REPORT No. 136

DAMPING COEFFICIENTS DUE TO TAIL SURFACES IN AIRCRAFT

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CONDENSED AND MODIFIED BY EDWARD P. WARNER

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INTRODUCTION.

The object of the experiments described in this report, submitted to the National Advisory Committee for Aeronautics for publication, was to compare the damping coefficients of an airfoil as calculated from a knowledge of the static characteristics of the section with those obtained experimentally with an oscillator. The damping coefficients so obtained, according to the conventional notation, can be considered either as due to pitching or due to yawing, the oscillator in these experiments being so arranged that the surface oscillates about a vertical axis. This is in reality the case when the airplane is yawing about the standard Z-axis, but it can also be considered as a pitching motion when the model is so rigged that its standard Y-axis becomes vertical. This horizontal oscillation has the advantage of eliminating the gravity action and avoiding the use of counterweights, whose presence in the wind tunnel is undesirable because of their interference with the air flow. The experimental work was all done in the four-foot wind tunnel at the Massachusetts Institute of Technology, in connection with the preparation by the writer of a thesis in the course in aeronautical engineering at that institution.

The apparatus used in the experiments is essentially a bifilar suspension. In designing this apparatus great difficulty was encountered in making it so as to obtain moment of inertia large enough to keep the system oscillating for a time long enough to be measured accurately and without elaborate special apparatus. After several trials it was found necessary to employ adjustable counterweights outside the tunnel.

The real point of these experiments was to separate the damping due to rotation from that due to translation. Consider the motion of a surface situated at a distance behind the center of rotation about which it oscillates. That motion can be considered as the resultant of two component motions: First, a rotation about the center of pressure of the surface; second, a translation perpendicular to the wind direction. The larger part of the damping is due to the translational motion, and only this part can readily be calculated from static tests. By varying the distance between the center of pressure and the center of rotation on the oscillator, the variation of damping moment can be observed, and the rotational and translational effects can be separately determined.

SUMMARY.

In the first part of the present work, dealing with theoretical damping coefficients, a brief discussion of the method of calculation is given. Owing to the limited amount of time no attempt was made to test a large series of models. An 8 by 2 inch flat plate was first tested, followed by a tail piece with two elevator settings. Static tests were first performed on these surfaces for three speeds, 30, 20, and 10 miles per hour, and the corresponding damping coefficients calculated. Owing to the inaccuracy of force measurements and the slow damping at very low speeds the 10-mile runs for the tailpiece were omitted.

Dynamic tests were made and experimental damping coefficients were calculated for the same settings used in the statical tests. These tests are included in Part II of the thesis. A comparison of results from Parts I and II is given in Part III with brief discussion and conclusion.

The results were such as to encourage the continued use of the conventional method of calculation of damping coefficients. The agreement between the experimental and calculated values was extremely good for all positions which would be likely to be occupied by an actual tail surface, the maximum difference for such positions being less than 10 per cent. In general, although not in all cases, the experimental value was a very little below the calculated.

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PART I.

THEORETICAL DAMPING COEFFICIENTS.

Consider a tail surface situated at a distance behind the center of gravity of the airplane and rotating about the C. G. If the machine has a forward velocity V and an angular pitching velocity q, the resultant wind velocity relative to the tail surface is V_r as shown in the figure. The angle between V_r and the direction of flight, measured in radians, is $\tan \frac{-iql}{V}$ or $\tan \frac{-iql}{V} \times 57.3$ in degrees.

The change in lift due to the rotation is:

$$-\frac{dC_{\rm L}}{d\alpha} (57.3) \frac{ql}{V} \frac{\rho}{2} S V^2,$$

where α is the angle of attack of the tail surface and S is its area.

