REPORT No. 117

THE DRAG OF ZEPPELIN AIRSHIPS

By MAX M. MUNK
National Advisory Committee for Aeronautics

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INTRODUCTION.

This report was prepared for the National Advisory Committee for Aeronautics, and is a discussion of the results of tests with Zeppelin airships, in which the propellers were stopped as quickly as possible while the airship was in full flight. Some details of these tests are described in a paper by V. Soden and Dornier.[1] They were continued after that publication and cover a series of interesting types. In this paper I intend to refer to the theory involved in these tests and to one scientifically interesting fact which can be derived from them and which has not yet been noted.

The chief general question concerning these tests is, of course: Does the negative acceleration of an airship with stopped propellers supply proper data for determining the drag of the airship when in uniform flight? This can not absolutely be answered in the affirmative, the two phenomena not being identical in principle. We believe, however, that in this particular case the agreement is sufficient and that the data obtained from the test are the true or, at least, the approximate quantities wanted. We have several strong reasons for our opinion and will proceed to discuss them.

MOTION IN A NONVISCOUS FLUID.

Consider in the first place what motion of the airship is to be expected. It is generally believed—and the following tests confirm the belief to a certain degree—that the drag of an airship can be represented by an expression of the form

(1) $$D = A \cdot V^2 \rho/2$$

where A is a constant which has the dimension of an area and may therefore be called the area of drag; V is the velocity of flight; and $\rho/2$ is half the density of the surrounding air. The mass of the floating ship is equal to the mass of the displaced fluid and is therefore

(2) $$M = v \cdot \rho$$

where v is the displacement of the ship. Hence, according to the general law of mechanics, the motion after the propellers stop is determined by the equation

(3) $$-d/dt(V \cdot v\rho) = A V^2 \cdot \rho/2$$

in which, however, the influence of the retardation on the drag itself is not yet taken into consideration. By integrating (3) two times we obtain successively

(4) $$V = (2v/A)/(t + c_1)$$

(5) $$\frac{X + c_2}{(2v/A)} = \log \frac{t + c_1}{c_3}$$

where c_1, c_2, and c_3 are three constants of integration determinable by the initial conditions.

[1] V. Soden and Dornier, Mitteilungen des Luftschiffbau Zeppelin in Friedrichshafen, Die Bestimmung des Schiffswiderstandes durch den Fahrtversuch, Zeitschrift für Flugt. und Motorl., 1911.

We chose the situation of $x=0$ and the origin of time $t=0$ so that c_1 and c_2 are zero and c_3 is the unity of time; and we have then

(6*) $$V=\frac{(2v/A)}{t}$$

(7*) $$\frac{X}{(2v/A)}=\log t$$

$(2v/A)$ has the dimension of a length, characteristic of the motion of the ship. In this paper we will call it the "characteristic length" of the ship and denote it s.

(8) $$s=(2v/A) \qquad \text{(Definition)}.$$

Then we obtain

(6) $$V=s/t$$

(7) $$X=s \log t.$$

At any moment the ship moves with a velocity such as would be necessary to cover the constant length s in the time elapsed from a constant origin of time.

We proceed now to take into account the difference between the drag in uniform flight and that in retarded flight; and shall consider, in the first place, the conditions of flight in a nonviscous fluid.

In such a fluid a uniformly moving body would in general have no drag at all.[1] When a solid is moving in a fluid, the latter possesses kinetic energy proportional to the square of the velocity of the solid. Consequently, when the solid is retarded, the fluid itself must lose kinetic energy; and this means that the force opposing the motion of the fluid must have an equal reaction on the solid. This reaction is in such a direction as to oppose the change, i. e., it is a negative drag, tending to accelerate the solid. This energy is given the fluid by the force necessary to put the body into motion and is given back if the body is being retarded. The effect of this kinetic energy of the fluid is the same as if the body had a constant increment of mass, additional to its own mass. The force in question is perfectly taken into account if the displacement v in (8) is increased by a corresponding increment of volume.

