REPORT No. 97

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GENERAL THEORY OF THE STEADY MOTION OF AN AIRPLANE

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REPORT No. 97

GENERAL THEORY OF THE STEADY MOTION OF AN AIRPLANE

IN SEVEN PARTS

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By GEORGE DE BOTHEZAT
National Advisory Committee for Aeronautics

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INTRODUCTION.

I hope it may be interesting to the reader to learn briefly, as it were, the history of the method here proposed for the study of steady motion, one which is different from other methods used. In his course of 1909–1910 at the "École Supérieure d'Aeronautique," M. Paul Painlevé showed how convenient the drag-lift curve was for the study of airplane steady motion. His treatment of this subject can be found in "La Technique Aeronautique" No. 1, January 1, 1910. In my book "Etude de la stabilité de l'aéroplane," Paris, 1911, I had already added to the drag-lift curve, the curve I call speed curve, which permits a direct checking of the speed of the airplane under all flying conditions. But the speed curve was still plotted in the same quadrant as the drag-lift curve. Later, with the progressive development of the new aeronautical science, with the continual increasing knowledge about engines and propellers, when seeking a convenient method of airplane design that really took account of all the particulars of the subject, I was brought to add the three other quadrants to the original one quadrant, and thus was obtained the steady motion chart described in detail in this paper, a method which I have been using since 1914. This chart is the most convenient method I know for the complete representation of the airplane steady motion performance. This method allows an easy survey of all the mutual interrelations of all the quantities involved in the question and this is accomplished—the chart once plotted—without any computations or graphical tracings. The chart, therefore, permits one to read directly, for a given airplane, its horizontal speed at any altitude, its rate of climb at any altitude, its path inclination to the horizon at any moment, its ceiling, its propeller thrust, revolutions, efficiency, and power absorbed—that is, the complete set of quantities involved in the subject, and to follow the variations of all these quantities both for variable altitude and for variable throttle. At the same time, one can follow the variation of all of the above quantities in flight, as a function of the lift coefficient and of the speed. It is the possibility of doing this that constitutes the most important property of my steady motion chart and makes its use so convenient for any purpose or question connected with steady motion.

I have considered it necessary not to limit myself in this paper to the general exposure of the method proposed, but to give at the same time a general discussion of the main principles connected with the subject, about which so many misunderstandings are still widespread.

Thus, the question of the interreaction of the airplane and propeller through the slip stream will be found discussed here. Several authors have talked much about the great increase of airplane drag produced by the slip stream. The trouble is that the additional pressure on the airplane due to the slip stream is an interior force for the airplane system, and it thus can not be purely and simply added to the airplane drag, which in our statement of the problem is an exterior force. The way in which the momentum theorem is applied to the airplane must be well remembered in the present case. The airplane in flight is considered inclosed in a closed surface invariably connected to the airplane and it is the component along the flying speed of the fluid momentum that flows out of this surface that measures the drag of the whole airplane. But in the value of this momentum the additional pressure on the fuselage due to the slip stream and the additional thrust of the propeller, which is the direct reaction to the last additional pressure, appear with opposite signs, and thus only their difference affects the drag. I hope that those who will carefully follow the general treatment of this problem here given, will not have the slightest doubt about the real nature of the question.

[1] The following report on the Steady Motion of an Airplane was prepared by Dr. George de Bothezat, aerodynamical expert for the National Advisory Committee for Aeronautics, with the assistance of the technical staff and the approval of Major T. H. Bane, of the Engineering Division, Air Service of the Army, McCook Field, Dayton, Ohio.

The question of the properties of the engine-propeller system and its dependence upon the properties of the engine considered alone and of the propeller considered alone will be found treated here in the generality demanded by actual aeronautical engineering practice. When a given propeller is considered by itself, its characteristics are functions of the ratio of its translational speed to its revolutions. When an engine is considered by itself, its power characteristics are functions of the revolutions and throttle opening. But when a propeller is connected to a certain engine the propeller's revolutions have to adjust themselves to the translational speed of the engine-propeller system and its characteristics will be functions only of the translational speed and throttle opening.

These preliminaries to the study of airplane steady motion is completed by the discussion of the question of the standard atmosphere. It is the opinion of the author that this last question has, in general, been greatly misunderstood. The entire performance of an airplane depends upon the density and temperature of the air in which the airplane flight takes place. It is a property of the airplane to be able to reach a certain limiting atmospheric layer specified by a certain density, above which the airplane can not fly any more, which is called its ceiling. The altitude at which this atmospheric layer can be found is very variable with the meteorological conditions. Thus the airplane ceiling can not be specified by an altitude value, but only by a density value. The forces of air resistance depend only upon the density and are independent—in practical limits—of temperature; the lift, the drag, and propeller thrust depend only upon density; it is the power of the airplane engine alone that is affected by temperature. Thus at constant density only the engine power will be influenced by the temperature; and, when selecting a standard law connecting atmospheric temperatures with atmospheric densities, it is only the selection of standard working conditions for the engine that will be concerned. The temperature acts on the engine somewhat as a throttle variation. The last fact understood, it is clear that there is no reason for adopting a fantastic relation between temperature and densities for engine standard working conditions, and the adoption of a constant standard temperature for all densities becomes quite natural. It is in such a way that we are brought to the general conclusion that, for the standardization of airplane performance, it is the isothermic atmosphere that should be adopted. It is the proposition of the author to adopt the isothermic atmosphere of zero degrees centigrade as standard atmosphere. The tremendous advantages and great simplicity that result from such a selection will be found discussed in this paper. The isothermic atmosphere of zero degrees centigrade has also in its favor the fact that it satisfies all demands quite as well as any other "standard atmosphere." (See fig. 13.) The public has curiosity about the height at which an airplane is flying; but, from an engineering standpoint, we can only speak about the density reached by an airplane.

It is thus beyond discussion that, from the standpoint of aviation engineering, the isothermic atmosphere of zero degrees centigrade is the only one that can be reasonably adopted as the standard atmosphere.

For some special purposes we need to know the actual altitude at which an airplane is flying. But this is a totally different question, and no "standard atmosphere" can help us in such a case to obtain an accurate determination of the altitude. The question of the altitude determination from the knowledge of the atmospheric pressure and temperature is a special question in itself, totally independent of the conditions adopted for the standardization of airplane performances. The foregoing questions are discussed in the first three parts.

In Part IV the general theory of the steady motion of an airplane is developed. After the basic equations have been established and the method to be used for their discussion described, a general survey of the properties of an airplane in steady motion is given. I call attention to the detailed discussion of climbing phenomenon that will be found here and to the general formulae established for the rate of climb and time of climb, which quantities, under the simplest assumptions, appear as hyperbolic functions of the ceiling. It is also shown as a consequence of what conditions one can derive the law of linear variation of the rate of climb with altitude as practically observed. The influence of throttle variation on airplane performance is also submitted to a detailed study and the influence of the mechanical losses of the engine on the airplane when gliding is discussed.

The complete study of the properties of an airplane in steady motion is made by the same uniform method, and the complete representation of the entire performance is reached. It is the last fact that constitutes the main advantage of the method developed.

In Part V is discussed the question of the first checking of airplane performances, starting with a minimum of data available concerning the airplane considered. This question is of great practical interest, but certainly the performance is predicted only as a first approximation.

Part VI gives the general outlines of the author's method of airplane free flight testing, which permits the most complete and rigorous airplane tests. The whole system of airplane characteristics, including the separate determination of the engine and propeller characteristics as given by free flights, is obtained from a set of climbs and glides made at constant indicated air speeds. The horizontal speeds at all altitudes, the best rates of climbs, and the ceiling are found with great accuracy without the pilot having to fly under these conditions, which practically can never be reached with complete certainty. On the contrary, the flying at nearly constant indicated air speeds can be realized by the pilots fairly well and with ease. That is why the present method of free flight testing is so convenient in practice.

A last part is devoted to the study of the problem of soaring. This question of soaring has been since long a matter of great interest and discussion. The phenomenon is a direct consequence of the existence in the atmosphere of ascending currents of air, and all other explanations of it are devoid of any serious foundation. Soaring is only possible if the upward vertical wind component is equal to or greater than the glider's rate of descent. Gliders of very small rate of descent can be built with ease; special attention has only to be paid to their stability and maneuverability. On the other hand, as is explained in this paper, it is the opinion of the author that ascending winds in the atmosphere must be considered as a common occurrence; this being a result of the instability of the vortex sheets formed between air layers of different velocities, and which must break into the Karman stable system of quincunx vortex rows. Between such vortices, traveling in space, we must meet at equal intervals ascending and descending currents. Direct computations show that the vertical components of these air currents are sensible fractions of the speed difference between the atmospheric layers which have originated these quincunx vortex rows. We are thus brought to a general understanding of the soaring phenomenon and the possibility of its practical utilization. The great interest of the practical realization of soaring airplanes is, I hope, beyond discussion.

At the end of this report is added a sheet of drawings giving a general survey of some fundamental characteristics of the atmosphere. I owe to the amiability of Dr. C. F. Marvin the remarkably complete data concerning the constitution of the atmosphere with altitude.

It is a special pleasure for me to address my best thanks to Mr. W. F. Gerhardt, aeronautical engineer at McCook Field, and to express my appreciation of the critical judgment he has shown in preparing most of the figures for this report. This last has given me the opportunity to discuss with him many details of this paper, which has helped me to clarify several of them.

Figure 13, relating to the computation of the standard atmospheres has been prepared by Mr. C. V. Johnson, aeronautical engineer at McCook Field, and I also address him my most sincere thanks for his kind assistance.

This paper has been written during my stay at McCook Field, when introducing my method of airplane free flight testing. I am specially pleased to have this opportunity to address my heartiest thanks to Maj. T. H. Bane, chief of McCook Field, for the interest he has always shown in my work and for all the necessary assistance he has placed at my disposal for its successful achievement.

G. de Bothezat,

Aerodynamical Expert, National Advisory Committee for Aeronautics.

Dayton, Ohio, *July, 1920.*

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PART I.

PRELIMINARY.

Let us consider an airplane of any type or system, which, like most actual airplanes, has a plane of symmetry and constitutes a rigid system. By considering the airplane to be rigid, we only mean that we neglect the variations in the distribution of weight produced in the airplane by its small deformations and by the displacements of its rudders. The main influence of these last factors is to produce variations in the forces of air-resistance.

We will say that the airplane considered has reached one of its *states of steady motion* when the motion of the airplane proceeds with *a speed constant in magnitude and direction,* the plane of symmetry of the airplane being vertical, and the machine maintaining an invariable orientation relative to its rectilinear trajectory.

Let us consider the airplane moving in a mass of uniform air which in general may have the velocity $\bar{v}$ relative to the earth. The velocity $\bar{v}$ is the wind velocity in that part of the atmosphere where the airplane is actually flying.

The velocity of the airplane relative to the ground will be designated by $\bar{W}$ and called *ground speed* or *absolute speed,* because the earth can be considered, with sufficient approximation, as an absolute reference system in the present case.

The velocity of the airplane relative to the air mass containing it will be designated by $\bar{V}$ and called the *air-speed* or *self-speed.*

The velocities $\bar{v}$, $\bar{W}$ and $\bar{V}$ are vector quantities and are therefore characterized by their magnitudes, directions, and senses. Their magnitudes will be designated by v, W, and V. Between the velocities $\bar{v}$, $\bar{W}$, and $\bar{V}$ there always exists the relation

$$\bar{W} = \bar{V} + \bar{v} \qquad \text{(geometrical sum)}$$

which expresses the fact that the airplane, so to say, flies in the wind with its self-speed $\bar{V}$ and is carried by the wind with the velocity $\bar{v}$. In case of no wind,

$$\bar{v} = 0; \quad \bar{W} = \bar{V}$$

The airplane will move with a self-speed of translation $\bar{V}$, constant in magnitude and direction, when all the forces acting on the airplane have a resultant equal to zero, and when the resulting moment of these forces, relative to the center of mass, are also equal to zero. The last conditions are direct consequences of the theorems of momentum and moments of momentum.

The forces acting on an airplane are: The weight, $\bar{P}$; the propeller thrust, $\bar{Q}$; the total air resistance, $\bar{R}$. The foregoing forces include all the forces acting on the airplane.

The first condition of steady motion of an airplane is expressed by the relation:

$$\bar{P} + \bar{Q} + \bar{R} = 0 \qquad \text{(geometrical sum)}$$

Let us designate by $\bar{M}$ the resulting moment of all the forces acting on the airplane. The second condition of steady motion of the airplane is expressed by the relation

$$\bar{M} = 0$$

As we consider only those motions of the airplane for which its plane of symmetry is vertical, the moment $\bar{M}$ is always normal to the plane of symmetry.

In the discussion of the conditions that make $\bar{M}=\bar{0}$, two cases must be distinguished:

The first case is when the thrust $\bar{Q}$ of the propeller passes through the center of mass of the airplane considered. In this case, as the weight P always passes through the center of mass, *the moment $\bar{M}$ reduces itself to the moment of the forces of air-resistance.* These last forces are proportional to the square of the self-speed $\bar{V}$ and the angle of attack α of the airplane, and for a given state of steady motion of the airplane can be changed only by the displacement of the elevator, the orientation of which will be supposed fixed by an angle β. We can thus write

$$M = m V^2 \varphi(\alpha,\beta)$$

The angle of attack for which $M=0$ will thus be fixed by the condition:

$$\varphi(\alpha,\beta)=0$$

The function $\varphi(\alpha, \beta)$ in the flying interval must be a uniform function; thus to each value of β, i. e., for each position of the elevator, there must be a corresponding value of the angle of attack α for which $M=0$. The curve of α plotted against β can be called the curve of the elevator sensitivity. We are thus brought to the fundamental conclusion:

When the propeller thrust of an airplane passes through its center of mass—provided the action of the slipstream on the elevator can be neglected and the mass distribution considered as invariable—the angle of attack, for a state of steady motion of the airplane, can be changed only by displacement of the elevator. Any other conditions that can change in the flight can not alter the value of the angle of attack of the state of steady motion under consideration.

That is why I say that the angle of attack is the variable which the pilot is holding in his hand.

The second case is when the thrust $\bar{Q}$ of the propeller does not pass through the center of mass. This case is far more complicated than the first one. For a discussion of it, I will refer to my investigations of the question [1] and will mention here only the following: *In the case of the propeller decentration, a change in the angle of attack may be produced by acting on the throttle of the engine, as well as by changing the position of the elevator.*

I shall first give a general survey of the forces acting on the airplane. I shall afterwards deduce the consequences which follow from the condition that the resultant of the forces acting on an airplane is equal to zero when it has reached a state of steady motion. This will bring us to those fundamental references without which the understanding of airplane testing is impossible.

We shall use the metric units exclusively. Their use has been authorized in the U. S. Army by an act of Congress, and in practice tremendous advantages result from the use of these units.

We shall use the engineering metric units, i. e.,

kilogram-weight; meter; second

In these units, considering the gravitational acceleration as equal to $g=9,81$ mt/sec², a body having a weight equal to 9.81 kg. has a mass equal to unity. For,

$$1 \textit{ kilogram-weight} = \textit{mass of a kg.} \times g.$$

and accordingly,

$$\textit{mass of a kilogram-weight} = \frac{1}{g}$$

Thus a body of g kilogram-weight will have a mass equal to unity. We shall call this last unit of mass the *Newton.*

[1] See Dr. G. de Bothezat's "Etude de la Stabilité de l'Aéroplane," Paris, 1911, p. 164, and "Revue de Mécanique, août, 1913." "Théorie Générale de l'Action Stabilisatrice des Empennages Horizontaux ......" Also, "Introduction to Airplane Stability," p. 137 (in Russian), Petrograd, 1912.

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PART II.

THE FORCES ACTING ON AN AIRPLANE.

1. THE WEIGHT.

We shall designate by P the total normal *weight* that a given airplane is supposed to lift. The total weight of an airplane always acts vertically and passes through the center of mass of the airplane. The total normal weight is constituted of the following parts

The structural weight of the airplane.	P_a
The weight of the engine.	P_m
The weight of the fuel.	P_c
The useful weight.	P_u

The sum of the first two constituent weights will be designated by P_{am}. We thus have

$$P_{am} = P_a + P_m \tag{1}$$

The weight P_{am} is the minimum limit of the total weight of the airplane considered.

The sum of the last two constituents weight will be designated by P_{cu}. We thus have.

$$P_{cu} = P_c + P_u \tag{2}$$

The total normal weight is thus equal to

$$P = P_{am} + P_{cu} \tag{3}$$

For each airplane tested it it useful to note the value of the ratios:

$$P_a/P; \ P_m/P, \ P_c/P, \ P_u/P \text{ and } P_{am}/P = p_{am}, \ P_{cu}/P = p_{cu}.$$

For large weight-carrying and low-ceiling airplanes, p_{cu} is close to 50 per cent, and for small high-speed and high-ceiling airplanes, p_{cu} is around 25 per cent.

2. THE FORCES OF AIR RESISTANCE.

We will resolve the total air resistance $\bar{R}$ of the whole airplane into two components:

The drag R_x directed along the self-speed $\bar{V}$ of the machine, but always in the inverse sense, and the lift R_y perpendicular to its direction. We have

$$R^2 = R_x^2 + R_y^2$$

All experimenters in aerodynamics fully agree that for the flying range of variation of the speed V, the drag and the lift can be considered as being of the form

$$R_x = k_x \delta A V^2 \tag{4}$$

$$R_y = k_y \delta A V^2 \tag{5}$$

where A is the area, δ is the air density (expressed in Newtons), and k_x and k_y are the drag and lift coefficients, which are functions of the angle of attack only. The angle of attack measured from any fixed reference line in the plane of symmetry of the airplane will be designated by α. The angle of attack measured from the *zero lift direction* will be designated by i. To a first approximation, for the flying range of variation of i, the coefficients k_x and k_y may be considered as being of the form

$$k_x = k(ai^2 + bi + c) \tag{6}$$

$$k_y = ki \tag{7}$$

The empirical coefficients k, a, b, and c have to be determined from the empirical curves for k_x and k_y by the method of least squares. Thus, to a first approximation, we may consider the drag R_x and the lift R_y as being of the form

$$R_x = k\delta A V^2(ai^2 + Ci + c) \tag{8}$$

$$R_y = k\delta A V^2 i \tag{9}$$

The air resistance $\bar{R}$ here considered, components of which are the drag R_x and the lift R_y, is the total air resistance of the whole airplane, the propeller or propulsive system excluded.

For all the fundamental conceptions relating to the laws of air resistance, the reader is referred to the author's "Introduction into the Study of the Laws of Air Resistance of Aerofoils," published by the National Advisory Committee for Aeronautics, Washington, D. C., Report No. 28.

3. THE PROPELLER THRUST.

In modern airplanes the propeller thrust is produced by a blade-screw propeller driven by a gas engine.

We shall call the system composed of the propeller and the engine the *engine-propeller system.* Its properties, which are a result of the combined properties of the propeller and engine used are, however, different from the properties of the propeller considered alone and of the engine considered alone.

I shall first give a short survey of those properties of the propeller and the engine, the knowledge of which is necessary for a complete understanding of the properties of the engine-propeller system.

A. PROPERTIES OF THE PROPELLER.

Let us consider a given propeller of a diameter D. When this propeller makes N revolutions per second, i. e., when it has the angular velocity $\Omega = 2\pi N$, and moves with the uniform velocity $V^{mt}/sec.$ along its axis, it will produce a thrust of Q *klg* when a torque of C *klg. mt.* is applied to its axis.

The thrust power L_u, or useful power developed by the propeller, is equal to

$$L_u = QV \tag{10}$$

The torque power L_a, or power absorbed by the propeller, is equal to

$$L_a = C\Omega \tag{11}$$

The efficiency of the propeller is equal to

$$\eta = \frac{L_u}{L_a} \tag{12}$$

We will designate by μ, and call it the *advance per turn,* or shorter, *advance,* the ratio

$$\mu = \frac{V}{N} \tag{13}$$

The thrust Q of a propeller has for its general expression

$$Q = \delta V^2 F_1(\mu) = \delta N^2 F''_1(\mu) \tag{14}$$

The torque power developed by a propeller has for its general expression

$$L_a = \delta V^3 F_2(\mu) = \delta N^3 F_2''(\mu) \tag{15}$$

In the last expressions, the quantities

$$F_1''(\mu) = \mu^2 F_1(\mu), \quad F_2''(\mu) = \mu^3 F_2(\mu)$$

are functions of the advance μ, only. These functions can be considered either as explicit functions which can be calculated from the screw dimensions[1] and its aerodynamical characteristics, or can be considered as empirical functions determined by direct experiment.

Using the values (14) and (15) for Q and L_a, it is easy to see that η has for its general expression

$$\eta = \frac{F_1(\mu)}{F_2(\mu)} \tag{16}$$

i. e., the efficiency η is a function of the advance μ only.

The thrust Q_o produced and the power L_o absorbed by a propeller working at a *fixed* point have for their general expressions

$$Q_o = \delta N^2 C_1' \tag{17}$$

$$L_o = \delta N^3 C_2' \tag{18}$$

where C_1' and C_2' are two constants that represent the limiting values which F_1'' and F_2'' take when V tends toward zero.

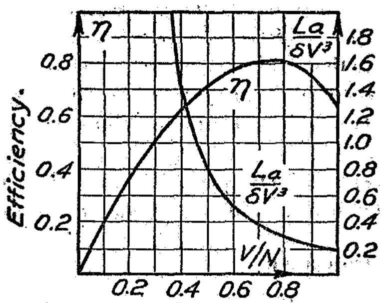


Fig. 1. Advance.

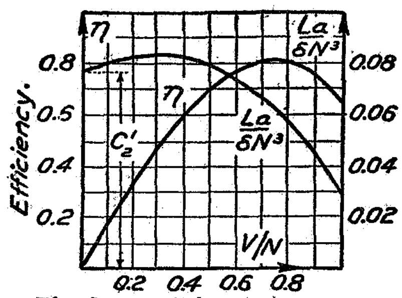


Fig. 2. Advance.

The deformation of the propeller blades and the deviation of the fluid resistance from the square law produce some departures from the foregoing laws. But these laws hold perfectly well when the variations of V and N are limited to certain intervals for which the constants appearing in expressions (14), (15), (16), (17), and (18) have been specially determined.

For the complete specification of the properties of a given propeller, two characteristic curves are necessary. We will use as such either

$$\eta(\mu) \text{ and } \frac{L_a}{\delta V^3} = F_2(\mu) \tag{19}$$

or

$$\eta(\mu) \text{ and } \frac{L_a}{\delta N^3} = F_2''(\mu) \tag{20}$$

which have the advance μ as argument.

The general courses of these curves are represented on figure 1 and figure 2, which correspond to a propeller of the Dorand type tested by G. Eiffel.[2]

All the foregoing refers to a propeller working in free air. In some cases different bodies disposed in the neighborhood of the propeller can interfere with the working of the propeller. As the neighborhood conditions require a slight generalization of the ordinary conceptions relating to propellers, I shall consider somewhat in detail the relations that hold in this case.

[1] For the explicit expressions of these functions, and methods of their calculations, see pp. 58 and 59 of "The General Theory of Blade Screws," by Dr. G. de Bothezat. Report No. 29, published by the National Advisory Committee for Aeronautics, Washington, D. C.

[2] G. Eiffel, "Nouvelles Recherches sur la Résistance de l'Air et l'Aviation," Paris, 1914. Atlas, plate XXXIII, propeller No. 11.