 $\begin{aligned} \text{Moment} &= -\frac{dC_{\text{L}}}{d\alpha} \times 57.3 \times \frac{gl}{V} \times \frac{\rho}{g} S \ \nabla^2 \times l \\ &= -\frac{dC_{\text{L}}}{d\alpha} \times 57.3 \ \frac{\rho}{g} S \times l^2 \times \nabla \times q \\ \\ \frac{dM}{dq} &= -\frac{dC_{\text{L}}}{d\alpha} \times 57.3 \ \frac{\rho}{g} S \ l^2 \nabla \\ &= \frac{d(C_{\text{L}} \ \rho}{d\alpha} \times S \ V^2) \times \frac{l^2}{V} \times 57.3 \\ &= -57.3 \ \frac{dL}{d\alpha} \times \frac{l^2}{V} \text{ ft. lbs./radian/sec.} \end{aligned}$

Therefore

where $\frac{dL}{d\alpha}$ is the slope of the lift curve plotted against angle of attack in degrees. The value of $\frac{dL}{d\alpha}$ is very nearly constant in the neighborhood of zero angle, as the lift curve there is approximately a straight line. With this value obtained from the statical tests of the section, the theoretical damping coefficients due to the translational component can be calculated.

The statical tests on the flat plate were made at three speeds: 30, 20, and 10 miles per hour. A complete set of characteristic curves are given for 30 miles per hour, but only the lift curves are given for the 10 and 20 mile runs, as required for the purpose of computing $\frac{dM}{dq}$. The value $\frac{dL}{d\alpha}$

of course varies very closely as the square of the speed.

The mean position of center of pressure for the flat plate is taken at one-fourth of the chord from the leading edge—that is, 0.5 inch. The mean position of center of pressure for the tailpiece with -30° elevator setting is taken at the hinge, and that for the zero degree setting is taken 0.3 inch forward of the hinge. With these mean positions of center of pressure the value of *l* is properly taken and the theoretical value of damping coefficients can be calculated. The results are tabulated in Table I and plotted in figures 1, 2, and 3.

STATICAL TESTS ON FLAT PLATE AND TAILPIECE.

The tests were made in the usual manner on the wind tunnel balance, lift, drag, and moment readings being taken. The results for the tail surfaces are fully shown in the curves which follow.

I.

Theoretical damping coefficients of tailpiece.

[Elevator setting -30 and 0°. Wind speeds 20 and 30 miles per hour.]

	1		1			_	1	
Elevator angl	e1	80°.	8	0°.	. 0'	•	0*	•
$\begin{array}{c} V, \text{ miles per hot} \\ \frac{dL}{d\alpha} \\ \frac{d\alpha}{d\alpha} \\ l, \text{ inches} \\ \frac{dL}{d\alpha} \\ \frac{dL}{d\alpha$	17 80 15 0	0120 1355 1244 10081	20 .0 15 .0 .0	051 588 156 0078	30 .0 14.7 .0 .0	115 341 225 0075	20 .00 14.7 .01 .01)49 510 143 1072
V, miles per hour. dL	30 .0120 12 .0227 .0156 .00052	20 .(12 .(.(0051 0341 0995 0050	80 .0 .0 .0	1115 216 142 0047	20 .0 11.7 .0 .0	049 326 0916 0046	• •-
V, miles per hour. dL da L inches. HV Mg Mg	30 .0120 8 .0101 .00693 .00023	20 ,0 8 .0 .0	051 1520 0442 0022	30 01 7.7 .00 .00	15 935 816 021	20 .00 7.7 .01 .00 .00	49 41 397 020	
V, miles per hour da l, inches D/V Mg Vg.	30 . 0120 4 . 00252 . 00173 . 000058	20 .00 4 .00 .00	61 379 1105 0053	20 .01 & 7 .00 .00	15 216 142 0047	20 .00 8.7 .00 .00	49 325 092 0048	

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PART II.

EXPERIMENTAL DAMPING COEFFICIENTS.

METHOD OF EXPERIMENTING AND DESCRIPTION OF APPARATUS.

The general scheme employed in testing was to mount the model in such a way so that it oscillated about a fixed vertical axis. As mentioned in the introduction, the use of a vertical axis has the advantage of avoiding the action of gravity and the use of counterweights. For minimum interference, a minimum of frictional damping, and simplicity of contruction a bifilar suspension was chosen.