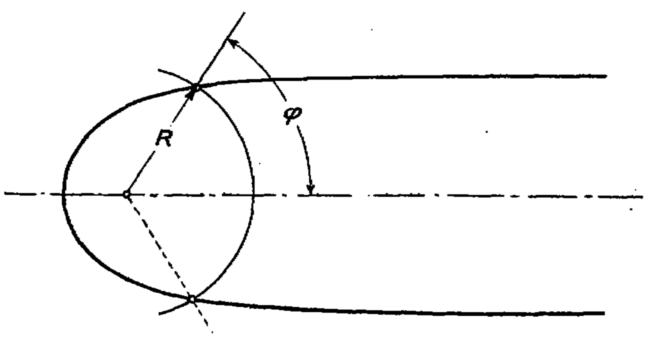


FIG. 1.

There is no difficulty in calculating this increment as exactly as desired. We will, however, confine ourselves to the simplest proper assumption, believing this to be quite sufficient for the present purpose, and for the conclusions we are about to make. We will limit ourselves to the case of a very long airship, so that the influence of each end on the other is small and may be neglected. Moreover, we shall assume the ends to be so shaped as to be capable of representation by the combination of the flow from a point source and the constant velocity V^2. The intensity of the point source must be

(8) $$I=r^2\pi V$$

where r is the radius of the greatest section of the ship. Let the point source be situated at the origin of a system of polar coordinates R, φ. The fluid passing in unit of time through a spherical segment, $R=$constant, within the cone $\varphi=$constant is composed of two parts, one due to the constant velocity V, and the other to the point source. The first part is $(R \sin \varphi)^2\pi . V$ and the second part is $\frac{1}{2}(1-\cos\varphi) . r^2\pi . V$. If the edge of the spherical segment coincides

[1] Compare Report No. 114.

with the surface of the airship in question, the entire fluid passing, that is to say, the sum of these two expressions is equal to the intensity of the source $r^2 \pi V$., whence we obtain the equation of the airship body.

$$(9) \qquad r^2\pi V = (R \sin \varphi)^2\pi V + \frac{1}{2}(1-\cos\varphi)r^2\pi V$$

or, transformed,

$$(10) \qquad R = r \cdot \frac{\cos \varphi/2}{\sin \varphi}$$

As was to be expected, the shape is independent of the velocity.

Now we proceed to calculate the kinetic energy of the fluid outside the airship body, assuming the airship moving in air at rest. The motion of the air is entirely represented by the point source. At the distance R the velocity is

$$\frac{r^2\pi V}{4\pi R^2}$$

and the potential is

$$\Phi = -\frac{r^2\pi V}{4\pi R}.$$

Through a spherical zone between the two cones $\varphi = \varphi_1$ and $\varphi = \varphi_1 + d\varphi$, with the area $2R^2\pi \sin \varphi_1 d\varphi$; the fluid passing in unit of time is $d\psi = \frac{1}{2} r^2\pi V \sin \varphi_1 d\varphi_1$.

The space integral of the kinetic energy can easily be transformed into a surface integral.[3] Twice the kinetic energy can be represented by the integral

$$(11) \qquad 2T = \rho \int\int \Phi d\psi$$

which is to be performed over the surface of the body. Substituting in (11) the expressions Φ and ψ before mentioned, and replacing R by the right side of (10), it appears that

$$(12) \qquad 2T = \frac{1}{8} r^3\pi V^2 \cdot \rho \int_0^\pi \frac{\sin^2 \varphi}{\cos \varphi/2} d\varphi$$

The integrant in (12) can be transformed into $4\,\frac{\sin^2 \varphi/2\ .\ \cos^2 \varphi/2}{\cos \varphi/2} d\varphi$ the integral of which is $\frac{4}{3} \sin^3 \varphi/2$. Hence

$$(13) \qquad 2T = \frac{1}{6} r^3\pi V^2 \rho$$

Each end of the airship gives rise to the same kinetic energy, so that 2 T is the total energy. This equals one-fourth of the energy which a sphere of the fluid would have if moving with velocity V, whose radius is the radius of the largest cross section of the airship. This gives us the apparent increment of mass of the airship, and is equivalent to about 2½% of the entire volume.