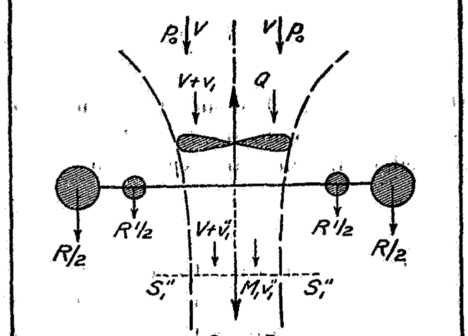
Let us consider any vehicle of locomotion, in our case an airplane flying under an angle of attack i and a speed V. In order to secure the flight of the airplane, i. e., to overcome the air resistance, it is necessary to supply a certain amount of power, which we will call power utilized L_u by the vehicle. For a given airplane with invariable load, at constant altitude and with motor throttle kept at constant opening, the power L_u is a function of the flying speed V only. This is because, as will be seen later, the angle of attack i under such conditions is a function of the speed V only. The power L_u is delivered to the vehicle by a propulsor, in our case a screw blade propeller. It is self-evident that the power delivered by the propulsor to the vehicle is the same thing as the *power utilized by the vehicle.* But in order to make the propulsor able to deliver the power L_u to the vehicle, we must always deliver to the propulsor a power L_a greater than L_u, called *power absorbed by the propulsor.*

It is the ratio

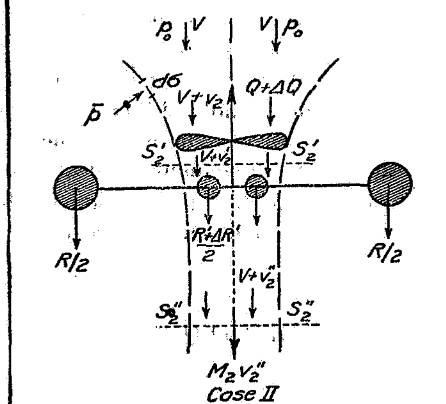
$$\eta=\frac{L_u}{L_a} \tag{21}$$

which we shall call the efficiency of the *propulsor.*

It is easy to understand that the same propulsor applied to different vehicles will generally show different efficiencies on account of the neighborhood conditions interfering with the work of the propulsor. A propeller must be especially adapted to the vehicle under consideration in order to give a high efficiency. In order that we may have a complete understanding of the circumstances that here occur, let us compare the working conditions of two identical screw-blade propellers, applied to two airplanes, identical from the standpoint of air resistance, but in one case with an unobstructed slipstream and in the other case with some of the parts of the airplane disposed in the slipstream created by the propeller. These two cases, which we will call Case I and Case II, are represented schematically on Figure 3.



Case I
With unobstructed slip stream.
Fig. 3.



Case II
With obstructed slip stream.
Fig. 3.

In Case I, when the airplane has reached a speed V, the total drag is equal to $R+R'$, where R' is the resistance of those parts which in Case II are in the slipstream, and the thrust is equal to Q. When a state of steady motion is reached, applying the momentum theorem *to the airplane*, we find

$$Q=R+R' \tag{22}$$

If we designate by L^1_a the power absorbed by the propulsor, the efficiency of the propulsor is equal to

$$\eta_1=\frac{L_u}{L^1_a}=\frac{(R+R')\,V}{L^1_a}=\frac{QV}{L^1_a} \tag{23}$$

Applying the momentum theorem to the slipstream, we find that the momentum M_1v_1'' communicated to the fluid that crosses the propeller, measured in the section $S_1''S_1''$ where the exterior pressures on the boundary surface of the slipstream balance, is equal to

$$M_1v_1''=Q=R+R' \tag{24}$$

In Case II the bodies having the air resistance R' have been brought inside the slipstream. Their resistance changes then to $R' + \Delta R'$. When the same speed V is reached, the resistance of the bodies outside the slipstream will be unaffected, but the propeller thrust Q, on account of changed neighborhood conditions, will have been changed to a certain value equal to $Q + \Delta Q$. When a state of steady motion is reached, we have

$$Q + \Delta Q = R + R' + \Delta R' \tag{25}$$

and on account of (22)

$$\Delta Q = \Delta R' \tag{26}$$

Designating by L^{II}_a the power absorbed by the propeller in this second case, we have

$$\eta_2 = \frac{L_u}{L_a^{II}} = \frac{(R+R')V}{L_a^{II}} \tag{27}$$

If in Case II we had measured the thrust of the propeller in free flight by a thrust meter we should have found for its value $Q + \Delta Q$. It is very tempting to take as a measure of the propeller efficiency the expression

$$\eta_2' = \frac{(Q+\Delta Q)V_1}{L_a^{II}} \tag{28}$$

but this, as it is easy to see, will give an overestimate of the propeller efficiency, because $(Q + \Delta Q) > (R + R')$. There is nothing astonishing in this, because it must not be forgotten that $(Q + \Delta Q)$ is in reality only an interior force in relation to our airplane system. It expresses only the stress state between engine-propeller set and airplane fuselage, and not the resultant exterior force securing the propulsion.

Let us calculate the momentum $M_2\, v_2''$ that crosses a section $S_2''\, S_2''$ of the slipstream taken behind the body of resistance R'. In this case we can not in general assume that the exterior pressure on the outside boundary surface of the slipstream, counted up to the section $S_2''\, S_2''$, balances, and will therefore designate by $\Sigma''\, \bar{p}\, d\sigma$ the resultant of this exterior pressure which on account of symmetry is necessarily directed along the slipstream axis. In the last expression $d\sigma$ is an element of the slipstream boundary surface counted up to section $S''_2\, S''_2$ and $\bar{p}$ the outside vector pressure considered normal to each corresponding surface element $d\sigma$. Applying the momentum theorem to the slipstream counted up to the section $S_2''\, S_2''$ we find:

$$M_2\, v_2'' = Q + \Delta Q - (R' + \Delta R') + \Sigma''\, \bar{p}\, d\sigma \tag{29}$$

or, since $\Delta Q = \Delta R'$ and $Q = R + R'$

$$M_2\, v''_2 = R + \Sigma''\, \bar{p}\, d\sigma \tag{30}$$

In general $\Sigma''\, \bar{p}\, d\sigma < R'$, thus $M_2\, v_2'' < M_1\, v_1''$.

It is natural to try to find out in what relation the power L_a^{II} stands to the power L_a^{I}. The whole thing depends upon the values of the efficiencies η_2 and η_1. We have

$$L_u = \eta_1\, L_a^I = \eta_2\, L_a^{II} \tag{31}$$

thus,

$$\frac{L_a^I}{L_a^{II}} = \frac{\eta_2}{\eta_1} \tag{32}$$

I immediately remark that by no means is it necessarily true that $\eta_2 < \eta_1$ and it can even happen that for a given propeller we may get $\eta_2 > \eta_1$. An examination of the comparative losses that take place in both cases will show the nature of the question.

Applying in Case I the theorem of kinetic energy to the slipstream, we find

$$\tfrac{1}{2}\, M_1(V+v_1'')^2+\tfrac{1}{2}\, I_1''\, \omega_1''^2-\tfrac{1}{2}\, M_1\, V^2=Q(V+v_1)+C_1\omega_1$$

or, since $Q=M_1\, v_1''$

$$Q(V+v_1)+C_1\, \omega_1=VQ+\tfrac{1}{2}\, M_1\, v_1''^2+\tfrac{1}{2}\, I_1''\, \omega_1''^2 \tag{33}$$

where:

M_1 = fluid mass that crosses the propeller disk in a unit of time.
v_1'' = slip velocity in section $S_1''\, S_1''$
I_1'' = moment of inertia of the fluid mass M_1 in section $S_1''\, S_1''$
ω_1'' = race rotation in section $S_1''\, S_1''$
v_1 = slip velocity in the plane of propeller rotation.
C_1 = torque acting on the propeller axis.
ω_1 = race rotation in the plane of the propeller.

The flow conditions in the slipstream are assumed uniform for sake of simplicity.

On the other hand we have,

$$L_a^I=C_1\, \Omega_1=Q(V+v_1)+C_1\, \omega_1+p_r^I \tag{34}$$

where

Ω_1 = angular velocity of propeller rotation.
p_r^I = losses by impact and friction of the fluid against the propeller blades.

We thus finally find

$$L_a^I=VQ+\tfrac{1}{2}\, M_1\, v_1''^2+\tfrac{1}{2}\, I_1''\, \omega_1''^2+p_r^I \tag{35}$$

and

$$\eta_1=\frac{L_u}{L_a}=\frac{L_a^I-[\tfrac{1}{2}\, M_1\, v_1''^2+\tfrac{1}{2}\, I_1''\, \omega_1''^2+p_r^I}{L_a^I} \tag{36}$$

Applying in Case II the theorem of kinetic energy, we find

$$\tfrac{1}{2}\, M_2\,(V+v_2'')^2+\tfrac{1}{2}\, I_2''\, \omega_2''-\tfrac{1}{2}\, M_2\, V^2-S_2''(V+v_2'')\,(p_0-p_2'')+(Q+\Delta Q)\,(V+v_2)+C_2\, \omega_2-(R'+\Delta R')\,(V+v_2')$$

or since $Q+\Delta Q=M_2\, v_2''+(R'+\Delta R')-\Sigma''\, \overline{p}\, d\sigma$ and considering $\Sigma_1''\, p\, d\sigma \cong S_2''\,(p_o-p'')$ where S_2'' is the area of the section $S_2''\, S_2''$ of the slipstream, p_0 the outside pressure, and p_2'' the pressure in section $S_2''\, S_2''$ we get:

$$(Q+\Delta Q)\;(V+v_2)+C_2\, \omega_2=V(Q+\Delta Q)+\tfrac{1}{2}\, M_2\, v_2''+\tfrac{1}{2}\, I_2''\, \omega_2''^2-S_2''(p_0-p_2'')v_2''+(R'+\Delta R')v_2' \tag{37}$$

where M_2, v_2'', I_2'', ω_2'', C_2, ω_2 have meanings analogous to Case I, $S_2''\;(V+v_2'')\;(p_0-p_2'')$ represents the work of the resultant exterior pressure $\Sigma_1''\, \overline{p}\, d\sigma$ considered as built up from the work of the pressures in a section far in front of the propeller and in section $S_2''\, S_2''$, where p_0 and p_2'' are the corresponding pressures; $(V+v_2')$, a mean velocity included between $(V+v_2)$ and $(V+v_2'')$ whose product by $(R'+\Delta R')$ represents the work corresponding to that resistance.

But as:

$$L_a^{II}=C_2\, \Omega_2=(Q+\Delta Q)\;(V+v_2)+C_2\, \omega_2+p_r^{II} \tag{38}$$

(with Ω_2 and p_r^{II} having meanings analogous to Case I) we finally find:

$$L_a^{II}=V\,(Q+\Delta Q)+\tfrac{1}{2}\, M_2\, v_2''^2+\tfrac{1}{2}\, I_2''\, \omega_2''^2-S_2''\,(p_0-p_2'')\, v_2''+(R'+\Delta R')\, v_2'+p_r^{II},\ \text{or, since } \Delta Q=\Delta R'$$

$$L_a^{II}=VQ+\tfrac{1}{2}\, M_2\, v_2''^2+\tfrac{1}{2}\, I_2''\, \omega_2''^2+R'\, v_2'+\Delta R'\;(V+v_2')-S_2''\,(p_0-p_2'')\, v_2''+p_r^{II} \tag{39}$$

and

$$\eta_2 = \frac{L_u}{L_a^{II}} = \frac{L_a^{II} - [{}^1/_2 \; M_2 \, v_2''^2 + {}^1/_2 \, I_2 \, \omega_2''^2 + R' \, v_2' + \Delta R' \; (V + v_2') - S_2'' \; (p_0 - p_2'') \; v_2'' + p_r^{II}]}{L_e^{II}} \tag{40}$$

For the ratio of L_a^I to L_a^{II} we find the value

$$\frac{L_a^I}{L_a^{II}} = \frac{Q \; (V + v_1) + C_1 \, \omega_1 + p_r^I}{(Q + \Delta Q) \; (V + v_2) + C_2 \, \omega_2 + p_r^{II}} \tag{41}$$

In general $(C_1 \; \omega_1 + p_r^I)$ and $(C_2 \; \omega_2 + p_r^{II})$ are of the same order of magnitude; $(Q + \Delta Q) > Q$, but generally $v_2 < v_1$. We thus can not decide *a priori* between $L_a^I \gtreqless L_a^{II}$.[3]

I would warn those who think that the losses $[R' \, v_2' + \Delta R'(V + v_2')]$ can be estimated easily. As a matter of fact: First, the velocity in the slipstream, when some bodies are introduced into it, is totally changed in comparison with a free slipstream; second, the slipstream is not a uniform current, but a current of variable velocity along its axis; third, as the slipstream is a stream with free boundaries, the formulae and coefficients of fluid resistance deduced from experiments in fluids of infinite boundaries can not be applied to it, especially when the bodies considered do not have small cross sections in comparison with the cross section of the slipstream.

The efficiency η_1 will be called *free efficiency,* and designated in the following by η_f. The efficiency η_2 will be called *propulsive efficiency* and designated by η_p. We shall designate by f, and call it *neighborhood factor,* the ratio of the propulsive efficiency to the free efficiency.

We thus write:

$$\eta_p = f \eta_f \tag{42}$$

It is understood that the neighborhood factor f can be

$$f \gtreqless 1$$

As has been mentioned already, the free efficiency η_f is a function of the advance $\mu = V/N$ only. But as the slipstream created by a given propeller is also a function of μ only, the neighborhood factor, for a given propeller and given neighborhood conditions, can be a function of μ only. *Thus the propulsive efficiency must be a function of the advance μ only.*

In airplane testing, it is the propulsive efficiency η_p that has to be measured in order to evaluate the propeller in the actual working conditions.

One could raise the following two questions:

a. In what relation does the thrust $(Q + \Delta Q)$ of Case II stand to the momentum $M_2 \, v_2'$ in section $S_2' \, S_2'$ (see fig. 3)? It is easy to see that

$$Q + \Delta Q = M_2 \, v_2' + \Sigma' \; \overline{p} d\sigma$$

where $\Sigma' \; \overline{p} d\sigma$ is the resultant of the outside pressure on the whole boundary of the slipstream counted up to the section $S_2' \, S_2'$. Between the momentum $M_2 \, v_2'$ and $M_2 \, v_2''$ we have the relation:

$$M_2 \, v_2'' + (R' + \Delta R') - \Sigma'' \; \overline{p} d\sigma = M_2 \, v_2' + \Sigma \; \overline{p} d\sigma$$

b. What would the momentum in section $S_2'' \, S_2''$ be if the whole resistance $R + R^1$ had been put in the slipstream? It is easy to see from relation (30) that we simply have:

$$M_2 \, v_2'' = \Sigma'' \; \overline{p} d\sigma$$

because the resistance left outside the slipstream is in this case equal to zero.

I shall not go into a more detailed study of this important question of the propulsive efficiency. This would carry us too far into the propeller theory. Those who would like to

[3] For experimental data referring to the slipstream effect see "The design of screw propellers," London, 1920, pp. 192-196, by Henry C. Watts.

have a deeper understanding of the foregoing discussion are referred to the author's "General Theory of Blade Screws," previously mentioned.

B. PROPERTIES OF THE ENGINE.

Many discussions have been brought about by the question of how the brake horsepower L_m of a given gasoline engine, as actually used on airplanes, varies with the altitudes. Such discussions are rather a misunderstanding, because the power L_m does not depend on the altitude, but depends only upon:

1. The number of revolutions N at which the engine is running.
2. The throttle opening x.
3. The density δ and temperature T of the air in which the engine is working.
4. The quality of the gasoline used.

The question as to how density and temperature are connected with altitude depends exclusively upon meteorological conditions, which as known, are variable through the day, as well as through the year. In the following chapter the question of the standard atmosphere will be discussed briefly.

Since for a given mass of air, its pressure p, density δ and absolute temperature T are connected by the Claperyon relation $p/\delta = gRT$ where R is the gas constant, the brake horsepower can be as well considered as a function of the pressure p and temperature T. But since the propeller thrust and the forces of air resistance depend on the density δ, it is more convenient to relate the power L_m directly to the density δ.

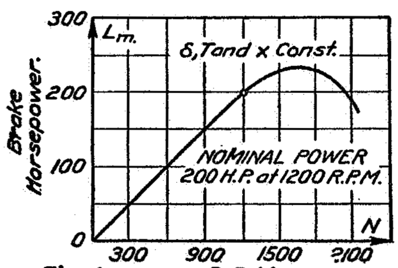


Fig. 4.

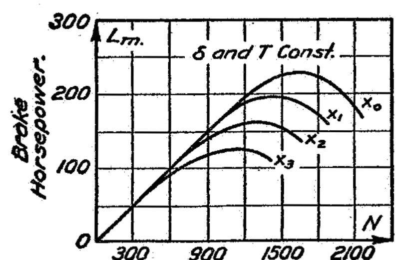


Fig. 5.

In figure 4 is represented the general course of the brake horsepower curve of a gas engine as a function of the revolutions N for δ, T and x constant. The power L_m generally first increases very closely proportionally to the revolutions, but afterwards, when the piston speed becomes too high, the power begins to drop, mainly on account of an incomplete filling of the cylinders by the carburetted air, whose flow speed is limited by the size of the suction pipes. This phenomenon is expressed generally by saying that the volumetric efficiency begins to drop, starting from a certain number of revolutions. The family of brake horsepower curves for δ and T constants, but for variable throttle openings x—a typical set of which is represented in figure 5—show well that the drop of power starting from a certain value of the engine revolutions is due to the drop of volumetric efficiency, because the smaller the throttle opening the earlier the power drop starts. In the figure, $x_0 > x_1 > x_2 > x_3$-----

In figure 6 is represented the general course of the power curve of a gas engine as a function of the density δ for N, T and x constants, and which for most gasoline engines is very close to a straight line. The density δ' (see fig. 6) is the small density at which the motor delivers just enough indicated power to compensate the mechanical losses, so that the brake horsepower is zero. For densities less than δ' power has to be applied to the engine in order to keep it rotating at a constant number of revolutions; p_0 indicates the mechanical losses of the engine when $\delta = 0$.

The general shape of the last power curve finds its explanation in the fact that the indicated horsepower is very closely proportional to the density δ; and, if the mechanical losses are considered as depending only slightly upon the density, *the linear dependence of the brake horsepower upon the density*, for N, T and x constants, follows.

In figure 7 is represented a family of brake horsepower curves as a function of N, for different values of the density δ, but for T and x constants, with $\delta_0 > \delta_1 > \delta_2 > \delta_3$..... (Compare with fig. 5.)

It follows from the foregoing that as a first approximation the power of a gasoline engine can be represented by the formula

$$L_m = mN(c\delta - c_0) \tag{43}$$

for the range of variation of N and δ that we meet in aviation practice. This last formula assumes that the engine is used in the interval of the linear variation of the power with the revolutions, and that the mechanical losses are proportional to the revolutions. For actual aviation engines the coefficients c and c_0 have for mean values

$$c = 11; \ c_0 = 0.1$$

so that for average computations, we can write

$$L_m = mN(11\delta - 0.1) \tag{44}$$

where the coefficient m is fixed by the value of the nominal power of the engine.[4]

We do not possess actually sufficient experimental data on the question of variation of the power L_m with the temperature T, the density δ being kept constant. In some tests the

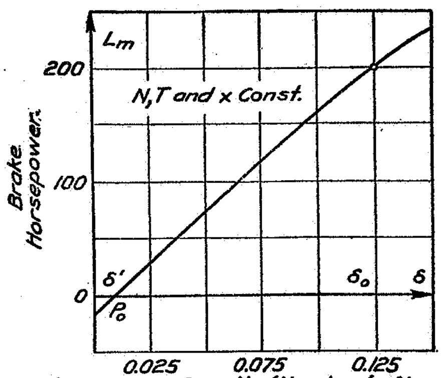


Fig. 6.

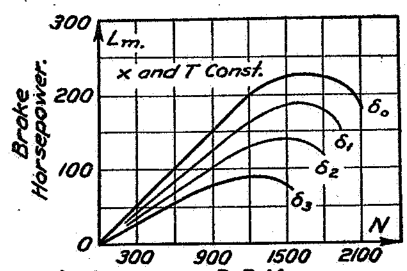


Fig. 7.

variation of the power L_m with T was observed for constant pressures. But in this case the main effect is the change of power produced by the change of density resulting from the temperature variation. It is the change of power with temperature at constant density that solely interests the aeronautical engineer for the study of altitude flight.

Sometimes engines are submitted to the following test: A propeller is fixed on the engine and this is run at different throttle openings x up to the full throttle x_0, and the curve of the power L_0 delivered by the engine is plotted as a function of the revolutions. The curve thus obtained has the general shape represented in figure 8. The main fact to be noted is that this curve is not the characteristic of the engine, but the characteristic of the propeller used, as tested at a fixed point. It is the curve that corresponds to the equation

$$L_m = L_0 = C_2' \delta N^3$$

which is a cubic parabola in N. It is possible by such tests to obtain the characteristic of the engine if it is tested either with a set of different propellers or with a variable-pitch propeller.

Suppose we run a first test with a propeller No. 1 and get the curve L_0^{I} on which we have carefully marked the different throttle openings x. Afterwards we run a test with a propeller No. 2 and get the curve L_0^{II} and so on. If we now join all the points of equal throttle openings

[4] The last formula assumes that at sea level the mechanical losses constitute around 7.5 per cent of the brake horsepower. For an engine giving at sea level 200 horsepower, at 25 revolutions per second, m turns out to be equal to 6.2.

we will get the engine characteristics L_m at constant density and different throttle openings x (See fig. 8.)

The foregoing explanations have been given only in order to recall briefly to mind those engine properties the knowledge of which is necessary for the study of our airplane steady-motion problem.

C. PROPERTIES OF THE ENGINE-PROPELLER SYSTEM.

As we have seen, the characteristics of a propeller are functions of the "advance" V/N. The characteristics of an engine are functions of the revolutions N for δ and x constant. It is easy to show that the characteristics of an engine-propeller system are functions of the flying speed V alone (for δ and x constant).

Let us consider an aviation engine with its propeller put on a railroad car and made to move along the axis of the engine-propeller system in air of density δ. Let us start by considering the car at *rest*. For a given *throttle opening*, if the engine is now set in motion it will reach a steady working condition at a certain number of revolutions N_0, at which the propeller will give a thrust Q_0. So far as we do not touch the throttle, the revolutions N_0 and the thrust Q_0 will remain constant. Let us now allow the car to run at a speed V. For each different value of the speed the engine will run at a different number of revolutions N and the propeller will give a different thrust Q, but if the car speed and throttle opening are kept unchanged, N and Q will remain constant. We thus see that for a given throttle opening x and air density δ, the revolutions N and the thrust Q of an engine propeller set are functions of the translation speed V alone. The main fact to be noted is that for an engine propeller set the revolutions have to adjust themselves to the speed V, which makes the thrust Q, for δ and x constant, depend upon the speed V alone.

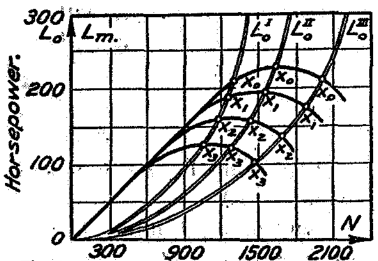


Fig. 8.

Let us now solve the following problem. A propeller is given to us by its characteristic curves:

$$\eta = F(V/N), \ L_a = \delta N^3 F_2''$$

and an engine, by its characteristic curve:

$$L_m = f(N)$$

for δ and x constant. The characteristics of the engine-propeller set have to be found.

Let us first deduce from the $L_m = f(N)$ curve, the curve of $L_m/\delta N^3$ as a function of N for the given δ and x. This last curve plotted, the following table is computed:

In column I we write selected values of $L_m/\delta N^3$.

In column II we write the corresponding values of N taken from the $L_m/\delta N^3$ curve plotted as a function of N.

I.	II.	III.	IV	V.	VI.	VII.	VIII.
$L/m\delta N^3$	N	V/N	η	V	$L_u/\delta N^3$	Q/δ	Q
–							
–							
–							

When a propeller is fixed to a given engine, the power absorbed by the propeller is equal to the power delivered by the engine and both run at the same number of revolutions (in case of gearing, the gearing constant has only to be introduced) that is, we have

$$\frac{L_m}{\delta N^3} = \frac{L_a}{\delta N^3}$$

Let us thus read from the $L_a/\delta N^3$ curve as a function of V/N the values of the advance V/N that correspond to the selected $L_m/\delta N^3$ values in Column I and write these values of the advance V/N in Column III.

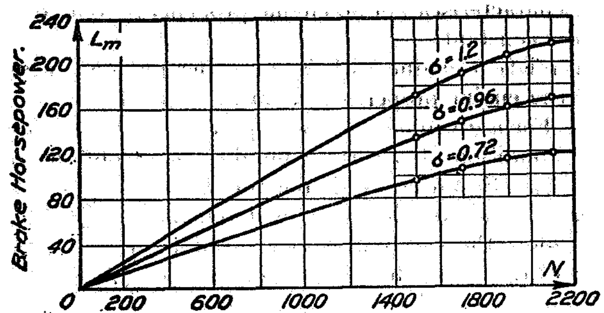


Fig. 9.

In column IV we shall write the values of the efficiencies η that correspond to these same advances V/N.

Multiplying Column II by Column III, we find the value of V (written in Column V) that correspond to the $L_m/\delta N^3$ values of Column I.

Multiplying Column I by Column IV, we find the value of $L_u/\delta N^3$ (L_u=useful power) that correspond to the values of V of Column V

Finally, multiplying Column VI by the corresponding values of N of Column II and dividing by the corresponding values of V in Column V, we find in Column VII the values of Q/δ as a function of the V of Column V. Finally, the values of the thrust Q are given in Column VIII.

Thus we are able to plot the curve of Q/δ or Q as a function of V for given δ and x.