Referring to the accompanying drawing (fig. 7), it is seen that two beams shaped to stream-line form constitute the principal structural members of the set-up. These beams are secured to the channel wall by inserting the tongues on the ends of the beams into the sheetmetal sockets on the wall, thus making it possible to erect or dismount the apparatus quickly. Two piano wires with one end hooked to the upper beam (1), passing through the hole bored in the lower beam and running over the pulleys (9), are led to the outside of the channel. It is the chief aim of the design of the apparatus to put as few as possible of the parts inside the tunnel. Two weights of about 10 pounds each are attached to the other ends of the wires. These two stretched wires therefore function as a spring under constant tension which supplies the restoring moment necessary to keep the model in regular periodic motion.

The turnbuckles (11) can be adjusted so that the wires pass through the small holes on the floor of the channel freely without touching the sides of the holes.

The oscillating bar (3) is made of two semicircular rods clamped together by screws. The bar is clamped to the piano wires by screws at one end, and the model is clamped between the halves at the other end. The method of mounting the model is illustrated in the drawing.

The damping and statical moments in these experiments were both unusually large on account of the long lever arm (in the extreme case more than 15 inches). This necessitated the use of heavy inertia weights. As shown in the drawing, these weights are placed outside the tunnel and are clamped to the rod (18), which is connected rigidly to the oscillating bar (3) through the vertical rod (13). At the intersection of the vertical rod 13 and the oscillating bar 3 a universal joint is provided, so that the stretched wires and the rod impose no lateral constraint on each other and there is no side thrust on the bearing at the bottom of the rod except that due to friction in the universal and to the air resistance of the rod itself.

At the lower end of the vertical rod 13 a pivot rests in a socket. This bearing is intended only for constraining the motion of the vertical rod and not for taking any load, as virtually all the weight is carried by the piano wires.

A pointer is attached to one end of the rod (18), and the angular displacements are read directly on the dial (20), which is graduated in degrees.

The apparatus was set up about 4 feet upwind from the balance, so that the stand (23) was placed very near to the motor rheostat. Only one observer was thus needed for the experiment, regulating the wind speed and taking the reading at the same time.

SOLUTION OF THE DIFFERENTIAL EQUATION OF MOTION.

The mathematical principles involved in these experiments are simple and need only be summarized.

The damped harmonic motion.—In this oscillating system the damping is due to two causes, the mechanical friction of the mechanism and the action of the air on the system. The resisting

moments due to these two causes, together with restoring moment of the piano wires at any instant of the motion, must be equal to the product of the mass moment of inertia by the angular acceleration:

$$\frac{I_{\rm m} d^2\theta}{g dt^2} = -E\theta - b\frac{d\theta}{dt} \quad (I)$$

where $\frac{l_m}{g}$ is the moment of inertia of the entire oscillating system measured in slug units; $E\theta$ is the restoring moment due to the elasticity of the wires and to the actual change of angular position of the model, and the term $b \frac{d\theta}{dt}$ is the moment due to wind and frictional damping. It is the purpose of the following experiments to determine the value of b under various conditions.

Before the value of b can be determined the differential equation of motion must first be solved. This equation is of the type of second order with constant coefficients, because, after reducing,

$$\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + C\theta = 0 \quad \text{(II)}$$

where

$$B = \frac{bg}{I_{\rm m}}$$
, and $C = \frac{Eg}{I_{\rm m}}$

The solution of this type of equation is known to be of the form

$$\theta = e \left(-\frac{B}{2} \pm \sqrt{\frac{B^2}{4} - C} \right) t$$

In order that the motion may be oscillatory the expression under the radical must be negative Then,

$$\theta = e^{-\frac{B}{2}t} \times e^{t} \sqrt{\frac{B^{2}}{4} - C} t = e^{-\frac{B}{2}t} \left(\cos \sqrt{C - \frac{B^{2}}{4}} t + i \sin \sqrt{C - \frac{B^{2}}{4}} t \right)$$

• The period of the oscillation is

and the time to damp the motion to
$$\frac{1}{n}$$
 times the original amplitude is $\frac{\log e^n}{B}$.

