REPORT No. 82

AIRPLANE STRESS ANALYSIS

IN FIVE PARTS

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WITH PROBLEMS AND DIAGRAMS

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REPORT NO. 82.

PREFACE.

With each delivery of a new type or size of airplane the manufacturer furnishes (1) the data and (2) the computations by which to judge the performance and safety of the machine.

Under "data" may be classed the geometrical dimensions of the entire craft and its parts; the physical properties of its materials, both raw and as treated for final use, and a description of the method of treatment of these materials; the measured strengths of the whole structure and of its elements; the weight schedule, giving the mass and position of the structural parts and of the carried loads; the aerodynamic properties of the sustaining and control surfaces, the body, the undercarriage, and the craft as a whole; the records of inspection and full-scale test, etc. To this must be added much special information about the engine, the propeller, the navigating instruments, etc.

Under "computations" may be furnished information as to the stresses and performance of (1) the motor; (2) the screw; (3) the craft as a whole.

The present work, intended as a handbook, covers primarily the theory of airplane stress analysis, but ignores, as foreign to its scope, the forces within the engine and propeller. It presents analytical methods and formulas with little if any argument, assuming the reader can supply the proofs or will not require them. All the formulas are illustrated by problems given immediately in the text and solved in Chapter IV.

Acknowledgment is here made to the Journal of the Franklin Institute for a part of the diagrams and subject matter which the writer previously had published in that periodical; to the Curtiss Aeroplane & Motor Corporation for practical data used in the problems; and to Mr. L. H. Crook, Mr. N. C. Luther, and Mr. R. H. Smith for assistance in revising the text and reading the proofs. The aerodynamic data have been taken partly from reports of the bureau of Construction and Repair partly from those of other laboratories. A portion of the wing-stress equation, taken from this work, will appear in the bureau's forthcoming book entitled "Aircraft Design Data."

A. F. ZAHM.

JANUARY, 1918.

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PART I.

GENERAL CONSIDERATIONS.

By A. F. Zahm.

INTRODUCTION.

This report was prepared at the Aerodynamical Laboratory, Navy Yard, Washington, D. C., by direction of Rear Admiral D. W. Taylor, Chief Constructor, U. S. Navy, and Member of the National Advisory Committee for Aeronautics, for publication in the committee's Fifth Annual Report.

1. TERMINOLOGY.

The mechanical terms in this text bear the same meaning as in standard works on applied mechanics; the aeronautical terms, where practicable, follow the nomenclature published in 1917 by the United States National Advisory Committee for Aeronautics (Report No. 25). Some words not so published are used of necessity, and when of uncertain meaning are defined upon their first appearance in the text. For example, "air force" for the resultant of air pressure and friction. As an abbreviation for "angle of incidence," the term "incidence" is sometimes used in this text, as is commonly done in aerodynamic works.

2. UNITS AND DIMENSIONS.

For the most part in this text ordinary British units are employed. The unit of mass is sometimes the pound; sometimes the slug, or g pounds, g being the acceleration of gravity.

The standard of air density is taken as 0.07635 pound, or 0.00238 slug, per cubic foot, or that of dry air at 760 m. m. and 15.6° C. Hence the familiar full impulse ρV^2, per unit cross section of a jet, is 0.00238 V^2 pound per square foot when V is feet per second, or 0.0051 V^2 when V is miles an hour. And the air force on a normal surface of A square feet is $R = 0.00238\ CAV^2$ pounds at V feet per second, C being a dimensionless multiplier, called the "shape coefficient," or "absolute coefficient"—a constant independent of the system of units.[1]

3. NORMAL AND ABNORMAL LOADS.

Structurally an airplane is under normal load in two notable cases: (1) On earth when resting naturally on a level surface; (2) in air when in steady straightaway flight at any incidence.[2] In these cases the external applied forces, whether due to air, earth, or motor, are constant and in algebraic sum equal to the weight of the craft. Some may be positive, others negative.

In all other conditions the loading is abnormal and may be either constant or not; either uniformly increased or not; either positive or negative. For example, an airplane in steady flight around a level circular course bears a constant load determined by the speed and curvature. For this case the actual load in terms of the normal is tabulated in figure 5. For the large path curves there shown the increase of loading is substantially uniform throughout the structure. Again the craft may be diving steeply at steady speed and incidence. Then its loading, both lift and drift, is constant; the first smaller than normal, the second larger. When the machine is standing on earth some of the loads are reversed from their direction in air; when flying inverted all "lift" loads are reversed.

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[1] In general, the same formula applies to an inclined flat surface whose face area is A, so long as its incidence and orientation remain constant.

[2] The loading due to the general air stream is illustrated in figs. 1, 2, 3, 4 for a typical monoplane surface.

4. SUDDEN LOADS.

Inconstant abnormal loads may arise from sudden changes of air speed or incidence, as in diving, or riding gusts; from motor jerking; from impact against earth or water, etc.; and their change may be uniform throughout the structure or localized; uniform or irregular in growth and decay. But frequently it suffices to assume that all parts of the structure sustain the same acceleration and hence the same change of load. If this acceleration be j at any part of the airplane, the masses in that part induce stresses j/g times those due to their weight alone, assuming j and g in the same direction.

When practicable in engineering tests, an accelerometer should be carried in the active machine to record the component accelerations in the air and on the earth. To find j throughout the craft, it is well to use several accelerometers distributed throughout the structure.

Sometimes the acceleration is estimated from observed or assumed data. For example, suppose that a craft, regarded as a single rigid mass in pure translation, lands with vertical velocity component v, whose "head" is $h = v^2/2g$, and comes to rest with *uniform* cushioning, of yield d.[1] Then the ratio of its average vertical acceleration to gravity is[2]

$$j/g = 1 + h/d. \tag{1}$$

It is twice this amount if, as rarely happens, the cushion resistance be directly proportional to the cushion deflection d.

Equation (1) is true not only of the machine as a whole, but of every part of it, however elastic the structure. If the craft approach land in pure translation, h is the same for all elements, but d varies throughout the structure, being least in the chassis parts and greatest in the parts remote from the impact points. A like treatment applies to longitudinal and lateral accelerations.

The assumed maximum acceleration to be provided for in the design of an airplane is usually specified by the purchaser.

Example 1.[3]—If an accelerometer fixed to an airplane records a maximum vertical acceleration of 48 feet per second, what is the ratio of the abnormal to the normal loading?

Example 2.—An airplane in landing has a vertical velocity component of 10 feet per second and a uniform cushion yield of 6 inches. Find the ratio of the abnormal to the normal stresses in the landing gear, assuming the machine to be a rigid structure.

5. SIMPLE STRESSES.

The *direct simple stresses* here treated are the common tensile, compressive, and shearing stresses. For each the intensity or unit stress is uniform over the cross section and is given by

$$S = P/A, \tag{2}$$

P being the load sustained by the structural cross section A. For each case the load and stress have the same direction. For torsion in a round shaft the intensity of direct shearing stress at different points of a cross section varies directly with the radius to those points and in the outermost fiber is

$$S = Mc/J \tag{3}$$

where M is the torsional moment, c the distance from the center of gravity to the outermost fiber, J the polar moment of inertia of the section.

6. INDIRECT SIMPLE STRESSES.

Indirect simple stresses perpendicular to the applied loads occur in transverse and in torsional loading. In both cases the longitudinal and the transverse shearing stresses at any point are equal.

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[1] The cushion force may be partly air lift, partly spring lift. The air lift diminishes slightly with v; the spring lift increases with d, though seldom directly as the deflection, depending on the nature of the cushioning mechanism. In general, the longer the yield, d, the less the shock, or mean vertical acceleration.

[2] To prove (1) equate the work of cushioning to the work of total fall:

thus $Mjd = Mg(h+d)$,

whence $j/g = 1 + h/d$,

in which M is the mass of the machine, j the average cushioning retardation.

[3] The solutions to all the examples cited in the text are given in Part IV.

In transverse loading the tension or compression is greatest in the outermost fibers of the beam and is

$$S = Mc/I, \qquad (4)$$

where M is the bending moment, I the moment of inertia of the cross section.

In a beam so loaded the longitudinal shearing stress varies over any cross section, being zero at the remotest fiber and increasing toward the neutral axis of that section. At any reference axis Y in the section, parallel to the neutral axis, the horizontal shear is

$$S = ac\ V/It \qquad (5)$$

where a is the sectional area outward from Y, c the distance of the centroid of a from the neutral surface, V the total shear over the entire beam section, and t the thickness of the beam at Y. In most, but not all, practical cases the horizontal shear is a maximum at the neutral surface. Its value there is, for a round beam, 4/3 the mean vertical shear V/A; for a rectangular one 3 $V/2\ A$.

Example 3.—Find the longitudinal shearing stress in a 4-inch square beam given the total vertical shear as 1,400 pounds; first with the diagonal vertical, then with it inclined 45°.

7. RESULTANT UNIT STRESS.

When several causes simultaneously produce like stresses at any point in a structure, these may be algebraically added.

(a) *Endwise stresses due to simultaneous endwise and transverse loads.*—Several endwise stresses may occur at a point simultaneously. Thus in the lower rear spar of an airplane the lift produces tension, the drag compression; the bending moment produces tension and compression. The algebraic sum of all these is the effective stress. If P be the aggregate endwise stress, the above operation is expressed by the following equation,

$$S = P/A \pm M/Z, \qquad (6)$$

in which M is the bending moment due to the running load only, z, the section modulus. The obvious physical meaning of this equation is that the first term represents the unit direct stress, the second term the unit bending stress, which may be taken either as tensile or compressive.

A more accurate formula for the total endwise unit stress here considered is

$$S = P/A + M/Z + k^1 Pd/Z, \qquad (7)$$

in which d is the deflection due to M only (computed as in figs. 13, 14, 15, 16), and k is a correction factor to be applied to d, because the latter is slightly increased by P.

If in (6) and (7) P/A is a tensile stress, the succeeding terms can be reduced to "equivalent tensile stress" by multiplying them by S_b/S_t, or the ratio of the bending to the tensile strength of the material. This is sometimes done when S_b differs materially from S_t. Similarly, when P/A is a compressive stress. The equation (6) then assumes the form

$$S = S_t + r\, M/Z \qquad (8)$$

where S is the equivalent tensile stress and r is the ratio of the tensile to the bending strength of the material. The equivalent compressive stress is similarly found.

Formulas (6), (7), may be used for wing spars and for non-tapering flat struts bearing a considerable side wind pressure. But if the strut be tapering, S can not be found by formula (7) here presented, since d is unknown. However, for normal flying conditions the deflection is ignored, since there is no appreciable side wind against the strut.

Example 4.—Find the resultant unit stress in a 2-inch square simple beam, due to an endwise load of 900 pounds, a maximum bending moment of 1,100 inch-pounds, and a maximum deflection of 0.05 inch at the place of maximum moment.

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[1] By a well-known approximate formula $k = \sec \frac{l}{2} \sqrt{P/EI}.$

(b) *Endwise stresses due to simple endwise loads accompanied by bending.*—For a uniform column or strut bearing an endwise load, but no transverse one, the unit actual stress in the outermost fiber is usually found by Rankine's formula

$$S = [1 + a(l/r)^2]P/A, \qquad (9)$$

in which P is the actual end load producing stresses lower than the elastic limit, l is the length of the strut, r its least radius of gyration at the center section, and a is a numerical constant depending on the material, strut form, and manner of constraining the ends.[1] Values of a for various materials, forms, and end conditions are given in books on applied mechanics.

If the endwise load is distant p from the column axis the unit stress is

$$S = [1 + a(l/r)^2 + cp/r^2]\,P/A \qquad (10)$$

where c is the distance of the remotest fiber, and r is the radius of gyration.

Example 5.—Find the unit stress in a pin-ended column 1 inch square and 30 inches long under an axial load of 200 pounds; also the unit stress when taking the eccentricity of the load as 2 inches.

(c) *Combined shearing and normal stress.*—If at any point of a section, S is the normal unit stress, and S_s the transverse or the equal longitudinal shearing unit stress, then at that point the maximum resultant shearing stress S_p, and maximum normal stress S_n are, respectively

$$S_p = \sqrt{S_s^2 + (S/2)^2} \qquad (11)$$

$$S_n = S/2 \pm S_p \qquad (12)$$

Example 6.—A beam is subject to a compressive unit stress of 200 pounds per square inch and at the same time to a longitudinal shearing stress of 250 pounds per square inch. Compute the maximum resultant stresses.

8. REPETITIVE AND EQUIVALENT STRESS.

When a variable load stresses a member frequently through a fraction m of its elastic limit, the *equivalent steady stress* may be taken as

$$S' = S(1 + m)^n \qquad (13)$$

where S is the allowable constant stress, and n is unity for very numerous stress fluctuations, zero for very few.[2]

Example 7.—If a member whose allowable constant stress is 20,000 is stressed frequently to 15,000 and has an elastic limit of 60,000, what is the equivalent stress?

9. MAXIMUM STEADY LOAD AND STRESS.

For a member subject to uniform simple stress the greatest possible load it will sustain is

$$P = SA \qquad (14)$$

S being the strength of the material, and A the sectional area of the member. Examples in airplane construction are: For tension, the stays; for compression, the short struts; for shear, the clevis pins. In all such cases the maximum load and maximum unit stress occur together.

For a structural element not subject to uniform simple stress the greatest possible load may exceed that causing the greatest stress. A long strut, for example, may bear a greater endwise load and sustain less fiber stress before much bending occurs than when bowed excessively.

For a pin-ended wooden strut having a slenderness ratio l/r, above 120, the maximum load is computed by Euler's formula

$$P_{max} = \pi^2 EI/l^2 \qquad (15)$$

where l is the length between pins, I the least moment of inertia of the middle section.

For pin-ended struts with a lower slenderness ratio Johnson's formula

$$P_{max} = AC(1 - Cl^2/40Ek^2) \qquad (16)$$

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[1] See Rankine's Applied Mechanics, section 328. The use of formula (9) for computing P_{max} is not recommended, formulæ (15), (16) giving better results.

[2] See Upton, Materials of Construction, section 215.

is used, where A is the middle section area, C the crushing strength of the material, k the least radius of gyration of the middle section. If the strut has very securely fixed ends, the above two values of P_{max} may be quadrupled.

The crippling load on any strut or column is sometimes given as a function of the slenderness ratio l/r, in tables or diagrams derived from laboratory tests of full scale test pieces.

Example 8.—Find the maximum load for a pin-ended spruce column of length 60 inches, cross section 3 square inches, and moment of inertia 0.3 inch⁴.

10. FACTORS OF SAFETY.

Given the resultant fiber stress intensity, this may be divided into an assumed limiting stress to find the strength-stress ratio, commonly called the "factor of safety." The limiting stress is determined from standard test pieces of the material and of the structural forms in question. For each material and form employed in the industrial arts the assumed limiting stress is commonly fixed by agreement between the constructor and the purchaser. For stays, turnbuckles, fastenings, etc., and sometimes for struts—also for entire truss members, wings, fuselages, etc.— the factor of safety is taken as the ratio of the greatest possible load to the greatest actual load of the member, the former load being found experimentally, the latter either by calculation or by instrumental test under working conditions either real or simulated.

Example 9.—Find the factors of safety in example 6 for a maximum shearing stress of 1,000 pounds per square inch, and a maximum compressive stress of 4,000 pounds per square nch; also the factor of safety in example 8 for an applied load of 250 pounds.

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PART II.

AIRPLANE WING STRESSES.

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11. SCOPE OF TREATMENT.

The study of wing stresses may cover in succession the fabric, the ribs, the ailerons, the spars as beam members, the lift and drag trussing. From the resultant stresses so found are computed the factors of safety for steady normal flight, taking account of the known strength of the individual members or of their dimensions and materials.

12. WING FABRIC LOADING AND STRESS.

The tensile stress in the fabric at any point of a wing surface may be computed from the given curvature and air-pressure distribution at that point. Typical external pressure distributions on a monoplane surface are shown in figures 1, 2, 3, and 4. The internal pressure is sensibly constant and unknown, but with impervious fabric may be made equal to the external surface pressure at any point by perforating the canvas there.

At any part of the surface, as in figure 6, let p be the resultant point pressure of the air in pounds per square foot, a the distance in inches between ribs, and c the depth in inches of the bulge in the canvas midway between the ribs; then the fabric tension t, in pounds per linear inch, neglecting the effect of the fore and aft curvature, can be shown to be approximately

$$t = 0.00087 \ pa^2/c. \tag{17}$$

Values of t for various air pressures and bulging of the fabric are given in figure 6.

13. RIB LOADING AND STRESS.

The usual airplane rib may be considered as a beam supported at two points (at the spars) and sustaining the air force on all the fabric lying nearer to itself than to the neighboring ribs. In figure 7 is shown the distribution of the air force normal to the rib surface and also the distribution of the components of this air force normal to the chord. The running load on the rib is not sufficiently uniform to make applicable the ordinary formulas for uniformly loaded beams.

By considering the average loading upon each element of length as a concentrated load we may compute the shear and moment for a number of points and plot them as in figure 8. This process, however, is laborious.

For approximate treatment we may divide the rib into three parts, the segment between the spars, the front segment, and the rear segment, and consider the total running load on each segment as a concentrated load. The magnitude and position of each such load may be found by well-known methods. From these concentrated loads the shear and moment diagrams can be readily drawn, as shown in full line in figure 9, where the dotted lines are superposed from figure 8. The maximum vertical shearing forces are practically the same in both cases, while the maximum negative bending moment in the case of such approximation is about twice that of the true moment and should be halved for the working approximation.

If the concentrated load on the front and rear segments, as shown in figure 9, be denoted for each by R and located at a distance l from the spar, the unit bending stress next to the spar is

$$S = Rl/Z, \tag{18}$$

Z being the section modulus of the cross section of the unsevered part of the rib segment where it meets the spar.

Again, if R, the concentrated load on the portion of the rib amid spars, is distant a and b from the front and rear spars, respectively, the unit bending stress at R is

$$S = Rab/(a+b)Z, \qquad (19)$$

Z being the section nodulus there. Dividing S in (19) by 2 gives a fair approximation to the true bending stress for the distributed load.

The rib shearing stress may be found by the shearing stress formulas presented in Part I.

Example 1.—A rib loaded as in figure 39 has the dimensions there specified; find the unit stresses for bending, maximum shearing, and average vertical shearing just outside the spars and just inside the spars; also the unit bending stress at R between the spars.

14. AILERON LOADING AND STRESSES.

The loads on the aileron are the control-wire pull and the air force. The normal pull may be taken as the greatest the pilot would care to exert regularly in flight and is measured with a spring balance when the pilot seated in a stationary machine vigorously plies the control. The aileron moment equals the control-wire pull times its distance from the aileron hinge axis; also equals the aileron air force times its distance from the hinge. If this latter distance be assumed of some reasonable magnitude and be divided into said moment, it gives the amount of the air force. Sometimes also the air force is estimated from the size, incidence, and forward speed of the aileron, taken with suitable aerodynamic data and with allowance for the propeller slip stream, if any. In practice, the aileron force may be assumed to be at the center of the surface and equal to PA, where P is the resultant pressure per unit area of the aileron surface A. The value of P is usually specified by the purchaser.

Having thus found either the control-wire pull or the moment of the air force on the aileron, the stresses may be readily computed from the frame diagram by the usual methods of statics. If the moment of the air force tends to twist the aileron's hinge rod, the unit stress in the latter is computed by the formula

$$S = M/Z \qquad (20)$$

where M and Z are, respectively, the given moment and the torsion section modulus. In this case the aileron ribs are simple cantilevers jutting out from the hinge rod, and are stressed like the trailing edge of a wing plane, and are treated by the foregoing rib analysis.

Sometimes the control wire pulls on an aileron lever from whose outer end several stays run to the rear edge of the aileron surface, as shown in figure 38. Each principal aileron rib then sustains a compressive component force due to the applied stay, a transverse running air force, and the transverse component forces of its outer and its inner end attachments. If only the transverse forces be considered, the aileron rib stress may be calculated by the formula for the mid segment of a wing rib, as already treated. The endwise force causes, at any cross section of the aileron rib, a compressive unit stress roughly equal to that force divided by the section area.

The leading and trailing edges of the aileron are treated as continuous beams supported and loaded as in figure 13.

Example 2.—An aileron bearing a uniform pressure of 20 pounds per square foot has the dimensions and structural form shown in figure 38. Find the moment about the hinge and the stress in the control wire. Find also the vertical reactions at the stay wire upon the trailing edge beam.

15. RUNNING LOAD ON WING PLANES [1] AND SPARS.

The air loading and weight of a wing plane is in general not uniform along the whole length of the plane, but may be taken as uniform for each small unit of the length. The actual distribution of the air loading throughout the length, in a uniform wind,[2] is illustrated in figure 4. For

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[1] In this text the term "wing" denotes a main supporting member on the right or left of the airplane. Thus, a biplane wing comprises two wing planes, the "top plane" and the "bottom plane," joined by "interplane" trussing. Similarly for a triplane, a quadruplane, a multiplane.

[2] In case a propeller slip stream washes the plane the wind is still less uniform over the plane. To be very accurate, this case would require special treatment.

practical computation this distribution may be taken as constant along each panel[1] or each spar length of the wing plane, and is so taken in the present treatment.

The manner of referring the running air load and running weight of the plane to the spars, and there resolving them in the plane of the lift trussing and drag trussing, is shown in figure 10. Let L,[2] D, W be the running lift, drag, and weight of the plane and al, bl the distances, respectively, of their points of application from the rear spar, a and b being numerical fractions and l the distance between spars. Then at the front spar the parallel components of L, D are aL, aD; and the parallel component of W is bW, which, taken from aL, gives the net running lift across wind at the spar in question. Compounding graphically this net lift, $aL-bW$, with the drag aD, gives the resultant spar running load R. This resultant is now resolved graphically into the components w_1, w_2 in the lift and drag truss planes; also into w, w^1 in the spar web plane and normal thereto.

In a similar manner the resultant running loads and their component running loads in the lift and drag truss planes may be found for the remaining spars. Frequently in practice the running load on the lower spar of a biplane is found from that of the upper by dividing by some simple ratio, say 1.2, as indicated by aerodynamic experiments.

Example 3.—A biplane wing has the form and loadings shown in figure 41; find the resultant loadings on the spars and their components in the planes of the lift and drag trussing for angles of incidence of 2° and 12°.

16. SPAR BENDING MOMENTS, SHEARS, PIN REACTIONS, DEFLECTIONS.

From the running load w_1 on a spar, figure 10, and from the position of the strut pins, or constraints, the bending moments, shearing forces, pin reactions, and deflections may be computed by the familiar formulas for loaded beams. In general, these four quantitaties can be computed by direct use of Clapeyron's original three-moment theorem, figure 11, but for usual cases are more conveniently found by the formulas derived therefrom and presented in figures 13, 14, 15, 16.

(a) *Bending moments and bending stresses.*—The bending moment diagram is usually a chain of parabolic curves, whose maxima are tabulated, for usual cases, in figures 13, 14, 15, 16. From these maxima and the tabulated joint moments the complete diagram is plotted, as in figure 17.

In some unusual cases the axes of the spar, strut, and stay do not pass through a common point. The increment of moment caused by such eccentricity of the stay attachment is treated analytically in figure 12 and applied graphically in figure 18.

The maximum bending moment in the strut plane, multiplied by w/w_1, gives that for the plane of the spar web, figure 10, from which may be computed the unit bending stresses in the spar. The unit stress is given by the equation

$$S = M/Z, \tag{21}$$

M being the moment and Z the section modulus of the cross section of the spar.

(b) *Shearing forces and shearing stresses.*—The shear diagram is drawn by plotting as ordinates the values of the transverse shearing force on each side of the pins, then joining the ordinates by straight lines, as shown in figure 17. Each line, as is well known, cuts the spar axis at a point of zero shear and of maximum moment.

The values of these shearing forces multiplied by w/w_1 give the shearing forces in the plane of the spar web, from which may be derived the corresponding shearing stresses by the shear formulas of Part I.

(c) *Pin reactions.*—From the two "vertical" shears at any spar joint the pin reaction is most readily found by simple subtraction as in figures 14, 15, 16. If the points of zero shear—that is, of maximum moment—are known, the pin reaction at any joint is taken as the distance between the neighboring maxima times the mean loading.

(d) *Deflection.*—To find the exact place and amount of the maximum spar panel bending, the deflection curve may be plotted from the elastic equation given in figures 14, 15, 16.

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[1] The panel of a spar or wing plane is its portion between consecutive lift struts.
[2] The determination of L and D is explained in the aerodynamic treatment of airplanes.

Otherwise, since the point of maximum deflection in any span is near that of maximum bending moment, the latter point may be taken as the place of maximum deflection. Then the approximate maximum deflection d is, by the formula of figures 14, 15, 16,

$$d(=y) = -x_0[12\,M(l-x_0) + 4\,V(l^2+x_0^2) + w(l^3-x_0^3)]/24\ EI, \qquad (22)$$

where x_0 is the tabulated abscissa of the point of maximum moment. As shown for an extreme practical case in figure 19, the difference between this approximate deflection and the true deflection is less than 2 per cent.

Example 4.—Find the shears, moments, and reactions for the upper and lower front spars of the biplane trussing shown in figure 40 due to the uniform running loads found in example 3; also the reactions of the spars in the planes of the lift and drag trussing and the shears and moments on each spar in the plane of the spar web.

Example 5.—Find and plot the resultant moments due to the uniform loading and the eccentric stay wire attachments shown in figure 18.

Example 6.—Compute the deflections in the plane of the spar web for all panels of the biplane trussing of figure 40, using the results of example 4.

17. CONCENTRATED LIFT AND DRAG ON WING TRUSSING.

The total lift component on any strut pin in the plane of the lift struts is equal and opposite to the pin reactions given by the formulas of figures 14, 15, 16 for a running load in the plane of the lift trussing. Multiplying this lift component by w_2/w_1, figure 10, and adding half the air resistance of the adjoining strut and stay wires, gives the drag component on the strut pin in the plane of the drag struts. The pin lifts and drags so found are taken as the applied loads on the lift trussing and drag trussing and are used to find the endwise stresses in their struts, stays, and spars. Convenient formulas for the concentrated drag loads are given in figure 20.

An alternative method for finding the force on the strut pin is to multiply the mean running load on the spar by the distance between the points of zero shear in the adjoining spar panels. Equivalent formulas for this operation are given in figure 21.

In applying this alternative method, if the points of zero shear have not previously been found they may be taken as at the centers of each panel except the inner or root panel. For the root or inner panel the point of zero shear is three-eighths the panel length from the body hinge of the "engine section." This is an approximate method sometimes used for brevity. Its accuracy may be judged by reference to the typical moment diagram of figure 19.

To the above concentrated lifts must be geometrically added the weights of the struts and stays and in some cases the weight of the motor, the force of the aileron, the thrust of the propeller, etc.