The error due to our two assumptions with respect to the shape of the ship and to its length is not great. The share of a particle of the fluid in contributing to the energy decreases as the third power of R, and is small to the same degree that R is great when compared with $\frac{1}{2}r$, the distance of the point source from the head of the ship. Nor do we believe that the influence of the shape of the ship is great. In any case, the increment of the mass is so small when compared with the entire mass of the ship that it little matters whether the error in the increment is a little smaller or greater. It is only the order of magnitude of the increment that we intended to calculate.

[3] See Lamb. Hydrodynamics, section 61.

The exact calculation of the increment for several forms, however, would be useful too. It could be based with advantage on a valuable paper of Fuhrmann[4] on a similar theme. The integration could be performed graphically.

MOTION IN A VISCOUS FLUID.

The preceding calculation shows the increment of mass to be about $2\frac{1}{2}$ per cent of the mass of the ship. Accordingly, the force on the ship due to the retardation of the surrounding flow is only $2\frac{1}{2}$ per cent of the drag due to the viscosity of the air. Hence also the change in the distribution of pressure is small. If the fluid motion near the surface is stable, we can not expect it to be affected by so small a change in the distribution of pressure. The tests on airships show that within certain limits the drag is proportional to the square of velocity. This points to stability of flow for these particular ships. In this case we do not even expect a small change of the flow and of the drag.

The quantity deduced above is, so to speak, the influence of the motion in frictionless air on the friction. In reality the matter is more complicated. The ship leaves behind it a stream of air, following with a velocity less than the velocity of flight; this may be called the "wake."

It is possible to obtain a certain notion of the magnitude of the effects. Generally the velocity of the air in the wake can not surpass the velocity of flight. But even if it were as great, the air would remain in the neighborhood of the ship, there would be no space for the new wake to be formed, steady motion could not occur, and the phenomenon would not agree with the facts. Let V be the velocity of flight and V' the average velocity of the following stream of air. This air occupies a cylinder with the radius r' behind the ship. Then the volume of this cylinder filled with air in the unit of time is $r'^2\pi(V-V')$, and its momentum is

$$M=r'^2\pi(V-V')V'\rho \tag{14}$$

The radius r' has a minimum if V' is $\frac{1}{2}V$. Let us assume this for the present so that $M=\frac{1}{4}V^2\rho\pi r'^2$. If the coefficient of drag with respect to the section πr^2 is about .08, as it appears to be in the following tests, we would obtain for the momentum given the wake $.08\,V^2\frac{\rho}{2}\pi r^2$. Hence,

$$.08\,V^2\frac{\rho}{2}\cdot r^2\pi=\frac{1}{4}V^2\rho r'^2\pi$$

and therefore,

$$\frac{r'}{r}=.40.$$

Tests with airship models show that the distribution of pressure agrees with the theoretical value about up to $r'=\frac{1}{2}r$ at the rear end. For this reason it is probable that the mean velocity of the air in the wake is indeed not very different from half the velocity of flight. It would be interesting and important to determine it more exactly by model tests. As far as we know such tests have not yet been published. For the present purpose the exactness of our assumption is sufficient.

If the ship is retarded, the air in the wake, owing to its momentum, meets air with a less velocity, and pushes it aside. At last the air overtakes the ship. We will calculate what velocity the ship has at this moment of meeting. The following air has traveled the distance

$$\frac{1}{2}V_1(t_2-t_1)$$

V_1 being the velocity of the ship at the time t_1, and t_2 being the instant of the meeting. According to equation (7) the ship has traveled in the same time

$$s\log\frac{t_2}{t_1}$$

[4] Georg Fuhrmann: Theoretische und experimentelle Untersuchungen an Ballonmodellen. Jahrbuch der Motorluftschiff Studienges. 1911/12.

