REPORT No. 20

AERODYNAMIC COEFFICIENTS AND TRANSFORMATION TABLES

By JOSEPH S. AMES
Member, National Advisory Committee for Aeronautics

REPORT NO. 20.

AERODYNAMIC COEFFICIENTS AND TRANSFORMATION TABLES.

By Joseph S. Ames.

The problem of the transformation of numerical values expressed in one system of units into another set or system of units frequently arises in connection with aerodynamic problems.

The following brief explanation, with tables of equivalents in various systems of units, has been prepared in order to facilitate such transformation.

FUNDAMENTAL AERODYNAMICAL FORMULA.

$$F = C\rho S V^2$$

where F is the total force acting on the aerofoil,
ρ is the density of the air,
S is the area of the aerofoil,

V is the velocity of the aerofoil relative to the air and C is an abstract number, varying for a given aerofoil with its angle of incidence, independent of the choice of units, provided these are consistently used for all four quantities (F, ρ, S, and V).

It follows that the pressure

$$p = F/S = C\rho V^2$$

This is often written

$$p = KV^2, \text{ i. e., } K = C\rho$$

If one set of units is used in the expression of p and another in that of V, the facts may be expressed by writing $p = K' V^2$.

The results of experiments are given in different ways in different countries. It is most desirable that they should all be given in terms of C.

In what follows, tables will be given for the calculation of C:

I. When K is given in the published results.

II. When K' is given in the published results.

Formulæ will then be given by which, knowing C, the pressure p or total force F may be calculated.

I.

1. If K is given in French Tables, it is understood, unless the contrary is stated, that the units are as follows:

Unit of pressure, "weight of a kilogram" per square meter.
Unit of density, based upon—
unit of mass, based upon—
unit of force, "weight of a kilogram;"
unit of length, meter;
unit of time, second;
unit of volume, cubic meter.
Unit of velocity, meter per second.

Hence ρ, the density, is equal to 0.125, provided the air is dry, at 15.6° C. (60° F.) and under 76 cm. of mercury pressure.

Hence $C=8K$.

If the air is at another temperature or pressure, correction must be made, as indicated in the section on such corrections.

2. If K is given in American Tables, it is understood, unless the contrary is stated, that the following units are used:

Unit of pressure, "weight of a pound" per square foot.
Unit of density, based upon—
unit of mass, based upon—
unit of force, "weight of a pound;"
unit of length, foot;
unit of time, second;
unit of volume, cubic foot.
Unit of velocity, foot per second.

Hence ρ, the density, is equal to 0.00238, provided the air is dry, at 15.6° C. and under 76 cm. of mercury pressure

Hence $C=420.2\ K$

II.

1. If K' is given in French Tables, it is understood, unless the contrary is stated, that the following units are used:

Unit of pressure, "weight of a kilogram" per square meter.
Unit of density, based upon—
unit of mass, based upon—
unit of force, "weight of a kilogram;"
unit of length, meter;
unit of time, second;
unit of volume, cubic meter.
Unit of velocity, kilometer per hour.

Hence $\rho = 0.125$ for "standard air."

Hence $K' = C \times 0.125 \times \frac{10^6}{3600^2} = 0.0096C.$

Hence $C = 104.2K'$

2. If K' is given in American Tables, it is understood, unless the contrary is stated, that the following units are used:

Unit of pressure, "weight of a pound" per square foot.
Unit of density, based upon—
unit of mass, based upon—
unit of force, "weight of a pound;"
unit of length, foot;
unit of time, second;
unit of volume, cubic foot.
Unit of velocity, mile per hour.

Hence $\rho = 0.00238$

Hence $K' = C \times 0.00238 \times \left(\frac{5280}{3600}\right)^2 = 0.00512C.$

Hence $C = 195.3\ K'.$

Summary.

	Unit of pressure.	Density.	Unit of velocity.	
French systems	"*Weight of a kilogram*" / square meter	0.125	*meter* / second	$C=8K.$
	"*Weight of a kilogram*" / square meter	0.125	*kilometer* / hour	$C=194.2K'$
American systems	"*Weight of a pound*" / square foot	0.00238	*foot* / second	$C=420.2K'$
	"*weight of a pound*" / square foot	0.00238	*mile* / hour	$C=195.3K'$[1]

[1] English and German Tables usually give C directly.

