REPORT No. 101

THE CALCULATED PERFORMANCE OF AIRPLANES EQUIPPED WITH SUPERCHARGING ENGINES

IN TWO PARTS

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PART I.

THE CALCULATION OF PERFORMANCE CURVES FOR AN AIRPLANE ENGINE FITTED WITH A SUPERCHARGING CENTRIFUGAL COMPRESSOR.

RÉSUMÉ, PART I.

The following report was prepared by Mr. E. C. Kemble at the request of the National Advisory Committee for Aeronautics and covers the theoretical discussion of the performance of an airplane as affected by the use of a supercharging engine, and also includes a very thorough discussion of the respective merits of the different types of superchargers that are considered.

The power developed by an aircraft engine under any given external conditions can be computed approximately if the normal power at the given speed is multiplied by appropriate temperature and pressure correction factors.

The temperature correction factor is given by equation (1), which is taken from Report No. 45.

When the intake and exhaust pressures are equal, it is best to use an equation based on the work of Report No. 46 for the pressure correction factor.

For unequal intake and exhaust pressures, the correction factor for a small range of values may be taken from figure 2, which is copied from Report No. 45.

The temperature rise in the compressor, which has an important part in determining the power output of the engine, can easily be computed when the pressure ratio, the shaft efficiency, and the heat "radiated" per pound of air by the compressor and discharge pipe are known. Under typical conditions, with the compressor exposed to the full force of the propeller slip stream, the computed value of the ratio of the actual temperature rise to the theoretical rise without heat "radiation" is 0.864.

The efficiency of the compressor and the power which it absorbs depend on the quantity of air handled per unit time. It therefore becomes necessary to discuss the variation of the volumetric efficiency of the engine with the intake temperature and the exhaust back pressure.

It is assumed that the compressor is designed for operation at a certain normal altitude and normal speed. The calculation of the net horsepower available at the propeller under these normal conditions is particularly simple. In a numerical example it is assumed that the Liberty engine is fitted with a gear-driven compressor designed to furnish sea-level carburetor pressure at 18,000 feet and an engine speed of 1,700 revolutions per minute. The shaft efficiency of the compressor is assumed to be 64 per cent. The computed horsepower is 371.

In calculating the power of an engine equipped with a turbine-driven compressor, it is assumed that the back pressure created by the turbine is equal to the increase in the carburetor pressure produced by the blower. The computed power to be expected from a Liberty engine fitted with a turbine-driven supercharger under the conditions of the preceding problem is 394.

In laying out performance curves showing the power to be expected from an engine-compressor unit at various speeds and altitudes, the variation in the efficiency of the compressor should be taken into account. The computation is somewhat involved, but can be carried through graphically.

Figure 11 shows comparative performance curves evaluated in this manner for the turbine-driven compressor, the gear-driven compressor, and for the engine operating without the compressor. The curves for the gear-driven installation are not carried to the highest altitudes on account of lack of data regarding the pressure correction coefficient for very low exhaust pressures. In carrying the computation through it was assumed that the maximum safe speed of the compressor was that required to give sea-level carburetor pressure to the engine at 18,000 feet when the crank-shaft speed was 1,700 revolutions per minute.

Curves showing the relative fuel consumption at different speeds and altitudes are easily obtained if it is assumed that the carburetor of the engine is adjusted for maximum power. (Cf. fig. 13.) They show an increase of about 20 per cent in the fuel economy at normal speed and an altitude of 20,000 feet. An even larger gain is to be expected in practice as a result of avoiding carburetor troubles due to the low temperatures which prevail at great altitudes.

1. INTRODUCTION.

This report is the outgrowth of a set of calculations made during the war on the probable performance characteristics of an airplane whose engine is equipped with a supercharging compressor of the gear-driven type. The discussion is here extended to the case of the turbine-driven type of compressor on the basis of the rough empirical rule that the exhaust back pressure created by the turbine is equal to the rise in the intake pressure due to the compressor.

The purpose of the report is twofold. It aims, in the first place, to outline a method of predicting the probable performance curves of an airplane fitted with a supercharging centrifugal compressor, and in the second place to apply this method to the case of a typical modern airplane in order to determine, as nearly as possible with the somewhat meager data now available, the gains which the use of a supercharger may be expected to bring in the near future.

Part I of the report is devoted exclusively to the discussion of the performance of the engine-compressor unit itself. This part is itself separable into two main divisions. In the first of these only so much of the theory is taken up as is necessary for the evaluation of the power which the engine and supercharger will deliver under the conditions for which the latter is designed. In the second division the variation in the efficiency of the compressor is considered, and a semigraphical method of laying out performance curves for all speeds and altitudes is evolved.

2. POWER DEVELOPED BY ENGINE WITH KNOWN INTAKE PRESSURE AND TEMPERATURE.

The computation of the power developed at various altitudes by an airplane engine operating with or without a supercharging compressor is greatly facilitated by the results of recent tests made at the Bureau of Standards and embodied in Reports No. 45 and No. 46 of the National Advisory Committee for Aeronautics.

The power developed by an airplane engine at any given speed depends on three externally variable quantities, viz, the temperature of the air entering the carburetor (intake temperature), the pressure of the air entering the carburetor (intake pressure), and the exhaust pressure. The variation in the power delivered by an engine with each of these quantities has been studied in the tests cited above.

In order to determine from the horsepower observed at the temperature t_0 (F.) the horsepower to be expected at the temperature t, we multiply by the correction factor.[1]

$$F_t = \frac{H.P.}{(H.P. \text{ at } t_0)} = \frac{920 + t_0}{920 + t}. \tag{1}$$

Wherever the intake and exhaust pressures of an engine are equal, the following formula may be used to determine the variation of the power with variation in the common value of these two pressures:

$$r_p = \frac{H.P.}{(H.P.)_{76}} = 1 - \frac{1}{\eta_{76}}\left[1 - \frac{P}{76}\right]. \tag{2}$$

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[1] Cf. Report No. 45, National Advisory Committee for Aeronautics, 1920, Part 3. It is unfortunately necessary to use the above factor for temperatures outside the range of its experimental verification.

Here p is the intake and exhaust pressure in centimeters of mercury; η_{76} and $(H. P.)_{76}$ are respectively the mechanical efficiency and the brake horsepower at 76 cm.[2]   The above equation follows directly from two hypotheses strongly supported by Report No. 46,[3] viz, (a) that the friction horsepower is independent of the intake and exhaust pressures, and (b) that the indicated indicated horsepower is directly proportional to the pressure at constant temperature. Figure 1 shows r_p plotted against p in accordance with (2) for three different values of the mechanical efficiency.

The experiments of Moss[4] show that when a centrifugal compressor is driven by an exhaust gas turbine of careful design, the pressure rise generated by the compressor under the best conditions is approximately equal to the back pressure created by the turbine.   In order to avoid excessive complication in the calculations it will be assumed throughout this report that the intake and exhaust pressures of an engine fitted with a turbine driven compressor are always equal.   In adopting this rough assumption we admittedly overestimate somewhat the per-
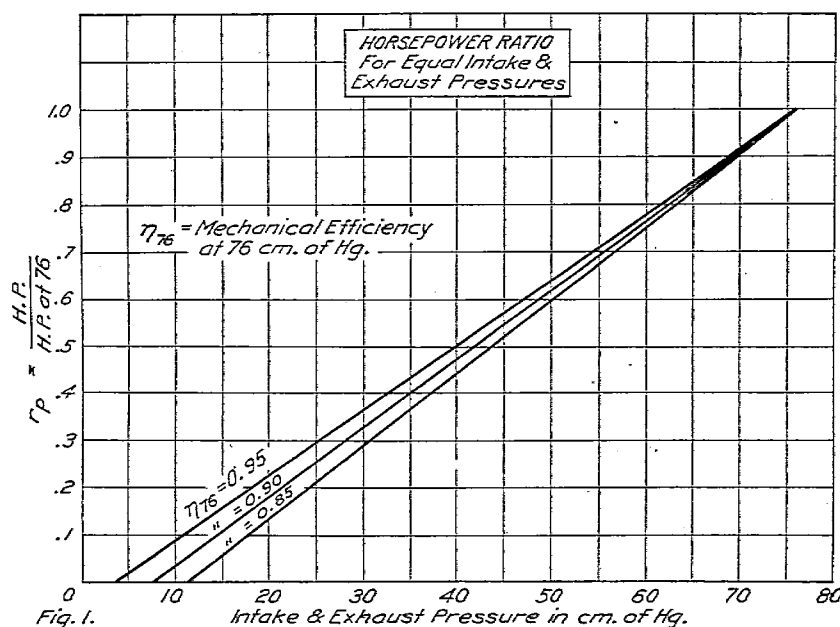

Fig. 1.   Intake & Exhaust Pressure in cm. of Hg.

formance to be expected under conditions which depart from the normal.   In the writer's opinion, however, the error involved is of a minor character.

In the case of a gear-driven compressor, on the other hand, the engine-exhaust pressure is less than the carburetor pressure, and the gross power output of the engine depends on this pressure difference as well as on the carburetor pressure.   In dealing with a problem of this type figure 2 may be used.   This diagram, which is taken from Report No. 45,[5] shows values of the ratio R_p of the horsepower developed with any given carburetor and exhaust pressures to the horsepower developed when the two pressures are each 76 cm. of mercury.   It is based on tests of a Hispano-Suiza 150-horsepower engine with a compression ratio 5.3 to 1 at 1,500 revolutions per minute.   At this speed the engine in question has mechanical efficiency of 92 per cent. Strictly speaking this set of curves is applicable only to engines having the same compression ratio and mechanical efficiency, but, in default of better information, it may be used as a first approximation for engines of other compression ratios and other mechanical efficiencies.   It will be observed that the variation of r_p (fig. 1) with the mechanical efficiency for equal carburetor and exhaust pressures increases as the pressure is lowered.   On this account the curves of

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[2] A summary of the notation used is given at the end of each part of the report.

[3] "A Study of Airplane Engine Tests", by Victor R. Gage, Report No. 46, National Advisory Committee for Aeronautics, 1920.

[4] "The General Electric Turbo Supercharger for Airplanes", by Sanford A. Moss, Aviation and Aeronautical Engineering, VIII, p. 147, 1920.

[5] "Effect of Compression Ratio, Pressure, Temperature, and Humidity, on Power," Part 2, by H. C. Dickinson and G. V. Anderson.

figure 2 may not be extrapolated to low pressures and used there for mechanical efficiencies other than 92 per cent without danger of serious error.

In order to predict the performance curves of an engine operating at various altitudes, a sea-level horsepower-speed curve (fig. 3) and curves showing the variation in the mean atmospheric temperature and pressure with altitude are needed. Figure 4 shows the relationship between temperature, pressure, and altitude recently agreed upon as standard by the French and Italian Governments. The temperature-altitude graph, from which the pressure-altitude graph is computed, diverges quite appreciably from the curves for the observed mean temperature both for very small and very great altitudes, but not enough to seriously affect the present computation.

Figure 5 shows the horsepower of an average Liberty engine as a function of altitude and speed, computed from figures 1, 3, and 4, together with the temperature correction factor of equation (1).

Fig. 2.
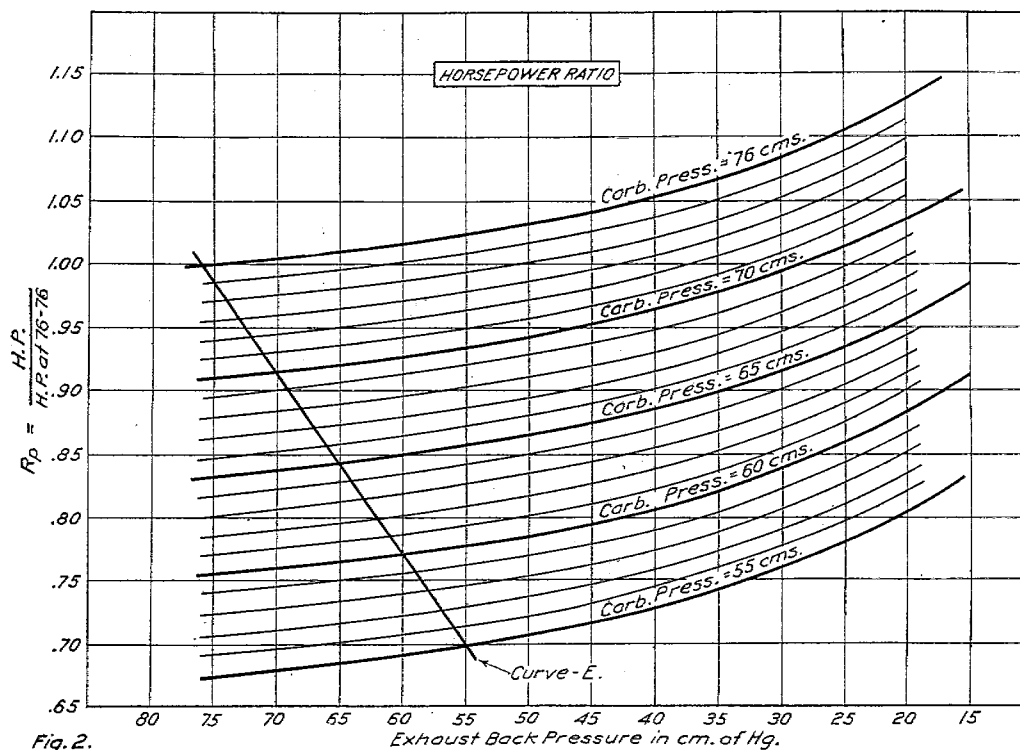
3. CALCULATION OF INTAKE PRESSURE AND TEMPERATURE FOR STANDARD ALTITUDE AND ENGINE SPEED: LIMITATIONS ON VALUE OF CARBURETOR PRESSURE.

In order to predict the performance of an engine operating with a supercharger, it is further necessary to know the increase in pressure and temperature of the air as it passes through the compressor. If the compressor is gear-driven, we must also know the power which it absorbs. Taking up first the rise in pressure, we observe that it is limited, in general, by two factors, viz, the necessity for avoiding "preignition" and the maximum safe speed of the compressor. The power available for compression is practically unlimited either in the case of a gear-driven compressor or of one driven by an exhaust-gas turbine.

The experiments of Moss have already shown that a high-compression engine, which is on the point of "preignition," or pinking, at sea level, may be operated with sea-level carburetor pressure at any altitude in spite of the high temperature of the compressed air. It is possible that higher carburetor pressures can be used with high carburetor temperatures than with low, but this point has yet to be settled, and in the present calculation it will be assumed that the carburetor pressure may not exceed a standard sea-level atmosphere. It is

worthy of note that a small net gain in power, even at sea level, is theoretically possible by using a reduced compression ratio and a carburetor pressure which is above normal sea-level value. This gain in power is accompanied by a loss of efficiency, however, and is probably of no practical importance.

The pressure ratio of a centrifugal compressor of given construction depends on the ratio of the speed to the volume of air handled, but is independent of the intake pressure. It follows that when the engine and compressor speeds are kept constant, the carburetor pressure is directly proportional to the pressure of the atmosphere. In any case there will be a certain maximum altitude for each engine speed at which the compressor can develop sea-level pressure. Above this altitude the carburetor pressure and the net available horsepower must drop off steadily as they do for an engine which is not equipped with a supercharging compressor.
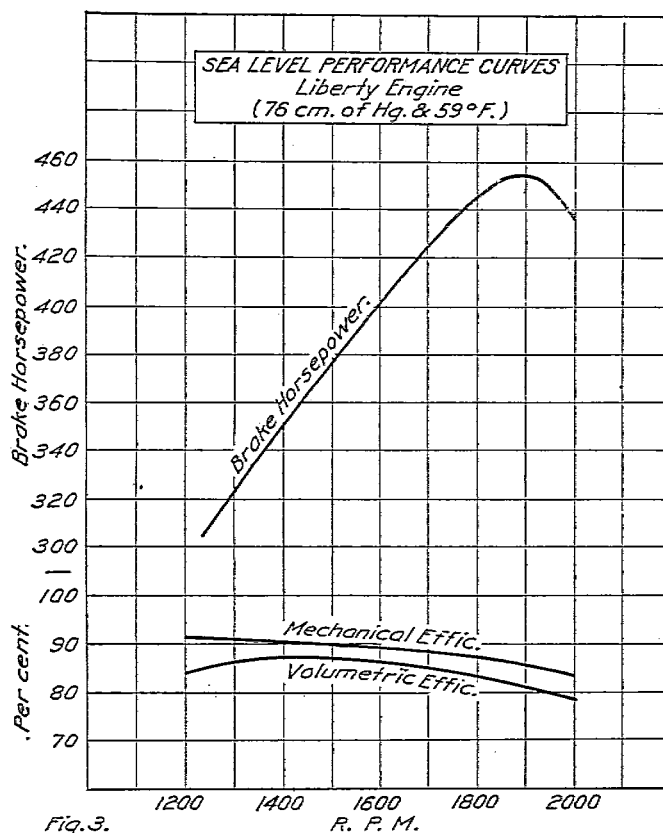

Fig. 3.

Below this altitude some means must be adopted for keeping the carburetor pressure from exceeding sea-level value. This may be done either by decreasing the compressor speed or by throttling the inlet to the compressor. The former method of control is the better, if practicable, since throttling the inlet leads to excessive heating of the air.

4. THE NORMAL ALTITUDE AND SPEED OF OPERATION.

It will be assumed that the compressor is designed to operate under the conditions prevailing at a certain definite altitude, which we will designate as the "normal" altitude. It will further be assumed that when engine and compressor operate at their normal speeds at this normal altitude, the compressor will just develop sea-level carburetor pressure and will work with maximum, or nearly maximum, shaft efficiency. The calculation of the gain in power due to supercharging is particularly simple for this one set of conditions, since the hydraulic and shaft efficiencies of the compressor may be treated as known quantities.

The normal speed of the compressor may be equal to, or less than, the maximum safe speed. In the former case the normal altitude will be the maximum altitude for normal engine speed at which the compressor can develop sea-level pressure, and consequently the altitude at

which the compressor gives the maximum increase in power. At this altitude the airplane will attain the maximum possible horizontal flight speed consistent with normal engine speed.

In the numerical example discussed in this report, it is assumed that the normal compressor speed is its maximum safe speed.

We shall first consider the operation of the compressor under the normal conditions just described, taking up the general problem later on.

5. TEMPERATURE RISE IN COMPRESSOR.

The carburetor pressure is assumed to have its sea-level value, i. e., 76 cm. of mercury. The carburetor temperature can be computed from the shaft efficiency if the heat lost due to "radiation" is known. The method is as follows:

Let the subscripts 1 and 2 refer to the states of the air as it enters the compressor and enters the carburetor, respectively. (We assume that the carburetor is located between the compressor and the engine.)
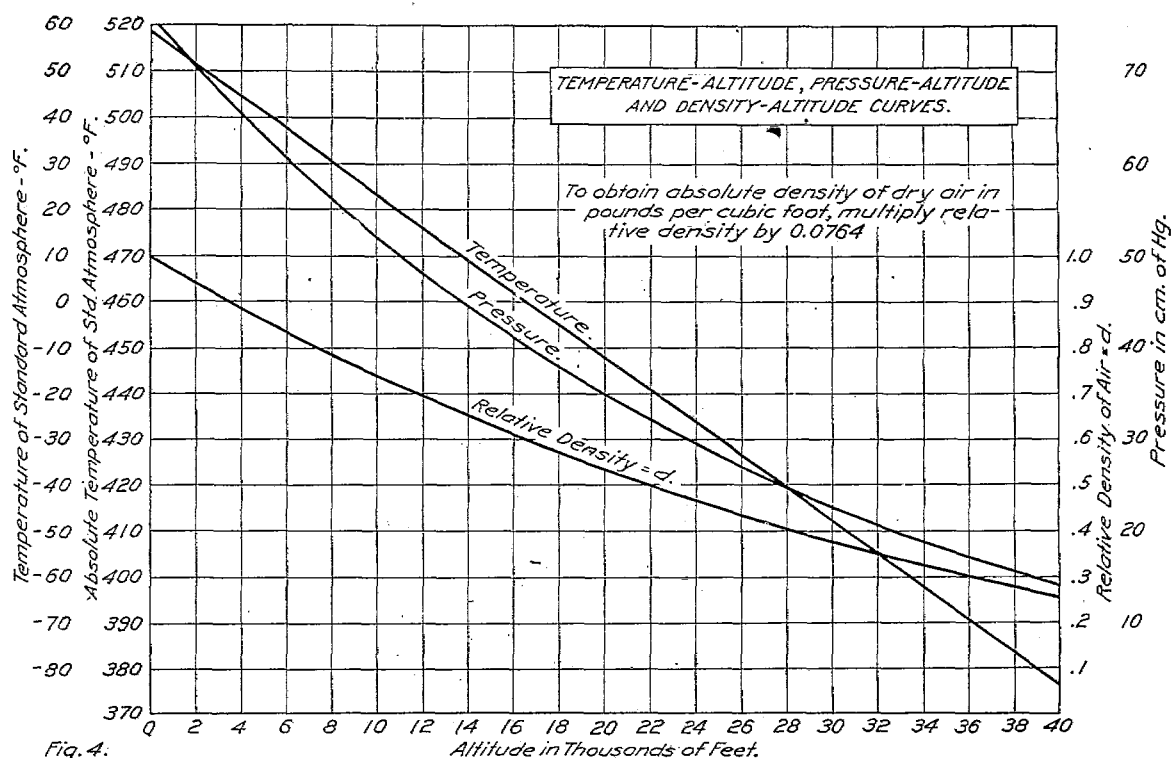

Fig. 4.