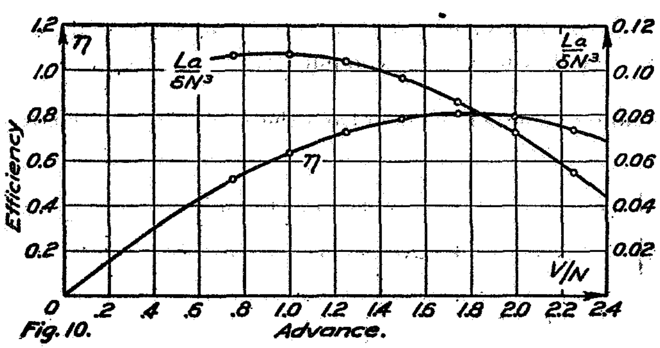


Fig. 10.

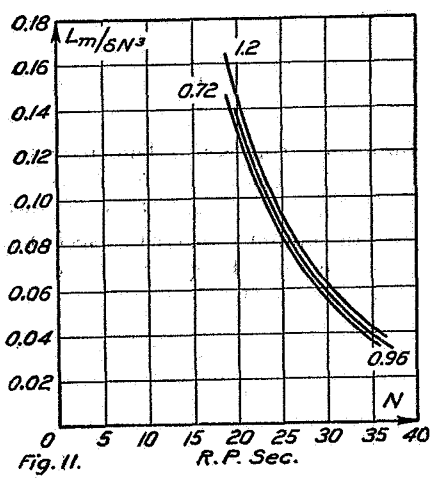


Fig. 11.

For each new values of δ and x, this computation has to be repeated. Columns I, III, IV, and VI do not change, and only Columns II, V, VII, and VIII have to be recomputed.

In such a manner, starting from the knowledge of the propeller characteristics as functions of the advance V/N and the engine characteristics as functions of N, δ, and x, the characteristics of the engine-propeller system, such as

$$Q,\ L_u,\ L_m,\ \eta, \text{ and } N$$

will be found as functions of V, δ, and x.

Following is an example of the application of this method. In figures 9 and 10 are represented the characteristics of the propeller and engine used. In figure 11 are represented the curves of $L_m/\delta N^3$ as a function of N.

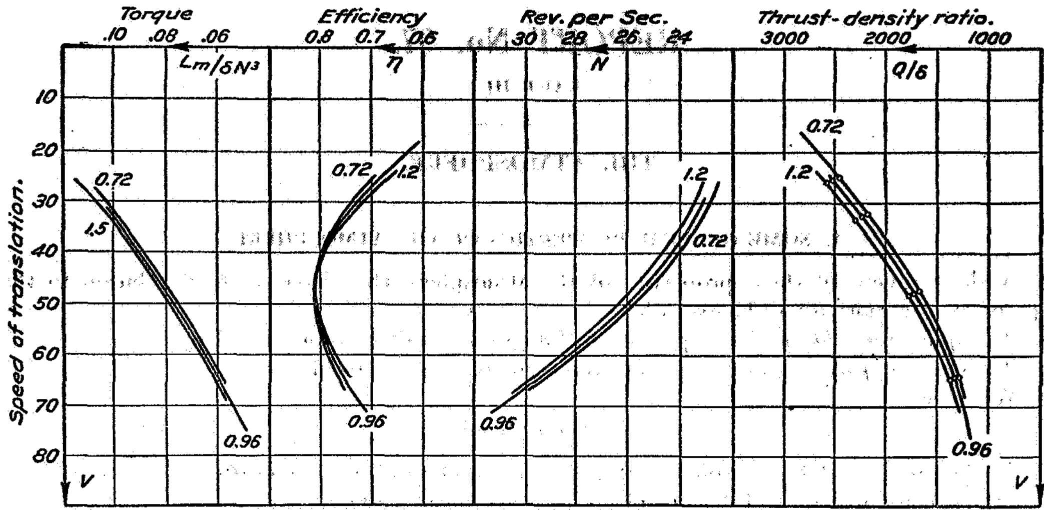


Fig. 12. — CHARACTERISTICS OF THE ENGINE-PROPELLER SET. —

In the following table are given the computations which are made for full throttle openings and for three different air densities. The characteristics obtained for the engine-propeller set are represented in figure 12, on which are plotted the curves of Q/δ, N, η, and $L_m/\delta N^3$ as functions of V for different densities. It can be easily seen that the sets of these curves can as a first approximation be reduced to one mean curve for each set, which is a consequence of the fact that the deviation of the motor power from proportionality to air density is not great.

We now see upon what factors the variation of the propeller thrust in flight depends and what laws it follows.

Computation of the characteristics of an engine-propeller system.

δ	$L_m/\delta N^3$	N	V/N	η	V	$L_m/\delta N^3$	Q	Q/δ
1.20	1.05	23.5	1.16	0.70	27.2	0.0735	325	2650
	1.00	24.0	1.40	.76	33.6	.0760	288	2350
	.080	26.5	1.87	.81	49.5	.0647	224	1830
	.060	30.0	2.20	.76	66.0	.0396	172	1400
.96	1.05	23.0	1.16	.70	26.7	.0735	247	2530
	1.00	23.5	1.40	.76	32.9	.0760	220	2250
	.080	26.0	1.87	.81	48.6	.0647	171	1750
	.060	29.6	2.20	.76	65.1	.0396	133	1360
.72	1.05	22.5	1.16	.70	26.1	.0735	177	2410
	1.00	23.0	1.40	.76	32.2	.0760	158	2150
	.080	25.5	1.87	.81	47.7	.0647	122	1660
	.066	29.2	2.20	.76	64.3	.0396	96	1315

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PART III.

THE ATMOSPHERE.

1. SOME GENERAL PROPERTIES OF THE ATMOSPHERE.

A short review of those properties of the atmosphere that have a direct relation to the airplane steady motion will be given here.

The specific weight of air—expressed in kilograms—will be represented by σ.

The density of air—expressed in newtons will be represented by δ.

We have

$$\sigma = g\delta \text{ with } g = 9,\ 81 mt/sec^2$$

At the pressure of 1 atmosphere$\cong$10330 klg/mt^2 and absolute temperature $T = 273° + 15° = 288°$, the specific weight and density of air have for mean values

$$\sigma = 1{,}225 \text{ klg}; \ \delta = 0{,}128 \text{ newton.}$$

At the same pressure of 1 atmosphere and zero degrees centigrade ($T = 273$) we find

$$\sigma_0 = 1{,}293 \text{ klg}; \ \delta_0 = 0{,}132 \text{ newton.}$$

Using the former values, the gas constant R, deduced from Clapeyron's relation

$$p = \sigma R T \tag{45}$$

has for its value

$$R = \frac{p}{\sigma T} = \frac{10330}{1{,}225 \times 288} = 29,\ 27 \tag{46}$$

Let us consider the atmosphere to be in a perfect static condition (no winds). If we rise in such an atmosphere through a distance dH, the pressure p will vary by an amount $-dp$ equal to

$$-dp = \sigma dH \tag{47}$$

or on account of (45)

$$\frac{-dp}{p} = \frac{dH}{RT} \tag{48}$$

(If in this last formula we consider $dp/p = 0.01$, $R = 29,\ 27$, $T = 273$ we find $dH \cong 80\ mt$. This means that a difference of pressure of 1 per cent in the atmosphere corresponds at 0° centigrade to a change of altitude of 80 *mt*.)

Integrating (48) we get

$$lg\frac{p}{p_0} = \frac{1}{R}\int_H^{H_0} \frac{dH}{T} \tag{49}$$

where p_0 is the pressure at the altitude H_0 and $H > H_0$. This last formula gives the value of the altitude H from the knowledge of pressure, when the law of variation of the temperature T with altitude is known. For $T = 273$; $p/p_0 \cong 0.5$ and $H_0 = 0$, we find $H \cong 5{,}000\ mt$. In an isothermic atmosphere of zero degrees centigrade, at the altitude of around 5000 *mt* the pressure is one-half of the ground level pressure.

2. DISCUSSION OF THE STANDARD ATMOSPHERE.

This question of the law of variation of temperature with altitude has been lately a matter of considerable discussion. Numerous so-called *standard atmospheres* have been proposed, which are supposed to be some kind of average deduced from different sets of meteorological observations. The Paris "Peace Conference of 1919" has even considered it necessary to fix by interallied agreement, some kind of standard atmosphere.

A careful examination of all the propositions made has brought me to the conclusion that this question of the standard atmosphere has been somewhat misunderstood.

Let us consider the whole question from a general standpoint and make clear for what purpose we need the standard atmosphere in aviation.

For each geographical position, at a given hour of a given day, there exists along the vertical drawn through the place considered, a certain distribution of pressures and temperatures. This distribution of pressures and temperatures depends upon the meteorological conditions and is variable through the whole year. The variations of this pressure and temperature are very important. It is well known that the same pressures and temperatures can be met at levels where altitude differences can amount to several thousands of meters. If a certain mean distribution of pressures and temperatures is adopted, the deviation from this mean distribution can also make up actual altitude differences of the order of a thousand meters.

On the other hand for an airplane, the forces of air-resistance, the propeller thrust, and the power of its engine are all functions of the air density and decrease with this. It is a property of the airplane to be able to reach a certain limiting small value of the air density, at which the airplane can still fly level, but is unable to climb any more. This limit of density is called the *ceiling density* δ_c. The aviation engineering problem consists in finding for each airplane its ceiling density. But the question, at what altitude this density δ_c is located is purely a meteorological question. The distribution of densities in the air is greatly different and the same density can be met on different days at very different altitudes. The question of the relation of densities and altitudes stand outside the aviation engineering problem and is merely a question of public curiosity. Technically speaking, we can only say that a given airplane has the ability to reach a certain density δ_c. The smaller this density δ_c, the greater is the climbing capacity of the airplane considered. There is no reason for expressing this density in altitude figures, because density already completely specifies the question. There is only one fact that must still be taken into account. The power of the airplane engine, at a given density, depends somewhat upon the temperature. When we speak of airplane performances, they must thus be referred to a certain temperature. In the selection of this temperature, we must be guided only by convenience and simplicity. There are no reasons to adopt a temperature variable with the altitude, but there are many reasons for adopting a constant temperature at all altitudes.

It is easy to see that, exactly speaking, it is rather standard conditions for engine work that we have to select than to adopt a standard atmosphere. If we make the temperature variable with altitude, in other words, with density, this would mean that the standard conditions adopted for engine work consist of a special temperature for each density. This introduces a very troublesome element in engine-power computations, which is neither necessary nor demanded by any reason. On the contrary, a constant temperature for all densities is a natural condition, demanded for the sake of simplicity of the standard conditions adopted.

We are thus brought to the conclusion that from the standpoint of aviation engineering the only standard atmosphere that can be reasonably adopted is the *isothermic atmosphere*.

The proposition of the author is to adopt for aviation engineering, as a standard atmosphere, an atmosphere of constant temperature in its whole mass equal to zero degrees centigrade.

The advantages of such a convention are as follows:

1. In all questions of design and performance prediction all temperature corrections are totally eliminated.

2. In airplane testing the only correction to be made is the temperature correction of the engine power and reduction of the performance to this corrected power. This correction is

quite simple, since we reduce each of the quantities to the same temperature independent of the altitude.

3. If we compare our isothermic atmosphere of zero degrees centigrade with the inter-allied standard atmosphere and with the atmosphere of an average winter and summer day, the isothermic atmosphere gives a general idea of the altitude quite as good as does the inter-allied standard in the sense that the altitude departures for the winter and summer day from the isothermic atmosphere and from the inter-allied standard are of the same order of magnitude. (See fig. 13.)

It is thus evident that everything speaks in favor of the isothermic atmosphere of zero degrees centigrade.

In all aviation engineering all data, computations, and performances should be expressed exclusively in densities of the isothermic atmosphere. The altitude language has to be used for the general public only. The altitude language is of no use to the aviation engineer and only creates unnecessary complications.[1]

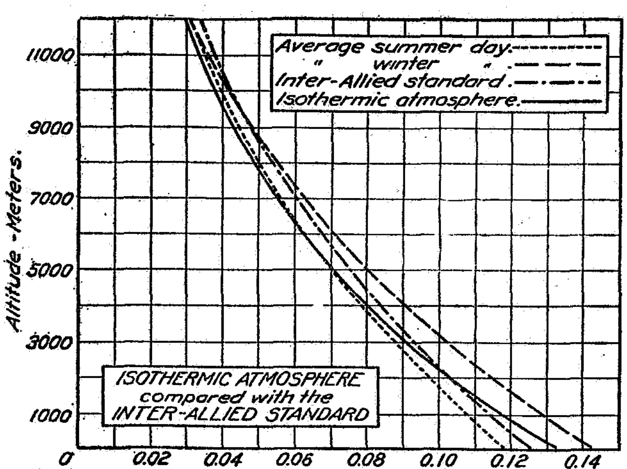


Fig. 13.

For some special problems we need to know the actual altitude of an airplane. But in such cases no standard atmosphere can be of any help to us. If we want to deduce with some accuracy the actual altitude from pressure measurements on an airplane, we have to record when climbing the laws of the actual variation of pressure and temperature with altitude.

Methods or instruments for quick computation, with a certain accuracy, of the actual altitude from such records can be developed.

As a general conclusion I will say: *It is fitted to the purpose to adopt for aviation engineering the isothermic atmosphere of zero degrees centigrade as standard atmosphere.*

3. CALCULATION OF THE RATE OF CLIMB FROM A BAROGRAM.

The vertical component U of the air speed V of a climbing airplane is generally called *rate of climb*. If in an element of time dt an airplane climbs a height dH, its rate of climb is equal to

$$U=\frac{dH}{dt} \tag{50}$$

or on account of formula (48)

$$U=\frac{RT}{p}\cdot\frac{-dp}{dt}=\sigma\,\frac{-dp}{dt} \tag{51}$$

By this last formula the rate of climb of an airplane can be found from the flight barogram, which gives the pressure p as a function of the time t. At each moment of time the slope of the tangent to the barogram curve at the point considered gives the value of $-dp/dt$ and the value of the specific weight follows from the corresponding values of pressure and temperature.

[1] The author is of the opinion that the scales of altimeters and barographs ought to be graduated in pressure units, pressure being the quantity that these instruments really measure. The author can not understand why it is considered "from a practical standpoint" preferable to have the pilot read the *wrong altitude* (the altitude scale of an altimeter being purely conventional) than to read the *exact pressure*. Very little practice would be required from pilots to accustom themselves to express the level reached in the atmosphere in pressure figures. This would be, physically, perfectly correct and would avoid a great deal of misunderstanding.

4. INFLUENCE OF WINDS AND SELF-SPEED ON COCKPIT PRESSURE.

Let us consider briefly what influence the atmospheric winds can have on the observed pressures. Consider two air masses at nearly the same altitude, in which the Bernouilli constant has the same value, one mass having no speed, the other having a speed of 10 meters per second which is itself a strong wind. We will find under such conditions that the pressures in these two air masses are related by the equation

$$p_1 = p_2 + \frac{\delta v^2}{2}$$

or

$$p_1 - p_2 = \frac{\delta v^2}{2} = \frac{0,125 \times \overline{20}^2}{2} = 6,25 klg/mt^2$$

At ground level with $p_1 = 10330$ klg/mt^2 this gives

$$p_2/p_1 = 1 - \frac{6,25}{10330} = 0,9994$$

The difference between p_2 and p_1 thus appears to be of the order of 0.05% of the atmospheric pressure, which corresponds at ground level to a difference of altitude of around 4 meters. At an altitude where the pressure would be 10330/2 (around 5,000 meters) the difference between p_1 and p_2 would be double and this would still correspond only to a difference of altitude of 8 meters. We are thus brought to the conclusion that ordinary winds will affect only slightly the calculation of altitude from pressure distribution.

A much more marked influence is that of the variation of the airplane speed V upon the measurements of pressures as made on an airplane. The difference between the static pressure p at the level where the airplane is flying and the pressure p' in the cockpit is very closely equal to

$$p - p' = \frac{\delta V^2}{2}$$

An airplane in climbing can have its speed reduced to about half of its horizontal self-speed, that would give for the cockpit pressures p'_1 and p'_2 corresponding to the two cases, a difference of

$$p_2' - p_1' = 0,75 \frac{\delta V^2}{2}$$

This is the difference between the "corrections" which are necessary in the two cases in order to determine, from the cockpit readings, the real static pressures. With $V \cong 50$ meters per second, at ground level, this can be about 1% of the atmospheric pressure, or 80 meters difference in altitude. Such differences have to be taken into account when pressure observations are made in the cockpit, at different values of the speed. The last circumstance may make it desirable to use special devices allowing the direct observation in the airplane of the static pressure, instead of the cockpit pressure.

REPORT No. 97.

PART IV.

THE THEORY OF STEADY MOTION.

1. THE BASIC EQUATIONS.

All of the properties of the airplane steady motion are a direct consequence of the fact that in *steady motion* all the forces acting on an airplane mutually balance.

In order to express the last condition let us consider an airplane in steady flight and draw from its center of mass vectors parallel to the main quantities involved in the problem. This will facilitate a determination of the angular relations. In this manner, on figure 13, have been represented:

G_f a reference line, invariably connected with the airplane wings, from which the angle of attack is measured.
V the self-speed.
i the angle of attack.
γ the angle of inclination of the flying path to the horizontal, counted positive for climbing.
P the total weight of the airplane.
R the total air-resistance.
R_x the drag—component of R along the speed V.
R_y the lift—component of R along the normal to the self-speed.
Q the propeller thrust.

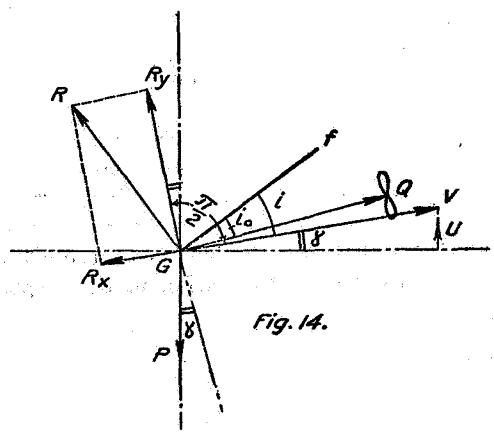


Fig. 14.

Airplanes are generally built in such a way that for horizontal flying in normal conditions, the thrust Q is directed along the speed V. The angle of attack that corresponds to those conditions will be designated by i_0.

The vertical component of V, designated by U in figure 14, is called the *rate of climb*.

$$U = V \operatorname{Sin} \gamma \tag{52}$$

Let us now project all the forces acting on the airplane on the direction of the speed V and the normal to it. The conditions of steady motion will then be expressed by

$$R_x = Q \operatorname{Cos}(i - i_o) - P \operatorname{Sin}\gamma \tag{53}$$

$$R_y = P \operatorname{Cos}\gamma - Q \operatorname{Sin}(i - i_o) \tag{54}$$

in which equations, we have:

$$R_x = k_x\, \delta\, A V^2 \tag{55}$$

$$R_y = k_y\, \delta\, A V^2 \tag{56}$$

where

k_x = drag coefficient.
k_y = lift coefficient.
δ = air density at flying level.
A = area of the airplane wings.

To these two equations (53) and (54) must be added the fundamental relation that connects the engine with the propeller.

$$L_m\,(x,\,\delta,\,N) = L_a = \delta N^3\, T_2''\left(\frac{V}{N}\right) \tag{57}$$

which expresses the fact that in steady flight the power L_m delivered by the engine—a function of x, δ and N—is always equal to the power L_a absorbed by the propeller—a function of N, δ and V/N.

The detailed discussion of the fundamental equations (53) and (54) is greatly complicated by the complex laws, fixed by the relation (57) governing the variation of the thrust Q in flight, which we have considered in full detail in the foregoing.

In order to allow a better survey of fundamental properties of the airplane in steady motion, without complicating the question by those factors that have only a slight influence on the quantitative value of the results and do not affect at all their general meaning, we shall make the following simplifications in equations (53) and (54).

I shall first remark that, on the one hand, we do not possess any reliable information as to the laws of variation of the propeller thrust for the case when the self-speed V makes a certain angle with the propeller axis, and on the other hand, since the angle $(i - i_o)$, as we shall see later, can take only small values in normal flying conditions, we shall consider it to be a sufficient approximation to assume

$$Q\ Cos\ (i - i_o) \cong Q$$
$$Q\ Sin\ (i - i_o) \cong O$$

It must be further noted, that in normal flying conditions, the angle of the flying path to the horizontal does not usually take large values. It seldom exceeds 15°, taking larger values only in steep dives and steep glides, which must be considered separately. We thus assume

$$Sin\ \gamma \cong \gamma;\quad Cos\ \gamma \cong 1.$$

Introducing these simplifications in the equations (53) and (54) we get:

$$R_x = k_x\, \delta\, A V^2 = Q - P\gamma \tag{58}$$

$$R_y = k_y\, \delta\, A V^2 = P \tag{59}$$

The simplifications we have made affect principally the value of the self-speed V, which we shall calculate from the equation $V = \sqrt{P/k_y\,\delta\,A}$ instead of the more exact relation $V = \sqrt{Cos\ \gamma}\sqrt{P/k_y\ \delta\ A}$. But it is easy to see that for $\gamma < 5°$ we have $Cos\ \gamma > 0,96$ and the error made in the speed will be less than $1 - \sqrt{0,96} \cong 0,02$, that is less than 2 per cent. In any case, when necessary, once the self-speed V obtained by equation (59) and the value of γ known, it is always possible to correct its value by taking it equal to $V\sqrt{Cos\ \gamma}$.

For the study of the airplane in steady motion, it is more convenient to consider, instead of the relations (58) and (59), the system of equations

$$k_y V^2 = \frac{P}{\delta A} \tag{60}$$

$$\gamma = \frac{Q}{P} - \frac{k_x}{k_y} \tag{61}$$

which follow directly from the last.

With the same approximation, the rate of climb U has for its value

$$U = V\gamma \tag{62}$$

It is from the system of equations (60) and (61) that we shall deduce all the properties of an airplane in steady motion. Once all of the mutual interrelations of all the quantities involved in the problem are perfectly established, all quantitative corrections, when demanded, can always be made post factum and in our statement of the problem, are only necessary in the case of steep climbing or gliding flights.

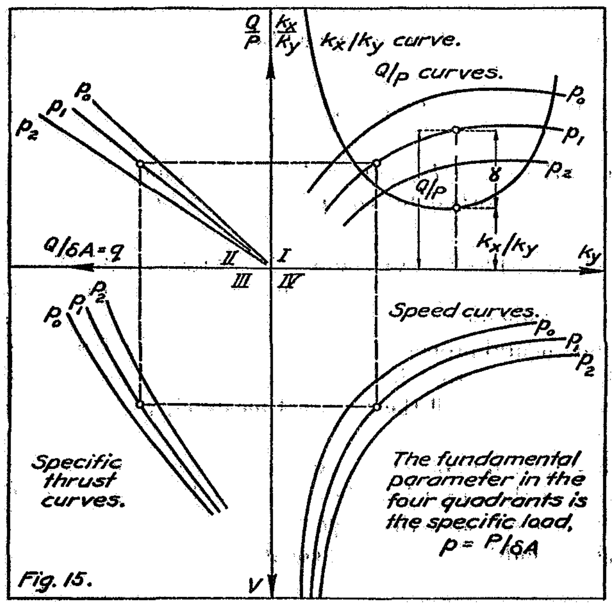


Fig. 15.

We shall start by the study of the steady motion of an airplane of constant total load weight P and constant wing area A, the engine working all the time at full throttle opening.

For this study we shall make use of a graphical interpretation of the equations (60) and (61) which consists of the following:

2. THE METHOD.

The lift coefficient k_y will be adopted as fundamental variable. As the lift coefficient k_y is a function of the angle of attack only, under the restrictions made, its value can be changed in flight only by a displacement of the control stick.

The four quadrants formed by two straight lines intersecting at right angles will be designated respectively by I, II, III, and IV (See fig. 15.)

In quadrant IV, the horizontal axis will be adopted as k_y axis, and the downward vertical axis as V axis, and the speed V will be plotted in this quadrant as a function of the lift coefficient k_y, according to equation (60). We obtain a system of hyperbola-like curves having $P/\delta A$ as parameter, which allow one to read directly the value of the speed V that corresponds to each value of the lift coefficient when the value of $P/\delta A$ is known.

It is important to remark that when the ratio $P/\delta A$ has the same value, independently of the special values that P, δ and A have, to each k_y corresponds the same speed V. On account of this we shall use the special symbol p to represent the ratio $P/\delta A$ and shall call it *specific load*.

$$p = \frac{P}{\delta A} \tag{62}$$

It will be seen from all that follows that the specific load p is the fundamental parameter of the whole problem of steady motion.