To obtain the Pressure, given C:

By fundamental formula $p = C\rho V^2$, *provided units on both sides of the equation are consistent.* Hence, in such a case, substitute the appropriate values of C, ρ, and V. If the units are not consistent, certain factors must be introduced. The following formulæ give the results of the substitution for ρ and this factor for those combinations of units generally used.

$$\frac{p \text{ "weight of a pound"}}{\text{square feet}} = C \times 0.00238 \left(V \text{ ft./sec.}\right)^2$$

$$\frac{p \text{ "weight of pound"}}{\text{square feet}} = C \times 0.00512 \left(V \frac{\text{miles}}{\text{hour}}\right)^2$$

$$\frac{p \text{ "weight of a pound"}}{\text{square inches}} = C \times 0.0000165 \left(V \frac{\text{feet}}{\text{second}}\right)^2$$

$$\frac{p \text{ "weight of a pound"}}{\text{square inches}} = C \times 0.0000355 \left(V \frac{\text{miles}}{\text{hour}}\right)^2$$

$$\frac{p \text{ "weight of a kilogram"}}{\text{square meters}} = C \times 0.125 \left(V \frac{\text{meters}}{\text{second}}\right)^2$$

$$\frac{p \text{ "weight of a kilogram"}}{\text{square meters}} = C \times 0.0096 \left(V \frac{\text{kilometer}}{\text{hour}}\right)^2$$

For other units of pressure, calculate pressure by one of these formulæ and use transformation tables for pressures.

Since the pressure is proportional to ρ, these formulæ apply only if the air is dry, at 15.6° C. and under 76 cm. of mercury pressure.

Correction for Temperature and Pressure of the Air.

The following formulæ give the values of ρ for conditions of pressure other than standard:

Temperature.	Pressure.	
$t°$ C.	h (cm. of mercury)	$\rho=\frac{3.79h}{t+273}\times$density at 15.6° C. and 76 cm.
$t°$ F.	h (inches of mercury)	$\rho=\frac{17.33h}{t+460}\times$density at 60° F. and 30 in.

Since the pressure on the aerofoil is proportional to ρ, if we know the pressure calculated for standard conditions, and wish to know its value under other conditions, we must multiply this calculated pressure by the ratio of the densities of the air in the two conditions.

That is, if we wish to calculate the pressure when the air is at h cm. of mercury at $t°$ C., we must multiply the value of the pressure obtained from the formulæ of the last section by $\frac{3.79h}{t+273}$;

Or, if we wish to calculate the pressure when the air is at h inches of mercury and $t°$ F., we must multiply the value of the pressure obtained from the formulæ of the last section by $\frac{17.33h}{t+460}$.

The approximate value of h for different heights above the earth's surface is given in tables.

If moisture is to be taken into account in the values of ρ, reference may be made to the Smithsonian Meteorological Tables.

Barometer and Altitude.

In Tables I and II are given for values of the barometer as argument the corresponding elevations, assuming for the intermediate barometric column a uniform temperature of 50° F. for English measures and 0° C. for metric measure; the average temperature to be anticipated at such elevation, and the elevation corrected for temperature, assuming for the latter a mean value between temperature, at the bottom and the stated value at elevation.

TRANSFORMATION TABLES.

I. Height above earth's surface determined by barometer, corrected for temperature. English units.
II. Height above earth's surface determined by barometer, corrected for temperature. Metric units.
III. Length equivalents.
IV. Area equivalents.
V. Volume equivalents.
VI. Capacity equivalents.
VII. Mass equivalents.
VIII. Density equivalents.
IX. Velocity equivalents.
X. Acceleration equivalents.
XI. Force equivalents.
XII. Couple equivalents.
XIII. Pressure equivalents.
XIV. Work equivalents.
XV. Power equivalents.
XVI. Temperature equivalents.

TABLE I.—*Height above earth's surface determined by barometer, corrected for temperature—English units.*

Barometer, inches of mercury.	Elevation, temperature of 50° F.	Average temperature at elevation.	Elevation (corrected).
	Feet.	*° F.*	*Feet.*
12	24,340	−31.6	22,750
13	22,640	−23.0	20,960
14	20,620	−15.0	19,250
15	18,750	− 7.2	17,660
16	17,000	0.0	16,130
17	15,350	6.5	15,670
18	13,790	10.1	13,230
19	12,320	14.8	11,870
20	10,930	18.8	10,580
21	9,600	23.0	9,330
22	8,340	27.0	8,140
23	7,130	30.5	6,990
24	5,970	34.2	5,880
25	8,460	37.5	4,820
26	3,900	41.0	2,770
27	2,770	43.5	2,760
28	1,750	46.0	1,740
29	830	48.0	828
29.9	0	50.0	0

