).

(b) The minimum useful power per 100 kilograms of weight carried is 5 to 6 horsepower for hydravions with floats and 4 to 5 horsepower for hydravions with a boat fuselage.

For the first it is necessary to provide 12 to 13 horsepower (on account of the surface tension which must be overcome as the floats leave the surface of the water) for the power developed by the engine on the shaft per 100 kilograms of weight carried, or 104 horsepower (say an engine of 120 horsepower) for an equipment weighing 800 kilograms. The weight of the engines in flying condition represents about 45 per cent of the total weight of the entire equipment.

For hydravions with a boat fuselage it is necessary to count on 13 or 14 horsepower per 100 kilograms of weight carried for the power developed by the engine on the shaft, or 560 horsepower (two engines of 300 horsepower) for an equipment weighing 4,000 kilograms (weight of engines = 45 per cent of the total weight of the equipment).

(5) Experiments made at the Institute of Saint-Cyr on a Blériot airplane.

At the Institute of Saint-Cyr a study has been made by the car method on a two-passenger Blériot monoplane (side by side). This airplane has a horizontal tail plane in form of V with the main plane, enlarging toward the tail. The characteristics are as follows:

| | | |
|--|---------------------------|----------------------|
| Total spread..... | 11.10 meters | (36.4 feet). |
| Length..... | 9.00 meters | (29.5 feet). |
| Area of the planes..... | 25.35 meters ² | (272.9 square feet). |
| Area of the projection of the fuselage (from its nose to the beginning of the tail plane)..... | 3.27 meters ² | (35.2 square feet). |
| Area of the tail plane..... | 7.76 meters ² | (83.5 square feet). |
| Area of the depth rudder..... | 1.68 meters ² | (18.1 square feet). |
| Angle of the chord of the planes with the tail plane..... | 6°. | |

This airplane has been studied between the incidences (angle of the chord of the plane near the fuselage with the horizontal) of $+20^\circ$ and -2° for three positions of the depth rudder, as follows:

- (1) Position in the prolongation of the tail plane.
- (2) Position of maximum turning downward, the rudder making then an angle of 18° with the tail plane.
- (3) Position of maximum turning upward, the rudder making then an angle of 51° with the tail plane.

There is determined, as a function of the incidences, the values of R_x and R_y and the distances from the leading edge of the planes to the point where the resultant cuts the chord of the planes (in the projection on the plane of symmetry of the airplane).

The following results have been obtained:

(a) The values of R_x are sensibly the same for all positions of the depth rudder. The propulsive resistance is sensibly independent of the position of this rudder.

(b) For a given value of the incidence, the force R_y increases continuously in passing from the rudder position for upward turning to that for downward turning.

The surface of the depth rudder intervenes then in the sustentation.

It should be noted that the quotient $\frac{R_y}{25.35}$ does not exactly represent the portance of the airplane. It is really necessary to take into account the surfaces of the tail plane and of the depth rudder, which, with the variation of the incidence, have incidences positive or negative relative to the horizontal, and thus intervene in a variable manner in the value of the portance.

(c) The position of what may be called the center of pressure (intersection of the resistance of the air with the chord) for a given value of the angle of incidence varies much with the inclination of the rudder.

For a given value of this angle, the center of pressure moves continuously from the leading edge as the change is made from the position for turning upward to that for turning downward.

For a given position of the depth rudder, for example, the position in the plane of the tail plane and for near-by positions, the center of pressure moves continuously toward the leading edge for a decreasing incidence, or moves from the leading edge for an increasing incidence. This is the opposite of what takes place with an isolated plane: The variation observed here shows the influence due to the tail plane and the depth rudder.

(6) Study of aeroplane in free flight.

Experiments have been made at the Institute of Saint-Cyr on a Maurice Farman biplane and on a Blériot. If we call S the net carrying-surface (17.65 meters² (190 square feet) in the Blériot) the quotient $\frac{R_y}{S}$ will measure the portance of the machine.

(a) The portance of any avion in volplane flight is less than that in normal flight, when the propeller blast acts on a carrying part of the avion (main plane, tail plane, or supporting tail).

For the Blériot this difference is shown to be 15 per cent.

(b) Whenever, in slowing up, the propeller operates as a brake, the head resistance in volplane flight is greater than the head resist-

ance of the avion without propeller. From this action as a brake there results an augmentation of the planing angle.

For the Blériot, fitted with a single screw (diameter 2.45 meters (8.05 feet); pitch, 1.53 meters (5.0 feet), with a rotative speed of 400 to 500 revolutions per minute there has been found 20 to 25 per cent increase in the resistance.