Let P, p = absolute pressure in pounds per square foot and centimeters of mercury, respectively;

V = volume in cubic feet;

T = absolute temperature in Fahrenheit degrees;

C_p = specific heat of air at constant pressure in B. t. u. per pound,
    = 0.241;

C_v = specific heat of air at constant volume in B. t. u. per pound,
    = 0.171;

$\gamma = C_p/C_v = 1.406$;

J = mechanical equivalent of heat,
    = 778 foot-pounds per B. t. u.;

I = input of mechanical energy per pound of air handled;

M = air flow through compressor in pounds per minute;

h = heat radiated per minute by the compressor and any cooling device which may be put between the compressor and the carburetor.

Neglecting the kinetic energy of the air in the discharge pipe of the compressor, we equate the net energy input per pound of air handled to the increase in the total heat of the air. Thus

$$\frac{I}{J} - \frac{h}{M} = C_p(T_2 - T_1). \tag{3}$$

Let $T_2' =$ the temperature to which the air would rise if the compression were adiabatic;
  $I' =$ corresponding energy input per pound;
and let the function A be defined by the equation

$$A(P_2/P_1) = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 = \left(\frac{P_2}{P_1}\right)^{0.289} - 1.$$

PERFORMANCE CURVES
Liberty Engine
Without Compressor.
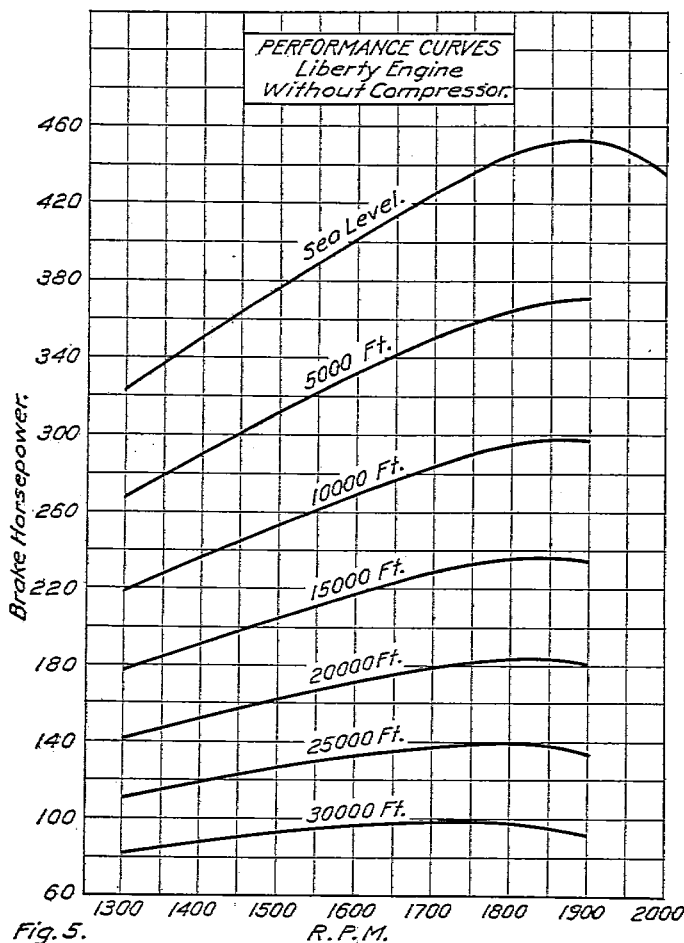
Fig. 5.

Then the familiar formula for the adiabatic compression of a perfect gas gives the relation

$$T_2' - T_1 = A(P_2/P_1)T_1,$$

and equation (3) yields

$$\frac{I'}{J} = C_p A(P_2/P_1)T_1. \tag{3a}$$

A graph of the function A is shown on figure 6. The ratio of I' to I is, by definition, the shaft efficiency of the compressor, which we denote by E_s. Therefore

$$\frac{1}{E_s} = \frac{h/M + C_p(T_2 - T_1)}{C_p A(P_2/P_1)T_1},$$

and

$$T_2 - T_1 = \frac{A(P_2/P_1)T_1}{E_s} - \frac{h}{MC_p}. \tag{4}$$

Since the heat radiated should be proportional to $T_2 - T_1$, we introduce the quantity k defined by the relation

$$h = k(T_2 - T_1).$$

Equation (3) then becomes

$$T_2 - T_1 = \frac{\mu A (P_2/P_1) T_1}{E_s},\qquad\qquad (5)$$

where

$$\mu = \frac{M C_p}{M C_p + k}.\qquad\qquad (6)$$

It is evident at once that μ is the ratio of the actual temperature rise to that which would occur if there were no radiation.

The radiation coefficient k will obviously vary a good deal with the installation and also with the conditions of operation. If the compressor is mounted behind the engine where it is exposed to little or no air current, the radiation may be nearly negligible. If it is placed at the front of the engine and exposed to the full propeller blast, the radiation may be quite important, while if a specially designed air-to-air radiator is employed the radiation coefficient k may be made as large as desired, but at the expense of increased head resistance.

An accurate theoretical evaluation of k for any given installation is not possible, but some idea of its order of magnitude and of the extent of its variation with external conditions can be obtained from theoretical considerations.

Let us set ourselves the problem of computing an approximate value of k for a supercharging compressor which is placed in front of the engine with 4 square feet of radiating area exposed to the full velocity of the propeller slip stream. Let the aeroplane have a speed of 150 miles per hour at 18,000 feet altitude, and let the compressor deliver to the engine 700 cubic feet of air per minute at sea-level pressure and at the temperature T_2. (This is approximately the volume of air required by the Liberty engine at 1,700 revolutions per minute.)

An analysis of the recent radiator tests made at the Bureau of Standards shows that the coefficient of heat transfer from a radiating surface to a stream of air is given with considerable accuracy by the empirical equation [6]

$$C_h = 29 \left(\frac{\rho v}{10} \right)^{0.83},\qquad\qquad (7)$$

where C_h = coefficient of heat transfer in B. t. u. per square foot per degree Fahrenheit mean temperature difference per hour.
ρ = density of air in pounds per cubic foot.
v = air speed in feet per second.

This equation shows that the rate of heat transfer increases rapidly with the air speed and air density. Now in practice the speed and density of the air inside the compressor casing and discharge pipe will generally be a good deal larger than the speed and density outside the casing. Consequently it is to be expected that the mean temperature difference between the casing and the external air will be a good deal greater than the mean temperature difference between the compressed air and the casing. With this fact in mind, but without making a detailed computation of the rate of heat transfer from the compressed air to the casing, we make the arbitrary assumption that the mean temperature difference between the exposed surface of the casing and the external air is three-fourths of the net temperature rise, $T_2 - T_1$. We take the air speed v to be the full air speed of the slip stream, or about 1.2 times the speed of advance of the plane. Thus

$$v = 1.2 \times 150 \times \frac{88}{60} = 264 \text{ feet per second.}$$

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[6] From data privately communicated to the writer by Mr. R. V. Kleinschmidt, formerly of the Bureau of Standards.

The relative density of the air (fig. 4) is 0.57, and the absolute density in pounds per cubic foot is 0.0436. Hence

$$C_h = 29 \ (11.5)^{0.83} = 35.6.$$

The radiation coefficient is therefore

$$k = \frac{h}{T_2 - T_1} = \frac{4 \times 35.6 \times (\frac{3}{4})}{60} = 1.78.$$

The weight of air flowing through the compressor in pounds per minute is

$$M = \frac{700 \times P_2}{R T_2},$$

where R is the gas content for air. Inserting the numerical values of R and P_2, we obtain

$$M = \frac{700 \times 144 \times 14.7}{53.3 \ T_2} = \frac{27,800}{T_2}.$$

Hence equation (6) becomes

$$\mu = \frac{6,700}{6,700 + 1.78 \ T_2}. \tag{8}$$

The carburetor pressure is exactly twice the intake pressure (see fig. 4), and the corresponding value of A (fig. 6) is 0.2218. Let the shaft efficiency of the engine be 0.64. Then (5) becomes

$$T_2 = T_1 \left[1 + \frac{0.2218}{0.64} \left(\frac{6,700}{6,700 + 1.78 T_2} \right) \right]. \tag{9}$$

The absolute intake temperature is $-5°$ F. Hence

$$T_1 = 460 - 5 = 455°,$$

and equation (9) is transformed into

$$T_2 = 455 + \frac{1,057,000}{6,700 + 1.78 T_2}.$$

The root of this equation is 591. Hence

$$\mu = \frac{6,700}{6,700 + 1.78 \times 591} = 0.864.$$

This value of μ will be used throughout the remainder of the present paper. The reader should bear in mind the fact, however, that μ will vary in practice with the installation and with the external conditions, i. e., with the values of v and M.

6. POWER ABSORBED BY COMPRESSOR.

Before making a specific application of the theory to a gear-driven compressor it is necessary to determine the power absorbed by the compressor. The theoretical input per pound of air is given by (2). To get the actual input we divide by the shaft efficiency. Thus

$$I = \frac{J C_p A (P_2/P_1) T_1}{E_s}. \tag{10}$$

The horsepower absorbed by the compressor is accordingly

$$H_c = \frac{J M C_p A (P_2/P_1) T_1}{33,000 \ E_s}. \tag{11}$$

In order to evaluate the air flow M exactly, we introduce the following notation:

$D =$ total piston displacement of engine in cubic feet;

$\epsilon =$ volumetric efficiency of engine;

$N_e =$ engine speed in revolutions per minute of crank shaft;

and

$\rho_2 =$ density of the air as it enters the carburetor.

Then

$$M = \frac{D N_e \epsilon \rho_2}{2} = \frac{D N_e \epsilon P_2}{2RT_2} \qquad (12)$$

(11) now becomes

$$H_c = \frac{J C_p A (P_2/P_1)}{66,000\ E_s} \times \frac{D N_e \epsilon P_2}{R} \times \frac{T_1}{T_2}. \qquad (13)$$

In order to apply the above formula, an estimate of the volumetric efficiency of the engine must be made. It is desirable in making supercharging calculations to have an experimental curve showing the relationship between volumetric efficiency and speed at sea level. Figure 3 shows such a curve for the Liberty engine. If experimental data are not available, a volumetric efficiency curve must be "fudged" with the aid of the curve for the brake, or, better, the indicated, mean effective pressure.

7. VARIATION OF VOLUMETRIC EFFICIENCY WITH INTAKE TEMPERATURE AND EXHAUST BACK PRESSURE.

Experiments recently made at the Bureau of Standards altitude laboratory, and privately communicated to the writer by Mr. S. W. Sparrow, show that the volumetric efficiency increases with the intake temperature and also with the ratio of the carburetor pressure to the exhaust back pressure. The ratio of the volumetric efficiency of the Hispano-Suiza 150-horsepower engine at $+10°$ C. to that at $-10°$ C. is 1.022. If the volumetric efficiency is assumed to be a linear function of the intake temperature, the following equation is easily deduced:

$$\epsilon = \epsilon_{59} + 0.00054\ (t - 59). \qquad (14)$$

Here t is the intake temperature in degrees Fahrenheit, and ϵ_{59} is the volumetric efficiency for $59°$ F.

The experimental data available (see table below) on the variation in the volumetric efficiency with the ratio of the intake to the exhaust pressure are too meager to be of service for our present purpose without the help of theoretical considerations. We will therefore proceed to derive a theoretical formula containing one adjustable constant which can be fitted to the available experimental results.

When the intake pressure P_i exceeds the exhaust pressure P_e, the volumetric efficiency of the engine will be increased, owing to the fact that there is less residual exhaust gas left in the cylinder at the end of each exhaust stroke. When the inlet valve is opened, the residual exhaust gas will be compressed from the pressure P_e to the pressure P_i. The new volume of these gases being less than the volume of the compression space, the volume left to be filled by the incoming charge is greater than normally.

To a first order approximation, the mass of the charge which enters the cylinder is independent of the heat exchange which takes place between it and the residual exhaust gas. This is because the decrease in the density of the incoming charge due to heat absorption is offset by the increase in volume available due to the cooling and shrinkage of the residual gas. If it were not for the wiredrawing which occurs when the charge begins to enter the low pressure cylinder, we might compute the effective volume of the residual gas as if it were compressed adiabatically from the pressure P_i. On account of the wiredrawing the rise in temperature which accompanies the compression will be somewhat greater than for adiabatic compression. This can be taken into account by assuming polytropic compression with an appropriate index.

Let V_s = stroke volume of one cylinder;

$\quad$ V_c = compression volume of one cylinder;

$\quad$ r = compression ratio.

Then

$$V_c = \frac{V_s}{r-1}.$$

This is the volume occupied by the residual gas at the pressure P_e. The volume occupied at P_i will be

$$V_c \left(\frac{P_e}{P_i}\right)^{\frac{1}{m}} = \frac{V_s}{r-1}\left(\frac{P_e}{P_i}\right)^{\frac{1}{m}},$$

where m is the index of compression, which would be 1.4 if there were no wiredrawing. The volume to be filled by the incoming charge will then be

$$V_s + V_c - \frac{V_s}{r-1}\left(\frac{P_e}{P_i}\right)^{\frac{1}{m}} = V_s\left[\frac{r}{r-1} - \frac{1}{r-1}\left(\frac{P_e}{P_i}\right)^{m}\right].$$

But with equal intake and exhaust pressures this volume would be simply V_s. Hence the ratio of the volumetric efficiency, when P_e and P_i are different, to the normal volumetric efficiency is

$$\sigma = \left[\frac{r}{r-1} - \frac{1}{r-1}\left(\frac{P_e}{P_i}\right)^{\frac{1}{m}}\right], \tag{15}$$

The value of m might, perhaps, be computed theoretically, but it is easier to treat it as a constant to be determined empirically. The accompanying table shows the results of a short series of tests on the variation of the volumetric efficiency of a Hispano-Suiza engine with a $7\frac{1}{2}$ to 1 compression ratio. In the fourth column are tabulated the theoretical values of σ as computed from (15), using 2 for the value of m. The agreement is within the limits of experimental error and establishes 2 as an approximate and convenient value for the index m.

TABLE No. 1.

P_i.	$P_i - P_e$.	Relative air flow.		Remarks.
Inches of mercury.		Exp.	Theor.	
25	1.73	1.005	1.006	Engine driven by dynamometer.
25	−1.73	.99	.995	Do.
17.5	1.73	1.01	1.008	Do.
17.5	−1.73	.99	.992	Do.
14.5	1.73	1.005	1.01	Do.
14.5	−1.73	.995	.991	Do.
14.5	1.73	1.015	1.01	Engine driven by own power.
14.5	−1.73	.991	.991	Do.

In the case of an engine fitted with a gear-driven centrifugal compressor, P_i is equal to the compressor exhaust pressure, P_2. Hence (15) may be rewritten in the form

$$\sigma = \left[\frac{r}{r-1} - \frac{1}{r-1}\sqrt{\frac{P_e}{P_i}}\right]. \tag{15a}$$

In dealing with turbine-driven compressors we shall use equation (14) for the volumetric efficiency. For gear-driven compressors this is to be replaced by

$$\epsilon = \sigma\left[\epsilon_{59} + 0.00054\,(t-59)\right]. \tag{16}$$

8. APPLICATION TO LIBERTY ENGINE, GEAR-DRIVEN COMPRESSOR.

Let us assume a gear-driven centrifugal compressor having a maximum shaft efficiency of 64 per cent and capable of doubling the carburetor pressure of the Liberty engine when engine and compressor are working at their normal speeds. We inquire regarding: (a) The volume of air which the compressor must be designed to handle; (b) the horsepower required to drive the compressor under normal conditions; and (c) the net gain in power of the engine when coupled with the compressor at the "normal altitude."

(a) The conditions are the same as those used in determining the value of k (cf. latter part of art. 5). The normal pressure ratio being 2:1, the normal altitude must be that for which the barometric pressure has half its sea level value, i. e., 18,000 feet. The corresponding intake temperature is $-5°$ F., or 455° absolute (cf. fig. 4.) The value of $A(P_2/P_1)$ is given by figure 6. Inserting numerical values into equation (5), we obtain 136° as the net temperature

$$A\left(\tfrac{P_2}{P_1}\right) = \left(\tfrac{P_2}{P_1}\right)^{0.289} - 1$$

Fig. 6. P_2/P_1
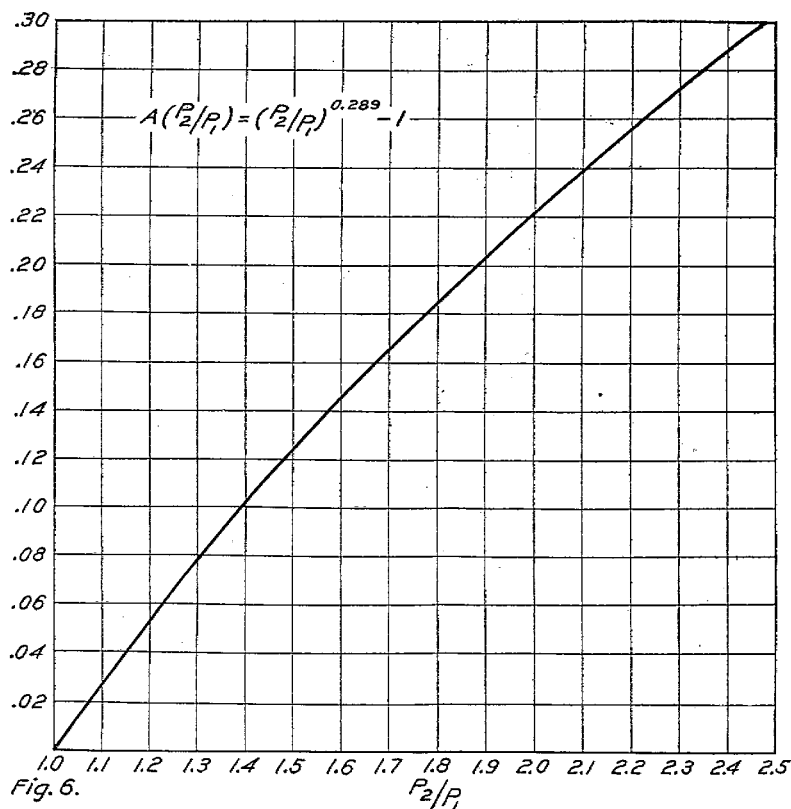
rise in the compressor. Hence the temperature of the air entering the carburetor is 131° F., or 591° absolute. The normal speed of the Liberty engine is 1,700 revolutions per minute and its piston displacement is 0.96 cubic foot. With the aid of these numerical values we reduce (12) to the form

$$M = \frac{0.96 \times 1700 \times 14.7 \times 144\epsilon}{2 \times 53.3 \times 591} = 54.8\epsilon.$$

The normal value of the volumetric efficiency at 1,700 revolutions per minute, and with a carburetor temperature of about 59° F., is 0.85. The compression ratio of the engine is 5.42. Hence the corrected volumetric efficiency for 131° F. and a 2 to 1 pressure ratio is (cf. equation (16)):

$$\epsilon = (0.85 + 0.00054 \times 72)(1.226 - 0.226 \sqrt{1/2}),$$
$$= 0.95$$

The corresponding value of M is 52 pounds per minute.   This is a little greater than the value given by the approximate formula (7).

Let Q denote the intake volume for the compressor in cubic feet per minute. Q is equal to M divided by the density of the air entering the compressor.   Figure 4 gives 0.57 as the relative density at 18,000 feet.   The corresponding absolute density is 0.0436 pounds per cubic foot. Hence

$$Q = \frac{52}{0.0436} = 1{,}193 \text{ cubic feet per minute.}$$

The above intake volume, the normal speed, and normal pressure ratio are the three fundamental quantities which determine the design of the compressor.

(b) All the quantities which enter into the right-hand member of (13) are now known. The numerical value of H_c computed from this formula is 46.5 horsepower.