TABLE II.—*Height above earth's surface determined by barometer, corrected for temperature—metric units.*

Barometer, mm. of mercury.	Elevation, temperature of 0°.	Average temperature at elevation.	Elevation (corrected).
	Meters.	*° C.*	*Meters.*
300	7,430	−34.4	7,100
350	6,200	−25.4	6,020
400	5,130	−18.0	5,060
450	4,190	−12.1	4,160
500	3,350	− 7.4	3,390
550	2,550	− 2.8	2,580
600	1,860	− 0.7	1,930
650	1,250	+ 4.4	1,290
700	657	+ 7.2	678
750	106	+ 9.7	109
760	0	+10.0	0

TABLE III.—*Length equivalents.*

Units.	Inches.	Feet.	Yards.	Miles.	Centimeters.	Meters.	Kilometers.	Nautical miles.
1 inch	1	0.08333	0.027.8	0.0_415783	2.540	0.0254	0.0_4254	0.0_413701
1 foot	12	1	.333	$.0_418939$	30.480	.30480	$.0_43048$	$.0_41644$
1 yard	36	3	1	$.0_45682$	91.440	.9144	$.0_49144$	$.0_44933$
1 mile	63,360	5,280	1,760	1	160,934	1,609.34	1.609	.8683
1 centimeter	.3937	.03281	.01094	$.0_46214$	1	.01	10^{-5}	.000005395
1 meter	39.37	3.281	1.0936	$.0_46214$	100	1	.001	.0005395
1 kilometer	39,370	3,281	1,093.6	.6214	100,000	1,000	1	.5395
1 nautical mile	72,962	6,080.2	2,026.7	1.15155	185,325	1,853	1.8532	1

TABLE IV.—*Area equivalents.*

Units.	Square inches.	Square feet.	Square yards.	Square miles.	Square meters.
1 square inch	1	0.006944	0.007716	$0.0_{9}2491$	$0.0_{3}6452$
1 square foot	144	1	.111	$.0_{3}3587$	.09290
1 square yard	1,296	9	1	$.0_{6}3228$	.8361
1 square mile	.....	27,878,400	3,097,600	1	2,589,998
1 square meter	1,549.9	10.764	1.196	$.0_{6}3861$	1

TABLE V.—*Volume equivalents.*

Units.	Cubic inches.	Cubic feet.	Cubic yards.	Cubic centimeters.	Cubic meters.
1 cubic inch	1	$0.0_{3}5787$	$0.0_{4}2143$	16.39	$0.0_{4}1638$
1 cubic foot	1,728	1	.03704	28,317	.02832
1 cubic yard	46.656	27	1	764,559	.7645
1 cubic centimeter	.06102	.03531	.001307	1	.001
1 cubic meter	61,023	35.314	1.3079	1,000,000	1

TABLE VI.—*Capacity equivalents.*

Units.	Cubic inches.	Fluid ounces.	Gills.	Liquid pints.	Liquid quarts.	Gallons (U. S.).	Gallons (Imperial).	Liters.
1 cubic inch	1	0.5541	0.1385	0.3463	0.01732	0.04329	0.0036046	0.01639
1 fluid ounce	1.8046	1	.25	.0625	.03125	.007813	.006506	.02957
1 gill	7.2187	4	1	.25	.125	.03125	.002602	.118292
1 liquid pint	28.875	16	4	1	.5	.125	.10408	.473167
1 liquid quart	57.75	32	8	2	1	.25	.20833	.9463
1 gallon (U. S.)	231	128	32	8	4	1	.83265	3.785
1 gallon (Imperial)	277.42	153.718	38.423	9.608	4.804	1.201	1	4.5458
1 liter	61.025	33.814	8.453	2.113	1.0577	.2642	.21975	1