11. PROPELLERS AT A FIXED POINT.

Col. Charles Renard had stated, for propellers geometrically similar, the following law:

$$\text{The ratios } \alpha_o = \frac{\Pi_o}{n^2 D^4}, \beta_o = \frac{P_e^{(o)}}{n^3 D^5} \dots \dots \dots (3)$$

are constant.

Researches undertaken at the Institute of Saint-Cyr have led to the following results:

The coefficients α_o and β_o of the formulæ of Renard increase, in general, a little with the rotative speed; however, for certain propellers these coefficients decrease slightly as n is increased, and then increase with further increase of n . The variations in the values of these coefficients are, however, so small that for ordinary values of the rotative speed they may be considered constant.

Col. Renard had also introduced the idea of the quality of a sustentation propeller. This is defined as

$$Q = \frac{\alpha_o^2}{\beta_o^2} \times \frac{4}{0.08\pi}$$

This quantity depends especially on the pitch of the propeller. It is smaller as the pitch is larger. The product of the pitch by the quality is sensibly constant for propellers geometrically similar.

12. PROPELLERS ADVANCING RELATIVE TO THE MEDIUM.

(1) For propellers geometrically similar, the magnitudes

$$\left. \begin{aligned} \alpha &= \frac{\Pi}{n^2 D^4} \\ \beta &= \frac{P_e}{n^3 D^5} \\ \rho &= \frac{\alpha\gamma}{\beta} \end{aligned} \right\} \dots \dots \dots (5)$$

are functions of $\gamma = \frac{V}{nD}$, $\epsilon = nD$, that is to say, of functions of the speed of advance V and of the peripheral speed πnD .

If on two rectangular axes we lay off as abscissæ the values of γ and as ordinates the values, either of α , of β , or of ρ , the points representing the properties of a type of propeller are distributed on curves $nD = \text{constant}$ in the planes (α, γ) , (β, γ) and (ρ, γ) .

However, for large values of V (of the order of 27 to 28 meters (88 to 92 feet) per second (about 62 miles per hour)) and of nD (of the order of 25 to 30), the curves $\epsilon = nD$, corresponding to variations of ϵ of 10 units, are sensibly the same. As these conditions are found

in the values used in practice, we may take for practical purposes α , β , ρ as functions of the quantity γ . In each of the planes (α, β) , (β, γ) , (ρ, γ) the properties of a given type of propeller may be represented by a single curve.

In the same way the researches carried out at the Institute of Saint-Cyr have shown that, for a wide field of values and comprising the conditions of practice, the ratios $\frac{\alpha}{\alpha_0}$ and $\frac{\beta}{\beta_0}$ are also functions of γ for a given type of propeller.

(2) The ratio $\frac{\alpha}{\alpha_0}$ decreases regularly and quite rapidly as the value of γ increases.

For a given number of revolutions of the propeller, α_0 has a determinate value.

For a given number of revolutions of the propeller, the traction decreases as the speed increases.

(3) In the experiments at Saint-Cyr, the values of $\frac{V}{nD}$ did not exceed 0.90, a value for which $\frac{\alpha}{\alpha_0}$ is not zero. Let us assume it justifiable to extrapolate the curve $(\frac{\alpha}{\alpha_0}, \gamma)$ to its intersection with the axis of γ , and below this axis. Let us further assume that α_0 has a constant value, whatever may be the revolutions of the propeller. We may then state the following proposition, which, however, is only approximate.

Above a certain value of $\frac{V}{nD}$, the propeller acts as a brake (traction negative); below this value, it acts as a propeller (traction positive).

According to this, the number of revolutions beyond which the propeller becomes propulsive is the greater as the speed V is greater. For a certain propeller of 2.40 meters (7.88 feet) it has been found that as the value of V increases from 4 meters (13.1 feet) per second to 12 meters (39.4 feet) per second, the number of revolutions for which the traction becomes zero passes from 300 to 566.

(4) The values of $\frac{\beta}{\beta_0}$, for a part of the propellers studied, continually decrease with increasing values of γ ; for others, $\frac{\beta}{\beta_0}$ first increases slightly with γ and then decreases. In any case, the decrease of $\frac{\beta}{\beta_0}$ is less rapid than that of $\frac{\alpha}{\alpha_0}$.

In considering, as above, what develops for a given velocity of rotation of the propeller, it is seen that the traction π decreases more rapidly than the power P_e . The latter is, in those conditions, proportional to the couple transmitted to the propeller shaft. The traction and the engine torque are then very far from being proportional.

