REPORT No. 139

INFLUENCE OF MODEL SURFACE AND AIR FLOW TEXTURE ON RESISTANCE OF AERODYNAMIC BODIES

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PREFACE.

The following text, submitted for publication to the National Advisory Committee for Aeronautics, is a slightly revised form of the unpublished Report No. 160 from the Aerodynamical Laboratory, Bureau of Construction and Repair, Navy Department, written in December, 1920.

RESISTANCE OF SMOOTH MODELS IN A SMOOTH STREAM.

General formula.—Following the lead of Newton,[1] Stokes,[2] and Helmholtz,[3] Lord Rayleigh[4] expresses the drag of a body of fixed shape and presentation moving uniformly through a viscous incompressible fluid, or a series of geometrically similar models so moving, by the theoretically derived formula—

$$D=\rho L^2 V^2 f\,(LV/\nu) \qquad (1)$$

in which ρ denotes the fluid density, ν the kinematic viscosity, L a linear dimension of the body, V the speed of translation. In the process of derivation, which is too well known to require treatment here, it is shown that the relative movement of the fluid and model, for varying values of L, V, ν, remains geometrically similar if $f\,(LV/\nu)$ remains constant. External forces, such as gravity, are assumed not to be influencing the motion.

Physical significance of ρV^2.—The quantity ρV^2 is the well-known "impulse" of hydromechanics. For example, a jet of one square unit cross-section issuing horizontally from a tank requires a force ρV^2 to maintain it, and reacts with the force $-\rho V^2$. Also an obstacle in a Newtonian stream of inelastic particles sustains a force ρV^2 per unit of normally exposed area, and a total force $\propto \rho L^2 V^2$. Calling $\rho A V^2$ the "standard impulsive drag," due to such an ideal fluid, and $C\rho A V^2$ the "actual drag" in any fluid of the same density, makes C the ratio of the actual to the standard drag. A like formula can be shown to apply to an inclined plane or the front of a solid surface.[5]

If said unit jet is of a continuous fluid and strikes a normal plane, flattening without rebound, it exerts a push ρV^2 and a maximum point-pressure $\rho V^2/2$; that is, the impulse of the unit jet is twice its greatest dynamic pressure per unit area. These relations are well known.[5]

If an obstacle in a continuous stream sustains a mean pressure $C'\rho V^2/2$ per unit of frontally projected area, its total drag is $\frac{1}{2}C'\rho A V^2$; wherefore $C'=2C$. But if the body is surrounded by a guard ring its front pressure is everywhere $\rho V^2/2$, and its corresponding drag is $\frac{1}{2}\rho A V^2$. Calling this the "standard pressural drag" makes C' the ratio of the actual to the standard drag.

It appears then that both the foregoing expressions, $C\rho A V^2$ and $\frac{1}{2}C'\rho A V^2$, for the actual drag have some physical meaning; the one having reference to a distributed impulse, the other to a distributed pressure; while the coefficient for each is the ratio of the actual drag to an ideal standard drag.

Absolute coefficient.—Denoting by C the dimensionless multiplier, or "absolute coefficient," (1) may be written

$$C=f\,(LV/\nu)=D/\rho L^2 V^2, \qquad (2)$$

an equation commonly used in plotting fluid resistance data, in which the single quantity LV/ν, called "Reynolds number," is the independent variable.

[1] Principia, Book II, Proposition 32.
[2] Mathematical and Physical Papers, Vol. III, p. 117.
[3] Wissenschaftliche Abhandlungen, Vol. II, p. 158.
[4] Phil. Mag. XXXIV, p. 59 (1892); Scientific Papers, p. 575.
[5] See Journ. Frank. Inst. Mar., 1912, article "Aerodynamics."

Plain graphs.—For very accurate experimental values of V, L, ν, plots of (2) commonly portray C as a one-valued function of the single variable LV/ν, in which either or all of the three component quantities may be variable. Usually the plot is a curve, on plain section paper; sometimes it is a practically straight line for a considerable range of LV/ν, indicating that the drag increases as the square of the speed.