(c) Let H_G denote the gross horsepower developed by the engine with the intake pressure and temperature P_2 and T_2, respectively, and with the exhaust pressure P_1.   The temperature correction factor, equation (1), is

$$F_t = \frac{920 + 59}{920 + 131} = 0.931.$$

The pressure correction factor for a 76 cm. intake pressure and a 38 cm. exhaust pressure is 1.06.   (See fig. 2.)   The normal power developed by the engine at 1,700 revolutions per minute is 423.   Hence the value of H_G is

$$H_G = 423 \times 1.06 \times 0.931 = 417.5 \text{ horsepower.}$$

Subtracting the power required to drive the compressor, we obtain the net power available for driving the propeller, which is

$$H = 417.5 - 46.5 = 371 \text{ horsepower.}$$

9. TURBINE-DRIVEN COMPRESSOR.

As previously stated, we assume equal intake and exhaust pressures for the turbine-driven compressor.   Under the normal working conditions just considered, these pressures will have the common value 76 cm.   The pressure correction factor drops out, and the net power becomes equal to the gross power.   Thus

$$H = H_G = 423 \times 0.931 = 394 \text{ horsepower.}$$

Since the pressure correction factor for the volumetric efficiency drops out, the normal volume of air which the turbine-driven compressor must be designed to handle is a little less than that for the gear-driven compressor.   The numerical value is

$$Q = 1{,}115 \text{ cubic feet per minute.}$$

We may estimate the weights of the gear-driven and turbine-driven compressors at 75 and 100 pounds, respectively.   The weight of the engine without the compressor, dry, is 844 pounds.   Hence the weight per horsepower at 18,000 feet with the compressor, works out to be 2.48 pounds per horsepower and 2.40 pounds per horsepower, for the gear-driven and turbine-driven jobs, respectively.   The weight per horsepower without the compressor at this altitude is 4.5.   Thus the reduction in the weight per horsepower ratio due to the compressor is 44.9 per cent and 45.7 per cent for the gear-driven and turbine-driven jobs, respectively.

10. CALCULATION OF PERFORMANCE CURVES: COMPRESSOR THEORY.

In order to extend the calculation to other than normal conditions and to draw up performance curves for the engine compressor unit at all altitudes and speeds, it is necessary to make use of the characteristic curves for the shaft and hydraulic efficiencies of the compressor.[7]

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[7] See article on Centrifugal Compressors by Dr. L. C. Loewenstein in Marks's "Mechanical Engineers' Handbook."

Experiment shows that these efficiencies depend primarily on the ratio of the volume of intake air to the speed of the compressor. This ratio is called the quantity coefficient, and will be designated by the symbol q.

$$q = Q/N.$$

It is convenient to plot the efficiencies as ordinates against values of q as abscissae, as in the characteristic curves shown in figure 7.
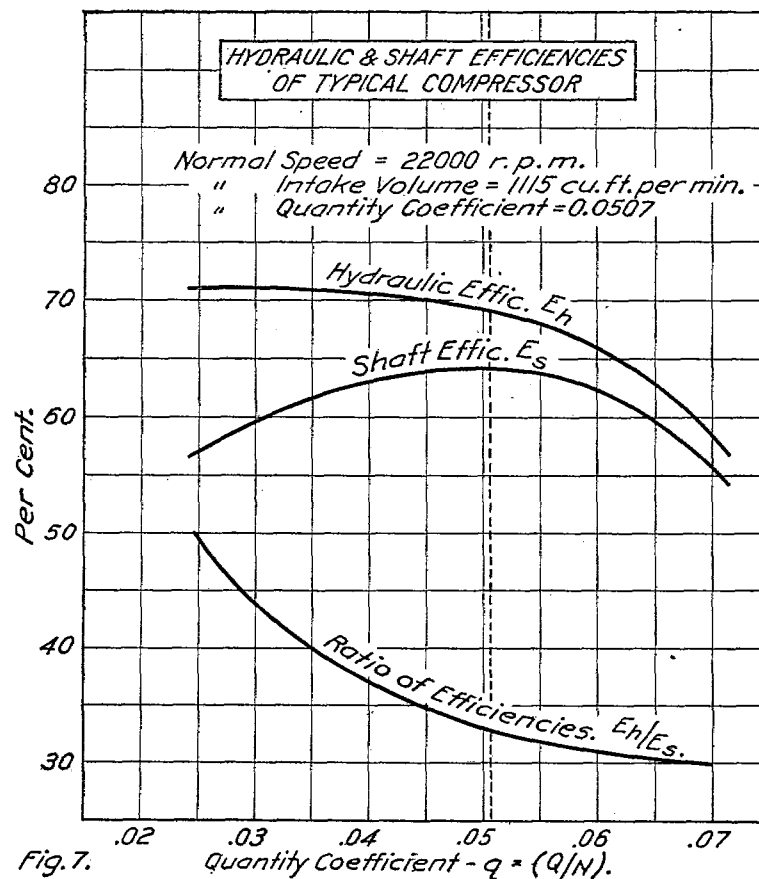

HYDRAULIC & SHAFT EFFICIENCIES OF TYPICAL COMPRESSOR

Normal Speed = 22000 r.p.m.
" Intake Volume = 1115 cu.ft. per min.
" Quantity Coefficient = 0.0507

Hydraulic Effic. E_h

Shaft Effic. E_s

Ratio of Efficiencies. E_h/E_s.

Per Cent.

Fig. 7.   Quantity Coefficient - $q = (Q/N)$.

The fundamental formula which determines the pressure rise is

$$A(P_2/P_1) = \frac{E_h(q)}{g}\left(\frac{\gamma-1}{\gamma}\right)\frac{u_a^2}{RT_1}. \tag{17}$$

where $E_h(q)$ = hydraulic efficiency;

g = acceleration of gravity;

u_a = peripheral speed of compressor impeller in feet per second.

The other symbols have already been defined. Since the peripheral speed is proportional to N, the above equation can be rewritten as

$$A(P_2/P_1) = \frac{\alpha E_h(q)N^2}{T_1}, \tag{18}$$

where α is a constant for any given compressor. Solving for P_2/P_1, we obtain

$$P_2/P_1 = \left[\frac{1 + \alpha E_h(q)N^2}{T_1}\right]^{3.46} \tag{19}$$

In order to use the above equation for the determination of P_2/P_1, or to use (5) to evaluate the carburetor temperature, it is necessary to calculate the value of the quantity coefficient, q. To this end we divide (12) by $N\rho$, where ρ_1 denotes the density of the air entering the compressor, and obtain

$$q = \frac{M}{\rho_1 N} = \frac{D}{2} \frac{N_e}{N} \frac{P_2}{P_1} \frac{T_1}{T_2} \epsilon.$$ (20)

The right-hand member involves unknown quantities which must be eliminated before (20) can be solved for q.

In carrying out this elimination we have two cases to consider.   (I) In the case of a turine-driven compressor working at an altitude below its normal altitude of operation, the speed of the compressor will be adjusted to give sea-level carburetor pressure. In this case, the intake pressure being known, the pressure ratio is known, and the speed N must be eliminated from equations (18) and (20) in order to solve for q.   (II) In the case of a gear-driven compressor operating at any altitude, or of a turbine-driven compressor operating above the normal altitude, the speed of the compressor is determined either by the gear ratio, or by the maximum safe speed of the compressor, and the pressure ratio is the unknown quantity to be eliminated from equations (18) and (20).

11. CASE I. TURBINE-DRIVEN COMPRESSOR: PRESSURE RATIO GIVEN.

Consider first the case where the pressure ratio is known. Eliminating N between (18) and (20), we obtain

$$q = \frac{D N_e}{2} \frac{P_2}{P_1} \frac{T}{T_2} \epsilon \sqrt{\frac{\alpha F_h(q)}{A T_1}}.$$

Eliminating $\frac{T_1}{T_2}$ by means of (5) the above becomes

$$q = \frac{D N_e}{2} \frac{P_2}{P_1} \epsilon \sqrt{\frac{\alpha F_h(q)}{A T_1}} \frac{1}{\left[1 + \frac{\mu A}{F_s(q)} \right]}$$

or

$$\frac{2\sqrt{A T_1}}{D N_e \epsilon \sqrt{\alpha} P_2} \frac{P_1}{P_2} = \frac{F_s(q) \sqrt{F_h(q)}}{q[F_s(q) + \mu A (P_2/P_1)]}.$$ (21)

The right-hand side is a function of q and (P_2/P_1). The left-hand side is sensibly independent of q, and its value is easily calculated. In computing the value of ϵ, we combine (16) with (5), and obtain

$$\epsilon = \sigma \left[\epsilon_{59} + 0.00054 \left\{ T_1 \left(1 + \frac{\mu A}{E_s} \right) - 519 \right\} \right]$$ (22)

In this equation F_s may be treated as a constant without serious error.

It is convenient to introduce the notation

$$\phi(q, P_2/P_1) = \frac{F_s(q) \sqrt{F_h(q)}}{q[F_s(q) + \mu A (P_2/P_1)]}.$$ (23)

$$\psi(P_2/P_1, T_1, N_e) = \frac{2\sqrt{A(P_2/P_1)} T_1}{\sqrt{\alpha} D N_e \epsilon} \frac{P_1}{P_2}$$ (24)

Equation (21) then becomes

$$\psi \left(\frac{P_2}{P_1}, T_1, N_e \right) = \phi \left(q, \frac{P_2}{P_1} \right)$$ (25)

To solve this equation, a set of curves showing ϕ as a function of q for various constant values of P_2/P_1, may be drawn, as in figure 8. The value of ψ can be computed directly from (24). To find q we simply follow the line $y = \psi$ horizontally across to the point which corresponds to the appropriate value of P_2/P_1 and note the corresponding abscissa.

Having calculated the quantity coefficient in this manner, the shaft efficiency can be found from figure 7, and equation (5) employed to determine the temperature at the carburetor.

Fig. 8.
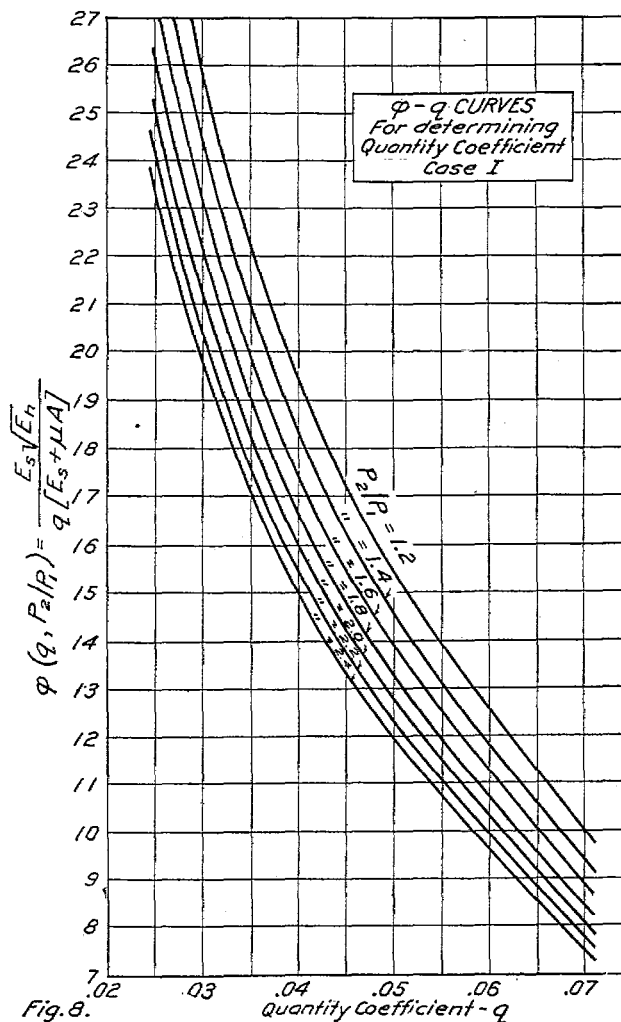
The remainder of the computation for the horsepower of the engine-compressor unit is similar to that already carried out for the normal conditions of operation in article 8.

12. NUMERICAL APPLICATION.

· To illustrate with a numerical example, we assume the turbine-driven engine compressor unit of the problem of article 8. The constant α of equation (18) can be computed from the data already assumed for normal operation at 18,000 feet altitude. The normal speed of the compressor is 22,000 revolutions per minute. The normal intake volume is 1,115 cubic feet per minute. Hence the normal value of the quantity coefficient is 0.0507 and the normal hydraulic efficiency is 0.69. The normal pressure ratio is 2, and the corresponding value of A (fig. 6) is 0.2218. The normal absolute intake temperature (fig. 4) is 455°. Solving (18) for α, we obtain

$$\alpha = \frac{AT_1}{E_h N^2} = \frac{0.2218 \times 455}{0.69 \times (22,000)^2} = 0.303 \times 10^{-6}.$$

Inserting the numerical values of α and D in (24), we obtain

$$\psi = \frac{3.788\sqrt{AT_1}}{N_e\epsilon} \cdot \frac{P_1}{P_2} \tag{24'}$$

Giving E_s the constant value 0.64, putting σ equal to unity, and giving μ the value previously computed, viz, 0.864, we reduce (22) to the form

$$\epsilon = \epsilon_{59} + 0.00054[T_1(1 + 1.35A) - 519]. \tag{25}$$

Figure 8 shows the ϕ—q curves plotted from (23) for $\mu = 0.864$.

Let us apply these formulas and curves to the problem of the determination of the power delivered by the engine and turbine-driven compressor at 1,800 revolutions per minute and an altitude of 10,000 feet.

The intake pressure (fig. 4) is 52.1 cm. of mercury, and the pressure ratio required to give sea-level carburetor pressure is accordingly

$$P_2/P_1 = 76/52.1 = 1.46.$$

From figure 6

$$A\left(\frac{P_2}{P_1}\right) = 0.1156.$$

The volumetric efficiency at 59° F. and 1,800 revolutions per minute (fig. 3) is 0.83, and the compressor intake absolute temperature at 10,000 feet is 483.4°. Then by (25)

$$\epsilon = 0.83 + 0.00054[483(1 + 1.35 \times 0.1156) - 519] = 0.8505.$$

Equation (24') yields

$$\psi = \frac{3.788\sqrt{0.1156 \times 483}}{1800 \times 1.46 \times 0.8505} = 12.67.$$

The corresponding value of q (fig. 8) is 0.0561. The compressor speed, equation (18), is 16,810 revolutions per minute. Its shaft efficiency (fig. 7) is 0.6345, and the absolute carburetor temperature, equation (5), is

$$T_2 = 483(1 + 0.864 \times 0.1156/0.6345) = 559°.$$

Hence the temperature correction factor, equation (1), is

$$F_t = \frac{920 + 59}{920 + 99} = 0.961.$$

The sea-level horsepower at this speed is 445. Hence the power at 1,800 revolutions per minute and 10,000 feet altitude is

$$H = 0.961 \times 445 = 427.5.$$

13. CASE II. TURBINE-DRIVEN COMPRESSOR: ROTATIONAL SPEED GIVEN.

Consider next the case where the compressor speed is known and the pressure ratio is unknown. In this case P_2/P_1 must be eliminated from (20) by means of (19). Then

$$q = \frac{D}{2} \frac{N_e}{N} \frac{T_1}{T_2} \epsilon \left[1 + \alpha\, E_b(q)\, \frac{N^2}{T_1}\right]^{3.46}. \tag{26}$$

Combining equations (5) and (18), we obtain

$$T_2 = T_1\left[1 + \mu \alpha \frac{E_h(q)}{E_s(q)} \frac{N^2}{T_1}\right]. \tag{27}$$

Eliminating T_1/T_2 from (26) by means of (27) leads to the following equation:

$$q = \frac{D}{2} \frac{N_e}{N} \epsilon \frac{\left[1 + \alpha E_h(q) \frac{N^2}{T_1}\right]^{3.46}}{\left[1 + \mu\alpha \frac{E_h(q)}{E_s(q)} \frac{N^2}{T_1}\right]} \cdot \qquad (28)$$

In order to calculate ϵ, we combine (16) with (27), and obtain

$$\epsilon = \sigma \left[\epsilon_{59} + 0.00054\left(T_1 - 519 + \mu\,\alpha N^2 \frac{E_h}{E_s}\right)\right]. \qquad (29)$$

Here again it is sufficiently accurate to treat E_h/E_s as a constant. A further simplification results from the fact that in the case of the turbine-driven compressor, now under consideration, σ is always unity. It is convenient to introduce the notation

$$x\left(q, \frac{N^2}{T_1}\right) = \frac{\left[1 + \alpha E_h(q) \frac{N^2}{T_1}\right]^{3.46}}{q\left[1 + \mu\alpha \frac{E_h(q)}{E_s(q)} \frac{N^2}{T_1}\right]}; \qquad (30)$$

$$\zeta(N_e, N, T_1) = \frac{2N}{D N_e\left[\epsilon_{59} + 0.00054\left(T_1 - 519 + \mu\alpha N^2\frac{E_h}{E_s}\right)\right]} \cdot \qquad (31)$$

Then (28) reduces to

$$\zeta(N_e, N, T_1) = x\left(q, \frac{N^2}{T_1}\right). \qquad (32)$$

To solve this equation a set of curves showing χ as a function of q for various constant values of N^2/T_1 is drawn up, as on figure 9. The value of ζ for any particular case can be computed directly from (31), since ζ does not involve q. To find q, we proceed as in Case I. Follow the line $y = \zeta$ horizontally across to the point which corresponds to the appropriate value of N^2/T_1 and note the corresponding abscissa.

When q is known, the pressure ratio can be found at once from (18) and figure 6. Combining equations (5) and (18), we obtain

$$T_2 = T_1 + \mu\,\alpha\,N^2 \frac{E_h\,(q)}{E_s\,(q)}. \qquad (33)$$

Equation (33) serves for the determination of the temperature rise, and the calculation of the horsepower is carried through as before.

14. NUMERICAL APPLICATION OF THEORY FOR CASE II.

We illustrate the above theory with a numerical example. Consider the operation of the engine-compressor unit of the preceding examples above the altitude of normal operation. For maximum power the compressor will operate at its maximum safe speed, 22,000 revolutions per minute, under all conditions. The values of μ and α are 0.864 and 0.303×10^{-6}, respectively. Figure 9 shows the $\chi - q$ curves obtained from (30). Giving E_h/E_s the value 1.077, equation (31) reduces to

$$\zeta = \frac{45,800}{N_e[\epsilon_{59} + 0.00054\,(T_1 - 383.5)]}. \qquad (34)$$

Equation (33) becomes

$$T_2 = T_1 + 126.7\,E_h\,(q)/E_s(q), \qquad (35)$$

and (18) takes the form

$$A = 146.8\,E_h(q)/T_1. \qquad (36)$$

Let us use the above equations to evaluate the power delivered by the engine at 40,000 feet and 1,900 revolutions per minute. From figure 4 the temperature T_1 is 376.5°. The value of ζ, equation (34), is 28.95. N^2/T_1 is 1.285×10⁶. Hence the quantity coefficient, q, (fig. 9) is 0.0554. E_h/E_s (fig. 7) is 1.066, and the carburetor temperature, equation (35), is 495° absolute. Th temperature correction factor, equation (1), is 1.025. The hydraulic efficiency

Fig. 9.
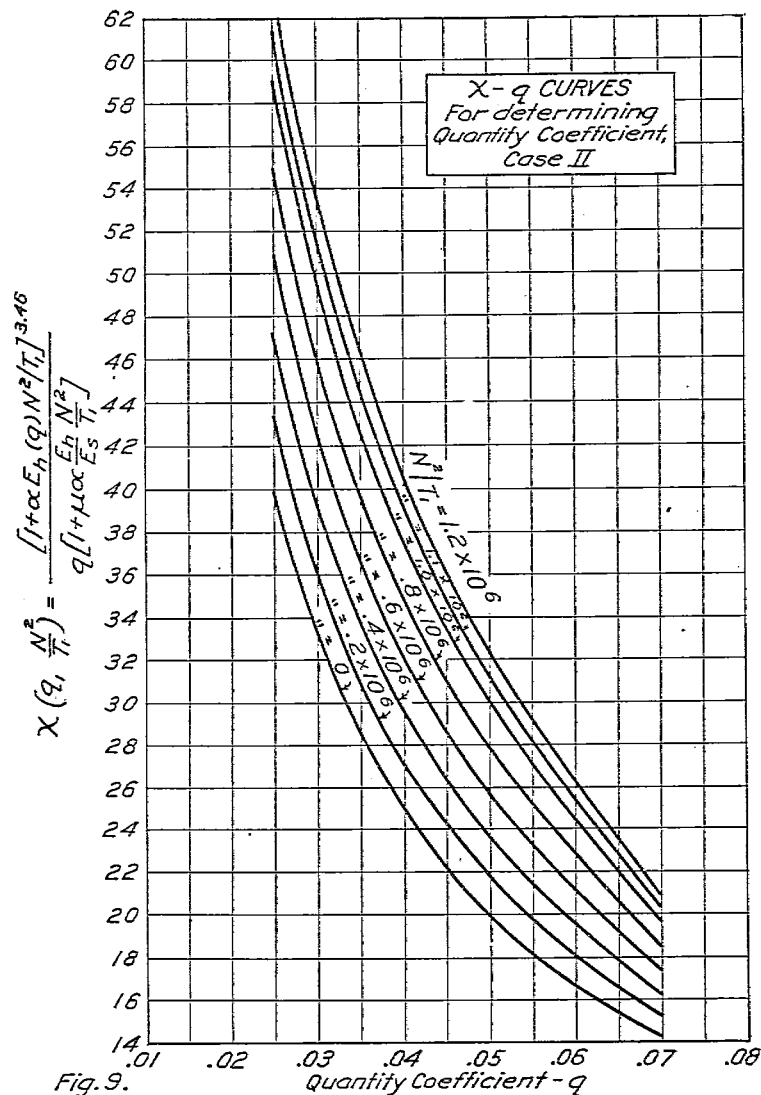
(fig. 7) is 0.678, and the corresponding value of A, equation (36), is 0.2645. The pressure ratio (fig. 6) is 2.251. The compressor intake pressure (fig. 4) is 14 cm. of mercury, and the carburetor pressure is accordingly 31.5 cm. of mercury. The mechanical efficiency at 76 cm. and 1,900 revolutions per minute is 0.855 (fig. 3), and the pressure correction factor (fig. 1) is 0.315. The sea-level horsepower is 453, and consequently the horsepower under the conditions assumed is

$$H = 453 \times 1.025 \times 0.315 = 146.$$

Complete performance curves for the Liberty engine fitted with a turbine-driven compressor, and worked out in the above manner, are shown on figure 10.