In the experiments at Saint-Cyr, the point on the axis of γ for which $\frac{\beta}{\beta_0} = 0$ was not determined. As above, let us assume as justified the extrapolation which consists in prolonging the curve $(\frac{\beta}{\beta_0}, \gamma)$ to

its intersection with the axis of γ . What has just been said shows that this point is farther removed from the origin on the axis of γ than the point of intersection of the curve $\left(\frac{\alpha}{\alpha_0}, \gamma\right)$ with this same axis.

When the motive power is zero, the traction is negative and the propeller functions like a windmill. It absorbs power furnished by the air, but does not transmit it to the engine; this power furnished by the air is absorbed by the resistance proper of the propeller, which turns without any manifestation of motive power on the shaft.

(5) For a given number of revolutions, the power absorbed by the propeller at a fixed point is, in general, greater than that absorbed when the propeller moves in the direction of its axis. For the same number of revolutions, it is necessary to supply at the fixed point a greater power than when the propeller advances in the direction of its axis.

For the same power absorbed by the propeller, the number of revolutions of the propeller at a fixed point is, in general, less than that when advancing in the direction of its axis.

Let us consider a propeller put into operation on an airplane at rest. It absorbs a certain power equal to that furnished by the engine. If the airplane is put into motion and if the number of revolutions of the propeller remains constant, the power absorbed by the propeller first decreases, while the power furnished by the engine tends to remain the same. In order that equality may obtain between the two powers, it is necessary that the revolutions of the propeller increase. Thus for a given opening of the throttle valve for the engine, the number of revolutions of the propeller with the airplane in flight is in general greater than when at rest. This increase in the number of revolutions per minute may range from 30 or 40 to 100. For a given engine, certain propellers, giving, with the airplane at rest, a suitable number of revolutions, may in free flight give a number too far above the normal regimen to permit of using such propellers.

In any case, it may be noted that there are certain propellers which require in flight a torque greater than at rest. Instead of speeding up the engine (with fixed throttle opening), they slow it down.

Following are the results of experiments made at the Institute of Saint-Cyr.

Blériot monoplane with Gnome motor, 60 horsepower.

Observations taken in free flight with four Chauvière propellers under the same conditions regarding the engine:

| Propeller. | Diameter. | Pitch. | Speed of horizontal flight. | Revolutions per minute. | |
|------------|----------------|----------------|------------------------------|-------------------------|------------|
| | | | | At rest. | In flight. |
| I..... | 2.45 meters.. | 1.53 meters... | 26.7 meters per second..... | 1,160 | 1,200 |
| | 8.05 feet..... | 5.02 feet..... | 27.5 feet per second..... | | |
| II..... | 2.40 meters.. | 1.75 meters... | 25.6 meters per second..... | 1,130 | 1,090 |
| | 7.90 feet..... | 5.75 feet..... | 24.0 feet per second..... | | |
| III..... | 2.50 meters.. | 1.60 meters... | 25.25 meters per second..... | 1,160 | 1,300 |
| | 8.20 feet..... | 5.25 feet..... | 23.3 feet per second..... | | |
| IV..... | 2.45 meters.. | 1.44 meters... | 27.5 meters per second..... | 1,180 | 1,220 |
| | 8.05 feet..... | 4.72 feet..... | 90.0 feet per second..... | | |

For the propeller of the greatest pitch there is decrease in the rotative speed; for the other three there is increase in this speed.

From the practical point of view, if in certain cases the number of revolutions of the propeller in free flight is for the same conditions at the motor nearly the same as with the airplane at rest, it must not necessarily be concluded that the power of the engine is decreasing; it may well be that with the propeller employed it can not be otherwise.

(6) The efficiency ρ increases, at first nearly linearly, passes through a maximum, and then decreases rapidly. All propellers have, then, a maximum efficiency corresponding to a determinate value of $\frac{V}{nD}$ peculiar to each type of propeller. This value is nearly independent of nD , at least for the values comprised between 30 and 40 (region of actual practice).

(7) Let us consider propellers which are not geometrically similar. We may say that these propellers form a "group" if the definition of their geometrical form contains a variable parameter with the different values of which they are designed. This parameter may be the pitch, the curvature of the blade, the variation of its width with the distance from the axis, etc. The designer, for example, passes from one propeller to another of the group by preserving the various sections of the blade, but in causing the pitch to vary.

If, then, we consider the propellers of a group differing, for example, only in the pitch, the maximum efficiencies and the corresponding values of $\frac{V}{nD}$ continuously increase with increase in the ratio of the pitch to the diameter.