When external stays are applied to the wing, such as lead wires or under struts, these may either be assumed severed or be taken as an integral part of the trussing. They are commonly assumed to be severed, so that the wing may be shown adequately strong without them and not liable to disaster in case of their accidental rupture. Then in turn the external stays are assumed to bear the whole lift or drag while the internal ones are severed.

If any sloping external stay, figure 24, of length r, whose three projections on the reference planes of the machine are x, y, z, sustain a tension R, whose components are X, Y, Z, then

$$R/r = X/x = Y/y = Z/z. \qquad (23)$$

Thus if any internal drag wire should fail, causing a forward pull X in the lead wire, the stress in this latter would be $R = rX/x$, entailing a compression in the spar $Y = yX/x$.

In case of cabane stays, figure 24, the y may be zero and $R = rX/x$. Similarly, the stress in a fore-and-aft diagonal wing stay, supporting a drag X on the top plane, is $R = rX/x$.

Example 7.—Find the concentrated loads on the lift trussing of example 4, given the weight of the front struts, stays, cabanes, etc.

Example 8.—Given the resistance of the front struts and stays and the running load, find the concentrated loads on the drag trussing of example 4, by the zero shear method.

Example 9.—Given the resistance of the struts and stays at high and low speeds, find the concentrated loads on the drag trussings for both speeds, using values of the pin reactions found in example 4.

Example 10.—An aileron bearing a uniform pressure of 20 pounds per square foot has the dimensions and structural form shown in figure 38. Find the stresses in the stay wires.

18. ENDWISE STRESSES IN MEMBERS OF NONMULTIPLANE WING TRUSSING.

(a) *Due to lift and drag.*—From the given concentrated loads and frame diagram of the wing, the aggregate endwise stresses, and thence the unit stresses, are found by well known analytic and graphic methods.

Convenient analytical methods of finding the aggregate stresses, i. e., in struts, stays, and spars, are presented in figures 22 and 23. The usual graphic method is illustrated in figures 26 and 27.

If the stress in but a single strut or stay of a biplane be desired, it can be obtained directly by an appropriate formula as a simple summation, or a product following a summation. In such truss analysis the following generalizations may be useful:

1. Any strut exerts a thrust equal to the sum of all the loads preceding it. Thus in figure 22 R is preceded by the loads G, G', H, H', I, and therefore exerts at I a thrust equal to their sum.

2. Any stay exerts a tension equal to the sum of the loads preceding it times its own length divided by the truss gap.[1] Thus for the stay Q the tension is $(H + H' + I + I')\ q/h$. Otherwise the vertical component of any stay tension equals the difference between the strut thrust and the concentrated load on the strut joint; also the horizontal tension component equals the difference of the spar thrusts on either side of the joint.

3. Any top spar panel exerts a thrust equal to the sum of the moments of the preceding loads about its inner end divided by the gap. Any lower spar panel exerts a tension equal to the sum of the moments of the preceding loads about its outer end divided by the gap. The tension in a lower spar panel equals the compression in the one obliquely above and out from it.

(b) *Endwise stresses in struts, stays, and spars due to wing torsion.*—The aileron lift PA exerts a torsional moment $M = PAl$ about the wing axis distant l from PA. The ensuing endwise stresses in the wing struts, stays, and spars can be calculated by the formulas for a twisted pyramidal truss given in figures 33 and 34.

Example 11.—Find the endwise stresses in the struts, stays, and spars of the front lift trussing, figure 40, for low speed and the rear lift trussing for high speed.

Example 12.—Find the stresses in the drag trussings, figure 40, due to the concentrated loads of example 9.

Example 13.—Find the stresses and factors of safety in the spars, struts, and stays in figure 40 for low and high speeds, respectively.

Example 14.—Find the stresses in the principal members of the wing trussing of figure 40 due to a uniform air pressure of 20 pounds per square foot on the aileron surface of figure 38.

19. ENDWISE STRESSES IN MEMBERS OF MULTIPLANE WING TRUSSING.

Figure 25 gives a general process for finding the stresses in a multiplane wing truss, and applies it to a triplane. Before employing this method the total lift on the strut exerted by all the planes is found by summing their individual lifts, as in article 17 on concentrated loads.

20. STRESSES IN REDUNDANT TRUSS MEMBERS.

The last article illustrates the case of a truss having redundant members, whose stresses are indeterminate by rigid statics, but determinate by elastic statics. To generalize this case, suppose a multiplane wing truss having initial stresses in both its load wires and its landing

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[1] "Truss gap" here means the distance between spar centers in either the lift or the drag plane.

wires. Here a lift L applied to a strut causes increase of tension in the load wires, decrease in the others. The increments of tension in both load and landing wires can be resolved vertically, summed and equated to the increment in L which causes them. The obvious expression for this is, by figure 25,

$$\Delta L = \Delta Pm/p + \Delta P'm'/p' + \Delta Qn/q + \Delta Q^1 n^1/q'$$

in which the primes refer to the landing wires which cross the load wires diagonally. In many practical instances of wing and body construction the unprimed and primed quantities are respectively equal. The formula then becomes:

$$\Delta L = 2 \ (\Delta Pm/p + \Delta Qn/q)$$

Example 15.—Given the data for figure 25, as below, solve for the tensions in the stay wires; $L = 145$ pounds; $m = 60$ inches; $n = 50$ inches; $p = 99.8$ inches; $q = 94.2$ inches; $A = .012$ square inch; $B = .012$ square inch; and $E = 30,000,000$.

21. GROUPING OF WING ANALYSIS DATA AND COMPUTED VALUES.

Figure 28 [1] shows synoptically for a typical wing, (1) the general aerodynamic data for its individual planes, (2) the load distribution on the surfaces and trussing at both high and low speeds, and the stress analysis for these conditions, (3) the tabulated dimensions of the truss members, their principal stresses and factors of safety. Such detail calculations as do not appear in the diagram are given in the solutions of the individual problems of this chapter.

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[1] This type of diagram, prepared by the writer, was published in part by the Franklin Institute Journal in December, 1914; entire in Aviation and Aeronautical Engineering in 1917.

REPORT NO. 82.

PART III.

AIRPLANE BODY STRESSES.

By A. F. Zahm.

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21. SCOPE OF TREATMENT.

The study of airplane body stresses may cover in succession: The tail unit; the tail skid; the chassis or landing carriage; the fuselage; each case compromising the applied loads, the induced reactions, the strength of the structural elements, and finally the factors of safety; the fuselage being treated last because stressed by all the other members thereto attached.

22. THE TAIL UNIT.

The tail unit comprises (a) the vertical tail surfaces; (b) the horizontal tail surfaces. The combined air forces on the tail surfaces—that is, the lateral force on the vertical surfaces and the vertical force on the horizontal surfaces—are taken as applied loads on the fuselage stern when in flight.

(a) *The vertical tail surfaces.*—The rudder may be hinged to the rear edge of a fixed vertical fin attached to the stern of the fuselage or other rearward projecting framework. The loads on the rudder are the air force and the tiller pull. In unbalanced rudders the normal tiller pull is usually taken as the greatest force a pilot would care to exert in regular flying, and is determined by measuring with a spring balance the force exerted by a pilot when seated in the stationary machine and plying the control vigorously. In common practice the force is assumed, as for the aileron in section 14, to be at the center of the surface and equal to PA, where P is the pressure per unit area of the rudder surface A.

The fixed vertical fin sustains both its own air force and the hinge forces exerted by the vertical rudder. The first may be taken as PA applied at the center of the fin area; the second may be computed by use of figure 13.

(b) *The horizontal tail surfaces.*—The horizontal rudder, or elevator, is usually hinged to the rear edge of a fixed tail plane, or stabilizer. The elevator stresses are found as for the cases already considered of the aileron and the vertical rudder. The stabilizer is treated very like a monoplane wing except that it sustains considerable force on its rear edge, due to the hinge pressures, and is aerodynamically influenced by the presence of the horizontal rudder, and commonly also by the propeller slip stream.

Example 1.—An elevator bearing a uniform pressure of 20 pounds per square foot has the dimensions and structural form shown in figure 37. Find the moment about the hinge and the stresses in the stays and the control wires; also the hinge reactions.

Example 2.—Find the vertical components of the pin reactions of the front and rear beams of the stabilizer or horizontal fin, figure 37, due to a uniformly distributed pressure of 20 pounds per square foot and the hinge reactions of example 1.

Example 3.—A rudder and vertical stabilizer bearing a uniform pressure of 20 pounds per square foot has the dimensions and structural form shown in figure 37; find the transverse loads on the upper and lower trussing of the fuselage and the transmitted couple about the normal axis of the various sections.

The tail skid.—When the craft is landing or resting on the earth the tail skid force is taken as a stern load on the fuselage.

The normal static or *dead load* on the tail skid is its ground pressure when resting on a level surface, or, in the case of certain waterplanes, it is the static lift on the rear float. The normal lift of the tail skid is $L = Wa/l$, figure 29, if W be the total weight of the craft, and a, l, the distances respectively of W and L from the tread, or forward support of the machine.

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The *live load* on the tail skid may be resolved into rectangular components which are found as in figure 31, when the craft's dimensions and accelerations, there assumed, are known. Otherwise the maximum tail skid force to be provided against may be specified by the purchaser. From the force so found or specified the stresses in the tail skid, tail float or superstructure may be approximately calculated by elementary statics, ignoring the accelerations within the structure.

Example 4.—Compute the normal lift on the tail skid from the data given in figure 29.

Example 5.—Given $I = 60{,}544$ pound-feet^2, $j_x = 16$ feet/sec.^{2}, $j_y = 8$ feet/sec.^{2}, $\alpha = 0.1$ rad. /sec.^{2}, in figure 31 and the dimensions of a machine, find the resultant live load on the tail skid.

23. UNDERCARRIAGE LOADS AND STRESSES.

The normal static or *dead load* at the bottom of the undercarriage equals in magnitude the total weight there supported when the craft is resting level. The supporting pressures are perfectly definite in some cases, as when the craft rests on two wheels and a skid, or on four cushioned wheels, etc.

The stresses in the under carriage may be computed first for the normal static load, then for a similar magnified load by multiplying by an assumed ratio, such as j/g in section 4.

When the airplane pitches, skids, or slews about, other considerations, which may be of great importance, enter the stress analysis. The case of an airplane skidding or abruptly canting is treated at the end of this article.

For a common airplane, figure 29, if l, a be, respectively, the distances aft the axle of the tail-skid toe and the weight W of the whole craft less wheels, then the upward pressure on the skid toe is Wa/l, and this taken from the weight gives twice the pressure R on each of the axles.

Resolving R parallel to the side-view projections of the struts, figure 29, gives, respectively, the force P sustained jointly by the front strut and stay and Q borne by the rear ones. These forces in turn resolved as shown give the stresses in the individual uprights of the undercarriage. The first resolution obviously gives

$$P = R \sin \alpha / \sin \gamma$$
$$Q = R \sin \beta / \sin \gamma, \tag{24}$$

where α, β, γ are, respectively, the angles opposite P, Q, R in the force triangle. Similar equations in turn give the components of P, Q in the uprights.

The stresses in the undertrussing of a typical seaplane at rest are treated in figure 30. The lift R resolved parallel to the front view projections of the struts and stays, as shown, give its components P, Q in the planes, respectively, of the stays and of the struts. Now resolving P in the true plane of the stays, Q in that of the struts, as shown, gives the stresses in the uprights.

As shown in figure 29, the strut pairs bear down each with the force R on the axle, here assumed to be a straight and single tube. This axle normally finds support at the centers of the wheels and sustains on its segment outward from the struts bending and shearing stresses computable by the simplest cantilever formulas as used for wing ribs. Between the strut bearings the axle sustains a constant bending moment equal to the maximum in the cantilever portion. On this latter portion the maximum vertical shearing stress equals the wheel lift R; the maximum moment is Rl if l be the distance from the lift R to the down pressure of the strut pair. From the shear and moment the factors of safety are found as usual.

The *live load* on the bottom of the undercarriage is treated in quite the same manner as that on the tail skid by the equations of figure 31.

When an airplane skids on the ground or rests with one wheel low, the ground reaction on one wheel can be resolved into two rectangular components, one parallel the other perpendicular to the axis. The parallel component F, say, exerts on the axle a bending moment, $M = FR$, where R is the radius of the wheel. If W be the weight, α the angle of cant, $F = 1/2\ W \sin \alpha$. The ensuing moment, $M = 1/2\ WR \sin \alpha$, may be quite formidable.

Example 6.—From the data of figure 29 compute the normal load at the wheels of the undercarriage; also the stresses in the undercarriage struts and stays and their factors of safety.

Example 7.—Find the stresses in the undercarriage trussing of the seaplane in figure 30 due to a lift of 600 pounds applied at a point one-third the distance from the front to the rear strut attachments.

Example 8.—An airplane weighing 3,000 pounds, with wheels 2 feet in diameter and 6 feet apart, rests with one wheel 10 inches lower than the other. Find the added bending moment on the axle, assuming each wheel to bear half the entire weight.

24. FUSELAGE LOADS AND STRESSES.

The fuselage may sustain gravitational, aerodynamic, and impact or acceleration loads. The ensuing stresses at each point of the structure are separately computed, then combined to determine the resultant stress there. A twofold analysis is usually made, (a) for flying conditions, (b) for landing or static conditions. In both cases the analysis may be made either for the fuselage as a unit or separately for the rear segment, the front segment, and the center segment.

(a) *Fuselage loads and stresses for flying conditions.*—If the loads *exert no torque* on the fuselage, then for a typical *rear segment*, figure 32 presents the analytical treatment, figure 42 the graphical. In practical computation the weights of the struts, stays, and longeron panels are regarded as all concentrated on the upper pins, rather than as part on the upper, part on the lower pins. The air force on the side of the segment is comparatively negligible; that on the stern, due to the tail unit, is given in paragraph 2 of this part.

The gravitational stresses in the *front segment* are found similarly to those treated in (a); the power stresses, due to propeller thrust and torque, engine vibration, etc., may be estimated separately, then combined with the former. The aerodynamic forces on the bow are usually negligible.

The stresses in the *center segment* of the fuselage, due to the gravitational and aerodynamic loadings, may be found separately, then combined with those due to the action of the attached members; i. e., wings, undercarriage, and front and rear fuselage segments.

When the applied loads *exert a torque* on the fuselage, endwise stresses ensuing from the latter are computed by the formulas of figures 33 and 34 for twisted trusses. For example, if the rudder force PA is distant l from any axis of the fuselage, it may be replaced by a force PA and a couple PAl, both applied at said axis, the force generating one set of stresses, the couple another, and each set separately computable by one or the other of the above formulas.

(b) *Fuselage loads and stresses for landing or static conditions.*—For static conditions the loads inside the fuselage are the same as those for flight. The external applied loads are the wing weights, the reactions of the tail skid and undercarriage. The stresses are found as shown in figures 32 and 42.

For kinetic conditions the applied loads on each part are computed as explained in figure 31. The stresses are then found as explained in the preceding paragraph.

Example 9.—Find the stresses in the struts, stays, and longerons of the rear segment of the fuselage due to a uniform pressure of 20 pounds per square foot upon the horizontal tailpieces; also those due to gravitational loads alone.

Example 10.—Find the stresses in the struts, stays, and longerons of the vertical trussing of the front segment of the fuselage shown in figure 35, due to gravitational loads.

Example 11.—From the data in the problems above find the stresses and factors of safety for the principal members of the fuselage for a steady, circular flight around a level curve of 200 feet radius at 80 miles per hour.

Example 12.—Find the stresses in the rear segment of the fuselage due to the torsional loads in example 3.

25. GROUPING OF BODY ANALYSIS DATA AND COMPUTED VALUES.

Figures 35 and 36 show synoptically for a typical airplane body the graphical analysis and the numerical results for both flying and static conditions. Such detail calculations as do not appear in this diagram are given in the solutions of the individual problems of this part of the text.

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REPORT NO. 82.

PART IV.

PROBLEMS IN AIRPLANE STRESS ANALYSIS.

By L. H. Crook.

PROBLEMS IN PART I.

Stresses in Materials.

Example 1.[1]—If an accelerometer fixed to an airplane records a maximum vertical acceleration of 48 feet per second per second, what is the ratio of the abnormal to the normal loading? Given $g = 32$ feet per second per second, $j = 48$ feet per second per second.
Then by section 4

$$r = j/g = 48/32 = 1.5 \text{ (ratio). Ans.}$$

Example 2.—An airplane in landing has a vertical velocity of 10 feet per second and a uniform cushion yield of 6 inches. Find the ratio of the abnormal to the normal stresses in the landing gear, assuming the machine to be a rigid structure.
Given $V = 10$ feet per second.
Then by section 4

Velocity "head" $h = v^2/2g$
$$= (10)^2/2(32) = 1.56 \text{ feet.}$$

By equation 1

$$r = 1 + h/d$$
$$= 1 + 1.56/0.5 = 4.12 \text{ (ratio). Ans.}$$

Example 3.—Find the longitudinal shearing stress in a 4-inch square beam, given the total vertical shear as 1,400 pounds, first with diagonal vertical, then with it inclined 45°.

I. DIAGONAL VERTICAL.

Shear at neutral axis.
Given $\quad a = 8$ sq. ins.; $V = 1,400$ lbs.; $I = 21.33$ ins.[4]
$\quad\quad\quad c = 0.942$ ins.

By formula, 5
$$S = ac\ V/I\ t$$
$$= 8(0.942)1400/21.33(5.656)$$
$$= 87.5 \text{ lbs. per sq. in. Ans.}$$

Shear at $1/4\ h$.[2]
Given $\quad a = 4.5$ sq. ins.; $V = 1,400$ lbs.; $I = 21.33$ ins.[4]
$\quad\quad\quad c = 1.414$ ins. $\quad\quad\quad\quad t = 4.242$ ins.
$$S = ac\ V/I\ t$$
$$= 4.5(1.414)1400/21.33(4.242)$$
$$= 98.3 \text{ lbs. per sq. in. Ans.}[3]$$

Shear at $1/2\ h$.
Given $\quad a = 2$ sq. ins.; $V = 1,400$ lbs.; $I = 21.33$ ins.[4]
$\quad\quad\quad c = 1.885$ ins. $\quad\quad\quad\quad t = 2.828$ ins.
$$S = ac\ V/I\ t$$
$$= 2\ (1.885)1400/21.33(2.828)$$
$$= 87.5 \text{ lbs. per sq. ins.}$$

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[1] Unless otherwise stated, all quantities are expressed in foot, pound, second, gravitational units.
[2] Distance from neutral axis to outermost fiber.
[3] Note that maximum shear is not at neutral axis.

Shear at 3/4 h.

$a = 0.50$ sq. ins.; $V = 1,400$ lbs.; $I = 21.33$ ins.[4]
$c = 2.357$ ins. $t = 1.414$ ins.

$S = ac\ V/I\ t$
$= 0.50(2.357)(1400)/21.33(1.414)$
$= 54.7$ lbs. per sq. in.   Ans.

Shear at outermost fiber.

Since $a = o$, $t = o$, then $S = o$.   Ans.

II. DIAGONAL INCLINED 45°.

Shear at neutral axis.

Given $a = 8$ sq. ins.; $V = 1,400$ lbs.; $I = 21.33$ ins.[4]
$c = 1$ in. $t = 4$ ins.

$S = ac\ V/I\ t$.
$= 8\ (1,400)/21.33(4)$
$= 131.2$ lbs. per sq. in.   Ans.

Shear at 1/4 h.

Given $a = 6$ sq. in.; $V = 1,400$ lbs.; $I = 21.33$ ins.[4]
$c = 1.25$ ins. $t = 4$ ins.

$S = ac\ V/I\ t$.
$= 6\ (1.25)\ 1,400/21.33(4)$
$= 123.1$ lbs. per sq. in.   Ans.

Shear at 1/2 h.

Given $a = 4$ sq. ins.; $V = 1,400$ lbs.; $I = 21.33$ ins.[4]
$c = 1.50$ ins.; $t = 4$ ins.

$S = ac\ V/I\ t$
$= 4\ (1.50)\ 1,400/21.33(4)$
$= 98.4$ lbs. per sq. in.   Ans.

Shear at 3/4 h.

$a = 2$ sq. ins.; $V = 1,400$ lbs.; $I = 21.33$ ins.[4]
$c = 1.75$ ins. $t = 4$ ins.

$S = ac\ V/I\ t$
$= 2\ (1.75)\ (1,400)/21.33(4)$
$= 57.4$ lbs. per sq. in.   Ans.

hear at outermost fiber. $S = o$.

Example 4.—Find the resultant unit stress in a 2-inch square simple beam, 80 inches long; due to an endwise load of 900 pounds, a maximum bending moment of 1,100 inch-pounds and a maximum deflection of 0.05 inches at the place of maximum moment.

Given, $Z = 1.33$ ins.³; $d = 0.05$ in.; $E = 1,500,000$.
$I = 1.33$ ins.⁴; $A = 4$ sq. ins.

Then by footnote, section 7.

$K = \sec. \dfrac{l}{2}\ \sqrt{P/EI}$

$= \sec. [40\sqrt{900/(1,500,000)\ (1.33)}]$
$= \sec. [.845] = 1.52$.

Then by formula 7 and computed value of K

$S = P/A + M/Z + K\ P\ d/Z$
$= 900/4 + 1,100/1.33 + 1.52\ (900)\ (0.05)/1.33$
$= 225 + 827 + 51 = 1103$ lbs. per sq. in.

Example 5.—Find the unit stress in a pin-ended column 1 inch square and 30 inches long under an axial load of 200 pounds; also the unit stress when taking the eccentricity of the load as 2 inches.

Given $l = 30$ ins.; $r = \sqrt{I/A} = 0.288$ in.; $P = 200$ lbs.
$A = 1$ sq. in.; $a = 4/3,000$

Then by formula 9.

$$S = [(1 + a(l/r)^2]P/A$$
$$= [1 + (4/3,000) \ (30/0.288)^2] \ 200/1$$
$$= 3,081 \text{ lbs. per sq. in.}$$

Given $l = 30$ ins.; $r = 0.288$ (ins.); $c = 1/2$ in.
$A = 1$ sq. in.; $a = 4/300$ (constant)
$P = 200$ lbs.; $p = 2$ ins.

Then by formula 10.

$$S = [1 + a(l/r)^2 + cp/r^2) \ P/A$$
$$= [1 + 3/4,000 \ (30/0.288)^2 + 0.5(2)/0.288^2] \ 200/1$$
$$= 5,460 \text{ lbs. per sq. in.}$$

Example 6.—A beam is subject to a compressive unit stress of 200 pounds per square inch, and at the same time to a longitudinal shearing stress of 250 pounds per square inch. Compute the maximum resultant stresses.

Given, $S_s = 250$ pounds per square inch. $S = 200$ pounds per square inch.

Then by formulas 11 and 12

$$S_p = \sqrt{(S_s)^2 + (S/2)^2}$$
$$= \sqrt{(250)^2 + (200/2)^2}$$
$$= 269 \text{ pounds per square inch.   Ans.}$$
$$S_n = \sqrt{S/2 \pm S_p}$$
$$= 200/2 + 269 = 369 \text{ pounds per square inch.   Ans.}$$

Example 7.—If a member whose allowable constant stress is 20,000 is stressed frequently to 15,000 and has an elastic limit of 60,000, what is the equivalent steady stress?

Given, $S = 20,000$ pounds per square inch.
$n = 1$ (constant).
$m = 15,000/60,000 = 1/4$ (ratio).

Then by equation 13
$$S' = S(1 + m)^n$$
$$= 20,000 \ (1 + 1/4)^1$$
$$= 25,000 \text{ pounds per square inch.   Ans.}$$

Example 8.—Find the maximum load for a pin-ended spruce column of length 60 inches, cross section 3 square inches, and moment of inertia 0.3 inch[4].

Given, $l = 60$ inches; $l/r = 60/0.31 = 193$ (slenderness ratio).
$A = 3$ square inches; $E = 1,500,000$ pounds per square inch.
$I = 0.3$ inch.[4]

Then by formula 15

$$P_{max} = \pi^2 \ EI/l^2$$
$$= 9.86 \ (1,500,000) \ 0.3/(60)^2$$
$$= 1,233 \text{ pounds.   Ans.}$$

Example 9.—Find the factors of safety in example 6 for a maximum shearing stress of 1,000 pounds per square inch and a maximum compressive stress of 4,000 pounds per square inch; also the factor of safety in example 8 for an applied load of 250 pounds.

By section 10,

From example 6

    Compressive stress = 369 pounds per square inch.
    Shearing stress   = 269 pounds per square inch.

Then, for compression

    $F. S. = 4,000/369 = 10.8.$ Ans.

For shear

    $F. S. = 1,000/269 = 3.7.$ Ans.

From example 8

    Maximum load carried by beam = 1,250 pounds.

Then,

    $F. S. = 1,250/250 = 5.$ Ans.

PROBLEMS IN PART II.

Airplane Wing Stresses.

Example 1.[1]—A rib loaded as in figure 39 has the dimensions there specified. Find the unit stresses for bending, maximum shearing, and average vertical shearing just outside the spars and just inside the spars; also the unit bending stress at R between the spars.

SECTION M.

(a) Consider the cap strips as carrying all the bending stresses.

Given $I = 1.22$ in.[4]; $Z = 1.22/1.93 = 0.632$ in.[3]
    $R = 4$ lbs.; $l = 2$ in.

Then by formula (18)

    $S = Rl/Z$
    $= 4[2]/0.632 = 12.6$ lbs. per sq. in.   Ans.

(b) Consider the cap strips and web as carrying the horizontal shearing stress.

Given $I = 2.01$ in.[4]; $c = 1.0$ in.; $a = 0.61$
    $t = 0.25$ in.; $V = 4$ lbs.

Then by formula (5) $S = ac V/It$
                $= 0.61(1)4/2.01(0.25) = 4.9$ lbs. per sq. in.   Ans.

(c) The total vertical shear at this section is not distributed as shearing stress over the web section, since the web of the rib ends at this section. The total vertical shear, however, is carried primarily by a compressive stress on the upper surface of the tongue that projects into the spar. The compressive stress in this case is

$S = V/A$
    $= 4/(.25 \times .50) = 32$ lbs. per sq. in.   Ans.

SECTION N.

(a) Consider the mid part of the rib as a simple beam.[2] The bending stress at section N is then zero.

(b) Consider the cap strips and web as carrying the horizontal shearing stresses.

Given $I = 2.36$ in.[4]; $c = 1.08$ in.; $a = 0.64$ sq. in.
    $t = 0.25$ $V = 12.22$ lbs.

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[1] The solutions given in this example must be looked upon only as approximations. An accurate theoretical solution must take into account the hollowed web, the glued strips, strength of glued surfaces, etc.

[2] Spar being free to twist.

Then by formula (5)

$$S = ac \ V/It$$
$$= 0.64(1.08)12.22/2.36(0.25)$$
$$= 7.76/0.59 = 13.1 \text{ lbs. per sq. in.} \quad \text{Ans.}$$

(c) Treat the total shearing force as in (c) of section M.