That is,

$$\frac{1}{2}V_1(t_2-t_1)=s\log\frac{t_2}{t_1}.$$

Hence from equation (7).

$$\frac{1}{2}V_1\left(\frac{s}{V_2}-\frac{s}{V_1}\right)=s\log\frac{V_1}{V_2}$$

or

$$\log\frac{V_1}{V_2}=\frac{1}{2}\left(\frac{V_1}{V_2}-1\right)$$

The solution of this equation is $\frac{V_1}{V_2}=3.5$. If the velocity has decreased to the 3.5th part of its value the air running after the ship overtakes it.

This air in the wake has the same direction as the ship, and therefore its momentum would decrease the drag of the ship and its retardation. The test, however, would show such an effect only if the pitot tube attached to the airship opens into air at rest, as in normal flight. In this case, when the air of the wake overtakes and surrounds the ship, the indicated velocity would cease to decrease before the velocity zero is reached.

If, on the other hand, the pitot tube is within the moving air, the test would show a more or less sudden increase of retardation, the following air having perhaps a greater velocity than the ship itself. This retardation, however, ceases soon, or at least decreases considerably.

We do not believe that the following air meets the ship at all. The distance the ship covers between the beginning of the retardation and the meeting with the following stream of air would be according to equation (7)

$$x_2-x_1=s\log\frac{t_2}{t_1}=s\log\frac{V_1}{V_2}=s\cdot\log 3.5.$$

The tests show s to be about 11,000 feet, and log 3.5 = 1.25; so that the distance would be about 13,700 feet = 2.5 miles. We can hardly imagine that in the greater number of tests the course of the ship was so exactly straight and the wind so uniform that after about 2 miles the ship has not left her path by half its diameter, i. e., by 40 feet. The stream of following air seems more likely to be dissolved during so long a distance. But even if it should meet the ship with undiminished velocity, the pitot tube would not be within it at each test. The cylinder of the radius $\frac{1}{2}r$ when distributed around a cylinder of the radius r occupies a tube with a thickness of wall $r(\sqrt{1.5}-1)=.22\,r$, or about 8 feet. The distance between the tube and the ship was in practice about 10 feet.

THE RESULTS OF THE TESTS.

We are now prepared to consider the results of the tests and to examine them with respect to the possibilities mentioned. Each of the curves represents a single test. $1/V$ is plotted against the time. According to equation (6) the tangent of the angle between the direction of the curve and the vertical axis of coordinates is the characteristic length of the ship. If the drag is proportional to the square of the velocity this length s is constant and the curve is a straight line. If the plotted points are lying on a uniformly curved line, the coefficient of drag changes continuously; if the curve has a sudden break, the coefficient changes discontinuously and suddenly from one value to another. On the right hand of each diagram a scale for the velocity in mi/h. is added.

L. Z. 10 is one of the oldest ships. Its velocity was only 19 *m/sec.* (*42.5 mi/h*).[5] All points lie as exactly on a straight line as can be expected. The same can be said of the test with *L 43*. *L 59* is the only ship the test with which gave a slightly curved line. The cofficient of drag was not constant during the test as with the two first-mentioned ships, but slowly increased as the velocity decreased.

[5] Jaray, in the paper "Studien zur Entwicklung der Luftfahrzeuge," in Zeitschr. f. Fl. u. Motorl. gives 21 m/s; we agree with V. Soden and Dornier in the above-mentioned papers. In the mentioned paper some other details of the ships can be found.

The remaining ships, six different ones, the tests of which were made at different times and independent of each other, show a very remarkable result. The points lie on two different straight lines, which cut each other at a definite angle. Look, for instance, at the curve of *L 44*. The first and the last points are somewhat irregular; and we think that at the first point the engines were not yet completely stopped and at the last point the dynamical pressure was too small to be obtained correctly. The other points coincide with the two straight lines mentioned; the first line is given by 7 points and the second by 4 points. We think that the genuineness of the broken line obtained with six ships out of nine can not be doubted. It is not improbable even that *LZ 10* also would have given a broken line if the test could have been begun at a higher velocity. We do not know whether *L 43* and *L 59* also have discontinuities outside the range of the test.