15. THE GEAR-DRIVEN COMPRESSOR.

The great advantage of the gear-driven type of compressor is that by its use the engine can be made to develop a high power at great altitudes with a low exhaust pressure. Such a low exhaust pressure involves a correspondingly low exhaust temperature and should materially

increase the life of the exhaust valves, which would be comparatively short in an engine equipped with a turbine-driven compressor and operated continuously with sea-level intake and exhaust pressures. It also seems not improbable that if the mechanical problem of designing a slipping clutch which will take excessive acceleration stresses from the gears can be solved, the gear-driven type of compressor will prove the more durable of the two.

On the other hand, the computation of article 9 predicts that the net power available per unit weight from a given engine operating with a gear-driven compressor at its normal altitude is about 3 per cent less than the net power available from the same engine under the same conditions, with a turbine-driven compressor. If this result be accepted as correct, the case for the gear-driven compressor would seem to be a poor one.

The simplest mechanical arrangement for a gear-driven compressor involves a single set of gears and a constant value for the ratio of the compressor speed to the engine crank-shaft speed.
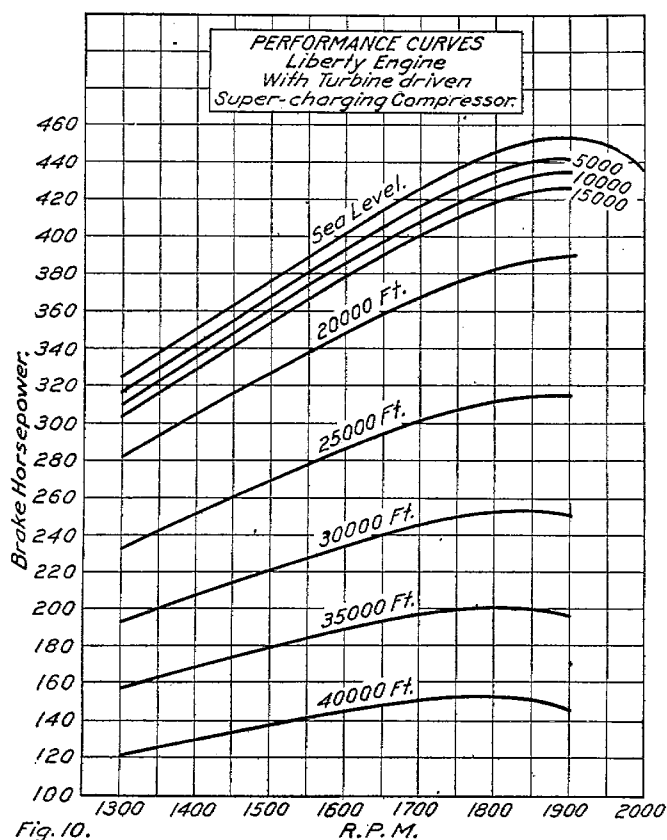

Fig. 10.

This lack of flexibility is an additional distinct disadvantage which might possibly be overcome, in part, by the use of a two-speed gear, or through the use of a constantly slipping clutch. The mechanical difficulties involved in these forms of speed control are great, however, and in view of the proved feasibility of the turbine-driven compressor, neither arrangement will be considered here.

In order to prevent the carburetor pressure from rising above sea-level value at low altitudes with a gear driven compressor, it is necessary to disconnect the compressor entirely, or to throttle the air at the inlet to the compressor. In our computation of performance curves, it will be assumed that means are provided for inlet throttling at moderate altitudes. The horsepower-altitude curve for each speed is then divided into three parts, viz, a portion ab (see fig. 11) for the lowest altitudes where the compressor can not be used to advantage at all, and is assumed to be disconnected, a portion bc over which the compressor is assumed to be throttled at the inlet in such a manner as to maintain the carburetor pressure at the constant value 76 cm., and

finally, a portion cd for the highest altitudes, where no inlet throttling is necessary, and where the carburetor pressure is less than at sea level.

Let us consider, first, the region over which inlet throttling is necessary. It will be assumed that the kinetic energy developed by the air as it passes through the throttle valve is immediately converted into heat. Then the temperature of the air as it enters the compressor will be sensibly equal to the temperature of the external atmosphere. The carburetor pressure is known, but the pressure of the throttled air P_1 is not, so that the pressure ratio must be treated as an unknown quantity. Since the speed of the compressor is known the method of computing the quantity coefficient is practically the same as for the turbine-driven compressor above the normal

Fig. 11.   Altitude in Thousands of Feet.
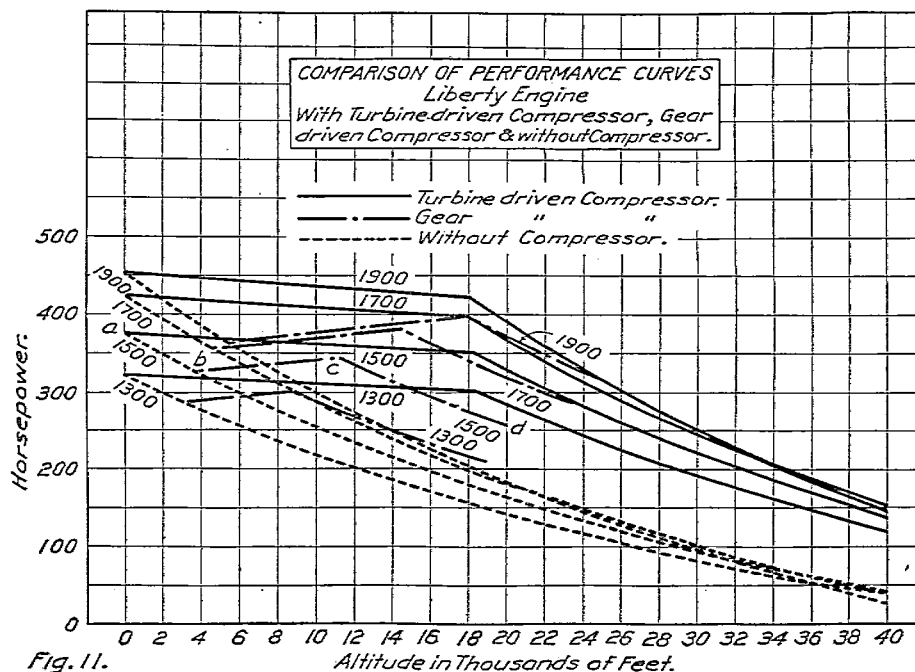
altitude of operation. The one difference is that the quantity σ, which gives the pressure correction to the volumetric efficiency, does not reduce to unity. Equation (32), which was used for the determination of the quantity coefficient q, is replaced by

$$\zeta_1 = \chi\left(q, \frac{N^2}{T_1}\right)$$ (37)

where

$$\zeta_1 = \frac{\zeta(N_e, N, T_1)}{\sigma}.$$ (38)

The value of σ is to be computed from (15a), P_e being identified with the pressure of the external atmosphere. When q is evaluated by means of (37) and a χ—q chart, such as figure 9, the compressor efficiencies are determined, and equation (18) is used to calculate the carburetor temperature. The temperature correction factor and the pressure correction factor are taken from equation (1) and figure 2, respectively.

The product of the normal sea-level power into the temperature and pressure correction factors is the gross horsepower developed by the engine. From this must be subtracted the power absorbed by the compressor, which may be computed from the following equation, derived by combining (13) and (18):

$$H_c = \frac{J C_p D \alpha N^2 N_e}{66,000 R} \frac{E_h}{E_s} \frac{\epsilon P_2}{T_2}.$$ (39)

In computing the performance curves for the region where inlet throttling is *not* necessary, P_2 is an unknown, and P_e is to be identified with P_1. σ can not be determined until the value of q is known. We therefore substitute from (19) into (15a) and obtain the following expression for σ in terms of q and N^2/T_1:

$$\sigma = \frac{r}{r-1} - \frac{1}{r-1}\left[1 + \alpha E_h(q)\frac{N^2}{T_1}\right]^{-1.73} \tag{40}$$

Equation (32) is replaced by

$$\zeta = \chi_1\left(q,\frac{N^2}{T_1}\right), \tag{41}$$

where

$$\chi_1\left(q,\frac{N^2}{T_1}\right) \equiv \chi\left(q,\frac{N^2}{T_1}\right)\left[\frac{r}{r-1} - \frac{1}{r-1}\left(1 + \alpha E_h(q)\frac{N^2}{T_1}\right)^{-1.73}\right]. \tag{42}$$

The computation of q and of the temperature correction factor then goes through as in the preceding case. The pressure ratio is determined from figure 6 and equation (18). Figure 2 is used to determine the pressure correction factor, and the gross horsepower, compressor horsepower, and net horsepower computed as before.

Figure 11 shows a set of predicted performance curves for the Liberty engine equipped with a gear-driven compressor, calculated by the method described above. In arriving at this set of curves (dot-and-dash lines) it was assumed that gear ratio was 11.6:1, making the compressor speed 22,000 revolutions per minute (maximum safe speed) when the engine runs at 1,900 revolutions per minute. For convenience in computation the same compressor was assumed as in the calculations for the turbine-driven job. As we have already shown (Art. 9) that the gear-driven compressor should have a slightly larger volume capacity than the turbine-driven compressor for normal operation on the same engine, it is evident that our method of procedure involves a slight handicap to the gear-driven job, when operating at its highest speed. This handicap, which consists in assuming compressor efficiencies which are somewhat smaller than the maximum obtainable efficiencies, is small, however, compared with the differences between the outputs of the gear driven and turbine driven arrangements, and does not materially affect the relative merits of the two schemes. For comparison the performance curves for the turbine-driven supercharging unit and for the engine without the compressor are also shown on figure 11. The curves of the gear-driven compressor are not carried to very great altitudes because the pressure correction factors involved lie outside the chart of figure 2.

Figure 11 shows that at low speeds for all altitudes, and at all speeds for medium altitudes, the gear-driven compressor without speed control produces much less power than the turbine-driven compressor. In view of this fact, and of the mechanical difficulties involved in the gear drive, the turbine-driven compressors alone will be considered in the remainder of this report.

16. FUEL CONSUMPTION.

The formulas for the temperature and pressure correction factors which we have used are based on tests in which the carburetor was adjusted for the maximum fuel economy consistent with maximum power. The corresponding fuel consumption curves must accordingly presuppose carburetor adjustment for maximum power at all altitudes. The investigations of Mr. P. S. Tice [8] at the Bureau of Standards show that the mixture ratio which gives maximum power is independent of the barometric pressure, and we therefore base our computation of fuel economy on the assumption of a constant air-fuel ratio. Power-altitudes and fuel-economy curves based on the assumption that the carburetor is adjusted for maximum fuel economy would be of value, but data for their computation are not available at present.
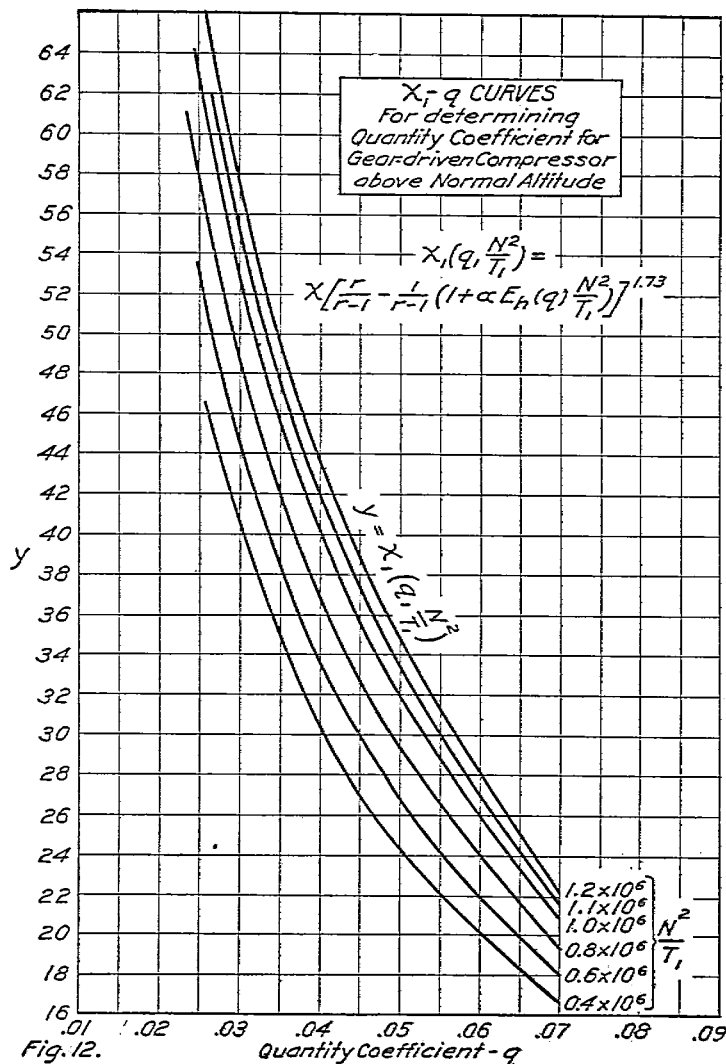
Let the subscript s indicate quantities pertaining to operation under standard sea level conditions (76 cm. pressure and 59° F.). The relative fuel consumption for a constant mixture ratio

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8 "Carbureting Conditions Characteristic of Aircraft Engines," by P.S.Tice, Report No. 48, National Advisory Committee for Aeronautics. 1920.

is equal to the quotient of the relative air flow divided by the relative horsepower output. Hence the relative fuel consumption is

$$r.\,f.\,c. = \frac{H_s}{H}\,\frac{N_e}{(N_e)_s}\,\frac{\epsilon}{\epsilon_s}\,\frac{P_2}{P_s}\,\frac{T_s}{T_2},\qquad (43)$$

where P_2 is the carburetor pressure in centimeters of mercury and T_2 is the absolute temperature of the air entering the carburetor.

X-q CURVES
For determining
Quantity Coefficient for
Gear-driven Compressor
above Normal Altitude

$$X_1(q, \tfrac{N^2}{T_1}) = X\left[\tfrac{r}{r-1} - \tfrac{1}{r-1}\left(1 + \alpha E_h(q)\tfrac{N^2}{T_1}\right)\right]^{1.73}$$

$y = X_1(q, \tfrac{N^2}{T_1})$

1.2×10^6
1.1×10^6
1.0×10^6
0.8×10^6 $\dfrac{N^2}{T_1}$
0.6×10^6
0.4×10^6

Fig. 12.   Quantity Coefficient - q

In some cases it is desirable to refer the fuel consumption for any given speed under high altitude conditions to the fuel consumption at the same speed on the ground. Then (43) becomes

$$r.\,f.\,c. = \frac{H_s}{H}\,\frac{\epsilon}{\epsilon_s}\,\frac{P_2}{P_s}\,\frac{T_s}{T_2}.\qquad (44)$$

As a check on the above formula we insert Table 2, which gives a comparison of computed and experimental values of the relative fuel consumption at low pressures. The experimental values are taken from Report No. 46 ("A Study of Airplane Engine Tests" by Victor R. Gage). The temperature was the same for all the tests and the intake and exhaust pressures were in all cases equal. Hence the factors T_s/T_2 and $\dfrac{\epsilon}{\epsilon_s}$ reduce to unity and drop out of the computation.

TABLE 2—*Computed and experimental values of relative fuel consumption of Hispano-Suiza 150 horsepower engine at low pressure.*

Engine speed.	Ground.		10,000 feet.		20,000 feet.		30,000 feet.	
	Com-puted.	Experi-mental.	Com-puted.	Experi-mental.	Com-puted.	Experi-mental.	Com-puted.	Experi-mental.
1,300	1.00	1.00	1.05	1.025	1.10	1.087	1.21	1.26
1,500	1.00	1.00	1.05	1.04	1.105	1.091	1.19	1.27
1,700	1.00	1.00	1.05	1.03	1.10	1.091	1.20	1.213
1,900	1.00	1.00	1.045	1.043	1.10	1.122	1.233	1.269
2,100	1.00	1.00	1.06	1.047	1.14	1.14	1.284	1.314

The very appreciable discrepancies in the above table are presumably to be attributed to the uncertainty of a carburetor setting for maximum power. It will be observed that the tendency is for the experimental values to exceed the theoretical ones for the lowest pressures

Fig. 13.
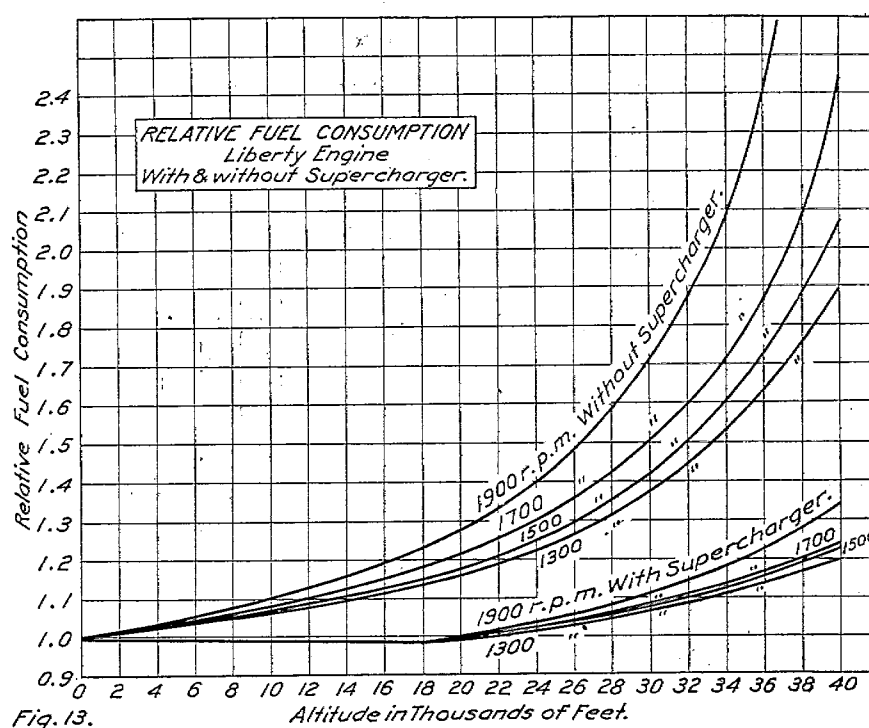
(greatest altitudes). This tendency would be greatly augmented under flying conditions at great altitudes as a result of the very low temperatures and consequent poor carburetion and distribution. It is to be expected, therefore, that in using (44) we will underestimate the fuel consumption at great altitudes for the engine without the supercharger. Since the carburetor pressures and temperatures for the supercharging engine are relatively high, however, the above remark does not apply to the estimated fuel consumption for the engine compressor unit.

Figure 13 shows curves giving the relative fuel consumption of the Liberty engine with and without the supercharging compressor at all altitudes, as computed from (44). The great waste of fuel at high levels is very evident. Its physical explanation lies in the fact that as the power drops off, the mechanical losses, which are assumed to be constant for each speed, use up a larger and larger percentage of the energy of the fuel. The supercharging compressor, by maintaining the power, maintains the mechanical efficiency.

The actual value of the supercharging device as a means for saving fuel is best seen, however, when we are in a position to plot curves showing the fuel consumption per mile instead of per brake horsepower hour. This subject will be taken up in the second part of the report.

SUMMARY OF NOTATION FOR PART 1.

The subscripts 1 and 2 refer to the state of the air as it enters the compressor and the carburetor, respectively.

F_t = temperature correction factor for horsepower of engine.

r_p = pressure correction factor for equal intake and exhaust pressures.

R_p = pressure correction factor for unequal intake and exhaust pressures.

p, P = air pressure in cm. of mercury and pounds per square foot, respectively.

P_i, P_e = engine intake and exhaust pressures, respectively.

t, T = Fahrenheit and absolute Fahrenheit temperatures, respectively.

V = volume in cubic feet.

R = gas constant for air in engineer's units (= 53.34).

ρ = density of air (dry) in pounds per cubic foot.

C_p = specific heat of air at constant pressure in B. t. u. per pound (= 0.241).

C_v = specific heat of air at constant volume in B. t. u. per pound (= 0.171).

$\gamma = C_p/C_v = 1.406.$

J = mechanical equivalent of heat = 778 foot pounds per B. t. u.

Q = compressor intake volume in cubic feet per minute.

M = air flow through compressor in pounds per minute.

I, I' = energy input of compressor per pound of air (actual and theoretical).

T'_2 = theoretical carburetor temperature (adiabatic compression).

h = heat radiated per minute.

C_h = coefficient of heat transfer in B. t. u. per square foot per degree Fahrenheit mean temperature difference per hour.

v = air speed in feet per second.