SECTION O.

(a) Consider the bending stress S as carried by the actual section.

Given $I = 2.84$; $c = 2.30$; $a = 14.20$; $b = 19.80$; $R = 20.41$
$$Z = I/c = 2.84/2.30 = 1.23$$

Then by formula (19),

$$S = R \ ab/2(a+b)z$$
$$= 20.41(14.20)(19.80)/2(34)(1.23) = 69 \text{ lbs. per sq. in.} \quad \text{Ans.}$$

(b) Total vertical shear and also the horizontal shearing stresses are zero at this section.

SECTIONS P AND Q.

Treat similarly to section N and M, respectively.

Example 2.—An aileron bearing a uniform pressure of 20 pounds per square foot has the dimensions and structural form shown in figure 38. Find the moment about the hinge and the stress in the control wire. Find also the vertical reactions of the stay wire on the trailing edge beam.

(A) TOTAL LOAD ON AILERON.

By section 14 and figure 38.

Given $P = 20$ lbs. per sq. ft. (assumed loading).

$A =$ area of aileron surface $= 20.7$ sq. ft.

Then total load $= P \ A$
$$= 20 \times 20.7 = 414 \text{ lbs.} \quad \text{Ans.}$$

(B) MOMENT ABOUT HINGE.

By figure 38. Center of gravity $C = 9.45''$ from hinge.
$$M = CPA = 9.45(414) = 3,910 \text{ lbs. in.} \quad \text{Ans.}$$

(C) STRESS IN CONTROL WIRE.

By figure 38, given arm $= 12.75$; moment $= 3,910$

Then tension $= 3,910/12.75 = 307$ lbs. Ans.

(D) VERTICAL REACTIONS OF TRAILING EDGE BEAM.

Total load distributed along trailing edge is approximately 1/2 aileron load. By figures 38 and 13,

$$2wl = 414/2 = 207 \text{ lbs. and running load} = 207/162 = 1.28 \text{ lbs. per linear in.}$$

Given $l = 81$ in.; $b = 70$ in.; $a = 11$ in.

$w = 1.28$ lbs. per in.

By formula in figure 13—

$$R_1 = w \ l \ (4 \ l^2 - b^2)/8 \ bl$$
$$= 1.28 \ (81) \ [4(81)^2 - 70^2]/8(70) \ 81$$
$$= 48.8 \text{ pounds.} \quad \text{Ans.}$$
$$R_2 = 2 \ (w \ l - R_1)$$
$$= 2 \ [1.28 \ (81) - 48.8] = 109.7 \text{ pounds.} \quad \text{Ans.}$$
$$R_3 = 48.8 \text{ pounds.} \quad \text{Ans.}$$

Compare these answers with those found from the table in figure 13.

The vertical reactions are then as follows:

For central stay $= 109.7$ pounds. Ans.

Outside stays (each) $= 48.8$ pounds. Ans.

The tension in each stay wire may be found from these reactions. See example 10.

Example 3.—A biplane wing has the form and loadings shown in figure 41; find the resultant loadings on the spars and their components in the planes of the lift and drag trussings for angles of incidence of 2° and 12°.

The running lift L, the running drag D, and the distances to their points of application (al) depend upon the aerodynamic characteristics of the wing, the angle of incidence, the stagger, the biplane effect, etc. The running lift L and weight W as determined for any given wing plane of a machine are constant for all angles of incidence in straightaway flight. In the present example, however, L, D, W, a, and b are given.

I. ANALYSIS FOR LOW SPEED.
(Incidence 12°.)

(a) Running loads on upper front spar.

Given $L = 2.256$ lbs./in $D = 0.275$ lb./in. $W = 0.313$ lb./in.

$a = 0.67$ $a = 0.67$ $b = 0.46$

Then $aL = 1.511$ lbs./in. $aD = 0.184$ lb./in. $bW = 0.144$ lbs./in.

$aL - bW = 1.367$ lbs./in.

By simple graphics.

$w_1 = 1.389$ lbs./in. $w_2 = 0.100$ lb./in. $w = 1.370$ lbs./in.

The running loads on lower front spar, upper rear spar, and lower rear spar are similarly found. Their values are given in the table below.

II. ANALYSIS FOR HIGH SPEED.
(Incidence 2°.)

(a) Running loads on upper front spar.

Given $L = 2.256$ $D = 0.141$ $W = 0.313$.

$a = 0.29$ $a = 0.29$ $b = 0.460$.

Then $aL = 0.654$ $aD = 0.041$ $bW = 0.144$ $aL - bW = 0.510$.

By graphics, $w_1 = 0.517$ $w_2 = 0.106$ $w = 0.510$.

The running loads on lower front spar, upper rear spar, and lower rear spar are similarly found. Their values are given in the table below.

Grouping of data and computations for running loads w_1, w_2, w, on spars for low and high speeds.[1]

LOW SPEED.

Symbols.	L	D	W	a	b	aL	aD	bW	$aL-bW$	w_1	w_2	w
Front spars:												
Upper	2.256	0.275	0.313	0.67	0.46	1.511	0.184	0.144	1.367	1.389	0.100	1.370
Lower	1.845	.225	.286	.67	.46	1.236	.151	.131	1.105	1.127	.095	1.112
Rear spars:												
Upper	2.256	.275	.313	.33	.54	.744	.091	.169	.575	.588	.059	.580
Lower	1.845	.225	.286	.33	.54	.609	.074	.154	.455	.467	.049	.462

HIGH SPEED.

Symbols.	L	D	W	a	b	aL	aD	bW	$aL-bW$	w_1	w_2	w
Front spars:												
Upper	2.256	0.141	0.313	0.29	0.46	0.654	0.041	0.144	0.510	0.517	0.106	0.510
Lower	1.845	.116	.286	.29	.46	.535	.033	.132	.403	.408	.084	.403
Rear spars:												
Upper	2.256	.141	.313	.71	.54	1.602	.100	.169	1.433	1.456	.282	1.436
Lower	1.845	.116	.285	.71	.54	1.309	.082	.154	1.155	1.172	.226	1.158

[1] This method of grouping of data and computed values has been used; first, to help the computer in distinguishing the running loads and their components at the spars for different speeds; second, as a method of grouping of data and computed values for ready reference. The table below gives the computed values only.

Grouping of computed values of running loads on spars.

Speed—Plane of:—	Low.						High.					
	Lift trussing.		Drag trussing.		Spar web.		Lift trussing.		Drag trussing.		Spar web.	
	Front.	Rear.	Front.	Rear.	Front.	Rear.	Front.	Rear.	Front.	Rear.	Front.	Rear.
Running load on upper spars	1.389	0.588	0.100	0.059	1.370	0.580	0.517	1.456	0.106	0.282	0.510	1.436
Running load on lower spars	1.127	.467	.095	.049	1.112	.462	.408	1.172	.084	.226	.403	1.158

Example 4.—Find the shears, moments, and reactions for the upper and lower front spars of the biplane trussing shown in figure 40, due to the uniform running loads found in example 3; also the reactions of the spars in the planes of the lift and drag trussing and the shears and moments on each spar in the plane of the spar web.

I. SHEARS, MOMENTS, REACTIONS FOR THE FRONT SPARS DUE TO RUNNING LOADS IN PLANE OF FRONT STRUTS (LOW SPEED).

1. *Upper wing spar.*—The upper wing spar is considered as a continuous beam supported at four points and having one overhanging end.

Given $A = B = C = D = 1.389$ [1] lbs./ins.

$a = 30$ ins.; $b = 50$ ins.; $c = 84$ ins.; $d = 79$ ins.

Then by the formulas of figure 15.

Auxiliary Symbols.

$l = 1.389[(50)^3 + (84)^3]/4 - 1.389(30)^2 50/2$	[2] 217,970.21
$m = 1.389[(84)^3 + (79)^3]/4$	377,024.20
$h = 2(50 + 84)$	268.00
$i = 2(84 + 79)$	326.00

Joint Bending Moments.

$M_1 = 1.389(30)^2/2$	625.05
$M_2 = [217970.21(326) - 377024.20(84)]/268(326) - (84)^2$	490.44
$M_3 = [217970.21 - 490.44(268)]/84$	1,030.14
$M_4 = 0$	0.00

Shear, Right of Pin.

$V_1 = (490.44 - 625.05)/50 - 1.389(50)/2$	-37.41
$V_2 = (1030.14 - 490.44)/84 = 1.389(84)/2$	-51.91
$V_3 = (0 - 1030.14)/79 - 1.389(79)/2$	-67.89

Shear, Left of Pin.

$U_1 = 30(1.389)$	41.67
$U_2 = -37.41 + 1.389(50)$	32.04
$U_3 = -51.91 + 1.389(84)$	64.76
$U_4 = -67.89 + 1.389(79)$	41.84

Pin Reactions.

$R_1 = 41.67 + 37.41$	79.08
$R_2 = 32.04 + 51.91$	83.95
$R_3 = 64.76 + 67.89$	132.65
$R_4 = 41.84 + 0$	41.84

Position of Maximum Moments.

$x_1 = 37.41/1.389$	26.93
$x_2 = 51.91/1.389$	37.37
$x_3 = 67.89/1.389$	48.87

Maximum Moments Between Joints.

$M'_1 = 625.05 - (37.41)^2/2(1.389)$	+121.27
$M'_2 = 490.44 - (51.91)^2/2(1.389)$	-479.55
$M'_3 = 1030.14 - (67.89)^2/2(1.389)$	-628.98

2. *Lower wing spar.*—The lower wing spar is considered as a continuous beam supported at three points and having one overhanging end.

Given $A = B = C = 1.127$ [3] lbs. per in.; $a = 27$ ins.; $b = 84$ ins.; $c = 79$ ins.

Then by formulas of figure 14.

---

[1] The running load may be taken as unity. In the present problem it is the running load in plane of front lift trussing for low speed. (See problem 3, Pt. II.)

[2] All calculations were carried out two decimal places to insure accuracy primarily for comparing other methods of calculation, as slide rule, graphical, etc. (See example 7, Pt. II, and fig. 28.)

[3] Running load on lower spar in plane of front lift trussing for low speed. (See example 3, Pt. II.)

Auxiliary Symbols.

$l = 1.127[(84^3 + 79^3)/4] - 1.127(27)^2 84/2$ 271,401.60

$h = 2(84+79)$ 326.00

Joint Bending Moment.

$M_1 = 1.127\ (27)^2/2$ 410.79

$M_2 = 271401.60/326$ 832.52

Shears, Right of Section.

$V_1 = (832.52 - 410.79)/84 - 1.127(84)/2$ .... -42.31

$V_2 = (0 - 832.52)/79 - 1.127(79)/2$ .... -55.04

Shears, Left of Section.

$U_1 = 27(1.127)$ 30.42

$U_2 = -42.32 + 1.127(84)$ 52.35

$U_3 = -55.04 + 89.03$ 33.99

Pin Reactions.

$R_1 = 30.42 + 42.31$ 72.73

$R_2 = 52.35 + 55.04$ 107.39

$R_3 = 33.99 + 0$ 33.99

Position of Maximum Moments Between Joints.

$x_1 = 42.31/1.127$ 37.54

$x_2 = 55.04/1.127$ 48.83

Maximum Moments Between Joints.

$M'_1 = 410.79 - (42.31)^2/2(1.127)$ .... -383.41

$M'_1 = 832.52 - (55.04)^2/2(1.127)$ .... -511.49

II. REACTIONS OF SPARS IN PLANES OF LIFT AND DRAG TRUSSING FOR HIGH AND LOW SPEEDS.

The reactions in the planes of the lift and drag trussings for front and rear spars at high and low speeds may be found from the known reactions on the front upper and lower spars, by multiplying by the ratio of the running loads.[1] In the table below the ratios of the running loads are given, as well as the running loads.

Data and computed values for reactions.

Speed	Low.				High.			
Trussing	Lift.		Drag.		Lift.		Drag.	
	Front.	Rear.	Front.	Rear.	Front.	Rear.	Front.	Rear.
Running load on upper spars	1.389	0.588	0.100	0.059	0.517	1.456	0.106	0.282
Ratio of loads	1	[1].424	.072	.043	.373	1.049	.077	.204
Reactions of upper spars R_1..	79.08	[1]33.52	[1]5.69	[1]3.40	[1]29.49	[1]82.95	[1]6.08	[1]16.13
R_2..	83.95	35.59	6.04	3.60	31.31	88.06	6.46	17.12
R_3..	132.65	56.24	9.55	5.70	49.47	139.14	10.21	27.06
R_4..	41.84	17.74	3.01	1.79	15.60	43.89	3.22	8.53
Running load on lower spars	1.127	.469	.095	.049	.408	1.172	.084	.226
Ratio of loads	1	.416	.085	.044	.363	1.040	.075	.200
Reactions of lower spars R_1..	72.73	30.25	6.18	3.20	26.40	75.63	5.45	14.54
R_2..	107.39	44.67	9.12	4.72	38.98	111.68	8.05	21.47
R_3..	33.99	14.13	2.88	1.49	12.33	35.34	2.54	6.79

[1] These values are found by multiplying the column on extreme left by the ratio of loads.

III. MOMENTS AND SHEARS IN PLANE OF SPAR WEB AS FOUND FROM THOSE IN PLANE OF LIFT TRUSSING FOR FRONT SPARS ONLY.

The shears and moments in plane of spar web may be found directly, by the equations of figures 14, 15, 16, etc.; using a running load in the plane of the spar web, or indirectly by ratio methods, that is, by ratio from a known analysis for unit loading or a running load in some other plane. The table below gives the moments and shears in plane of spar web as found from those in plane of lift trussing for front spars only.

[1] This is true only for similar spar lengths.

Data and computed values for moments and shears.

LOW SPEED.

Front spar	Upper.		Lower.	
Plane of	Lift trussing.	Spar web.	Lift trussing.	Spar web.
Running loads	1.389	1.370	1.127	1.112
Ratio of loads	1	.9864	1	.9867
Joint bending moments $\begin{cases}M_1 \\ M_2 \\ M_3 \\ M_4\end{cases}$	625.05	[1] 616.54	410.79	[1] 405.32
	490.44	483.77	832.52	821.44
	1,030.14	1,016.13	0	0
	0	0		
Maximum moments between joints $\begin{cases}M'_1 \\ M'_2 \\ M'_3\end{cases}$	+121.27	+119.62	−383.41	−378.31
	−479.55	−473.02	−511.49	−504.68
	−628.98	−620.42		
Shears, left of pins $\begin{cases}U_1 \\ U_2 \\ U_3 \\ U_4\end{cases}$	41.67	41.10	30.42	30.01
	32.04	31.60	52.35	51.65
	64.76	63.87	33.99	33.63
	41.84	41.27		
Shears, right of pins $\begin{cases}V_1 \\ V_2 \\ V_3\end{cases}$	−37.41	−36.90	−42.31	−41.74
	−51.91	−51.20	−55.04	−54.30
	−67.89	−66.96		

[1] Found from column on left by using ratio of loads.

IV. MOMENTS AND SHEARS IN PLANE OF SPAR WEB FOR ALL SPARS AT HIGH AND LOW SPEED, AS FOUND FROM DATA IN TABLE III.

Moments and shears for spars.

UPPER SPARS.

Speeds	Low.		High.	
Spar	Front.	Rear.	Front.	Rear.
Running load	1.370	0.580	0.510	1.436
Ratio of loads	1	.424	.373	1.048
Joint-bending moments $\begin{cases}M_1 \\ M_2 \\ M_3 \\ M_4\end{cases}$	616.54	[1] 261.41	[1] 242.30	[1] 626.13
	483.77	205.11	190.12	500.99
	1,016.13	430.83	399.33	1,064.90
	0	0	0	0
Maximum moments between joints $\begin{cases}M_1 \\ M_2 \\ M_3\end{cases}$	+119.62	+50.71	+47.07	+125.36
	−473.02	−200.56	−185.89	−495.72
	−620.42	−263.05	−243.82	−650.20
Shears, left of pin $\begin{cases}U_1 \\ U_2 \\ U_3 \\ U_4\end{cases}$	41.10	17.42	16.15	43.07
	31.60	13.39	12.41	33.11
	63.87	27.08	25.10	66.93
	41.27	17.49	16.21	43.25
Shears, right of pin $\begin{cases}V_1 \\ V_2 \\ V_3\end{cases}$	−36.90	−15.64	−14.50	−38.67
	−51.20	−21.70	−20.12	−53.65
	−66.96	−28.39	−26.31	−70.17

[1] Found from column on left by using ratio of loads.

LOWER SPARS.

Speeds	Low.		High.	
Spar	Front.	Rear.	Front.	Rear.
Running load	1.112	0.462	0.403	1.158
Ratio of loads	1	.416	.363	1.042
Joint-bending moments $\begin{cases}M_1 \\ M_2 \\ M_3\end{cases}$	405.32	168.61	147.13	422.34
	821.44	341.71	124.04	855.94
	0	0	0	0
Maximum moments between joints $\begin{cases}M_1 \\ M_3\end{cases}$	−378.31	−157.37	−57.12	−394.19
	−504.68	−209.94	−76.20	−525.87
Shears, left of pin $\begin{cases}U_1 \\ U_2 \\ U_3\end{cases}$	30.01	12.48	4.53	31.27
	51.65	21.48	7.79	53.81
	33.63	12.99	5.07	35.04
Shears, right of pin $\begin{cases}V_1 \\ V_2\end{cases}$	−41.74	−17.36	−6.30	−43.49
	−54.30	−22.58	−8.19	−56.58

A close examination of the foregoing results will show that the greatest stresses occur in the front spars for low speed and the rear spars for high speeds.

Example 5.—Find and plot the resultant moment due to the uniform loading and the eccentric stay wire attachments shown in figure 18.

I. GENERAL DATA.

$$b = 50 \text{ in.}; \quad K_1 = 0; \quad P = 0$$
$$c = 84 \text{ in.}; \quad K_2 = 0.023; \quad Q = 228$$
$$d = 79 \text{ in.}; \quad K_3 = 0.025; \quad R = 460$$

II. COMPUTATIONS.

Consider the vertical components of the stay forces as concentrated loads on the continuous beam and apply the theorum of Bresse as given in figure 11 and explained in figure 12.

Application of Theorems to Spans (b) and (c).

$$M_1 b + 2 M_2(b+c) + M_3 c = -Pb^2(K_1 - K_1^3) - Qc^2(2K_2 - 3K_2^2 + K_2^3)$$
$$0 + 2 M_2(50 + 84) + 84 M_3 = 0 - 228(84)^2(0.04628)$$
$$\therefore 268\, M_2 + 84 M_3 = -74,453.78$$

Application of Theorems to Spans (c) and (d).

$$M_2 c + 2 M_3(c+d) + M_4 d = -Qc^2(K_2 - K_2^3) - Rd^2(2K_3 - 3K_3^2 + K_3^3)$$
$$84 M_2 + 2 M_3(84 + 79) + o = -228(84)^2(0.024) - 460(79)^2(0.04812)$$
$$\therefore 84 M_2 + 326 M_3 = -176,756.21$$

By elimination

$$M_1 = 0; \quad M_2 = -117; \quad M_3 = -512; \quad M_4 = 0$$

SHEARS AT RIGHT OF PINS.

$$V_1 = (M_2 - M_1)/b + P(1 - K_1)$$
$$= (-117 + o)/50 + 0 = -2.34.$$
$$V_2 = (M_3 - M_2)/c + Q(1 - K_2)$$
$$= (-512 + 117)/84 + 228(1 - 0.024) = +217.83.$$
$$V_3 = (M_4 - M_3)/d + R(1 - K_3)$$
$$= (o + 512)/79 + 460(1 - 0.025) = +454.98.$$

SHEARS AT LEFT OF PINS.

$$U_1 = o.$$
$$U_2 = V_1 - P$$
$$= -2.34 - o = -2.34.$$
$$U_3 = V_2 - Q$$
$$= +217.83 - 228 = -10.17.$$
$$U_4 = V_3 - R$$
$$= 454.98 - 460 = -5.02.$$

PIN REACTIONS.

$$R_1 = V_1 - U_1$$
$$= -2.34 - o = -2.34.$$
$$R_2 = V_2 - U_2$$
$$= +217.83 + 2.34 = +220.17.$$
$$R_3 = V_3 - U_3$$
$$= +454.98 + 10.17 = +465.15.$$
$$R_4 = V_4 - U_4$$
$$= +o + 5.02 = +5.02.$$

Check.

$$R_1 + R_2 + R_3 + R_4 = 688.$$
$$P + Q + R = 688.$$

MOMENTS AT POSITIONS OF LOADS.

$$M'_2 = M_2 + V_2 x$$
$$= -117 + 217.83(2) = +318.66.$$
$$M'_3 = M_3 + V_3 x$$
$$= -512 + 454.98(2) = +397.96.$$

III. GRAPHICAL RESULTS.

Figure 18 shows separately the moment curve for the uniformly loaded beam, that for the eccentric stay wires, and their resultant.

Example 6.—Complete the deflections in plane of spar web for all panels of the biplane trussing of figure 40, using the results of Table IV, example 4.

I. UPPER SPARS.

(1) Data table for use in computing deflections on front upper spar. (See Table IV, example 4, and formula for deflection in fig. 14.)

Span	b.	c.	d.
Joint-bending moments (M)	617	484	1,017
Shears, right of pin (V)	−37	−52	−67
Distance to maximum moment (x)	27	38	49
Length of span (l)	50	84	79
Running load (w)	1.370	1.370	1.370
Moment of inertia (I)	2.82	2.82	2.82
Modulus of elasticity (E)	1,880,000	1,880,000	1,880,000

(2) Computations.

The general formula for deflection, figure 14, is

$$d \; (=y) = -x[12\,M(l-x) + 4\,V\,(l^2-x^2) + w(l^3-x^3)]/24\,EI.$$

(a) Computation for span b:

$$(l-x) = 23 \quad (l^2-x^2) = 1,771 \quad (l^3-x^3) = 105,317.$$

$$d = -27\,[12(617)(23) - 4(37)(1,771) + 1.37(105,317)]/24(1,880,000)(2.82) = -0.0111.$$

(b) Computations for span c:

$$(l-x) = 46 \quad (l^2-x^2) = 5,612 \quad (l^3-x^3) = 537,832.$$

$$d = -38[12(484)46 - 4(52)5,612 + 1.37(537,832)]/127,283,400 = 0.048 \text{ inches.}$$

(c) Computations for span d:

$$(l-x) = 30 \quad (l^2-x^2) = 3,840 \quad (l^3-x^3) = 375,390.$$

$$d = -49[12(1,017)30 - 4(67)3,840 + 1.37(375,390)]/127,238,400 = 0.057 \text{ inches.}$$

(3) Computation [2] table for deflections in upper spars at high and low speeds as found from upper front spar at low speed.

UPPER SPARS.

Speeds	High.		Low.	
Spars	Front.	Rear.	Front.	Rear.
Ratio of running, load	1	0.424	0.373	1.048
Ratio of moments of inertia [3]	1	1.215	1	1.215
Product of ratios	1	.515	.373	1.273
Spans {b..	.011	[4].005	[4].004	[4].014
{c..	.048	.024	.017	.061
{d..	.057	.029	.021	.072

[1] Deflections usually turn out +; this is an exception.
[2] All deflections in the upper spars, assuming all upper spars to have the same spans, may be found from those computed for the front spar for a given speed and loading by a simple ratio. This ratio is directly dependent upon the ratio of the running loads and the inverse ratio of the moments of inertia, i. e., their product.
[3] Inverse ratio of moments of inertia.
[4] Found from values in first column by using ratio figure directly above.

(1) Data table for use in computing deflections on front lower spar:

LOWER SPARS.

Span	c.	d.
Joint-bending moments (M)	406	822
Shears, right of pin (v)	−42	−55
Distance to maximum moment (x)	38	49
Length of span (l)	84	79
Running load (w)	1.112	1.112
Moments of inertia (I)	2.82	2.82
Modulus of elasticity (E)	1,880,000	1,880,000

(2) Computations.

The general formula for deflection is

$$d(=y) = -x[12\,M(l-x) + 4\,V(l^2-x^2) + w(l^3-x^3)]/24EI$$

(a) Computation for span c:

$(l-x) = 46;\ (l^2-x^2) = 5,612;\ (l^3-x^3) = 537,832$

$d = -38[12(406)46 - 4(42)5,612 + 1.112(537,832)]/127,238,400 = 0.036$ inch

(b) Computations for span d:

$d = -49[12(822)30 - 4(55)3,840 + 1.112(375,390)]/127,238,400 = 0.051$ inch

(3) Computation table for deflections in lower spars at high and low speeds as found from lower front spar at low speed.

LOWER SPARS.

Speeds	High.		Low.	
Spars	Front.	Rear.	Front.	Rear.
Ratio of running loads	1	0.416	0.363	1.042
Ratio of moments of inertia	1	1.215	1	1.215
Product of ratios	1	.505	.363	1.266
Spans {c..	.036	.018	.013	.045
{d..	.051	.025	.018	.064

Example 7.—Find the concentrated loads on the lift trussing of example 4, given the weight of the struts, stays, cabanes, etc.

I. PIN REACTION METHOD.

Computation table for concentrated loads on upper and lower pin joints as found in tables for spar reactions of example 4 and the given weights of attached struts, stays, wires, etc. (G′, H′, I′; G, H, I) are symbols for concentrated loads in figure 22.

FRONT UPPER PIN JOINTS.

Concentrated load symbol (fig. 22).	Beam reactions in plane of lift trussing.	Weights at pin joints.	Net load.	Practical working load.[1]
G′	79.08	0.55	78.53	79
H′	83.75	4.17	79.78	80
I′	132.65	4.22	128.43	129

FRONT LOWER PIN JOINTS.

G	0	0	0	0
H	72.73	4.72	68.01	69
I	107.39	4.77	102.62	103

REAR UPPER PIN JOINTS.

G′	82.95	0.55	82.40	83
H′	88.06	4.17	83.89	84
I′	139.14	4.22	134.92	135

REAR LOWER PIN JOINTS.

G	0	0	0	0
H	75.63	4.72	70.91	71
I	111.68	4.77	106.91	107

[1] See application in problem 11.

II. ZERO SHEAR METHOD.

The concentrated loads may also be found by a method similar to that used for the drag trussing in figure 21 and example 8; i. e., briefly, by multiplying the running loads by the distance between zero shears and subtracting the weights of struts and stays.

Example 8.—Given the resistance of the front struts and stays and the running load, find the concentrated loads on the drag trussing of example 4 by the zero shear method.