This was shown by M. le Commandant Dorand in his experiments at Chalais-Meudon on propellers of the same blade area in which the ratio of the pitch to the diameter continuously increased from 0.65 to 1.29.

M. Eiffel has developed the same results on models of the following propellers:

First group: Diameter 0.80 meter (31.5 inches); blades flat on working face; pitch sensibly constant for each propeller; width of blade, $\frac{1}{10}$ diameter. At equal distances from the axis the section of the blades is the same. The thickness decreases regularly from the hub to the tip of the blade.

| Pitch of propeller. | Pitch ratio. |
|--------------------------------|--------------|
| 0.42 meter (16.5 inches)..... | 0.53 |
| .64 meter (25.2 inches)..... | .80 |
| .78 meter (30.7 inches)..... | .97 |
| 1.04 meters (41.0 inches)..... | 1.30 |

Second group: Diameter, 0.80 meter (31.5 inches); width of blade, $\frac{1}{10}$ diameter; blades hollow on working face. The mean line of the section has a height of segment equal to $\frac{1}{12}$ the chord. Pitch constant for each propeller.

| Pitch of propeller. | Pitch ratio. |
|--------------------------------|--------------|
| 0.42 meter (16.5 inches)..... | 0.53 |
| .65 meter (25.6 inches)..... | .81 |
| .82 meter (32.3 inches)..... | 1.025 |
| 1.09 meters (40.0 inches)..... | 1.26 |

(8) For propellers of the same pitch and same diameter, but of varying widths of blade, the maximum efficiency passes through a maximum maximorum when the ratio between the greatest width of the blade and the diameter is approximately 1/10.

This ratio has become classical. It is found closely approximate in nearly all propellers.

(9) It is desirable to use a propeller in the vicinity of its maximum efficiency.

In fact, for values of $\frac{V}{nD}$ near the maximum efficiency, the curve (ρ, γ) is, in general, quite flat. It results that, in spite of the variations of regimen of the engine and of the speed of an airplane, the efficiency ρ is always near the maximum. A propeller which does not fulfill these conditions gives only mediocre results.

The practical result of the use of the propeller in the neighborhood of its maximum efficiency is an economy in fuel in horizontal flight, and the possibility of utilizing, more easily and more completely, the excess power of the engine for climbing or in traversing eddies.

Curves (ρ, γ) peaked near the maximum imply a rapid fall in efficiency in case of an acceleration of the engine. The practical consequence is that, in order to obtain a moderate increase in effective power, it is necessary to expend relatively a large amount of fuel and oil, and to risk overstraining the engine.

(10) It is desirable, in practice, in order to have a maximum efficiency high (between 0.70 and 0.80), that γ should be, for such maximum normally near the value 1.0, or equal, say, to 0.90.

In this case, if $nD=40$, the normal speed of horizontal flight will be equal to 36 meters (118 feet) per second, or 129.6 kilometers (80.5 miles) per hour.

If $n=16.66$ revolutions per second (1,000 revolutions per minute), $D=2.40$ meters (7.88 feet). If $n=20$ revolutions per second (1,200 revolutions per minute), $D=2$ meters (6.56 feet). If $n=8.33$ revolutions per second (500 revolutions per minute), $D=4.8$ meters (15.16 feet).

(11) Some writers have maintained that there is, for each type of propeller, a best value of the ratio of pitch to diameter, characteristic of this type of propeller. This is by no means certain. But it does not appear, as has been sometimes stated, that there is a best value of this ratio for all propellers, value independent of their form.

(12) Propellers have, in general, two or four blades. Four blades should be used in the following case.

Suppose that a propeller is required capable of absorbing a very considerable power. With two blades, there may result:

(a) A propeller of too great diameter.

(b) A propeller with a speed of rotation too high.

In these two cases, centrifugal force would have a value too high. It would then be advantageous to employ a propeller with four blades which would permit the reduction either of the diameter or of the number of revolutions, that is, to decrease the influence of centrifugal force.

It is necessary that the blades of a four-bladed propeller be designed so that the coefficients $\frac{P_c}{n^2 D^5}$ and $\frac{P_u}{n^2 D^5}$ shall be as nearly as possible equal to the sum of the values of these coefficients for two

propellers of two blades each, operating each as if alone. This is a matter to be examined specially for each case. Such examination is well adapted to the method by the use of models. It is thus that M. Eiffel has shown, for certain Drzewiecki propellers, that the reduction coming from the influence of the blades in a four-bladed propeller was minimum when the axes of the blades made, between themselves, angles of 75° and 105° .

