REPORT No. 96

STATICAL LONGITUDINAL STABILITY OF AIRPLANES

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This report is essentially a continuation and extension of Report No. 70 of the National Advisory Committee for Aeronautics, entitled "Preliminary Report on Free-Flight Testing," the last part of which was devoted to an elementary discussion of the statical stability characteristics of the JN4H and the DH4. Since the completion of Report No. 70 a large amount of experimental work has been done on the JN4H by the committee's staff at Langley Field, in addition to a little on several other types, and the results are presented here, together with a detailed theoretical analysis of statical stability, of the factors which affect it, and of the methods which can be employed for its modification. Some of the results obtained have been discussed in technical Note No. 1 of the National Advisory Committee for Aeronautics, "Notes on Longitudinal Stability and Balance," portions of which are reprinted in this report.

As in the earlier report, stability will be considered under the two entirely distinct heads of stability with locked controls and stability with free controls. The first depends solely on the control position, and is much simpler to analyze and easier to secure than is the second, which depends on the forces or, more accurately, the moments acting on the movable portion of the control surface.

THEORY OF STABILITY WITH LOCKED CONTROLS.

An airplane which is stable with the elevator locked in position so that it forms in effect a part of the fixed tail-plane will tend to return to its original attitude if the longitudinal equilibrium is disturbed by a change of the angle of attack in either direction. The pitching moment about the center of gravity, which is manifestly zero for the equilibrium condition, will therefore be positive for all angles of attack smaller than the equilibrium angle and negative for all angles in excess of that value. The stability with locked elevators is the only true inherent stability, the airplane acting absolutely as a rigid body with no moving parts.

If a stable airplane is in equilibrium at a given angle of attack and it is desired to change the equilibrium condition to a larger angle a stalling moment must be imposed to balance the negative pitching moment which would arise from any increase of the angle of attack. This stalling moment is secured by pulling up the trailing edge of the elevator, so that the algebraic value of the angle at which it meets the air is decreased, and then locking it in this new position. Similarly, in order that the equilibrium angle may be decreased, the angle at which the elevator is fixed must be increased. The direct criterion by which the degree of stability or instability with locked controls can be judged from free-flight tests is then that the angle at which the elevator is set, relative to some line fixed in the airplane, shall diminish as the equilibrium angle of attack increases and the speed of flight decreases. A curve of elevator angle against speed will therefore have a positive slope for a stable airplane, and the magnitude of the slope of such a curve is at once indicative of the degree of stability.

It has been pointed out that stability under any particular condition is assured if the curve of pitching moments crosses the horizontal axis once and only once, the moments being negative for all angles larger than that of equilibrium, positive for all angles smaller. If the angle of elevator setting be changed, everything else remaining as before, the curve of pitching moments is little changed in form, but is slid vertically, remaining approximately parallel to itself, since a change of elevator setting modifies the angle of attack of the tail as a whole and alters the lift coefficient by very nearly the same amount at all angles. If the curve did not change its form at all and remained exactly parallel to itself throughout it is evident that an airplane stable at all speeds would have a moment curve the slope of which would be negative at all points, since stability demands that the slope be negative where the curve crosses the horizontal axis and the curve can be so shifted, by adjustment of the elevator, as to cross the axis at any point of its length.

The curve of pitching moments would always move parallel to itself, and the criterion just mentioned would be perfect, if the tail as a whole always maintained the same form and if its lift coefficient curve were a straight line, so that a given change of angle would have the same absolute effect on the lift coefficient of the tail, whatever may have been the initial angle of attack. The second condition is very closely observed with all types of tail except when the tail is presented to the relative wind at an abnormally large angle, either positive or negative, and the first holds true when there is no fixed tail-plane, a change in the setting of the elevator therefore meaning a change in the angle of setting of the whole tail. In the much more usual design in which the tail is divided into fixed and movable portions, any change in elevator angle changes the sectional form of the tail, the effective camber becoming deeper as the elevator is turned in either direction away from the prolongation of the chord of the fixed portion. Since the slope of the lift curve is greater for a deeply-cambered tail than for one nearly flat, and since, as will be mathematically demonstrated a little later, the efficiency of the tail in producing stability depends primarily on the slope of the lift curve, it is clear that the stabilizing effect of a tail will be greatest when the angle between the tail-plane and elevator is considerable, or, in other words, when the tail-plane is set at such an angle that the machine is very nose-heavy or tail-heavy and that the elevator has to be held hard up or down in order to maintain equilibrium. While this has a good effect on stability with the controls locked, it makes the airplane very unpleasant to fly with free controls, and also decreases the efficiency of flight and the speed, the drag of the tail being much augmented, for a given lift, by setting the elevator at a considerable angle to the tail-plane. However, even though an airplane may be perfectly balanced under normal conditions, there is but one speed at which the elevator will lie exactly in line with the tail-plane, and at which as a result, the tail as a whole will have the designed section. The angle between the fixed and movable surfaces is greatest at very high and very low speeds, and the airplane is consequently liable to possess, under these extreme conditions, a higher degree of stability then would be prophesied from a wind tunnel test carried out, as such tests practically always are, with the elevator fixed parallel to the tail-plane for all angles of attack. This is especially true on those airplanes which have the gap between the tail-plane and elevator closed in some way. Another factor tending to give greater stability than that shown by model test is that, at extreme angles of attack, the fixed flat tail surfaces employed on wind-tunnel models meet the air at an angle approaching that of maximum lift, and exceeding that at which the lift curve begins to fall away from a straight line. The slope of the lift curve for the tail is therefore less than in steady free flight of the full-sized airplane, as the adjustment of the elevator with changing angle of attack is such, for a stable airplane, that the angle of the relative wind to the line connecting the leading edge of the tail-plane with the trailing edge of the elevator changes less rapidly as the angle of attack is varied than it would if the elevator remained fixed in one position relative to the airplane. Since both of these favorable effects (the effect of the elevator setting on camber of the horizontal tail surface as a whole and its effect on the true angle of attack of that surface) are most marked when the elevator angle varies most with changing speed and angle of attack, or, in other words, when the airplane is most stable with locked controls, it is evident that stability begets stability, and that the stability characteristics at high, low, and intermediate speeds are, to some extent, interdependent. The very act of increasing, by any means whatever, the degree of stability under normal conditions increases the stabilizing efficiency of the tail.

The pitching moment curve for any particular elevator setting can be studied and analyzed mathematically. If it be assumed, as a first approximation, that a negative slope of the pitching moment curve at all points is a sufficient condition of stability, the analysis can be confined to a single elevator setting.

The pitching moment under any conditions is equal to the sum of the moments due to the wings and tail (the effect of the body and chassis is small enough so that it can safely be neglected), and may be written:

$$M = M_1 + M_2$$

Developing each of the components,

$$M_1 = -(x-a) \times L_c \times A_1 \times V^2$$
$$M_2 = -(x'-a) \times L_{c2} \times A_2 \times V^2$$

where x = distance from leading edge of wings to C. P.
a = distance from leading edge of wings to C. G.
L_{c1} and L_{c2} = lift coefficients of wings and tail, respectively.
A_1 and A_2 = areas of wings and tail, respectively.
x' = distance from leading edge of wings to C. P. of tail surfaces.
V = speed of flight.

All distances are expressed, for convenience and uniformity, in terms of fractions of the wing chord.

The total moment is then:

$$M = -[(x-a) \times L_{c1} \times A_1 + (x'-a) \times L_{c2} \times A_2] \times V^2$$

Differentiating,

$$\frac{dM}{d\alpha} = -\left[(x-a) \times A_1 \times \frac{dL_{c1}}{d\alpha} + L_{c1} \times A_1 \times \frac{dx}{d\alpha} + (x'-a) \times A_2 \times \frac{dL_{c2}}{d\alpha} + L_{c2} \times A_2 \times \frac{dx'}{d\alpha}\right] \times V^2$$

The variation of V with regard to α can be neglected, as it would never result in changing the sign of the slope of the curve at its intersection with the horizontal axis.

Since it is the sign of the slope of the moment curve which is of primary interest, the factor V^2 can be disregarded for the present. In order that the airplane may be stable, $\frac{dM}{d\alpha}$ must be negative under all conditions, and the expression inside the brackets in the equation must therefore be positive. With wings and tail of ordinary section, the C. P. moving forward as the angle of attack is increased, the third of the four terms within the brackets is positive, while the second and fourth are negative. The sign of the first term depends on the location of the C. G. It is always positive at very small angles of attack, and some machines have the C. G. far enough forward so that the first term is positive under all conditions. If the lift curve be assumed to be a straight line, so that the value of L_c at any point is equal to the product of $\frac{dL_c}{d\alpha}$ by α (measured from the angle of zero lift) the factors $\frac{dL_{c1}}{d\alpha}$ and A_1 can be taken out of the equation just given for $\frac{dM}{d\alpha}$, and the expression inside the brackets can then be written, with all constant factors ignored:

$$(x-a) + \alpha \times \frac{dx}{d\alpha} + (x'-a) \times \frac{A_2}{A_1} \times \frac{dL_{c2}}{dL_{c1}} + \alpha' \times \frac{A_2}{A_1} \times \frac{dL_{c2}}{dL_{c1}} \times \frac{dx'}{d\alpha} =$$
$$(x-a) + \alpha \frac{dx}{d\alpha} + \frac{A_2}{A_1} \times \frac{dL_{c2}}{dL_{c1}} \times \left[(x'-a) + \alpha' \times \frac{dx'}{d\alpha}\right]$$

where α' is the angle of attack measured from the angle at which the lift of the tail is zero. Lanchester has given a construction[1] for the determination of a sufficient condition of stability, based on this assumption that the lift curve is a straight line and taking into consideration only the first two terms of the expression just given. It is, therefore, the construction for the condition under which the portion of the moment curve due to the wings alone has a negative

[1] The Flying Machine from an Engineering Standpoint, by F. W. Lanchester, N. Y., 1915.

sign, and is decidedly on the safe side. It is obvious that the expression inside the brackets must be positive, since x' is very large, and stability is therefore certain if the sum of the first two terms is positive.

If the tail had the same section, aspect ratio, and general efficiency as the wings $\frac{dL_{c2}}{dL_{c1}}$ would be approximately 0.6, as the rate of change of the angle at which the tail meets the air is diminished by the rate of change of the downwash angle, and this is about 0.4 as great as the rate of change of the angle of attack of the wings. As a matter of fact, however, the efficiency of the tail, as measured by the slope of the curve of lift coefficients, is about half that of the wings, and the value of $\frac{dL_{c2}}{dL_{c1}}$ is more likely to be 0.3 than 0.6.[2]

Since for any equilibrium condition the total pitching moment is zero,

$$(x-a)\times L_{c1}\times A_1 = -(x'-a)\times L_{c2}\times A_2.$$

It can then readily be shown that, still making the assumption that the lift curve is a straight line,

$$\alpha'\times\frac{dL_{c2}}{dL_{c1}}\times\frac{A_2}{A_1} = -\alpha\times\frac{(x-a)}{(x'-a)}$$

Substituting the second of these values for the first in the fourth term of the equation for slope of the moment curve, the variable part of that equation becomes:

$$(x-a)+\alpha\left[\frac{dx}{d\alpha}-\frac{x-a}{x'-a}\times\frac{dx'}{d\alpha}\right]+\frac{A_2}{A_1}\times\frac{dL_{c2}}{dL_{c1}}\times(x'-a) \qquad (1)$$

Examining each term of this equation in turn, it appears that the first term is always positive at small angles, when the center of pressure is far back, and may be either positive or negative at large angles. Its algebraic value can be increased to any desired extent by moving the center of gravity forward. The expression inside the brackets is always negative except in those rare cases where the wing cell itself has a "stable" center of pressure travel. The negative value can be reduced by moving the center of gravity forward, by shortening the fuselage, by using a wing section with a more stable center of pressure travel, or by using a tail surface with a more stable C. P. travel if the C. G. is back of the C. P. of the wings, so that $\frac{x-a}{x'-a}$ is negative. If $\frac{x-a}{x'-a}$ is positive it is disadvantageous to stability to have $\frac{dx'}{d\alpha}$ positive (i. e., to have a stable motion of the C. P. of the tail). Since $\frac{x-a}{x'-a}$ is usually positive at some angles of attack and negative at others it is rather difficult to tell what properties should be sought in a tail-plane to give the best stability. In any case, however, the effect of the second term inside the brackets is small as $\frac{x-a}{x'-a}$ is almost always less than 0.05, and the tail section may be chosen from considerations quite unconnected with stability. Finally, the last term in (1) is always large and positive, and can be increased by increasing the length of the body, the area of the tail, or its efficiency as defined by the slope of the curve of lift coefficients.