The curves show that the velocity decreases steadily at the end of each test, and that the retardation, but for one sudden discontinuity, is very regular and in accordance with equation (6). Neither does the velocity become constant at a definite value, nor can we find an increase of the retardation followed by a decrease. We are unable to find any indication of the following stream of air meeting the ship. We have but one explanation for the result of the test. The retarding ship suddenly changes its coefficient of drag, it being increased. Nor is this in disagreement with other experiments of aerodynamics. On the contrary, there is scarcely any body, if there is at all, the motion of the fluid around which or within which does not suddenly and discontinuously change under some particular conditions. We state that the airships investigated are no exceptions to this rule, but that such a discontinuity happens and the coefficient of drag suddenly changes if the ships are retarding and their velocity is 70 to 80 mi/h.

The reason is the same as in other cases, a sudden change of the motion of air, which has become unstable. It is not quite certain whether this change would also happen if the ships would be accelerated or if they were in uniform flight. We believe this change would occur either at the same velocity or at a velocity in the neighborhood.

Our opinion is supported by the particular values of the characteristic lengths obtained. These values are such that only the first ones, those obtained at the higher velocity, agree with the magnitude of the absorbed power of the ship.

According to the definition (8) $A=2v/s$, where v, the displacement of the ship is properly to be increased by $\pi/3\,r^3$, according to equation (13). This improvement is not very considerable. This area of drag A still contains the area of drag of the propellers, which we estimate to be 6.5 square feet per engine. After subtracting a corresponding value, we obtain an improved area of drag of the ship which may also be denoted by A. The required power then is

$$P=A\cdot q\cdot V\cdot \eta \tag{15}$$

where q is the dynamical pressure and η the efficiency of the propellers. The efficiency can be calculated by using this equation and its value is put into the table for the two characteristic lengths obtained in the columns headed η_1 and η_2.

The density of air was assumed to be that of sea level under mean conditions. It was not so, of course, the ship actually flying at some height. The publication of Soden and Dornier hints at a height of about 1,450 feet. But the power of the engines decreases as much as the density, and the result of the calculation remains unaltered. Only the last ship, *L 70*, is in exception, the engines of which are supercompressed. For this ship we assumed a height of flight of about 2,600 feet and put into a table a correspondingly increased value of the horsepower.

The values obtained for the efficiency, however, are rather uncertain. In (15) V occurs in the third power, the dynamical pressure q containing the square of V. Small differences of the velocity therefore considerably change the result, and the velocity of the ships is by no means very exactly known.

We think, however, that the values obtained from s_1, i. e., from the characteristic length at higher velocity, give better values. With the ships *L36*, *L44*, *L57*, and *L59* there are greater differences of the two lengths. In all these cases the efficiency obtained from the smaller characteristic length is too high.

The chief part of the theoretical loss of a propeller is [6]

$$1-\eta=\frac{\sqrt{1+c_p}-1}{\sqrt{1+c_p}+1};\quad c_p=\frac{T}{D^2\frac{\pi}{4}\cdot q}=\frac{N\cdot\eta}{D^2\frac{\pi}{4}\cdot q\cdot v}$$

where T denotes the thrust, N the required power, D the diameter, q the dynamical pressure, and η the efficiency. For one propeller of *L57*, for instance, we would obtain

$$c_p=\frac{240\text{ hp}\cdot 550\cdot .69}{75\text{ sq. ft.}\cdot 64.5^2\,(\text{mi/h})^2\cdot\frac{1}{350}\cdot 64.5\text{ mi/hr. }1.47}=1.77;$$

$$1-\eta=\frac{\sqrt{2.77}-1}{\sqrt{2.77}+1}=25\%$$

The loss as calculated from s_1 is $1-0.69=31\%$, which is in excess of the theoretical loss. The remaining loss of 14 per cent is well explained by the limited number of blades, by the rotation of the propeller stream, by the drag of the blades, and by the friction of the gear. The efficiency due to the second length s_2 is so high that it does not even account for the theoretical loss. The other ships give a similar result.