The curves of V as a function of k_y with p as parameter will be called *speed curves*. It is easy to see that the speed curves—in the approximation used in the problem—simply represent a mathematical relation independent of the special airplane considered, that is, the same family of speed curves corresponds to any airplane. The last property of the speed curves is a consequence of our selection of variables. A set of speed curves for p having the values p_o (ground level) p_1, p_2 . . . . . is schematically represented in quadrant IV of figure 15.

We have seen, when examining in the foregoing chapters the properties of the engine-propeller system, that the thrust Q given by such set is, for a constant throttle opening, a function of the speed V and density δ. In the study of the airplane in steady motion, for the sake of uniformity, we shall consider, instead of the thrust Q, the quantity $Q/\delta A$, which we shall call *specific thrust* and designate by q, i. e.,

$$q = Q/\delta A \tag{63}$$

It is easy to see that the specific thrust q will also be, for a constant throttle opening, a function of the speed V and density δ, or, in other words, a function of V and of the specific load $p = P/\delta A$ as P and A are constants of the airplane considered. We shall plot in quadrant III the curves of the specific thrust $p = P/\delta A$ as a function of the self-speed V with the specific load $p = P/\delta A$ as parameter, the horizontal axis being the q axis. A set of such specific thrust curves is represented in figure 15, with the parameter p having the same values p_o, p_1, p_2 as in quadrant IV. We have learned in previous chapters how to deduce this family of curves from the properties of the propeller and engine used on the airplane considered. The specific load curves allow us to deduce directly the value of the propeller thrust that corresponds to each flying speed.

Knowing the laws of variation of the thrust Q as a function of the speed V, it is easy to deduce the laws of variation of the thrust Q as a function of the lift coefficient k_y. For this purpose, let us consider the equation

$$\frac{Q}{P} = \frac{Q/\delta A}{P/\delta A} = \frac{q}{p} = y \tag{64}$$

and interpret it as a family of straight lines passing through the origin with q as abscissa, y as ordinate, and p as parameter, and plot these straight lines in quadrant II of figure 15, giving successively to p the values p_o, p_1, p_2 . . and using the vertical axis as the $y = Q/P$ axis. We shall call these last lines *transfer lines*.

This system of transfer lines once plotted in quadrant II, it is easy to trace directly in quadrant I, for each given value of the specific load p, the curve of the ratio Q/P as a function of k_y that corresponds to a given curve of $q = Q/\delta A$ as a function of V Each two corresponding points of two corresponding curves in quadrants III and I lie on the two diagonal vertices of a rectangle whose sides are parallel to the axes and whose two other vertices are located one on the speed curve in quadrant IV, the other on the transfer line in quadrant II, corresponding to the same value of the specific load p. (See fig. 15.) In such a way we can deduce in quadrant I, from the specific thrust curves of quadrant III, the curves of Q/P as a function of k_y with p as parameter, which give us the laws of variation of the thrust as a function of the lift coefficient. A set of such curves of Q/P as a function of k_y, for the same values p_o, p_1, p_2 . . of the specific load, is represented in quadrant I of figure 15. We now see that the chart represented on this figure has the property that *each four corresponding points in the four quadrants I, II, III and IV lie on the vertices of a rectangle with its sides parallel to the axes.*

Let us finally plot in quadrant I, in addition to the Q/P curves as a function of k_y, the curve of the drag-lift ratio k_x/k_y as a function of k_y. As the drag and lift coefficients are functions of the angle of attack only, the ratio k_x/k_y can be considered as a function of k_y only The general shape of the k_x/k_y curve as a function of k_y is represented in figure 15.

If we now remember equation (61), we see that the difference of the ordinates of the Q/P and k_x/k_y curves, for a given value of p, give directly the airplane path inclination γ (see figure 15).

We have thus succeeded in representing on the chart in figure 15 all the characteristics of the airplane in steady motion and their fundamental interrelations. The transfer lines of quadrant II and the speed curves of quadrant IV do not depend on the special type of airplane considered and thus are merely mathematical intermediaries. On the contrary, the curves of quadrant I and III constitute the characteristics of the airplane considered. Quadrant I with the k_x/k_y curve taken alone constitutes the characteristic of the airplane alone. Quadrant III, with the specific thrust curves, constitutes the characteristic of the engine-propeller system. Quadrant I, with the k_x/k_y and Q/P curves constitutes the characteristic of the airplane-engine-propeller system.

In Quadrant I, for a given value of k_y and p, we can read the values of k_x/k_y, Q/P and γ. In Quadrant IV we can read the corresponding values of the speed V; in Quadrant III we can read the corresponding value of the specific thrust $Q/\delta A$. We are at liberty to extend to the left Quadrant III, and plot, as had been done in figure 12, all the other characteristics of the engine-propeller set; and thus we shall obtain a complete graphical representation of the whole set of quantities involved in the steady motion of an airplane.

In order to make ourselves familiar with the above described chart, we shall discuss with its aid, in their general outline, the properties of the airplane in steady motion.

3. PROPERTIES OF STEADY MOTION.

All the curves of our basic chart represent quantities that can be measured directly, that is why all these curves can be considered as being deduced experimentally from direct tests. But for many purposes, it is convenient to have also analytical expressions, even if only to a first approximation, of all the curves of the four quadrants of the chart. That is why, deducing in the following the properties of an airplane in steady motion by the aid of the chart, we shall at the same time follow all the fundamental relations by the use of the following approximate equations.

We have already seen that the speed curves of Quadrant IV have for their equations

$$k_y V^2 = p \tag{65}$$

The shape usually obtained for the specific thrust curves of Quadrant III, allow us to use for their representation, with a sufficient approximation, an equation in the form of

$$\frac{Q}{\delta A} = q = q_0 - q_1 V^2 \tag{66}$$

the whole set of curves being represented by a single mean curve. The constant coefficients q_0 and q_1 are, as a first approximation, characteristic coefficients of the engine-propeller set. These coefficients q_0 and q_1 can be deduced from the mean specific thrust curve by the method of least squares. A justification of the last relation (66) will be found in the note at the end of this report.

By using equation (66) one sees that the ratio Q/P is equal to

$$\frac{Q}{P} = \frac{Q/\delta A}{P/\delta A} = \frac{q}{p} = \frac{q_0 - q_1 V^2}{p} \tag{67}$$

and on account of relation (65), we find for the Q/P curves of Quadrant I, the equation

$$\frac{Q}{P} = \frac{q_0}{p} - \frac{q_1}{k_y} \tag{68}$$

the specific load $p = P/\delta A$ being the parameter of this family of curves.

Rewriting equation (68) in the form

$$q_1 = k_y\left(\frac{q_0}{p} - \frac{Q}{P}\right) \tag{69}$$

it will be seen that the curves represent a family of equilateral hyperbolas having for asymptotes the Q/P axis and a line parallel to the axis, with its ordinate equal to

$$\frac{q_0}{p} = \frac{q_0 \delta}{P/A} \tag{70}$$

that is, proportional to the density δ. (See fig. 16.) When δ varies, the curves (69) merely move up or down, their ordinates changing proportionally to the density δ.

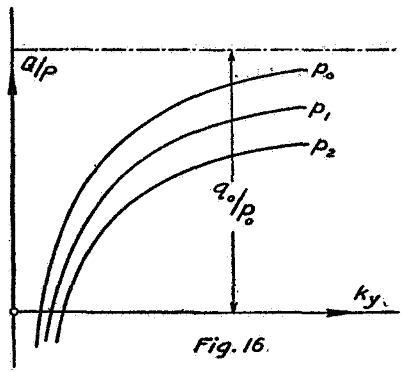


Fig. 16.

As, according to the approximate relations (8) and (9), the lift R_y is a linear function of the angle of attack, and the drag R_x is a quadratic function of the same angle, we can consider

$$R_x \cong k_y \;\delta\; AV^2\left(r\; k_y + t + \frac{\sigma}{k_y}\right); \tag{71}$$

that is, adopt for the $k_x/k_y = R_x/R_y$ curve, the approximate equation

$$\frac{k_x}{k_y} = r\; k_y + t + \frac{\sigma}{k_y} = y, \tag{72}$$

which represents a hyperbola, whose equation written in the form

$$r\; k_y^2 + (t-y)\; k_y + \sigma = f\;(y,\; k_y)$$

shows us that the center of this hyperbola has its coordinates given by

$$\frac{\partial f}{\partial k_y} = 2r\; k_y + t - y = 0$$

$$\frac{\partial f}{\partial y} = -k_y = 0$$

that is, is located at the point $k_y = 0$; $y = t$, and that the angular coefficients of the asymptotic directions are given by

$$r\; k_y^2 - y\; k_y = k_y\;(r\; k_y - y) = 0$$

that is, are equal to

$$k_y = 0 \text{ and } y/k_y = r$$

The hyperbola (72) is represented on figure 17, on which the coefficient t is assumed to be negative, as most generally is the case.

The minimum of k_x/k_y is given by

$$r\,k_y=\frac{\sigma}{k_y}$$

that is, takes place for

$$(k_y)_m=\sqrt{\frac{\sigma}{r}}$$

and is equal to

$$\left(\frac{k_x}{k_y}\right)_{min}=t+2\sqrt{r\sigma}$$

If we now remember that in the expression (71) it is the coefficient σ that depends mainly upon the parasite resistance of the airplane considered, we see that the center and the asymptotes of the hyperbola (72) are independent of the parasite resistance, and that with variable σ this hyperbola moves in and out between its invariable asymptotes.

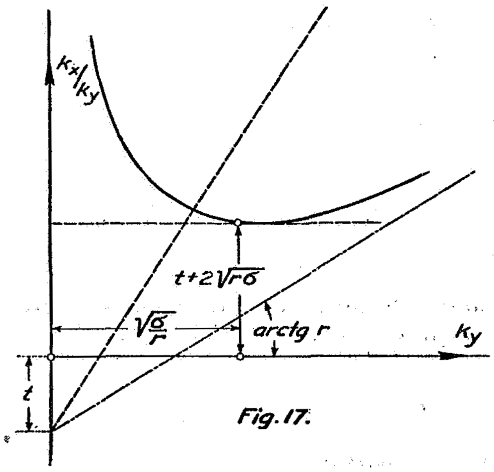


Fig. 17.

A. HORIZONTAL FLYING.

A horizontal flight of the airplane considered, at a level specified by a given value of the specific load $p=P/\delta A$ is only possible so far as the corresponding Q/P curve intersects in quadrant I with the k_x/k_y curve, because only for such points of intersection can we have $\gamma=0$. It is easy to see from figure 18 that there will be in general two such points of intersection, to which correspond two different values $(k_y)_1$ and $(k_y)_2$ of the lift coefficient. In quadrant IV we can read the two values V_1 and V_2 of the speed that correspond to these values of the lift. One of these speed values is greater than the other $(V_1>V_2)$. The greater value V_1 is called *high speed*, the smaller value V_2 *low speed*. In quadrant III we can read the corresponding values $Q_1/\delta A$ and $Q_2/\delta A$ of the specific thrust $(L_u)_1/\delta A$ and $(L_u)_2/\delta A$ of the power available; $(L_m)_1/\delta A$ and $(L_m)_2/\delta A$ of the power delivered; η_1 and η_2 of the propulsive efficiency and finally N_1 and N_2 of the revolutions. If desired, the power required can also be plotted, together with the power available.

Most airplanes can not fly at the low speed state of steady motion characterized by the values $(k_y)_2$; $(k_x/k_y)_2$; V_2; Q_2; N_2; η_2; $(L_u)_2$; $(L_m)_2$ because they do not have sufficient rudder areas to still keep control of the machine at this low speed.

When the airplane is considered flying at different increasing altitudes, the density δ decreases; that is, the value of the specific load p increases, the Q/P curve moves down and there will be a moment when it will become tangent to the k_x/k_y curve; that is, will have with the last only one point of intersection (see fig. 18). The value of the specific load that corresponds to this Q/P curve is the largest value p_c at which level flying of the airplane is still possible; and impossible for any larger value. The airplane has reached its *ceiling*, determined by the *ceiling specific load* p_c. In quadrant IV we can read the speed V_c at the ceiling, and in quadrant III, all the other ceiling characteristics.

It is easy to see and trace on the chart in quadrant IV the curve of the horizontal speeds at all altitudes. For this purpose it is sufficient to project on the corresponding speed curves the points of intersection of the k_x/k_y curve in quadrant I with the Q/P curves (see fig. 18). The speed curve to which this last curve is tangent gives again the value of the specific load p_c at the ceiling. The point at which these two curves are tangent separates the high speed states of steady motion from the low speed states.

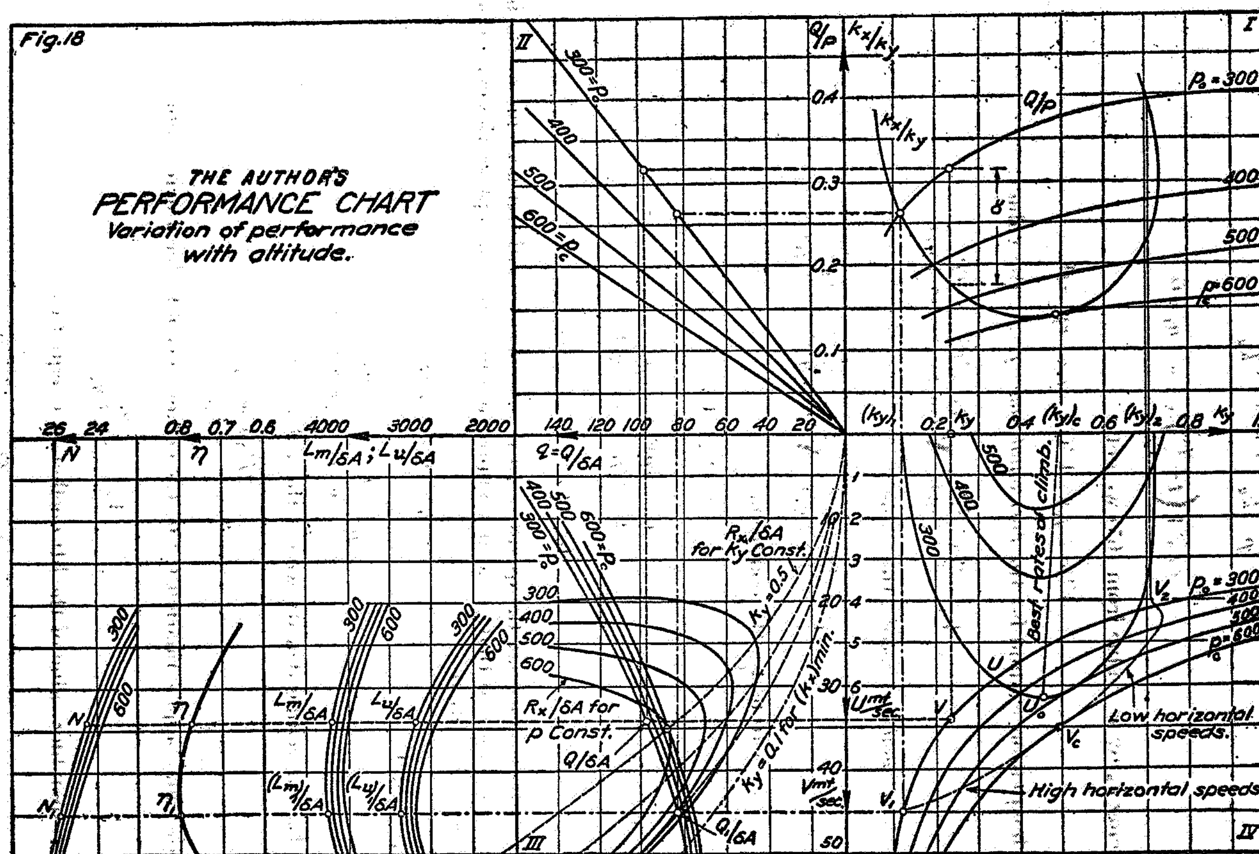


Fig. 18
THE AUTHOR'S
PERFORMANCE CHART
Variation of performance
with altitude.
I
II
III
IV
Q/p
kx/ky
Q/p
kx/ky
p0=300
400
500
p0=600
300=p0
400
500
600=p0
Lm/δA ; Lu/δA
q=Q/δA
(ky)1
ky
(ky)c
(ky)2
N
η
Rx/δA for Ky Const.
Ky=0.5
Ky=0.1 for (kx)min.
Rx/δA for p Const.
Q/δA
U mt/sec
V mt/sec
Best rates of climb.
Low horizontal speeds.
High horizontal speeds.
U
U0
V
V1
V2
Vc
N1
η1
(Lm)/δA
(Lu)/δA

Making use of our approximate equations (68) and (72), we may find the values of the horizontal speeds at all altitudes from the condition $Q/P=kx/ky$, that is

$$\frac{q_0}{p}-\frac{q_1}{k_y}=r\,k_y+t+\frac{\sigma}{k_y} \tag{73}$$

or

$$r\,k_y^2+\left(t-\frac{q_0}{p}\right)k_y+\sigma+q_1=0 \tag{74}$$

Replacing k_y by its value from (65) we get

$$V^4(\sigma+q_1)+V^2(tp-q_0)+rp^2=0 \tag{75}$$

From which equation we find for the *horizontal high speeds*, the values:

$$V_1^2=\frac{(q_0-tp)}{2(\sigma+q_1)}+\sqrt{\frac{(q_0-tp)^2}{4(\sigma+q_1)^2}-\frac{rp^2}{(\sigma+q_1)}} \tag{76}$$

and for the *horizontal low speeds*, the values:

$$V_2^2=\frac{(q_0-tp)}{2(\sigma+q_1)}-\sqrt{\frac{(q_0-tp)^2}{4(\sigma+q_1)^2}-\frac{rp^2}{(\sigma+q_1)}} \tag{77}$$

The ceiling is given to us by the condition

$$V_1=V_2 \tag{78}$$

that is,

$$\frac{(q_0-tp)^2}{4(\sigma+q_1)^2}=\frac{rp^2}{(\sigma+q_1)} \tag{79}$$

or

$$(q_0-tp)^2=4(\sigma+q_1)rp^2 \tag{80}$$

I shall remark here that the quantity tp is in general small (on account of t being small) in comparison with q_0, so that, as a first approximation, we can replace the condition (80) by the approximate condition

$$q_0^2=4(\sigma+q_1)rp^2 \tag{81}$$

from which we find:

$$p_0=\frac{P}{\delta_0 A}=\frac{q_0}{2\sqrt{r(\sigma+q_1)}} \tag{82}$$

and finally for the ceiling density we get the value

$$\delta_0=\frac{2P\sqrt{r(\sigma+q_1)}}{Aq_0} \tag{83}$$

With the same approximation ($t\cong 0$) the ceiling speed V_c has for its value

$$V_c^2=\frac{q_0}{2(\sigma+q_1)} \tag{84}$$

and the corresponding value of the lift coefficient is equal to

$$(k_y)_c=\frac{p_c}{V_c^2}=\sqrt{\frac{\sigma+q_1}{r}} \tag{85}$$

It is easy to see that this last ceiling lift value is somewhat larger than the value of the lift

$$(k_y)_m=\sqrt{\frac{\sigma}{r}} \tag{86}$$

at which k_x/k_y is a minimum.

It is worth notice that the power required for horizontal flying

$$R_x V = k_x \delta A V^3 = P p^{1/2} \frac{k_x}{k_y^{3/2}} \simeq P p^{1/2} (r k_y^{1/2} + \sigma k_y^{-3/2})$$

has a minimum for a lift value $(k_y)_M$ given by

$$\frac{\partial}{\partial k_y}\left(\frac{kx}{k_y^{3/2}}\right) = 1/2 r k_y^{-1/2} - 3/2 \sigma k_y^{-5/2} = O$$

and equal to

$$(k_y)_M = \sqrt{\frac{3\sigma}{r}} \tag{87}$$

Usually q_1 is of the same order of magnitude as σ, thus $(k_y)_o$, being greater than $(k_y)_m$, has a value close to $(k_y)_M$.

As has been mentioned already, the ceiling of an airplane is characterized by the value p_o of the specific load, but the ceiling density δ_o depends upon the weight P, that is, upon the loading of the airplane. The loading of each airplane can be increased up to such a value that its ceiling will be dropped to the ground level. The value of this *limiting load* P_0 is given by

$$p_o = \frac{P}{\delta_c A} = \frac{P_0}{\delta_0 A} = \frac{q_0}{2\sqrt{r(\sigma + q_1)}} \tag{88}$$

where δ_0 is the ground level air density. Hence.

$$P_0 = P \frac{\delta_0}{\delta_c} = \frac{q_0 \delta_0 A}{2\sqrt{r(\sigma + q_1)}} \tag{89}$$

It is of interest to add to the specific thrust curves of quadrant III, also the two families of the following curves.

In the first place, the curves of $R_x/\delta A$ as a function of V with p as parameter. Since $R_x/R_y = k_x/k_y$, we obtain these curves directly on the chart by transferring the k_x/k_y curve of quadrant I to quadrant III by the aid of the speed curves of quadrant IV, and transfer lines of quadrant II. Each speed curve and straight line corresponding to a given value of p will allow us to get one $R_x/\delta A$ curve in quadrant IV for the same value of p (see fig. 18). Since

$$R_x \simeq k_y \delta A V^2 \left(r k_y + t + \frac{\sigma}{k_y} \right) \text{ and } k_y V^2 = p,$$

these $R_x/\delta A$ curves have for their approximate equation

$$\frac{R_x}{\delta A} = V^2 (r k_y^2 + t k_y + \sigma)$$

or

$$\frac{R_x}{\delta A} = \frac{r p^2}{V^2} + pt + \sigma V^2 \tag{90}$$

With p as parameter, this equation represents a family of hyperbolas whose envelope is given by the relations

$$\frac{R_x}{\delta A} = \frac{r p^2}{V^2} + pt + \sigma V^2.$$

$$\frac{\partial (R_x/\delta A)}{\partial p} = \frac{2\, rp}{V^2} + t = O$$

and has for its equation

$$\frac{R_x}{\delta A} = V^2 \left(\sigma - \frac{t^2}{4r} \right) \tag{91}$$

In the second place, the curves of $R_x/\delta A$ as a function of V but with k_y as parameter. In order to plot these curves in quadrant III, it is sufficient to transfer the k_x/k_y curve of quadrant I to quadrant III, keeping for each traced curve k_y constant, and making use of all the speed curves of quadrant IV, and transfer lines of quadrant II for each value of k_y. It is in such a way that these curves have been traced on figure 18. These curves have for their approximate equation

$$\frac{R_x}{\delta A}=V^2(r\,k_y^2+t\,k_y+\sigma) \tag{92}$$

and represent a family of parabolas with the horizontal axis as the axis of symmetry and p as parameter. The parabola that corresponds to the minimum of $(r\,k_y^2+t\,k_y+\sigma)$, that is, to the minimum of R_x for a given V, which takes place for

$$k_y=\frac{-t}{2r}$$

has for its equation

$$\frac{R_x}{\delta A}=V^2\left(\sigma-\frac{t^2}{4r}\right) \tag{91}$$

and is the outermost of all the other parabolas of the family. This limiting parabola is the envelope of the family of hyperbolas defined by equation (90). (See fig. 18.)

The point in which a $Q/\delta A$ curve cuts a $R_x/\delta A$ curve with p as parameter gives us the horizontal speed for the corresponding value of p, and the $R_x/\delta A$ curve with k_y as parameter passing through that point gives the corresponding value of k_y.

B. CLIMBING.

If, starting from a given state of horizontal flying, the pilot by moving his stick varies k_y (see fig. 18) we can immediately see on the chart what value the path inclination γ will take. We shall be able easily to follow on the chart how γ, V, Q, L_u, L_m, η, and N will vary with variable k_y but constant p. A decrease of k_y will bring us to negative values of γ, the airplane will go down. An increase of k_y will cause the airplane to climb. The path inclination γ will pass through a maximum and decrease again until we reach the slow speed horizontal flight. Since for each value of k_y we know the corresponding values of γ and V, it is easy to compute the rate of climb

$$U=\gamma V$$

We can trace in quadrant IV the rate of climb curve as a function of k_y, plotting U on the V axis (but on a different scale). In such a way the U curves on the chart of figure 18 have been obtained for different values of p.

The maximum rate of climb decreases with increasing specific load until it becomes equal to zero at the ceiling.