The virtues, as a stabilizing agent, of a tail-plane set at a negative angle to the wings have been understood for many years, such a disposition of surfaces having been used by Penaud on his rubber propelled models about 1870. It is not so universally comprehended, however, that the inherent direct advantages of a negative tail-plane setting, are slight, arising only from the greater efficiency of the tail under those conditions, with the elevator held at a considerable angle to the tail-plane to give equilibrium, and that the great merit of such a setting is that it permits the center of gravity to be placed very far forward without throwing the airplane badly out of balance. The really crucial points in connection with stability with locked controls are the position of the center of gravity, the size and efficiency of the tail-plane, and the length of body, and that the first of these is by far the most important.

[2] It has been found at the Royal Aircraft Establishment that the tail efficiency for a B. E. 2E ranges from 0.5 at high speeds to 0.75 at low. (Full scale stability experiments on a B. E. 2E with R. A. F. 15. wing section: R. & M. (New Series) No. 326: 1917.

Lanchester's construction has been carried through for a number of representative aerofoil sections, and it has been found that the C. G. must lie (taking the average result for the several wings, which vary only slightly among themselves) not more than 0.23 of the way back on the mean wing chord if stability is to be secured without any assistance from the tail.

A wing having a "stable" center of pressure travel does not necessarily give an airplane complete stability, as shown by the curve of pitching moments. For example, an airplane with flat plate wings and no tail, and with the C. G. anywhere between the leading edge and the middle of the chord, would be stable at some definite angle of attack, so far as small disturbances and small excursions from that angle were concerned. It would, however, be subject to "catastrophic instability" in the event of large disturbances, the curve of pitching moments cutting the horizontal axis at three points within the range of possible flight angles, two of these points corresponding to stable conditions of flight, the third to an unstable condition. To completely insure against such instability it would be necessary to provide a tail and to move the C. G. forward at least to the leading edge of the wing, farther forward than is required with "unstable" wings of cambered section. In the case of an airplane in which the center of pressure of the wings approaches the leading edge as the angle approaches zero, as in the flat plate, the danger of getting into the inverted equilibrium position is greatest when the C. G. is far forward (but still back of the leading edge), as the angles of attack for normal flight and steady upside-down flight are then very close together. An airplane which flies normally at 8°, for example, and which has another point of equilibrium at $-8°$, is much less likely to be thrown into the inverted position by atmospheric disturbance or by an inadvertence on the part of the pilot than it would be if the angles were $+2°$ and $-2°$. In a certain sense, all airplanes are catastrophically unstable, since the curve of pitching moments, being continuous throughout 360°, must cut the horizontal axis at least twice if it cuts it at all. For a flat plate alone, with the C. G. anywhere along the chord, the curve cuts the axis four times during the complete circle. For a typical cambered wing, the curve cuts the axis twice if the C. G. lies in the first 30 per cent of the chord, four times if it lies between 0.3 and 0.5 of the way back. All wings, both flat and cambered, have a point of stable equilibrium at a small negative angle of attack, and it is the function of the tail to shift this point of equilibrium to an angle of positive lift. The other point of intersection, for a wing with the center of pressure far forward or for a complete airplane, is one of unstable equilibrium, and occurs at an angle of approximately 180°, corresponding to the conditions during a tail-slide. Catastrophic instability need then occasion no difficulty if the C. G. is located far enough forward, as it is easy to secure a moment curve which will have a negative slope at all angles from $-40°$ to $+40°$.

If, in (1) 0.3 be substituted for $\frac{dL_{c2}}{dL_{c1}}$, 0.13 for $\frac{A_2}{A_1}$, and 3.75 for $(x'-a)$, these values corresponding roughly to the average dimensions used at the present time, and if it be assumed that the motion of the C. P. of the tail is in the same direction and half as great as that for the wings, the formula becomes:

$$(x-a)+\alpha\times\frac{dx}{d\alpha}\times\left[1-\frac{x-a}{7.5}\right]+.146. \tag{2}$$

The factor inside the brackets is so nearly equal to 1 that it can safely be disregarded for purposes of approximation.

The mean values of the second term and of x determined by wind tunnel tests for a number of commonly-used wing sections are:

α	2°	3°	4°	5°	6°
$\alpha\times\frac{dx}{d\alpha}$	-0.333	-0.181	-0.142	-0.137	-0.128
x	.53	.46	.43	.40	.38

It should be borne in mind that these figures and the results deduced from them are only illustrative, relative to averages of wind tunnel tests and that they are subject to verification by free flight tests. Substituting these mean values in (2), the value which a must not exceed in order that the moment curve may have a negative slope can at once be found. This value is smallest for the smallest angle, where it is 0.35. In other words, the center of gravity of the airplane must lie not more than 35 per cent of the way back on the mean wing chord if stability is to be secured. If the tail area were decreased 50 per cent, the angle of setting being changed at the same time but the section and plan form remaining fixed, the C. G. would have to be moved forward until a became less than 0.28. If, on the other hand, the tail efficiency, as measured by $\frac{dL_{c2}}{dL_{c1}}$ be increased 50 per cent without change of area (a feat which should not be very difficult to accomplish in some present-day airplanes), the C. G. could be moved back to a point about 40 per cent of the chord from the leading edge without causing the airplane to become unstable. This backward movement of the C. G. decreases the load on the tail surfaces and improves the general efficiency of flight.

SLIP-STREAM EFFECTS.

The analysis so far has proceeded on the assumption that all parts have the same speed relative to the air through which they pass. This assumption is correct in the case of gliding flight, but it is very far from the truth with the throttle open, the mean air-speed at the tail being much higher than that over the wings, since nearly the whole tail lies in the slip-stream on most airplanes. If the slip-stream velocity varied in the same manner and proportion as the speed of the airplane relative to the undisturbed air the higher velocity would operate only to make the tail more effective and so to make the airplane more stable. Unfortunately, however, this condition does not prevail. It has been shown by theory and by experiment [3, 4] that the ratio of slip-stream velocity to air-speed increases as the air-speed decreases, and, in fact, that the speed in the slip-stream is almost independent of the air-speed.

The pitching moment equation with allowance for the slip-stream, if it be assumed that the whole of the tail, but no part of the wings, lie in the slip-stream, is:

$$M=-[L_{c1}\times A_1(x-a)\times V^2+L_{c2}\times A_2\times(x'-a)\times V_s^2]$$

where V_s is the slip-stream velocity. In this case the effect of a change in elevator setting is, as before, to slide the moment curve vertically, but it slides the curve parallel to itself only if the velocity across the tail does not change. It will be recalled that this same condition was laid down in the case where slip-stream effect was ignored, and that the analytical work was accordingly carried through on the assumption that the velocity remained constant. Similarly in the present instance it will be necessary to assume that the slip-stream velocity passing over the tail remains constant while the speed of flight varies. Differentiating M with respect to the angle of attack, treating V as a variable:

$$-\frac{dM}{d\alpha}=\left[\frac{dL_{c1}}{d\alpha}\times(x-a)\times V^2+\frac{dx}{d\alpha}\times L_{c1}\times V^2+2V\frac{dV}{d\alpha}\times L_{c1}\times(x-a)\right]\times A_1$$
$$+\left[\frac{dL_{c2}}{d\alpha}\times(x'-a)+\frac{dx'}{d\alpha}\times L_{c2}\right]\times A_2\times V_s^2$$

Strictly speaking, $\frac{V}{V_s}$, instead of V, should be taken as an independent variable, but, as has already been pointed out, V_s varies so little with airspeed in normal flight with wide-open throttle that it can be considered in an approximate treatment as actually remaining constant.

[3] Preliminary Report on Free-Flight Testing, by E. P. Warner and F. H. Norton: Report No. 70, National Advisory Committee for Aeronautics, Washington, 1920.

[4] Slip-Stream Corrections in Performance Computations, by E. P. Warner: Report No. 71, National Advisory Committee for Aeronautics, Washington, 1920.

Numerous reports of the British Advisory Committee for Aeronautics also deal with this subject.

In order that the airplane may be in equilibrium, the condition

$$(x-a)\times L_{c1}\times A_1\times V^2=-(x'-a)\times L_{c2}\times A_2\times V_s^2$$

must hold true. If, as before, the lift coefficient curves be assumed to be straight lines, so that

$$L_c=\alpha\times\frac{dL_c}{d\alpha},$$

the equation of equilibrium becomes:

$$\frac{dL_{c2}}{d\alpha}\times(x'-a)\times A_2\times\alpha'=\frac{-(x-a)\times A_1\times V^2\times\frac{dL_{c1}}{d\alpha}\times\alpha}{V_s^2}$$

Substituting the expression on the right-hand side of this equation for that on the left in the expression for the slope of the moment curve, so as to eliminate α',

$$-\frac{dM}{d\alpha}=\left[(x-a)\times V^2+\frac{dx}{d\alpha}\times\alpha\times V^2+2V\frac{dV}{d\alpha}\times\alpha\times(x-a)\right]\times A_1\times\frac{dL_{c1}}{d\alpha}$$
$$+\left[\frac{dL_{c2}}{d\alpha}\times(x'-a)\times A_2\times V_s^2\right]-\frac{x-a}{x'-a}\times A_1\times V^2\times\frac{dL_{c1}}{d\alpha}\times\alpha\times\frac{dx'}{d\alpha}$$

Taking A_1, $\frac{dL_{c1}}{d\alpha}$, and V_s^2 out as factors, the condition of statical stability becomes:

$$\left[(x-a)\times\left(\frac{V}{V_s}\right)^2\right]+\left[\frac{dx}{d\alpha}\times\alpha\times\left(\frac{V}{V_s}\right)^2\right]+\left[2\ \frac{V}{V_s^2}\times\frac{dV}{d\alpha}\times\alpha\times(x-a)\right]+$$
$$\left[\frac{dL_{c2}}{dL_{c1}}\times\frac{A_2}{A_1}\times(x'-a)\right]-\left[\frac{x-a}{x'-a}\times\left(\frac{V}{V_s}\right)^2\times\alpha\times\frac{dx'}{d\alpha}\right]>0.$$

If the airplane is in level or approximately level flight (inclination not in excess of 20°), as is usually the case when the throttle is open,

$$W=L_{c1}\times A_1\times V^2=\alpha\times\frac{dL_{c1}}{d\alpha}\times A_1\times V^2$$

neglecting the lift on the tail, where W is the total weight of the machine. Then

$$\frac{W}{\alpha}=\frac{dL_{c1}}{d\alpha}\times A_1\times V^2$$

$$-\frac{W}{\alpha^2}d\alpha=\frac{dL_{c1}}{d\alpha}\times A_1\times 2VdV$$

$$\frac{dV}{d\alpha}=-\frac{W}{\alpha^2\times\frac{dL_{c1}}{d\alpha}\times A_1\times 2V}=-\frac{V}{2\alpha}$$

Substituting this value for $\frac{dV}{d\alpha}$ in the stability equation, the third term exactly cancels the first, and:

$$\alpha\times\left(\frac{V}{V_s}\right)^2\times\left[\frac{dx}{d\alpha}-\frac{x-a}{x'-a}\times\frac{dx'}{d\alpha}\right]+\left[\frac{dL_{c2}}{dL_{c1}}\times\frac{A_2}{A_1}\times(x'-a)\right]$$

must be positive for stability. It will be noted that the term depending directly on the relation between the position of the C. G. and that of the C. P. of the wings does not appear in this equation, and that C. G. position has only a very slight effect on stability. In fact, the stability can only be increased, if the velocity of flow over the tail be constant, by increasing the area or efficiency of the tail surfaces or the length of the body.

It is evident that it is much more difficult to secure stability when the velocity across the tail is constant than when it varies in the same manner as that across the wings. The actual condition always lies somewhere between these two extremes, and the stability is improved as the tail is brought out of the slip-stream in whole or in part, thus approaching more nearly the second limiting condition.

It can be seen from physical reasoning that, if the slip-stream velocity is kept constant while the air-speed varies in such a manner as to keep the lift constant, the stability must be nearly independent of C. G. position, as the form of a curve of moments due to a series of parallel forces is independent of the position of the moment axis if the sum of the upward forces be equal to the weight of the machine at every angle of attack, and a shift of the C. G. therefore changes the wing moment by the same amount for all angles.

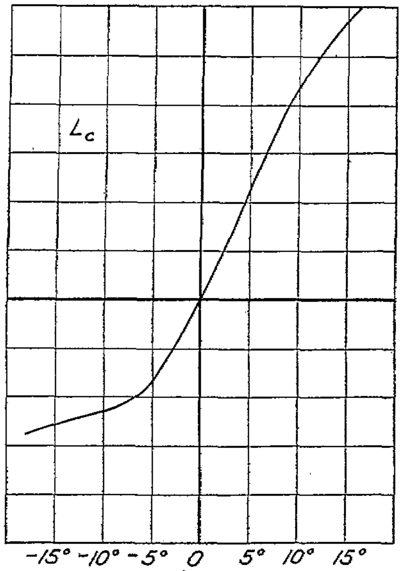

Fig. 1.