(a) Concentrated loads on upper front spar:

Given $A = B = C = D = 0.100$ lb. per in.

$$x_1 = 26.93; \quad x_2 = 37.37; \quad x_3 = 48.87$$
$$a = 30; \quad b = 50; \quad c = 84; \quad d = 79.$$

. Then by formulas of figure 21,

$$G = 0.13 + 0.100(30) + 0.100(26.93) = 5.82 \ [1]$$
$$H = 0.36 + 0.100(50 - 26.93) + 0.100(37.37) = 6.40$$
$$I = 0.53 + 0.100(84 - 37) + 0.100(48.87) = 10.07.$$

(b) Concentrated loads on rear upper spar:

Given $A = B = C = 0.059$ lb. per in.

a, b, c, x_1, x_2, x_3 as above.
$$G' = 0.13 + 0.059(30) + 0.059(26.93) = 3.48$$
$$H' = 0.36 + 0.059(50 - 26.93) + 0.059(37.37) = 3.92$$
$$I' = 0.53 + 0.059(84 - 37.37) + 0.059(48.87) = 6.16.$$

Example 9.—Given the resistance of the strut and stays at high and low speeds; find the concentrated loads on the drift trussings for both speeds, using values of the pin reactions found in example 4.

Computation table for concentrated loads on upper and lower drift trussings for low and high speeds.
LOW SPEED.

	Front spar.				Rear spar.			
	Load symbol.	Reaction.	Resistance.	Total.	Load symbol.	Reaction.	Resistance.	Total.
Upper drag trussing	G	5.69	0.13	[1] 5.82	G'	3.40	0.13	3.53
	H	6.04	.41	6.45	H'	3.60	.41	4.01
	I	.00	.00	.00	I'	.00	.00	.00
	J	9.55	.58	10.13	J'	5.70	.58	6.28
	K	.00	.00	.00	K'	.00	.00	.00
Lower drag trussing	G	6.18	.41	6.59	G'	3.20	.41	3.61
	H	.00	.00	.00	H'	.00	.00	.00
	I	9.12	.58	9.70	I'	4.72	.58	5.30
	J	.00	.00	.00	J'	.00	.00	.00

HIGH SPEED.

	Load symbol.	Reaction.	Resistance.	Total.	Load symbol.	Reaction.	Resistance.	Total.
Upper drag trussing	G	6.08	0.37	6.45	G'	16.13	0.37	16.50
	H	6.46	1.18	7.64	H'	17.12	1.18	18.30
	I	.00	.00	.00	I'	.00	.00	.00
	J	10.21	1.68	11.89	J'	27.06	1.68	28.74
	K	.00	.00	.00	K'	.00	.00	.00
Lower drag trussing	G	5.45	1.18	6.63	G'	14.54	1.18	15.72
	H	.00	.00	.00	H'	.00	.00	.00
	I	8.05	1.68	9.73	I'	21.47	1.68	23.15
	J	.00	.00	.00	K'	.00	.00	.00

[1] See application in example 12.

Example 10.—An aileron bearing a uniform pressure of 20 pounds per square foot has the dimensions and structural form shown in figure 38. Find the stresses in the stay wires.

I. BEAM REACTIONS.

The rear beam of the aileron is considered as continuous and supported by the stays at three points, with both ends overhanging. The vertical reactions of these stays have previously been found in example 2. Reactions as found in example 2.

$$R_1 = 48.8 \text{ lbs.}; \quad R_2 = 109.7 \text{ lbs.}; \quad R_3 = 48.8 \text{ lbs.}$$

[1] Compare with more general solution in example 9.

II. TENSIONS IN STAYS.

The tensions in the three stay wires are found from the reactions above and the dimensions given in figure 38. By the formula of figure 24.

(a) For center stay:

Given $r = 22.72$ ins.; $z = 13.37$ ins.; $Z = 109.7$.

Then by figure 24,

$$T_2 = R = Zr/z = 109.7(22.72)/13.37 = 186 \text{ lbs.}$$

(b) For each outer stay:

Given $r = 61.78$; $z = 13.37$; $Z = 48.8$.

Then by figure 24,

$$T_1 = T_2 = R = Zr/z = 48.8(61.78)/13.37 = 225 \text{ lbs.}$$

Example 11.—Find the endwise stresses in the struts, stays, and spars of the front lift trussing, figure 40 for low speed and the rear lift trussing for high speed.

I. STRESSES IN FRONT LIFT TRUSSING FOR LOW SPEED.

1. GENERAL DATA.[1]

$G = 0$	$G' = 79$	$b = 50$	$p = 80.3$
$H = 69$	$H' = 80$	$c = 84$	$q = 104.9$
$I = 103$	$I' = 129$	$d = 79$	$r = 100.9$
	$h = 62.9$		

2. ANALYTICAL SOLUTION.

By figure 22:

Strut Stresses.

$$P = G$$
$$= O = 0$$
$$Q = G + G' + H$$
$$= O + 79 + 69 = 148.00$$
$$R = G + G' + H + H' + I$$
$$= O + 79 + 69 + 80 + 103 = 331.00$$

Stay Stresses.

$$P' = p(G + G')/h$$
$$= 80.3 \ (0 + 79)/62.9 = 100.85$$
$$Q' = q(G + G' + H + H')/h$$
$$= 104.9 \ (0 + 79 + 69 + 80)/62.9 = 380.24$$
$$R' = r(G + G' + H + H' + I + I')/h$$
$$= 100.9 \ (0 + 79 + 69 + 80 + 103 + 129)/62.9 = 737.90$$

Spar Stresses.

$$P'' = (G + G')b/h$$
$$= (0 + 79) \ 50/62.9 = 62.79$$
$$Q'' = (G + G')(b + c) + c(H + H')/h$$
$$= (0 + 79) \ (50 + 84) + 84(69 + 80) \ 62.9 = 367.28$$
$$R'' = (G + G') \ (b + c + d) \ (c + d) \ (H + H') + d(I + I')/h$$
$$= (0 + 79) \ (50 + 84 + 79) + (84 + 79)(69 + 80) + 79$$
$$(103 + 129) \ 62.9 = 945.02$$

A second method may be used to find the same values by using only the equation on the right of figure 22.

---

[1] See example 7.

Strut Stresses.

$P = G = 0.00$

$Q = P + G' + H$
$\quad = 0 + 79 + 69 = 148.00$

$R = Q + H' + I$
$\quad = 148 + 80 + 103 = 331.00$

Factors.

$n_1 = (P + G)/h$
$\quad (0 + 79)/62.9 =$ [1] 1.255

$n_2 = (Q + H)/h$
$\quad (148 + 80)\ 62.9 = 3.624$

$n_3 = (R + I)/h$
$\quad (331 + 129)/62.9 = 7.313$

Stay Stresses.

$P' = pn_1$
$\quad = 80.3(1.255) = 100.77$

$Q' = qn_2$
$\quad = 104.9\ (3.624) = 380.15$

$R' = rn_3$
$\quad = 100.9(7.313) = 737.88$

Spar Stresses.

$P'' = bn_1$
$\quad = 50\ (1.254) = 62.70$

$Q'' = P + cn_2$
$\quad = 62.70 + 84(3.624) = 367.11$

$R'' = Q + dn_3$
$\quad = 367.11 + 79\ (7.313) = 944.83$

2. GRAPHICAL SOLUTION.

The graphical solution is similar to that shown in figure 27.

II. STRESSES IN REAR LIFT TRUSSING FOR HIGH SPEED.

GENERAL DATA.

$G = 0$	$G' = 83$	$b = 50$	$p = 80.3$
$H = 71$	$H' = 84$	$c = 84$	$q = 104.9$
$I = 107$	$I' = 135$	$d = 79$	$r = 100.9$
	$h = 62.9$		

Solve as in I above. Results given in table below.

III. REFERENCE TABLE FOR STRESSES IN LIFT TRUSSINGS FOR HIGH AND LOW SPEEDS.

LOW SPEED.

Symbol.	Strut stresses.	Stay stresses.	Upper spar stresses.	Lower spar stresses.[2]
P	0	101	63	0
Q	148	381	368	63
R	331	738	946	368

HIGH SPEED.

P	0	106	66	0
Q	154	397	384	66
R	345	770	987	384

[1] Carry out three decimal places.   [2] Found from stresses in upper spar.

Example 12.—Find the stresses in the drag trussings figure 40, due to the concentrated loads of example 9.

I. STRESSES IN UPPER DRAG TRUSSING FOR LOW SPEED.[1]

GENERAL DATA.

$G' = 3.53$	$G = 5.82$	$b = 50$	$p = 60.4$
$H' = 4.01$	$H = 6.45$	$c = 42$	$q = 54.0$
$I' = .00$	$I = .00$	$d = 42$	$r = 54.0$
$J' = 6.28$	$J = 10.13$	$e = 38.5$	$s = 51.3$
$K' = .00$	$K = .00$	$f = 38.5$	$t = 51.3$
		$h = 34$	

II. STRESSES IN LOWER DRAG TRUSSING FOR LOW SPEED.

GENERAL DATA.

$G = 6.59$	$G' = 3.61$	$b = 42$	$p = 54$
$H = .00$	$H' = .00$	$c = 42$	$q = 54$
$I = 9.70$	$I' = 5.30$	$d = 38.5$	$r = 51.3$
$J = .00$	$J' = .00$	$e = 38.5$	$s = 51.3$

III. STRESSES IN UPPER TRUSSING FOR HIGH SPEED.

GENERAL DATA.

$G = 6.45$	$G' = 16.50$	$b = 50$	$p = 60.4$
$H = 7.64$	$H' = 18.30$	$c = 42$	$q = 54$
$I = .00$	$I' = .00$	$d = 42$	$r = 54$
$J = 11.89$	$J' = 28.74$	$e = 38.5$	$s = 51.3$
$K = .00$	$K' = .00$	$f = 38.5$	$t = 51.3$

IV. STRESSES IN LOWER DRAG TRUSSING FOR HIGH SPEED.

GENERAL DATA.

$G = 6.63$	$G' = 15.72$	$b = 42$	$p = 54$
$H = .00$	$H' = .00$	$c = 42$	$q = 54$
$I = 9.73$	$I' = 23.15$	$d = 38.5$	$r = 51.3$
$J = .00$	$J' = .00$	$e = 38.5$	$s = 51.3$

Reference table for stresses in drag trusses for low and high speed.

STRESSES ON MEMBERS OF LOWER DRAG TRUSS.

Symbol.	Low speed.				High speed.			
	Struts.	Stays.	Rear spar.	Front spar.	Struts.	Stays.	Rear spar.	Front spar.
P	6.6	16.2	12.6	0	6.6	35.5	27.6	0
Q	10.2	16.2	25.2	12.6	22.4	35.5	55.2	27.6
R	19.9	38.0	53.7	25.2	32.1	83.3	117.7	55.2
S	25.2	38.0	82.2	53.7	55.2	83.3	180.2	117.7

STRESSES ON MEMBERS OF UPPER DRAG TRUSS.

P	5.8	16.6	13.8	0	6.5	40.8	33.8	0
Q	15.8	31.4	38.2	13.8	30.6	77.6	94.1	33.8
R	19.8	31.4	62.6	38.2	48.9	77.6	154.5	94.1
S	29.9	54.6	103.6	62.6	60.8	135.0	255.8	154.5
T	36.2	54.6	144.6	103.6	89.5	135.0	357.1	255.8

Example 13.—Find the stresses and factors of safety in the spars, struts, and stays shown in figure 40 for low and high speeds, respectively.

---

[1] The solutions for this example are similar to those in example 11. The arrangement of data for the different cases and tabulated results are given below.

144416—20——5

1. GENERAL DATA.

Table I below gives the principal data, stresses, and factors of safety for the spars, struts, and stays. All other data required in the solution of this example may be found in the separate problems or in the diagrammatic form in figure 28.

TABLE I.

Symbol.[1]	Member.	Material.	Strength (pounds per square inch).	Section area (square inches).	Moment of inertia.	Section modulus.	For low speed.				For high speed.			
							Fiber stress (pounds per square inch).			Factor of safety.	Fiber stress (pounds per square inch).			Factor of safety.
							Bend-ing.	Lift.	Drag.		Bend-ing.	Lift.	Drag.	
Spars:														
A...........	Front spar..	Spruce..	4,300	2.63	2.82	1.88	±328	24	0	21.2				
B...........	...do.......	...do.....	4,300	2.63	2.82	1.88	±540	140	−15	10.2				
C...........	...do.......	...do.....	4,300	2.63	2.82	1.88	±540	360	−24	6.8				
D...........	...do.......	...do.....	13,000	2.63	2.82	1.88	±437	− 24	− 5	17.3				
E...........	...do.......	...do.....	13,000	2.63	2.82	1.88	±437	−140	−10	14.9				
F...........	Rear spar..	...do.....	4,300	3.26	2.32	1.16					±539	20	11	13.2
G...........	...do.......	...do.....	4,300	3.26	2.32	1.16					±918	118	48	6.4
H...........	...do.......	...do.....	4,300	3.26	2.32	1.16					±918	303	79	4.8
I...........	...do.......	...do.....	13,000	3.26	2.32	1.16					±738	− 20	17	10.6
J...........	...do.......	...do.....	13,000	3.26	2.32	1.16					±738	−118	36	10.0
Struts:			Strength, pounds.			Length, inches.	Load, pounds.				Load, pounds.			
K...........	Front.......	...do.. ...	1,100	2.89	.279	6.29		148		7.4				
L...........	...do.......	...do.....	2,800	4.87	.792	6.29		331		8.4				
M...........	Rear.......	...do.....	1,100	2.89	.279	6.29						154		7.1
N...........	...do.......	...do.....	2,800	4.87	.792							345		8.1
Cables:				Number of cables.	Diame-ter.									
O...........	Front.......	Steel....	4,200	2	⅛			101		41.7				
P...........	...do.......	...do.....	6,400	2	₃₂			381		16.8				
Q...........	...do.......	...do.....	6,400	2	₃₂			738		8.6				
R...........	Rear.......	...do.....	4,200	2	⅛							106		39.6
S...........	...do.......	...do.....	6,400	2	₃₂							397		16.1
T...........	...do.......	...do.....	6,400	2	₃₂							770		8.3

Factors of safety are given for points of greatest resultant tensile or compressive stress.

2. SOLUTION FOR SPARS.

TABLE II.—*Values of the bending moments (±), the tensions (−), and compressions (+) on each spar at the points marked "X" in fig. 28.*

FOR LOW SPEED.

Symbol.	Bending moment.	Load due to lift truss.	Load due to drag truss.
	Inch-pounds.	*Pounds.*	*Pounds.*
A..	± 817	+ 63	0
B..	±1,016	+366	−38.2
C..	±1,016	+945	−62.6
D..	± 822	− 63	−12.6
E..	± 822	−366	−25.2

FOR HIGH SPEED.

F..	± 626	66	33.8
G..	±1,065	384	154.5
H..	±1,065	987	255.8
I..	± 856	− 66	· 55.2
J..	± 856	−384	117.7

[1] The values above may be found in examples 4, 11, 12, or in figure 28 in diagrammatic form.

TABLE III.—*Computations for resultant fiber stresses for low speed.*[1]

LOW SPEED (COMPRESSIVE STRESSES).

Member.	Bending stress compression. $S_b = M/Z$	Equivalent compressive stress due to bending. $NS_b = \dfrac{4,300}{7,900} S_b$	Compressive stress. Sc.	Resultant compressive stress. $S = S_c + \dfrac{4,300}{7,900} S_b$
A..............................	[2]328	178	24	202
B..............................	540	294	125	419
C..............................	540	294	336	630

LOW SPEED (TENSILE STRESSES).

Member.	Bending stress tension. $S_b = M/Z$	Equivalent tensile stress due to bending. $NS_b = \dfrac{13,000}{7,900} S_b$	Tensile stress. St.	Resultant tensile stress. $S = S_t + \dfrac{13,000}{7,900} S_b$
D..............................	−437	−719	− 29	−748
E..............................	−437	−719	−150	−869

[1] Spars A, B, C, D, E have the lowest factors of safety for low speed.
[2] An additional bending stress due to deflection is sometimes added to this stress.

Example for member A in table above. (See Tables I and II.)

$$S_b = M/Z$$
$$= 617/1.88 = 328 \text{ lbs. per sq. in.}$$

Equivalent compressive stress due to bending (see section 7 and formula 8).

$$NS_b = 4,300(328)/7,900 = 178 \text{ lbs. per sq. in.}$$

Compressive stress S_o is the resultant compressive stress as found from columns two and three, Table II, and cross-sectional area, Table I.

$$S_c = 63/2.63 = 24 \text{ pounds per square inch.}$$

Resultant compressive stress. (Add columns 2 and 3 above.)

$$S = S_c + 4,300(S_b)/7,900.$$
$$= 24 + 178 = 202 \text{ pounds per square inch.}$$

This value is then used to determine the factor of safety. Compressive strength is 4,300 pounds per square inch.

$$\text{Factor of safety} = 4,300/202 = 21.2.$$

TABLE IV.—*Computations for resultant fiber stresses for high speed.*[1]

HIGH SPEED (COMPRESSIVE STRESSES).

Member.	Bending stress compression, $S_b = M/Z$.	Equivalent compressive stress due to bending, $NS_b = \dfrac{4,300}{7,900} S_b$.	Compressive stress, Sc.	Resultant compressive stress, $S = S_c + \dfrac{4,300}{7,900} S_b$.
F..............................	539.	294	31	325
G..............................	918	499	166	665
H..............................	918	499	382	881

HIGH SPEED (TENSILE STRESS).

Member.	Bending stress ...sion, $S_b = M/Z$.	Equivalent tensile stress due to bending, $NS_b = \dfrac{13,000}{7,900} S_{bt}$.	Compressive tensile stress, St.	Resultant tensile stress, $S = S_t + \dfrac{13,000}{7,900} S_b$.
I..............................	−378	−1,214	− 3	−1,217
J..............................	−738	−1,214	−82	−1,296

[1] Spars F, G, H, I, J have usually the lowest factors of safety for high speed.

Example for member I in table above:

$$S_b = M/Z.$$
$$= 856/1.16 = 738 \text{ pounds per square inch.}$$

Equivalent tensile stress due to bending.

$$NS_b = 13,000 \ S_b/7,900.$$
$$= 13,000(738)/7,900 = 1,214 \text{ pounds per square inch.}$$

Tensile stress S_t is the resultant tensile stress as found from columns 2 and 3, Table III, and cross-question area in Table I.

$$S_t = -10.8/3.26 = -3 \text{ pounds per square inch.}$$

Resultant tensile stress (add columns 2 and 3).

$$S = S_t \ 13,000 \ S_b/7,900.$$
$$= -3 - 1,214 = -1,217 \text{ pounds per square inch.}$$

This value of stress is used in determining the factors of safety:

$$\text{Factor of safety} = 13,000/1,217 = 10.6. \quad \text{Ans.}$$

3. SOLUTION FOR STRUTS.

The data for the struts with the resultant loads upon them are given in Table I. The value for strength of struts is best found by actual test, in the case of design they are computed. In the table above it is the actual strength by test. The loading is taken from the lift truss diagram in figure 28 or from example 11. The factor of safety is found in the usual manner.

4. SOLUTIONS FOR CABLES OR STAYS.

The same analysis applies to cables as to struts. The strength of the cables is usually known from tests made by the manufacturer.

Example 14.—Find the stresses in the principal members of the wing trussing in figure 40 due to a uniform air pressure of 20 pounds per square foot on the aileron surface in figure 38.

1. GENERAL DATA.

(a) Eccentric load = 414 pounds; eccentricity = 9.45 + 17.
(b) Torque at each station = 414 × 26.45 = 10,950 in./lbs.
(c) Lengths of stays S, T, U, V. (See fig. 34.)

First panel.				Second panel.			
S_1	T_1	U_1	V_1	S_2	T_2	U_2	V_2
90.6	104.9	90.6	104.9	86	101	86	101

(d) Computation tables for direction cosines.

Symbol.	l	X	Y	Z	Direction cosines.		
					S_x	S_y	S_z
S_1................................	90. 6	−84	0	−62. 9	−0. 927	0	+0. 694
S_2................................	86. 0	−79	0	−62. 9	− .918	0	+ .731

Symbol.	l	X	Y	Z	Direction cosines.		
					t_x	t_y	t_z
T_1................................	104. 9	−84	−34	0	−0. 801	−0. 324	0
T_2................................	101. 0	−79	−34	0	− .782	− .336	0

Symbol.	l	X	Y	Z	Direction cosines.		
					u_x	u_y	u_z
U_1................................	90. 6	−84	0	+62. 9	−0. 927	0	+0. 694
U_2................................	86. 0	−79	0	+62. 9	− .918	0	+ .731

Symbol.	l	X	Y	Z	Direction cosines.		
					V_x	V_y	V_z
V_1................................	104. 9	−84	34	0	−0. 801	0. 324	0
V_2................................	101. 0	−79	34	0	− .782	.336	0

2. SOLUTION.

(a) Stresses in stays transmitting torque forces from station No. 1 to station No. 2. Given (see fig. 34):

$l = 35.8$ ins.; $a = 34$ ins.; $b = 62.9$ ins.
Torque $= 10,950$ in./lbs.

Then by formulas of figure 34:

$R = $ Torque $/4l$
$= 10,950/4(35.8)$ --- 76. 4
$P = Rb/2l$
$= 76.466 \ (62.9)/2(35.8)$ -- 67. 2
$Q = Ra/2l$
$= 76.466 \ (34)/2(35.8)$ -- 36. 3

Since this problem comes under special case I, right prismatic truss, then, the general equations of figure 34 reduces to

$$t_y T = -2P \qquad s_x S + S' = 0$$
$$u_z U = +2Q \qquad t_x T + T' = 0$$
$$v_y V = +2P \qquad u_x U + U' = 0$$
$$s_z S = +2Q \qquad v_x V + V' = 0$$

Solution is then

1. $T_1 = -2P/t_y.$
$T_1 = -2(67.2)/(-0.324)$ -- 414. 9
2. $U_1 = +2Q/u_z.$
$U_1 = +2(36.3)/(0.694)$ --- 104. 6
3. $V_1 = -2P/v_y.$
$V_1 = -2(67.2)/(-0.324)$ -- 414. 9
4. $S_1 = +2Q/s_z.$
$S_1 = +2(36.3)/(0.694)$ --- 104. 6

(b) Stresses in stays transmitting torque forces from station No. 2 to station No. 3. The values of l, a, b, P, and Q are the same as in the first panel.

1. $T_2 = -2P/t_y$
   $T_2 = -2(67.2)/(-0.336)$ --- 400. 0
2. $U_2 = +2Q/u_z$
   $U_2 = +2(36.3)/(0.731)$ --- 99. 3
3. $V_2 = -2P/v_y$
   $V_2 = -2(67.2)/(-0.336)$ --- 400. 0
4. $S_2 = +2Q/s_z$
   $S_2 = +2(36.3)/(0.731)$ --- 99. 3

Example 15.—Given the data for figure 25, as below, solve for the tensions in the stay wires;

$L = 145$ lbs.; $m = 60$ in.; $n = 50$ in.; $p = 99.8$ in. $q = 94.2$ in.; $A = 0.012$ sq. in.; $B = 0.012$ sq. in.; $E = 30,000,000$.

DATA.

$L = 145$ lbs.; $m = 60$ in.; $n = 50$ in.
$p = 99.8$ in.; $q = 94.2$ in.
$A = 0.012$ sq. in.; $B = 0.012$ sq. in.; $E = 30,000,000$.

SOLUTION.

By formulas in figure 25:

$P = LAmq^2/(Am^2q^2/p + Bn^2p^2/q)$
$P = [145(0.012)60(94.2)^2]/[(0.012)(60)^2(94.2)^2/99.8 + 0.012(50^2)(99.8)^2/94.2] = 132.1$ lbs.
$Q = LBnp^2/(Am^2q^2/p + Bn^2p^2/q)$
   $= 145(0.012)50(99.8)^2/[\text{constant}]\,[1] = 123.6$ lbs.

PROBLEMS IN PART III.

Airplane Body Stresses.

Example 1.—An elevator bearing a uniform pressure of 20 pounds per square foot has the dimensions and structural form shown in figure 37. Find the moment about the hinge and the stresses in stays and the control wires; also the hinge reactions.

(A) MOMENT ABOUT HINGE.

The moment about the hinge is equal to the product of the area, the uniform pressure, and the distance from the hinge axis to the centroid.

Given $A = 11$ square feet.; $P = 20$ pounds per square foot.

Distance to centroid $= 15$ in.

Then moment $= 11(20)15 = 3,300$ inch-pounds.

(B) TENSION IN CONTROL WIRE.

The control wire pull times its distance from the hinge axis is equal to the moment in (a) above.

Tension in control wire $= 3,300/7.75 = 425.8$ pounds.

(C) TENSION IN STAY WIRES.

The vertical reactions of the stays are approximately equal to one-half the total load on the elevator. Consider the stays as attached to a flexible rib, forming a continuous beam on three supports.

Then by table in figure 13, given $A \times P = 220$ pounds.; $b/l = 1$.

$wl = 220/4 = 55$ pounds.
$R_1 = R_3 = 0.375(55) = 20.62$ pounds.
$R_2 = 1.25(55) = 68.75$ pounds.
Check $R_1 + R_2 + R_3 = 110$ pounds.

---

[1] Use denominator as found directly above.

The total vertical hinge reactions of the rudders found in example 1 must be added to the reactions R_1 and R_2. The true pin reactions of the rear beam of the horizontal stabilizer then become:

$$R_1 = 67.23 - 292.20 = 359.43$$
$$R_2 = 9.04 - 0 \quad = 9.04$$
$$R_3 = 67.23 - 292.20 = 359.43$$

$$\text{Sum} \qquad = 727.90$$

The sum of all the vertical reactions of the stabilizer minus the vertical components of the stay wire pulls should equal the total load on stabilizer and elevator.

$$\text{Reactions of front beam} = 430.50$$
$$\text{Reactions of rear beam} = 727.90$$

$$\text{Sum,} \qquad 1,158.40$$

Minus vertical components of stay wires,   144.42

$$1,013.98$$

Again, Total load on stabilizer = 574 pounds.
　　　　Load on left elevator　　= 220 pounds.
　　　　Load on right elevator　= 220 pounds.

$$1,014$$

Example 3.—A rudder and vertical stabilizer bearing a uniform pressure of 20 pounds per square foot has the dimensions and structural form shown in figure 37; find the transverse loads on the upper and lower trussings of the fuselage in figure 36 and the applied couple about the normal axis of its various sections.

1. GENERAL DATA.

Member.	Area.	Centroid.	
		$^1\bar{z}$.	$^1\bar{x}$.
Rudder..	12.00	−17	19
Stabilizer......................................	3.80	13	10
Rudder and stabilizer.......................	15.80	− 9	17
Length of stern post, 12 inches.			

$^1\bar{z}$ is the distance (positive upward) from axis of upper longeron and $\bar{x}$ is the distance (positive forward) from axis of stern post.

2. WIND FORCE.

On rudder　　= PA = (20) (12)　= 240 pounds.
On stabilizer = PA = (20) (3.80) = 76 pounds.

Total,　　　　　　　　　　316 pounds.

3. TRANSVERSE LOADS ON HORIZONTAL TRUSSING AND COUPLES AT VARIOUS SECTIONS.

The total load at the centroid of the rudder and stabilizer may be replaced by an equal load at the center of the stern post and an equivalent couple.