Let us calculate the value of U using our approximate equations. We have (with $t\cong 0$)

$$\gamma=\frac{Q}{P}-\frac{k_x}{k_y}=\left(\frac{q_0}{p}-\frac{q_1}{k_y}\right)-\left(rk_y+\frac{\sigma}{k_y}\right) \tag{93}$$

or

$$\gamma=\frac{1}{k_y}\left[\frac{q_0}{p}k_y-rk_y^2-(\sigma+q_1)\right] \tag{94}$$

and with $k_y V^2=p$ the rate of climb is found equal to

$$\gamma=\frac{1}{k_y}\left[\frac{q_0}{p}k_y-rk_y^2-(\sigma+q_1)\right] \tag{95}$$

The rate of climb will be a maximum for

$$\frac{\partial U}{\partial k_y}=0$$

that is

$$k_y^2+\frac{q_0}{pr}k_y-\frac{3(\sigma+q_1)}{r}=0$$

which gives

$$k_y=-\frac{q_0}{2pr}+\sqrt{\frac{q_0^2}{4p^2r^2}+\frac{3(\sigma+q_1)}{r}}$$

or, on account of (82),

$$k_y=\frac{q_0}{2pr}\left[-1+\sqrt{1+3\frac{p^2}{p_c^2}}\right] \tag{96}$$

This value of k_y, when introduced in (95), will give the maximum value of the rate of climb U corresponding to each value of the specific load p.

The value of k_y at which the "best climb" takes place in general increases with the altitude and reaches its largest value at the ceiling. But the ceiling value of k_y is generally not greatly different from all the set of values given by (96); and since, on the other hand, a function does not change much near its maximum the rate of climb computed will not be greatly different from its maximum if it is assumed to take place at a constant lift equal to the ceiling value,

$$(k_y)_c=\sqrt{\frac{\sigma+q_1}{r}}$$

with which the ceiling will always be reached.

The rate of climb, with k_y having this value, is equal to

$$U=\frac{a}{p^{1/2}}-bp^{1/2} \tag{97}$$

where

$$a=\frac{q_0r^{1/4}}{(\sigma+q_1)^{1/}}=\frac{q_0}{\sqrt{(k_y)_c}};\ b=2r^{3/4}(\sigma+q_1)^{1/4}=2r\sqrt{(k_y)_c} \tag{98}$$

since, on account of (82) and (85), we have

$$q_0=2rp_c(k_y)_c, \tag{99}$$

we can put the expression (97) of the rate of climb in the form

$$U=\sqrt{2rq_0}(\sqrt{p_c/p}-\sqrt{p/p_c}) \tag{100}$$

The rate of climb at ground level is equal to

$$U_0=\sqrt{2rq_0}(\sqrt{p_c/p_0}-\sqrt{p_0/p_c}) \tag{101}$$

Let us calculate, under the condition of a climb with $k_y=(k_y)_c$ the time of climb t, from ground level, characterized by the value p_0 of the specific load, up to the level of specific load $p<p_c$.

According to the relations (45) and (47), we have for an isothermic atmosphere

$$dp=d\sigma RT=-\sigma dH$$

where here p is the atmospheric pressure. Thus

$$dH=-\frac{d\sigma}{\sigma}RT$$

But, since

$$U=\frac{dH}{dt} \text{ and } \frac{d\sigma}{\sigma}=\frac{d\delta}{\delta}=\frac{d\left(\frac{P}{pa}\right)}{\left(\frac{P}{pa}\right)}=-\frac{dp}{p}$$

where here p is again the specific load, we find

$$dt=\frac{dH}{U}=\frac{RTdp}{pU} \tag{102}$$

Making use of the value (97) for U and integrating, we get

$$t=RT\int_{p_0}^{p}\frac{dp}{ap^{1/2}-bp^{3/2}}$$

Substituting $p^{1/2}=z$ we get

$$t=2RT\int_{z_0}^{z}\frac{dz}{a-bz^2}$$

or

$$t=\frac{RT}{\sqrt{ab}}lg_e\frac{(\sqrt{ab}+bz)(\sqrt{ab}-bz_0)}{(\sqrt{ab}-bz)(\sqrt{ab}+bz_0)}$$

On account of (98) and (82) we have

$$\sqrt{ab}=bz_c=b\sqrt{p_c}=\sqrt{2rq_0}$$

and thus

$$t=\frac{RT}{\sqrt{2rq_0}}lg_e\frac{(1+\sqrt{p/p_c})(1-\sqrt{p_0/p_c})}{(1-\sqrt{p/p_c})(1+\sqrt{p_0/p_c})} \tag{103}$$

Taking account of (102) we finally have:

$$t=\frac{RT}{U_0}\left(\sqrt{p_c/p_0}-\sqrt{p_0/p_c}\right)lg_e\frac{(1+\sqrt{p/p_c})(1-\sqrt{p_0/p_c})}{(1-\sqrt{p/p_c})(1+\sqrt{p_0/p_c})} \tag{104}$$

The last two formulae give us the time of climb in an isothermic atmosphere of temperature T, from ground level up to the level of specific load p. For $p=p_c$ the time of climb turns out to be infinite. There is nothing atonishing in this last fact because the reaching of the ceiling is an asymptotic phenomenon.

Formula (104) applied to the isothermic atmosphere of zero degrees centigrade T=273° and the time t expressed in minutes, gives

$$t_{\min}=\frac{307}{U_0}(\sqrt{p_c/p_0}-\sqrt{p_0/p_c})\ lg_{10}\frac{(1+\sqrt{p/p_c})\ (1-\sqrt{p_0/p_c})}{(1-\sqrt{p/p_c})\ (1+\sqrt{p_0/p_c})} \tag{105}$$

Let us finally find the direct relation between the rate of climb U—with $k_y=(k_y)_c$—and the altitude H in the isothermic atmosphere.

According to (102) we have

$$dH=RT\frac{dp}{p} \tag{106}$$

Integrating this last relation, from ground level up to the altitude of specific load p, we get

$$H=RT\,lg\frac{p}{p_0} \tag{107}$$

The ceiling altitude is equal to

$$H_c = RT\, lg \frac{p_c}{p_o} \tag{108}$$

Subtracting (107) from (108) we find:

$$(H_c - H) = RT\, lg\; p_c/p = z \tag{109}$$

where z is the altitude below the ceiling. From this relation we get

$$\sqrt{p_c/p} = e^{\frac{z}{2RT}};\quad \sqrt{p/p_c} = e^{\frac{-z}{2RT}} \tag{110}$$

Substituting these last values in (100) we find

$$U = \sqrt{2rq_o}\left[e^{\frac{z}{2RT}} - e^{\frac{-z}{2RT}}\right] = 2\sqrt{2rq_o}\,\mathrm{Sinh}\frac{z}{2RT} \tag{111}$$

Developing Sinh in series, and keeping only the first term, which is a sufficient approximation even for the highest altitude actually reached, we find:

$$U \cong \frac{z\sqrt{2rq_o}}{RT} = \frac{(H_c - H)\;\sqrt{2rq_o}}{RT} \tag{112}$$

For the ground level

$$U_o = \frac{H_c\sqrt{2rq_o}}{RT} \tag{113}$$

We thus also have

$$U = U_o\left(1 - \frac{H}{H_c}\right) \tag{114}$$

The two formulae (114) and (112) show us that as a first approximation, the rate of climb is a *linear function of the altitude H.*

As a result of long experience in the measurement of rates of climb of airplanes, in free flight, it has always been observed that the rates of climb appeared to be linear functions of the latitude. This fact brings us to the conclusion that all the assumptions we have made in the foregoing really constitute an approximation practically fully sufficient and that, to the approximation with which we actually measure rates of climb, the climbing takes place as if the atmosphere was isothermic. One can thus see that the isothermic atmosphere appears to be quite as good as any other standard atmosphere, but in addition the isothermic atmosphere has all the advantages of being the simplest conventional atmosphere.

It is easy to deduce from formula (114) the time of climb as a function of the altitude. We have,

$$dt = \frac{dH}{U} = \frac{H_c}{H_o}\frac{dH}{H_c - H} \tag{115}$$

and integrating we get:

$$t = \frac{H_c}{U_o}\, lg_e \frac{H_c}{H_c - H} \tag{116}$$

The formula:

$$t_{\min} = 0{,}\,0384\,\frac{H_c}{H_o}\, lg_{10}\frac{H_c}{H_c - H} \tag{117}$$

gives the time of climb in minutes, from ground level up to the altitude $H < H_c$. If we take conventionally $H = 0{,}\,95\; H_c$ we find

$$t_{\min} = 0{,}\,05\frac{H_c}{U_o} \tag{118}$$

We will get the exact expression for the time of climb in an isothermic atmosphere, if we use in equation (115) the value (111) of the rate of climb. We thus obtain

$$dt=\frac{dH}{2\sqrt{2rq_o}\ \mathrm{Sinh}\frac{H_c-H}{2RT}}$$

and integrating we find:

$$t=\frac{-RT}{\sqrt{2rq_o}}\int_0^H\frac{d\left(\frac{H_c-H}{2RT}\right)}{\mathrm{Sinh}\frac{H_c-H}{2RT}}=\frac{RT}{\sqrt{2rq_o}}lg_e\frac{\mathrm{tgh}\left(\frac{H_c}{4RT}\right)}{\mathrm{tgh}\left(\frac{H_c-H}{4RT}\right)}$$

or, since

$$U_o=2\sqrt{2rq_o}\ \mathrm{Sinh}\ \frac{H_c}{2RT}$$

we finally find:

$$t=\frac{2RT\,\mathrm{Sinh}\frac{H_c}{2RT}}{U_o}lg\ \frac{\mathrm{tgh}\left(\frac{H_c}{4RT}\right)}{\mathrm{tgh}\left(\frac{H_c-H}{4RT}\right)} \tag{119}$$

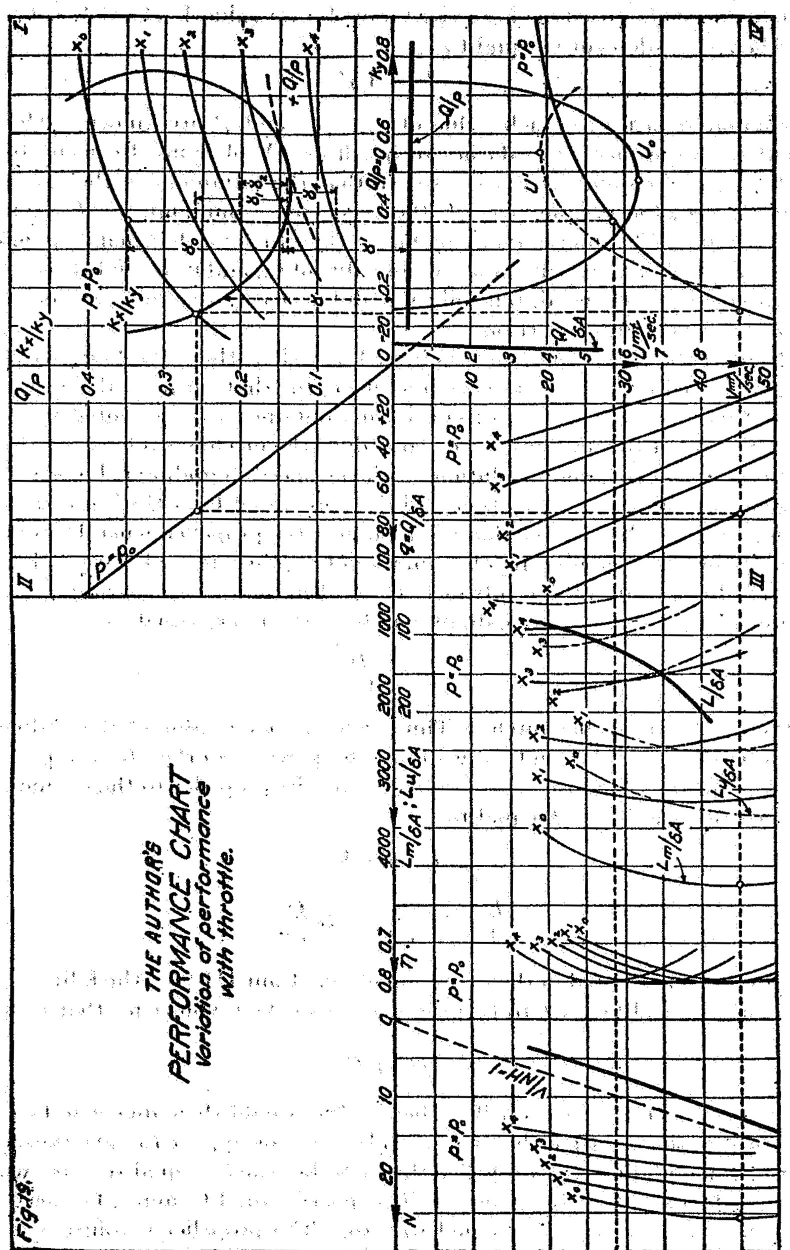
This last formula gives the exact value of the time of climb in an isothermic atmosphere of absolute temperature T, from ground level up to the altitude $H<H_c$ where U_o is the rate of climb at the ground, and the whole climb is supposed made at a constant value of the lift coefficient k_y equal to its ceiling value $(k_y)_c$.

C. ENGINE THROTTLING AND GLIDING.

Until now we have considered the flight of the airplane at full throttle opening and variable altitude. Let us now consider the flight of the airplane at constant altitude, for example close to the ground level, but with variable throttle. Returning to our chart, we see that the k_x/k_y curve of quadrant I is not affected by the throttle; the speed curves of quadrant IV and transfer lines of quadrant II are also unaffected by the throttle; but the specific thrust curves of quadrant II depend directly upon the throttle opening. Proceeding as was indicated in the chapter dealing with the engine-propeller system, it will be easy to compute the curves of specific thrust as a function of the speed for different throttle openings: x_0 (full throttle), x_1, x_2, x_3 ------. A set of such specific thrust curves have been represented in figure 19. A variation of the throttle means a variation of the engine power, which brings with it a shifting of the specific thrust curves. For a given speed the specific thrust drops when the throttle is reduced. In the extension of quadrant III have been plotted the curves of $L_u/\delta A$, $L_m/\delta A$, η, and N, corresponding to the different throttle openings x_0, x_1, x_2, x_3, x_4 ------. Quadrant III and extension thus give us now a complete representation of the engine-propeller system characteristics for variable throttle openings.

Once the set of specific thrust curves with variable throttle is established, it is easy to plot in quadrant I the corresponding Q/P curves, making use of the speed curve and transfer line of the altitude considered, the throttle opening being now our parameter. The chart thus completed, the influence of the engine throttling on the flight of the airplane becomes self-evident.

Let us consider the airplane first flying at full throttle opening x_0, at a certain value of k_y, the inclination of the flying path to the horizontal having the value γ. (See fig. 19.) The corresponding value of the speed is given by the speed curve of quadrant IV, and the other flying characteristics can be read from quadrant III and extension. If we now begin to throttle the engine, the path inclination will successively take the values γ_1, γ_2; and for the throttle opening x_3, for example, the flight will already be horizontal. The speed will *not be affected*—provided the action of the slip-stream on the rudders can be neglected, as has been already explained—because for invariable k_y and the same altitude, characterized by the value p_0 of the specific load, we read the same speed on the same speed curve. But all the other flying characteristics vary with the throttle, as can be seen directly from quadrant III and extension. For the throttle opening x_4 the path inclination γ_4 will be negative; that is, we will be descending with motor on. For each altitude there is a throttle opening, for which



the Q/P curve is tangent in quadrant I to the k_x/k_y curve (curve in dashes on fig. 19). At this throttle opening, the altitude considered is the ceiling. For any smaller throttle opening, the airplane will always be descending.

Let us now consider the engine power to be cut off, the airplane can only be descending; it is said to be *gliding*. When gliding, the propeller generally works as a windmill and thus will give us a negative thrust. We can plot for the altitude considered the $-Q/P$ curve on the negative extension of the axis of quadrant I (see fig. 19); we shall then be able to read, for each value of k_y, the exact inclination γ' of the airplane gliding trajectory, between the k_x/k_y curve and this $-Q/P$ curve.[1] The gliding speed will be read in quadrant IV on the same speed curve. For large values of γ', corresponding to small values of k_y, we shall get a more accurate value of the speed in gliding by taking for its value $V\sqrt{\text{Cos}\ \gamma'}$, as has been already explained. The rate of descent is equal to

$$U' = V\gamma'$$

and can be computed easily for each value of k_y, as V and γ^1 are known. The curve of the rate of descent has been plotted in dashes in quadrant IV of figure 19, using for it the same scale as for the rate of climb. It is easy to see that the minimum of the rate of descent U' does not coincide with the minimum of γ', the last being a minimum for k_x/k_y minimum; the value of k_y that makes U' a minimum being larger than the one that makes γ' minimum. We now see that, if an airplane is gliding at a certain value of k_y, and if we slightly open the throttle, it is not the speed that is changed, but only the gliding path; the angle γ is decreased, and the rate of descent is reduced in proportion.

We can transfer the $-Q/P$ curve of Quadrant I as well as the $-Q/\delta A$ curve of Quadrant III, by the aid of the speed curve and transfer line, using for that purpose the negative extension of the specific thrust axis. The $-Q/\delta A$ curve thus obtained is represented by a thick line on figure 19. By the aid of this last curve we can deduce the mechanical losses of the engine when the law of variation of the engine revolutions with the speed in gliding is known. Such a curve of the revolutions in gliding as a function of V is represented by a thick line at the end of the extended Quadrant III. The fact is, that when gliding, the propeller generally rotates at a much less number of revolutions than its regular number of revolutions and thus works as a windmill, with a relatively high value of the advance V/N. But under such conditions, the efficiency of η' of the propeller, working as a windmill, will be very closely equal to

$$\eta' = \frac{NH}{V} \tag{120}$$

where H is the *effective* propeller pitch.[2] Thus, if for a given value of the gliding speed V we know the corresponding $-Q/\delta A$ and N, we have the power absorbed by the propeller working as a windmill equal to $Q\,V$, and the power delivered by the propeller to the engine and absorbed by the last as mechanical losses L equal to

$$L = \eta'\,QV \tag{121}$$

or

$$\frac{L}{\delta A} = \eta' V \frac{Q}{\delta A} = NH \frac{Q}{\delta A} \tag{122}$$

The curve of $L/\delta A$ is represented in the extension of Quadrant III by a thick line. If in addition we assume the mechanical losses L to be proportional to the revolutions, that is, we put $L = fN$, we get

$$Q = f/H \tag{123}$$

The negative thrust given by the propeller when gliding would then appear to be constant at all speeds. Practically, the negative thrust in a glide does not appear to vary greatly.

If we wish, when gliding, the propeller thrust to be exactly equal to zero, we must adjust our throttle in such a way that the ratio V/NH is nearly equal to unity, because, as is known,[3] for $V/NH \cong 1$ the propelller thrust is equal to zero. The propeller revolutions will then vary proportionally with the speed according to the relation

$$N = V/H \tag{124}$$

[1] I owe this last remark to my assistant, Mr. W. F. Gerhardt, aeronautical engineer at McCook Field.
[2] See "General Theory of Blade Screws," by Dr. G. de Bothezat, Chapter II.
[3] For an exact discussion of this question see: "General Theory of Blade Screws," above mentioned, Chapter II.

where H is the propeller pitch. The curve of N as a function of V will be a straight line—represented in dashes at the end of Quadrant III extension of figure 19. When gliding under such conditions, our Q/P curve will be reduced to the k_y axis, as its ordinates will all be equal to zero. The inclination of our gliding path will now be measured simply by the ordinates of the curve (see fig. 19). We thus see that when gliding, the propeller acts as if the airplane drag was increased, and power has to be applied in order to eliminate this effect.

One can now see how complete a picture of the properties of an airplane in steady motion is given by the chart developed in this paper, and with what ease this chart allows us to follow the variations and mutual inter-relations of all the quantities involved in the problem.

The approximate equations we have deduced for all the curves of the chart may be used in many cases for a first checking, but attention must be paid to all the assumptions made in deducing these approximations. The approximate equations applied under carefully considered limitations give very good results for some problems, but for any general and broad discussion connected with the study of an airplane in steady motion the general method of the chart must be used.

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PART V.

PERFORMANCE PREDICTION.

Prediction of airplane performance is at present based more or less on wind tunnel tests. The purpose of this chapter is to show how such performance prediction can be based chiefly on data obtained from those free flight tests to which airplanes are usually submitted. I shall thus, in the first place, show how to deduce from regular free flight tests the data necessary for performance prediction, and, in the second place, show how to use these data in order to predict the performance.

1. COLLECTING THE NECESSARY DATA.

We have seen in the foregoing chapter that, as a first approximation, considering $t \cong 0$, the whole airplane performance depends upon the four coefficients r, σ, q_0, and q_1. Two of these coefficients, r and σ, characterize the airplane itself; the two other coefficients, q_0 and q_1 characterize the engine-propeller system. All the free flight characteristics can be expressed a, functions of these four coefficients.

Among the relations deduced in the foregoing chapter, we had:

The high horizontal speeds at all altitudes

$$V^2 = \frac{q_0}{2(\sigma+q_1)} + \sqrt{\frac{q_0^2}{4(\sigma+q_1)^2} - \frac{rp^2}{(\sigma+q_1)}} \tag{125}$$

or

$$V^2 = \frac{q_0}{2(\sigma+q_1)}\left[1+\sqrt{1-\frac{4rp^2\,(\sigma+q_1)}{q_0^2}}\right] \tag{126}$$

where $p = P/\delta A$ is the specific load which defines the altitude considered.

The ceiling value for the specific load

$$p_0 = \frac{P}{\delta_0 A} = \frac{q_0}{2\sqrt{r(\sigma+q_1)}} \tag{127}$$

The rate of climb

$$U = U_0\,\frac{\sqrt{p_0/p} - \sqrt{p/p_0}}{\sqrt{p_0/p_0} - \sqrt{p_0/p_0}} = U_0\left(1-\frac{H}{H_0}\right) \tag{128}$$

where U_0 is the rate of climb at ground level, equal to

$$U_0 = \sqrt{2rq_0}\,\left(\sqrt{p_0/p_0} - \sqrt{p_0/p_0}\right) \tag{129}$$

From relation (127) we get

$$\frac{p^2}{p_0^2} = \frac{4(\sigma+q_1)rp^2}{q_0^2} \tag{130}$$

Substituting in (126), and since

$$V_0^2 = \frac{q_0}{2(\sigma+q_1)} = \frac{2rp_0^2}{q_0} \tag{131}$$

we find:

$$V^2 = V_0^2\,\left[1+\sqrt{1-(p/p_0)^2}\right] \tag{132}$$

For a given airplane the minimum information that we can have about it is the knowledge of its horizontal speed V_0 at ground level, its rate of climb U_0 at ground level, and its ceiling H_0.

Let us try to find the expressions for the four coefficients r, σ, q_0, and q_1 as functions of these last quantities V_0, U_0, and H_0. We shall then be able to compute the four characteristic coefficients r, σ, q_0, and q_1 from the ordinary free flight tests.

When the ceiling H_c is known, we can compute the ceiling specific load p_c from the relation

$$H_c = RTlg\frac{p_c}{p_0} \tag{133}$$

which gives

$$p_c = p_0 \, e^{\frac{H_c}{RT}} \tag{134}$$

Knowing p_c, we find

$$V_c^2 = \frac{V_0^2}{1+\sqrt{1-(p_0/p_c)^2}} \tag{135}$$

Knowing V_c, we find

$$(k_y)_c = \frac{p_c}{V_c^2};\ \ (k_y)_0 = \frac{p_0}{V_0^2} \tag{136}$$

Knowing p_c and U_0 from (129) and (131), we find

$$\sqrt{2rq_0} = \frac{U_0}{\sqrt{p_c/p_0} - \sqrt{p_0/p_c}};\ \ \sqrt{\frac{2r}{q_0}} = \frac{V_0}{p_0}$$

and solving these two equations in r and q_0, we find

$$r = \frac{U_0 V_0}{2p_0[\sqrt{p_c/p_0} - \sqrt{p_0/p_c}]} \tag{137}$$

and

$$q_0 = \frac{U_0 p_0}{V_0[\sqrt{p_c/p_0} - \sqrt{p_0/p_c}]} \tag{138}$$

and knowing q_0 from (131), we get

$$(\sigma+q_1) = \frac{q_0}{2V_0^2} = \frac{p_0 U_0}{2V_0^3[\sqrt{p_c/p_0} - \sqrt{p_0/p_c}]} \tag{139}$$

It is clear that if only three conditions are given us which we have assumed to be the values of V_0, U_0, and H_c we can find only three relations containing r, σ, q_0, and q_1 as functions of V_0, U_0, and H_c, and we have just found the expressions for r, q_0, and $(\sigma+q_1)$ in terms of V_0, U_0, and p_c. A fourth condition has thus to be put forward in order to specify fully the problem. But it must be remarked that so far as we intend to predict only the horizontal self-speeds at all altitudes, the rates of climb at all altitudes, the ceiling and the time of climb, we do not need to know separately the coefficients σ and q_1 because, as can be seen from the relations (125), (128), (133), and (117), the quantities V, U, H_c, and t_{min} are functions only of r, q_0, and $(\sigma+q_1)$. But as soon as the propeller efficiency η_0 for horizontal flying at ground level is known, we shall immediately be able to find q_1 and thus know the separate values of σ and q_1. In effect we have

$$\eta_0 L_0 = (q_0 - q_1 V_0^2)\delta_0 A V_0 \tag{140}$$

where L_0 is the power delivered by the engine for horizontal flight at ground level, which in any case must be considered as a known quantity. From the last relation we get directly

$$q_1 = \frac{q_0}{V_0^2} - \frac{\eta_0 L_0}{\delta_0 A V_0^3} = \frac{1}{V_0^2}\left(q_0 - \frac{\eta_0 L_0}{PV_0} p_0\right)$$

or, since $\eta_0 L_0 = Q_0 V_0$, where Q_0 is the propeller thrust for horizontal flying at ground level, and using the notation

$$y_0 = \frac{(k_x)_0}{(k_y)_0} = \frac{Q_0}{P}$$

we finally get

$$q_1 = \frac{q_0 - y_0 p_0}{V_0^2} \tag{141}$$

The value of the coefficient q_1 once found, we can find the value of σ by the aid of relation (139).