It is also possible to estimate the effective area of wing of the cars, struts, ropes, and, last but not least, radiators. This drag was considerably higher with the older ships than with the newer ones. In spite of it we assumed a drag of 6.5 square feet per engine, and subtracted this amount from the area of drag. The remainder still contains a part of these resistances with the older ships. The percentage, however, is not high. The area of the drag of the airship body, so obtained, is divided by the two-thirds power of the deplacement. An absolute coefficient for the drag of the airship body results. It is put into the last column of the table.

The coefficients obtained look reasonable. The first ship, *L10*, has a very old-fashioned shape and accordingly a high drag. The next three ships, *L33*, *L36*, and *L43*, are similar and also belong together with respect to their coefficients. Their coefficients differ indeed, but their mean, .028, forms a distinct difference from the other ships. The next two ships, *L44* and *L46*, are similar to *L70*. They all have the same coefficient, .020. *L57* and *L59* have a more slender form, they are longer, and accordingly their coefficient is somewhat higher. This fact indicates also that the skin friction is an essential part of the drag. The value of s obtained for *L33* seems less reasonable than the values obtained for the other ships.

TABLE OF TESTS.

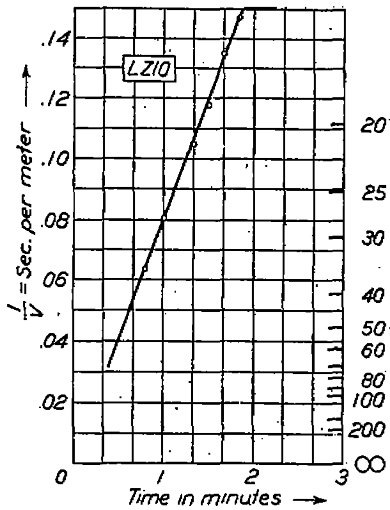
	No. of airship.	Name.	Displacement.	Diameter.	Increment.	Length.	s_1	s_2	Discontinuity at—	Number of engines.	Horsepower.	Maximum velocity.	A	η_1	η_2	D_e
			Cu. ft.	*Feet.*	*Cu. ft.*	*Feet.*	*Feet.*	*Feet.*	*mi/h.*			*mi/h.*	*Sq. ft.*			
1	10	L Z 10	706,000	45.9	12,600	460	2,460	2,460	....	3	450	42.5	565	67	....	0.069
2	76	L 33	2,140,000	78.3	62,800	645	9,800	9,340	21	6	1,440	63.0	409	49	53	.025
3	82	L 36	2,140,000	78.3	62,800	645	7,900	4,240	22	6	1,440	63.0	519	62	120	.029
4	92	L 43	2,140,000	78.3	62,800	645	7,900	7,900	....	5	1,200	60.5	525	56	....	.030
5	93	L 44	2,140,000	78.3	62,800	645	10,900	7,800	26	5	1,200	64.0	371	56	80	.020
6	94	L 46	2,140,000	78.3	62,900	645	11,100	8,560	25	5	1,200	65.0	364	58	77	.029
7	102	L 57	2,640,000	78.3	62,800	745	11,300	9,720	29	5	1,200	64.5	445	69	81	.022
8	104	L 59	2,640,000	78.3	62,800	745	11,800	7,590	Steady.	5	1,200	64.4	425	66	105	.021
9	112	L 70	2,400,000	78.3	62,800	694	10,800	9,160	38	7	2,000	77.3	405	65	78	.020

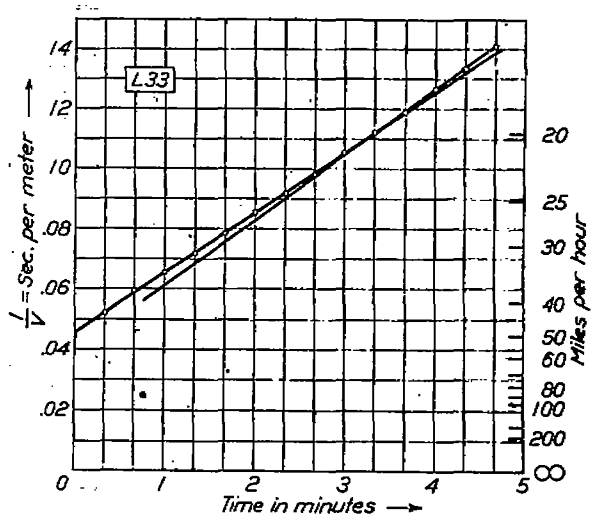
CONCLUSION.