Then,
　　(a) Load at center of stern post = 316 pounds.
　　(b) Couple in vertical plane of stern post 316 (17 + 6) = 7,268 pounds per sq. in.

From these reactions may be found the tensions in the stay wires by a treatment similar to that used to find the tensions in aileron stays. (See example 10, Pt. II.)

(D) HINGE REACTIONS.

The vertical hinge reactions may be considered as due to a uniformly loaded beam supported at three points and bearing one-half the total load on the elevator. The reactions are approximately equal to those found in (c) above. To the middle reaction must be added the vertical reaction of the mast, which is equal to the total vertical reactions of the stays, or one-half the total load on the elevator, plus the vertical component of the control wire pull.

Control wire pull.

Given $R = 425.80$ pounds.; $z = 7.25$ inches; $r = 42.75$ inches.

Then component of control wire pull

$Z = Rz/r = 425.80(7.75)/42.75 = 72.21$ pounds.

Hinge reactions

$R_1 = 20.62$
$R_2 = 68.75 + 110 + 72.21 = 250.96$
$R_3 = 20.62.$

Example 2.—Find the vertical components of the pin reactions of the front and rear beams of the stabilizer or horizontal fin, figure 37, due to a uniformly distributed pressure of 20 pounds per square foot and the hinge reactions of example 1.

(A) TOTAL LOAD EQUALS THE UNIT PRESSURE TIMES THE AREA.

Given, $B = 28.70$ square feet; $P = 20$ pounds per square foot.
Then, total load $= 28.70 (20) = 574$ pounds.

(B) DISTRIBUTION OF LOADING ON RIBS AND BEAMS.

The ribs may be considered as beams supported at two points (the position of front and rear beams, see m and n in figure 37). Approximately $0.75 W$ and $0.25 W$ is carried by the front and rear beams, respectively.

Front beam, $0.750(574) = 430.50$ pounds.
Rear beam, $0.250(574) = 143.50$ pounds.

(C) PIN REACTIONS DUE TO THE LOAD ON THE FRONT BEAM.

Consider as a continuous beam supported at three points and having both ends overhanging.
By table in figure 13 $W = wl = 430.50/2 = 215.20$
$$b/l = 28/50 \qquad = \quad 0.56$$
Then
$$R_1 = 0.822\ W = 177.11$$
$$R_2 = 0.354\ W = 76.28$$
$$R_3 = 0.822\ W = 177.11$$

$$\text{Sum} \qquad = 430.50$$

(D) PIN REACTIONS ON REAR BEAM.

Consider the rear beam as a continuous beam supported at the stern post and two stays.

By figure 13, given $W = wl = 143.50/2 = 71.75$
$$b/l = 28/56 = 0.50$$
Then
$$R_1 = 0.927(71.75) = 67.23$$
$$R_2 = 0.126(71.75) = 9.04$$
$$R_3 = 0.937(71.75) = 67.23$$

$$\text{Sum} \qquad = 143.50$$

(c) The couples at the various sections may be conveniently arranged in the following table. (See fig. 36 for dimensions of members.)

Station No.	Transverse load.	Eccentricity.[1]	Torque L (couple).
1	316	23	7,268
2	316	24.5	7,742
3	316	26	8,216
4	316	27.7	8,753
5	316	29.6	9,354
6	316	31.5	9,954
7	316	33.1	10,459

Computations for stresses due to the applied couples alone are given in example 12.

[1] Vertical distance of load above axis of sections.

Example 4.—Compute the normal lift on the tail skid from the data given in figure 29. Given $W = 1,890$ lbs.; $a = 19.13$ ins.; $l = 205.8$ ins.

$$L = Wa/l$$
$$= 1,890(19.13)/205.8$$
$$= 175.6 \text{ lbs.}$$

Example 5[1].—Given $I = 60,544$ lb. $-$ ft.^{2}, $j_x = 16$ ft./sec.^{2}, $j_y = 8$ ft./sec.^{2}, $\alpha = 0.1$ rad./sec.^{2} in figure 31 and the dimensions of a machine, find the resultant live load on the tail skid.

Given $M = 1,890$ lbs. $j_x = 16$ ft./sec.^{2} $\bar{x} = 1.59$ ft.
$\quad\quad I = 60,544$ lb. $-$ ft.^{2} $j_y = 8$ ft./sec.^{2} $\bar{y} = 2.91$ ft.
$\quad\quad \alpha = 0.1$ rad./sec.^{2} $\beta = 3° 37'$
$\quad\quad p = 17.14$ ft. $g = 32$ ft./sec.^{2}

Then, by figure 31,

$$P_y = \{ M[j_y \bar{x} - j_x \bar{y} + g \cos \beta. \bar{x}] + 1_o \alpha \}/p.$$
$$P_y = \{1890[8(1.59) - 16(2.91) + 32(.998)(1.59)] + 60,544(.1)\}/17.14.$$
$$P_y = 2,221 \text{ poundals, or } 69.4 \text{ pounds of force.}$$

Example 6.—From the data in figure 29 compute the normal load at the wheels of the undercarriage; also the stresses in the struts and stays and their factors of safety.

1. LOAD ON WHEELS.

Load carried by each wheel $= 857.$[2]

2. STRESSES IN STRUTS AND STAYS (ANALYTICAL).

Applying formula 24 to the triangles A, B, C in figure 29, we have—.
(a) For component in plane of front trussing,

$$P = R \sin \alpha/\sin \gamma$$
$$= 857 \sin (68°23')/\sin (76°14')$$
$$= 820.5.$$

In a similar manner the stresses are found.

(b) For component in plane of rear trussing 511.0
(c) For tension in front stay 619.7
(d) For compression in the front strut 1,307.1
(e) For tension in rear stay 301.7
(f) For compression in rear strut 778.9

3. STRESSES IN STRUTS AND STAYS (GRAPHICAL).

The graphical solutions are shown in figure 29. Figure 29 shows the lift components in planes of front and rear trussings, both being in a plane normal to axle. From these components the stresses in the front and rear struts and stays are found by simple graphics as shown.

---

[1] Solved by using absolute F. P. S. units.   [2] Total weight of machine minus load on tail skid and weights of wheels and axle.

4. FACTORS OF SAFETY.

The factors of safety are found by dividing the standard strength in pounds by the load in pounds.

The table below gives the complete data and results.

Member.		Material.	Standard strength.	Section area.	Moment of inertia.	Length in inches.	Load in pounds	Stress in pounds per square inch.	Factor of safety.
Struts	{front..	Spruce	5,600		0.418	33.3	1,307		4.2
	{rear...	...do	1,990		.418	55.7	779		2.5
Stays	{front..	Cable	4,700	$1\frac{7}{16}$			−620		7.5
	{rear...	...do	4,700	$1\frac{3}{16}$			−302		15.5

[1] Inches in diameter.

Example 7.—Find the stresses in the undercarriage trussing of the seaplane in figure 30, due to a lift of 600 pounds, applied at a point one-third the distance from the front to the rear strut attachments.

The resultant force is resolved along three axes, passing through its point of application; the X axis, which in the present case may be conveniently taken parallel to the line of the fore-and-aft lower pin connections of the trussing to the pontoon; the Y and Z axes normal thereto. The stresses in the trussings are then determined separately for each component and then algebraically added.

1. STRESSES DUE TO THE Z COMPONENT.

The Z component in the present problem acts in a vertical plane through the fore-and-aft pin connections of the trussing.[1] This vertical load is resolved, as shown in figure 30, into components in the planes of the fore-and-aft side struts and fore-and-aft stays, both components being in the YZ plane. From each of these components the stresses in the stays and struts may be found as shown. The component in the plane of the stays causes a drag in the strut plane, which must be determined before solving for the strut plane stresses. These drag forces (see forces (169) and (24) in diagram for true plane of struts, figure 30) are caused by the tension in the stays. Figure 30 (A) shows the resolution of lift on vertical plane (YZ); B, the diagram for the lengths and distances in true plane of stays with the graphical solution for stresses and strut reactions; (c), diagram for true plane of struts and graphical solution for stresses in struts.

2. STRESSES DUE TO THE X AND Y COMPONENTS.

The stresses in the struts and stays due to X, Y components of the applied load on the pontoon are in general solved by a method similar to 1.

Example 8.—An airplane weighing 3,000 pounds with wheels 2 feet in diameter and 6 feet apart rests with one wheel 10 inches lower than the other. Find the added bending moment on the axle, assuming each wheel to carry one-half the entire weight.

By section: 23

Given $W = 3{,}000$ lbs. $\alpha = 7°54'$ $R = 12$ ins.

By formula:

$$M = 1/2\ W R \operatorname{Sin} \alpha$$
$$= 1/2\ (3000)\ 12\ (0.1388)$$
$$= 208\ \text{ft. lbs.}\quad \text{Ans.}$$

Example 9.—Find the stresses in the struts, stays, and longerons of the rear segment of the fuselage due to a uniform pressure of 20 pounds per square foot upon the horizontal tail pieces; also those due to gravitational loads alone.

[1] In case the Z component is not in a vertical plane through the pin connections, an equivalent force and couple must be considered.

I. STRESSES DUE TO AIR FORCE ONLY.

1. GENERAL DATA.

$n_1 = 3.00$; $m_1 = 0$; $b = 19.00$; $p = 12.00$; $p_1 = 22.48$; $p_2 = 19.24$; $p_3 = b$

$n_2 = 3.00$; $m_2 = 0$; $c = 20.50$; $q = 15.00$; $q_1 = 25.40$; $q_2 = 20.72$; $q_3 = c$

$n_3 = 3.50$; $m_3 = 0$; $d = 24.00$; $r = 18.00$; $r_1 = 30.00$; $r_2 = 24.26$; $r_3 = d$

$n_4 = 3.87$; $m_4 = 0$; $e = 26.00$; $s = 21.50$; $s_1 = 33.74$; $s_2 = 26.29$; $s_3 = e$

$n_5 = 3.75$; $m_5 = 0$; $f = 29.00$; $t = 25.37$; $t_1 = 38.53$; $t_2 = 29.24$; $t_3 = f$

$n_6 = 3.19$; $m_6 = 0$; $g = 30.87$; $u = 29.12$; $u_1 = 42.44$; $u_2 = 31.08$; $u_3 = g$

$v = 32.32$;

$G = 507$ on each truss. (See example 2 part II.)

2. ANALYTICAL SOLUTION.

By formulas of figure 32:

UPPER LONGERON STRESSES.

$P''' = O$ --- 0. 00

The formulas of figure 32 apply to all cases and may be reduced to simple forms, for cases of parallel or horizontal longerons, by cancellation. Always rewrite the formulas before solving.

$Q''' = Gb/q$
   $= 507(19)/15$ -- [1] 642. 19

$R''' = G(b+c)/r$
   $= 507(19 + 20.5)/18$ -- 1, 112. 58

$S''' = G(b+c+d)/s$
   $= 507(19 + 20.5 + 24)/21.50$ -------------------------------------- 1, 497. 41

$T''' = G(b+c+d+e)/t$
   $= 507(19 + 20.5 + 24 + 26)/25.37$ -------------------------------- 1, 788. 15

$U''' = G(b+c+d+e+f)/u$
   $= 507(19 + 20.5 + 24 + 26 + 29)/29.12$ --------------------------- 2, 062. 80

$V''' = G(b+c+d+e+f+g)/v$
   $= 507(19 + 20.5 + 24 + 26 + 29 + 30.87)/32.32$ ------------------- 2, 343. 40

LOWER LONGERON STRESSES.

$P'' = p_2 Q'''/b$
   $= 19.24(642.19)/19$ -- 650. 30

$Q'' = q_2 R'''/c$
   $= 20.72(1,112.58)/20.5$ -- 1, 124. 52

$R'' = r_2 S'''/d$
   $= 24.26(1,497.41)/24$ -- 1, 513. 63

$S'' = s_2 T'''/e$
   $= 26.29(1,788.15)/26$ -- 1, 808. 10

$T'' = t_2 U'''/f$
   $= 29.24(2,062.80)/29$ -- 2, 079. 88

$U'' = u_2 V'''/g$
   $= 31.08(2,343.40)/30.87$ --- 2, 358. 96

STAY STRESSES.

$P' = p_1[G - P''n_1/p_2]/(q - n_1)$
   $= 22.48[507 - 650.30(3)/19.24]/(15 - 3)$ ------------------------- 759. 84

$Q' = q_1[G - Q''n_2/q_2]/(r - n_2)$
   $= 25.40[507 - 1,124.52(3)/20.72]/(18 - 3)$ ---------------------- 582. 82

$R' = r_1[G - R''n_3/r_2]/s(-n_3)$
   $= 30[507 - 1,513.63(3.5)/24.26]/(21.50 - 3.5)$ ------------------ 481. 05

[1] Solved two decimal places for comparing methods of solution. Use slide rule for engineering results.

$S' = s_1[G - S''n_4/s_2]/(t - n_4)$
$= 33.74[507 - 1,808.10(3.87)/26.29]/(25.375 - 3.87)$ _   377.41

$T' = t_1[G - T''n_5/t_2]/(u - n_5)$
$= 38.53[507 - 2,079.88(3.75)/29.24]/(29.12 - 3.75)$ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _   364.67

$U' = u_1[G - U''n_6/u_2]/(v - n_6)$
$= 42.44[507 - 2,358.96(3.81)/31.08]/(32.32 - 3.18)$ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _   386.80

STRUT STRESSES.

$P = G$
$= 507$ _   507.00

$Q = P'(q - n_1)/p_1$
$= 759.84(15 - 3)/22.48$ _   405.60

$R = Q'(r - n_2)/q_1$
$= 582.82(18 - 3)/25.40$ _   344.18

$S = R'(s - n_2)/q_1$
$= 481.05(21.50 - 3.5)/30$ _   288.63

$T = S'(t - n_4)/s_1$
$= 377.41(25.375 - 3.875)/33.74$ _   240.49

$U = T'(u - n_5)/t_1$
$= 36.467(29.12 - 3.75)/38.53$ _   240.16

$V = U'(v - n_6)/u_1$
$= 386.80(32.32 - 3.18)/42.44$ _   265.43

3. GRAPHICAL SOLUTION.

The graphical solution is given in figure 35 II.

II. STRESSES DUE TO GRAVITATIONAL LOADS ONLY.

1. GENERAL DATA.

$n_1 = 3.00;$	$b = 19.00;$	$p = 12.00;$	$p_1 = 22.48;$	$p_2 = 19.24;$	$p_3 = b$
$n_2 = 3.00;$	$c = 20.50;$	$q = 15.00;$	$q_1 = 25.40;$	$q_2 = 20.72;$	$q_3 = c$
$n_3 = 3.50;$	$d = 24.00;$	$r = 18.00;$	$r_1 = 30.00;$	$r_2 = 24.26;$	$r_3 = d$
$n_4 = 3.87;$	$e = 26.00;$	$s = 21.50;$	$s_1 = 33.74;$	$s_2 = 26.29;$	$s_3 = e$
$n_5 = 3.75;$	$f = 29.00;$	$t = 25.37;$	$t_1 = 38.53;$	$t_2 = 29.24;$	$t_3 = f$
$n_6 = 3.18;$	$g = 30.87;$	$u = 29.12;$	$u_1 = 42.44;$	$u_2 = 31.08;$	$u_3 = g$
$n_7 = 2.93;$	$h = 41.56;$	$v = 32.31;$	$v_1 = 52.64;$	$v_2 = 41.66;$	$v_3 = h$

$$w = 34.75$$

$G = 30.00;$ $H = 10.00;$ $I = 9.00;$ $J = 7.50;$ $K = 8.00;$ $L = 8.50;$ $M = 100$

In the case of loads G, H, I, J, K, L, M, etc., on a long truss it is convenient to find their moments about different sections before substituting in the equation.

The use of the table below can be readily seen when substituting in the equations of figure 32.

Problems in Part III.

Sum of distances.		Moment.
		Distance$\times G=30$
b	19.00	570.00
$b+c$	39.50	1,185.00
$b+c+d$	63.50	1,905.00
$b+c+d+e$	89.50	2,685.00
$b+c+d+e+f$	118.50	3,555.00
$b+c+d+e+f+g$	149.37	4,481.25
$b+c+d+e+f+g+h$	190.93	5,727.90
		Distance$\times H=10$
c	20.50	205.00
$c+d$	44.50	445.00
$c+d+e$	70.50	705.00
$c+d+e+f$	99.50	995.00
$c+d+e+f+g$	130.37	1,303.75
$c+d+e+f+g+h$	171.93	1,719.35
		Distance$\times I=9$
d	24.00	216.00
$d+e$	50.00	450.00
$d+e+f$	79.00	711.00
$d+e+f+g$	109.87	988.87
$d+e+f+g+h$	151.43	1,362.87
		Distance$\times J=7.5$
e	26.00	195.00
$e+f$	55.00	412.50
$e+f+g$	85.87	644.06
$e+f+g+h$	127.43	955.72
		Distance$\times K=8$
f	29.00	232.00
$f+g$	59.87	479.00
$f+g+h$	101.42	811.36
		Distance$\times L=8.50$
g	30.87	262.43
$g+h$	72.43	615.65
		Distance$\times M=100$
h	41.56	4,156.00

2. ANALYTICAL SOLUTION.

UPPER LONGERON STRESSES.

$P''' = 0$ 0.00

$Q''' = Gb/q$

$= 570\ ^1/15$ 38.00

$R''' = [G(b+c) + H(c)]/r$

$= [1,185 + 205]/18$ 77.22

$S''' = [G(b+c+d) + H(c+d) + Id]/s$

$= [1,905 + 445 + 216]/21.50$ 119.34

$T''' = [G(b+c+d+e) + H(c+d+e) + I(d+e) + Je]/t$

$= 2,685 + 705 + 450 + 195]/25.37$ 159.00

$U''' = [G(b+c+d+e+f) + H(c+d+e+f+) + \text{etc.}]/u$

$= [3,555 + 995 + 711 + 412.50 + 232]/29.12$ 202.75

$V''' = [G(b+c+d+e+f+g) + H(c+d+e+f+g+) + \text{etc.}]/v$

$= [4,481.25 + 1,303.75 + 988.87 + 644.06 + 479 + 262.43]/32.31$ 252.51

$W''' = [G(b+c+d+e+f+g+h) + \text{etc.}]/w$

$= [5,727.90 + 1,719.35 + 1,362.87 + 955.72 + 84.36 + 615.65,\ + 4,156.00]/34.75$ 409.00

LOWER LONGERON STRESSES.

$P'' = p_2 Q'''/b$

$= 19.24(38)/19$ 38.48

$Q'' = q_2 R'''/c$

$= 20.72(77.22)/20.5$ 78.04

$R'' = r_2 S'''/d$

$= 24.26(119.34)/34$ 120.63

$S'' = s_2 T'''/e$

$= 26.29(159.00)/26$ 160.00

[1] See data table above.

$T'' = t_2 U'''/f.$

$\quad = 29.24\ (202.75)/29$ -- 204. 42

$U'' = u_2 V'''/g.$

$\quad = 31.08\ (252.51)/30.875$ -- 254. 18

STAY STRESSES.

$P' = p_1\ [G - P''n_1/p_2]/(q - n_1).$

$\quad = 22.48\ [30 - 38.48(3)/19.24]/(15 - 3)$ -------------------------------- 44. 96

$Q' = q_1\ [G + H - Q''n_2/q_2]/(r - n_2).$

$\quad = 25.40\ [40 - 78.04(3)/20.72]/(18 - 3)$ ------------------------------ 48. 61

$R' = r_1\ [G + H + I - R''n_3/r_2]/(s - n_3).$

$\quad = 30\ [49 - 120.63(3.5)/24.26]/(21.50 - 3.5)$ --------------------------- 52. 66

$S' = s_1\ [G + H + I + J - S''n_4/s_2]/(t - n_4).$

$\quad = 33.74\ [56.50 - 160.00(3.87)/26.29]/(25.37 - 3.87)$ ------------------- 51. 66

$T' = t_1\ [G + H + I + J + K - T''n_5/t_2]/(u - n_5).$

$\quad = 38.53\ [64.50 - 204.42(3.75)/29.24]/(29.12 - 3.75)$ ------------------- 58. 14

$U' = u_1\ [G + H + I + J + K + L - U''n_6u_2]/(v - n_6).$

$\quad = 42.44\ [73 - 254.18(3.18)/31.8]/(32.31 - 3.18)$ --------------------- 69. 33

STRUT STRESSES.

$P = G.$

$\quad = 30$ -- 30. 00

$Q = H + P'\ (q - n_1)/p_1.$

$\quad = 10 + 44.96(15 - 3)/22.48$ -------------------------------------- 34. 00

$R = I + Q'(r - n_2)/q_1.$

$\quad = 9 + 48.61(18 - 3)/25.40$ --------------------------------------- 37. 71

$S = J + R'\ (s - n_3)r_1.$

$\quad = 7.50 + (52.66)\ (21.50 - 3.5)/30$ ------------------------------------ 39. 09

$T = K + S'\ (t - n_4)/s_1.$

$\quad = 8.00 + 5166(25.37 - 3.87)/33.74$ ------------------------------- 40. 91

$U = L + T'\ (u - n_5)/t_1.$

$\quad = 8.50 + 50.65(29.12 - 3.75)/38.53$ ------------------------------ 41.85

III. GRAPHICAL SOLUTION.

The graphical solution is shown in figure 35.

IV. REFERENCE TABLE FOR STRESSES IN LONGERONS.

Symbol.	Longeron stresses.			
	Upper.	Lower.	Stay stresses.	Strut stresses.
P	0. 0	38. 5	45. 0	30. 0
Q	38. 0	78. 0	48. 6	34. 0
R	77. 2	120. 6	52. 7	37. 7
S	119.3	160. 0	51. 7	39. 1
T	159. 0	204. 4	....	40. 9
U	202. 8	254. 2	....	41. 9
V	252. 5	....	....	....
W	....	....	....	....

Example 10.—Find the stresses in the struts, stays, and longerons of the vertical trussing of the front segment of the fuselage shown in figure 35, due to gravitational loads.

I. GENERAL DATA.

$b = 19.00;\quad m_1 = 6.12;\quad n_1 = 7.50;\quad p = 10;\quad p_1 = 24.92;\quad p_2 = 20.43;\quad p_3 = 19.96;$

$c = 19.50;\quad m_2 = 1.87;\quad n_2 = 5.50;\quad q = 23.62;\quad q_1 = 32.10;\quad q_2 = 20.26;\quad q_3 = 19.59;$

$d = 13.66;\qquad\qquad\qquad\qquad\qquad r = 30.94;\qquad\qquad\qquad\qquad\qquad r_3 = 13.67;$

$G = 165;\quad H = 11.$

II. ANALYTICAL SOLUTION.

Solution similar to that in example 9.

III. RESULTS.

Symbol.	Longeron stresses.			
	Upper.	Lower.	Stay stresses.	Strut stresses.
P...	0.0	142.5	174.2	165.0
Q...	133.3	220.5	130.5	103.9
R...	212.4	....	....	....

Example 11.—From the data in the problems above find the stresses and factors of safety for the principal members of the fuselage for a steady circular flight around a level curve of 200 feet radius at 80 miles per hour.

The stresses in the front and rear sections of the fuselage may be found from the gravity stresses for steady level flight in figure 36 by multiplying by a simple ratio. For a velocity of 80 miles per hour and 200 feet radius, formula ——, the resultant loading is 2.133 times the gravity loading.

The table below refers to figure 35, and gives the resultant stresses and factors of safety.

Symbol.	Standard strength, pounds.	Stress for steady flight.	Stress for 80M/HR 200 feet RAD.	Factor of safety.
Ba...	15,000	0.00	....	....
Cc...	15,000	− 71	−151	99
De...	15,000	−114	−243	61
Eg...	15,000			
Fi...	15,000	−147	−314	47
Ge...	13,000	−165	−352	37
Hn...	13,000	−140	−298	43
Ip...	13,000	−115	−245	53
Jn...	13,000	− 82	−174	74
Kt...	13,000	− 45	− 96	135
Lv...	13,000	0.00	....	....
	Strength, pounds.			
Ab...	10,500	144	307	34
Ad...	10,600	222	473	22
Af...	10,700	354	754	14
Oh...	9,600			
Ni...	8,100	442	942	8.5
Nk...	4,800	254	541	8.8
Nm...	1,300	204	435	9.8
No...	1,300	160	341	12.7
Nq...	3,800	121	258	14.7
Ns...	3,500	78	166	21
Nu...	3,500	39	83	42
Aa...				
bc...	6,000	112	239	25
de...	12,900	302	646	20
fg...	10,500			
hi...	10,200			
jk...	5,600	149	317	17.6
lm...	4,500	49	104	43
no...	4,600	41	87	53
pq...	3,800	39	83	45
rs...	3,800	38	81	47
tn...	3,800	34	72	52
vm...				
ab...	4,200	−175	−373	11.2
cd...	4,200	−131	−279	15
ef...	4,200	−328	−699	6
gh...	4,200			
ij...	2,100	−236	−503	4.1
kl...	2,000	− 69	−147	13.6
mn...	2,000	− 59	−125	16
op...	2,000	− 53	−113	17
qr...	2,000	− 52	−111	18
st...	2,000	− 49	−104	19.2
uv...	2,000	− 45	− 96	20.8

Example 12.—Find the stresses in the rear segment of the fuselage due to the torsional loads of example 3.

1. GENERAL DATA.

Table for applied torques as found in example 3.

Station.	1	2	3	4	5	6	7
Torque......................	7,268	7,742	8,216	8,753	9,354	9,954	10,459

Table for length of panels, struts, and stays.

Length of panels.		Length of struts.			Length of stays.		
Station.	Length.	Station.	Upper.	Side.	Station.	Upper.	Side.
1 to 2..........	19	1..............	0.00	12	1 to 2..........	....	22.48
2 to 3..........	20.5	2..............	7.75	15	2 to 3..........	23.4	25.40
3 to 4..........	24	3..............	14.25	18	3 to 4..........	29.2	30.00
4 to 5..........	26	4..............	18.50	21.50	4 to 5..........	32.9	33.70
5 to 6..........	29	5..............	21.37	25.37	5 to 6..........	36.7	38.50
6 to 7..........	30.87	6..............	23.00	29.12	6 to 7..........	39.0	42.40
7 to 8..........	41.60	7..............	24.00	32.3	7 to 8..........	....	....

Table for lengths of struts and stays in the transverse trussing at the different stations.

Station.	Upper strut.[1]	Side strut.	Diagonal.
2...	7.75	15.00	8.44
3...	14.25	18.00	11.45
4...	18.50	21.50	14.18
5...	21.37	25.37	16.58
6...	23.00	29.12	18.55
7...	24.00	32.30	21.11

[1] See fig. 34.

Computation tables for direction cosine.