13. STUDY OF THE MEDIUM SURROUNDING A SCREW PROPELLER.

M. Eiffel has studied, by means of a fan, a certain number of models of screw propellers. He has undertaken to investigate the variation in the velocities of the current air, both in front of and behind a propeller.

The measurements were made in a plane situated, either in front or behind the propeller, at distance equal to $1/5$ the diameter.

The velocities were determined (by means of a Pitot tube) at distances from the axis of rotation equal to $1/5$, $1/3$ approximately, $2/5$, $1/2$ approximately, and a little more than $1/2$ the diameter of the propeller. The next to the last position is near the tip of the blade. The last is a little outside of the cylinder circumscribing the propeller.

Values of $\gamma = \frac{V}{nD}$ are made to vary over a threefold range by varying either V or n , but the former by preference. To these values of γ correspond values of the efficiency ρ .

(1) There is acceleration in the velocity of the current of air, whether in front of or behind the propeller.

(2) The acceleration is greater behind than in front.

(3) Acceleration increases from the hub outward to a distance from the axis between $1/3$ and $2/5$ the diameter; it then decreases as the tip of the blade is approached.

This decrease is more rapid behind than in front.

(4) The value of the maximum of the acceleration depends on the direction of the relative velocity γ at the tip of the blade. Let γ_m be the value of γ for which the efficiency is maximum. If we then vary from γ_m in the direction of increasing γ , the maximum value of the increase of velocity diminishes; it increases, on the contrary, if we vary from γ_m in the direction of decreasing γ .

(5) The turbulent zone extends very little beyond the cylinder whose base is the circle swept by the tips of the blades. This result shows that the ratio $1/3$ adopted between the similar dimensions of a model and the full-sized propeller is sufficient to envelop the model with a surrounding cushion of quiet air sufficiently thick to permit of considering the model as moving in an indefinite mass of air.

(6) The increment of velocity between the forward and rear faces of the propeller is accompanied by a slight contraction in the size of the moving column of air.

(7) The augmentation of velocity due to the propeller has an influence on the operation of an airplane. The sustentation and the propulsive resistance are increased. At the same time this influence does not seem to be very important. Suppose that the blast from the propeller acts on $1/8$ of the spread of the airplane and that the increment of velocity is 50 per cent (a rather high value); the mean

velocity of the wind meeting the wing is then increased in the ratio $\frac{7}{8} + \frac{1.50}{8} = 1.065$. Such an increment is of no special importance.

(8) These experiments are an illustration of the hypothesis of the "preliminary dynamic condition," due to M. Soreau.

When the propulsive speed of the propeller is less than the circumferential speed of the tips of the blades, as is usually the case, the periodic and rapid movements impressed by the blades on the mass of air surrounding the propeller produce a condition of steady flow. This is characterized by the existence of a fluid vein having the same axis as the propeller which accompanies it in propulsion; this vein remains unchanged so long as the conditions of operation (V, n) remain unchanged. It is in this fluid vein in movement, independent of the position of the blades at any given instant, that the latter operate. M. Soreau gives to this fluid vein the name of "propeller vein."

At the same time there is formed around the blade in movement a sort of fluid prow and stern, on which glide the particles of air in such manner as to constitute a wake, which accompanies the blades without, however, entraining the particles of air. To these wakes or secondary veins, produced in the line of motion of the blades, M. Soreau gives the name of "blade veins."

Taking as a point of departure this hypothesis, M. Soreau has been led to represent certain experiments of M. Eiffel by a formula of the form

$$\alpha = A - B \left[\frac{V+w}{nD} \right]^2 \dots \dots \dots (5)$$

w being the mean axial velocity of the "propeller vein," while A and B are constant for geometrically similar propellers.

Certain experiments of M. Eiffel are well represented by the equation

$$\alpha = 0.0196 - 0.022 \left[\frac{V}{nD} + \frac{0.46}{V} \right]^2 \dots \dots \dots (6)$$

which may be applied from the value $\frac{V}{nD} = 0.3$.

Equation (6) shows that the axial velocity w is of the form

$$w = 0.46 \frac{nD}{V} \dots \dots \dots (7)$$

In the region of maximum efficiency of the family of propellers considered, γ is comprised between 0.5 and 0.7; w is then comprised between 0.92 and 0.66 meter (3.02 and 2.16 feet) per second. In this region the ratio $\frac{w}{V}$ is then small for values of the speed of propulsion higher than 10 meters (32.8 feet) per second. In this case α becomes a function of γ . Reference has been made above to this fact. The larger the value of V the less distinct are the curves $nD = \epsilon$.