It has been shown that it is always advantageous to increase as much as possible the efficiency of the tail surfaces as measured by the slope of the curve of lift coefficients. If a section flat on one side and cambered on the other be tested at both positive and negative angles (measuring angles from the zero lift position and defining their signs on the assumption that the cambered surface is uppermost), it is found that the curve of lift coefficient against angle of attack has the general form shown in figure 1, and that the slope of the curve at the point corresponding to any given positive lift coefficient (except a very small one) is materially greater than that at the point where there is a negative L_c of the same absolute magnitude. The tail-plane should therefore be so set, for best efficiency, as normally to work at a positive angle. Since the C. P. of the wings is behind the C. G. at all times on some airplanes, and at all except very low speeds on all, the load on the tail-plane is normally downward. In order that there may be a downward force while the tail is set at a positive angle of attack, using the term positive angle to denote the condition in which the flat surface of the tail experiences a larger normal pressure (algebraic value) than the cambered surface, the tail must be inverted, with the flat surface on top. It appears from the analysis that this disposition, which has been employed in the Pfalz and numerous other machines, possesses distinct advantages. The increase in stability by inversion of the tail should be greatest at high speeds, as it is at high speeds that the normally placed tail-plane meets the air at a large negative angle where the slope of the lift curve is small, and there is consequently more room for improvement under those conditions than under any others. At intermediate speeds, where the load on the tail is small and where there is little difference in the form and slope of the lift curve for equal positive and negative coefficients, it should make little difference whether the upper or lower surface is the cambered one. The position of the C. G. and the tail area can therefore be modified to change the stability of the airplane at all speeds, while the relation between stability at high speeds and at low can be controlled to some extent by altering the sectional form.

EXPERIMENTS ON STABILITY WITH LOCKED CONTROLS.

The experiments made to verify the above theory and to determine the degree of stability fall into two classes. In the first the airplane was flown under the control of the pilot, the elevator angles were dertermined for several air-speeds and a constant-throttle setting, and the angles thus measured were plotted against air-speed. In the second series of experiments the airplane was actually flown with the elevator locked in several different positions and the nature of its motion was observed. In order that the experiments might not be complicated by any interaction of longitudinal and lateral stability the locking device was so arranged as to permit the stick to be moved from side to side while keeping it in the same fore-and-aft position. The pilot was thus able at all times to make use of the rudder and ailerons to keep the airplane on an even keel laterally. The measurement of elevator angles and the curves based on those measurements admittedly are not very accurate in form, as the constant fluctuations in position due to minute air disturbances are large in comparison with the changes of angle as the speed changes, the stability being very near to neutral in most instances. However, while the accuracy is not great enough to permit of the making of delicate determinations of the point at which the instability appears or of refined analysis of the effect of changes in the airplane, it is still sufficient to show any large variations in stability and to check the analysis approximately.

The variables which were changed in these experiments were:

(*a*) Horizontal position, or X-coordinate, of the C. G.
(*b*) Stagger.
(*c*) Angle of setting of the tail-plane.
(*d*) Sectional form of tail-plane.
(*e*) Vertical position, or Z-coordinate, of the C. G.

A JN4H was used in all these experiments. Assembly drawings of this airplane were given in Report No. 70. The DH4 is the only other type of airplane on which any full-scale experiments on stability with locked controls or on elevator positions have been carried on in America. (*b*) has the same effect as (*a*) in that it moves the C. G. relative to the wings, but changing the stagger also has a direct effect, as it modifies the travel of the center of pressure of the wings. It is necessary to reduce the stagger below normal if the C. G. is to be brought very far forward on the mean chord, as the attachment of enough weight at the nose to move the C. G. to a distance of less than one-third of the chord from the leading edge of the mean chord would bring the C. G. so close behind the axle as to entail serious danger of nosing over. The tail-plane angle was modified by placing blocks under the leading or trailing edge, the fin being cut away at the bottom to provide clearance. The sectional forms tested were three in number. The standard tail was tested both in normal position and inverted, and the third arrangement was a tail of symmetrical section made by attaching convex fairings on the flat lower surfaces of the ribs of the standard tail. Only a single alteration was made in the vertical position of the C. G., a weight being attached to the axle during one test.

The data permitting direct study of the effect of C. G. position without the introduction of any other complicating factors are unfortunately rather sparse. As already noted, the C. G. can not be moved far forward of its normal position without danger of nosing over, and movement to the rear through more than 3 or 4 per cent of the chord length makes the airplane tail-heavy and tiresome, if not dangerous, to fly. Tests with the C. G. position coefficient (the ratio of the distance between the leading edge of the mean chord and a line through the C. G. perpendicular to the thrust line to the chord length) at 0.365 and 0.335, the stagger being 13 inches in both cases, show an improvement of stability with the throttle closed as the C. G. is moved forward. With the throttle open, the difference between the two cases is negligible.

This is strictly in accordance with theoretical deduction. As in the cases detailed in Report No. 70, the airplane is stable at large angles of attack and unstable at all speeds beyond a certain point. Instability with the throttle closed appears at a mean speed of 78 m. p. h. with a C. G. coefficient of 0.365, at 82 m. p. h.when the coefficient is 0.335. This difference, while distinct, is much smaller than might have been predicted. It is probable that one reason for the small apparent effect of the C. G. position is that the tail-plane in this series of tests was blocked up to a negative angle (2.9° to the top longeron), that the elevators had to be pulled down to maintain equilibrium, and that they were pulled down farther when the C. G. was back and the machine was tail-heavy than when it was forward. The combination of tail-plane and elevator then acts roughly as a cambered surface, and the camber is deepest and the effectiveness of the surface in producing lift is highest when the C. G. is farthest back. This increased

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effectiveness of the tail unit partially counterbalances the less stable form of the curve of moments due to the wings. Not even with a C. G. coefficient of 0.29, this being the farthest forward position that was tried, was there complete stability at high speeds. The prediction from the model test that stability at all speeds would be obtained with a C. G. coefficient of 0.35 is thus shown to be incorrect for this machine, and it is evident either that the travel of the C. P. of the wings in free-flight is different from that found from a model test or that the efficiency of the tail is less than was estimated. The latter is very probable, as the aspect ratio of the tail is low and the section thin.

Changes in stagger, like those in C. G. position, had very little direct effect. It is necessary to use a positive stagger of at least 50 per cent of the chord if any improvement in stability is to be secured by modification of the nature of the center of pressure travel, and the maximum stagger used in any of these tests on the JN4H was the normal amount, 27 per cent.

The effect of modification in the tail-plane angle was much larger than had been anticipated. As a concrete instance, three tests made with different tail-plane settings may be compared. With the tail plane set flat on the top longerons, the airplane became unstable at 57 m. p. h. with throttle open and at 62 m. p. h. when gliding. With the tail-plane set at $-1.4°$ to the top longerons the corresponding figures were 67 and 72 m. p. h., and when the angle of setting was increased to $-2.9°$ the critical speed was 75 m. p. h., both with open and with closed thr ttle. It can not be claimed that these speeds are correct to any high order of accuracy, but they are probably good to within a maximum error of 6 m. p. h. and a probable error of 3 m. p. h. The apparent change of stability with change of tail-plane setting is large enough so that, despite the considerable errors which may be present, the general trend of the variation, at least, is fairly certain. It will be seen that the range of speed in which the airplane is stable constantly increases as the rear of the tail-plane is raised. This points to a considerable indirect advantage in moving the C. G. forward, as the elevator angle for zero force on the stick (a condition always to be sought for when in equilibrium at normal speeds, even if stability must be sacrificed to obtain it) probably is nearly independent of the tail-plane setting, and the angle between tail-plane and elevator when properly balanced is therefore greatest when the C. G. is far forward and when the tail-plane has to be set at a large negative angle to keep the nose up. It has already been pointed out that setting the two portions of the surface at a large angle to each other improves the tail efficiency. Study of the results of wind tunnel tests on wings with hinged rear portions set at various angles does not indicate a change in slope of the lift coefficient curve sufficient to account for the magnitude of the effects observed in the present experiments, and it is probable that the direct effect of tail-plane setting would be much less if the tail were of reasonably thick and efficient section than it was in the present case where the section of the tail approximated to that of a flat plate. The effect of tail-plane setting on stability was much less marked in the DH4 than in the JN (see Report No. 70).

The effect of sectional form is to change the relative stability at different speeds, as was predicted from the theoretical analysis. The building up of the tail-plane to a symmetrical form increases the stability at high speeds while decreasing it at low, and the inversion of the tail-plane has the same effect in a still more marked degree. The inversion of the tail-plane raised the speed at which instability appeared by 5 m. p. h., and made the instability much less marked when it did appear, the elevator angle with throttle closed, with 6 inches stagger, and with the tail-plane at $-1°$, varying through a total range of less than $\frac{1}{2}°$ at all speeds from 60 to 91 m. p. h.

The next experiments dealt with the effect of lowering the C. G., the object being to reduce the difference between the balance with throttle open and with throttle closed by bringing the C. G. below the thrust line and so causing the thrust to produce a diving moment counterbalancing the stalling moment due to the action of the slipstream on the controls. The C. G. was lowered about 1¼ inches in the only experiment of this series so far conducted, and no effect was apparent, presumably because the change was not large enough. Calculations of a necessarily very approximate nature indicate that the C. G. would have to be lowered about a foot to bring the curves for all throttle settings into coincidence.

Summarizing the results of these experiments, it may be said that they check extremely well with the theory except that the effect of C. G. position is less, that of tail-plane setting more, than was expected.

As an index of the magnitudes of the total effects of these modifications, the curves for the JN with the standard rigging are plotted in figure 2 together with those for the most stable arrangement tried (6 inches stagger, coefficient of C. G. position 0.290, tail-plane inverted and set with chord at $-1°$ to top longerons).

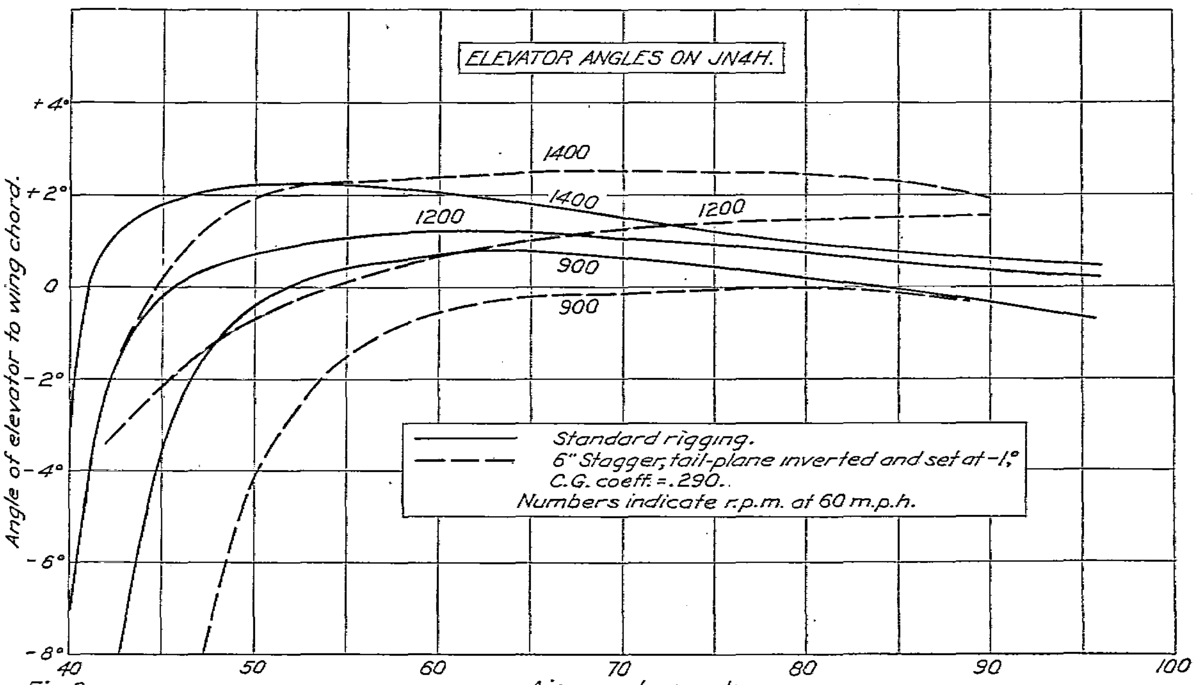

Fig. 2.

EXPERIMENTS ON ACTUAL FLIGHT WITH LOCKED CONTROLS.