Symbol.	Left side stays.						
	l	X	Y	Z	Direction cosines.		
					s_x [1]	s_y [1]	s_z [1]
S_2......................	27.4 [2]	20.5	3.10	18	−0.748	−0.113	−0.657
S_3......................	32.3	24	2.4	21.5	− .743	− .074	− .666
S_4......................	36.3	26	1.4	25.4	− .716	− .038	− .699
S_5......................	41.1	29	1.0	29.1	− .706	− .024	− .708
S_6......................	44.7	30.9	.6	32.3	− .691	− .013	− .723
S_7......................	50.1	41.6	0	34.8	− .831	−0.000	− .695

Symbol.	Upper stays.						
	l	X	Y	Z	Direction cosines.		
					t_x	t_y	t_z
T_2......................	23.4 [2]	20.5	10.9	0	−0.884	−0.470	0
T_3......................	29.0	24	16.7	0	− .828	− .576	0
T_4......................	32.7	26	19.9	0	− .795	− .608	0
T_5......................	36.6	29	22.4	0	− .792	− .612	0
T_6......................	38.8	30.9	23.6	0	− .796	− .608	0
T_7......................	48.0	41.6	24.0	0	− .867	− .500	0

[1] The direction cosines are found from the projections along X, Y, Z and the true lengths of the struts, stays, etc. The signs are given from observation in fig. 34.
[2] Inches.

Computation tables for direction cosine—Continued.

Symbol.	Right side stays.						
	l	X	Y	Z	Direction cosines.		
					u_x	u_y	u_z
U_2	25.5	20.5	3.10	15	−0.804	+0.121	+0.588
U_3	29.9	24	2.40	18	− .802	+ .080	+ .602
U_4	33.7	26	1.4	21.5	− .771	+ .041	+ .638
U_5	38.6	29	1.0	25.4	− .751	+ .026	+ .658
U_6	42.5	30.9	.6	29.1	− .728	+ .014	+ .685
U_7	52.6	41.6	0	32.3	− .791	+0	+ .614

Symbol.	Lower stays.						
	l	X	Y	Z	Direction cosines.		
					v_x	v_y	v_z
V_2	23.3	20.5	10.9	3.0	−0.880	+0.468	−0.129
V_3	29.1	24.0	16.7	3.4	− .825	+ .574	− .117
V_4	32.8	26.0	19.9	3.9	− .792	+ .607	− .118
V_5	36.7	29.0	22.4	3.7	− .790	+ .612	− .101
V_6	38.9	30.9	23.6	3.2	− .794	+ .606	− .082
V_7	43.9	41.6	24.0	2.3	− .948	+ .546	− .052

Symbol.	Upper longerons, left.						
	l	X	Y	Z	Direction cosines.		
					s_x'	s_y'	s_z'
S_2'	20.8	20.5	3.10	0	+0.986	+0.149	0
S_3'	24.1	24	2.4	0	+ .997	+ .099	0
S_4'	26.1	26	1.4	0	+ .998	+ .053	0
S_5'	29.1	29	1.0	0	+ .999	+ .034	0
S_6'	30.9	30.9	.6	0	+1.000	+ .019	0
S_7'	41.6	41.6	0	0	+1.000	+0	0

Symbol	Upper longerons, right.						
	l	X	Y	Z	Direction cosines.		
					t_x'	t_y'	t_z'
T_2'	20.8	20.5	3.10	0	+0.986	−0.149	0
T_3'	24.1	24	2.4	0	+ .997	− .099	0
T_4'	26.0	26	1.4	0	+ .998	− .053	0
T_5'	29.1	29	1.0	0	+ .999	− .034	0
T_6'	30.9	30.9	.6	0	+1.000	− .019	0
T_7'	41.6	41.6	0	0	+1.000	0	0

Symbol.	Lower longerons, left.						
	l	X	Y	Z	Direction cosines.		
					u_x'	u_y'	u_z'
U_2'	20.9	20.5	3.10	3.0	+0.981	−0.149	+0.143
U_3'	24.2	24	2.4	3.4	+ .992	− .099	+ .140
U_4'	26.1	26	1.4	3.8	+ .994	− .053	+ .146
U_5'	29.2	29	1.0	3.7	+ .999	− .034	+ .126
U_6'	30.9	30.9	.6	3.2	+1.00	− .019	+ .103
U_7'	41.6	41.6	0	2.0	+1.00	0.000	+ .048

Computation tables for direction cosine—Continued.

Symbol.	Lower longerons, right.						
	l	X	Y	Z	Direction cosines.		
					v_x'	v_y'	v_z'
V₂′	20.9	20.5	3.10	3.0	−0.981	−0.149	−0.143
V₃′	24.2	24	2.40	3.4	− .992	− .099	− .140
V₄′	26.1	26	1.4	3.9	− .994	− .053	− .149
V₅′	29.2	29	1.0	3.7	− .999	− .034	− .126
V₆′	30.9	30.9	.6	3.2	−1.00	− .019	− .103
V₇′	41.6	41.6	0	2.0	−1.00	0.000	− .048

II. SOLUTION.

FOR TORQUE OF 7,742 INCHES-POUNDS AT STATION NO. 2.

Given torque = 7,742 ins.-lbs.; $l = 8.44$ ins.; $b = 15$ ins.; $a = 7.74$ ins.

Then by figure 34:

$R = \text{torque}/4l$

$= 7,742/4\ (8.44)$ _____ 229.32 lbs.

$P = R\ b/2l$

$= 229.32(15)/2(8.44)$ _____ 203.77

$Q = Ra/2l$

$= 229.32\ (7.75)/2(8.44)$ _____ 105.28

Substituting these values in formulas of figure 34:

$s_y S + s_y' S' + t_y T + t_y' T' + 2\ P = 0 \quad -0.113S + 0.149S' - 0.470\,T - 0.149\,T' + 2(203.77) = 0$ (1)

$t_z T + t_z' T' + u_2 U + u'^1 U_2 - 2Q = 0 \quad +0 + 0.588\ U + 0.143\ U' - 2(105.28) = 0$ (2)

$u_y U + u_y' U' + v_y V + v_y' V' - 2P = 0 \quad +0.121\ U - 0.149\ U' + 0.468\ V + 0.149\ V' - 2(203.77) = 0$ (3)

$v_z V + v_z' V' + s_z S + s_z' S' + 2\ Q = 0 \quad -0.129\ V + 0.143\ V' - 0.657S + 0 + 2(105.28) = 0$ (4)

$s_x S + s' S^1 = 0 \quad -0.748S + 0.986S' = 0;\quad S' = 0.759S$ ( )

$t_z T + t_z' T' = 0 \quad -0.884T + 0.986\,T' = 0;\quad T' = 0.897T$ (6)

$u_x U + u_x' U' = 0 \quad -0.804\ U + 0.981 U' = 0;\quad U' = 0.819U$ (7)

$v_x V + v_x' V' = 0 \quad -0.880\ V + 0.981V' = 0;\quad V' = 0.897V$ (8)

Substituting values of S', T', U', V' from equations 5, 6, 7, 8, in equations 1, 2, 3, 4 we find—

$S = 320.7.$

$T = 675.0.$

$U = 298.7.$

$V = 678.0.$

Again by equations 5, 6, 7, 8—

$S' = 243.6.$

$T' = 605.5.$

$U' = 243.9.$

$V' = 608.2.$

In a similar manner the stresses may be determined for stations 3, 4, etc. The table below gives these results for the given torsion loads.

Stations.	S	T	U	V	S′	T′	U′	V′
2	320.7	675.0	298.7	678.0	243.6	605.5	243.9	608.2
3	334.4	426.1	311.5	427.5	249.2	354.0	251.8	355.8
4	288.0	360.0	268.1	360	206.8	286.9	208.1	286.9
5	256.0	337.0	240.7	337	181.0	267.2	181.2	266.6
6	230.0	338.0	218.8	339	148.9	269.0	159.3	269.1
7	223.1	417.3	238.2	382	185.4	361.8	188.4	362.1

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PART V.

ILLUSTRATIONS FOR PARTS I, II, III, IV.

By A. F. Zahm and L. H. Crook.

FIG. 1.—Contour maps of pressure on top and bottom surfaces of typical aerofoil (model R. A. F. 6, aspect ratio 6 to 1, air at standard density).
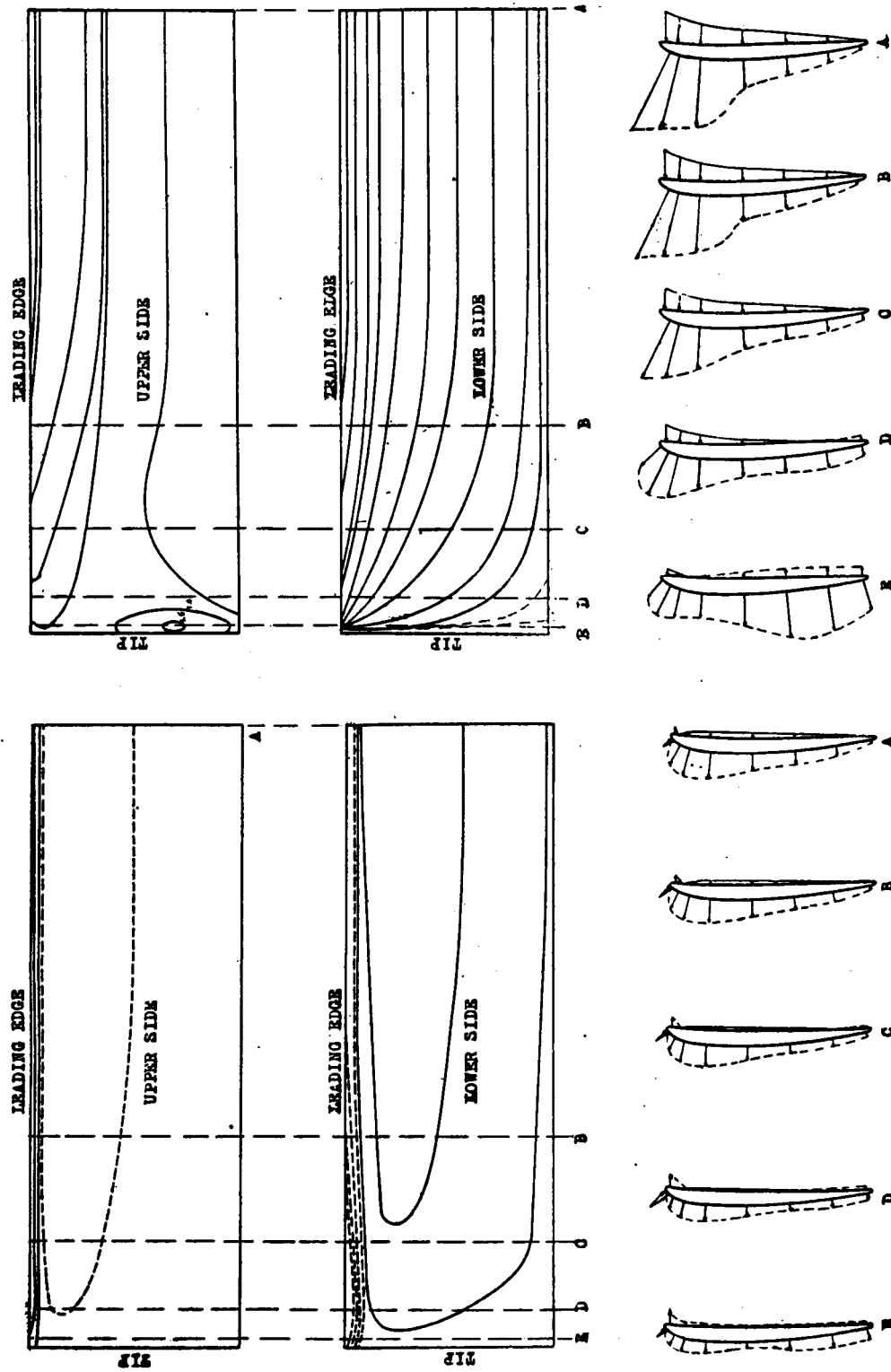
FIG. 2.—Pressure distribution on median section of R. A. F. 6 aerofoil of 1 to 6 aspect ratio at 30 feet per second, air at standard density.
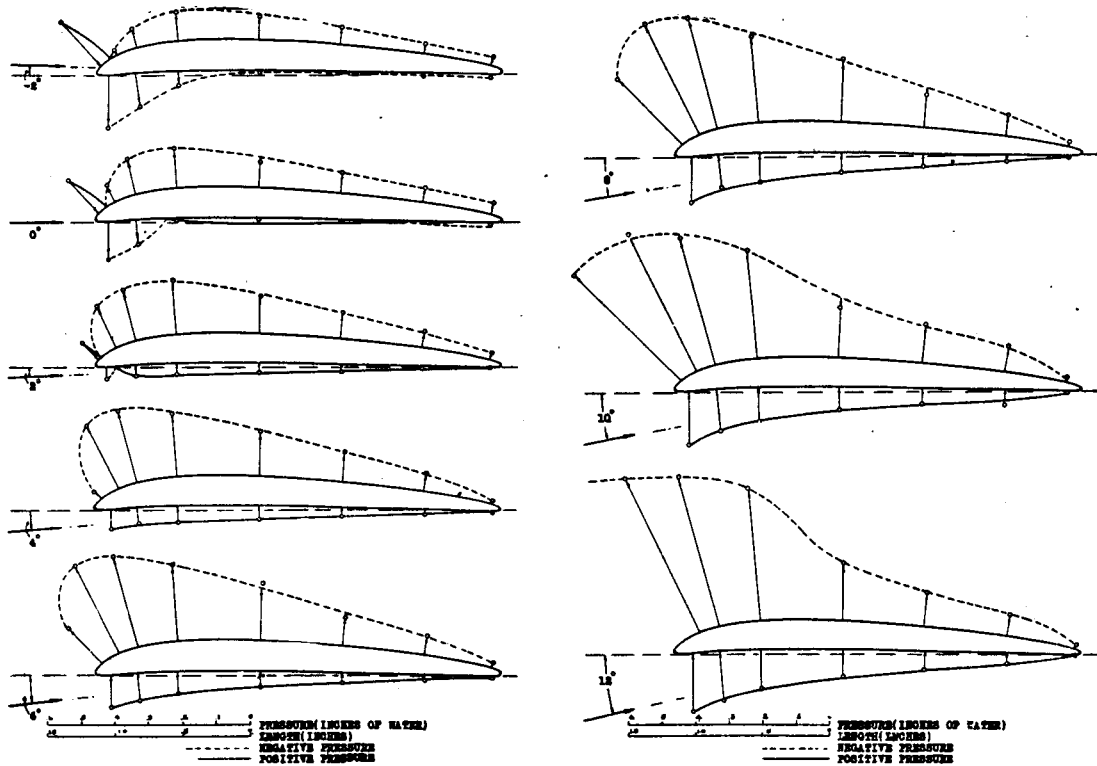
FIG. 3.—Pressure distribution on typical aerofoil with hinged rear margin.   (Model R. A. F. 6, aspect ratio 1 to 6, standard air density, speed 40 m. p. h.)
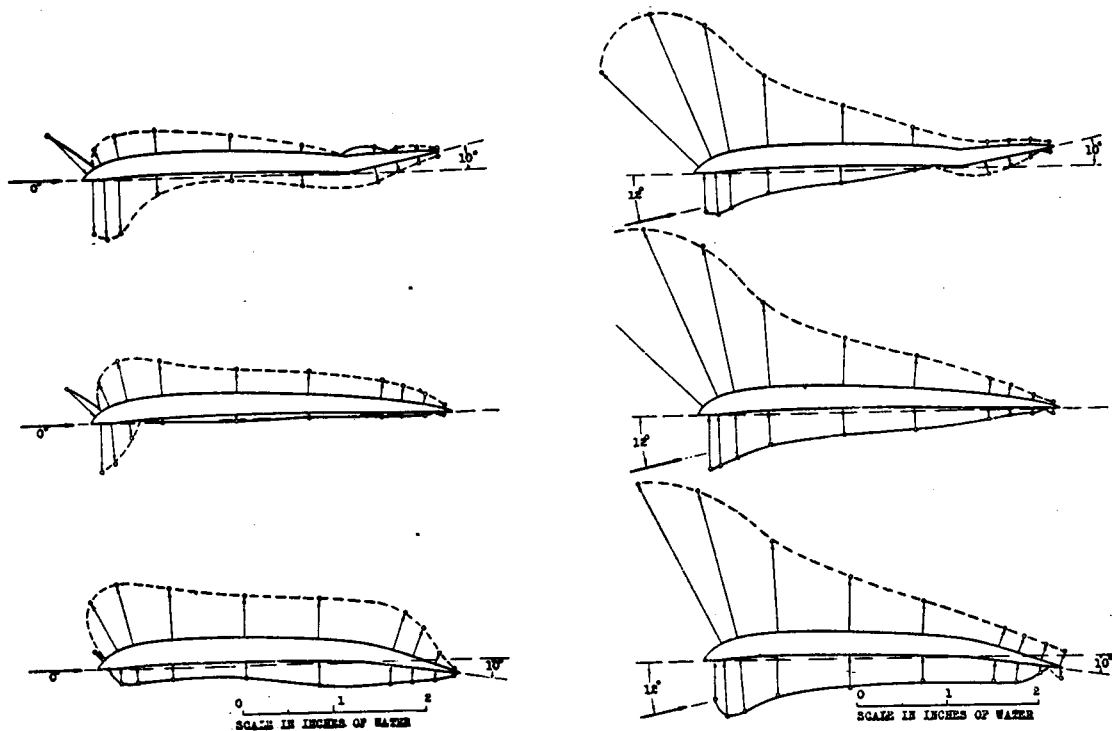
FIG. 4.—Lift, drag, and center of pressure for typical aerfoil at various cross sections and angles of incidence. (R. A. F. 6, 6 to 1 aspect ratio, 30 feet per second air speed.)
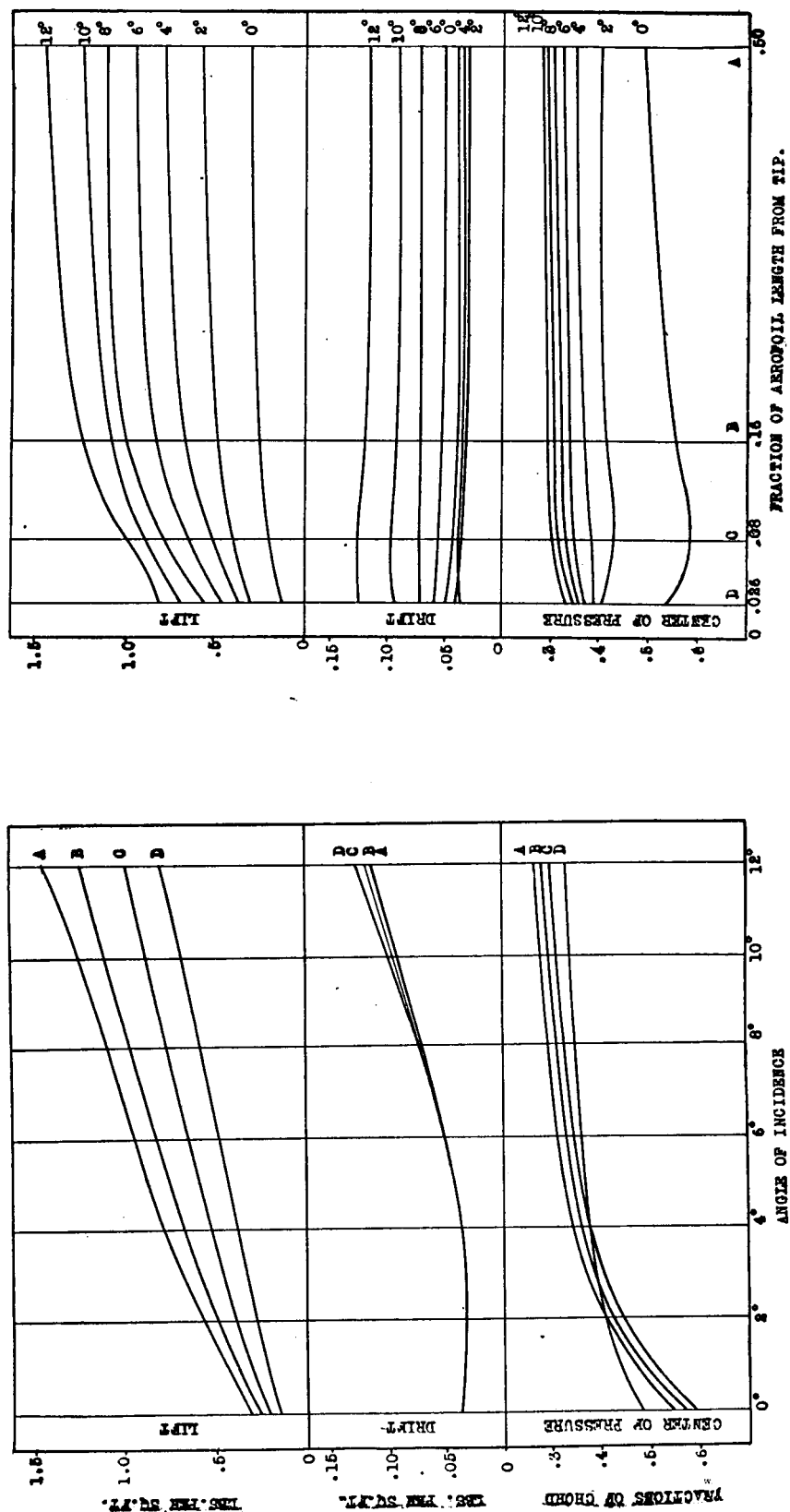
FIG. 5.—Values of the natural angle of bank α degrees, centrifugal force F, and whole force P perpendicular to the wings, for an airplane in steady circular flight around a level curve of radius R feet. F and P expressed as a fraction of the whole weight W.

Flight speed.	R=100 feet.			R=200 feet.			R=300 feet.			R=400 feet.			R=500 feet.			R=600 feet.			R=800 feet.		
Miles per hour.	α	F	P	α	F	P	α	F	P	α	F	P	α	F	P	α	F	P	α	F	P
20	14 56	0.267	1.035	7 30	0.133	1.009	5 5	0.089	1.004	3 49	0.067	1.002	3 9	0.053	1.002	2 33	0.044	1.001	1 55	0.033	1.001
30	30 58	.600	1.166	16 42	.300	1.044	11 19	.200	1.020	8 32	.150	1.011	6 51	.120	1.007	5 43	.100	1.005	4 18	.075	1.003
40	46 51	1.067	1.462	28 4	.533	1.133	19 34	.356	1.061	14 56	.267	1.035	12 3	.213	1.022	10 5	.178	1.016	7 36	.133	1.009
50	59 2	1.667	1.944	39 48	.833	1.302	29 3	.556	1.144	22 37	.417	1.083	18 26	.333	1.054	15 31	.278	1.038	11 46	.208	1.021
60	67 23	2.400	2.600	50 12	1.200	1.562	38 40	.800	1.281	30 58	.600	1.166	25 38	.480	1.109	21 48	.400	1.077	16 42	.300	1.044
70	72 59	3.267	3.416	58 31	1.633	1.915	47 26	1.089	1.478	39 14	.817	1.291	33 9	.653	1.195	28 34	.544	1.139	22 13	.408	1.080
80	76 49	4.267	4.383	64 53	2.133	2.356	54 53	1.422	1.739	46 51	1.067	1.462	40 29	.853	1.315	35 25	.711	1.227	28 4	.533	1.133
90	79 31	5.400	5.492	69 41	2.700	2.879	60 57	1.800	2.059	63 28	1.350	1.680	47 12	1.080	1.472	41 59	.900	1.345	34 1	.675	1.206
100	81 28	6.667	6.741	73 18	3.333	3.480	65 46	2.222	2.437	59 2	1.667	1.944	53 8	1.333	1.667	48 1	1.111	1.495	39 48	.833	1.302
110	82 56	8.067	8.128	76 5	4.033	4.155	69 36	2.689	2.869	63 37	2.017	2.251	58 12	1.613	1.898	53 21	1.344	1.675	45 14	1.008	1.420
120	84 3	9.600	9.652	78 14	4.800	4.903	72 39	3.200	3.353	67 23	2.400	2.600	62 29	1.920	2.165	58 0	1.600	1.887	50 12	1.200	1.562
130	84 56	11.27	11.31	79 56	5.633	5.721	75 5	3.755	3.886	70 27	2.817	2.989	66 4	2.253	2.465	61 58	1.878	2.127	54 37	1.408	1.727
140	85 37	13.07	13.10	81 18	6.533	6.609	77 4	4.356	4.469	72 59	3.267	3.416	69 4	2.613	2.798	65 20	2.178	2.396	58 31	1.633	1.915
150	86 11	15.00	15.03	82 24	7.500	7.566	78 41	5.000	5.099	75 4	3.750	3.881	71 34	3.000	3.162	68 12	2.500	2.693	61 56	1.875	2.125
175	87 12	20.42	20.44	84 24	10.21	10.26	81 38	6.806	6.879	78 55	5.104	5.201	76 14	4.083	4.204	73 37	3.403	3.546	68 36	2.552	2.741
200	87 51	26.67	26.69	85 43	13.33	13.37	83 35	8.889	8.945	81 28	6.667	6.741	79 23	5.333	5.426	77 19	4.444	4.556	73 18	3.33	3.480

FIG. 6.—Tension in airplane fabric in terms of pressure, bulge, and rib spacing.
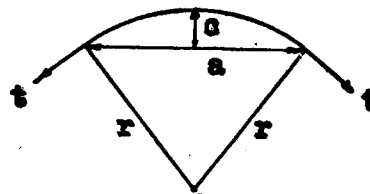
$$t = pa^2/144(8c).$$