Equation (6) may be written

$$\alpha = 0.0196 \left[1 - \frac{0.022}{0.0196} \left(\frac{V}{nD} + \frac{0.46}{V} \times \frac{1}{nD} \right)^2 \right] \dots \dots \dots (8)$$

This is of the form

$$\alpha = a \left[1 - \left(\frac{V}{\lambda' n D} + \frac{k}{\lambda' V} \right)^2 \right] \dots \dots \dots (9)$$

We should have in the same way

$$\beta = b \left[1 - \left(\frac{V}{\lambda'' n D} + \frac{k}{\lambda'' V} \right)^2 \right] \dots \dots \dots (10)$$

The coefficients a , b , λ' , λ'' , k are constant for propellers geometrically similar, so long as the ratio of similar dimensions does not exceed a certain limit. It does not appear that such relation can be admitted for a propeller and its model when the latter has dimensions too much reduced in relation to those of the propeller.

(9) The ratio $1/3$ adopted by M. Eiffel for propellers of airplanes seems to be an upper limit. It leads to rotative speeds of the model of 2,400 and 3,000 revolutions per minute, figures which it seems prudent not to exceed.

When the problem is concerned with the study of the propellers of a dirigible (diameter 4.5 meters (14.75 feet)), this ratio requires the use of models of 1.5 meters (4.9 feet) in diameter. These models seem a little large for the cylinder of air 2 meters (6.56 feet) in diameter employed at Auteuil by M. Eiffel. In this case it would be preferable to employ a model on the scale of $1/4$ (diameter, 1.125 meters (3.69 feet)), turning at 2,000 revolutions per minute, corresponding to 500 revolutions per minute for the propellers of the dirigible.

14. INFLUENCE ON THE OPERATION OF A PROPELLER OF A CURRENT OF AIR PERPENDICULAR TO THE AXIS OF ROTATION.

If we call W the velocity of the current of air and if we consider the ratio $\frac{W}{\pi n D}$, the influence on the traction and on the power absorbed seemed to depend on this ratio.

The ratios $\frac{\alpha}{\alpha_0}$ and $\frac{\beta}{\beta_0}$ increase with this ratio, at first very rapidly, then more and more slowly.

These conclusions result from calculations made at the Institute of Saint-Cyr, based on experiments made on small propellers by M. Riabouchinsky, director of the Aerotechnic Institute of Koutchino.

Suppose that for a propeller of the order of size suited for aviation, the action of a wind perpendicular to the axis depends on the relation $\frac{W}{\pi n D}$ in the same proportions as for the small propeller studied by M. Riabouchinsky. We can then estimate the traction which would be realized by a helicopter with vertical axis carried by an airplane in flight.

Suppose a propeller 2.5 meters (8.2 feet) diameter with vertical axis turning at 1,200 revolutions per minute and carried by an airplane with a horizontal velocity of 25 meters (82 feet) per second. The

peripheral speed of the propeller is equal to 157 meters (515 feet) per second and the ratio $\frac{V}{\pi n D}$ has a value 0.16. Referring to the calculations of M. Maurain, director of the Institute of Saint-Cyr, it is seen that the traction of this helicopter would be increased by about 1/3 of its value as a result of the relative current of air due to the movement of the airplane; but the power to be supplied would itself be increased by about 1/4.

It would be interesting to apply such conclusions to the results of experiments on propellers larger than those studied by M. Riabouchinsky.

CONCLUSION.

AERODYNAMIC STUDIES IN FRANCE DURING THE LAST TEN YEARS.

Ten years ago there was only one laboratory in France in which researches on the resistance of air were carried on in a systematic manner. This was the laboratory installed at Chalais-Meudon by Col. Charles Renard. An engineer of great talent and, at the same time, a remarkable scholar, our fellow countryman must be considered as one of the founders of experimental aerodynamics. His studies on the resistance of air upon bodies of different forms and his experiments upon supporting screw propellers have become classic.

Other experimenters had, to be sure, undertaken at this very time interesting researches upon the resistance of the air. We may cite the studies of Marey upon the flight of birds; the experiments with disks in free flight made by the Abbé Le Dantec at the Conservatoire des Arts et Métiers; those of Cailletet and Colardeau upon orthogonal disks thrown from the second story of the Eiffel Tower. Certain engineers, Ricour, Desdouits, Le Grain, Nadal, had taken up the study of the effects of air resistance upon bodies moving at a high rate of speed. But all these tests carried out under unlike conditions were not susceptible of affording a serious basis for studies in aerodynamics and did not furnish engineers with information which was adequate for the carrying through of their designs.