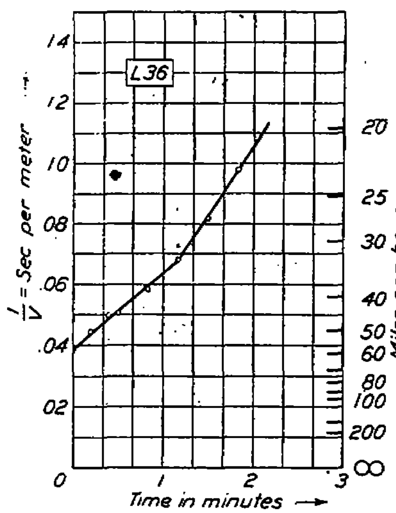
These are special results, and the tests show also one general result. If the drag coefficient changes considerably during the tests, it is almost sure that it would change on enlarging a model to its hundredfold linear dimension or to the millionfold volume. Under these circumstances it is quite useless to make model tests for the determination of the drag of particular airships, if in some way the effect of the change of scale can not be eliminated. It

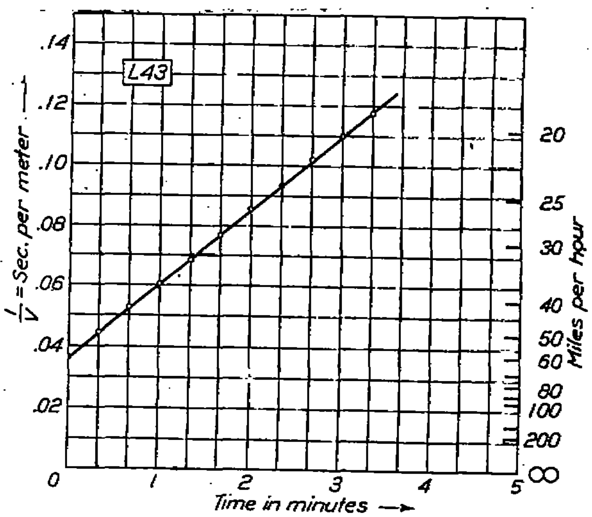
[6] See Report No. 114.

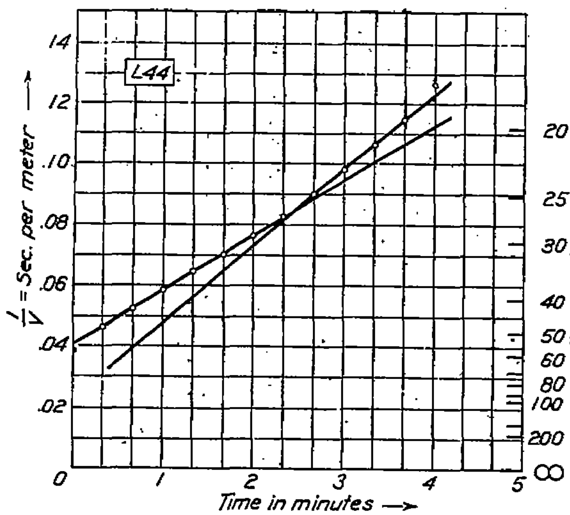
may be that in particular cases the coefficient obtained by the model test happens to agree with the full-sized coefficient; but that proves nothing. Different motions of the air may produce the same coefficient of drag. In consequence of the scale effect, it never is certain that one airship form is better than another, if the corresponding model gives a smaller drag at a Reynolds number 100 times as small.