Logarithmic graphs.—Wind tunnel data frequently give, when V alone varies,

$$D = aV^n, \quad (3)$$

in which a and n are positive constants. Consequently (2) becomes

$$C = bV^{n-2}, \quad (4)$$

and the plots on logarithmic paper delineate both (3) and (4) as straight lines. When $n=2$, the line (4) is horizontal; when less than 2, it slopes downward. On plain section paper (3) is a parabola, (4) an hyperbola.

For moderate speed ranges many kinds of models have straight-line drag-versus-speed diagrams on logarithmic paper. Struts round and faired, aerofoils at fixed incidences, airship hulls, are examples. When such forms are blunt, or so presented as to produce turbulence, n is close to 2; when they are more and more faired n diminishes and approaches its value for skin-friction planes. These facts have been known many years. Thus in an article published in the Philosophical Magazine for May, 1904, the present writer presented such straight-line diagrams, and stated that (3) applies to all the shapes tested at the limited speeds available in his tunnel—5 to 40 feet per second—but might not be extended to considerably higher speeds. Both statements are well illustrated, for limited speed ranges, by the data since obtained in various other aerodynamical laboratories.

RESISTANCE AS A FUNCTION OF SURFACE TEXTURE.

Symbol for surface roughness.—In the derivation of (1) true geometrical similarity, both in form and texture of the boundary surface of the models, is assumed and is supposed to be expressed by L. Now let L refer only to the general size of the surface, and let l/L denote a measure of the comparative roughness, l being a measure of the roughness. Then if the model remains perfectly similar to itself, while changing size, l/L must remain constant. Incidentally it is noteworthy that, since geometrical similarity in the stream-and-model system requires the size and disposition of the disturbances in the stream to bear a constant ratio to the size of the model, the equation $l/L=$ constant may express also that relation, where l now denotes a measure of said size or disposition. If the several kinds of influence coexist, they may be symbolized by as many different letters.

Monoline graphs for constant surface texture.—In this case it can be expected that (1) will plot as a single locus so long as VL/ν is constant, and provided other influences, such as compressibility and gravity, can be ignored. Likewise when l/L is negligible, that is when l is sufficiently small or L sufficiently large, the resistance should be one-valued so long as VL/ν is kept constant.

Multiform graphs varying with surface texture.—In other cases, i. e., when l/L is not constant and not negligible, D must be a many-valued function and may be written—

$$D = \rho L^2 V^2 f(VL/\nu, l/L) \quad (5)$$

This formula indicates that with VL/ν constant a multiform graph is obtained in two cases: (1) when the roughness l is varied while the size of the model is unaltered; (2) when the size of the model is varied while the surface texture remains unaltered; and further that such multiform graphs have as their limit the single locus of D for $l/L=0$.

For example, in the experiments cited above the writer found that when L and μ were kept constant while V varied, a great variety of graphs of D were obtained with a given skin-friction plane by merely altering the texture of the surface. The plane, which was held lengthwise of the air stream, measured 4 feet long by 2 feet wide by 1 inch thick and bore a smooth streamline prow and stern. On plane section paper its drag-velocity graphs all passed through a common point, the origin, and had the general form (3) in which n varied from 1.85 for a quite smooth surface to 2 for a comparatively rough one.

RESISTANCE A FUNCTION OF FLOW TEXTURE.

Symbol for flow roughness.—As the adding of an independent surface-roughness variable l, to the surface-size variable L made (1) a many-valued function, so may the adding of a flow-roughness variable v to the smooth-flow variable V. For if V represents the steady velocity, relatively to the model, of the smoothly flowing distant fluid and v is a measure of the velocity vectorially superposed on V to roughen the flow, then v/V is a geometrical measure of the comparative flow roughness. Hence in order that the fluid roughness shall remain perfectly similar to itself while changing its general stream velocity V, its comparative roughness v/V must be constant.

Monoline graphs for constant flow texture.—In this case (1) plots as a single graph so long as VL/ν remains constant. Likewise when v/V is negligible, that is when v is sufficiently small or V sufficiently large, the resistance should be one-valued so long as VL/ν is kept constant and other influences remain immaterial.