At this epoch they were still teaching in certain engineering schools, regarding the resistance of air upon planes inclined to the direction of the wind, the law of the square of the sine of the angle of incidence, although it had long since been demonstrated that this law, applied to the flight of birds, led to absurd conclusions.

The résumé which we have just made shows the progress which has been accomplished during the past 10 years.

There exist to-day four great laboratories which are chiefly devoted to the study of aerodynamics.

The military laboratory of Chalais-Meudon, under the learned direction of M. le Commandant Dorand, continues the fine traditions of Col. Renard. It is there that the complicated problem of screw propellers is beginning to be cleared up; it is there that important researches upon the gliding flight of avions, and upon the coefficient of safety which should be adopted in the construction of these machines, have been taken up.

M. de Guiche has devoted himself specially to the delicate problem of the distribution of pressure on the wings of airplanes. He has sub-

jected the actions exercised by the air on the surfaces of aerofoils to a minute and precise analysis; he has created a sort of topography of these surfaces which is of the greatest importance for the determination of the laws of aerodynamics.

The constructors of airplanes find effective aid in the laboratories of M. Eiffel, at Auteuil, so remarkably well supplied with equipment, and also at the Aerotechnic Institute of Saint-Cyr.

The experiments of M. Eiffel on models have been carried out with the constant purpose of furnishing constructors with coefficients which are reliable. After studying aerofoils, this eminent engineer has devoted his efforts to a precise determination of the influences which these exert upon each other when they are assembled to form actual flying machines. He has determined the relative coefficients for various parts of the avions, the cables and tension wires, the wheels of the landing frames, the fuselage. He has, finally, for the whole apparatus, studied the different conditions of flight.

The question of screw propellers is beginning to be well understood. We know, in particular, what the conditions are under which a model must be tried out in order to give information applicable to a propeller of normal size. The logarithmic diagram proposed by M. Eiffel facilitates the choice of a propeller which will suit a machine of given character.

Parallel with the studies of M. Eiffel on models, the Aerotechnic Institute of Saint-Cyr, under the energetic direction of its director, M. Maurain, and of its subdirector, M. Toussaint, makes use of its elaborate equipment to study avions or parts of avions in normal size. This laboratory, at the present moment the most important in the world, puts at the disposal of inventors numerous pieces of apparatus for measurements which enable them to determine *a priori* the qualities of the machines which they have under design. In collaboration with military aviation pilots, M. Toussaint has been able to install on the avions ingenious registering devices which make it possible to determine, during a flight, the effects of the air on the different parts and the pilot's maneuvers.

This ensemble of researches, executed by the different French laboratories, researches which complement each other, have already led to the series of results of which we have given an idea in Chapter IV of this report. These experimental results derive their importance from this fact, viz, that they have been obtained by means of a large number of careful experiments susceptible of giving them a high degree of reliability.

Aviation has, moreover, derived a great benefit from these laboratory experiments. I will cite here only one confirmatory example. In spite of certain ideas put forward by M. Rateau in regard to screw propellers, the constructors of airplanes made little of the influence of the back of the wings on the value of the supporting force; they believed that the whole effect came from the air pressure upon the face directly exposed to the wind. But certain researches carried out at the laboratory of M. Eiffel on the distribution of air effects on the two surfaces of an aerofoil showed that there were negative pressures on the back and that these were much more important than the pressures on the surface directly exposed to the wind. Wherefore, contrary to the mode of construction in practice, the necessity arose

of fixing solidly the canvas on the back of the wing in order to avoid accident.

The study of the conditions of flight for airplanes by means of registering instruments standardized in the laboratories has, as M. Toussaint has shown, a great importance from the point of view of safety. It is of prime importance to put within the hands of pilots instruments capable of controlling the quality of their evolutions. This is of special importance for the pupil; it is no less so for the experienced pilot. Statistics, in fact, show that a goodly number of accidents are to be imputed to mistakes in piloting. Such false maneuvers are often unconscious and result from the ignorance of the pilot as to the limits of safety in which he can maneuver his avion. By means of appropriate instruments these limits can be determined for each type of machine and even for each machine on the aviation fields by experienced pilots. The rôle of the aerodynamic laboratories is to combine such registering apparatus so as to simplify the installation on board the machines and to standardize these instruments. The Institute of Saint-Cyr has commenced to do this work with success.

Our aerodynamic laboratories are concerned, then, not only with the solving of problems which are a part of the science of aerodynamics but they strive also to come to the assistance of our constructors, and they have their share in the evolution of a weapon which is just now rendering such great services in the war where the destiny of the country which saw its birth is at stake.