$k = h/(T_2 - T_1).$

$\mu = MC_p/(MC_p + k).$

$A\left(\dfrac{P_2}{P_1}\right) = \left(\dfrac{P_2}{P_1}\right)^{0.289} - 1.$

E_s = shaft efficiency of compressor.

E_h = hydraulic efficiency of compressor.

N = compressor speed in revolutions per minute.

$q = Q/N$ = quantity coefficient.

u_a = peripheral speed of impeller of compressor in feet per second.

g = acceleration of gravity.

H_c = horsepower absorbed by compressor.

η = mechanical efficiency of engine.

ϵ = volumetric efficiency of engine.

σ = ratio of volumetric efficiency for actual intake and exhaust pressures to volumetric efficiency for equal intake and exhaust pressures.

r = compression ratio of engine.

V_s = displacement volume for one piston in cubic feet.

D = total piston displacement of engine in cubic feet.

V_c = compression volume in cubic feet.

N_e = crankshaft speed in revolutions per minute.

H_0 = gross horsepower developed by engine.

H = net horsepower available for driving propeller.

$\alpha = \dfrac{1}{gR}\left(\dfrac{\gamma - 1}{\gamma}\right)\left(\dfrac{u_a}{N}\right)^2.$

For the definitions of the remaining symbols see the equations indicated in the table below:

Symbol	φ	ψ	χ	ζ	ζ_1	χ_1
Equation	(23)	(24)	(30)	(31)	(38)	(42)

REPORT No. 101.

PART II.

THE CALCULATION OF AIRPLANE PERFORMANCE FROM THE ESTIMATED PERFORMANCE CURVES OF ENGINE AND COMPRESSOR.[1]

RÉSUMÉ, PART II.

If the heat leak from the gas turbine and exhaust pipes to the water jackets is prevented, and if the cooling system is kept under a constant pressure independent of that of the atmosphere, no additional radiator equipment should be required when a supercharging compressor is fitted to an airplane engine.

The total additional weight of the propelling plant due to the use of a supercharger is estimated at about 120 pounds.

A method of estimating airplane performance at altitudes with the aid of curves for the "reduced" thrust horsepower available and required, is developed. This method simplifies the graphs of the thrust horsepower required at altitudes, and is particularly useful in comparing the performance of planes of different sizes, wing loadings, and propelling plant characteristics, which have the same lift and drag coefficients.

Two methods for drawing curves of the thrust horsepower available with a variable pitch propeller are indicated.

Horizontal flight speed and maximum climbing speed curves for the LePere two-seater fighter when equipped with supercharging and nonsupercharging engines, and with both fixed blade and variable pitch propellers, are worked out with the aid of the estimated performance curves for the Liberty engine with turbine-driven supercharging compressor shown on figure 10, part 1.

Altitude-time curves at maximum climbing rate are plotted for the LePere when equipped with each of the four types of propelling plant just mentioned.

Curves showing the relative fuel economy (i. e., relative distance traversed per pound of fuel) with the engine wide open at all altitudes, are plotted and discussed.

A supercharging installation suitable for commercial use is described, and it is shown that with the aid of the compressor a great saving in fuel and a considerable increase in carrying capacity can be effected simultaneously.

The outcome of the investigation is distinctly favorable to the use of a supercharging compressor as a means for obtaining better performance for both military and nonmilitary airplanes. The variable pitch propeller would be a valuable adjunct to the supercharger, but is not essential to its utility.

In an appendix the writer derives a theoretical formula for the correction of the thrust coefficient of an airscrew to offset the added resistance of the plane due to the slip stream effect.

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[1] A summary of the notation for Part II will be found at the end of this part.

1. INTRODUCTION.

In this second part of the report, the estimated performance curves for the engine with turbine-driven supercharger are applied to the discussion of airplane performance at great altitudes. The methods of calculating altitude performance curves are of general application, although they are particularly convenient for the treatment of the specific problem here under discussion.

The reader who is not interested in methods of computation can omit articles 4, 5, 6, and the major portion of article 8.

The appendix on "The correction of the propeller thrust coefficient for the slip stream resistance" has only an indirect connection with the supercharging problem.

2. RADIATOR EQUIPMENT FOR AIRPLANES WITH SUPERCHARGERS.

In the recent trials of the General Electric turbine-driven supercharger on the LePere airplane, additional radiator equipment was needed, over and above that required for the engine without the compressor. Calculation shows, however, that in the future we may expect to dispense with this excess radiator equipment. A primary reason for the use of a large radiator with the General Electric supercharger has been that there is a considerable heat leak from the gas turbine and the exhaust manifolds to the water jackets. Suitable heat insulation should reduce this leak to negligible magnitude and so greatly decrease the amount of heat to be disposed of in the radiator.

The heat radiated per square foot of radiator area per unit time is proportional to the mean temperature difference between the water and the air, and to the 0.83 power of the product of the density of the air and the speed of advance.[2] The area required under any given conditions should be roughly proportional to the power of the engine and should vary inversely as the rate of heat transmission per unit area. We have already seen that the power of the engine remains nearly constant from sea level up to the maximum altitude at which sea-level carburetor pressure is obtainable. At this altitude the ratio of the density of the air to the power developed by the engine has its least value. Hence if the radiator is large enough to take care of the heat to be dissipated at sea level and also at this critical altitude, it should be large enough for all altitudes.

Let us therefore compute the ratio of the radiator area S required at the maximum altitude for sea level carburetor pressure, to the area S_o required at the ground. It will readily be seen from the preceding paragraph that this ratio is

$$\frac{S}{S_o} = \frac{H}{H_o} \left(\frac{d_o V_o}{dV}\right)^{0.83} \times \left(\frac{t_o' - t_o}{t' - t}\right) \tag{1}$$

where H = horsepower of engine,
   t = temperature of air,
   t' = mean temperature of water,
   V = speed of advance,
   d = relative density of air,
and the subscript $_o$ indicates sea level values.

In radiator calculations for altitude work it is customary to assume that the difference between the mean temperature of the water and the boiling point of water is to be kept constant as the atmospheric pressure changes. The lowering of the boiling point as the pressure drops off then largely compensates for the decrease in the air temperature, and greatly reduces the available temperature difference at great altitudes. It is possible, however, to put the cooling system under a constant pressure, so that the boiling point of the water will not vary with the pressure of the external atmosphere, and our calculation will be based on the assumption of the existence of such a fixed radiator pressure.

[2] Cf. article 5, Part I.

Consider the special case of the Liberty engine and turbine-driven compressor discussed in Part 1. The critical altitude is 18,000 feet. According to figure 4 the mean free-air temperatures at sea level and 18,000 feet are 59° F. and −5° F., respectively. We assume that the mean water temperature is 182° F., or 30° less than the sea level boiling point. Then

$$\frac{t_0'-t_0}{t'-t}=\frac{182-59}{182+5}=0.658.$$

The relative density of the air at 18,000 feet is 0.57, while that at sea level is by definition unity. We assume the speeds of advance at sea level without the compressor and at 18,000 feet with the compressor to be 138 miles an hour and 160 miles an hour, respectively. (This assumption will be justified later on.) The horsepowers at sea level and 18,000 feet for normal speed (fig. 11, 1700 revolutions per minute) are 425 and 395 respectively. Then

$$\frac{S}{S_o}=\frac{395}{425}\times0.658\left(\frac{138}{0.57\times160}\right)^{0.83}=0.862.$$

This shows that under the conditions of the above calculation the required radiator area should not be increased by the use of a supercharging compressor unless there is a heat leak from the turbine to the water jackets.

If the radiator is kept at the pressure of the external atmosphere, on the other hand, a small increase in radiator area is needed. The temperature factor becomes

$$\frac{t_o'-t_o}{t'-t}=0.796,$$

and the relative radiator area required at 18,000 feet is

$$\frac{S}{S_o}=1.043.$$

Such an increase of a little more than 4 per cent in the radiator area would not be a serious handicap to the performance of an airplane. In the calculations which follow we will assume, however, that the radiator area is *not* increased by the use of the supercharging compressor.

3. ADDITIONAL WEIGHT DUE TO SUPERCHARGER.

The one handicap to airplane performance involved in the use of a supercharger is additional weight. We estimate the weight of the compressor, turbine, and mountings at 100 pounds for the special case considered in part 1. The increase in the weight of the propeller would be about 20 pounds, making a total additional weight due to the supercharger of about 120 pounds.

The weight of the compressor would not vary greatly with the size of the engine to which it is fitted. Consequently the greatest increase in the ratio of the horsepower to the weight is to be expected in supercharging with *large* engines.

4. CALCULATION OF AIRPLANE PERFORMANCE CURVES FOR GREAT ALTITUDES: REDUCED THRUST HORSEPOWER AND REDUCED SPEED OF ADVANCE.

The method which we shall employ for computing altitude performance curves is new in part.

The simplest means for finding the maximum horizontal flight speed and maximum climbing speed at sea level is to draw up curves showing the thrust horsepower available and required for propulsion as functions of the speed of advance. The high speed intersection of these two curves gives the maximum horizontal flight speed of the airplane, and the maximum difference of the ordinates of the two curves is usually taken to be the maximum power available for climbing. In order to use this method for altitude performance calculations it is necessary to draw up a pair of curves for the horsepower required and available at each altitude

considered. If a comparison of the performances of the same airplane when equipped with two, or more, different propelling plants is desired, a further complication arises from the fact that any alteration in the weight of the machine modifies the set of curves for the required horsepower. In order to avoid this undue multiplication of graphs, we will substitute for the actual thrust horsepower and actual speed of advance two quantities which we shall call the "reduced thrust horsepower" and the "reduced speed of advance." When this is done the whole set of curves for the thrust horsepower required at different altitudes by similar machines of different weights collapses into one. A simple slide rule calculation suffices for the determination of the actual maximum horizontal flight speed under any given conditions when the reduced maximum horizontal flight speed is known.

Let U, V = speed of advance (relative to air) in miles per hour and feet per second, respectively;

A = wing area in square feet;
α = angle of attack;
θ = angle of climb;
ρ = density of air in pounds per cubic foot;
g = acceleration of gravity = 32.16 feet per second per second;
Y = lift of machine in pounds;
X = drag of machine in pounds;
$K_y(\alpha)$ = lift coefficient for entire airplane;
$K_x(\alpha)$ = drag coefficient for entire airplane.

Then from the definition of the lift and drag coefficients we have

$$Y = \frac{\rho V^2 A}{g} K_y(\alpha) \tag{2}$$

$$X = \frac{\rho V^2 A}{g} K_x(\alpha). \tag{3}$$

For our present purpose it will be sufficiently accurate to assume that the propeller thrust is opposite in direction to the speed of advance.

Let W = total weight of airplane in pounds;
T = propeller thrust in pounds.

Then

$$Y = W \cos \theta. \tag{4}$$
$$T = X + Y \tan \theta. \tag{5}$$

Substituting $Y K_x(\alpha)/K_y(\alpha)$ for X in (5), we reduce it to the form:

$$T = Y[K_x(\alpha)/K_y(\alpha) + \tan \theta]. \tag{6}$$

The elimination of Y between (4) and (6) then gives

$$T = W\left[\frac{K_x(\alpha)}{K_y(\alpha)} \cos \theta + \sin \theta\right]. \tag{7}$$

Let H_t = thrust horsepower *required* for steady flight with the angle of attack α and the angle of climb θ.

Then

$$H_t = TV/550 = \frac{WV}{550}\left[\frac{K_x(\alpha)}{K_y(\alpha)} \cos \theta + \sin \theta\right]. \tag{8}$$

This is the first of the two fundamental equations of our theory. The second is obtained by combining (2) amd (4). It is

$$V = \sqrt{\frac{Wg \cos \theta}{\rho A K_y(\alpha)}}. \tag{9}$$

The above equations in the parameter α can be used to plot the curve showing the relationship between the thrust horsepower required and the speed of advance for any specified angle of climb.

Substituting from (9) into (8) and rearranging, we obtain

$$\frac{H_t}{W}\sqrt{\frac{\rho A}{W}} = \frac{1}{550}\sqrt{\frac{g\cos\theta}{K_y(\alpha)}}\left[\frac{K_x(\alpha)}{K_y(\alpha)}\cos\theta + \sin\theta\right]. \tag{10}$$

Rearrangement of (9) itself yields

$$V\sqrt{\frac{\rho A}{W}} = \sqrt{\frac{g\cos\theta}{K_y(\alpha)}}. \tag{11}$$

The right hand side of each of the equations (10) and (11) is a function of α and θ independent of the density of the air and the weight of the machine. This fact suggests the definitions of the "reduced thrust horsepower" and the "reduced speed of advance" which we shall adopt.

Let w denote the wing loading (W/A) in pounds per square foot. We define the "reduced thrust horsepower required" h_t and the "reduced speed of advance" u by means of the equations

$$h_t = \frac{H_t}{W}\sqrt{\frac{\rho}{w}}, \tag{12}$$

$$u = U\sqrt{\frac{\rho}{w}} = 0.6818\,V\sqrt{\frac{\rho}{w}}. \tag{13}$$

Here H_t = horsepower required for propulsion at the altitude and speed of advance under consideration

Then equations (10) and (11) yield

$$h_t = \frac{1}{550}\sqrt{\frac{g\cos\theta}{K_y(\alpha)}}\left[\frac{K_x(\alpha)}{K(\alpha)_y}\cos\theta + \sin\theta\right] \tag{14}$$

$$u = 0.6818\sqrt{\frac{g\cos\theta}{K_y(\alpha)}}. \tag{15}$$

(14) and (15) might be used as they stand for the determination of the maximum speed of advance at different climbing angles. We shall be interested, however, only in the case of horizontal flight for which they reduce to

$$h_t = \frac{0.0103\,K_x(\alpha)}{[K_y(\alpha)]^{3/2}} \tag{16}$$

$$u = \frac{3.867}{\sqrt{K_y(\alpha)}}. \tag{17}$$

It will readily be seen from (16) and (17) that the relationship between h and u for horizontal flight depends only on the lift and drag coefficients of the airplane, and is independent of its size and weight, and also of the density of the air in which it flies.

The h_t, u curve for any given machine or family of machines may be plotted from the parametric equations (16) and (17), or it may be derived from the H_t, U curve for any given set of conditions if one is obtainable. For example, figure 14 shows the thrust horsepower required for the propulsion of the LePere two-seater fighter in horizontal flight at sea level as a function of the speed of advance U. Inserting in (12) and (13) the numerical values of ρ, W, and w applicable to this particular case, we obtain

$$h_t = 2.48 \times 10^{-5} H_t,$$
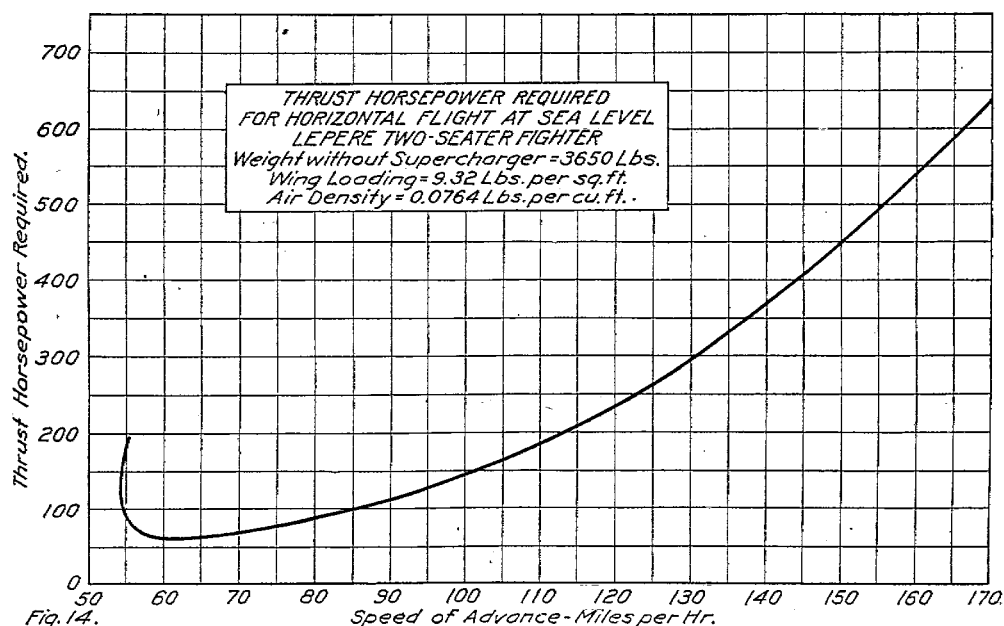
and

$$u = 0.0905\,U.$$

Figure 15 shows the curve for the reduced thrust horsepower required derived from figure 14 by means of the foregoing equations.

Let H_t' denote the maximum thrust horsepower *available* at the speed of advance U when the density of the air is ρ, and let h_t' denote its reduced value, i. e., let

$$h_t' = \frac{H_t'}{W}\sqrt{\frac{\rho}{w}}. \tag{18}$$

h_t' differs from h_t in that it depends on the density of the air, the weight of the machine, etc. Its value for any given airplane, propelling plant, and air density can be calculated, however, and a series of curves can be laid out showing h_t' as a function of u for various altitudes. Such a set of curves for the Liberty engine with turbine-driven supercharging compressor on the Le Pere plane is shown on figure 15. The method of plotting these curves will be discussed in the next article. For the present we will concern ourselves only with their use.

The maximum speed of horizontal flight is that speed for which the maximum thrust horsepower available is equal to the thrust horsepower required. But when H_t equals H_t' it is

THRUST HORSEPOWER REQUIRED
FOR HORIZONTAL FLIGHT AT SEA LEVEL
LEPERE TWO-SEATER FIGHTER
Weight without Supercharger = 3650 Lbs.
Wing Loading = 9.32 Lbs. per sq. ft.
Air Density = 0.0764 Lbs. per cu. ft.

Fig. 14.　　Speed of Advance - Miles per Hr.

obvious that h_t must equal h_t'. Hence the reduced value of the maximum horizontal flight speed at any altitude is the value of u for the point where the curve, $y = h_t'(u)$, for the given altitude crosses the curve, $y = h_t(u)$. To get the actual horizontal flight speed from its reduced value, we make use of (13).

For example, the reduced value of the maximum horizontal flight speed at 30,000 feet given by figure 15 is 8.5. The weight of the machine as modified by the addition of the compressor is 3,770 pounds. The wing loading is 9.625 pounds per square foot. The density of the air at 30,000 feet is 0.02866 pounds per cubic foot. Hence the actual maximum horizontal flight speed at 30,000 feet is

$$U = 8.5\sqrt{\frac{9.625}{0.02866}} = 155.5 \text{ mi./hr.}$$

The maximum climbing speed is also easily calculated with the aid of the curves for the reduced thrust horsepower available and required for horizontal flight. In accordance with common practice we make the approximate assumption that the maximum horsepower available for climbing is equal to the maximum difference between the ordinates of the curves for

the thrust horsepower available and required for horizontal flight.   Let H_c denote the horsepower available for climbing, and let V_c denote the maximum climbing speed in feet per minute.   Then

$$H_c = W \sqrt{\frac{w}{\rho}} (h_t{}' - h_t)_{max} = W V_c / 33,000.$$

Hence

$$V_c = 33,000 \sqrt{\frac{w}{\rho}} (h_t{}' - h_t)_{max}. \tag{19}$$

Fig. 15.
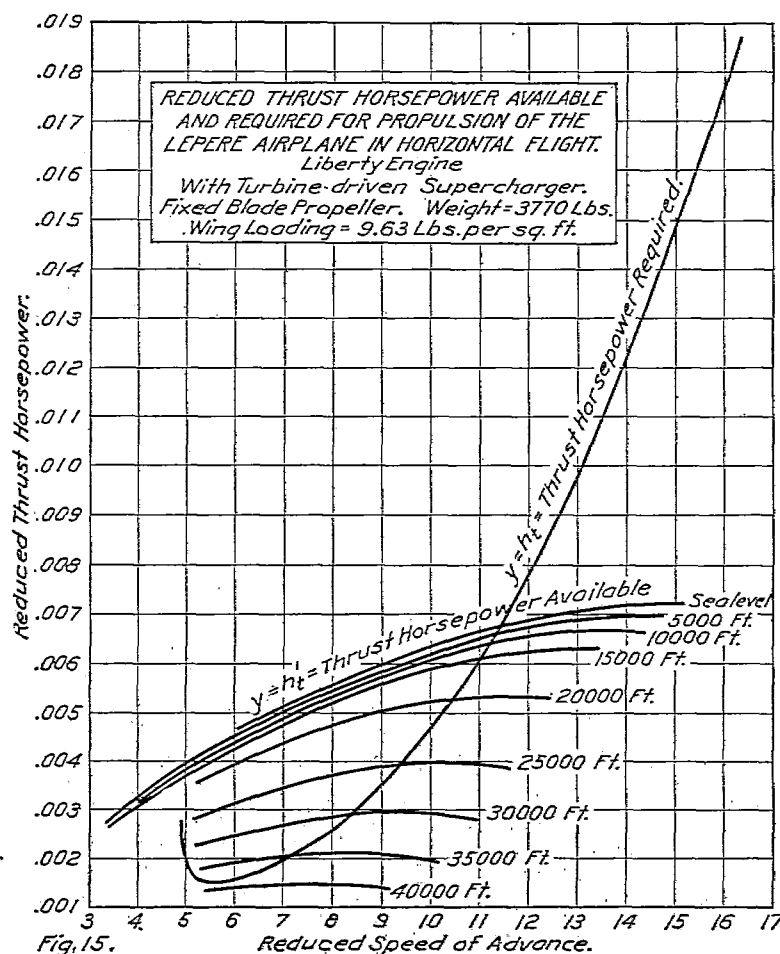
As an example of the application of (19) let us again consider the airplane and power plant of figure 15, at 30,000 feet.   The maximum value of $(h_t{}' - h_t)$ is 0.00099.   Hence

$$V_c = 33,000 \times 0.00099 \sqrt{\frac{9.625}{0.02866}} = 599 \text{ feet/minute.}$$

5. METHOD OF PLOTTING CURVES FOR THE THRUST HORSEPOWER AVAILABLE: 5. FIXED PROPELLER BLADES.

The form of the curves showing the reduced thrust horsepower available as a function of the reduced speed of advance is conveniently calculated by a method described by Bairstow and Coales.[3]

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[3] "Notes on the Prediction and Analysis of Aeroplane Performance," by L. Bairstow and Lieut J. D. Coales, British Advisory Committee or Aeronautics, Reports and Memoranda No. 474, May, 1918.