Thus, when we know:

$$V_0,\ U_0,\ H_0,\ \eta_0,\ \text{and}\ L_0$$

we can deduce immediately the values of the characteristic coefficients r, σ q_0 and q_1 that correspond to the airplane considered.

Unfortunately the propeller efficiency is the quantity that most generally is the least known among the quantities affecting airplane performance. That is why it may be of interest to attempt to check this efficiency, when unknown, even only to a first approximation.

We have seen in the foregoing that the value of the lift coefficient at which the ceiling is reached

$$(k_y)_o=\sqrt{\frac{\sigma+q_1}{r}}$$

is generally included between

$$(k_y)_m=\sqrt{\frac{\sigma}{r}}\ \text{and}\ (k_y)_M=\sqrt{\frac{3\sigma}{r}}$$

that is, included between the lift value at which k_x/k_y is minimum and the lift value at which $k_x/k_y^{3/2}$ is a minimum ("power required minimum"). It is clear on the other hand that the value of q_1 depends upon the propeller of the engine-propeller system considered. There is advantage in selecting such a propeller as would give us $q_1=2\sigma$, because we would then have $(k_y)_o=(k_y)_M$, the power required would be a minimum at the ceiling and the highest ceiling would be reached with the power available. But it is difficult to expect that each airplane is fitted with the best climbing propeller and that is why in general $q_1<2\sigma$. Let us designate by n the ratio

$$\frac{q_1}{\sigma}=n \tag{142}$$

It is easy to see that for high ceiling airplanes the ratio n will be close to the value 2, and for average ceiling machines closer to 1. By making in (139) the coefficient $q_1=n\sigma$ we get

$$\sigma=\frac{p_o U_0}{2V_0^3(n+1)[\sqrt{p_o/p_0}-\sqrt{p_0/p_o}]} \tag{143}$$

$$q_1=\frac{np_o U_0}{2V_0^3(n+1)[\sqrt{p_o/p_0}-\sqrt{p_0/p_o}]} \tag{144}$$

It is by estimating the value of n that one can decide to a first approximation upon the relative values of σ and q_1. It must be remarked that it is only the value of the propeller efficiencies that will be affected by the value adopted for n, because all other performance characteristics, as has already been mentioned, depend only upon $(\sigma+q_1)$ and thus are independent of the value of n.

Let us designate by N', η', and L'_m the number of revolutions, the propeller efficiency and power delivered at ground level for $k_y=(k_y)_o$—at that moment the airplane is climbing with the rate of climb U_0 and has the self-speed V; let us designate by N'', η'' and L''_m the number of revolutions, the propeller efficiency and power delivered at the ceiling; and by N_0 the number of revolutions for horizontal flying at ground level; and let us try to find the expressions for η_0, η', and η'' in terms of the characteristics of the airplane performance. We have:

$$\eta_0 L_0=(k_x)_o\,\delta_0\,AV_o^3=\frac{(k_x)_o}{(k_y)_o}(k_y)_o\,\delta_0\,AV_o^2\cdot V_o=\frac{(k_x)_o}{(k_y)_o}PV_o$$

$$\eta' L'_m=(k_x)_o\,\delta_o\,AV^3+P\,\gamma\,V=\frac{(k_x)_o}{(k_y)_o}PV_o\frac{p_o}{p_0}\frac{V^3}{V_o^3}+PU_0$$

$$\eta'' L''_m=(k_x)_o\,\delta_o\,AV_o^3=\frac{(k_x)_o}{(k_y)_o}P\,V_o$$

On account of

$$(k_y)_0=\frac{p_0}{V_0^2};\quad (k_y)_c=\frac{p_c}{V_c^2}=\frac{p_0}{V^2};\quad V^2=V_c^2\frac{p_0}{p_c};\quad \frac{(k_x)_0}{(k_y)_0}=y_0;\quad \frac{(k_x)_c}{(k_y)_c}=y_c$$

and

$$L'_m\cong\frac{N'}{N_0}L_0;\quad L''_m\cong\frac{N''L_0\,p_0}{N_0\,p_c}$$

we find

$$\eta_0=\frac{PV_0y_0}{L_0} \tag{145a}$$

$$\frac{N'}{N_0}\eta'=\frac{PV_cy_c\sqrt{p_0/p_c}+P\,U_0}{L_0} \tag{146a}$$

$$\frac{N''}{N_0}\eta''=\frac{PV_c\,y_c}{L_0\left(\frac{p_0}{p_c}\right)} \tag{147a}$$

On account of

$$y_0=r(k_y)_0+\frac{\sigma}{(k_y)_0}=\frac{rp_0}{V_0^2}+\frac{\sigma\,V_0^2}{p_0};\quad y_c=r(k_y)_c+\frac{\sigma}{(k_y)_c}=\frac{rp_c}{V_c^2}+\frac{\sigma V_c^2}{p_c}$$

and replacing r and σ by their values (137) and (143) we find:

$$\eta_0=\frac{P\,U_0}{L_0}\rho_0 \tag{145b}$$

$$\frac{N'}{N_0}\eta^1=\frac{P\,U_0}{L_0}\rho' \tag{146b}$$

$$\frac{N''}{N_0}\eta''=\frac{P\,U_0}{L_0}\rho'' \tag{147b}$$

where

$$\rho_0=\frac{(1+\sqrt{1-z_0^4})^{1/2}}{2z_0(1-z_0^2)}\left[\frac{z_0^4}{1+\sqrt{1-z_0^4}}+\frac{1+\sqrt{1-z_0^4}}{1+n}\right]$$

$$\rho^1=\frac{1-\frac{n}{2(1+n)}z_0^2}{1-z_0^2}$$

$$\rho''=\frac{\frac{n+2}{2(1+n)}}{z_0(1-z_0^2)}$$

and

$$z_0=\sqrt{p_c/p_0}$$

The curves of ρ_0, ρ', and ρ'' as functions of z_0^2 for $n=1$ and $n=2$ have been represented on figure 20. It is easy to see from this figure that in the foregoing formulæ the value of n has a sensible influence on the value of ρ_0, a somewhat less influence on ρ'', but that ρ' turns out to depend only very slightly upon n, especially for small values of z_0^2, that is for airplanes of high ceilings. Thus by the aid of formula (145b) it is difficult to check the efficiency η_0; by the aid of formula (147b) the efficiency η'' can be checked only to a rough approximation; but the efficiency η' can be fairly well predicted by the aid of the formula (146b).

Thus from the general knowledge of the airplane performance it is only the propeller efficiency η' at the best rate of climb that we are able to check.

Some engines show abnormal deviations for their power from the proportionality to the revolutions and density. For such engines, in the expression of the powers L'_m and L''_m as depending upon L_o, special correction factors have to be introduced in order to take account of these abnormal deviations.

We thus see that in the question of airplane performance prediction, it is the quantities connected with the power unit that will be predicted with less accuracy than all the other flying characteristics and this exclusively on account of the fact that exact information concerning the engine-propeller system is in general lacking.

A last remark has to be made concerning the coefficients r and σ. The drag of the airplane wings considered alone can, to a first approximation, be taken equal to

$$\delta A V^2 (r k_y^2 + s)$$

where r and s are two characteristic coefficients of the wings that can be deduced by the method of least squares from the experimental drag-lift curve. The drag of the airplane parasite

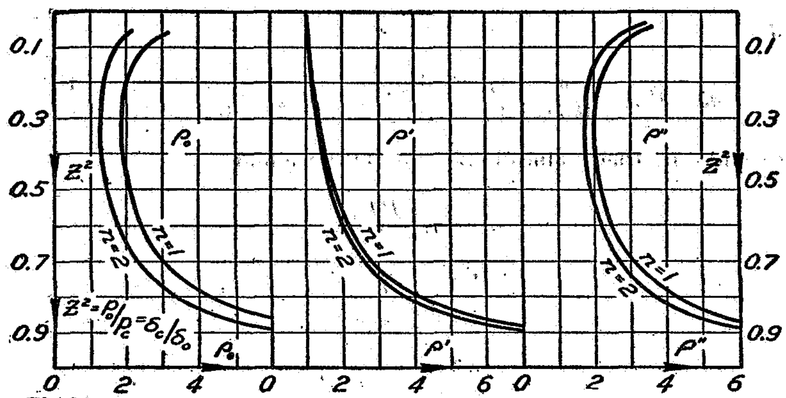

Fig. 20.

resistance considered alone can be taken equal to $k\delta a V^2$ where a is the *equivalent area* of the parasite resistance and $k = 0{,}64$ the coefficient of air resistance for the orthogonal motion of a flat plate. The drag R_x of the whole airplane will then appear to us as equal to

$$R_x = \delta A V^2 (r k_y^2 + \sigma) = \delta A V^2 (r k_y^2 + s) + k\delta a V^2$$

$$= \delta A V^2 \left(r k_y^2 + s + k\frac{a}{A} \right)$$

We thus see that

$$\sigma = s + 0{,}64\frac{a}{A} \tag{147}$$

The coefficient r thus appears to depend chiefly, to a first approximation, upon the wing shape alone, the coefficient σ to depend chiefly upon the ratio

$$\frac{a}{A} = \frac{\text{equivalent area of parasite resistance}}{\text{wing area}}$$

Knowing σ we can deduce the value of the ratio a/A, when the value of s will be known.

$$\frac{a}{A} = 1{,}56(\sigma - s) \tag{148}$$

Summing up the foregoing, we can say: The data obtained from average airplane tests, which usually are

The horizontal speed at ground level V_o,

The rate of climb at ground level U_o,

The ceiling H_o,

allow us to deduce the values of the four characteristic coefficients of the airplane in steady motion, when the efficiency η_o of its propeller is known. Otherwise, only by estimation can we evaluate the separate values of σ and q_1. These characteristic coefficients are equal to

$$r=\frac{U_oV_o}{2p_c[\sqrt{p_o/p_o}-\sqrt{p_o/p_o}]};\quad q_o=\frac{U_op_o}{V_c[\sqrt{p_o/p_o}-\sqrt{p_o/p_o}]}$$

$$(\sigma+q_1)=\frac{p_cU_o}{2V_o^3[\sqrt{p_o/p_o}-\sqrt{p_o/p_o}]}$$

$$q_1=\frac{1}{V_o^2}(q_o-y_op_o)$$

with

$$p_o=\frac{P}{\delta_oA};\quad y_o=\frac{(k_x)_o}{(k_y)_o}=\frac{\eta_oL_o}{PV_o};\quad p_o=p_oe^{\frac{\mathrm{He}}{\mathrm{RT}}};\quad V_o^2=\frac{V_o^2}{1+\sqrt{1-(p_o/p_o)^2}}$$

and

$$\frac{a}{A}=1{,}56(\sigma-s)$$

Making use of the notations:

$$z_o=\sqrt{p_o/p_o}=\sqrt{\delta_c/\delta_o} \tag{149}$$

$$r_1=\frac{z_o^3}{2(1-z_o^2)(1+\sqrt{1-z_o^4})^{1/2}} \tag{150}$$

$$q_{1o}=\frac{(1+\sqrt{1-z_o^4})^{1/2}}{z_o(1-z_o^2)} \tag{151}$$

$$\sigma_1+q_1=\frac{(1+\sqrt{1-z_o^4})^{3/2}}{2z_o(1-z_o^2)} \tag{152}$$

we can write

$$r=\frac{V_oU_o}{p_o}r_1;\quad q_o=\frac{U_op_o}{V_o}q_{1o};\quad (\sigma+q_1)=\frac{U_op_o}{V_o^3}(\sigma_1+q_1) \tag{153}$$

In order to simplify the computation of the coefficients r, q_o and $(\sigma+q_1)$ we have drawn on figure 21 the curves of r_1, q_{o1} and (σ_1+q_1) as functions of $z_o^2=p_o/p_o=\delta_o/\delta_o$, which allow us to read directly the values of these coefficients once the ceiling to which the airplane considered can climb is known.

Proceeding as above described, we can compute from observed performances the characteristic coefficients of the airplane's steady motions and thus collect values of these coefficients deduced from actual free flight tests.

Having deduced from tested airplanes the values that the characteristic coefficients can actually take, the prediction of performances is made as follows.

2. PREDICTING THE PERFORMANCE.

Two cases have to be distinguished:

In the first case, the airplane is considered as already built and tested, and the values of V_o, U_o and H_o experimentally found. The prediction of the complete performance is requested?

Knowing H_o we compute

$$z_o^2=p_o/p_o=e^{\frac{-\mathrm{H}_o}{\mathrm{RT}}}=e^{\frac{-\mathrm{H}_o}{8000}}$$

Having found the value of z_0^2 we compute the ceiling flying speed

$$V_c^2 = \frac{V_0^2}{1+\sqrt{1-z_0^4}}$$

The horizontal speeds at all altitudes are then found to be equal to

$$V^2 = V_c^2\,(1+\sqrt{1-z^4}) = V_0^2\,\frac{1+\sqrt{1-z^4}}{1+\sqrt{1-z_0^4}}$$

with $z^2 = p/p_c$ each altitude considered being defined by the value of the corresponding specific load

$$p = \frac{P}{\delta A}$$

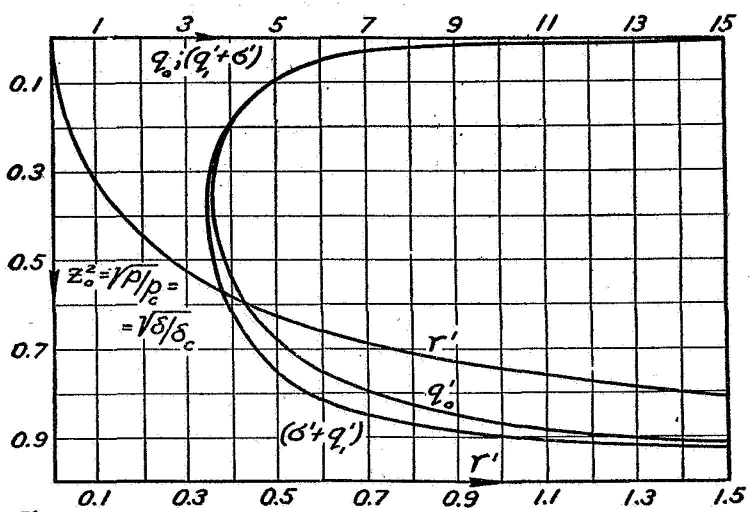

Fig. 21.

The rates of climb at all altitudes are equal to

$$U = U_0\,\frac{(1-z^2)z_0}{(1-z_0^2)z} \cong U_0\left(1-\frac{H}{H_c}\right)$$

The time of climb up to any altitude is equal to

$$t_{\,\min} \cong 0,\,0384\;\frac{H_c}{H_0}\,lg_{10}\,\frac{H_c}{H_c-H}$$

Afterwards, by the aid of the formulæ (141) and (153) the four coefficients σ, r, q_0 and q_1 will be computed.

The propeller efficiencies η' and η'' are computed by the aid of the formulæ (146a) and (147a).

The propeller's efficiency for the best climb at ground level is equal to

$$\frac{N''}{N_0}\eta' = \frac{PV_c}{L_0}\left(y_c z_0 + \frac{U_0}{V_c}\right)$$

The propeller efficiency at the ceiling is equal to

$$\eta'' = \rho''\,\frac{PV_c}{L_0}$$

Finally, the whole performance chart can be traced to a first approximation. The speed curves of quadrant IV and transfer lines of quadrant II are only geometrical intermediaries and can be traced at once. The coefficients r and σ being known, the k_x/k_y curve of quadrant I can be plotted. The coefficients q_0 and q_1 being known, the specific thrust curves of quadrant III can be plotted, the whole strip of curves being replaced to a first approximation by a single curve. Further, by the aid of the transfer lines and speed curves, the Q/P curves of quadrant I can be traced. Afterwards, in extension of quadrant III, the $L_u/\delta A$ curve can be plotted. Finally, by three points (η_o, V_o), (η', V), (η'', V_c) the efficiency curve can be traced and thus the $L_m/\delta A$ curve deduced. In such a way, from the knowledge of V_o, U_o, H_c, L_o, and η_o all the possible conclusions concerning the steady motion of the airplane considered will have been drawn.

In the second case, only drawings of the airplane considered are supposed to be available. It can be either an airplane in the process of design, or an airplane about which flying data are not available. The prediction of the complete performance is requested.

The values of the coefficients r and σ are first estimated by comparison with other similar airplanes. Two airplanes having the same wing-profile will have very closely the same values of r. The value of σ will be taken equal to

$$\sigma = s + 0,\ 64\frac{a}{A}$$

where the equivalent area a of the parasite resistance has to be estimated and s taken from data concerning the wings used on the airplane considered.

Further, the power L_o and the airplane weight P must be known and one must decide upon the value of the efficiency η_o. As we have

$$\eta_o L_o = (k_x)_o\ \delta_o A\, V_o^3 \text{ and } p_o = \frac{P}{\delta_o A} = (k_y)_o V_o^2$$

we find:

$$\frac{\eta_o L_o}{P p_o^{1/2}} = \frac{(k_x)_o}{(k_y)_o^{3/2}} = (k_y)_o^{-1/2}\left[r\ (k_y)_o + \frac{\sigma}{(k_y)_o}\right] \tag{154}$$

This last relation will give us the value of $(k_y)_o$ and thus the value of the self-speed V_o that is compatible with the power available and drag offered by the airplane considered.

The easiest way to get a solution of equation (154) is to plot first the k_x/k_y curve as function of k_y—as we can do from the knowledge of the coefficients r and σ—and to plot afterwards the curve of $k_x/k_y^{3/2}$ as function of k_y, by dividing the ordinates of the k_x/k_y curve by the corresponding values of $\sqrt{k_y}$.

The smallest abscissæ of the $k_x/k_y^{3/2}$ curve corresponding to the ordinate equal to

$$\frac{\eta_o\, L_o}{P p_o^{1/2}}$$

will give us the value of $(k_y)_o$ to which corresponds the high horizontal speed V_o, which we will be able to read directly if only previously in our so-called quadrant IV, the speed curve $p_o = k_y\ V^2$ has been traced (see fig. 22).

Having found V_o by the aid of the $k_x/k_y^{3/2}$ curve, let us consider the relation

$$\eta' L'_m \cong \frac{N'}{N_o}\,\eta' L_o = P\ V y_c + P\, U_o$$

from which we find

$$U_o = \frac{\frac{N'}{N_o}\,\eta' L_o - P\, V y_c}{P} \tag{155}$$

As has been explained, there is advantage in taking $(k_y)_c \cong (k_y)_M$, that is $(k_y)_c$ equal to the k_y that corresponds to the minimum of the $k_x/k_y^{3/2}$ curve. Making this last selection, we have

$$V^2 = \frac{p_o}{(k_y)_M};\ y_c = y_M = \frac{(k_x)_M}{(k_y)_M}$$

and adopting a certain value for η' we can calculate U_o by the aid of (155).

When flying at the ceiling, we have

$$\eta'' L_m'' \cong \frac{N''}{N_o} \eta'' L_o \frac{p_o}{p_c} = (k_x)_c\, \delta_c\, A V_c^3 \text{ and } p_c = (k_y)_c V_c^2$$

and we find

$$\frac{\frac{N''}{N_o} \eta'' L_o p_o}{P p_c^{3/2}} = \frac{(k_x)_c}{(k_y)_c^{3/2}} \tag{156}$$

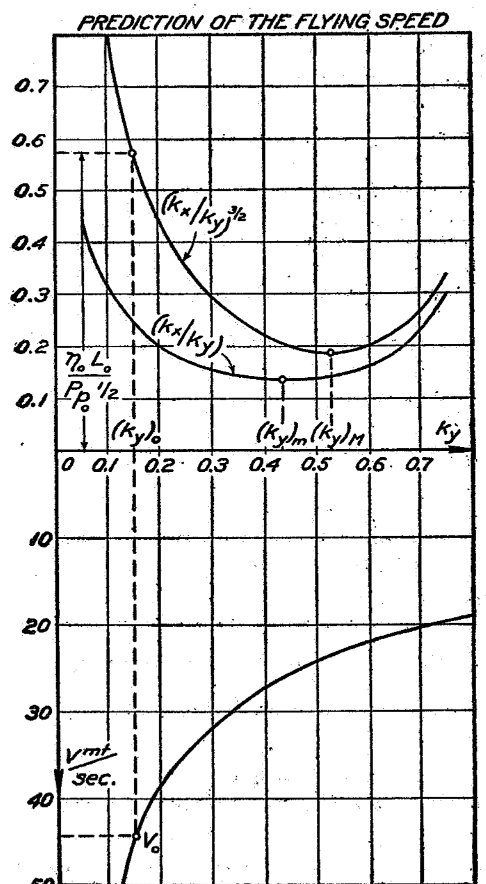

Fig. 22.

Assuming $(k_y)_c = (k_y)_M$ and deciding upon the value the efficiency η'' may reach, we get

$$\frac{p_o}{p_c} = \frac{P^{2/3} p_o^{1/3}}{\left(\frac{N''}{N_o}\right)^{2/3} \eta''^{2/3} L_o^{2/3}} \left[\frac{(k_x)_M}{(k_y)_M^{3/2}}\right]^{2/3} \tag{157}$$

which relation gives us the value of the ceiling. Knowing p_c, we find the ceiling self-speed, V_c, since

$$V_c^2 = \frac{p_c}{(k_y)_M} \tag{158}$$

It is in such a way that an estimation of the values of V_0, U_0 and p_0 can be reached. The characteristic coefficient q_0 can now be easily found by the aid of formula (153), and the value of q_1 by the aid of the formula (141). Finally, we have to verify, by the aid of the formula

$$(k_y)_0 = \sqrt{\frac{\sigma \times q_1}{r}}$$

how far the assumption $(k_y)_0 = (k_y)_M$ holds.

The checking of the rate of climb U_0 and ceiling p_0 by the aid of the last method gives good results because we have to deal with values of functions close to their minimum, where they do not vary much, the differences between y_M and y_0, and between $(k_x)_M/(k_y)_M^{3/2}$ and $(k_x)_0/(k_y)_0^{3/2}$ being in fact only very slight.

In all the preceding discussion, I had chiefly in view to point out the real nature of the problem of the performance prediction and to show by what concatenation of ideas we can be brought to its solution. Special attention must be paid to the rôle the $k_x/k_y^{3/2}$ curve plays in the finding of the self-speed V_0 from the knowledge of the power available $\eta_0 L_0$, and the meaning of the minimum $(k_x)M/(k_y)M^{3/2}$ of the $k_x/k_y^{3/2}$ curve for the ceiling of the airplane considered.

The standpoint adopted in all this chapter was the prediction of the performance, starting with the knowledge of the smallest amount of data available concerning the airplane considered. But when for a given airplane, we know its k_x/k_y curve and possess all the data necessary in order to plot the specific thrust curve of the airplane's engine-propeller system; then the simplest way to predict the performance is just to draw, for the case considered, our performance chart, which will give the most complete performance prediction of the airplane considered. It is this question of finding from free flight tests these two fundamental curves—the k_x/k_y curve and the specific thrust curve—that we will consider in the next chapter.

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PART VI.

FREE FLIGHT TESTING.

The performance chart we have developed in the foregoing gives a complete representation of the performance of an airplane in steady motion. A complete free flight test of an airplane must thus consist in getting all the data necessary in order to establish such a chart. The speed curves of quadrant IV and transfer lines of quadrant II being only geometrical intermediaries, it is only the curves of one of the quadrants I or III that we have to establish, because the curves of these quadrants mutually correspond to one another by the aid of quadrants II and IV. We shall show how to obtain from actual free flight tests the k_x/k_y curve and the Q/P curves of quadrant I.