Depth of bulge c, inches.	Tension t in pounds per linear inch for various air pressures, p pounds per square foot.							
	p=5.	p=8.	p=10.	p=15.	p=20.	p=25.	p=50.	p=100.
	For rib spacing a=12 inches; t=p/8c.							
0.1	6.25	10.00	12.50	18.75	25.00	31.25	62.50	125.00
.2	3.12	5.00	6.25	9.37	12.50	15.62	31.25	62.50
.3	2.08	3.33	4.16	6.25	8.33	10.41	20.83	41.66
.4	1.56	2.50	3.12	4.68	6.25	7.81	15.62	31.25
.5	1.25	2.00	2.50	3.75	5.00	6.25	12.50	25.00
	For rib spacing a=13 inches; t=.1465 p/c.							
.1	7.32	11.72	14.65	22.00	29.30	36.60	73.25	146.50
.2	3.66	5.86	7.32	11.00	14.65	18.30	36.62	73.25
.3	2.44	3.91	4.88	7.33	9.76	12.20	24.41	48.80
.4	1.83	2.93	3.66	5.50	7.32	9.15	18.31	36.60
.5	1.46	2.34	2.93	4.40	5.86	7.32	14.65	29.30
	For rib spacing a=14 inches; t=.1701 p/c.							
.1	8.50	13.61	17.01	25.51	34.02	42.52	85.05	170.10
.2	4.25	6.80	8.50	12.75	17.01	21.26	42.52	85.00
.3	2.83	4.53	5.67	8.50	11.34	14.17	28.35	56.70
.4	2.12	3.40	4.25	6.37	8.50	10.63	21.26	42.52
.5	1.70	2.72	3.40	5.10	6.80	8.50	17.01	34.00
	For rib spacing a=15 inches; t=.1953 p/c.							
.1	9.76	15.62	19.53	29.29	39.06	48.82	97.65	195.3
.2	4.88	7.81	9.76	14.64	19.53	24.41	48.82	97.6
.3	3.25	5.20	6.51	9.76	13.02	16.27	32.55	65.1
.4	2.44	3.90	4.88	7.32	9.76	12.20	24.41	48.8
.5	1.95	3.12	3.90	5.85	7.81	9.76	19.53	39.0
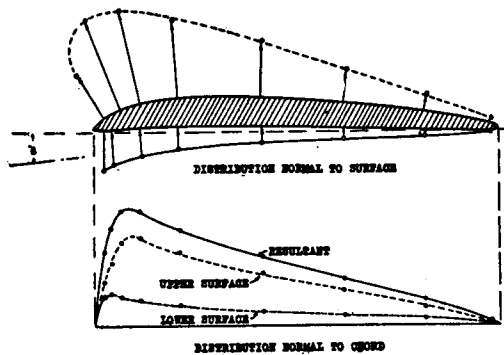
FIG. 7.—Typical pressure distribution on wing plane resolved normal to chord.
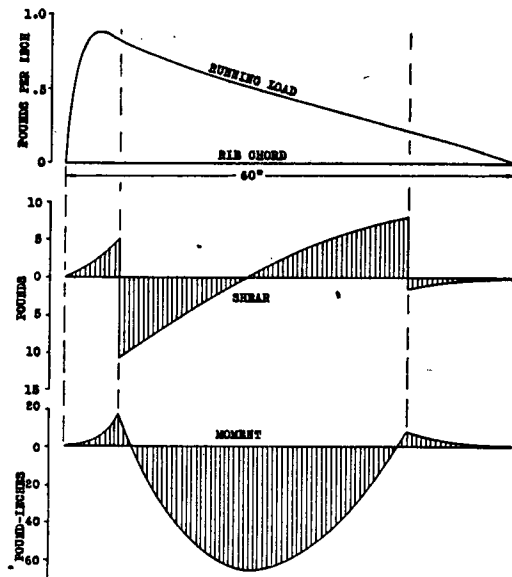
FIG. 8.—Shear and moment on rib of a wing plane due to loading normal to chord.
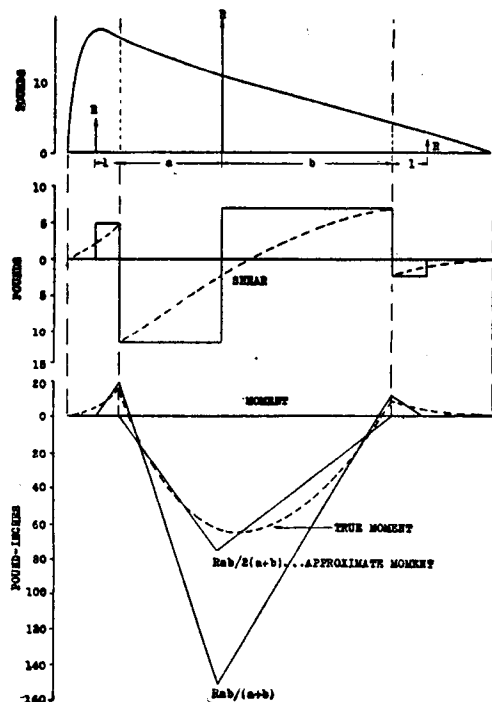
FIG. 9.—Shearing force and bending moment on wing rib due to three concentrated loads approximately representing the loading normal to chord.
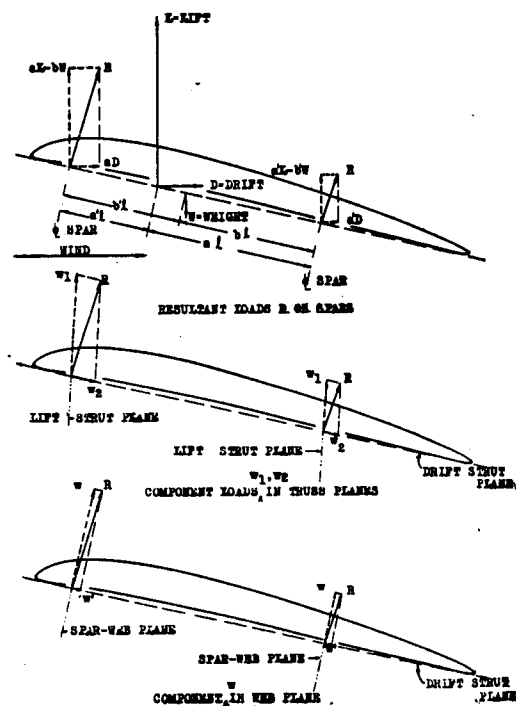
FIG. 10.—Resolution of wing plane loads into wing spar loads (pictured in sequence).
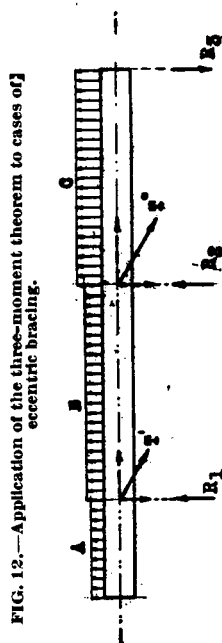
FIG. 12.—Application of the three-moment theorem to cases of] eccentric bracing.

For centrally attached stays, form 1, Fig. 11, is applied,
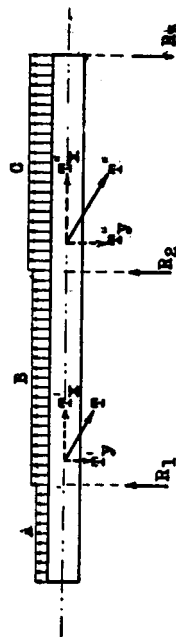
For stay aside the strut axis the tension is replaced by its horizontal and vertical components T_x T_y the latter being regarded as a concentrated load and treated by form 2, Fig. 11.

FIG. 11.—Forms of the three-moment theorem.

I FOR RUNNING LOADS ONLY (CLAPEYRON).
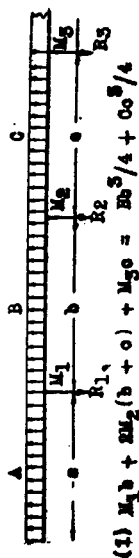
$$(1)\ M_1 b + 2M_2(b + c) + M_3 c = Bb^3/4 + Cc^3/4$$

II FOR CONCENTRATED LOADS ONLY (BRESSE).
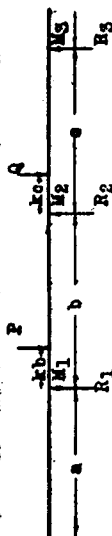
$$(2)\ M_1 b + 2M_2(b + c) + M_3 c = -Pb^2(k_1 - k_1^3) - Qc^2(2k_2 - 3k_2^2 + k_2^3)$$

FOR RUNNING LOADS WITH SUPPORTS AT DIFFERENT LEVELS.

$$1)\ M_1 b + 2M_2(b + c) + M_3 c = +Bb^3/4 + Cc^3/4 + 6EI\left[(h_2-h_1)/b+(h_2-h_3)/c\right]$$

FIG. 13.—Bending moments and reactions for a continuous beam symmetrically supported.

FIG. 14.—Bending moments, shears, and reactions for a continuous beam supported at three points and having one end overhanging. Uniformly loaded on each panel.

Fig. 14 table

Auxiliary symbols	$l=(Bb^3+Cc^3)/4-Aa^2b/2$; $h=2(b+c)$.		
Joint bending moments	$\{ M_1=Aa^2/2. \quad M_2=l/h. \}$		
	Shear left of pin.	Shear right of pin.	Pin reactions.
Joint shears and pin reactions.	$U_1=aA.$	$V_1=(M_2-M_1)/b-Bb/2.$	$R_1=U_1-V_1.$
	$U_2=V_1+Bb.$	$V_2=(0-M_2)/c-Cc/2.$	$R_2=U_2-V_2.$
	$U_3=V_2+Cc.$		$R_3=U_3-o.$
	Magnitude.		Position.
Maximum moments between joints.	$\{ M_1'=M_1-V_1^2/2B.$		$X_1=-V_1/B.$
	$M_2'=M_2-V_2^2/2C. \}$		$X_2=-V_2/C.$
Gen. eqs. for any panel	B. M. curve, $M_x=M+Vx+wx^2/2.$ Shear curve, $V_x=V+wx.$ Elastic curve, $y=-x[12M(l-x)+4V(l^2-x^2)+w(l^3-x^3)]/24\,EI.$		

NOTE.—For briefer analysis, let A, B, C=w, the uniform running load.

Fig. 13 table

Bending moments	$\{ M_1=wa^2/2=M_3. \quad M_2=wb^2/8. \}$		
	Shear left of pin.	Shear right of pin.	Pin reactions.
Joint shears and pin reactions.	$U_1=wa.$	$V_1=(M_2-M_1)/b-wb/2.$	$R_1=w(4l^2-b^2)/8b.$
	$U_2=V_1+wb.$	$V_2=(M_3-M_2)/b-wl/2.$	$R_2=2(wl-R_1).$
	$U_3=V_2+wb.$	$V_3=(0-M_3)/a-wa/2.$	$R_3=R_1.$
	Magnitude.		Position.
Maximum moments between joints.	$\{ M_1'=M_1-V_1^2/2w.$		$X_1=-V_1/w.$
	$M_2'=M_2-V_2^2/2w. \}$		$X_2=-V_2/w.$
Gen. eqs. for any panel	B. M. curve, $M_x=M+Vx+wx^2/2.$ Shear curve, $V_x=V+wx.$ Elastic curve, $y=-x[12M(l-x)+4V(l^2-x^2)+w(l^3-x^3)]/24\,EI.$		

R_1 and R_2 as fractions of wl.

b/l	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3
R_1	.375	.443	.525	.627	.758	.937	1.20	1.638
R_2	1.250	1.114	.950	.746	.484	.126	−.40	−1.276

FIG. 16.—Bending moments, shears, and reactions for a continuous beam supported at five points and having one end overhanging. Uniformly loaded on each panel.

	Shear left of pin.	Shear right of pin.	Pin reactions.
Auxiliary symbols......	$l = (Bb^3+Cc^3)/4 - bAa^2/2;$ $m = (Cc^3+Dd^3)/4;$ $n = (Dd^3+Ee^3)/4;$	$h=2(b+c).$ $i=2(c+d).$ $j=2(d+e).$	
Joint bending moments	$M_1=Aa^2/2.$ $M_2=[l(ij-d^2)-c(jm-nd)]/[h(ij-d^2)-c^2i].$ $M_3=(l-hM_2)/c.$ $M_4=(n-M_3d)/j.$		
Joint shears and pin reactions.	$U_1=Aa.$ $U_2=V_1+Bb.$ $U_3=V_2+Cc.$ $U_4=V_3+Dd.$ $U_5=V_4+Ee.$	$V_1=(M_2-M_1)/b-Bb/2.$ $V_2=(M_3-M_2)/c-Cc/2.$ $V_3=(M_4-M_3)/d-Dd/2.$ $V_4=(o-M_4)/e-Ee/2.$	$R_1=U_1-V_1.$ $R_2=U_2-V_2.$ $R_3=U_3-V_3.$ $R_4=U_4-V_4.$ $R_5=U_5-0.$
	Magnitude.		Position.
Maximum moments between joints.	$M_1'=M_1-V_1^2/2B.$ $M_2'=M_2-V_2^2/2C.$ $M_3'=M_3-V_3^2/2D.$ $M_4'=M_4-V_4^2/2E.$		$X_1=-V_1/B.$ $X_2=-V_2/C.$ $X_3=-V_3/D.$ $X_4=-V_4/E.$
Gen. eqs. for any panel.	B. M. curve, $M_x=M+V_x+wx^2/2.$ Shear curve, $V_x=V+wx.$ Elastic curve, $y=-x[12M(1-x)+4V(l^2-x^2)+w(l^3-x^3)]/24$ EI.		

NOTE.—For briefer analysis, let A, B, C—w, the uniform running load.

FIG. 15.—Bending moments, shears, and reactions for a continuous beam supported at four points and having one end overhanging. Uniformly loaded on each panel.

	Shear left of pin.	Shear right of pin.	Pin reactions.
Auxiliary symbols.....	$l = (Bb^3+Cc^3)/4 - Aa^2/2;$ $m = (Cc^3+Dd^3)/4;$	$h=2(b+c).$ $i=2(c+d).$	
Joint bending moments	$M_1=Aa^2/2.$ $M_2=(li-mc)/(hi-c^2).$ $M_3=(l-M_2h)/c.$		
Joint shears and pin reactions.	$U_1=aA.$ $U_2=V_1+Bb.$ $U_3=V_2+Cc.$ $U_4=V_3+Dd.$	$V_1=(M_2-M_1)/b-Bb/2.$ $V_2=(M_3-M_2)/c-Cc/2.$ $V_3=(o-M_3)/d-Dd/2.$	$R_1=U_1-V_1.$ $R_2=U_2-V_2.$ $R_3=U_3-V_3.$ $R_4=U_4-0.$
	Magnitude.		Position.
Maximum moments between points.	$M_1'=M_1-V_1^2/2B.$ $M_2'=M_2-V_2^2/2C.$ $M_3'=M_3-V_3^2/2D.$		$X_1=-V_1/B.$ $X_2=-V_2/C.$ $X_3=-V_3/D.$
Gen. eqs. for any panel.	B. M. curve, $M_x=M+Vx+wx^2/2.$ Shear curve, $V_x=V+wx.$ Elastic curve, $y=-x[12M(1-x)+4V(l^2-x^2)+w(l^3-x^3)]/24$ EI.		

NOTE.—For briefer analysis, let A, B, C—v, the uniform running load.

FIG. 17.—Shear and moment on upper and lower spars in plane of front struts.

FIG. 19.—Comparison of distances to maximum deflection and maximum bending moment for typical spar panel.

FIG. 18.—Bending moments for a uniformly loaded continuous beam with pin supports and stays attached aside from strut axis.
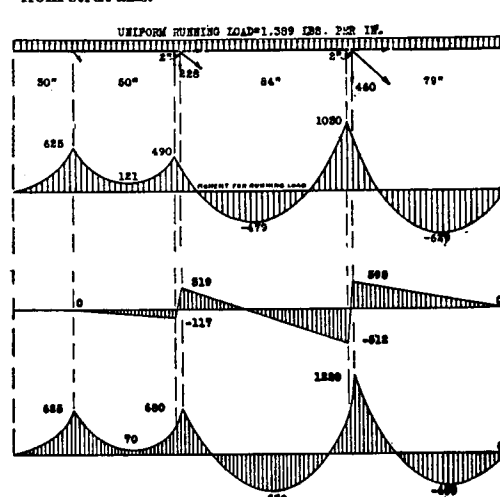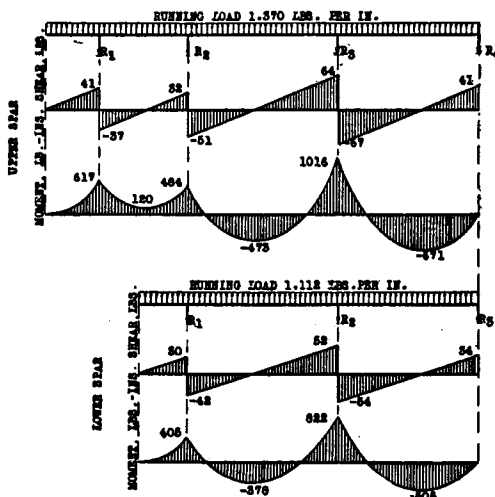
FIG. 20.—Concentrated loads on drag trussing in terms of pin reactions R., and strut and stay resistance.

I. BIPLANE TRUSSING.

(a) *For upper drag trussing—*
1. Loads due to *front* upper spar plus resistance of front struts and stays.
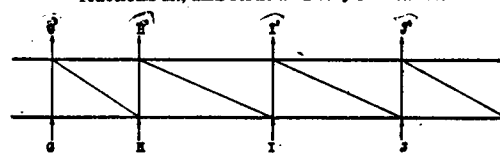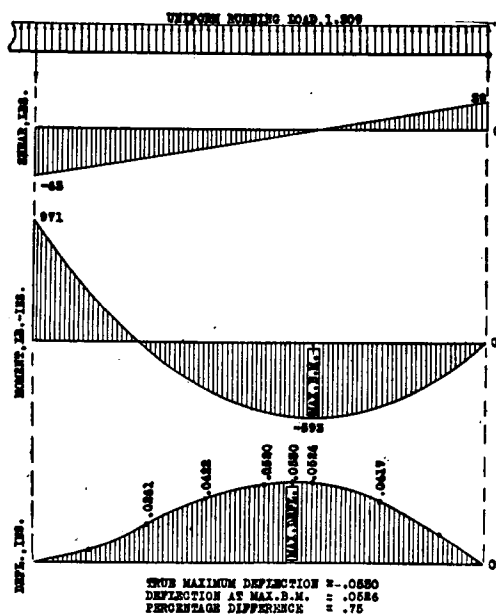
$$G = r_1 + R_1\ N.$$
$$H = r_2 + R_2\ N.$$
$$I = r_3 + R_3\ N.$$
$$J = r_4 + R_4\ N.$$

2. Loads due to *rear* upper spar plus resistance [1] of rear struts and stays.

$$G' = r_1 + R_1\ N_1.$$
$$H' = r_2 + R_2\ N_1.$$
$$I' = r_3 + R_3\ N_1.$$
$$J' = r_4 + R_4\ N_1.$$

Where r_1, r_2, r_3, etc., equal one-half the air resistance of the struts and stays adjacent to the pins; $N = w_2/w_1$, the ratio of the running load on front spar in plane of drag trussing to the running load on front spar in plane of lift trussing; $N_1 = w_2/w_1$, the ratio of the running load on rear spar, in plane of lift trussing to running load on rear spar in plane of drag trussing;

(b) *For lower drag trussing.*—For concentrated loads on lower drag trussing change the word *upper* into *lower* in (a) above.

II. ANY TRUSSING.

The formulas above apply also to drag trussing in monoplanes or multiplanes.

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[1] Consider the resistance as acting only in the plane of drag trussing.

FIG. 21.—Concentrated loads on drag trussing in terms of strut and stay resistance and running loads on spars.
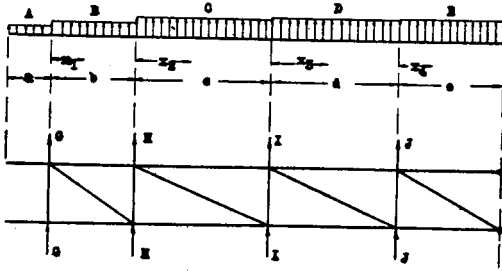
I. BIPLANE TRUSSING.

(a) For upper drag trussing.—

1. Loads due to the *front* upper spar plus one-half the resistance of the *front* struts and stays.

$$G = r_1 + Aa + Bx_1$$
$$H = r_2 + B(b-x_1) + Cx_2$$
$$I = r_3 + C(c-x_2) + Dx_3$$
$$J = r_4 + D(d-x_3) + Ex_4$$

2. Loads due to the *rear* upper spar plus one-half the resistance of the *rear* struts and stays.

$$G' = r'_1 + Aa + Bx_1$$
$$H' = r'_2 + B(b-x_1) + Cx_2$$
$$I' = r'_3 + C(c-x_2) + Dx_3$$
$$J' = r'_4 + D(d-x_3) + Ex_4$$

Where r_1, r_2, r_3, etc., are the loads due to one-half the air resistance* of the front struts and stays; r_1', r_2', r_3', etc., are the loads due to one-half the air resistance* of the rear struts and stays; x_1, x_2, etc., the distances to the points of zero shear.

*(b) For lower drag trussing.—*Treatment similar to above.

II. ANY TRUSSING.

The formulas above apply also to the drag trussing in monoplanes or multiplanes.

FIG. 23.—Endwise stresses in terms of loads and size in any monoplane wing truss. Analytical treatment.
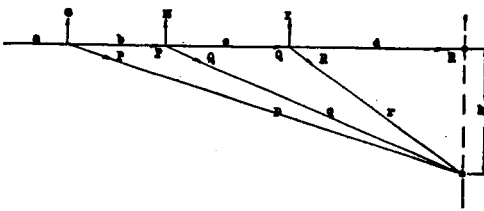
STAY STRESSES.

$$P' = pG'/h$$
$$Q' = qH'/h$$
$$R' = rI'/h$$

SPAR STRESSES.

$$P'' = (b+c+d)G/h$$
$$Q'' = (b+c+d)G/h + (c+d)H/h$$
$$R'' = (b+c+d)G/h + (c+d)H/h + dI/h$$

*Consider the resistance as acting only in the plane of the drag trussing.

FIG. 22.—Endwise stresses in a biplane wing truss with parallel spars. Analytical treatment.
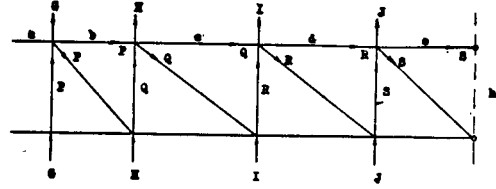
STRUT STRESSES.

$$P = G = G$$
$$Q = G+G'+H = P+G'+H$$
$$R = G+G'+H+H'+I = Q+H'+I$$
$$S = G+G'+H+H'+I+I'+J = R+I'+J$$

STAY STRESSES.

$$P' = p(G+G')/h = p(P+G')/h = pn_1$$
$$Q' = q(G+G'+H+H')/h = q(Q+H')/h = qn_2$$
$$R' = r(G+G'+H+H'+I+I')/h = r(R+I')/h = rn_3$$
$$S' = s(G+G'+H+H'+I+I'+J+J')/h = s(S+J')/h = sn_4$$

SPAR STRESSES.

$$P'' = (G+G')b/h = b(P+G')/h = bn_1$$
$$Q'' = [(G+G')(b+c) + c(H+H')]/h = P'' + c(Q+H')/h$$
$$= P'' + cn_2$$
$$R'' = [(G+G')(b+c+d) + (c+d)(H+H') + d(I+I')]/h$$
$$= Q'' + d(R+I')/h = Q'' + dn_3$$
$$S'' = [(G+G')(b+c+d+e) + (c+d+e)(H+H') + \text{etc.}]/h$$
$$= R'' + e(S+J')/h = R'' + en_4$$

FIG. 24.—Three-component resolution of stay tension.
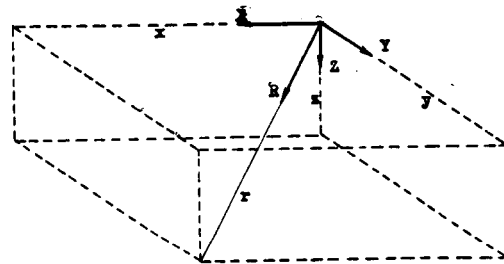
I. GENERAL CASE.

$$R/r = X/x = Y/y = Z/z$$
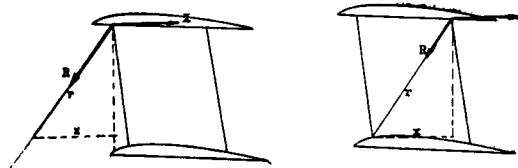
II. SPECIAL CASES.

(a) For cabane stays,
$$R = rX/x$$
(b) For cross diagonal,
$$R = rX/x$$

FIG. 25.—Endwise stresses in terms of loads and size in a multiplane wing truss. Analytical treatment.
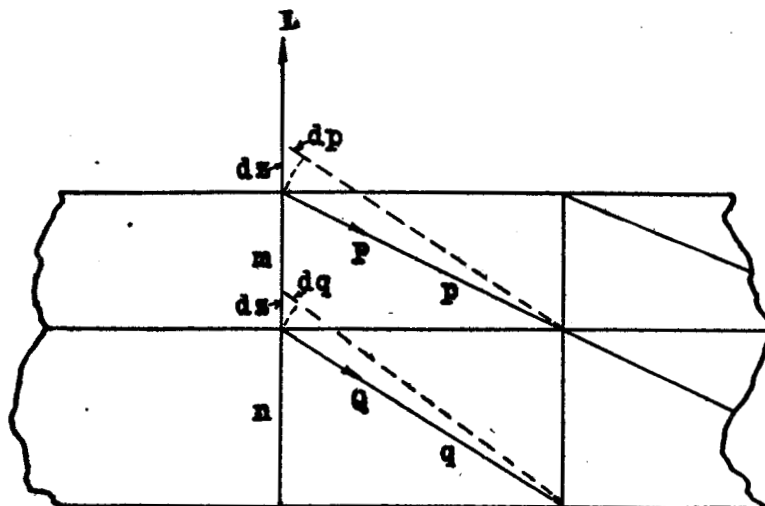
GENERAL THEORY.

Let L be the total lift on a multiplane strut, as shown,

P, Q, tensions in the stays p, q,

dp, dq, stretches of the lengths p, q, for vertical strain dz of truss,

m, n, strut lengths between joints,

A, B, E, cross sectional areas of stays and modulus of elasticity.

Then $L = Pm/p + Qn/q + $ etc., for more planes, if any,

$P = AEdp/p$,

$Q = BEdq/q$,

$R = $ etc., for more planes, if any.

$dz = qdq/n = pdp/m = $ etc., for more planes, if any.

$P/Q = Amq^2/Bnp^2$, $Q/R = $ etc., for more planes, if any.

FOR A TRIPLANE.

$P = LAmq^2/(Am^2q^2/p + Bn^2p^2/q)$.

$Q = LBnp^2/(Am^2q^2/p + Bn^2p^2/q)$.

FOR A QUADRUPLANE.

$P = LAmr^2/(Am^2r^2/p + Bn^2r^2p^2/q^3 + C\ o^2p^2/r)$.

$Q = LBnp^2/(Bn^2p^2/q + Co^2p^2q^2/r^3 + Am^2q^2/p)$.

$R = LCoq^2/(Co^2q^2/r + Am^2q^2r^2/p^3 + Bn^2r^2/q)$.

NOTE.—In a similar way the equation for any multiplane may be written.

Having thus obtained the strut and stay stresses in a multiplane, the spar stresses follow by ordinary statics.

FIG. 27.—Endwise stresses in a biplane wing truss with parallel spars. Graphical treatment.[1]

[1] Read anticlockwise.

FIG. 28.—Endwise stresses in a monoplane wing truss. Graphical treatment.