Let Q = propeller torque in pounds-feet;
  n = propeller speed in revolutions per minute;
  D = propeller diameter in feet;
  P = experimental mean pitch in feet.

The torque coefficient q_c is defined by the following equation:[4]

$$q_c = -\frac{gQ}{\rho n^2 D^5}. \tag{20}$$

The dimensionless quantity q_c is a function of V/nD, or of V/nP, which is to a first approximation independent of the propeller size for a family of similar propellers. The propeller efficiency, which we denote by η, is also a function of V/nP. In order to make the computation by the method here described, it is necessary to have curves showing q_c and η as functions of V/nP for the propeller employed. (In the future the quantity V/nP will be denoted by the single symbol σ).

The power absorbed by the propeller, i. e., the brake horsepower of the engine H, is related to the torque coefficient by the following simple equation:[5]

$$q_c = \frac{550gH}{2\pi\rho n^3 D^5}. \tag{21}$$

It is easy to compute from this equation, and the curves for q_c and η as functions of σ, the propeller efficiency and the speed of advance corresponding to any given set of values for n, H, ρ, and D. To do this we first calculate the value of q_c from (21). The q_c, σ curve gives value of σ, and the speed of advance is then computed from

$$V = nP\sigma. \tag{22}$$

The propeller efficiency for the same value of σ is taken from the η, σ curve. The thrust horsepower available at the speed V is then

$$H_t' = \eta H. \tag{23}$$

We are interested in the computation of the reduced speed of advance and the reduced thrust horsepower available. In order to get these reduced values directly, we substitute from (22) and (23) into (12) and (13). The resulting expressions are

$$u = 0.682nP\sigma\sqrt{\frac{\rho}{w}}, \tag{24}$$

$$h_t' = \frac{\eta H}{W}\sqrt{\frac{\rho}{w}}. \tag{25}$$

Assuming a series of different values for n, it is easy to determine the form of the u, h_t' curve from equations (21), (24), and (25).

Example.—Let us compute the h_t', u curve for the Liberty engine with turbine-driven supercharger, as installed on the Le Pere two-seater fighter, for a propeller with fixed blades at an altitude of 30,000 feet. We assume the propeller efficiency and torque coefficient curves shown on figure 16.[6] The ratio of the experimental mean pitch (in flight) to the diameter

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[4] Note that this definition differs from that adopted by Durand in Reports 14 and 30 (National Advisory Committee for Aeronautics, 1917 and 1919). The relation between q_c and the Durand coefficient is shown by equation (26), q. v.

[5] Cf. Bairstow and Coales, loc. cit., equation 14.

[6] These curves were not derived from experimental tests on any definite propeller. They were drawn up from the Bairstow-Coales-Betts empirical formulas for the torque and thrust coefficients (British Advisory Committee for Aeronautics, Reports and Memoranda No. 474, Appendix) with the aid of the meager data available on the propeller used in the test of the Le Pere two-seater fighter on Aug. 15, 1918. The assumption of a maximum efficiency of a little over 80 per cent is pure guesswork.

The curves are intended to show the *effective* values of the torque coefficient and of the propeller efficiency in flight. The determination of these effective values involves corrections for body interference and for the additional resistance of the body due to the slip stream. The first of these corrections consists in a simultaneous increase in effective pitch and in the efficiency of the propeller. The correction for the slip stream consists in a scaling down of the thrust coefficient curve in a manner described in the appendix to this report.

for the family of propellers specified by these curves is assumed to be 1.06. In order that a propeller of the family specified by these curves shall hold the speed of the supercharging engine down to 1,800 revolutions per minute at 18,000 feet altitude with a speed of advance of 162 miles per hour (cf. fig. 22) it is necessary that its diameter shall be 10.54. This is large for practical purposes, but that fact need not concern us in the present theoretical discussion.

The density of the air at 30,000 feet being 0.02866 pounds per cubic foot, equation (21) reduces to

$$q_c(\sigma) = 0.756 H/n^3. \tag{21'}$$

The pitch of the propeller is 11.17 feet, and equations (24) and (25) become

$$u = 0.4165 n\sigma, \tag{24'}$$

$$h_t' = 1.446 \times 10^{-5} \eta H. \tag{25'}$$

– The computation of the u, h_t' curve for this altitude is summarized in the accompanying table.

TABLE 3.

N_e. R. P. M.	n. R. P. S.	H. (Fig. 10.)	q_c. (Equation 21'.)	σ (Fig. 16.)	u. (Equation 24'.)	η (Fig. 16.)	h_t' (Equation 25'.)
1,600	26.67	234	.00933	0.519	5.76	0.712	24.1
1,700	28.33	246	.00818	.66	7.78	.795	28.3
1,800	30.00	253	.00708	.752	9.4	.805	29.4
1,900	31.67	252	.0060	.825	10.9	.761	27.7

6. THRUST HORSEPOWER AVAILABLE WITH VARIABLE PITCH PROPELLER.

There are two methods of drawing up curves for the reduced horsepower available with a variable pitch propeller. The first and more accurate method requires experimental curves for the torque coefficient and efficiency at various blade settings, such as those given by Durand for propeller 96 in Report No. 30 of the Fourth Annual Report of the National Advisory Committee for Aeronautics. The second method makes use of approximate empirical formulas due to (Miss) Betts, Mettam, Bairstow, and Coales, and requires only a minimum of data regarding the characteristics of the propeller at the particular blade setting which gives maximum efficiency.

Method I. A complete set of curves for the efficiency and torque coefficients at various blade settings available.--We assume that the propeller is to be adjusted so that under all conditions the engine is permitted to revolve at the speed of maximum power. The diameter being chosen, let it be required to draw up a graph of the reduced horsepower available as a function of the reduced speed of advance for some definite altitude. Equations (21), (24), and (25) still hold, but the propeller setting and pitch for any given speed of advance is unknown.

The method is as follows: The torque coefficient q_c is computed from (21). The torque coefficient Q_c, as defined by Durand, is related to q_c by the equation

$$Q_c = \frac{1000 q_c}{g (V/nD)^2}. \tag{26}$$

(We assume that the available data is in the form of curves for Q_c and η as functions of $V/(nD)$). Q_c is calculated from (26). Since V/nD is a known quantity, the Q_c curves (see Plate XIX, Report No. 30, Fourth Annual Report of the National Advisory Committee for Aeronautics) together with the value of Q_c for the given conditions fix the blade setting. The propeller efficiency can then be determined by interpolation from the η, $\frac{V}{nD}$ curves. (See Plate XIII, Report No. 30.) Finally the reduced thrust horsepower available and the reduced speed of advance can be computed from (25) and (13), respectively.

Method II. Use of empirical formulas for thrust and torque coefficients.—The method of computation here briefly described is due to Miss A. D. Betts and H. A. Mettam.[7] It has the advantage over the method described above that it requires a minimum of information regarding the propeller. The only data needed are the absolute maximum of efficiency for the propeller, the experimental mean pitch corresponding to the maximum efficiency and the torque coefficient corresponding to the maximum efficiency. .
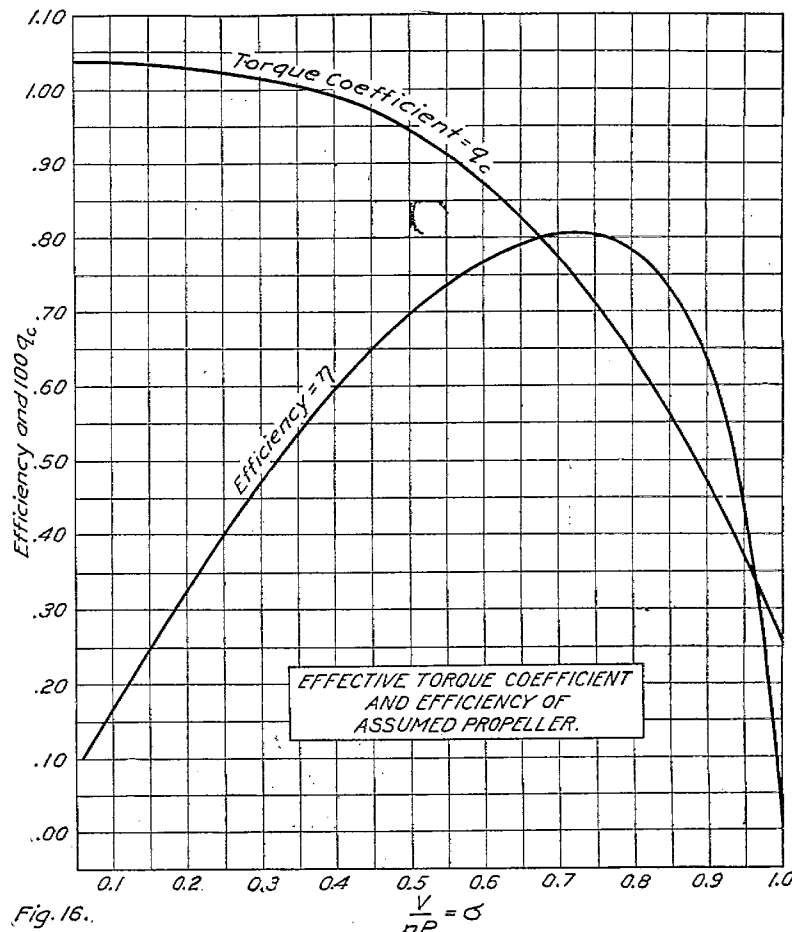

Fig. 16.

The method is based on the following empirical formulas for the thrust coefficient, the torque coefficient, and the efficiency of any fixed blade propeller:

$$K_t t_c = \frac{4}{3}(1 - \sigma^2), \tag{27}$$

$$K_q q_c = 1.1042 - 0.833\sigma^3, \tag{28}$$

$$\eta = \frac{1}{2\pi} \frac{P}{D} \frac{K_q}{K_t} F'(\sigma). \tag{29}$$

Here t_c is the thrust coefficient as defined by

$$t_c = \frac{gT}{n^2 D^4}, \tag{30}$$

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[7] Miss A. D. Betts and H. A. Mettam, "Empirical Formulæ for a Variable Pitch Airscrew, with Applications to the Prediction of Aeroplane performance," British Advisory Committee for Aeronautics, Reports and Memoranda, No. 474, February, 1919.

K_q and K_t are constants, while $F(\sigma)$ is defined by the equation

$$F(\sigma) = \frac{\sigma - \sigma^3}{0.828 - 0.625\sigma^3}.$$ (31)

Figure 17 shows curves of $F(\sigma)$ and $K_q q_c$.

The above formulas hold for variable pitch propellers, if it be understood that K_t and K_q depend on the pitch. Let η_m, P_m, and K_{q_m} denote, respectively, the absolute maximum of efficiency, the corresponding pitch, and the corresponding value of K_q. Betts and Mettam have established approximate empirical relationships between P/P_m and the ratios K_q/K_{q_m} and η/η_m, which we indicate by the following equations:

$$K_q = K_{q_m} \phi(P/P_m)$$ (32)

$$\eta = \eta_m \zeta(P/P_m) F(\sigma).$$ (33

Graphs of the empiracally determined functions ϕ and ζ are shown on figure (18).
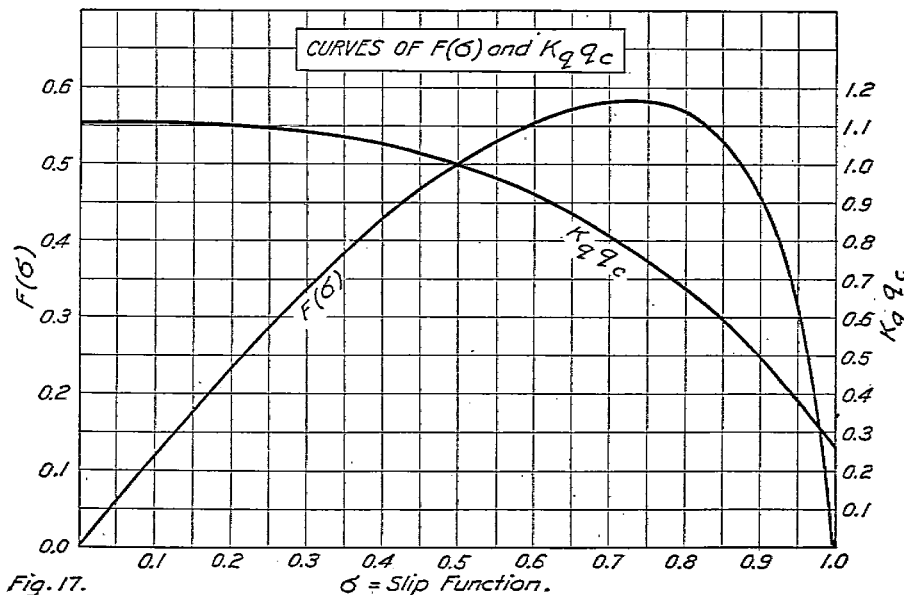

Fig. 17. σ = Slip Function.

The function ζ may be expressed by the formula

$$\zeta = \frac{1}{0.583} \frac{P}{P_m}^{1-P/P_m}.$$ (34)

Let q_{c_m} denote the torque coefficient corresponding to the maximum efficiency. Since the maximum efficiency occurs when $F(\sigma)$ is a maximum, or when σ is 0.725, it is easy to calculate K_{q_m} from q_{c_m}. The substitution of 0.725 for σ in (28) yields

$$K_{q_m} = 0.787/q_{c_m}.[8]$$ (35)

When η_m, P_m, and K_{q_m} are known, the determination of the curve for the reduced horse-power available is comparatively straightforward. Assume a number of values of P. For each, calculate the value of P/P_m and determine K_q with the aid of (32) and figure 18. The torque coefficients q_c are calculated from equation (21), and the values of $F(\sigma)$ are taken from the $K_q q_c$ curve of figure 17. Equation (33) in conjunction with the graphs of ζ and $F(\sigma)$

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[8] An error in the paper of Betts and Mettam, loc. cit., p. 11, states that $K_{q_m} q_{c_m} = 1$. As a matter of fact, the product $K_q q_c$ is equal to unity when σ is 0.5, which is not the point of maximum efficiency. See figure 1 of the Betts and Mettam report.

yields the propeller efficiencies. The computation is completed with the application of (24) and (25) to the determination of the values of u and h'_t corresponding to the several assumed values of P.

Example.—Let us compute a set of u, h'_t curves for the Liberty engine with supercharger as installed on the Le Pere, the propeller being of variable blade angle, but otherwise similar to the propeller of the problem of article 5.

Our first task is to fix the values of P_m, η_m, and K_{q_m}. This can be done with the aid of the information we already possess regarding the torque coefficient and efficiency of the propeller of article 5, together with one additional assumption. Referring to the efficiency curves for

Fig. 18.
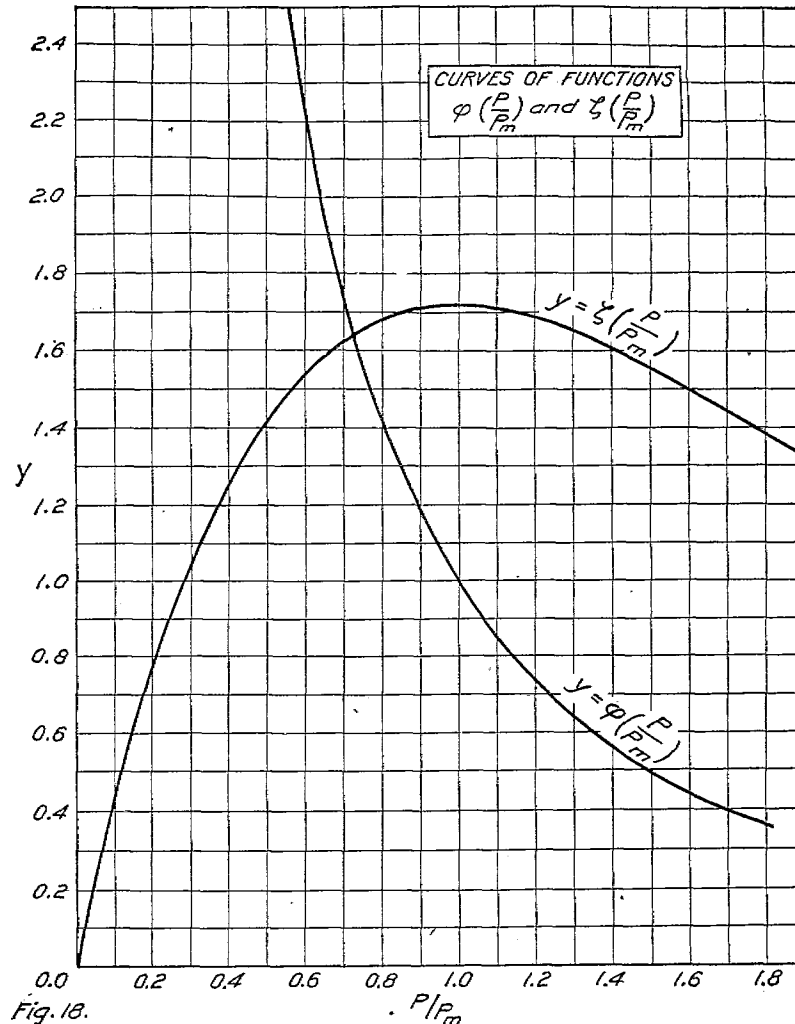
propeller No. 96 (Plate XIII, Report No. 30), it will be observed that the maximum efficiency of the propeller increases as the blade setting is advanced from its normal position until the pitch/diameter ratio (given by the intersection of the efficiency curve with the $V/(nD)$ axis) reaches the value 1.3. This is in accord with the general observation that the efficiency of fixed blade propellers tends to increase with the pitch/diameter ratio at least up to values as great as 1.2. We therefore assume that the efficiency of the variable pitch propeller now under consideration reaches its absolute maximum when the pitch/diameter ratio is 1.3.

Since the propeller diameter is 10.54 feet, the above assumption fixes the value of P_m as 13.7 feet. The maximum efficiency for the normal pitch (11.17 feet) is 80.7 per cent. The corresponding values of P/P_m and ζ (P/P_m) are 0.815 and 1.683, respectively. The value of

$F(\sigma)$ for maximum efficiency is 0.583. Hence equation (33) can be solved for the absolute maximum efficiency η_m. It yields

$$\eta_m = \frac{0.807}{0.583 \times 1.683} = 0.823.$$

The constant K_{q_m} can be evaluated in similar fashion. The torque coefficient q_c for the maximum efficiency consistent with the normal pitch of 11.17 feet is 0.00743. (Cf. fig. 16.) The corresponding value of $K_q q_c$ (fig. 17) is 0.785. Hence K_q is 105.6 when the pitch is 11.17 feet. Equation (32) and figure 18 now yield the desired value of K_{q_m}.

$$K_{q_m} = \frac{105.6}{1.385} = 76.3.$$

The remainder of the computation will be summarized for a single altitude only, viz, 30,000 feet. Let it be assumed that the propeller is adjusted at all speeds of advance to allow the engine to turn up to 1,800 revolutions per minute. The horsepower absorbed at 18,000 feet will then be 253. (Fig. 10.) The density of the air is 0.02866 pounds per cubic foot. Hence the torque coefficient q_c for this altitude is 0.00708 (equation (21)). (24) reduces to the form

$$u = 1.116\, P\sigma. \tag{24''}$$

while (25) unites with (33) to give

$$h'_t = 0.00301\, \zeta\, F(\sigma). \tag{25''}$$

The rest of the calculation is condensed into Table 4.

TABLE 4.