In these experiments the time to damp the amplitude to one half of the initial displacement was observed. Therefore

$$\log_{e} 2 = \frac{B \cdot t}{2} = \frac{b \cdot g \cdot t}{2I_{\rm m}} = b = \frac{2 \log_{e} 2 \times I_{\rm m}}{gt} = \frac{1.386 I_{\rm m}}{gt}$$

If I_m and t are known b can be calculated. This b contains three parts, the mechanical damping b_0 , the damping due to the wind on the apparatus and the damping on the model itself, b_m . To find the friction damping on the apparatus it is allowed to oscillate with no wind blowing, Next the wind damping on apparatus is found by allowing the apparatus to oscillate in the wind with the model removed, which gives $b_0 + b_a$; b_a is obtained by subtraction. Finally the damping on the model itself is obtained by allowing the model to oscillate in the wind, which gives $b = b_0 + b_a + b_m$; by subtracting $(b_a + b_0)$ from b, b_m is determined.

The moment of inertia of the system is calculated from direct measurements of the modulus of torsion and the period of oscillation without any wind. This neglects the effect of friction on the period, which effect, however, is certainly less than 0.1%. If this factor be neglected

$$T = \frac{2\pi}{\sqrt{C}} = \frac{2\pi}{\sqrt{\frac{Eg}{I}}} \text{ and } \frac{I}{g} = \frac{T^2 E}{4\pi^2}$$

The direct measurement of E is performed by applying weights at a known distance from the center of rotation and observing the angular displacement. The value of E found in this case is .765 ft. lbs./radian. The moments of inertia of the entire oscillating system for different positions of model are calculated and tabulated in Table II. The weight of the flat plate and the tailpiece are very nearly equal, so that the moments of inertia for same position of the two surfaces are the same. These values as tabulated in Table II are used in calculating the M_q .

Π.

Determination of moments of inertia of apparatus and model (at different positions). [NOTE.-I-distance between center of rotation and center of gravity of model.]

I	Time for 10 oscilla- tions (sec.).	Period T.	L, Slugs-ft.*	
15 inches 12 inches 8 inches 4 inches 0 inches Without model.	52 51.4 51.2 51 51 51	5.20 5.14 5.12 5.10 5.10 5.10	. 525 . 514 . 510 . 505 . 505 . 505 . 505	

THE DAMPING ON THE APPARATUS.

The values of b_a in these experiments are determined for different positions of the spindle. The results are tabulated in Table III and plotted in figure 4. It is seen that the damping on the apparatus follows very closely the law of linear variation with speed, and varies as the square of the distance l, measured from center of rotation to spindle.

THE OSCILLATOR TESTS.

On the flat plate two trial runs were made before the final experiments, the results of which are here recorded. For the tailpiece two readings were taken for every different condition of testing; namely, for different speed and positions of the models. The average values are used in the computation of M_q , the values of which are tabulated in Tables IV to VI. Since M_q is directly proportional to speed, a simple basis of comparison is obtained by dividing the damping coefficient by the wind speed. The values of $\frac{M_q}{V}$ are plotted against distance from the center of rotation in figures 5, 6, and 7.

Ш.

Determination of damping on apparatus. [NOTE.--! = distance from center of rotation to spindle.]

.			30	miles per ho	air.
1	La	0.e	t	$b_{\bullet} + b_{\star}$	b.
15 inches . 12 inches . 8 inches . 0 inch	375 339 283 255	.00193 .0021 .0025 .00274	245 sec. 248.5 230 219	.00256 .00284 .00305 .00319	.00093 .00074 .00055 .00045
			20	miles per ho	ar.
15 inches. 12 inches. 8 inches. 0 inch			978 272 243 235	. 00256 . 00258 . 00258 . 00258 . 00298	.00063 .00048 .00038 .00024
		ĺ	10	miles per ho	ur.
15 inches. 12 inches. 8 inches. 0 inch			316 302 258 241	.00222 .00232 .00270 .00290	.00029 .00022 .00020 .00016

IV.

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Experimental damping coefficients for flat plate.

[]- 14.75 inches; from center of rotation to center of pressure.]

v]	<i>b</i> ∙		ð,	bo + b	
30	.0015		00093	.002	19.8
20	.0015		00063	.002	31.8
10	.0015		00029	.001	58.9
V	$b = b_{\bullet} + b_{m}$	b.	b _m =	$b - b_o b_a$	bm ▼
30	. 0360		0.	336	.00112
20	. 0241		0.	220	.00110
10	. 0134		0.	116	.00116

[l=12 inches.]