To secure the most direct possible check on all of this theoretical and experimental work, a number of attempts to fly the airplane with the elevator controls locked were made. It was found that, as indicated by the angle curves, the standard JN was quite unstable except at very low speeds. When the stagger was decreased to 13 inches and the tail-plane set at $-2°$ to the top longeron the airplane was stable with locked controls throughout the range of normal flight speeds when the throttle was closed and at speeds up to 65 m. p. h. with open throttle. The "peak" of the curve of elevator angles, supposed to represent the point where instability begins, is at approximately 72 m. p. h., but the curve is so flat between this point and 65 m. p. h. that a "bump" or other disturbance is likely to throw the airplane over the "peak," to an angle corresponding with a speed greater than 72 m. p. h. When this happens, the speed continues to increase, and the machine would presumably ultimately go over on its back if the pilot did not resume control. This smallness of the reserve of statical stability for the purpose of resisting atmospheric or other disturbing factors is an objectionable feature of all arrangements in which the stability curve is very nearly horizontal for a considerable distance (as, for example, in the case of the inverted tail-plane, already discussed).

The airplane could be set oscillating, with the controls locked, by quickly opening and closing the throttle. When this was done anywhere within the stable range of speeds the airplane started oscillating with a period of about 20 seconds, the motion being well damped and dying out quickly. In normal weather the flight with locked elevator, aside from such artificially-produced irregularities, is very steady, comparing favorably in that respect with fully-controlled flight by the average pilot.

The conclusions to be drawn from all this experimental work are that the C. G. should be far forward, certainly not over 30 per cent of the mean chord back of the leading edge of that chord, that the tail-plane should be at a negative angle such that the machine balances without any force on the stick at the best climbing speed, that the C. G. should be as low relative to the thrust line as is possible without disarranging the essentials of the design or decreasing its usefulness in any way, and that the tail should be of thicker section than is the common practice at present, with at least a part of the convex camber on the lower surface. It is impossible to be sure until more data on numerous different types of airplanes have become available, but it is not believed that it will ordinarily be advisable to go to the extreme of using a flat upper surface with a convex lower.

STABILITY WITH FREE CONTROLS.

Stability with free controls is much more difficult to treat theoretically than is that with controls fixed, but it is easier to secure accurate experimental data for the first condition than for the second.

As pointed out in Report No. 70, it is impossible to predict accurately the behavior of an airplane with free controls except after an exhaustive series of tests on the pressure distribution over the tail, as the moment about the hinge is governed by the position of the center of pressure of the elevator and the motion of the center of pressure on a surface hinged to the rear of another surface is a very uncertain quantity, especially when the elevator is set at an angle close to that of zero lift, as is usually the case with a properly balanced airplane. An approach to the theory of stability with free controls can best be made by considering separately several simplified cases.

The simplest possible case is that in which, as on the old Grade and Wright model B, the Salmson, and some of the Halberstadts and Moranes, there is no tail-plane, the whole horizontal tail-surface moving as one piece (or, in the Wright and Grade, flexing). The air load on such an elevator must be downward, thus acting with the weight of the elevator, whenever the center of pressure of the wings is behind the C. G. With the C. G. located in accordance with the present practice, the total moment about the hinge is likely to be such as to require a pull on the stick at all times, the airplane not being truly balanced at any speed unless a spring or elastic is attached to the stick to hold it back and reduce the effort required from the pilot. This expedient is employed on the Salmson. Obviously, since the center of pressure moves farther to the rear of the C. G. as the angle of attack decreases, the download carried by the elevator increases as the speed increases. On the other hand, since, as shown in Report No. 70, the criterion of stability with free controls is that the pull on the stick must decrease, and ultimately become a push, as the speed increases, the moment about the hinge must decrease with increasing speed. If the moment is to decrease while the force increases, it is evident that the center of pressure on the elevator must move forward as the speed rises.

At this point in the analysis three cases must be recognized and treated separately, two relating to gliding conditions and one to flight with the throttle open. It has just been seen that $L_{c2}\,A_2 V^2$, where L_{c2} is, as before, the lift coefficient for the tail, increases with increasing speed. The rate of change of the force on the tail, or the amount of change for a given alteration in speed, is independent of the position of the C. G., but the relative change, the ratio of the forces for any given pair of speeds, is governed entirely by that factor. Writing the complete equation for force on the elevator, the symbols having the same significance as in the part of the report dealing with locked controls:

$$L_{c2}\,A_2 V^2 = -L_{c1}\,A_1 V^2 \times \frac{x-a}{x'-a} = -W \times \frac{x-a}{x'-a}$$

$$L_{c2} = -\frac{W}{A_2 V^2} \times \frac{x-a}{x'-a} = L_{c1} \times \frac{A_1}{A_2}\,\frac{x-a}{x'-a}$$

Differentiating this with respect to the angle of attack, neglecting variations in x':

$$\frac{dL_{c2}}{d\alpha} = -\frac{A_1}{A_2 \times (x'-a)} \times \left[\frac{dL_{c1}}{d\alpha} \times (x-a) + \frac{dx}{d\alpha} \times L_{c1}\right]$$

Assuming $L_{c1} = \alpha \times \frac{dL_{c1}}{d\alpha}$, as before:

$$\frac{dL_{c2}}{d\alpha} = -\frac{A_1}{A_2} \times \frac{1}{x'-a} \times \frac{dL_{c1}}{d\alpha} \times \left[(x-a) + \alpha \times \frac{dx}{d\alpha}\right]$$

If the expression inside the brackets is positive, the lift coefficient of the elevator must decrease in absolute value as the angle of attack decreases and the speed of flight increases, while the change is in the reverse direction if the sum of the bracketed terms is negative. It is therefore necessary for stability with free controls that the center of pressure of the tail move forward as the lift coefficient decreases if the C. G. is forward of a certain critical point, the location of which depends on the characteristics of the wing, and must move forward with an increasing L_c if the C. G. is behind that point. In other words, the C. P. travel on the elevator must be "stable" in the first case, "unstable" in the second. In wings of normal form for which the calculation has been made, the farthest forward location of the point just alluded to ranged from 20 per cent to 25 per cent of the way back on the mean chord (assuming the strict applicability of wind tunnel results). At low speeds the point lies about 30 per cent of the chord length from the leading edge. If the C. G. were exactly coincident with the critical point at any instant, the angle between the elevator and the relative wind would not change at all as the speed changed slightly. The center of pressure on the tail therefore could not move, and it would be utterly impossible to secure stability with free controls as shown by the curve of stick forces (see Report No. 70).

There remains to be considered the case of flight with open throttle. This, as for locked controls, will be treated on the extreme assumption of a constant velocity in the slip-stream. Since the load on the tail increases with increasing speed of flight, it is evident that the lift coefficient for the tail must increase if the speed in the slip-stream remains constant. The reasoning is then the same as for the case with throttle closed and C. G. back, and the travel of the center of pressure must be "unstable." If the C. G. is forward of the critical point, then, the requirements for stability with open and with closed throttle are diametrically opposed and absolutely incompatible. In this type of machine (one with no fixed tail-plane) the stability with free controls is actually injuriously affected by moving the C. G. forward beyond a certain point. It will be shown later that other arrangements are not subject to this disadvantage, or at least not in the same degree, and the use of an elevator without a tail-plane is therefore to be avoided if stability is desired, entirely apart from its disadvantages in respect of ease of control, other things being equal.

The next illustrative case to be analyzed is that in which there are a separate tail-plane and elevator working entirely independently of each other, being placed side by side, as in the Bleriot XI bis, instead of in tandem, as is the present practice. In this case the tail-plane carries a down load at high speeds and an up load at low. If the tail-plane be made large enough, and be set at a large enough negative angle, to give statical stability by the locked-control criterion and to balance the airplane at some angle in the normal flying range without any assistance from the elevators (that is, if a wind tunnel test of a model with the elevators removed gives a curve of pitching moments which has a negative slope everywhere and which cuts the axis of zero moment somewhere between 0° and 12°), it is evident that, if the elevator section be assumed to be symmetrical about its center line and if the effect of the elevator's weight be neglected or be assumed balanced by a spring or counterweight, the airplane will fly with no force on the stick at the same angle and speed at which it was found to be in equilibrium with the elevators removed. Furthermore, the maintenance of equilibrium at any higher speed will require that the elevator furnish a diving moment to counteract the stalling moment due to the inherent stability without the elevators, and there will therefore be an upward load on the elevators and a push on the stick. At all speeds lower than the normal trimming speed, similarly, there must be a pull on the stick. It is then certain that the airplane will be completely stable with free controls at the normal trimming speed, but it does not follow that the slope of the stick force curve is everywhere negative, as would be necessary if there were to be stability at all speeds. If the elevator section is not symmetrical, but is of aerofoil form, the elevators will take up a position for which the moment about the hinge is zero. There will then be a downward force if the upper surface of the elevator is convex and the lower one flat or at least less convex, as the lift coefficient for an aerofoil section is always negative when the moment about the leading edge is zero. The elevator will therefore give a stalling moment, and the airplane will fly in equilibrium with free controls at a larger angle of attack than that at which the moment is zero with the elevators removed.

If the weight of the elevators be taken into account it is clear that they will hang down at such an angle that the moment about the hinge due to the air forces is equal to that due to the weight. In general, this means that there will be a small upward force on the elevators

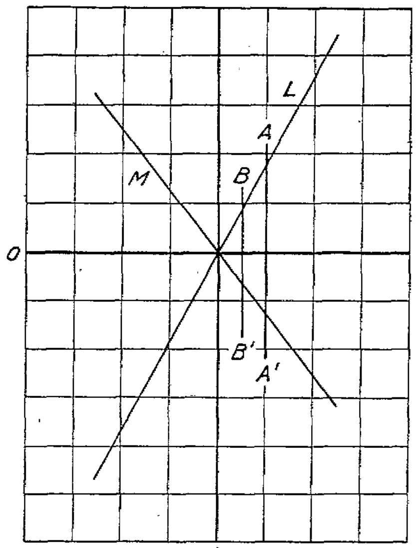

Fig. 3.

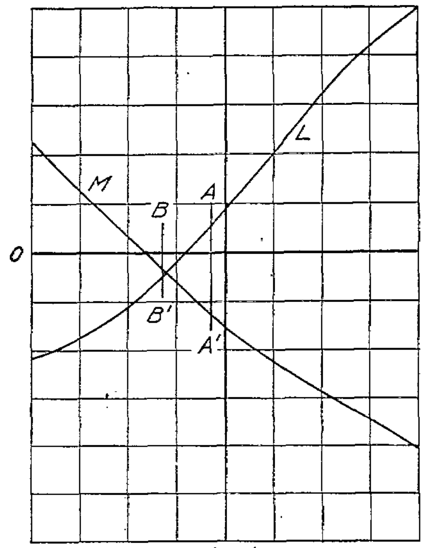

Fig. 4.

and that a diving moment will act on the airplane as a result. The effect of elevator weight on stability and on the form of the stick force curves can best be shown with the aid of a graph. The curves of lift coefficient and of moment coefficient (moment about the leading edge) for a symmetrical section are diagrammatically shown in figure 3, and those for a representative aerofoil section are similarly shown in figure 4. All of these curves are straight lines, to a first order of approximation, in the neighborhood of the zero lift angle. Let it be supposed that on an airplane with an elevator of symmetrical section the elevators hang at the angle represented by the line AA' when flying at the speed V_1, and that, at the higher speed V_2, they would hang freely at the angle indicated by BB' (of course there can actually be only one trimming speed for steady flight with free elevators), the moment coefficients at the two angles being inversely proportional to the squares of the corresponding speeds since the total moment must be constant and equal to the moment of weight. Since the curves of L_c and M_c are both straight lines passing through the origin, the ratio of the two is the same at all angles of attack. Then, since M_c is inversely proportional to V^2 the total lift on the elevators, or product of L_c and V^2, is constant. The lift is the same at BB' as at AA', the diving moments in the two cases are therefore the same, and the analysis carried through for the weightless elevator holds good without change. It is still true that an increase of speed requires a diving moment from the elevator, that this in turn exacts an increase in the up load on the surface, and that an increasing up load means an increasing moment about the hinge and a pull on the stick, so that there is stability with free controls at least at the trimming speed. It is now, however, apparent, since each successive increase of lift entails a further increase of moment about the hinge, that the curve of stick forces has a positive slope and a stable form throughout the whole range of flight speeds.

Passing now to the case of the aerofoil section, where the curves of L_c and M_c no longer intersect at the origin, it is evident that it is no longer true that total lift and total moment are in a fixed ratio to each other. In passing from AA' to BB' in figure 4, choosing BB' so that $M_c V^2$ will be constant, L_c actually changes sign. For an aerofoil section right side up, as the speed increases and M_c decreases the algebraic value of $\frac{L}{M}$ grows less and the diving moment due to the free-hanging elevator decreases and becomes at very high speeds a stalling moment. This is desirable from the standpoint of stability, as the free elevator works with the tail-plane to return the machine to its original attitude, and the force which must be exerted on the stick to fly the airplane at any speed other than its trimming speed is thereby increased. If the elevators were flat above and cambered below the condition would be reversed, and the lift for constant hinge moment would change in a manner disadvantageous for stability.