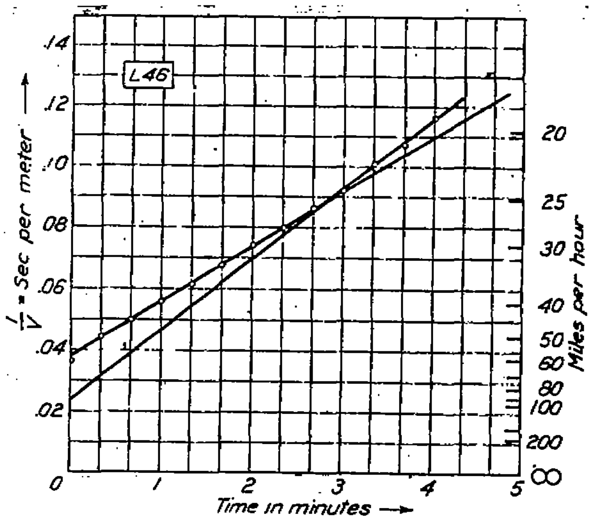


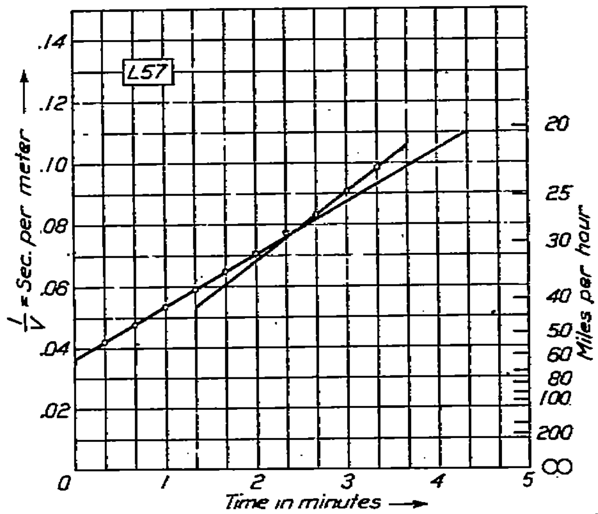











L57
$\frac{1}{V}$ = Sec. per meter
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.08
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.04
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1
2
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4
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Time in minutes
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30
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80
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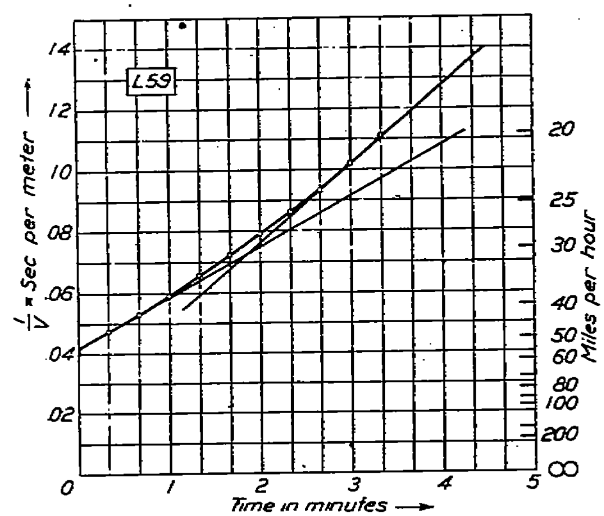
200
∞
Miles per hour


L59
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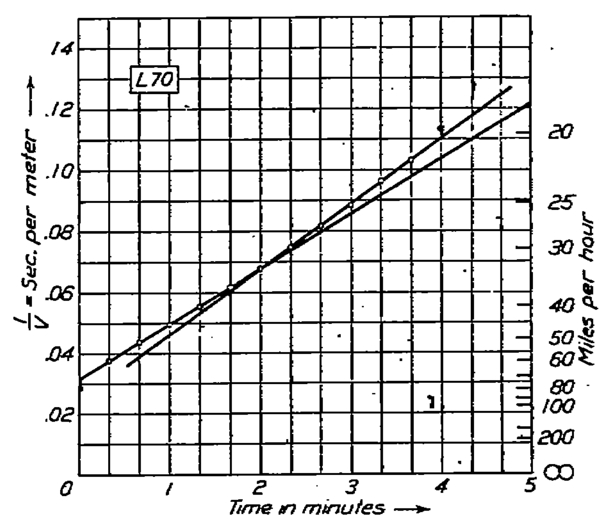
Miles per hour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Miles per hour