Multiform graphs varying with flow texture.—But in case v/V is not constant in repeated tests of the same model the drag

$$D=\rho L^2 V^2 f_2(VL/\nu,\ v/V) \qquad (6)$$

will plot as a multiform graph determined by the variation of comparative flow roughness v/V. For example, if V is held fixed while a variety of screens in turn are placed before the model, a great variety of values of D are found, that is, a great variety of coefficients of resistance of the same model in the same medium.

To illustrate further, suppose a fine strut or double-cambered aerofoil, set at zero pitch and yaw in a large uniform air stream, to have just before it an ample honeycomb capable of translation across current. If then the steady wind speed is V, the angle of incidence against the plane of symmetry of the model is zero for the honeycomb stationary, but $\tan^{-1}(v/V)$ for it moving across stream with the velocity v normal to said plane. This angle, depending on v/V, is fixed only when v/V is fixed, and varies when v or V varies independently. The same holds if the honeycomb oscillates to and fro across stream so as to cause a wavy current. It is obvious, therefore, that such a wavy stream, if begot in any other way will cause a variable resistance coefficient unless its flow roughness v/V remains constant. Indeed, for a thin strut the drag coefficient may even become negative when $\tan^{-1}(v/V)$ slightly exceeds 10°, so that while this quartering stream lasts the strut may be actually pulling upstream.

Negative drag in pulsating wind.—To illustrate this last phenomenon, let the wavy stream for an instant meet the strut set at zero pitch and yaw. Then the downstream drag along the unyawed direction is—

$$D_x=D\cos\alpha-L\sin\alpha \qquad (7)$$

where L and D are the lift and drag referred to the instantaneous wind course, making the angle α to the steady direction. From (7) it is seen that the net drag is zero when—

$$L/D=\cot\alpha \qquad (8)$$

and is upstream or downstream according as $L/D \gtrless \cot\alpha$.

The following cotangents comprise ordinary L/D values:

Angle	4°	6°	8°	10°	12°	14°
Cotan	14.3	9.5	7.1	5.67	4.7	4.0

For instance, in British R. and M. Report No. 183 the following values af L/D are found for a fairly flat strut at various angles of yaw:

Incidence	5°	7.5°	10°
L/D	10.07	11.2	10.65

From these tables it appears that such a strut can have a negative resistance in side winds of 6° to 10° referred to its plane of symmetry.

Such winds may be due to side slip, skidding, sudden gusts, etc. So also a wing beating up and down without torsional rotation in still air, or soaring stiff in an atmosphere beating up and down, may have some amount of upstream propulsion. It would therefore be interesting to determine experimentally what shapes are best adapted to self-propulsion in gusts or wavy streams, and how much energy may in practice be extracted in this manner from aerial turbulence by aircraft or bird.

MORE GENERAL RESISTANCE FORMULAS.

Drag formula involving both surface and flow texture.—From what precedes it appears that formula (1), when revised to take account of the texture both of the model's surface and of the fluid stream passing it, takes the general form—

$$D = L^2 V^2 f_3(VL/\nu,\ l/L,\ v/V) \qquad (9)$$

Formula involving gravity and compressibility.—As is well known, when gravity and compressibility are taken into account the resistance formula (1) may be written

$$D = \rho L^2 V^2 f_4(VL/\nu,\ V/a,\ V^2/Lg) \qquad (10)$$

where a is the velocity of sound in the medium, and g the acceleration of gravity.

CONCLUSION.

From the foregoing treatment, which may also be extended to the other force components and the moment, it appears that the shape coefficient C, given in (2), may at times be a function of many other variables besides Reynolds number VL/ν. Hence it is not surprising that various experimenters in fluid dynamics, using the same model and same value of VL/ν, should discover materially different coefficients.

If the formula

$$C = f(VL/\nu,\ l/L,\ v/V,\ V/a,\ V^2/lg) \qquad (11)$$

were kept in mind, or something still more complex, it would serve as a warning not to expect the same results from apparently similar hydrodynamic measurements without a close scrutiny of the attendant circumstances. One of the pressing tasks now before aerodynamic experimenters is to ascertain what agreement can be found among the values of C in (9) for the same model tested in various laboratories.