Let us consider an airplane equipped with the following instruments: An air speed meter, a barograph, a strut thermometer. The barograph will be consisdered to be calibrated in pressure units, which is the only reasonable calibration of these instruments when used for free flight testing. In order to control, to a certain measure, the power of the engine, a tachometer must also be available. The test can be made either at full throttle or at any reduced throttle.

The airplane so equipped must make two or three climbs, at different indicated air speeds, but the last must be kept constant in each case all through the climb; also, on the way down, after each climb, it must make two or three glides. Each glide must also be made at different indicated air speeds, but constant for each glide. The glides will be done partly with the throttle completely closed and partly with the throttle so adjusted that

$$\frac{V}{NH} \cong 1$$

In these last glides the propeller thrust will be practically equal to zero.

The indicated air speed is proportional to the quantity

$$\frac{\delta V^2}{2}$$

But since $P \cong k_y \delta A V^2$, we have

$$k_y = \frac{2P}{A\frac{\delta V^2}{2}}$$

So that, if we keep $\delta V^2/2$ constant, that means that our glides or climbs take place at a constant value of the lift coefficient, and this is independent of our altitude.

Let us first consider the data furnished by the glides, made under the condition $V/NH = 1$. The barograms obtained from those glides will give us the value of the pressure at each moment, and taking account of the corresponding temperatures, we can find the values of the densities δ for each moment of the glide and thus can deduce the actual self speeds V from the knowledge of the indicated air speeds and the calibration curve of the speed meter used. Further, the rates of descent can be deduced from the glide-barograms, which, as has been shown in Chapter III, are equal to

$$U = -\sigma \frac{dp}{dt} \tag{51}$$

where $\sigma=\delta g$ is the corresponding specific weight of the air and dp/dt is the angular coefficient of the tangent to the glide barogram curve at the point considered. Knowing V and δ, or the specific load $p=P/\delta A$, we find from the relation $k_y=P\ Cos\ \gamma/V^2$, the corresponding value of k_y. Since, for the glides under the condition $V/NH=1$, we have $Q=O$ and thus $-\gamma=k_x/k_y=-U/V$, the point in quadrant I with the coordinates k_y and $-U/V$ will be a point of the k_x/k_y curve (see figs. 18 and 19). Proceeding in the same way for glides made at different indicated air speeds, we find a set of points of the k_x/k_y curve. The author has convinced himself by the actual use of the above described method that it is easy to get points of the k_x/k_y curve for values of $k_y>(k_y)_m$ by making glides at sufficiently low self-speeds. These glides have only to be made at heights sufficient for the safety of the pilot.

When proceeding, as above described, with the glides made with the throttle completely closed, we get a certain k_x'/k'_y curve. The difference of the ordinates of this last curve and the k_x/k_y curve will give us the $-Q/P$ curve, which transferred in quadrant III by the aid of the speed curves and transfer line will give us the $-Q/\delta A$ curves, by the aid of which we can estimate the mechanical losses of the motor, as has been already shown in the foregoing. (See fig. 19.)

If now we proceed in a similar manner with the climb barogram, and recorded indicated air-speeds; that is, deducing from them the corresponding δ, V and γ we obtain from each barogram a set of values of γ corresponding to a constant value of k_y, for different values of the specific load $p=P/\delta A$ for which we can adopt a set of standard values. If now we plot these values of γ in quadrant I, starting from the k_x/k_y curve and join all the points that correspond to equal values of the specific load, we get the family of the Q/P curves, with γ as parameter, since $Q/P=k_x/k_y+\gamma$ in each climb. These Q/P curves, transferred in quadrant III by the aid of the speed curves and transfer lines will give us the set of specific thrust curves.

By tracing the rate of climb curves in quadrant IV the ceiling will be checked, and, as described in the foregoing, the whole airplane performance can be deduced with ease from the knowledge of the k_x/k_y curve and the specific thrust curves.

If when making the last tests the airplane were equipped with a torque meter, then by recording the torque and the revolutions we would know the power delivered at each moment by the engine and we then could trace in extension of the quadrant III, of our chart, the $L_m/\delta A$ curve. As the $L_u/\delta A=QV/\delta A$ curve can be directly deduced from the $Q/\delta A$ curve of quadrant III, the knowledge of the $L_m/\delta A$ curves will allow us to immediately deduce the efficiency curves. It is in such a way that from free flight tests the propulsive efficiencies can be deduced. It is easy to deduce from the efficiency curves and the $L_m/\delta A$ curves, by the aid of the revolution curves as function of the self-speed V, the efficiency curve as well as the $L_a/\delta N^3$ curve as function of V/N, and thus to get from the free flight test the complete characteristics of the propeller. It is also from the $L_m/\delta A$ curve that the engine power characteristics as function of N for different values of the density δ and throttle x can be deduced.

One can now realize how important it is to use a torque meter in free flight tests. A torque meter, giving us a continuous control of the power, will make the test perfectly reliable in the sense of knowledge of the power really developed by the engine; and besides, the torque meter will allow us to obtain, in addition to the complete characteristics of the airplane, the complete and separate characteristics of the propeller and of the engine.

The chart (fig. 26) annexed at the end of this paper gives the characteristics of a Vought airplane as actually obtained from free flight tests by the above described method. For all the details concerning such test the reader is referred to the McCook Field (Dayton, Ohio) Report No. 1242, "*A report showing the use of the de Bothezat performance chart for expressing the performance of the VE-7 airplane P-113, from data obtained in actual flights,*" by Mr. C. V. Johnson and W. F. Gerhardt.

I wish finally to call attention to one more important question connected with free-flight testing. The power of the engine is affected by the air temperature, and it is thus necessary to reduce the power of the engine, and thus the whole performance, to some standard temperature, if we wish to get results that can be compared with other tests. For reasons that have been discussed in Chapter IV, it is the isothermic atmosphere of zero degrees centigrade that we adopt as standard, and thus the whole performance has to be reduced to zero degrees centigrade. It is evident that one has to take account of the temperature to find the value of the densities from the pressures given by the barograph, but how must we take into account the influence on the performance of the power variation due to the temperature?

At a constant density, the engine power depends upon temperature. That is, at the same density but at the temperature of zero degrees centigrade the engine would give a slightly different power from that in the actual flight. Let ΔL_m be this positive or negative increment of the power due to temperature difference at constant density. On account of the fact that the drag and lift of the airplane depend only upon density, neither k_x nor k_y nor V—because $P=k_y\delta A V^2$—will be affected by the temperature; that is, neither the k_x/k_y curve, nor the speed curves. The increment of power ΔL_m will act on the performance as a slight change of throttle and it is only the values of Q/P or $Q/\delta A$ that will have to be corrected. As δ and V remain the same, the efficiency η will remain the same, and the variation ΔL_u of the power available will be proportional to the variation of the power delivered, but as $L_u = VQ$ and V remain the same, we have

$$\Delta L_u = \eta \Delta L_m = V \Delta Q$$

The correction to be applied to the thrust thus simply turns out to be equal to

$$\Delta Q = \frac{\eta \Delta L_m}{V}$$

and the correction to be applied to the Q/P values turns out to be equal to

$$\frac{\Delta Q}{P} = \frac{\eta \Delta L_m}{PV}$$

The power correction ΔL_m due to temperature at *constant density* has to be determined by special tests of the engine.

Those who have followed carefully the methods and questions of principles discussed in this paper will not meet the slightest trouble in making the most complete and rigorous airplane free-flight tests.

REPORT No. 97.

PART VII.

SHORT DISCUSSION OF THE PROBLEM OF SOARING.

We have until now, paid exclusive attention to the airplane self-speed $\bar{V}$. This means that we have considered the airplane flight from a system of coordinates that had, relatively to the ground, a speed constant in magnitude and direction—and equal to the wind speed $\bar{v}$. Let us now follow the airplane flight from a system of coordinates invariably connected to the ground. As we have already mentioned, at the beginning of Chapter I, the ground or absolute speed $\bar{W}$ of the airplane is at each moment equal to

$$\bar{W} = \bar{V} + \bar{v} \tag{159}$$

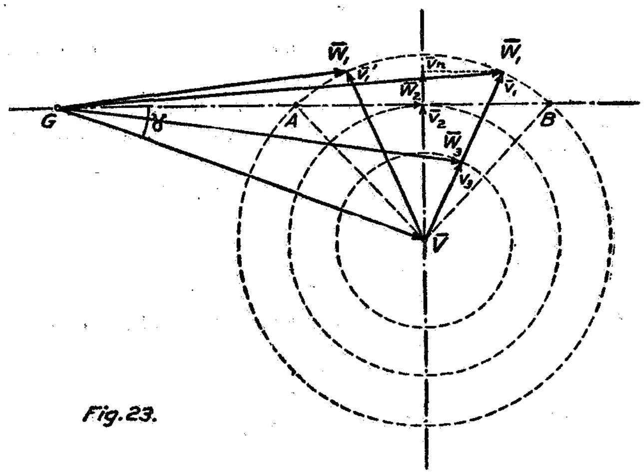

Fig. 23.

INFLUENCE OF WIND ON THE AIRPLANE GLIDING TRAJECTORIES

Let us consider a gliding airplane and for the sake of simplicity neglect the negative propeller thrust. Let us draw from the center of mass G of the airplane a vector $\bar{V}$ equal to its gliding self-speed at the moment considered and making the angle γ (the actual airplane path inclination) with the horizontal. (See fig. 23.) If the wind speed $\bar{v}$ at the moment considered and the point of the atmosphere where the airplane is actually gliding is equal to zero, then

$$\bar{W} = \bar{V} \tag{160}$$

But, since for gliding the angle γ turns out to be always negative (see equation (61), Chap. IV), the absolute speed $\bar{W}$ can under those conditions only be a descending speed. We are thus brought to the general conclusion:

In those parts of the atmosphere where there is no wind a glider can only be descending.

Let us now consider the airplane gliding in a wind having a magnitude equal to v. Let us draw from the end of the vector $\bar{V}$ (see fig. 23) a circumference having a radius equal to v. Three cases can be encountered.

In the first case the described circumference cuts the horizontal in two points A and B. In this case when the wind of magnitude v has a direction included in the angle $\widehat{AVB}$, the absolute speed $\overline{W}$ of the airplane will be either horizontal or ascending. For any other wind direction the absolute speed $\overline{W}$ will be descending.

In the second case the described circumference is tangent to the horizontal. In this case only for the wind blowing directly upwards can the absolute speed $\overline{W}$ be horizontal.

In the third case the described circumference is disposed entirely below the horizontal. In this case the absolute speed $\overline{W}$ will always be descending independent of the direction of the wind.

We are thus brought to the following fundamental conclusion:

The absolute speed $\overline{W}$ of any glider in a state of steady motion, be it an airplane or a bird, can be ascending or horizontal only in ascending wind, and provided the vertical component v_n of the last is larger than the rate of descent U.

The so-called phenomenon of soaring is thus only possible in an ascending wind, for which

$$v_n > U \tag{161}$$

The smaller the rate of descent U the smaller may be the vertical wind component v_n necessary for soaring.

Let us discuss briefly those conditions that make the rate of descent a minimum.

One can see from equation (95) of Chapter IV that for $Q=0$ the rate of descent is equal to

$$U = -p^{1/2}(rk_y^{1/2} + \sigma k_y^{-3/2}) = -p^{1/2}\frac{k_x}{k_y^{3/2}} \tag{162}$$

and has a minimum given by the condition

$$\frac{\partial U}{\partial k_y} = -p^{1/2}\frac{\partial}{\partial k_y}\left(\frac{k_x}{k_y^{3/2}}\right) = -1/2p^{1/2}(rk_y^{-1/2} - 3\sigma k_y^{-5/2}) = 0$$

which gives

$$k_y = (k_y)_M = \sqrt{\frac{3\sigma}{r}} \tag{163}$$

and

$$U_{\min} = -p^{1/2}\left[r\left(\frac{3\sigma}{r}\right)^{1/4} + \frac{\sigma}{\left(\frac{3\sigma}{r}\right)^{3/4}}\right] \tag{164}$$

The rate of descent U is thus a minimum for the same value $(k_y)_M$ of the lift coefficient for which the $k_x/k_y^{3/2}$ curve has a minimum.

We shall introduce the notation

$$S = \left[r\left(\frac{3\sigma}{r}\right)^{1/4} + \frac{\sigma}{\left(\frac{3\sigma}{r}\right)^{3/4}}\right] = 4/3r\left(\frac{3\sigma}{r}\right)^{1/4} \tag{165}$$

and call it the *soaring constant* of a glider, because it depends only upon the aerodynamical properties of the glider consider. We can thus write

$$U_M = S\sqrt{\frac{P}{\delta A}} \tag{166}$$

where U_M represents the magnitude of the rate of descent at its minimum.

The rate of descent U_M will be the smaller, the smaller P/A is, that is, the wind loading of the considered glider, and the smaller the soaring constant is.

As has already been remarked, the coefficient r depends chiefly upon the wing profile and its value is included in narrow limits. But the ratio σ/r depends upon the value of the ratio a/A and can be greatly reduced by reducing the drag of the parasite resistance and increasing the wing area. In figure 24 has been represented the curve of

$$S' = \frac{S}{r} = 4/3\left(\frac{3\sigma}{r}\right)^{1/4} \tag{167}$$

as function of the ratio σ/r, which allows a quick checking of the value that the soaring constant

$$S = r\,S' \tag{168}$$

can have.

Taking for average values $\sigma/r = 0,2$ and $r \cong 0,1$ from figure 24, we find $S' \cong 1,2$ and $S \cong 0,12$, and if we take $P/A \cong 8$ *klg/mt*² this would make, with $\delta = 1/8$, the rate of descent equal to

$$U_M = 0,12\sqrt{64} \cong 0,96 \ mt/sec$$

It is thus quite possible to realize gliders with a rate of descent less than 1 meter per second, especially on account of the fact that the ratio σ/r can be made still less than the average value we have adopted. A rising wind with a vertical component equal to 1 meter per second would thus be sufficient to secure the soaring of such glider.

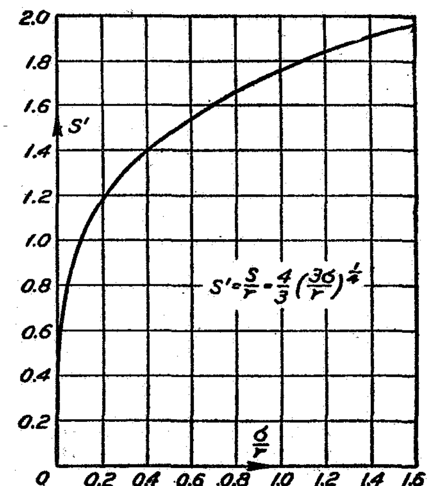

Fig. 24.

The gliding of such glider would take place at a value of

$$k_y = (k_y)_M = \sqrt{\frac{3\sigma}{r}} \cong 0,77.$$

and its self-speed would be equal to

$$V = \sqrt{\frac{P}{k_y\,\delta\,A}} = \sqrt{\frac{64}{0,77}} \cong 9,2 \ mt/sec$$

This would be the low speed of the glider; its high speed could be made around 20 *mt/sec* which was the speed of the early airplanes. High cambered aerofoils can give lift values up to $(k_y) \cong 0,8$.

We are thus brought to the conclusion that it is quite possible to build gliders having a very low rate of descent. Such gliders must have a high cambered aerofoil, a low self-speed, a small drag, and a small loading per unit of area. Special attention must only be paid to secure the complete stability and maneuverability of the glider at its lowest speed, by the aid of sufficient stabilizing surfaces and rudders. Such a glider, having a low rate of descent, will soar in any ascending wind whose vertical component is equal to or greater than the minimum rate of descent of the glider.

The fact that the soaring of birds is very often observed in some regions shows that in those regions ascending winds, whose vertical component has a sufficient value to secure soaring, must be a common phenomenon.

It is the opinion of the author that the main reason for the frequent occurrence of ascending winds is the following:

As is known, winds are generally variable with altitude, that means the different layers of the atmosphere have different velocities. It even sometimes happens that two air layers have opposite speeds; as a result of this speed difference, a vortex sheet must be formed between them.[1] But such vortex sheets being unstable, as is known, they must break into a system of vortex rows.

v. Karman[2] has shown that among all possible vortex rows, it is the system of quincunx vortex rows that constitute a stable configuration and the unstable vortex sheets seem most generally to break into such quincunx vortex rows. In figure 25 a system of such quincunx vortex rows is diagrammatically represented.

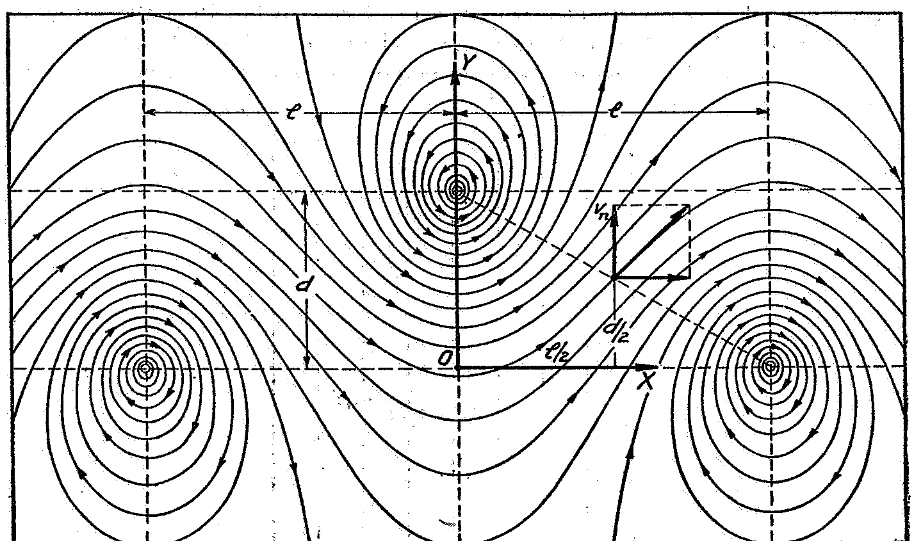

Fig. 25. THE STRUCTURE OF THE WIND BUILDING OF QUINCUNX VORTEX ROWS BETWEEN AIR LAYERS OF DIFFERENT VELOCITY

We are thus naturally brought to the conclusion that as a consequence of the speed difference between air-layers, a formation of vortex rows in quincunx must take place between such layers. The ordinary atmospheric wind thus appears to us in its structure to be made up of wind layers separated by quincunx vortex rows traveling between the air layers.

It seems also that the unequal heating of the ground by the sun rays acts greatly in favor of the formation of such traveling qiuncunx vortex rows.

Once the formation of such atmospheric-quincunx vortex rows is admitted, it is easy to conceive that we must meet in the atmosphere in some places ascending currents, in other places descending currents. It is the ascending waves of the atmospheric quincunx vortex rows that makes soaring possible, and it is in these waves that birds soar when they meet them. It is by remaining in the boundaries of such ascending wave, or by gliding from one ascending column into another, that birds can maintain soaring.

It is of interest to check the mean value that the vertical wind component in the ascending column produced by a system of quincunx vortex rows can have. Let us consider such a

[1] See the author's "Introduction to the Study of the Laws of Air Resistance of Aerofoils," Chapter III, Report No. 28 of the National Advisory Committee for Aeronautics. Washington, D. C.

[2] See the above-mentioned Report No. 28, p. 46, note IV.

quincunx vortex row and adopt as mean value of the vertical wind component the vertical wind value at the middle of the line joining two consecutive vortices taken one in each row (see fig. 25). If we refer one of the rows to a system of $X\,O\,Y$ axes, the origin being in the middle between two of the vortices, the X axis directed along the vortex row and the Y axis perpendicular to the last, if $2l$ is the distance between two consecutive vortices in one row and d the distance between the two rows, then the point adopted as the one having the mean vertical wind component will have the coordinates $(l/2, d/2)$, and the vertical wind component produced by one row will be found equal to:[3]

$$v = \frac{\pm I}{4l} \frac{tg\frac{\pi}{4}\left(2tgh^2\frac{\pi d}{4l} - 1\right)}{1 + 4tg^2\frac{\pi}{4}\, tgh^2\frac{\pi d}{4l}} \tag{169}$$

Where I is the intensity of each of the vortices of the row. The vertical wind component produced by both rows and which we will designate by v_n will have a double value and this will be equal to

$$v_n = \frac{\pm I}{2l} \frac{tg\frac{\pi}{4}\left(2tgh^2\frac{\pi d}{4l} - 1\right)}{1 + 4tg^2\frac{\pi}{4}\, tgh^2\frac{\pi d}{4l}} \tag{170}$$

But according to Karman for the stable quincunx vortex row system the ratio $d/2l$ has the value

$$\frac{d}{2l} = 0.283 \tag{171}$$

Substituting this last value in (170) we get

$$|v_n| = 0.24\frac{I}{2l} \tag{172}$$

Such would be the mean value of the vertical wind component produced by a system of atmospheric quincunx vortex rows. By the aid of this last relation, when two of the three quantities v_n, $2l$ or I are known, the third one can be estimated. If for example we know the values of v_n and $2l$, we can find the value of

$$I = \frac{2l}{0.24} v_n$$

of the intensity of the vortices of the rows; in other words, the value of the circulation around each vortex.

When a vortex sheet breaks into vortices, the intensity of each vortex is very closely equal to the speed differences in the two layers between which the vortex sheet was formed, multiplied by the distance between the vortices. When the vortex sheet between two atmospheric layers breaks into a quincunx vortex row, we evidently first have the formation of one row, with vortices at a distance l, but this soon goes over into the stable quincunx vortex system by the upward or downward displacement of one-half of the vortices of the row, with a distance between vortices in each row equal to $2l$. If we thus call w the original speed difference in the two vortex layers which have given rise to a system of quincunx vortex rows, the intensity I of each of such vortices will thus be equal to

$$I \cong lw$$

and substituting in (172) we find

$$v_n = 0.24\frac{lw}{2l} = 0.12w$$

[3] See the author's "Introduction to the Study of Laws of Air Resistance of Aerofoils," p. 51.

Another case can also happen. We can imagine the quincunx vortex rows formed by an air current appearing in an air mass, having the same velocity in the whole current. The quincunx vortex rows will then be formed from two vortex sheets. The vortices formed from each sheet will remain in the same level and we will have

$$I = 2lw$$

where w is the speed difference between the air current and the rest of the surrounding atmosphere. In such a case, we have

$$v_n = 0.24w$$

We are thus brought to the remarkable conclusion that *the mean vertical wind component produced by a system of quincunx vortex rows, resulting from the breaking of the vortex sheets between atmospheric layers, can have values from one-eighth to one-quarter of the speed difference between the atmospheric layers that have originated these quincunx vortex rows.*

As speed differences of a few meters per second are easy to conceive between atmospheric layers, ascending wind currents of somewhat smaller values must be a frequent phenomenon, as the soaring of birds undoubtedly prove.

As a general conclusion of this discussion, one can see that *the realization of gliders able to soar in average atmospheric conditions must be considered as perfectly possible and as presenting the greatest interest.*

Such a glider must be conceived, as has already been explained, as an airplane well streamlined, with high cambered wings, small wing loading and small speed and thus small power. By the aid of its engine the airplane will reach that altitude where the formation of the system of quincunx vortex rows has taken place, and once in the ascending current will soar in it and by continuously turning around will remain in it. In an airplane specially built for soaring, the pilot will very easily feel the ascending current by the upward acceleration that it will communicate to the airplane. Even in actual high-speed airplanes, pilots have a very clear feeling of the upward and downward currents. When strong enough, the pilots describe them as the so-called "bumps," and "air holes." The bumps are, exactly speaking, strong ascending currents and the holes strong descending ones. But I mention once more that the building of such soaring airplanes will be met by complete failure, if the conditions of their maneuverability and stability are not considered with sufficient attention.

REPORT No. 97.

NOTE 1.

THE APPROXIMATE EQUATIONS OF THE CHARACTERISTICS OF THE ENGINE-PROPELLER SYSTEM.

In the foregoing has been given the method of deducing the characteristics of the engine-propeller system from the empirical curves of the engine and propeller characteristics. It is desirable in several instances to possess also approximate equations of the characteristics of the engine-propeller system because they allow a better survey, even if only to a first approximation, of the relations that held between the quantities involved in the question. The approximate equations of the different characteristics of the engine-propeller system can be easily deduced when approximate equations for the characteristics of the propeller and engine have been properly selected.