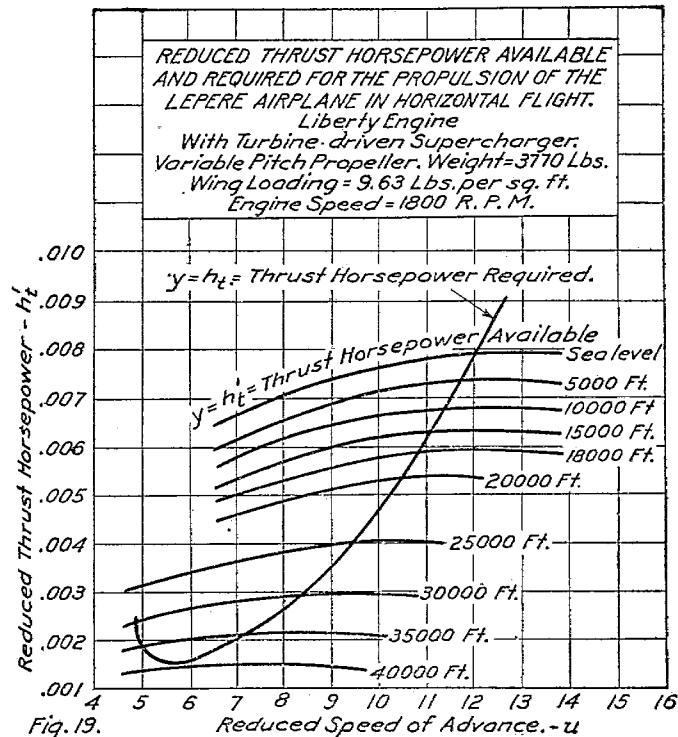| P.	P/P_m.	φ. (Fig. 18.)	ζ. (Fig. 18.)	K_q. Equation 32.	$K_q q_c$.	σ. (Fig. 17.)	$F(\sigma)$. (Fig. 17.)	h'_t. Equation 25''.	u. Equation 24''.
9	0.657	1.933	1.588	147.5	1.044	0.413	0.438	0.00209	4.15
10	.73	1.649	1.649	125.8	.891	.633	.566	.00281	7.06
11	.803	1.415	1.677	108.0	.765	.740	.583	.00294	9.1
12	.876	1.239	1.702	94.6	.670	.804	.566	.00290	10.78
13	.949	1.089	1.713	83.1	.589	.851	.528	.00272	12.36

7. THEORETICAL PERFORMANCE CURVES FOR AIRPLANE WITH SUPERCHARGING COMPRESSOR.

Figure 15 shows a complete set of curves for the reduced thrust horsepower available and required for the propulsion of the Le Pere two-seater fighter in horizontal flight when equipped with a turbine-driven supercharger and a fixed blade propeller. Figure 19 shows a similar set of curves for the case where the propeller is of variable pitch. Figures 20 and 21 show curves for the reduced thrust horsepower available and required for the same plane with fixed and variable propeller blades, but without the supercharger. The blade form of the airscrew is the same for all four sets of curves.

The resulting horizontal flight speed and maximum climbing speed curves, calculated in the manner described at the end of article 4, are shown on figures 22 and 23. The very great gain in ceiling, and in the horizontal flight speed and rate of climb at considerable altitudes, due to supercharging even without the variable pitch propeller, is the outstanding feature of these charts. At sea level, to be sure, the horizontal flight speed and the maximum climbing speed are reduced considerably by the use of the supercharger with a fixed blade propeller, but at an altitude of a little over 4,000 feet the supercharging plane is on even terms with the nonsupercharging plane, and at altitudes above 15,000 feet the gain due to supercharging is enormous.

The increase in the height of the ceiling due to the use of the variable pitch propeller is small, particularly in the case of the supercharging job. It should be remembered in this connection that the gain in ceiling due to the use of a variable pitch propeller depends essentially on the particular fixed pitch propeller with which the comparison is made. A suitably chosen fixed pitch propeller (i. e., an altitude propeller) would give as high a ceiling as one of variable pitch, but at the cost of sea level performance.

The chief advantage of the variable pitch airscrew for the supercharging military airplane is in the great increase in climbing speed which it gives, especially for the first 18,000 feet.

REDUCED THRUST HORSEPOWER AVAILABLE AND REQUIRED FOR THE PROPULSION OF THE LEPERE AIRPLANE IN HORIZONTAL FLIGHT.
Liberty Engine
With Turbine-driven Supercharger.
Variable Pitch Propeller. Weight=3770 Lbs.
Wing Loading = 9.63 Lbs. per sq. ft.
Engine Speed = 1800 R. P. M.

Fig. 19.

This gain in ability to climb is best studied, however, with the aid of altitude-time curves, which will be discussed in the next article.

8. ALTITUDE-TIME CURVES FOR MAXIMUM RATE OF CLIMB.

Let z denote the altitude in feet, and let τ denote the time in minutes.

The theoretical prediction of an altitude-time curve for maximum climb requires the integration of the right-hand member of the equation

$$\tau - \tau_0 = \int_{z_0}^{z} \frac{dz}{V_c(z)}. \tag{36}$$

In the absence of a simple mathematical formula for V_c as a function of z, some method of approximation must be resorted to in order to evaluate the above expression. It so happens that the maximum climbing speed curves are usually approximately rectilinear. Consequently a close approximation to the true curve for climbing speed can usually be obtained by means of a broken line of two or three segments, drawn in by eye with a straight edge. In replacing the computed curve for maximum climbing speed by such a broken line it should be observed that the smaller the value of V_c, the more important a small discrepancy in its value becomes.
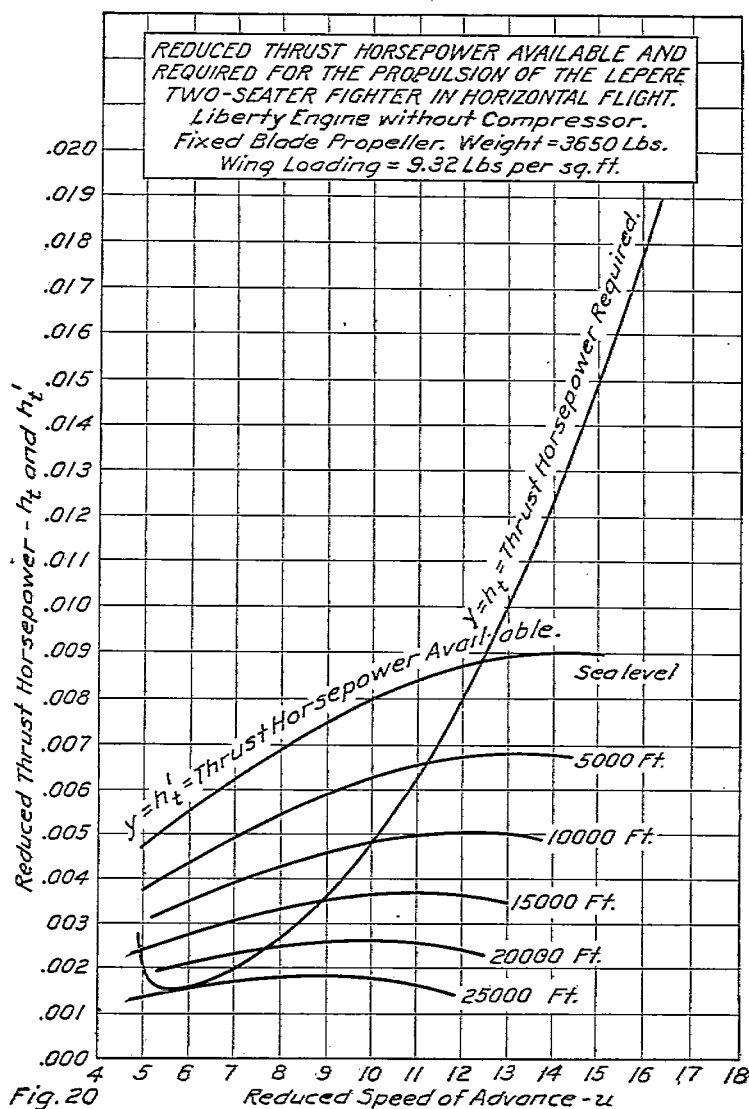
The problem of determining the time-altitude curve is thus reduced to the rather simple one of evaluating the right-hand member of (36) when the relation between V_c and z is given

by a broken line. Let the segments be numbered (fig. 23a) 0, 1, 2, etc., and let the coordinates of the end points of the rth segment be z_r, $(V_c)_r$ and z_{r+1}, $(V_c)_{r+1}$. The equation of this segment will then be

where

$$V_c(z) = (V_c)_r + m_r(z - z_r), \tag{37}$$

$$m = \frac{(V_c)_{r+1} - (V_c)_r}{z_{r+1} - z_r}. \tag{38}$$

REDUCED THRUST HORSEPOWER AVAILABLE AND REQUIRED FOR THE PROPULSION OF THE LEPERE TWO-SEATER FIGHTER IN HORIZONTAL FLIGHT. Liberty Engine without Compressor. Fixed Blade Propeller. Weight = 3650 Lbs. Wing Loading = 9.32 Lbs per sq.ft.

Fig. 20

Let τ_r denote the time corresponding to the altitude z_r. Then (36) yields

$$\tau - \tau_r = \frac{2.3026}{m_r} \log_{10} \left[\frac{V_c(z)}{(V_c)_r} \right]. \tag{39}$$

But evidently,

$$\tau_r - \tau_0 = 2.3026 \sum_{p=0}^{p=r-1} \frac{1}{m_p} \log_{10} \left[\frac{(V_c)_{p+1}}{(V_c)_p} \right]. \tag{40}$$

The addition of equations (39) and (40) gives the following simple expression for the time required to reach an altitude z in the rth segment.

$$\tau - \tau_0 = 2.3026 \left\{ \frac{1}{m_r} \log_{10} \left[\frac{V_c(z)}{(V_c)_r} \right] + \sum_{p=0}^{p=r-1} \frac{1}{m_p} \log_{10} \left[\frac{(V_c)_{p+1}}{(V_c)_p} \right] \right\}. \tag{41}$$

When the values of V_o are taken from the graph, it is easy to evaluate the right-hand member of (41) if the number of segments is small.

Figure 24 shows approximate altitude-time curves obtained in the manner described above from the four maximum climbing speed curves of figure 23. In the evaluation of the curves for the supercharging airplane, broken lines of two segments only were used, while single straight lines were employed for the other two cases.

Comparison of the two altitude-time curves for the supercharging airplane shows that the plane with the variable pitch airscrew climbs 24,000 feet while that with the fixed blade propeller climbs 20,000. Since the former plane would also be able to outmaneuver the latter
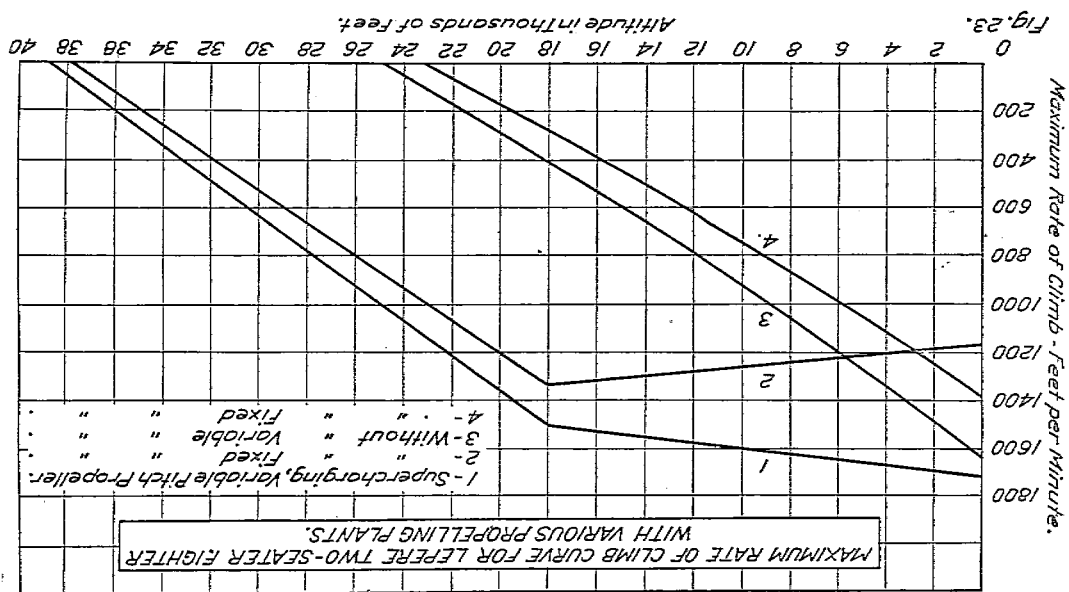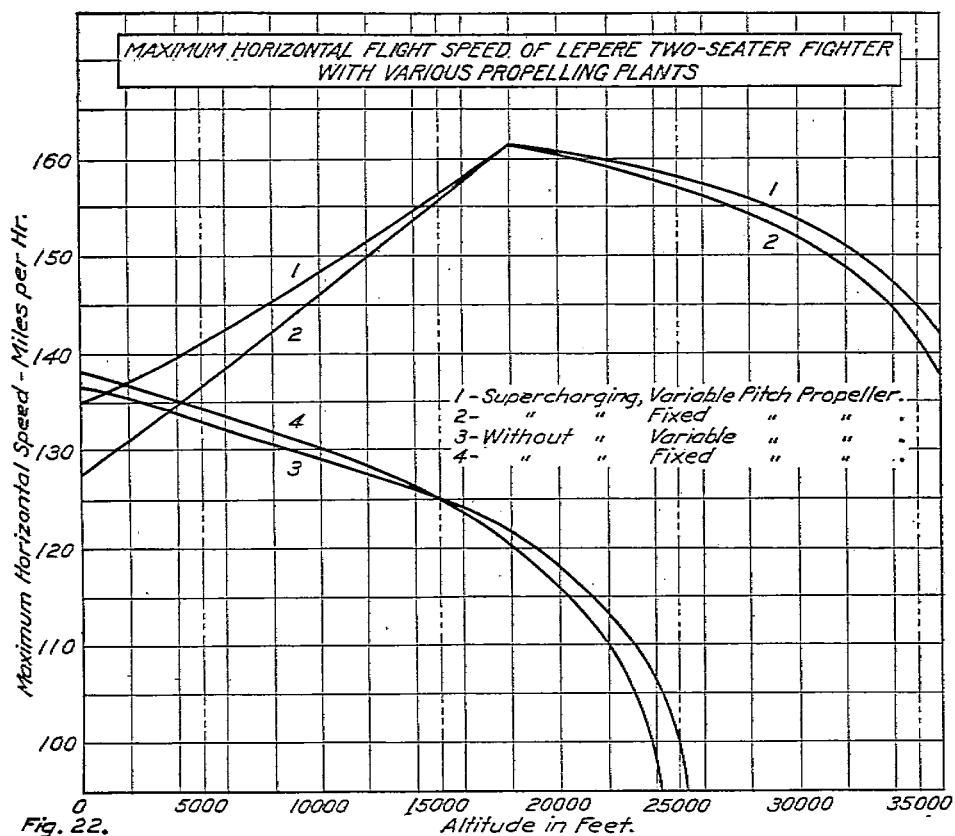

Fig. 21.   Reduced Speed of Advance.- u
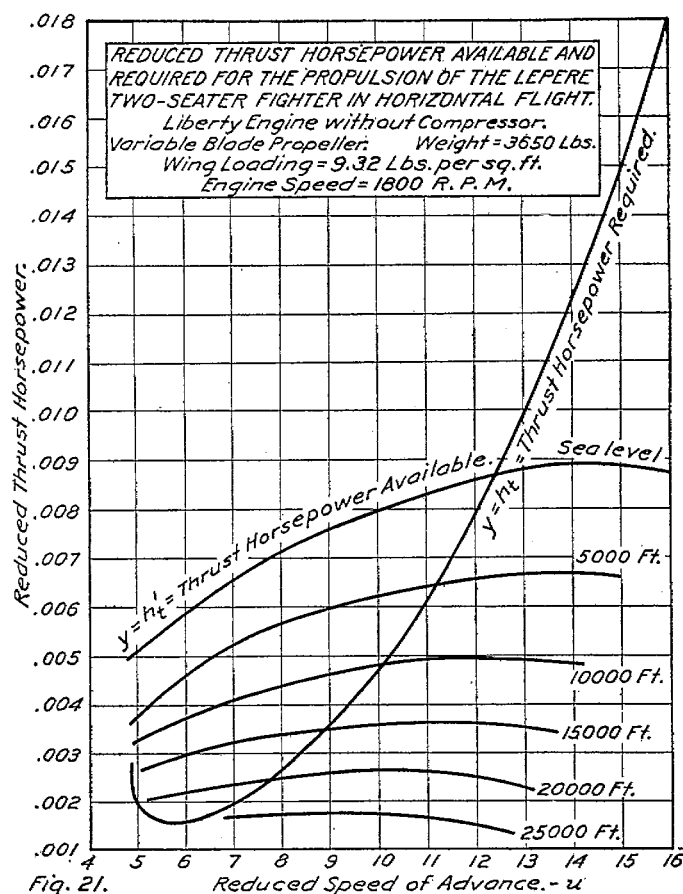
completely at all altitudes below 15,000 feet, the prime importance of the variable pitch airscrew for fighting military planes is evident.

9. FUEL ECONOMY: COMMERCIAL APPLICATIONS.

Airplane transportation will always be high speed transportation, and the commercial aeronautical engineer will always be interested in horizontal flight speeds, but he will always have to consider the question of fuel economy at the same time. It is therefore important to discover to what extent the high speeds which the supercharger offers are to be obtained at the cost of fuel waste.

Figure 25 shows theoretical curves for the relative fuel economy (relative distance traversed per pound of fuel) of the LePere plane equipped with the four different propelling plants discussed in the preceding articles. These curves were worked out with the aid of the graphs of figures 13 and 22. The computation was based on the assumption that the engine is wide open at all altitudes, and that the carburetor is adjusted for maximum power. The small variation in the fuel consumption of the engine with speed at sea level was neglected.
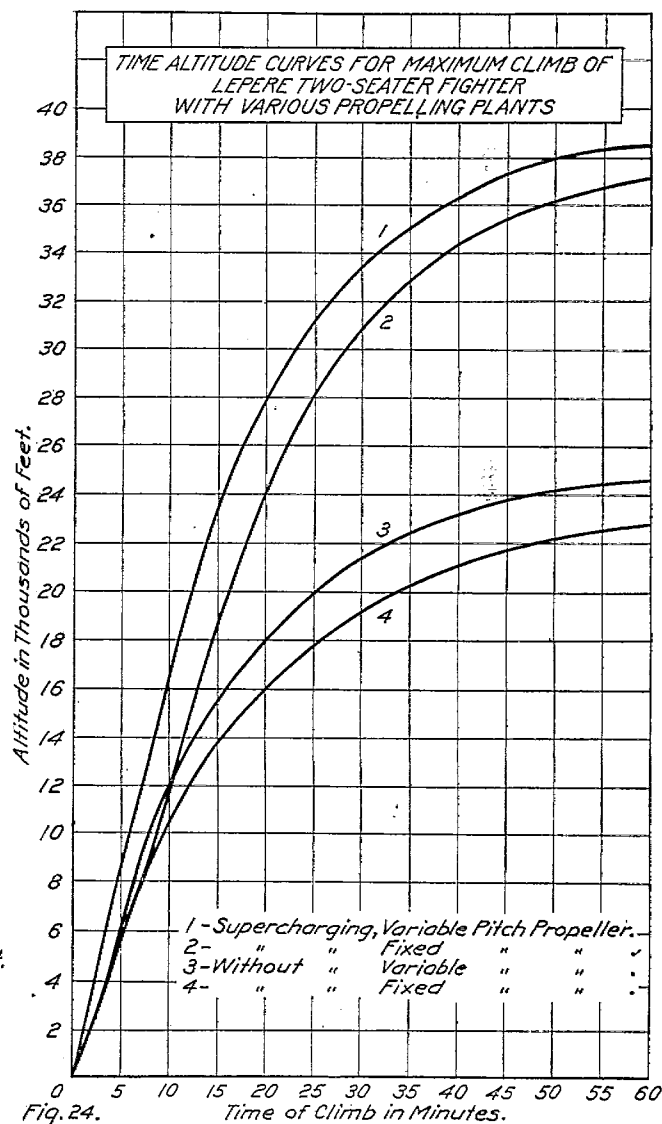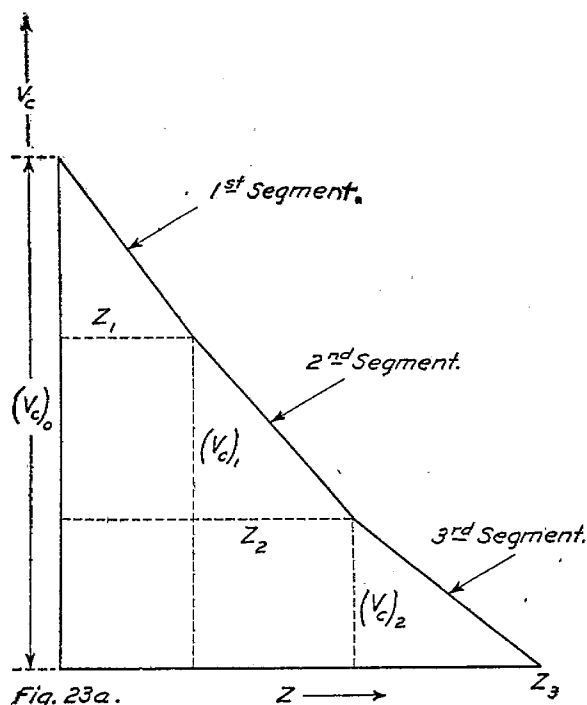
The graphs on figure 25 emphasize the fact that the prime controlling factor in determining the fuel economy is the angle of attack of the plane.   The fuel economy is proportional to the product of the lift over drag ratio of the airplane, the efficiency of the propeller, and the recip-

MAXIMUM HORIZONTAL FLIGHT SPEED OF LEPERE TWO-SEATER FIGHTER WITH VARIOUS PROPELLING PLANTS

1 - Supercharging, Variable Pitch Propeller.
2 -   "   "   Fixed   "   "
3 - Without   "   Variable   "   "
4 -   "   "   Fixed   "   "

Maximum Horizontal Speed - Miles per Hr.