ANALYSIS FOR HIGH SPEED—75 MILES PER HOUR

AIRPLANE WING ANALYSIS

ANALYSIS FOR LOW SPEED—44 MILES PER HOUR
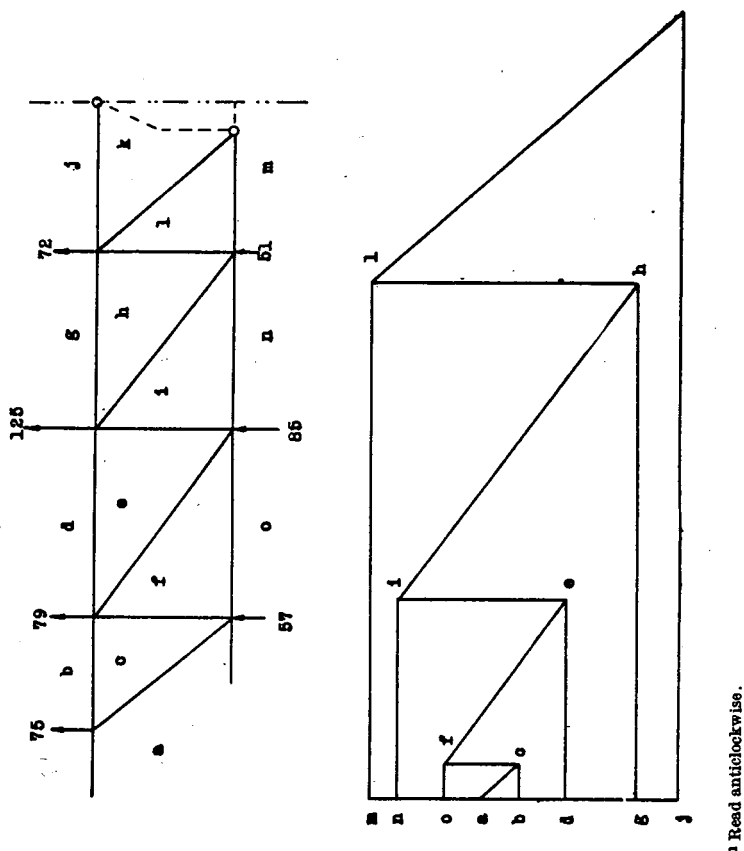
FIG. 28.
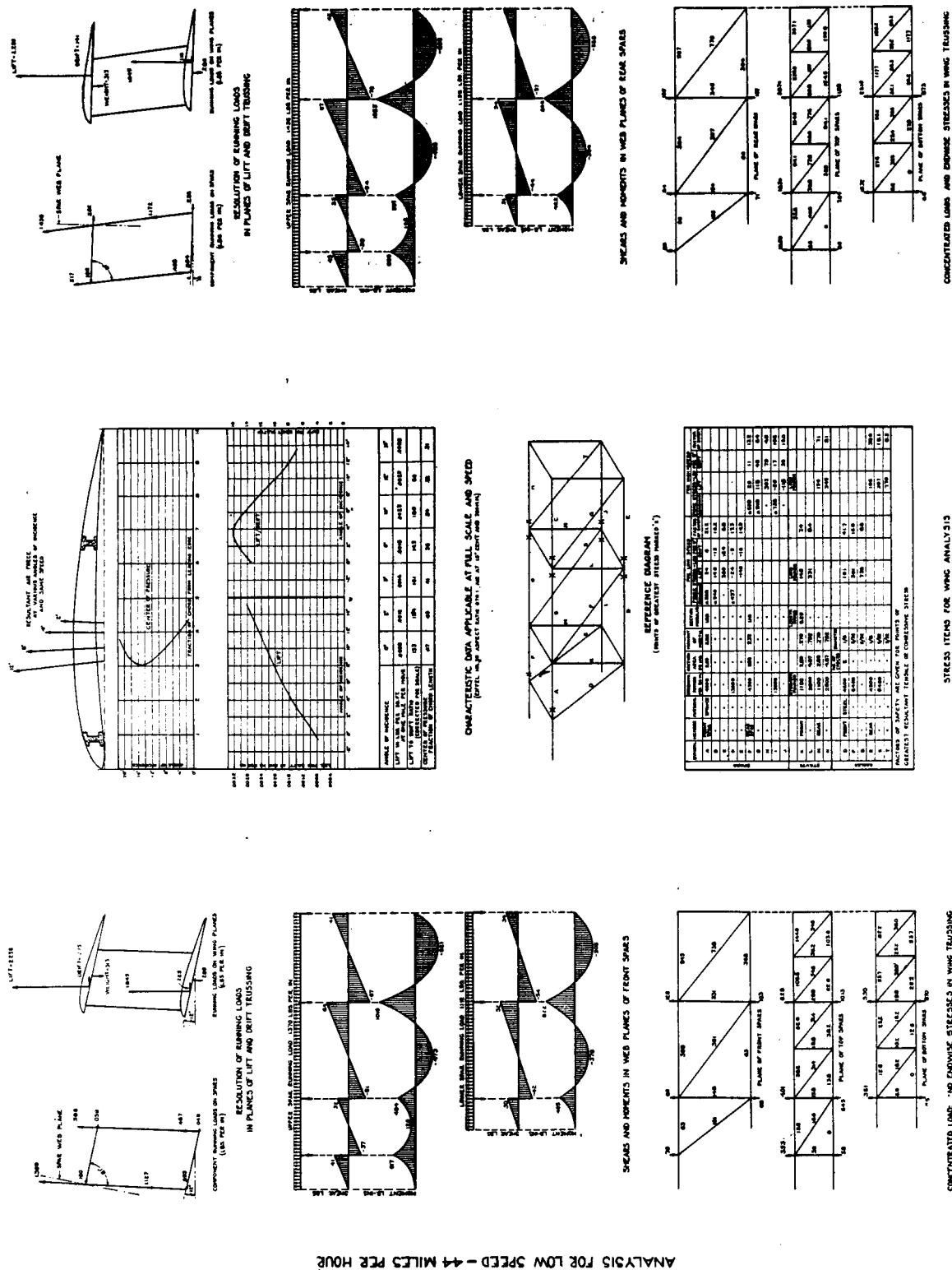
FIG. 29.—Graphical analysis for airplane undercarriage.
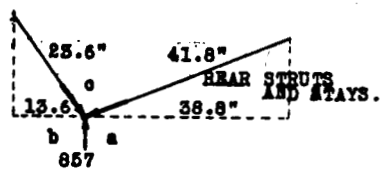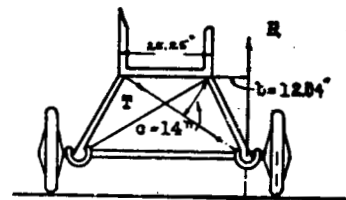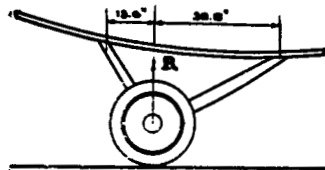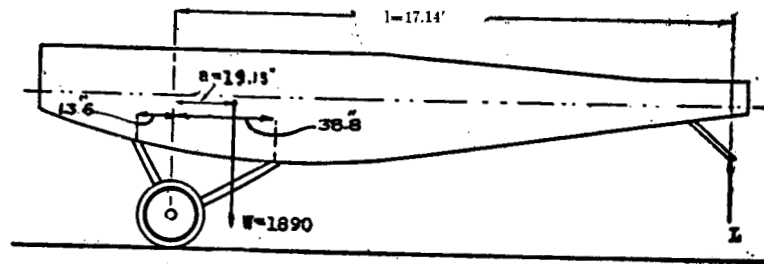
SIDE VIEW OF UNDERCARRIAGE.
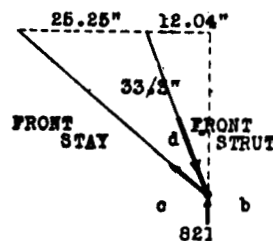
TRUE PLANE OF FRONT TRUSSING
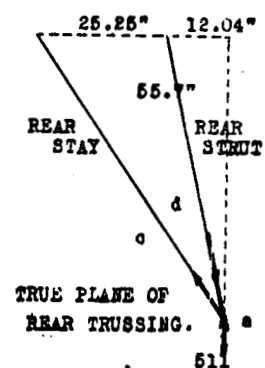
TRUE PLANE OF REAR TRUSSING.
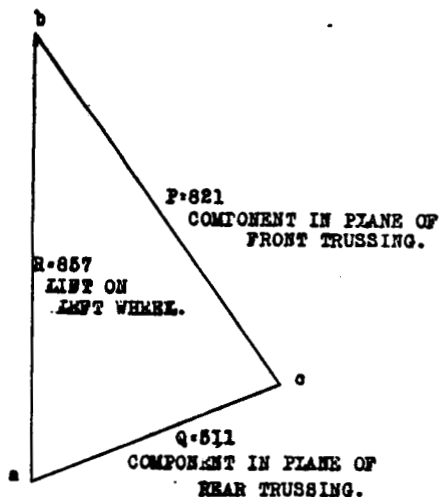
(A) RESOLUTION OF WHEEL LIFT IN PLANE NORMAL TO AXLE.
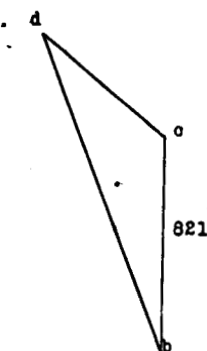
(B) STRESSES IN FRONT TRUSSING.
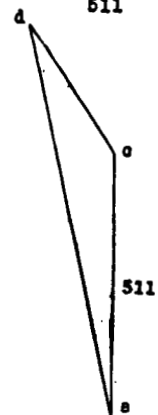
(C) STRESSES IN REAR TRUSSING

FIG. 30.—Graphical analysis for seaplane undercarriage.

(A) RESOLUTION OF LIFT IN VERTICAL PLANE.

(B)—STRESSES IN STAYS[1]

(C)—STRESSES IN STRUTS[1]
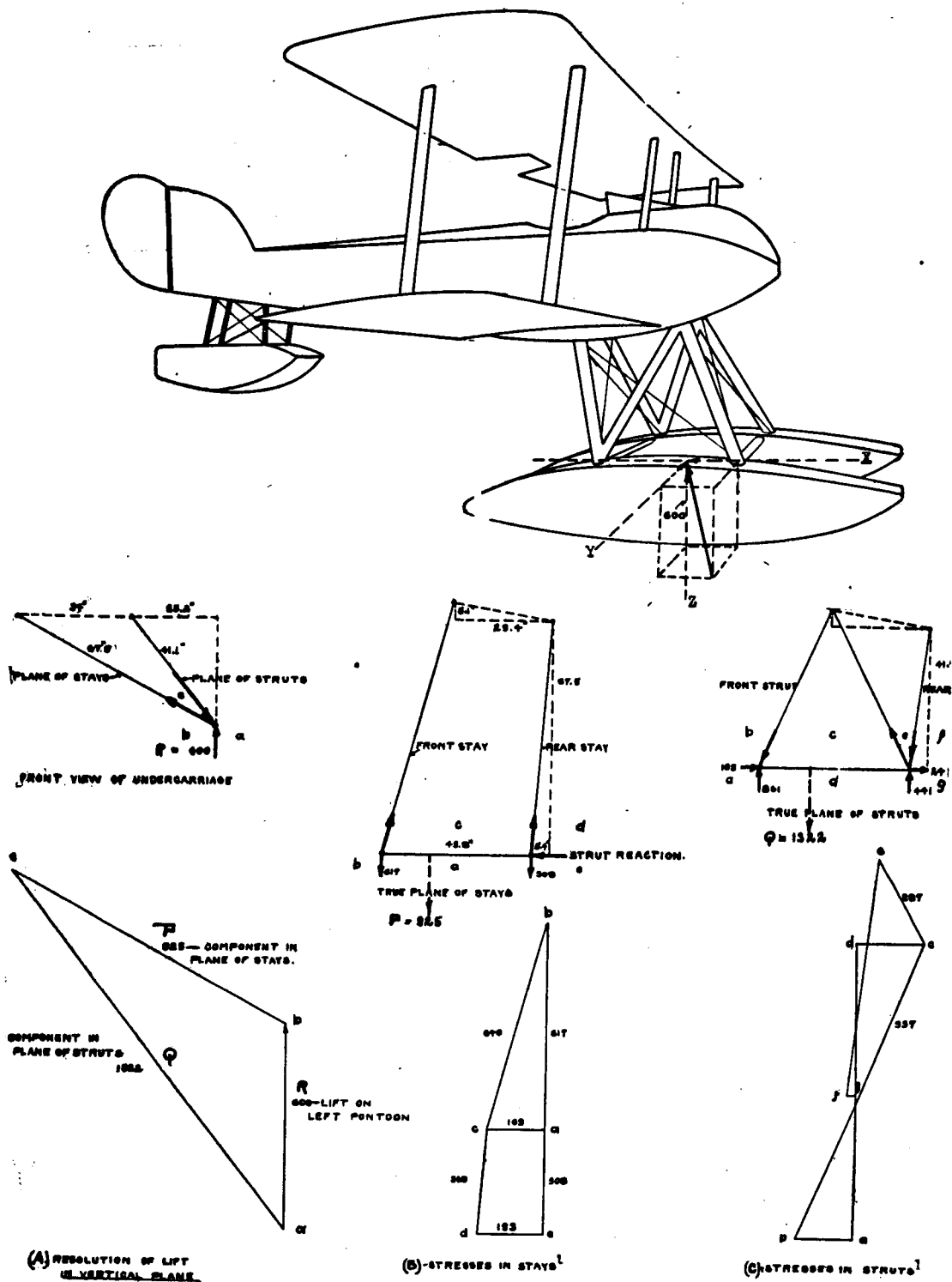
[1] Read anticlockwise.

FIG. 31.—Gravitational plus dynamical load on skid and axle.

FIG. 32.—Endwise stresses in body trussing with struts vertical and loads concentrated. Analytical treatment.

UPPER LONGERON STRESSES.

$$P'''=0$$
$$Q'''=q_3*(G+G')b/cq$$
$$R'''=r_3[(G+G')(b+c)+(H+H')c]/dr$$
$$S'''=s_3[(G+G')(b+c+d)+(H+H')(c+d)+(I+I')d]/es$$

LOWER LONGERON STRESSES.

$$P''=p_2cQ'''/bq_3$$
$$Q''=q_2dR'''/cr_3$$
$$R''=r_2eS'''/ds_3$$
$$S''=s_2fT'''/et_3$$

STAY STRESSES.

$$P'=p_1[(G+G')-P''n_1/p_2-P'''m_1/p_3]/(q-n_1)$$
$$Q'=q_1[(G+G'+H+H')-Q''n_2/q_2-Q'''m_2/q_3]/(r-n_2)$$
$$R'=r_1[(G+G'+H+H'+I+I')-R''n_3/r_2-R'''m_3/r_3]/(s-n_3)$$
$$S'=s_1[(G+G'+H+H'+I+I'+J+J')-S''n_4/s_2-S'''m_4/s_3]/(t-n_4)$$

STRUT STRESSES.

$$P=G$$
$$Q=H+P'(q-n_1)p_1+P'''m_1/p_3-Q'''m_2/q_3$$
$$R=I+Q'(r-n_2)/q_1+Q''m_2/q_3-R'''m_3/r_3$$
$$S=J+R'(s-n_3)/r_1+R''m_3/r_3-S'''m_4/s_3$$

$$M(j_y+gcos\beta)$$

$$P_y=M[+j_x\ddot{x}-(j_x\ddot{y}+gcos\beta\cdot\bar{x})+I_o\alpha]/p$$
$$Q_y=M(j_y+gcos\beta)-P_y$$
$$P_x=aP_y$$
$$Q_x=bQ_y$$

Where M is mass of craft less wheels and axle,

j_x, j_y, are component accelerations of M parallel to X, Y;

I, α, are the angular inertia and acceleration about the axis o;

$\bar{x}$, $\bar{y}$ are the coordinates of craft centroid referred to o;

p is the distance from skid contact to axle.

NOTE.—If j, j' be the simultaneous vertical accelerations at points on the longitudinal axis l units apart, then $\alpha=(j-j')/l$.

*g, r, etc., p, q, r, etc., represent the lengths of the members whose stresses are Q''', R'', etc., P', Q', R', etc., respectively. Interchange m's and n's in Fig. 2.

FIG. 33.—Stresses in twisted pyramidal truss, triangular type.

FIG. 34.—Stresses in twisted pyramidal truss, rectangular type.
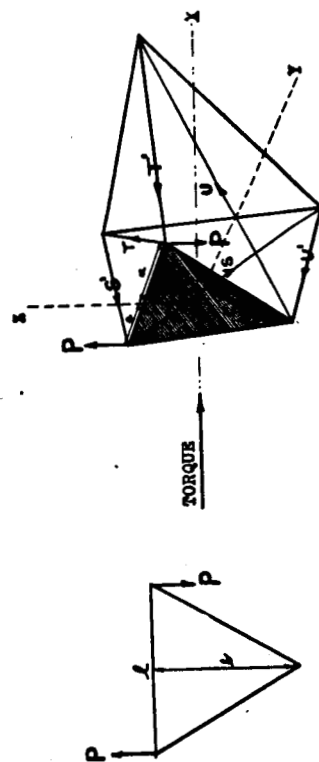
$P = \text{torque}/l$

General case.—Applied couples in plane of truss base. Use notation similar to that of figure 34.

Then $s_s S + s_s' S' + t_s T + t_s' T' + u_s U + u_s' U' = 0$

$s_s S + s_s' S' + t_s T + t_s' T' + u_s U + u_s' U' = 0$

$(s_s S + s_s' S')c + (t_s T + t_s' T')a + (u_s U + u_s' U')b + P(a+c) = 0$

Also $s_x S + s_x' S' = 0$

$t_x T + t_x' T' = 0$

$u_x U + u_x' U' = 0$

Special case 1.—Right prismatic truss.

$s_x = t_x' = u_x' = 1;\ s_y',\ t_y',\ u_y',\ s_s,\ t_s',\ u_s',\ u_x' = 0;\ t_s = 0$

Special case 2.—Pyramidal wedge truss.

If $b=0$, the wire U can be dropped.

NOTE.—When some edges are normal to the bases, some oblique, use the more general equations.

$R = \text{torque}/4l;\ P = Rb/2l;\ Q = Ra/2l$

General case.—Applied couples in plane of truss base.

Denote stay trusses by S, T, U, V.

Denote stay direction cosines by $s_x,\ s_y,\ s_s;\ t_x,\ t_y,\ t_s;\ u_x,\ u_y,\ u_s;\ v_x,\ v_y,\ v_s$.

Denote longeron cosines by the same letters primed.

Then $s_s S + s_s' S' + t_s T + t_s' T' + 2P = 0$. Also $s_x S + s_x' S' = 0$

$t_s T + t_s' T' + u_s U + u_s' U' + 2Q = 0;\qquad t_x T + t_x' T' = 0$

$u_s U + u_s' U' + v_s V + v_s' V' + 2P = 0;\qquad u_x U + u_x' U' = 0$

$v_s V + v_s' V' + s_s S + s_s' S' + 2Q = 0;\qquad v_x V + v_x' V' = 0$

Special case 1.—Right prismatic truss.

$s_x' = t_x' = u_x' = v_x' = 1;\ s_y',\ t_y',\ u_y',\ v_y',\ s_s,\ t_s,\ u_s',\ v_s' = 0;\ s_f,\ u_x,\ v_s = 0$

Special case 2.—Pyramidal wedge truss.
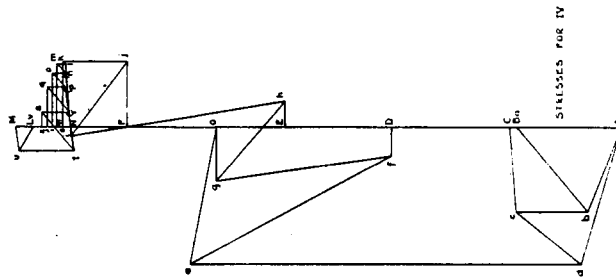
If $a=0$, then $Q=0$, and the wires S, U can be dropped.

NOTE.—When some edges are normal to the bases, some oblique, use the more general equations.

68 c

68 a.

STRESSES FOR V

V FUSELAGE WITH TAIL SKID FORCE ONLY
IV FUSELAGE WITH COMPLETE STATIC LOAD

STRESSES FOR IV

STRESSES FOR III

III UPPER TRUSSING OF FUSELAGE WITH PART OF TRANSVERSE AIR LOAD (V)
DUE TO 20 LBS PER SQ FT ON VERTICAL SURFACES   ONE HALF OF (V)
CARRIED BY UPPER TRUSSING

REAR STRUT
REAR STAY
TRUE PLANE OF REAR TRUSSING

STRESSES IN REAR TRUSSING

FRONT STRUT
FRONT STAY
TRUE PLANE OF FRONT TRUSSING

STRESSES IN FRONT TRUSSING

REAR STRUT AND STAY
SIDE VIEW OF UNDERCARRIAGE

COMPONENT IN PLANE OF FRONT TRUSSING
LIFT ON LEFT WHEEL
COMPONENT IN PLANE OF REAR TRUSSING

RESOLUTION OF WHEEL LIFT IN PLANE NORMAL TO AXLE

ANALYSIS FOR UNDERCARRIAGE

AIRPLANE BODY ANALYSIS
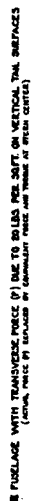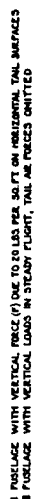
68 B

I FUSELAGE WITH VERTICAL AIR LOAD (V) DUE TO 20 LBS PER SQ FT ON HORIZONTAL TAIL SURFACE
II FUSELAGE WITH VERTICAL GRAVITY LOADS IN STEADY FLIGHT TAIL AIR FORCES OMITTED

STRESSES FOR I

STRESSES FOR II

68 c

FIG. 38.

144410—30. (Face page 68.) No. 1

FIG. 36.

AIRPLANE BODY ANALYSIS

STRESS ITEMS FOR I–FUSELAGE WITH VERTICAL FORCE (P) DUE TO 20 LBS PER SQ FT ON HORIZONTAL TAIL SURFACES

STRESS ITEMS FOR II–FUSELAGE WITH TRANSVERSE FORCE(P) DUE TO 20 LBS PER SQ FT ON VERTICAL TAIL SURFACES

STRESS ITEMS FOR III–FUSELAGE WITH COMPLETE STATIC LOAD

STRESS ITEMS FOR IV–UNDERCARRIAGE WITH STATIC LOADING

STRESS ITEMS FOR II–FUSELAGE WITH VERTICAL LOAD IN STEADY FLIGHT, TAIL AIR FORCES OMITTED

I FUSELAGE WITH VERTICAL FORCE (P) DUE TO 20 LBS PER SQ FT ON HORIZONTAL TAIL SURFACES

II FUSELAGE WITH VERTICAL LOADS IN STEADY FLIGHT, TAIL AIR FORCES OMITTED

III FUSELAGE WITH TRANSVERSE FORCE (P) DUE TO 20 LBS PER SQ FT ON VERTICAL TAIL SURFACES
(ACTUAL FORCE P) REPLACED BY EQUIVALENT FORCE AND TORQUE AT STERN CENTER)

IV FUSELAGE WITH COMPLETE STATIC LOAD

V UNDERCARRIAGE WITH STATIC LOADING

14416—20. (Face page 68.) No. 2

FIG. 37.—Specifications for typical tail unit.

A-11.00 Sq.Ft.   x-15.00°
B-28.70   "   4 - 7.75°
C-12.00   "   s - 7.25°
D - 3.80 "   r-42.75°

HORIZONTAL SURFACES

VERTICAL SURFACES

FIG. 38.—Specifiations for typical aileron and connections.

Area A-20.7 Sq.Ft.   a-11"   b-70"   c-18.75"
d-12.75"   x-18.75"   z-13.37"   r-22.72"   r-61.78

FIG. 39.—Specifications for typical wing rib.

SECTION   HEIGHT h
m 3.56
n 3.60
o 4.10
p 3.10
q 2.90

FIG. 40.—Specifications for typical biplane wing trussing.

LIFT TRUSSING
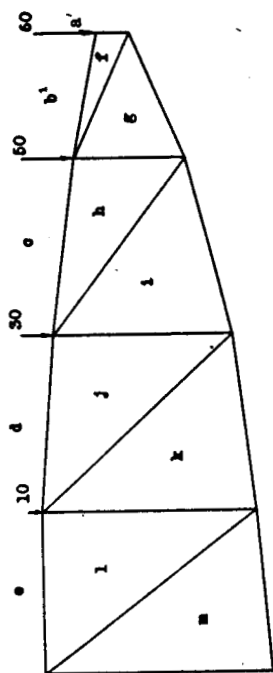
UPPER DRIFT TRUSSING
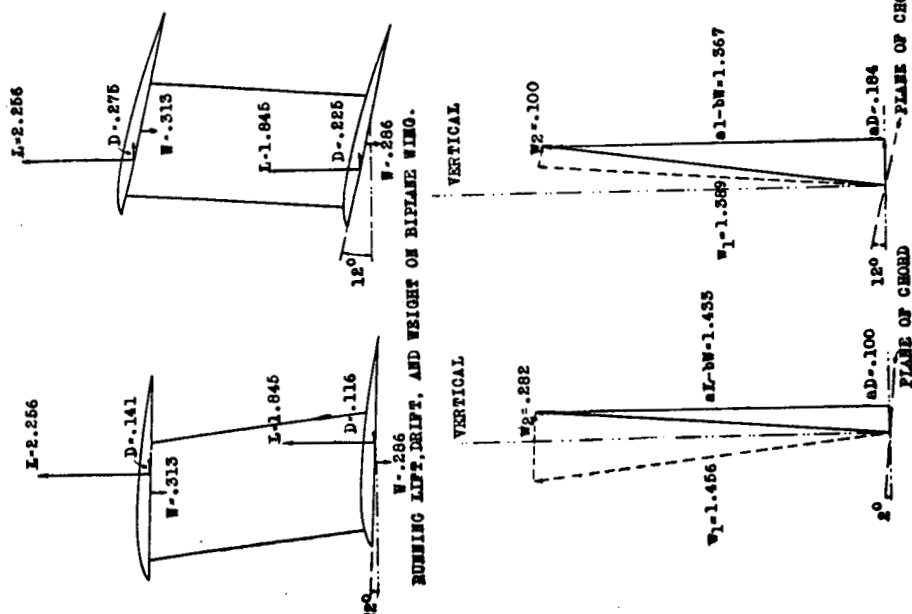
LOWER DRIFT TRUSSING.

FIG. 42.—Endwise stresses in body trussing with struts vertical and loads concentrated. Graphical treatment.

FIG. 41.—Wing loads and spar loads.

RUNNING LIFT, DRIFT, AND WEIGHT ON BIPLANE WING.

1 Read anticlockwise.