In the case of an airplane where the elevator is hinged to the rear of a tail-plane it is only possible to reason by analogy from the simpler type of tail surface just discussed. The relationship between the moment and lift coefficients is now dependent in a rather indeterminate manner on the angle of attack. In general, however, it is sufficient for stability that the curve of pitching moments should have a negative slope at all points when tested without the elevators and that a curve of coefficients of moment about the elevator hinge plotted against lift coefficient should have a negative slope at all points for all angles of the tail-plane. If the first of these conditions is observed both with throttle open and with throttle closed, the airplane will be stable with free controls under both of these conditions of operation. These specifications are not absolutely rigorous, as the force on the tail-plane and its effect on stability are somewhat affected by the presence of the elevator, especially if the elevator is a heavy one. The efficiency of the tail is, as already noted, greatest when the elevator is set at a considerable angle to the tail-plane. A heavy elevator, which hangs down below the line of the relative wind and which requires that the tail-plane be set at a larger negative angle to maintain equilibrium at any given speed than would be necessary with a lighter control member, offers some advantage in this respect. Other things being equal, and neglecting the direct effect of C. G. position on stability, a tail-heavy airplane would have a more "stable" curve of stick forces than would one properly balanced, as the elevator has to be pulled down to preserve equilibrium on the tail-heavy machine and this increases the efficiency of the tail-plane. To secure a true measure of the effect of a change in C. G. position the tail-plane should be adjusted, after the change, to such an angle that the airplane will trim with free controls at the same speed as before, and it should be found, if this is done, that there nearly always is an improvement of stability by moving the C. G. forward, the exceptions being machines with very small tail-planes.

Another reason, in addition to that just mentioned, for the increasingly stable form of the stick force curve as the negative angle of tail-plane setting is increased, is that a given change of setting means a change of lift coefficient for the tail-plane which is approximately the same for all angles of attack. The total stalling moment due to the change is then proportional to the square of the speed, and the additional upward force on the elevator and decrease (algebraic) of stick force necessary to produce a diving moment to balance this stalling moment is accordingly greater at high speed than at low. The effect therefore is to move the stick force curve downward, as a whole, but also to tilt it so that negative slopes are increased, positive slopes decreased. This phenomenon was discussed in Report No. 70, already referred to on several occasions, in connection with the testing of a DH-4 with several different settings of the adjustable tail-plane. The condition connecting L_c and M_c for the elevator is observed on the BE-2A[5], BE-2C[6], and the JN-2[7], the only machines for which hinge moment tests are available.

[5] Report of British Advisory Committee for Aeronautics, 1912-13, Rep. No. 74, p. 123.
[6] Full Scale Experiment on the Moment about the Hinge of the Air Forces on an Elevator; British Advisory Committee for Aeronautics, R. & M. No. 284, 1916.
[7] Bulletin of the Airplane Engineering Department, U. S. A.: Dec., 1918.

In fact, any elevator which did not have a lift-moment curve with a negative slope at all points would be overbalanced. The chief deduction to be drawn from this analysis is that models should be tested for stability in the wind tunnel with the elevators removed, and that, if stability with free controls is desired, the tail-plane should be large enough so that the curve of pitching moments from such a test will have a negative slope at all points, both with and without the slipstream effect. This points directly to the advantage to be gained by the use of a large tail-plane and small elevators. If possible, the tail-plane and elevator should be of such sectional form that there is a downward force on the elevator when the moment about the hinge is zero.

Heavy elevators are to be avoided for several reasons, chief among which is the effect of accelerations on the stick force required. To give a concrete instance, the pull on the stick required to balance the weight of the elevators in a JN is 8½ pounds. In pulling out of a loop with an acceleration of 3g, the stick force would be 25 pounds, even if there were no air load at all on the elevator. In the VE–7 the pull under the same conditions would be only about 8 pounds. A heavy elevator increases both the natural period of oscillation of the airplane with free controls and the damping of the motion, as the accelerations of the elevator turn it down during the lower part of an oscillation, up during the upper part, always moving so as to oppose the existing pitching motion of the airplane.

Before passing on to the discussion of experiments on stick forces something should be said with regard to balanced controls. Overbalance may be defined as the condition in which the curve of coefficient of hinge moment against lift coefficient for the elevator has a positive slope at some points for some tail-plane settings, and it is quite possible that some types of elevator may be overbalanced when hinged at the leading edge, although such a state of affairs would be rare. If an elevator is much overbalanced the airplane is usually unpleasant, although not necessarily dangerous, to fly. Curiously enough, the best stability with free controls if the elevator hinges are too far back is obtained if the airplane is extremely deficient in stability with locked controls. If the machine is statically unstable when tested without the elevators there must be an upward force on those members at speeds below, a downward force at those above, the equilibrium speed. The elevator being overbalanced at all speeds, this gives a push on the stick at high speeds and a pull at low. Actual flight with free controls would hardly be possible, however, as the stick has no equilibrium position when the controls are overbalanced, but moves quickly to one or the other of its extreme limits of travel as soon as released. Flight with free controls would be possible only if the elevators were fitted with stops confining their oscillations between very narrow limits.

Intentional and extreme overbalancing forms the basis of the "automatic rudders" invented by Col. Crocco. The "automatic rudder" consists of a tail plane hinged at the rear and with the leading edge free to move vertically but restrained by springs. If the aircraft noses down the top load on the tail plane is increased and the leading edge moves downward, still further increasing the downward force on the tail plane and the righting moment derived therefrom. The efficiency of the tail plane as a stabilizing factor can be trebled or quadrupled in this way. This device has been successfully employed on some Italian airships, and it theoretically is equally applicable to airplanes, but it would probably be rendered unsatisfactory in service by excessive vibration of the tail plane and because of the relatively short natural periods of oscillation of an airplane. Overbalanced surfaces of any sort should in general be avoided at all costs.

EXPERIMENTS ON STABILITY WITH FREE CONTROLS.

The methods of conducting these experiments were explained in report No. 70. The results obtained with free controls are relatively more accurate than those with locked controls, largely because no communication between pilot and observer is required and because the personal equation of only one individual enters into the result. Also, more data have been obtained with free controls because stick-force measurements can be made on any machine without making the slightest change or installing any special equipment, and such measurements were therefore made on several airplanes on which there was no opportunity to install an angle indicator. In all the tests on the JN the machine was piloted by Mr. R. G. Miller.

The tests made on the JN4H included a series dealing systematically with the effects of C. G. position and tail-plane setting. The curves for three different C. G. positions and a tail-plane angle of $-2.4°$ to the top longerons are plotted, both for open and for closed throttle, in figure 5. It will be observed that, as prophesied from the theory, the machine is most stable with the C. G. back when the throttle is open. The position of the C. G. with the throttle closed seems to have very little effect, much less than would be expected.

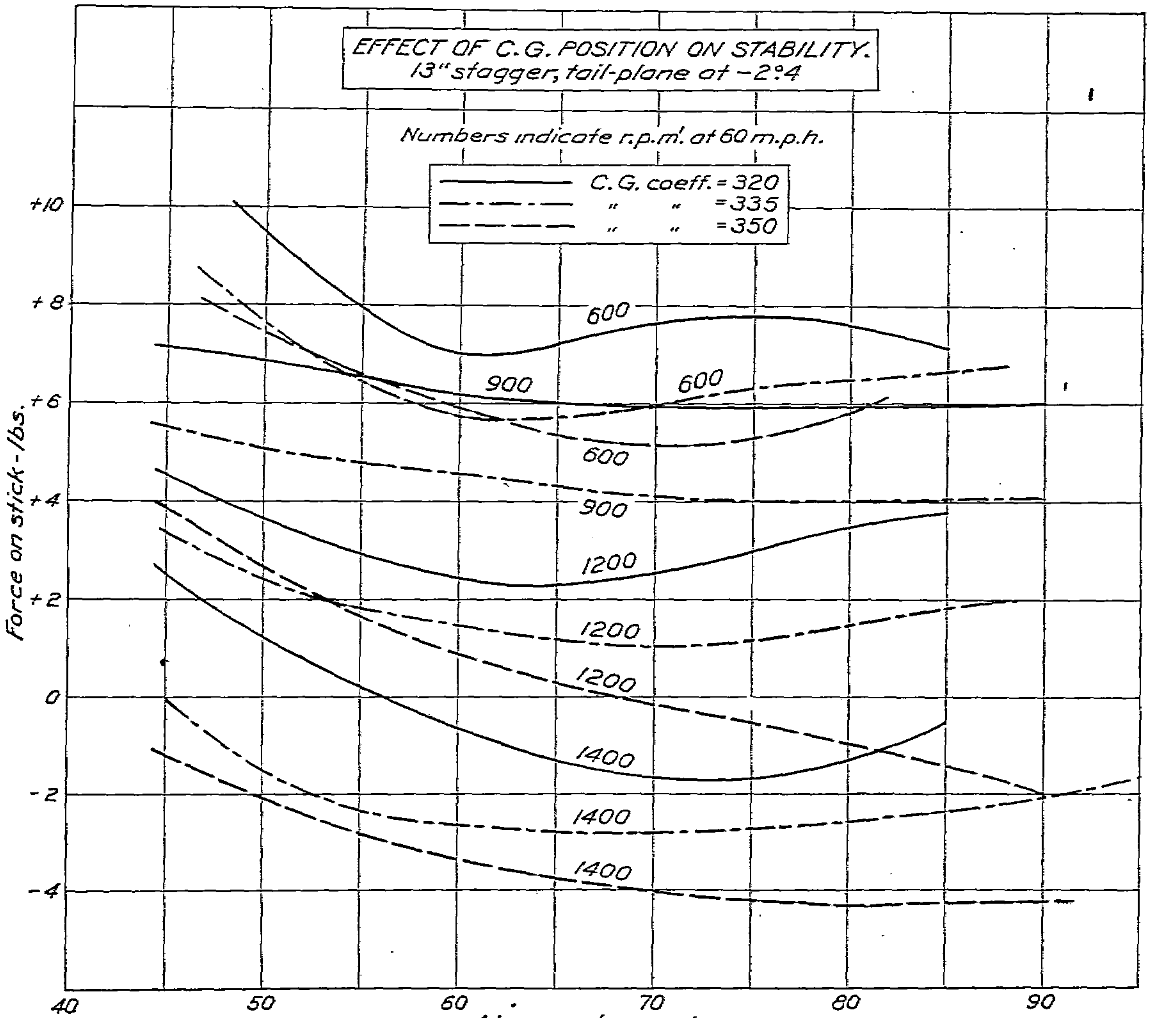

Fig. 5.

The effect of tail-plane setting is shown in figure 6, where the curves for three different settings with a constant C. G. position are given. The curves show that the stability is much better with the tail plane at $-2.4°$ to the top longerons than with it set at $-1.4°$, and that a further increase of angle of setting to $-2.9°$ produced still further improvement when gliding, but had comparatively little effect when the throttle was more than half opened. Apparently the most efficient camber for the tail as a whole is nearly if not quite reached when the tail plane is set at $-2.4°$ and the elevators are pulled down enough to balance the machine with the C. G. 35 per cent of the way back on the mean chord.

The next group of tests dealt, as in the case of locked controls, with the effect of sectional form of the tail. It is rather difficult entirely to separate the effect of sectional form from such complicating factors as angle of setting. It is obvious that data for tails of different types can not be made directly comparable by simply setting the tail planes in all cases with their chords at the same angle to the wings. The best means of obtaining a comparison appears to be to set the several tails at such angles that the force on the stick at economical speed will be the same in all cases. This has been done approximately for the standard, the inverted, and the symmetrical tails in figure 7. The curves show, as was deduced from theory, that the cambering of the lower surface of the tail increases the stability at high speed while decreasing that at low. The range of stability is not increased, but the curve is flattened.