V	b o .	b'n	b.+b.	t
80	.0015	.00074	.0022	29.9
20	.0015	.00048	.0020	45.7
10	.0015	.00022	.0017	78.5

v	b=b_+b_+b_m	$b_m = b - b_o - b_n$	<u>b</u> <u>∎</u> ▼
30	.02384	.0216	.00072
20	.01558	.0136	.00068
10	.00908	.0074	.00074

[l=8 inches.]

V	ba		b _a	bo+b	
30 20 10	.0015 .0015 .0015		00055 00038 00020	.0021 .0019 .0017	67.6 79 150
v	$b=b_0+b_0$	+ bm	b _m =b-	-b _o -b _a	$\frac{b_{m}}{V}$
30 20 10	.010)15 398 170		.0081 .0071 .0030	.00027 .00035 .00030

[i=0; center of rotation and center of gravity coincide.]

v	b.	ba	bo+b	t
30	.0014	.00045	.00185	285
20	.0014	.00024	.00164	382.5
10	.0014	.00016	.00156	430
v	b.			• 💆
30	.000.	127	00042	.000014
20	000.	83	00019	.000009
10	001.	59	00008	.000003



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V

b.....b.

bm..... bm V

v.

Experimental damping coefficients of tailpiece.

VI.

Experimental damping coefficients of tailpiece. [Elevator setting--30°. Wind speeds, 30 and 20 miles per hour. Fric-tional damping--.000927. Distance from C. R. to C. P.-15 inches.]

[Elevator setting 0°. Wind speeds 20 and 30 miles per hour. Frictiona damping-b_o-.000778. Distance from C. R. to C. P.-14.7 inches.]

V	··· · 30	20	
ba	.000930	.000630	
ba+bo	.001708	.001408	
t	35	46	
b	.02065	.01570	
bm	.01834	.01429	
v	.00063	.00071	

ba ba+bo b b b bm bm T	. 00093 . 00136 29 . 0259 . 0240 . 00030	.00063 .00156 45 .0161 .0145 .00078

Elevator setting-30°. Wind speeds, 30 and 20 miles per hour. Fric-tional damping-.000927. Distance from C. R. to C. P.-12 inches.]

30

.00074 .00167 45

.01585

.00047

20

.00049 .00142 70.4 .01020 .00878

.00014

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[Elevator setting, 0°. Wind speeds, 30 and 20 miles per hour. Frictional damping-b_- 000778. Distance from C. R. to C. P.-11.7 inches.]

v	80	20
b_{a} b_{a} + b_{o} b b_{a} b_{a} b_{a} b_{a}	. 000740 . 001518 48 . 01482 . 01380 . 000 <u>14</u>	.000490 .001268 .65.6 .01087 .00960 .00048

[Elevator setting, 0°. Wind speeds, 30 and 20 miles per hour. Frictional damping-bo-.000778. Distance from C. R. to C. P.-7.7 inches.]

v	30	20
ba ba+bo t b ban ban V	.000565 .001343 96.4 .00734 .00600 .00020	.000380 .001158 148 .00476 .00380 .00018

[Elevator setting-30°. Wind speeds, 30 and 20 miles per hour. Fric-tional damping-.000927. Distance from C. R. to O. P.-8 inches.]

•••	v	30	20	
	ba ba+bo t b baa baa baa baa baa baa baa baa baa baa baa baa baa baa baa baa t baa	. 00056 . 00149 72 . 00982 . 00833 . 00028	.000380 .00131 125 .00582 .00431 .00022	

[Elevator setting, 0*. Wind speeds, 30 and 20 miles per hour. Frictional damping-.000778. Distance between O. R. and O. P.-0 inch.]

v	30	20
ba br.+bo 5 5 5 5 5 5 5 5 7	• 000450 • 001228 450 00155 • 00033 • 000011	.000240 .001018 486 .00144 .00042 .000021

[Elevator setting-30°. Wind speeds, 30 and 20 miles per hour. Frio-tional damping-.000927. Distance from O. R. to O. F.-O inch.]