TABLE VII.—*Mass equivalents.*

Units.	Kilograms.	Grains.	Ounces.		Pounds.		Tons.		
			Troy.	Avoirdupois.	Troy.	Avoirdupois.	Short.	Long.	Metric.
1 kilogram	1	15,432	32.150	35.273	2.6792	2.2046	0.001102	$0.0_{3}9842$	0.001
1 grain	$0.0_{4}6479$	1	.002083	0.02286	$.0_{4}1736$	$.0_{4}1429$	$.0_{7}7143$	$.0_{7}6378$	$.0_{7}6450$
1 ounce (troy)	.0311	480	1	.10971	.08333	.06857	$.0_{4}3429$	$.0_{4}3061$	$.0_{4}3110$
1 ounce (avoirdupois)	.02835	437.5	.9115	1	.07595	.0625	$.0_{4}3125$	$.0_{4}2790$	.02835
1 pound (troy)	.3732	5,760	12	13.17	1	.8229	.04114	.02673	.03732
1 pound (avoirdupois)	.4536	7,000	14.583	16	1.2152	1	.0005	$.0_{4}4464$	$.0_{4}4536$
1 ton, short	907.18	140_{6}	29,167	32,000	2,431	2,000	1	.8929	.9072
1 ton, long	1,016	$15,680_{4}$	326_{8}	35,840	2,722	2,240	1.12	1	1.016
1 ton, metric	1,000	15,432,356	32,151	35,274	2,679	2,205	1.102	.9842	1

Mass units used by engineers.

A. English systems:
Unit of mass=g pounds, where g is the acceleration due to gravity.
Hence, on foot-second system, unit of mass=32.14 pounds; give it arbitrary symbol U_1.
Hence, on mile-hour system, unit of mass=78,900 pounds; give it arbitrary symbol U_2.

B. French systems:
Unit of mass=g kilograms.
Hence, on meter-second system, unit of mass=9.80 kilograms; give it arbitrary symbol U_3.
Hence, on kilometer-hour system, unit of mass=127,000 kilograms; give it arbitrary symbol U_4.

TABLE VIII.—*Density equivalents.*

Units.	Grams per cubic centimeter.	Pounds per cubic inch.	Pounds per cubic foot.	Kilograms per cubic meter.	Pounds per United States gallon.
1 gram per cubic centimeter	1	0.03613	62.43	1,000	8.345
1 pound per cubic inch	27.68	1	1,728	277.02	231
1 pound per cubic foot	.01602	.0$_5$5787	1	16.02	.1337
1 kilogram per cubic meter	.09998	.003612	.06243	1	.008345
1 pound per U. S. gallon	.1198	.004329	7.481	119.845	1

Using engineering units of mass.

$$1\ \frac{\text{lb.}}{\text{ft.}^3}=0.0311\ \frac{U_1}{\text{ft.}^3};\quad 1\ \frac{U_1}{\text{ft.}^3}=32.14\ \frac{\text{lb.}}{\text{ft.}^3}$$

$$1\ \frac{\text{kg}}{\text{m.}^3}=0.1020\ \frac{U_2}{\text{m.}^3};\quad 1\ \frac{U_2}{\text{m.}^3}=9.80\ \frac{\text{kg.}}{\text{m.}^3}$$

TABLE IX.—*Velocity equivalents.*

Units.	Centimeters per second.	Meters per second.	Meters per minute.	Kilometers per hour.	Feet per second.	Feet per minute.	Miles per hour.	Knots.
1 centimeter per second	1	.01	0.6	0.036	0.03281	1.9685	0.02237	0.01942
1 meter per second	100	1	60	3.6	3.281	196.85	2.237	1.942
1 meter per minute	1.667	.01667	1	.06	.05468	3.281	.03728	.03237
1 kilometer per hour	27.78	.2778	16.67	1	.9113	54.68	.6214	.53960
1 foot per second	30.48	.3048	18.29	1.097	1	60	.6818	.59209
1 foot per minute	.5080	.00508	.3048	.01829	.01667	1	.01136	.00987
1 mile per hour	44.70	.4470	26.82	1.609	1.467	88	1	.86839
1 knot	51.497	.51497	30.898	1.8532	1.68894	101.337	1.15155	1

TABLE X.—*Acceleration equivalents.*

	$\frac{\text{cm.}}{\text{sec.}^2}$	$\frac{\text{ft.}}{\text{sec.}^2}$	$\frac{\text{mi.}}{\frac{\text{hour.}}{\text{sec.}}}$
1 centimeter per second, per second	1	0.3281	0.02237
1 foot per second, per second	30.48	1	0.6818
1 mile per hour, per second	44.70	1.467	1

TABLE XI.—*Force equivalents.*

1 megadyne=10^6 dynes=72.33 poundals.
1 poundal =0.013825 megadynes.
Engineering units:
1 kilogram= 0.980 megadynes.
=70.88 poundals.
= 2.2046 pounds.
1 pound = 0.45359 kilograms.

TABLE XII.—*Couple equivalents.*

1 kilogram-meter=7.233 pound-feet.
1 pound-foot=0.1383 kilogram-meter.