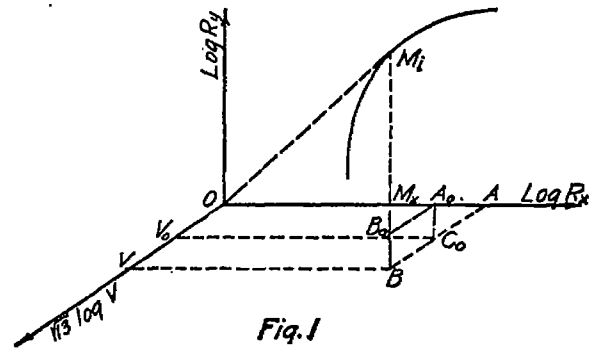


Fig. 1

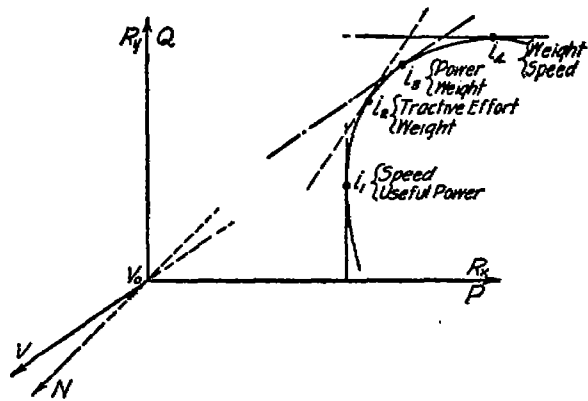
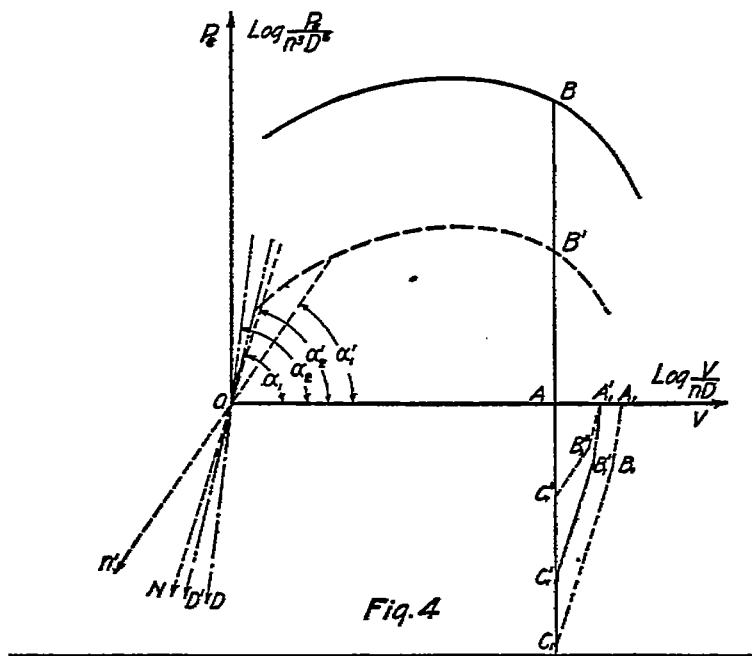
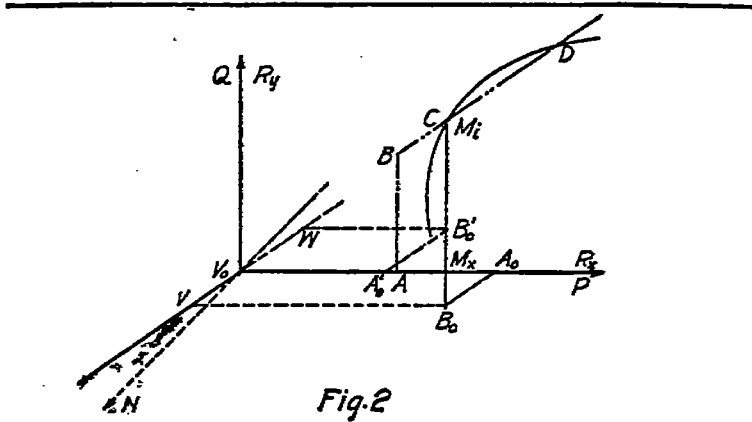


Fig. 3



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- Note.—The works of Col. Charles Renard have been published in the Comptes Rendus de l'Académie des Sciences, in the Revue du Génie Militaire, and in the Revue de l'Aéronautique. References will be found in le Cours d'Aéronautique de la Faculté des Sciences de Paris (L. Marchis), 1ère Partie.