For the propeller, as a very good approximation of the characteristics, for the range of the flying interval, the following equations can be adopted:

For the thrust

$$Q = h_0 \delta N^2 H^2 D^2 \ (1 - x) \tag{173}$$

For the power

$$L_a = h_0^1 \delta N^3 H^3 D^2 \ (1 - h^2 x^2) \tag{174}$$

where h_0, h_0^1 and h^2 are three constants that characterize the propeller considered; N the number of revolutions per second; H the *zero thrust* pitch; D the propeller diameter; $x = V/NH$ the *relative pitch*[1]).

We will call h_0 the *thrust coefficient*, h_0^1 the *power coefficient*, and h^2 the *pitch coefficient*, the last named being selected for reasons that will appear later.

The *zero thrust pitch* H considered above is defined by the condition that $V/NH = 1$ for $Q = 0$, that is H is taken equal to the advance V/N, for which the thrust Q disappears. The value of H has to be deduced from the $Q/\delta \ N^2$ curve plotted against V/N, which curve intersects the V/N axis at the point $V/N = H$.

The coefficients h_0, h_0^1, and h^2 must be deduced from the empirical curves of

$$\frac{Q}{\delta N^2 H^2 D^2} = h_0 \ (1 - x) \tag{175}$$

and

$$\frac{L_a}{\delta N^3 H^3 D^2} = h_0^1 \ (1 - h^2 x^2) \tag{176}$$

plotted against $x = V/NH$ by the method of least squares.

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[1] In my general theory of blade screws I have established the following formulæ (see relation (114) p. 48):

$$\Delta Q = 2\delta \Delta \delta \Omega^2 \frac{\delta v^2}{az} r^2 tg^2 \ (\phi + \beta)$$

where ΔQ is the partial thrust. δ the air density. ΔS the annulus to which corresponds the thrust ΔQ. Ω the angular velocity of the propeller rotation $= 2\pi N$. ρ_v^2/az a dimensionless quantity function of $x = V/NH$ only. $r^2 tg^2 \ (\phi + \beta)$ a quantity nearly equal to the pitch of the blade section considered.

Integrating the above relation it will be easy to see that the result must be of the form

$$Q = h_0 \delta \ N^2 H^2 D^2 f(x).$$

On the other hand the function $f(x)$ turns out to be, in general, very closely a linear function of x, with $f(x) = 0$ for $x = 1$. We thus can write:

$$Q \cong h_0 \delta \ N^2 H^2 D^2 \ (1 - x).$$

The formula (173) is thus justified. It is from similar considerations that the formula (174) also follows.

It will be easy to convince oneself that the equations (175) and (176) will generally be able to represent the experimental curves with good accuracy.

Making use of the equations (173) and (174) we find for the efficiency η of the propeller the expression (177)

$$\eta=\frac{QV}{L_a}=\frac{h_0 x\ (1-x)}{h_0^1\ (1-h^2x^2)}. \tag{177}$$

The efficiency η is equal to zero for

$$x=0, \text{ and } x=1$$

and reaches its maximum η_m for

$$\frac{\partial\eta}{\partial x}=\frac{h_0\ (1-2x)\ (1-h^2x^2)+2h_0h^2x^2\ (1-x)}{h_0^1\ (1-h^2x^2)}=0$$

that is

$$h^2x^2-2x+1=0$$

which relation gives

$$h^2=\frac{2x_m-1}{x_m^2} \tag{178}$$

and

$$x_m=\frac{1-\sqrt{1-h^2}}{h^2} \tag{179}$$

where x_m is the value of x that corresponds to $\eta=\eta_m$.

It is the last relation (179) that has to be used for finding the value of x_m, the coefficient h^2 being found by the method of least squares from the empirical curves (175) and (176) of the thrust and power. One must avoid checking the value of x_m from the curve of the efficiency η, because it is always difficult to find accurately from an empirical curve the value of the abscissa that corresponds to the maximum of the relation (178) which shows that h^2 is a function of x_m alone. That is why I have called h^2 the pitch coefficient.

The maximum of $\eta=\eta_m$ is thus equal to

$$\eta_m=\frac{h_o x_m}{2\ h'_o} \tag{180}$$

and we also have

$$\frac{h_o}{h'_o}=\frac{2\ \eta_m}{x_m} \tag{181}$$

and finally

$$\eta=\frac{2\ \eta_m}{x_m}\frac{x\ (1-x)}{(1-h^2x^2)} \tag{182}$$

Let us find the expressions of the thrust and power coefficients h_o and h'_o as function of the power absorbed by the propeller. We have

$$L_a=h'_o\delta N^3H^3D^2\ (1-h^2x^2)$$

For the propeller working at its maximum efficiency we will have

$$(L_a)_m=h'_o\delta N_m{}^3H^3D^2\ (1-h^2x^2{}_m) \tag{183}$$

where $(L_a)_m$ and N_m are the power absorbed by the propeller and its number of revolutions for $\eta=\eta_m$. It must be remembered that in general, $(L_a)_m$ as well as N_m, are functions of the translational speed of the propeller because, when $\eta=\eta_m$ we have $x=x_m$; that is, $V_m/N_m=Hx_m$ which relation fixes only the ratio of the translational speed V_m to the number of revolutions N_m corresponding to the condition of the maximum efficiency η_m. But when the propeller considered is connected to a given engine, then the characteristics of the engine-propeller system as it has been already shown in this report, are functions—for each value of the density and throttle opening—of the translational speed V of the engine-propeller system alone. In such case there will be only a single value $N=N_m$ of the revolutions and $L_a=(L_a)_m$ of the power absorbed at which—for a given density and throttle opening—we will have $\eta=\eta_m$ if only the maximum efficiency can be reached in the given working conditions of the engine-propeller system.

From (183) we find

$$h'_o=\frac{(L_a)_m}{2\ (1-x_m)\ \delta N^3{}_m H^3 D^2} \tag{184}$$

and on account of (181)

$$h_o=\frac{\eta_m (L_a)_m}{x_m(1-x_m)\ \delta N^3{}_m H^3 D^2} \tag{185}$$

We also have

$$L_a\cong(L_a)_m\frac{N^3}{N^3{}_m}\frac{(1-h^2x^2}{2\ (1-x_m)} \tag{186}$$

For the engine power we will adopt as a first approximation,

$$L_m\cong m\ N\ \delta \tag{187}$$

considering that the engine is used in the interval at which its power is still proportional to the revolutions and giving one the liberty of making when necessary a correction for the deviation of the power from its proportionality to the density.

When a given propeller is connected to a given engine for each state of steady conditions the power absorbed by the propeller must be equal to the power delivered by the engine; that is, we must have

$$L_a=L_m$$

or

$$\frac{L_a}{\delta N^3}=h'_oH^3D^2(1-h^2X^2)=\frac{L_m}{\delta N^3}=\frac{m}{N^2}$$

From the last relations we find the law of variation of the revolutions N as function of the speed V for the engine-propeller system under consideration. We thus find—

$$N^2\left(1-h^2\frac{V^2}{N^2H^2}\right)=\frac{m}{h'_oH^3D^2} \tag{188}$$

or

$$N^2=\frac{m}{h'_oH^3D^2}+\frac{h^2V^2}{H^2}=\frac{h^2V^2}{H^2}\left(1+\frac{m}{h^2h'_oHD^2V^2}\right)$$

or finally putting

$$c=\frac{m}{h^2h'_oHD^2} \tag{189}$$

we get

$$N^2=\frac{h^2V^2}{H^2}\left(1+\frac{c}{V^2}\right) \tag{190}$$

and

$$N=\frac{hV}{H}\sqrt{1+\frac{c}{V^2}} \tag{191}$$

we also have

$$x^2=\frac{V^2}{N^2H^2}=\frac{1}{h^2\left(1+\frac{c}{V^2}\right)};\quad x=\frac{1}{h\sqrt{1+\frac{c}{V^2}}} \tag{192}$$

The expression (191) is the equation of the strip of the revolution curves of figure 12. When no allowance is made for the deviation of the engine power from its proportionality to the density the whole strip of curves is replaced by one mean curve.

Let us now find the equation of the specific thrust curve.

We have

$$Q=h_0\delta N^2H^2D^2\ (1-x)$$

or

$$Q=h_0\delta D^2\frac{V^2}{x^2}\ (1-x)$$

Substituting for x its value (192) we find,

$$Q=h_0\delta D^2h^2V^2\left(1+\frac{C}{V^2}\right)\left(1-\frac{1}{h\sqrt{1+\frac{c}{V^2}}}\right)$$

or

$$Q=h_0\delta D^2V^2\left[h^2\left(1+\frac{c}{V^2}\right)-h\sqrt{1+\frac{c}{V^2}}\right] \tag{193}$$

The last relation gives us the law of variation of the thrust Q of the motor-propeller set in function of the speed V.

On account of the fact that for the flying interval the quantity $\frac{c}{V^2}$ varies between comparatively narrow limits, we can develop the radical $\sqrt{1+\frac{c}{V^2}}$ in serie neglecting the terms of higher order and thus simplify the relation (193). We can thus take

$$\sqrt{1+\frac{c}{V^2}}\cong\alpha+\beta\frac{c}{V^2}$$

where α and β are two constants. On account of (187); (178) and (184) the value of (189) of c can be written

$$c=\frac{2(1-x_m)}{2x_m-1}\ V^2_m$$

and thus

$$\frac{c}{V^2}=\frac{2(1-x_m)}{2x_m-1}\ \frac{V^2_m}{V^2}$$

For most propellers x_m is included between 0.7 and 0.8, and in the flying range the ratio $\frac{V_m}{V}$ can hardly come out of the limits

$$1/2<\frac{V_m}{V}<2$$

Consequently the ratio c/V^2 will be usually included between the limits

$$1/2<\frac{c}{V^2}<8$$

For the last interval of variation of c/V^2 one can take with a good approximation $\alpha=1.3$; $\beta=0.21$ and thus

$$\sqrt{1+\frac{c}{V^2}}\cong1.3+0.21\ \frac{c}{V^2}$$

Making use of the approximate expression of the radical $\sqrt{1+\frac{c}{V^2}}$ the relation (193) can be written

$$Q \cong h_0 \delta D^2 V^2 \left[h^2\left(1+\frac{C}{V^2}\right)-h\left(\alpha+\beta\frac{c}{V^2}\right)\right]$$

or

$$Q \cong h_0 \delta D^2 [ch(h-\beta)-h\ (\alpha-h)V^2] \tag{194}$$

We thus find for the specific thrust the expression

$$q=\frac{Q}{\delta A}=\frac{h_0 D^2}{A}[ch(h-\beta)-h\ (\alpha-h)V^2] \tag{195}$$

The approximation, adopted in this report for the specific thrust as being of the form

$$q=q_0-q_1 V^2 \tag{196}$$

is fully justified, and we find

$$q_0=ch_0 D^2\frac{h(h-\beta)}{A} \tag{197}$$

$$q_1=h_0 D^2\frac{h(\alpha-h)}{A} \tag{198}$$

since the relations (175) and (176) constitute a good approximation for the propeller thrust and power characteristics, the possible deviation of the specific thrust curve—for a given density and throttle opening—from the law (196) must be chiefly due to the deviation of the engine power from its proportionality to the revolutions.

Substituting in the last expressions of q_0 and q_1 for the constants C, h_0 and h their values (189), (185) and (178) we find

$$q_0=\frac{C_0}{\delta A}\frac{\eta_m (L_a)_m}{V_m};\ q_1=\frac{C_1}{\delta A}\frac{\eta_m (L_a)_m}{V_m^3} \tag{199}$$

with

$$C_0=\frac{2\sqrt{2x_m-1}-\beta x_m}{\sqrt{2x_m-1}};\ C_1=\frac{(\alpha x_m-\sqrt{2x_m-1})\ \sqrt{2x_m-1}}{(1-x_m)} \tag{200}$$

In order to easily check c_0 and c_1 curves of these coefficients as functions of x_m can be traced.

We thus find for the specific thrust curve the general equation

$$q=\frac{Q}{\delta A}=q_0-q_1 V^2=\frac{\eta_m (L_a)_m}{\delta A V_m}\left(c_0-c_1\frac{V^2}{V_m^2}\right) \tag{201}$$

The thrust curve of the engine-propeller, represented in figure 12, for the approximation of a single strip, has for equation

$$\frac{Q}{\delta}=\frac{Q_m}{\delta}\left(c_0-c_1\frac{V^2}{V_m^2}\right) \tag{202}$$

with

$$Q_m=\frac{\eta_m (L_a)_m}{V_m} \tag{203}$$

Let us further find the equation of the efficiency curve of the engine-propeller set. We have:

$$\eta = \frac{QV}{L_a} = \frac{h_0 x\,(1-x)}{h_0^1\,(1-h^2x^2)}$$

Substituting for x its value (192) we get

$$\eta = \frac{h_0 V^2}{h^1_0 c}\left[\frac{1}{h}\sqrt{1+\frac{c}{V^2}} - \frac{1}{h^2}\right]$$

and on account of $\frac{c}{V^2}$ being small we find

$$\eta = \frac{h_0}{h^1_0}\left[\frac{\beta}{h} - \frac{1-\alpha h}{ch^2}V^2\right] \tag{204}$$

Substituting for h_0, h_0^1, h and c their values (185), (184), (178) and (189) we find

$$\eta = \frac{\eta_m}{\sqrt{2x_m-1}}\left[2\beta - \frac{(x_m - \alpha\sqrt{2x_m-1})\,\sqrt{2x_m-1}}{1-x_m}\cdot\frac{V^2}{V_m^2}\right] \tag{205}$$

setting

$$\frac{(x_m - \alpha\,\sqrt{2x_m-1})\sqrt{2x_m-1}}{1-x_m} = c_1'. \tag{206}$$

we finally get

$$\eta = \frac{\eta_m}{\sqrt{2x_m-1}}\left(2\beta - c_1'\frac{V^2}{V_m^2}\right)$$

The last equation gives the important law of variation of the efficiency η of the engine-propeller system as function of the speed V.

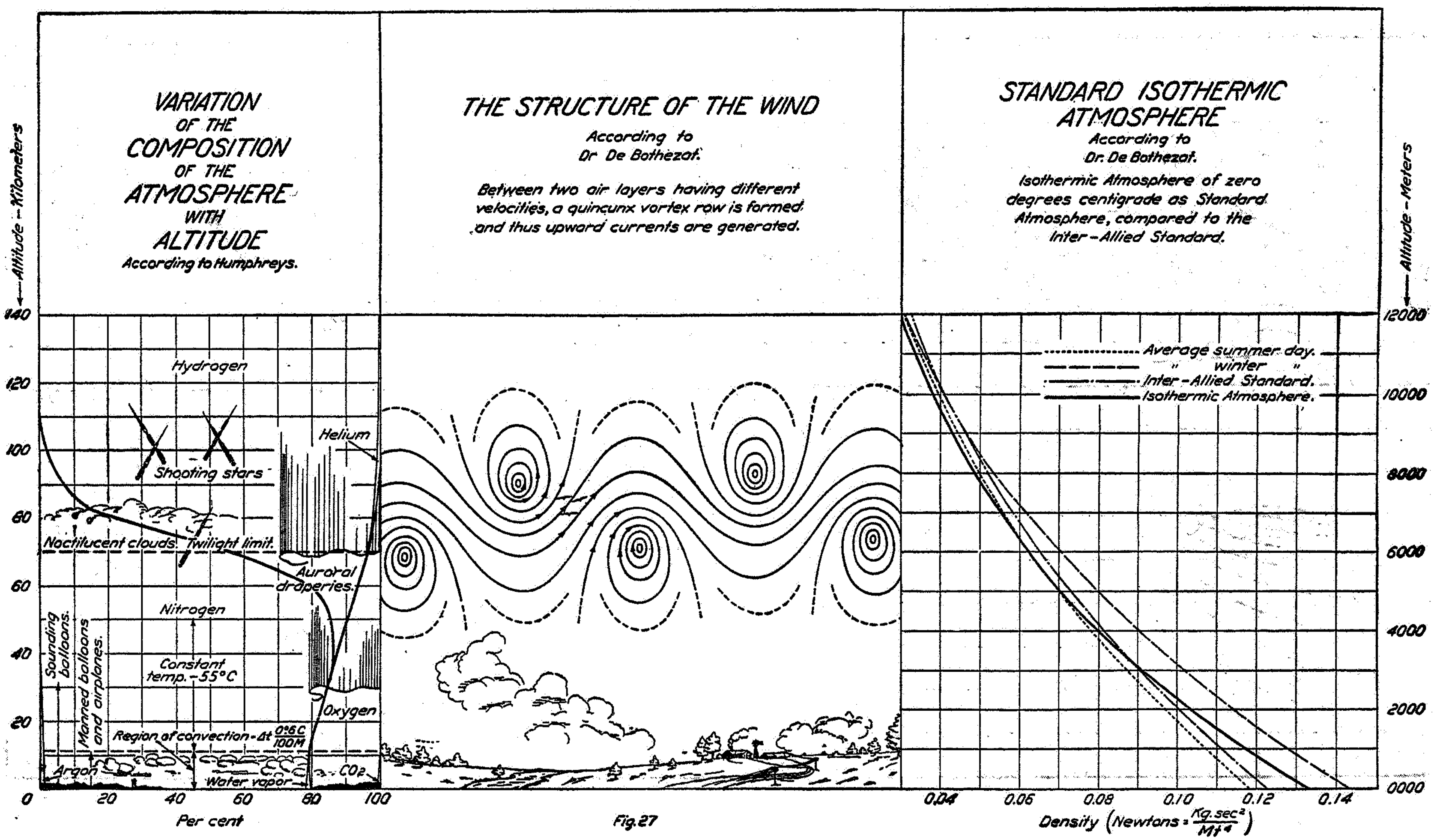
Let us finally find the equation of the power of the engine-propeller system. We have

$$L_a = \frac{N^3\,(1-h^2x^2)\,(L_a)_m}{2N_m^3\,(1-x_m)}$$

After corresponding substitutions we find

$$L_a = (L_a)_m\left[\alpha\,\sqrt{2x_m-1}\cdot\frac{V}{V_m} + \frac{2\beta\,(1-x_m)}{\sqrt{2x_m-1}}\cdot\frac{V_m}{V}\right] \tag{207}$$

The relations (191), (202), (206) and (207) are to first approximations, the equations of the main characteristics of the engine-propeller system.



VARIATION OF THE COMPOSITION OF THE ATMOSPHERE WITH ALTITUDE
According to Humphreys.
Altitude - Kilometers
140
120
100
80
60
40
20
0
20
40
60
80
100
Per cent
Hydrogen
Helium
Shooting stars
Noctilucent clouds.
Twilight limit.
Auroral draperies.
Nitrogen
Constant temp. -55°C
Sounding balloons.
Manned balloons and airplanes.
Oxygen
Region of convection - Δt 0.5°C/100M
Argon
Water vapor
CO2
THE STRUCTURE OF THE WIND
According to Dr De Bothezat.
Between two air layers having different velocities, a quincunx vortex row is formed and thus upward currents are generated.
STANDARD ISOTHERMIC ATMOSPHERE
According to Dr. De Bothezat.
Isothermic Atmosphere of zero degrees centigrade as Standard Atmosphere, compared to the Inter-Allied Standard.
Altitude - Meters
Average summer day.
" winter "
Inter-Allied Standard.
Isothermic Atmosphere.
12000
10000
8000
6000
4000
2000
0000
0.04
0.06
0.08
0.10
0.12
0.14
Density (Newtons = Kg.sec²/Mt⁴)

Fig.27

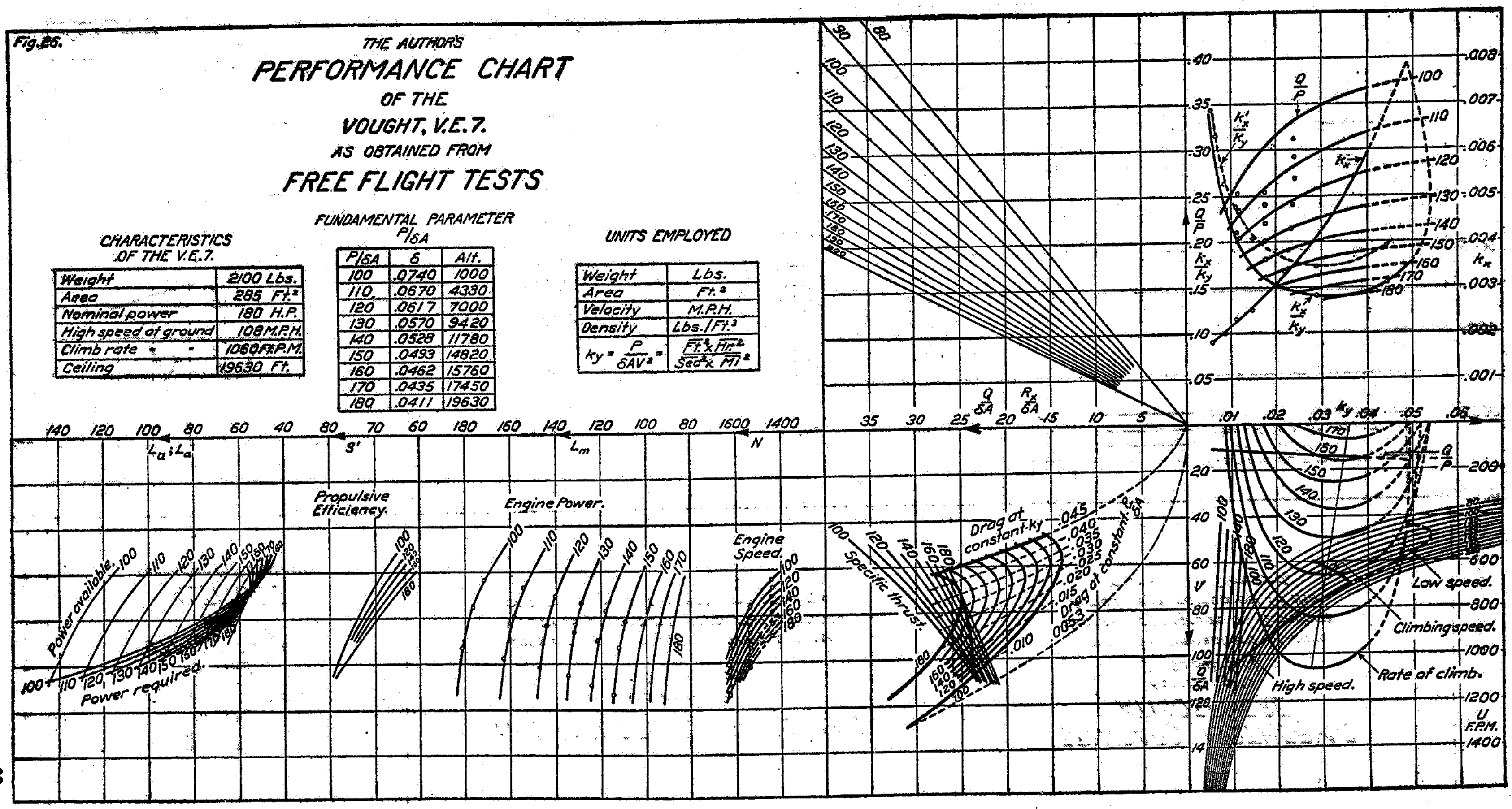


Fig. 26.
THE AUTHOR'S
PERFORMANCE CHART
OF THE
VOUGHT, V.E.7.
AS OBTAINED FROM
FREE FLIGHT TESTS
CHARACTERISTICS OF THE V.E.7.

Weight	2100 Lbs.
Area	285 Ft.²
Nominal power	180 H.P.
High speed at ground	108 M.P.H.
Climb rate " "	1060 Ft. P.M.
Ceiling	19630 Ft.

FUNDAMENTAL PARAMETER $P/\delta A$

$P/\delta A$	δ	Alt.
100	.0740	1000
110	.0670	4330
120	.0617	7000
130	.0570	9420
140	.0528	11780
150	.0493	14820
160	.0462	15760
170	.0435	17450
180	.0411	19630

UNITS EMPLOYED

Weight	Lbs.
Area	Ft.²
Velocity	M.P.H.
Density	Lbs./Ft.³
$k_y = \frac{P}{\delta A V^2}$ =	$\frac{Ft.^2 \times Hr.^2}{Sec.^2 \times Mi.^2}$

Power available.
Power required.
Propulsive Efficiency.
Engine Power.
Engine Speed.
Specific thrust.
Drag at constant k_y
Drag at constant $\frac{R_x}{\delta A}$
Low speed.
Climbing speed.
Rate of climb.
High speed.
L_a; L_a
S'
L_m
N
$\frac{Q}{\delta A}$
$\frac{R_x}{\delta A}$
$\frac{Q}{P}$
$\frac{k_x'}{k_y}$
$\frac{k_x}{k_y}$
k_x
k_y
V
U F.P.M.