TABLE XIII.—*Pressure equivalents.*

Units.	Megabars or megadynes per square centimeter.	Kilograms per square centimeter.	Kilograms per square meter.	Pounds per square inch.	Pounds per square foot.	Long tons per square inch.
1 megabar (=10^6 dynes per square centimeter)	1	1.0197	10,197	14.50	2,088	.....
1 kilogram per square centimeter	.9807	1	10,000	14.22	2,047.6	0.000348
1 kilogram per square meter	.....	.....	1	.....	.2048	.....
1 pound per square inch	.06895	.07031	703.1	1	144	.....
1 pound per square foot	.....	.....	4.882	.00694	1	.....
1 long ton per square inch	.....	157.5	1,575,000	.....	.....	1
1 long ton per square foot	.....	.....	.....	.....	.....	.....
1 atmosphere	1.0133	1.0333	10,333	14.70	2,116.8	.....
Mercury 1 meter	1.333	1.3596	13,596	19.34	2,784.9	.....
Mercury 1 inch	.03386	.03453	345.3	.4912	70.732	.....
Water 1 meter	.09798	.09991	999.1	1.421	204.62	.....
Water 1 inch	.002489	.002538	25.4	.03613	5.204	.....
Water 1 foot	.02936	.03045	304.5	.4332	62.380	.....

Units.	Long tons per square foot.	Atmospheres.	Columns of mercury at 15° C.		Columns of water at 15° C.		Feet.
			Meters.	Inches.	Meters.	Inches.	
1 megabar (=10^6 dynes per square centimeter)	.....	0.9869	0.7500	29.53	10.21	401.8	33.48
1 kilogram per square centimeter	.....	.9678	7,355	28.96	10.01	394	32.84
1 kilogram per square meter	.....	.....	.....	.....	.....	.03937	.....
1 pound per square inch	.....	.06804	.05171	2.036	.7037	27.70	2.309
1 pound per square foot	.....	.....	.....	.....	.....	.1922	.01602
1 long ton per square inch	.....	.....	.....	.....	.....	.....	.....
1 long ton per square foot	.....	.....	.....	.....	.....	.....	.....
1 atmosphere	.....	.....	.76	29.92	10.34	407.2	33.93
Mercury 1 meter	.....	.....	1	39.37	13.61	535.7	44.64
Mercury 1 inch	.....	.....	.0254	1	.3456	13.61	1.134
Water 1 meter	.....	.....	.07349	2.893	1	39.37	3.281
Water 1 inch	.....	.....	.001868	.07349	.02540	1	.08333
Water 1 foot	.....	.....	.02240	.8819	.3048	12	1