Experiments on the effect of the vertical coordinate of the C. G. were not carried far enough to be conclusive. Theoretically, lowering the C. G. relative to the thrust line should decrease the effect of opening the throttle and should increase the stability, since the thrust is largest at low speeds and a lowering of the C. G. produces the largest additional diving moment and re-

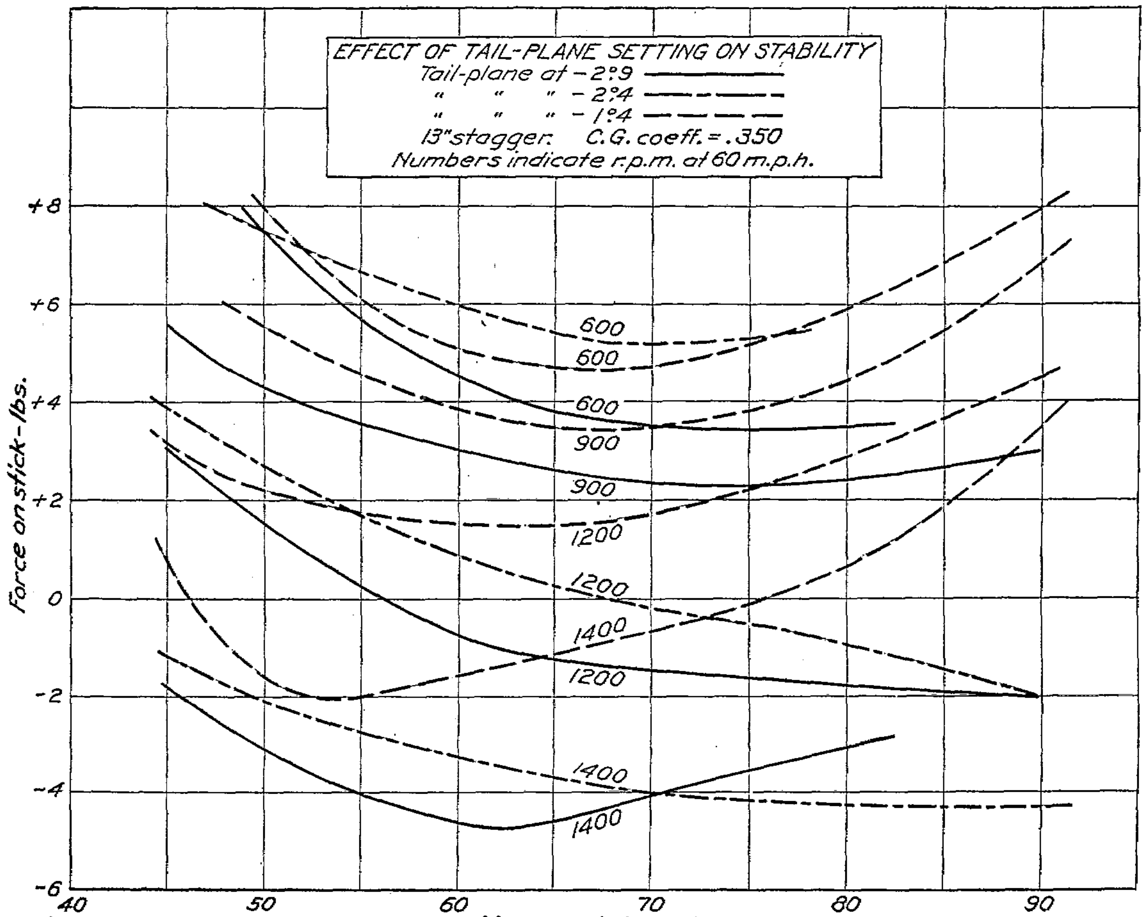

Fig. 6.

quires the largest additional pull on the stick under those conditions. Actually, however, neither of these effects appeared when the C. G. was lowered about an inch by the attachment of 50 pounds of lead to the axle. The propeller thrust on a JN4H at 60 m. p. h. is 470 pounds (calculated from a wind-tunnel test of the propeller). A lowering of the C. G. by 1 foot then produces a diving moment of 470 pounds-feet. Then, assuming the center of pressure of the tail to be distant 18 feet from the C. G., the down load on the tail is increased by 26 pounds. Part of this additional load comes on the tail-plane, as the pulling up of the elevator "banks up the air" on the tail-plane and increases its lift coefficient. Assuming that the additional force is equally distributed between the fixed and movable portions of the surface, and that the center of pressure of the elevator alone lies at 32 per cent of its chord behind its leading edge, this being the value determined in wind-tunnel tests on the pressure distribution on a JN tail,[8] the change in moment about the elevator hinge due to lowering the C. G. by 1 foot would be 139

[8] Bulletin of the Airplane Engineering Department, U. S. A., December, 1918, p. 38.

pounds-inches, corresponding to a change in stick force of 5.7 pounds. Since the mean separation between the curves for open and closed throttle at 60 m. p. h. is 8.1 pounds it would theoretically be necessary to lower the C. G. by 1.4 feet in order to bring the curves to coincidence. This would manifestly be impossible without a complete change in the type of the airplane.

Another possible method, suggested by Mr. F. H. Norton, for reducing slip-stream effect on the controls is to tip the engine down at the front so that the slip-stream makes a smaller angle with the tail-plane than does the relative wind when there is no slip-stream. This was tried out by placing tapered blocks between the engine and its bearers so as to incline the thrust line at 2° to the top longerons. Some improvement resulted from this change, but the gain

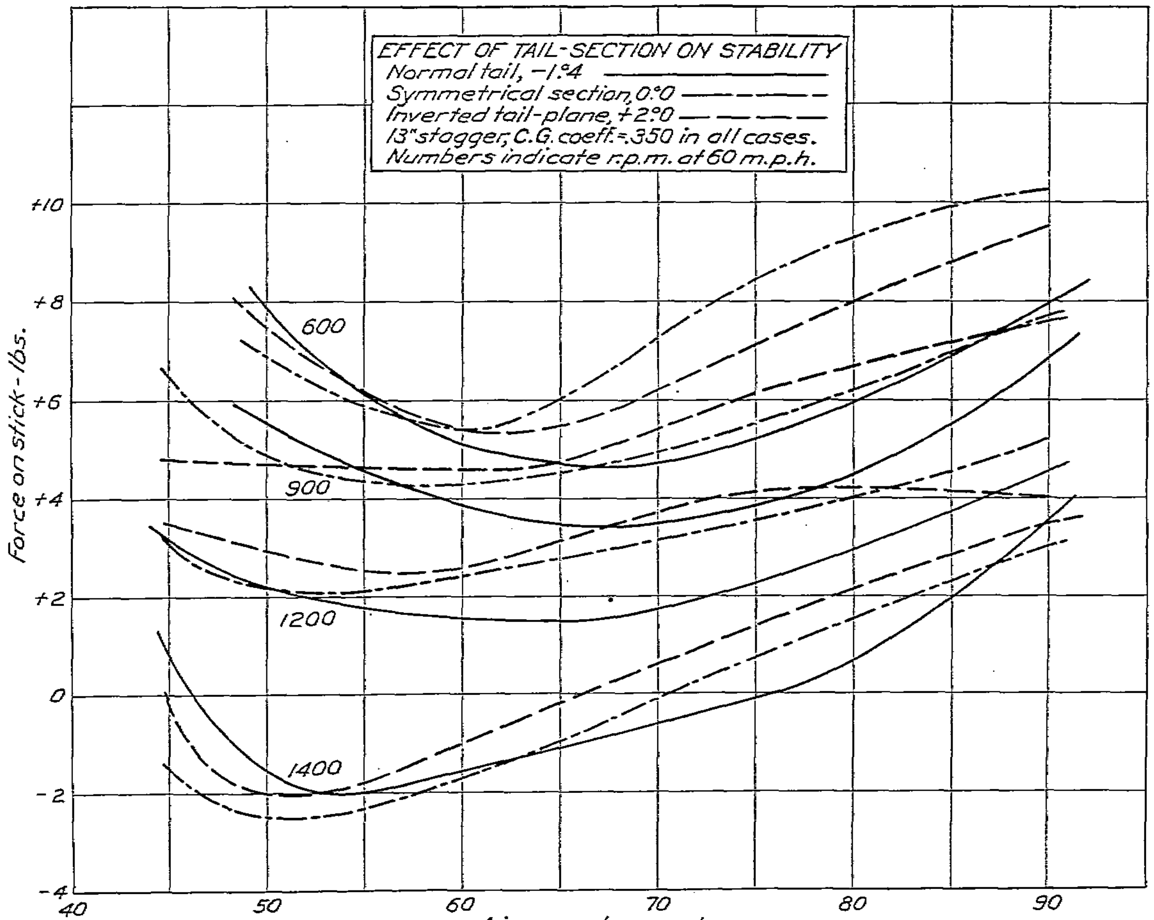

Fig. 7.

was not marked enough to justify the recommendation of such an inclination of the engine as a regular feature of design. An inclination large enough to be of much use in neutralizing the slip-stream effect on the controls would be distinctly detrimental to efficiency at maximum speed. The best way that has yet appeared to reduce slip-stream effect on a single-engined machine is to use a tail of large aspect ratio so that a considerable portion of it will lie outside of the slip-stream.

In closing the treatment of the experiments on the JN, as an indication of the net improvement of stability which has resulted from all this work, there are plotted in figure 8 the curves for the standard JN and for the best arrangement finally arrived at (6 inches stagger, tail-plane inverted and at −2° to the top longerons). It will be observed that there is a great improvement in stability, especially at high speeds, and that the danger of the stick force in a dive increasing to a point where it would be impossible to pull the machine out has entirely disappeared.

TESTS ON OTHER AIRPLANES.

In addition to the DH4, the results for which were discussed in report No. 70, stick-force determinations have been made, through the courtesy of the Airplane Engineering Department at McCook Field and particularly of Col. T. H. Bane and Lieut. Col. V. E. Clark, on the VE7 (Vought), U. S. A. C11 (Lepere Biplane), and Martin Transport. The assembly drawings of these three airplanes are reproduced in figures 9, 10, and 11. All three of these airplanes were flown during the tests by Lieut. H. R. Harris. The stick-force curves for the three machines

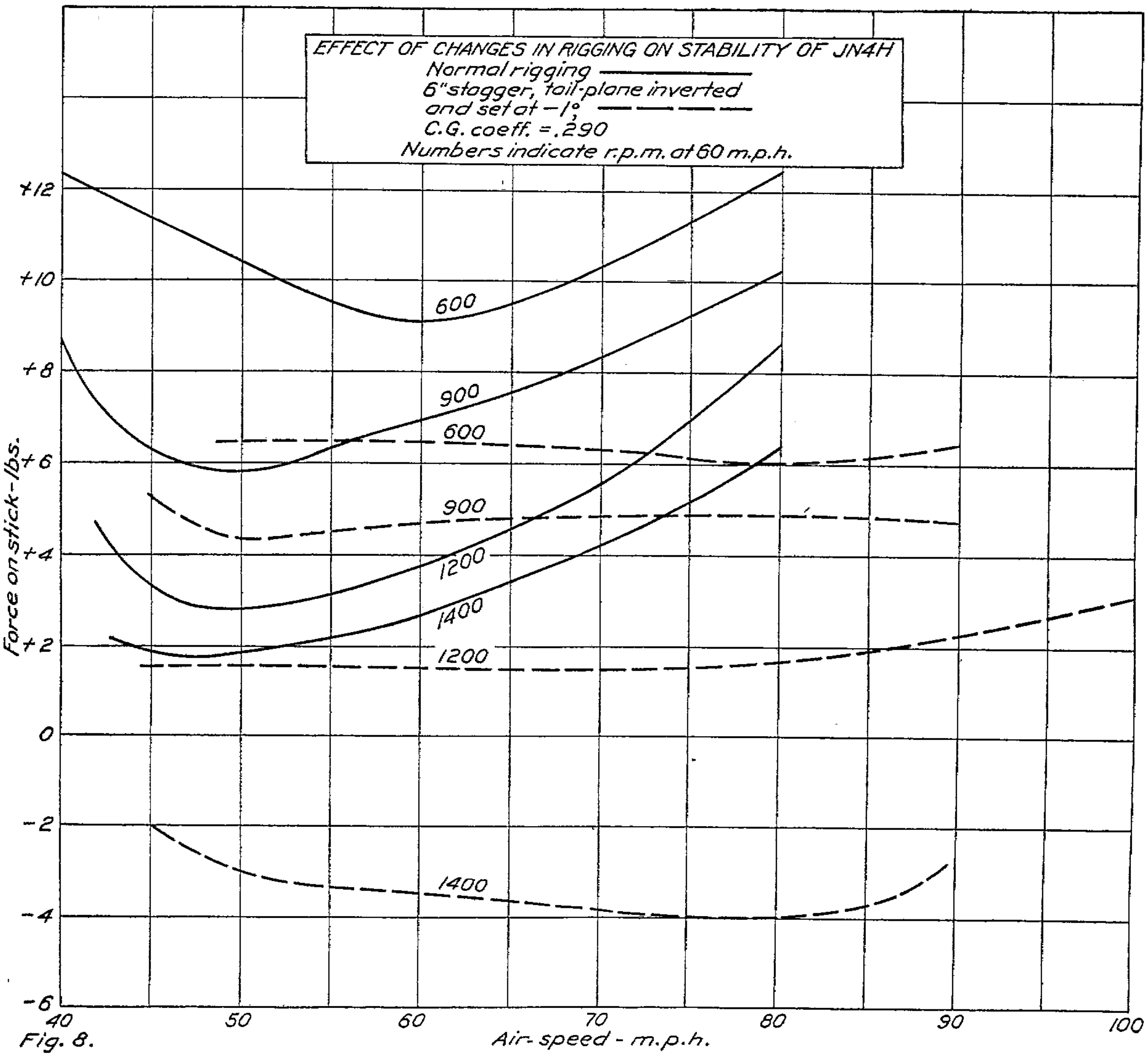

Fig. 8.

are given in figures 12, 13, and 14. The curves for the Martin must be regarded with some suspicion, as the friction in the control system (of the column type) was so great as to make it impossible to be sure of the forces within 2 or 3 pounds.