Altitude in Feet.

Fig. 22.

MAXIMUM RATE OF CLIMB CURVE FOR LEPERE TWO-SEATER FIGHTER WITH VARIOUS PROPELLING PLANTS.

1 - Supercharging, Variable Pitch Propeller.
2 -   "   "   Fixed   "   "
3 - Without   "   Variable   "   "
4 -   "   "   Fixed   "   "

Altitude in thousands of Feet.

Maximum Rate of Climb - Feet per Minute.

Fig. 23.

rocal of the specific fuel consumption of the engine.   As the airplane climbs to greater and greater altitudes, the angle of attack becomes larger and larger, and the increase in the lift over drag ratio causes a decided increase in the fuel economy in spite of the steadily increasing

fuel consumption of the engine per brake horsepower hour. The variation in the propeller efficiency plays a relatively small part in the variation of the over-all fuel economy.

The lift over drag ratio has practically the same value at the ceiling for all cases, and consequently the difference between the maximum fuel economy for the plane without the supercharger, and with the supercharger, is due entirely to the differences in the mechanical efficiencies of the engines and in the propellers for the two cases.[9] The somewhat higher maximum fuel economy which the graphs indicate for the supercharging arrangement is due

Fig. 23a.

Fig. 24.

TIME ALTITUDE CURVES FOR MAXIMUM CLIMB OF LEPERE TWO-SEATER FIGHTER WITH VARIOUS PROPELLING PLANTS

1 – Supercharging, Variable Pitch Propeller.
2 – " " Fixed " "
3 – Without " Variable " "
4 – " " Fixed " "

Altitude in Thousands of Feet.

Time of Climb in Minutes.

primarily to the fact that the power output near the ceiling, and hence the mechanical efficiency, is greater when the compressor is used.

As already stated, the curves of figure 25 are based on the assumption that the engine is wide open at all times. It is to be understood that fuel economy can be gained at any altitude below that of the ceiling at the expense of speed by throttling the engine or slowing down the compressor.
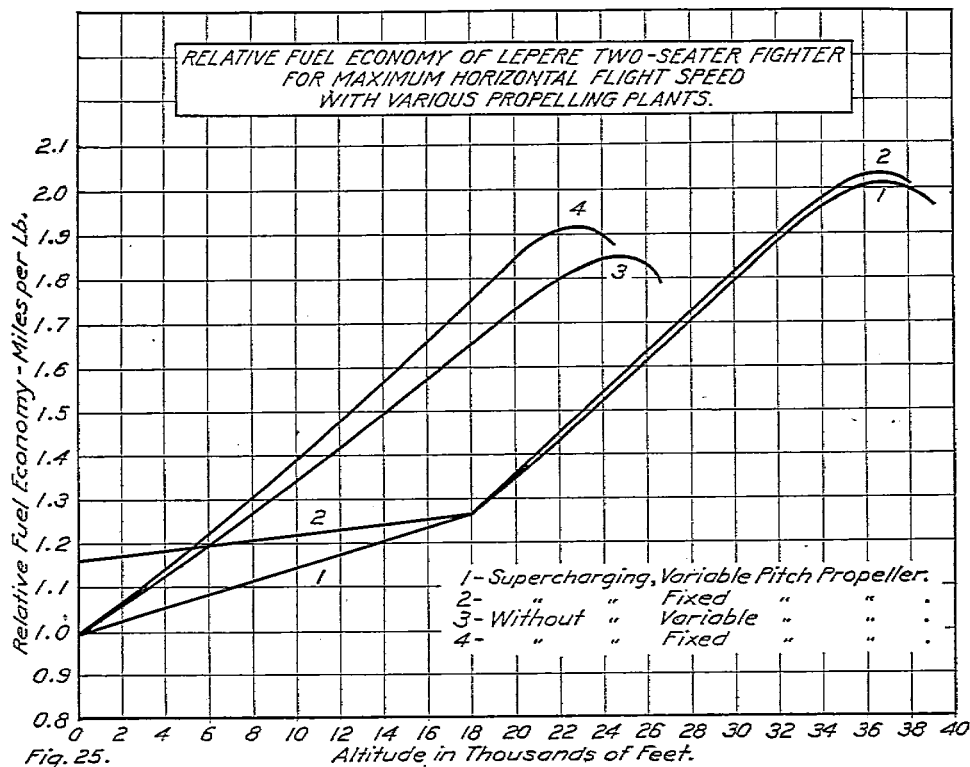
While the supercharging installation considered thus far is excellent for a military fighting plane, it very much overpowers the machine for commercial or military transportation. It is

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[9] Here we neglect the fuel losses due to poor carburetion and distribution which must commonly occur without the compressor as a result of the very low intake temperatures at great altitudes.

therefore desirable, in conclusion, to consider an installation adapted to the transportation of a load at a moderate speed with as great a fuel economy as possible.

In order to obtain an absolute maximum of fuel economy, the engine should drive the airplane at its most economical angle of attack while developing as large a percentage as possible of its sea level power, or mean effective pressure. Obviously this means that the most economical way to fly is near the ground with an engine which is barely able to lift the plane, but to obtain maximum economy in this manner would involve a large loss of speed, for the horizontal flight speed at any given angle of attack is inversely proportional to the square root of the air density.[10] To obtain maximum economy with a given plane and wing loading without sacrificing speed, the plane should operate at as great an altitude as is practicable.

The device of feeding warm compressed air from the supercharger to the aviators will in all probability make it possible to operate airplanes in the future at much greater altitudes

RELATIVE FUEL ECONOMY OF LEPERE TWO-SEATER FIGHTER
FOR MAXIMUM HORIZONTAL FLIGHT SPEED
WITH VARIOUS PROPELLING PLANTS.

1 - Supercharging, Variable Pitch Propeller.
2 -   "   "   Fixed   "   "   .
3 - Without   "   Variable   "   "   .
4 -   "   "   Fixed   "   "   .

Fig. 25. Altitude in Thousands of Feet.

that at present. There will be a practical upper limit in any case, however, and we may, for the purposes of argument, set it at 25,000 feet. In order to see what the real commercial advantage of the supercharging compressor is, we therefore compare the weights and fuel economies of two engines developing equal power at 25,000 feet, one with the supercharger, the other without.

Let us assume that an airplane A, having the same lift and drag coefficients and the same wing loading as the Le Pere two-seater fighter, but larger and heavier, is to be driven with a horizontal flight speed of 120 miles an hour at 25,000 feet by the Liberty engine with supercharging compressor "all out."[11] The curve for the reduced thrust horsepower required (fig. 26) is the same as for the Le Pere. The ordinates of the new curves for the reduced thrust horsepower available are to be obtained from those of figure 18 (assuming a fixed blade propeller) through multiplication by the ratio of the weight of the Le Pere to the weight of A. The reduced

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[10] Compare equations (13) and (15), which combine to give $V = \sqrt{\dfrac{w\,g\,\cos\theta}{\rho\,K_y\,(\alpha)}}$. The speed of advance for any given plane and angle of attack can not be increased by increasing the wing loading on account of the necessity for preserving a moderate landing speed.

[11] The comparison would be essentially the same if the engines were assumed to operate partially throttled at 25,000 feet.

speed of advance corresponding to 120 miles an hour at 25,000 feet is 7.45. The corresponding reduced horsepower required is 0.0227. But the reduced horsepower available for the same propelling plant when installed on the Le Pere is 0.0455. (Fig. 18.) Hence the ratio of the weight of A to the weight of the Le Pere is 0.0455/0.0277, or 2.005. The weight of the Le Pere with the supercharger being 3,770 pounds, the weight of A works out to be 7,560 pounds. The reduced thrust horsepower available for the nonsupercharging Liberty engine when installed on a plane weighing 3,650 pounds is 0.0175 pounds at the speed and altitude in question. (Cf.
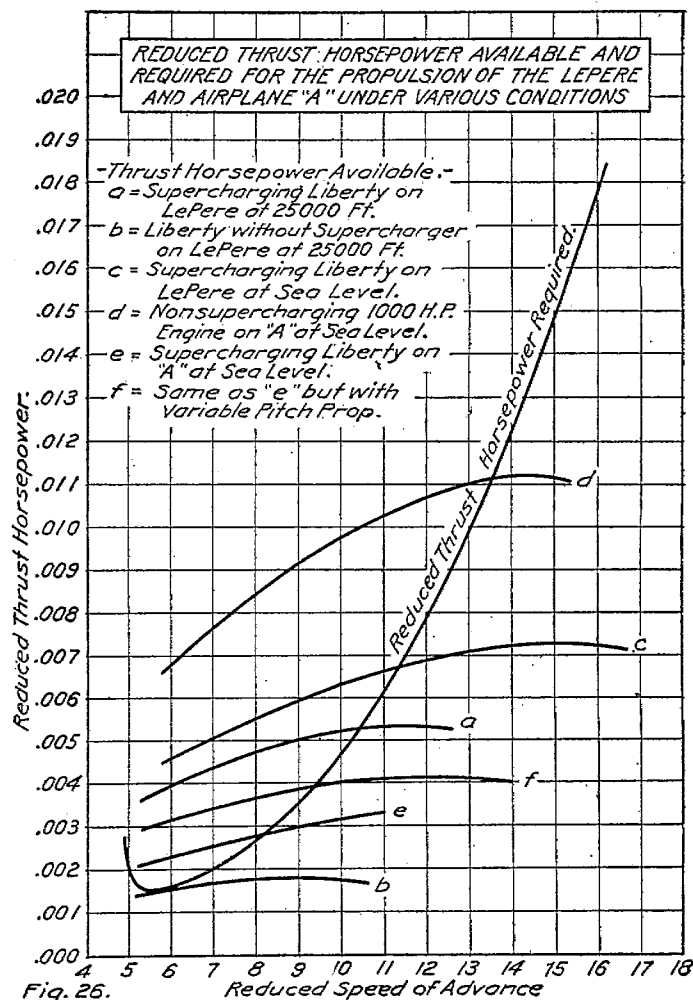

Fig. 26.

fig. 20.)   Hence the reduced horsepower available for the nonsupercharging engine on plane A would be 0.0175×3650/7560, or 0.00844. This is 1/2.69 times the reduced horsepower required. Calling the nominal power of the Liberty engine 400, it is evident that the nominal power of a supercharging engine, capable of driving this plane at the assumed speed of 120 miles an hour at 25,000 feet, would be 400×2.69, or 1,075 horsepower. This comparison is somewhat unfair to the nonsupercharging engine, however, since the propellers assumed would make the engine speeds 1,670 revolutions per minute and 1,570 revolutions per minute for the supercharging and nonsupercharging cases, respectively. Assuming the same speed for both (at 25,000 feet), the nominal horsepower of the required nonsupercharging engine would be a trifle over 1,025. Thus the use of the compressor would increase the carrying capacity of plane A by an amount equal to the difference between the weight of a 1,000 horsepower engine and that of a 400 horsepower engine, minus 100 pounds, the weight of the turbine and compressor. This may be roughly estimated at from 900 to 1,000 pounds.

At the same time the curves of figure 13 show that the nonsupercharging engine will use 33 per cent more fuel per brake horsepower hour, and per mile. The one drawback of the small engine and supercharger as compared with the large engine would be in the excessively low climbing speed at sea level. This works out to be 257 feet per minute as compared with 2,145 feet per minute for the 1,000 horsepower engine. A variable pitch propeller would increase the sea-level climbing speed for the smaller engine and compressor to 563 feet per minute (a gain of 119 per cent) and the horizontal flight speed at sea level from 92 miles an hour to 105 miles an hour. This ability to more than double the sea-level climbing speed of a heavy plane with a high-power loading would be of great use in getting the machine off the ground and points to an important commercial application of the variable pitch propeller.

The fuel consumption per mile for the same airplane operating at sea level with a speed of 120 miles an hour would be 62 per cent greater than at 25,000 feet with the supercharging compressor.

It may be observed in conclusion that on account of the meagerness of the data available, the probable error involved in the present estimate of the performance of an airplane equipped with an engine and supercharging compressor is considerable. The calculated gains are so large, however, that there can be little doubt of the great value of the compressor both for military and commercial purposes.

REPORT No. 101.

APPENDIX.

NOTE ON THE CORRECTION OF THE PROPELLER THRUST COEFFICIENT CURVE FOR THE SLIP STREAM RESISTANCE.

Bairstow and Coales (British Advisory Committee for Aeronautics, Reports and Memoranda 474) have shown on the basis of an empirical formula for the resistance of the parts of an airplane in the slip stream, that it is possible to correct for the extra head resistance due to the slip stream effect by merely scaling down the thrust coefficient curve by a constant factor. In the following treatment of the slip stream effect (cf. note 6, part 2 of this report) the writer employs Dr. Warner's theoretical expression for the slip stream velocity [1] to derive a theoretical expression for the effective thrust coefficient. It turns out that the correction factor is not quite constant, but is nearly so for a consierable range of values of V/np.

Let R_s = resistance of portion of machine in slip stream;

R = resistance of portion outside slip stream;

R' = resistance which the entire machine would have if the slip stream velocity were equal to the speed of advance;

V_s = slip stream velocity in feet per second.

Substituting $R + R_s$ for X in equation (5) (part 2), we obtain

$$T = R + R_s + Y \tan \theta. \tag{a}$$

But

$$R_s = k_s \rho V^2_s / g, \tag{b}$$

where k_s is an easily calculable coefficient, and

$$R' = R + k_s \rho V^2 / g. \tag{c}$$

Combining (b) and (c) with (a), we obtain

$$T - \frac{k_s \rho V^2}{g}\left[\left(\frac{V_s}{V}\right)^2 - 1\right] = R' + Y \tan \theta. \tag{d}$$

The left-hand member is equal to the thrust which would be required if there were no slip stream effect, and can properly be called the *effective* thrust. We denote it by the symbol T' thus:

$$T' = T - \frac{k_s \rho V^2}{g}\left[\left(\frac{V_s}{V}\right)^2 - 1\right]. \tag{e}$$

The substitution of the value of T in terms of the torque coefficient (equation 30, part 2) yields

$$T' = \frac{\rho n^2 D^4}{g}\left\{t_c - \frac{k_s}{D^2}\left(\frac{V}{nD}\right)^2\left[\left(\frac{V_s}{V}\right)^2 - 1\right]\right\} \tag{f}$$

Let

$$t'_c = t_c - \frac{k_s}{D^2}\left(\frac{V}{nD}\right)^2\left[\left(\frac{V_s}{V}\right)^2 - 1\right] = t_c - \frac{k_s \sigma^2}{D^2}\left[\left(\frac{V_s}{V}\right)^2 - 1\right]\left(\frac{P}{D}\right)^2. \tag{g}$$

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[1] "Slip Stream Velocities," by E. P. Warner, Report No. 71 of the National Advisory Committee for Aeronautics.

By definition, the torque coefficient is a quantity which, when multiplied by $\rho n^2 D^4/g$, gives the true thrust. But when t'_c is multiplied by $\rho n^2 D^4/g$, it gives the effective thrust. Hence t'_c plays the part of an effective thrust coefficient. It remains to show that t'_c like t_o is a function of σ only, for a given propeller and airplane.

Warner's momentum formula for the slip stream velocity is

$$T = 0.636 \frac{\rho}{g} D^2 V_s (V_s - V),$$

or

$$t_c n^2 D^2 = 0.636 \ V_s \ (V_s - V). \tag{h}$$

(h) is easily thrown into the form

$$\frac{V_s}{V}\left(\frac{V_s}{V} - 1\right) = \frac{t_c}{0.636 \ \sigma^2}\left(\frac{D}{P}\right)^2. \tag{i}$$

Solving for V_s/V, we obtain

$$\frac{V_s}{V} = \frac{1}{2}\left[1 + \sqrt{1 + \frac{4t_c}{0.636}\left(\frac{D}{P}\right)^2 \cdot \frac{1}{\sigma^2}}\right] \tag{j}$$

Since t_c is a function of σ, it is evident that V_s/V, and therefore t'_c, is a function of σ. Equations (j) and (g) can be used for the evaluation of t'_c when the relationship between t_o and σ is known.

If the effective value of the thrust coefficient T_c as defined by the equation

$$T_c = \frac{100 \ T}{\rho V^2 D^2} \tag{k}$$

is desired, equation (g) should be replaced by

$$T'_c = T_c - \frac{100 k_s}{g \ D^2}\left[\left(\frac{V_s}{V}\right)^2 - 1\right]. \tag{l}$$

Through the range of values of σ which are used in practice, the velocity ratio V_s/V is generally less than 1.5, and consequently the following approximation should be useful. Treating $(V_s/V - 1)/2$ as a quantity small in comparison with unity, we can write:

$$\left(\frac{V_s}{V}\right)^2 - 1 = 2\frac{V_s}{V}\left(\frac{V_s}{V} - 1\right) \tag{Approx.}$$

Substituting this value for $\left(\frac{V_s}{V}\right)^2 - 1$ into (g), we obtain

$$t'_c = t_o \ (1 - 3.15 \ k_s / D^2). \tag{m}$$

Thus to a first rough approximation, the effective torque coefficient can be obtained from the true torque coefficient through multiplication by a constant correction factor.

Since the above approximate expression for $(V_s/V)^2 - 1$ is somewhat too large for all values of V_s/V, better results are obtained by reducing the coefficient of k_s/D^2 in (m) to the value 2.9. Thus

$$t_c' = t_o \ (1 - 2.9 \ k_s/D^2). \tag{m'}$$

This equation also holds if T_c' and T_o are substituted for t_c' and t_o, respectively.

The accompanying table shows the percentage error in the correction to the thrust and thrust coefficient for various values of σ due to the above approximation, as computed for the propeller of figure 16. The percentage error in the thrust coefficient itself would, of course, be much smaller.

$\sigma = \dfrac{V}{nP}$	$V_a/V.$	Per cent error in $(V_a/V)^2 - 1.$	Propeller efficiency.
0.3	1.836	19.5	0.47
.4	1.523	11.1	.59
.5	1.340	5.6	.70
.6	1.221	1.4	.77
.7	1.139	−1.7	.805
.8	1.080	−3.9	.78
.9	1.035	−5.8	.63

The effective efficiency of the propeller for any value of σ, or of $V/(nD)$, is decreased in the same ratio as the corresponding thrust coefficient.

SUMMARY OF NOTATION FOR PART 2.

H = brake horsepower of engine.

H_t = thrust horsepower of propeller.

U, V = speed of advance in miles per hour and feet per second, respectively

V_c = maximum climbing speed in feet per minute.

ρ = density of air in pounds per cubic foot.

d = relative density of air.

S = radiator area.

t = atmospheric temperature (Fahrenheit).

t' = mean temperature of water in radiator.

α = angle of attack.

θ = angle of climb.

g = acceleration of gravity.

Y = total lift of airplane in pounds.

X = total drag of airplane in pounds.

T = propeller thrust in pounds.

W = weight of airplane in pounds.

A = wing area in square feet.

w = wing loading in pounds per square foot.

$K_y(\alpha)$ = lift coefficient for entire machine.

$K_x(\alpha)$ = drag coefficient for entire machine.

h_t = reduced thrust horsepower required.   (Cf. Equation (12).)

h_t' = reduced thrust horsepower available.   (Cf. Equation (18).)

u = reduced speed of advance.   (Cf. Equation (13).)

n = propeller speed in revolutions per second.

D = propeller diameter in feet.

P = propeller experimental mean pitch in feet.

η = propeller efficiency.

Q = propeller torque in pounds-feet.

q_c = torque coefficient as defined by equation (20).

Q_c = torque coefficient as defined by equation (26).

t_c = thrust coefficient as defined by equation (30).

$\sigma = V/(nP)$.

K_q, K_t = constants defined by equations (27) and (28).

$F(\sigma)$ = function defined by equation (31).

$\varphi\,(P/P_\mathrm{m})$, $\zeta\,(P/P_\mathrm{m})=$ functions defined by the graphs of figure (18).

$\tau=$ time of climb in minutes.

$z=$ altitude in feet.

$V_\mathrm{s}=$ slip stream velocity, in feet per second.

$R=$ resistance of portion of plane outside slip stream.

$R_\mathrm{s}=$ resistance of portion of plane in slip stream.

$R'=$ resistance which entire machine would have if slip stream velocity were equal to V.

$k_\mathrm{s}=$ constant defined by (c) (Appendix).

$T'=$ effective thrust.

$t_\mathrm{c}'=$ effective thrust coefficient.