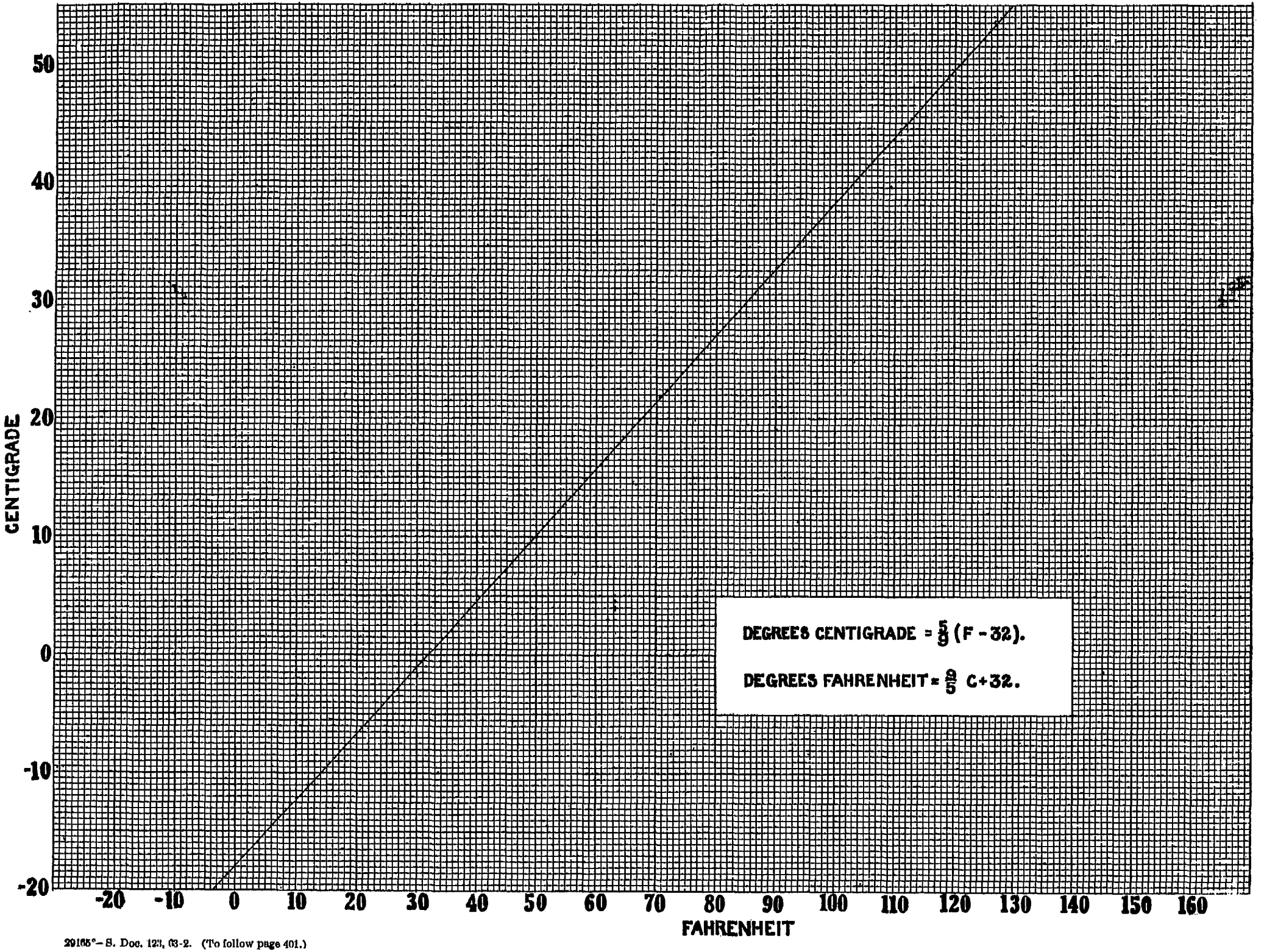
TABLE XIV.—*Work or energy equivalents.*

Units.	Joules=10^7 ergs.	Kilogram meters.	Foot-pounds.	Kilowatt hours.	Cheval vapeur hours.	Horsepower hours.	Calories.	Kilogram calories.	British thermal units.
1 joule	1	0.10197	0.7376	0.0_62778	0.0_63777	0.0_63725	0.2390	0.0_32390	0.0_39486
1 kilogram meter	9.80665	1	7.233	$.0_32724$	$.0_37037$	$.0_33653$	2.344	.002344	.00930
1 foot-pound	1.356	.1383	1	$.0_43766$	$.0_51206$	$.0_50505$	.3240	$.0_33241$	.00128
1 kilowatt hour	$3.6x10^6$	$3.671x10^5$	$2.655x10^6$	1	1.3596	1.341	860,500	860.5	3,415
1 cheval vapeur hour	$2.648x10^6$	270,000	$1.9529x10^6$	.7355	1	.9863	632,900	632.9	2,512
1 horsepower hour	$2.6845x10^6$	$2.7375x10^5$	$1.98x10^6$	.7457	1.0139	1	641,700	641.7	2,547
1 calorie	4.183	.4266	3.086	$.0_41162$	$.0_41653$	$.0_41558$	1	.001	.003958
1 kilogram calorie	4,183	426.6	3,086	.001162	.00165	.001558	1,000	1	3.968
1 British thermal unit	1,054	107.5	777.52	$.0_32925$	$.0_43931$	$.0_43927$	252.2	.25200	1

TABLE XV.—*Power equivalents.*

Units.	Horsepower	Kilowatts.	Cheval vapeur metric horsepower.	Meter kilograms per second.	Foot-pounds per second.	Kilogram calories per second.	British thermal units per second.
1 horsepower	1	0.7457	1.014	76.04	550	0.1783	0.7074
1 kilowatt	1.341	1	1.360	102.0	737.6	.2390	.9486
1 cheval vapeur metric horsepower	.9863	.7355	1	75	542.3	.1758	.6977
1 meter kilogram per second	.01315	.009807	.01333	1	7.233	.002344	.009303
1 foot pound per second	.00182	.001356	.00184	.1383	1	.0₃3241	.001286
1 kilogram calorie per second	5.610	4.183	5.688	426.6	3,086	1	3.968
1 British thermal unit per second	1.414	1.054	1.433	107.5	777.5	.2520	1

29165°—S. Doc. 123, 65-2——26



29165°—S. Doc. 121, 63-2. (To follow page 401.)