The stability of the VE7 is virtually ideal. This machine had the C. G. 30 per cent of the way back on the mean chord and one-half inch below the thrust line. The tail-plane is convex on both surfaces, the upper camber being about twice the lower. Comparisons between different machines show a remarkable divergency in the location of the point of maximum stability, and that location seems to be largely controlled by the section of the tail. The DH4 and Lepere have tails of virtually symmetrical section and are much more stable at high speeds than at low. The JN has all its camber on the upper surface and is much more stable at low speeds than at

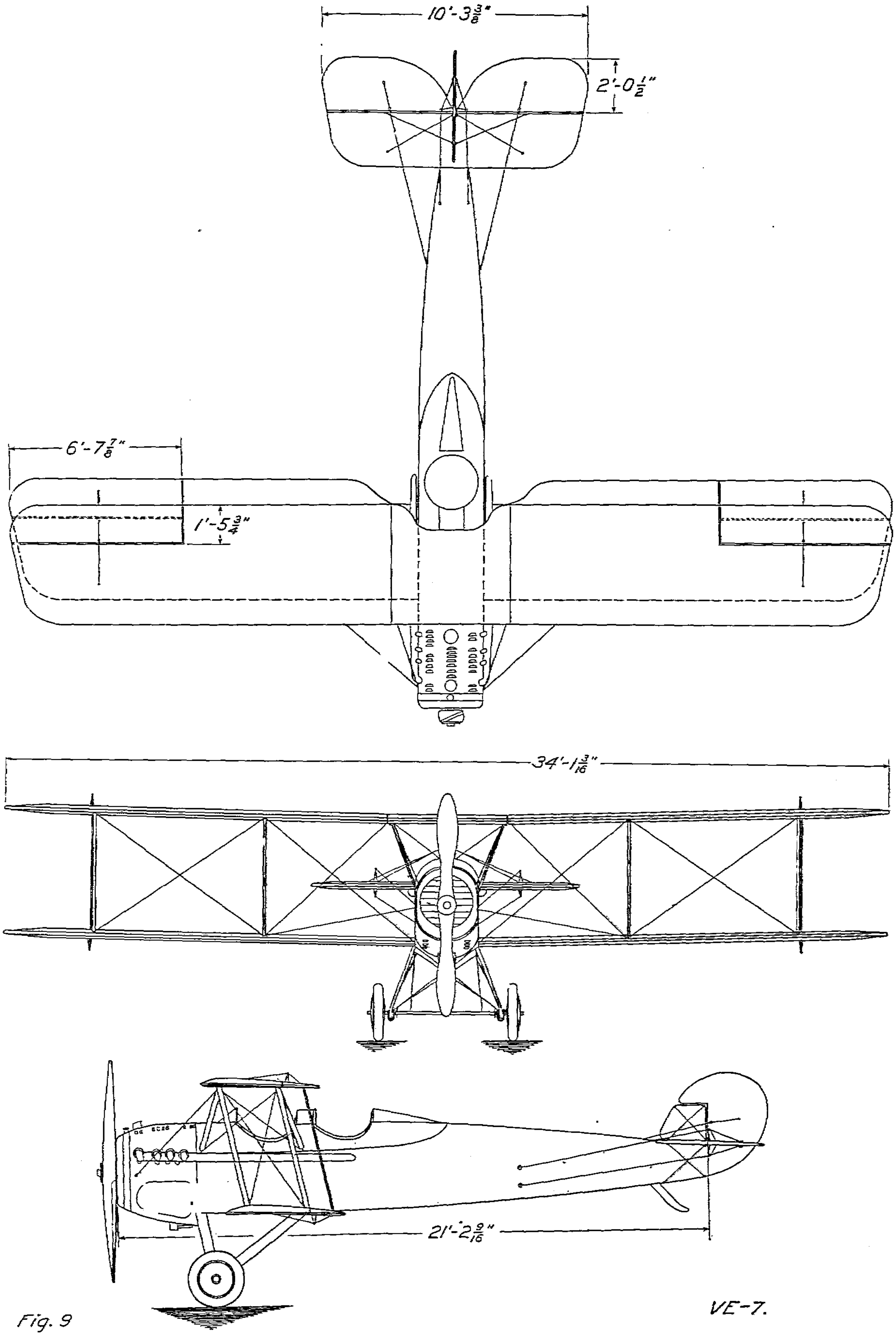

Fig. 9

VE-7.

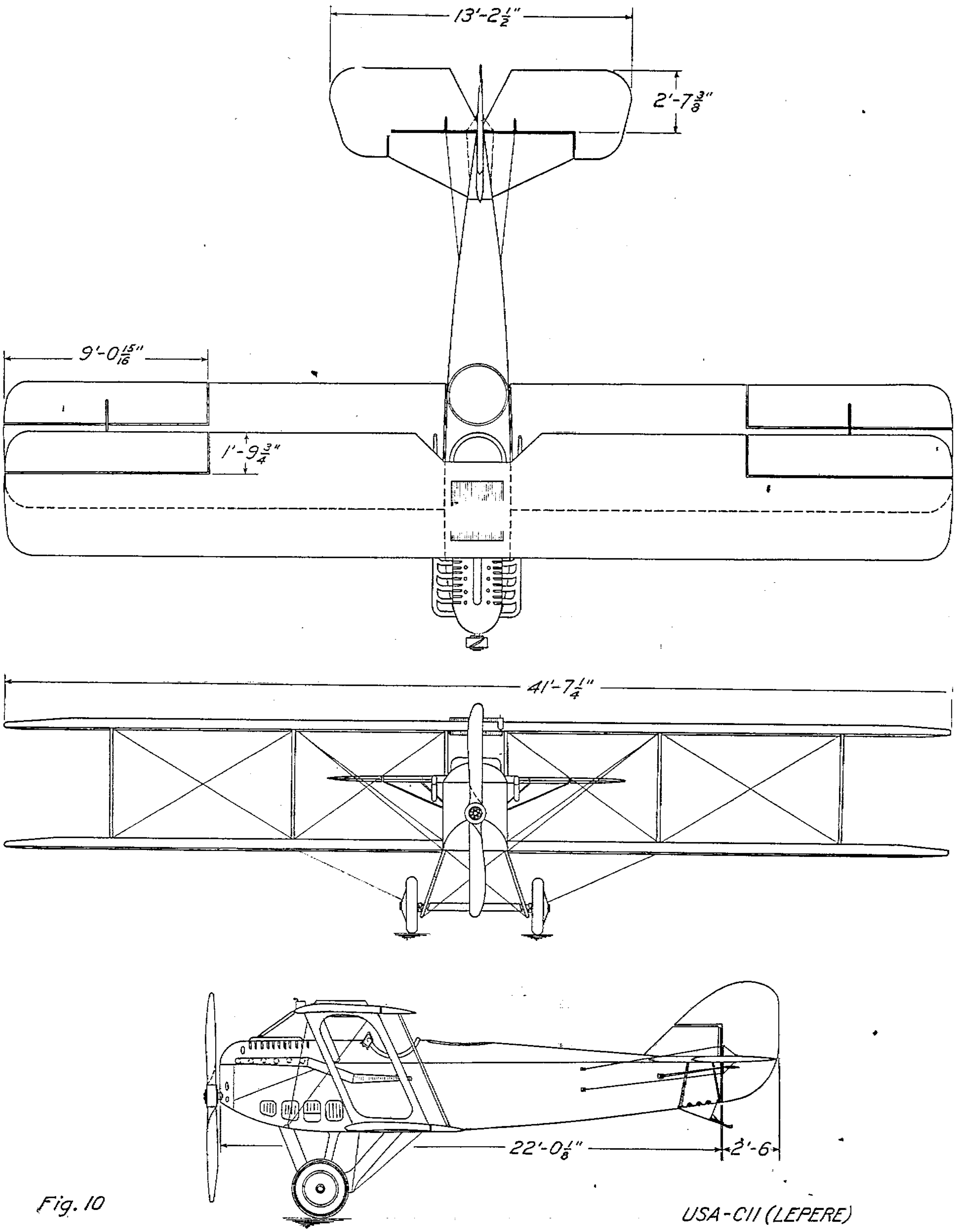

Fig. 10

USA-C11 (LEPERE)

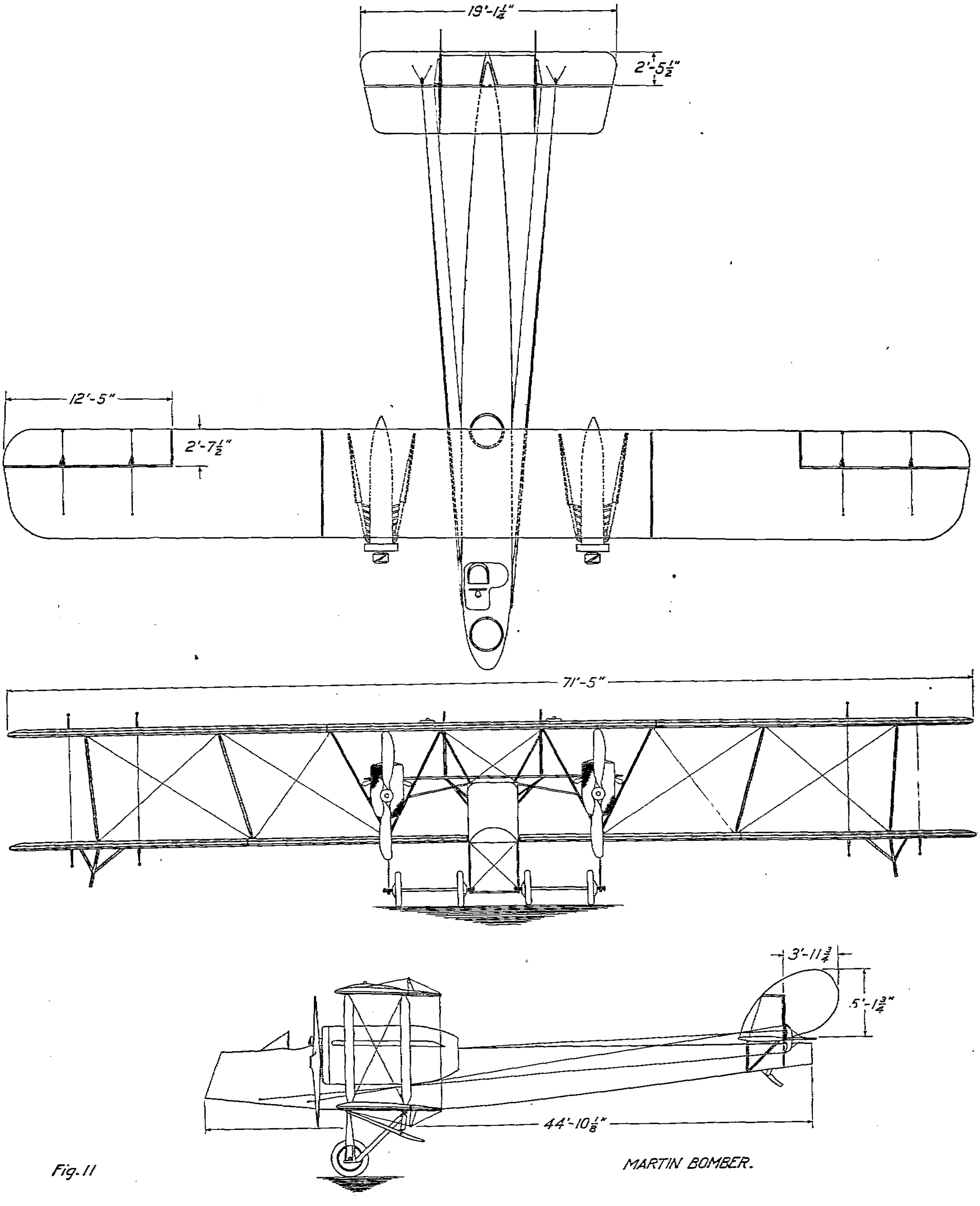

Fig. 11 MARTIN BOMBER.

high, while the VE7, which has twice as much camber on the upper surface as on the lower is equally stable at all speeds. That the section should exercise an influence in the general direction that it does is of course predictable from theory, but the magnitude of the effect found in comparing those four machines is much greater than would be expected either from theory or from the experiments on the effect of sectional form of the tail in the JN. At least it is possible to say definitely that the tail should not have a flat lower surface. All experiments and theories agree on that point.