Ŷ	30	20	
ba ba.+bo b bm bm. bm. P	.00045 .00138 432.4 .00162 .00024 .00008	.00024 .00117 575 .00122 .00005 .000003	

DAMPING COEFFICIENTS DUE TO TAIL SURFACES IN AIRCRAFT.

PART III.

COMPARISON OF CALCULATED AND EXPERIMENTAL DAMPING COEFFICIENTS.

In order to facilitate the comparison of the calculated and experimental values they have been tabulated side by side. In place of giving the actual values of the damping coefficients in this tabulation, the average ratio of damping coefficient to speed has been used, the assumption being made that the actual deviations from strict proportionality to wind speed are smaller than the experimental errors, and that the average is therefore nearer to the true ratio for all speeds than any particular observed value.

J. Experimen- tal M _q .		Calculated M_q .	Experimen- tal M_q -cal- colated M_q .	
14.75	.09113	.00110	+.00003	
12	.00071	.00072	00001	
8	.00031	.00032	00001	
0	.000003	.0	+.00001	

I.	Elevator at 0°.		Elevator at30°.			
	Experi- mental.	Calcu- lated.	Experimen- tal-calcu- lated.	Experi- mental.	Calcu- lated.	Experiman- tai-calcu- lated.
15 12 8 0	.00057 .00046 .00019 .000016	.00074 .00047 .00021 0	$\begin{array}{c}00007 \\00001 \\00002 \\ +.00002 \end{array}$.00077 .00046 .00025 .000026	. 00080 . 00051 . 00023 0	00003 00005 +.00002 +.00001

TAILPIECE.

The check between the experimental and calculated values is quite remarkably good. On the average, the experimental values are a little smaller than those obtained by calculation, but the difference seldom exceeds the probable experimental error.

There is a remarkable difference between these results and those obtained in another series of experiments made at Massachusetts Institute of Technology in 1917¹, where the conclusion was that the damping due to the tail surfaces on a complete airplane model is 50 per cent more than the calculated amount. The difference can only be attributed to a systematic error in one set of experiments or to some fundamental difference between the conditions of the two tests. If any systematic error exists it is more likely to be in the first set of experiments than in those described in this report, as the new apparatus is a decided improvement over the oscillator originally employed at the Massachusetts Institute of Technology. It is improbable, however, that any of these experiments would permit of an error so large as the difference between the results of the two sets. As for differences in surrounding conditions, the only important one is the difference between a complete model and an isolated plate. The presence of the wings

¹ An investigation of the elements which contribute to statical and dynamical stability, by A. Klemin, E. P. Warner, and G. M. Denkinger: Third Annual Report National Advisory Committee for Aeronautics. undoubtedly affects the damping action of the tail surfaces, because of the reduction of air speed behind the wing cell, the down wash, and the increased turbulence in the stream, and this effect may be important. Its magnitude can not, however, be determined in general, even by an approximate rule, until much more extensive tests have been made.

It is difficult even to predict from the data now available the nature of the effect of interference on damping. The reduction in air speed would naturally be expected to reduce the effectiveness of the tail in respect of its contribution to dynamic, as well as to static, stability, but the question of downwash is more difficult. In so far as downwash is a function of angle of attack alone, it should not affect damping if the axis of oscillation of the model passes close to the center of pressure of the wing, so that the true angle of the wing to the air is not affected by rotation. It should be remembered, however, that various points along the chord of a wing so mounted are moving in different directions, and the result is much the same as that of changing the camber of the wing section, the effective curvature being deeper when the pitching rotation is positive, less when it is negative. The lift coefficient and downwash at a given angle therefore appear to depend to some extent on the angular velocity, and their variation with angular velocity must produce a secondary effect, probably very small, on the damping due to the tail. It has recently been pointed out by Cowley and Levy that the time lag before the downwash reaches the tail may have an important effect on the damping coefficient.

In short, while these experiments leave much to be settled by future research, they do at least justify the methods hitherto employed for an approximate calculation of damping, and they show that the damping directly due to rotation is negligible by comparison with that due to translational motion of surfaces as far away from the axis of oscillation as are the tail surfaces on an airplane of conventional design.