The control surfaces both in the VE-7 and in the Lepere are much lighter than in the JN, a pull of only 2½ pounds on the stick being required to hold up the elevator on the VE-7 when at rest. In a loop or a tight spiral the pull required on the stick would then be about 18 pounds less on the Vought than on the JN, from this cause alone, and this factor contributes in no small degree to the remarkable controlability of the former machine.

The slip-stream effect on the controls still appears in the Martin, notwithstanding the fact that it is a twin-engined machine with the thrust line high relative to the C. G. It is probable, although direct experiments on the point have not yet been made, that the slip-streams on a twin-engined machine tend to approach each other and to draw along by viscous drag the air which lies between them, and that the portion of the tail which lies in the slip-streams is therefore actually larger than is usually assumed. Two possible methods of reducing slip-stream effect in a twin-engined airplane are to "toe in" the engines, setting them at an angle to the plane of symmetry so that the slip-streams will diverge and miss the tail, and to turn the propellers in opposite directions, the upper blade of each propeller moving away from the center line of the machine so that there is an upward component of race rotation in that portion of the slip-stream which strikes the tail, thus reducing or annulling the additional downward force due to the slip-stream.

The experiments on the Vought, Martin, and Lepere have been discussed at much greater length in Technical Note No. 1 of the National Advisory Committee for Aeronautics. Certain interesting points in connection with the balancing of the Lepere tail surfaces have been treated in that note and need not be repeated here.

FLIGHT TESTS WITH CONTROLS FREE.

Needless to say, the final test of stability with free controls is to release them while in flight and observe the subsequent motion, and this has been tried with two of the five machines for which stick force measurements were made. In order that lateral and longitudinal motions might be kept entirely separate, as in the case of locked controls, a short vertical stick was mounted directly on the longitudinal tube to which the regular control-stick is pivoted and which carries at its ends the sectors to which the aileron cables are attached. This secondary stick then permits the pilot to operate the ailerons without any possibility of affecting the elevator.

With 13 inches stagger and with the tail-plane set at from $-2°$ to $-3°$ to the top longerons the JN airplane would fly indefinitely with elevator control free for a small range of engine speeds. The factor limiting the range of r. p. m. was not the appearance of instability but the large separation of the force curves for different throttle settings. With the throttle closed there is a pull on the stick for all speeds at which it was considered safe to dive, and the airplane would therefore go into an approximately vertical dive, if not actually over on its back, if the controls were released and left free for a long enough time with the throttle closed. From about 1,000 to 1,300 r. p. m., however, the flight was more steady than with locked controls and more steady than it could be held by the use of the controls by any pilots except those of the most exceptional skill. In fairly smooth air (not ideal, but not unduly bumpy) the elevator moved continuously through a total angular range of about 0.5°. Most of the trials were started by releasing the stick while the airplane was diving steadily at about 80 m. p. h. With the throttle wide open the nose began to come up at once and continued to rise until the longitudinal axis was vertical, at which time the pilot resumed control. The machine still had plenty of speed and it is possible that, if left to itself, it would complete a loop with free controls. With the throttle partly opened the nose rose to a definite point and then began to drop again, com-

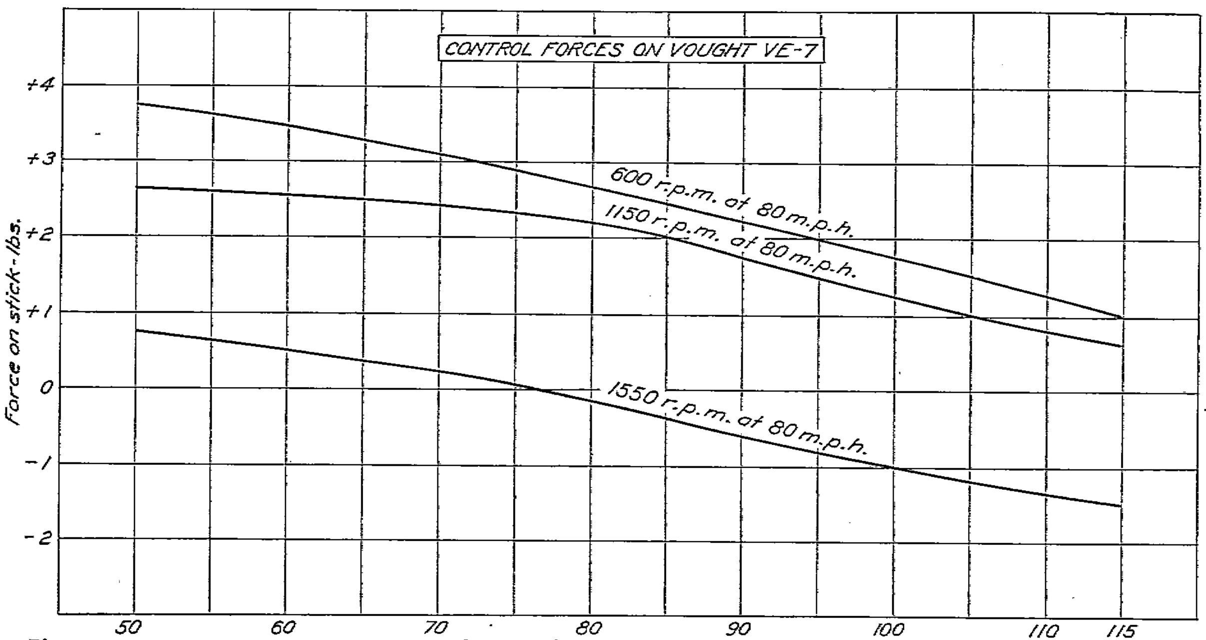

Fig. 12.

ing to an equilibrium position after two or three oscillations. The airplane was also flown in a circular path with angles of bank up to 15° and with the elevator control entirely free. The steadiness of flight when circling, although sufficient, was inferior to that with free controls.

The subject of dynamical stability will be treated at length in a subsequent report, but a few observations will be noted here. The dynamical stability of the JN proved to be excellent, the oscillations being heavily damped except in a few instances. The periods measured ranged from 25 to 28 seconds, and the oscillations were by no means simple harmonic in form, the nose rising much more slowly than it dropped, and seeming to creep gradually up to the most stalled position, hang there for two or three seconds, and then drop abruptly.

The VE–7 was also flown with free controls and was also found to be very steady, although not quite so good as the JN in this respect. The period of oscillation was from 14 to 17 seconds, being shorter than on the JN chiefly because of the smaller moment of inertia.

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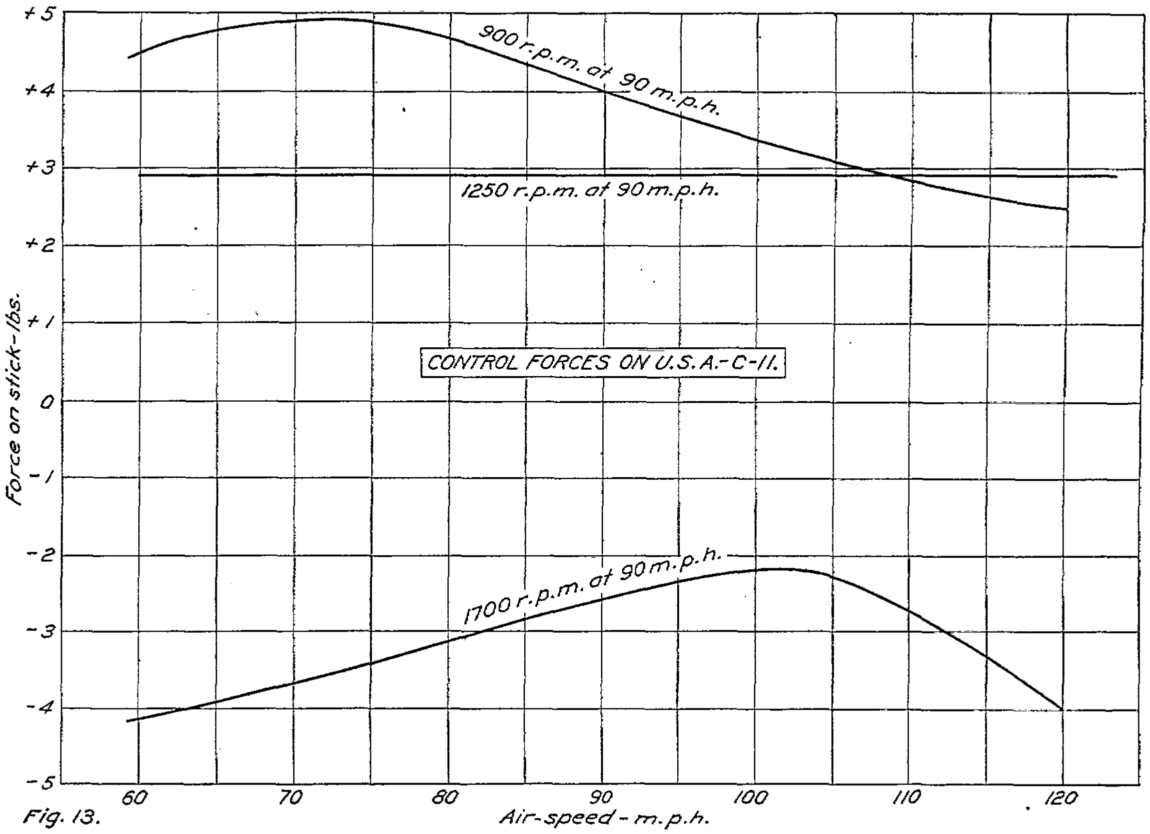
CONTROL FORCES ON U.S.A.-C-11.
900 r.p.m. at 90 m.p.h.
1250 r.p.m. at 90 m.p.h.
1700 r.p.m. at 90 m.p.h.
Force on stick-lbs.
+5
+4
+3
+2
+1
0
-1
-2
-3
-4
-5
60
70
80
90
100
110
120
Air-speed - m.p.h.

Fig. 13.

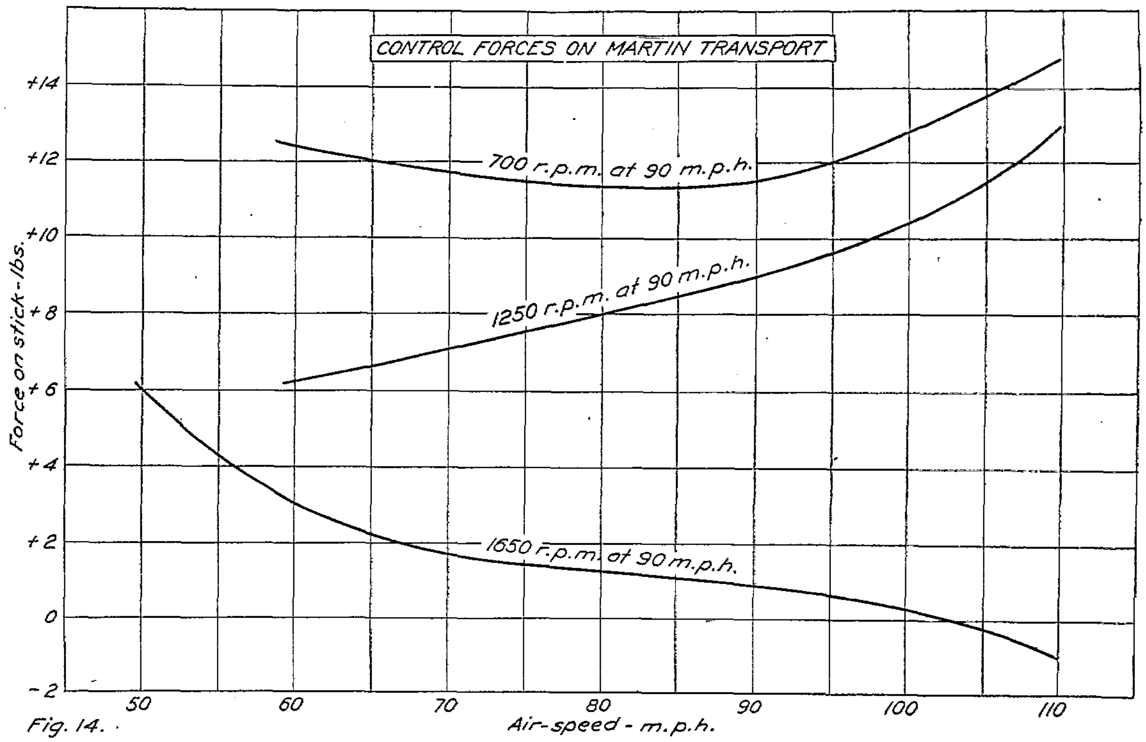
CONTROL FORCES ON MARTIN TRANSPORT
700 r.p.m. at 90 m.p.h.
1250 r.p.m. at 90 m.p.h.
1650 r.p.m. at 90 m.p.h.
Force on stick-lbs.
+14
+12
+10
+8
+6
+4
+2
0
-2
50
60
70
80
90
100
110
Air-speed - m.p.h.

Fig. 14.