NACA-TR-151

REPORT No. 151

GENERAL BIPLANE THEORY

IN FOUR PARTS

BY

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National Advisory Committee for
Aeronautics

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SUMMARY.

The following report deals with the air forces on a biplane cellule.

The first part deals with the two-dimensional problem neglecting viscosity. For the first time a method is employed which takes the properties of the wing section into consideration. The variation of the section, chord, gap, stagger, and decalage are investigated, a great number of examples are calculated and all numerical results are given in tables. For the biplane without stagger it is found that the loss of lift in consequence of the mutual influence of the two wing sections is only half as much if the lift is produced by the curvature of the section, as it is when the lift is produced by the inclination of the chord to the direction of motion.

The second part deals with the influence of the lateral dimensions. This has been treated in former papers of the author, but the investigation of the staggered biplane is new. It is found that the loss of lift due to induction is almost unchanged whether the biplane is staggered or not.

The third part is intended for practical use and can be read without knowledge of the first and second parts. The conclusions from the previous investigations are drawn, viscosity and experimental experience are brought in, and the method is simplified for practical application. Simple formulas give the drag, lift, and moment. In order to make the use of the simple formulas still more convenient, tables for the dynamical pressure, induced drag, and angle are added, so that practically no computation is needed for the application of the results.

1. INTRODUCTION.

The appearance of a treatise on the aerodynamics of the biplane cellule, including the monoplane as a particular case, needs hardly any apology at the present time. For the wings, which primarily enable the heavier-than-air craft to fly, are its most important part and determine the dimensions of all the other parts. The knowledge of the air forces produced by the wings is of great practical use for the designer, and the understanding of the phenomenon is the main theme of the aerodynamical physicist. In spite of this the present knowledge on the subject is still very limited. The numerous empirical results are not systematically interpreted. The only general theory dealing with the subject, that is, the vortex theory of Dr. L. Prandtl and Dr. A. Betz, gives no information concerning the influence of different sections, nor on the position of the center of pressure. This theory is indeed very useful, by giving a physical explanation of the phenomena. But the procedure is not quite adequate for obtaining exact numerical information nor is it simple enough. The theory of the aerodynamical induction of biplanes, on the other hand, is developed only so far as to give the induced drag, but not the individual lift of each wing.

I hope, therefore, that the following investigation will be favorably received. I try in it to explain the phenomena, to calculate the numerical values, and to lay down the results in such a form as to enable the reader to derive practical profit from the use of the given formulas, tables, and diagrams without much effort.

The problem of the motion of the fluid produced by a pair of aerofoils moving in it is a three-dimensional problem and a very complicated one. The physical laws governing it are simple, indeed, in detail, as long as only very small parts of the space are concerned. But the effect on the fluid at large can not be predicted with safety without reference to experience. The viscosity of the fluid plays a strange part, though not quite without analogy with the friction between solid bodies gliding along each other or with the behavior of structural members. For under certain conditions the forces produced by a mechanical gear can be calculated without paying much attention to the friction. But often this can not be done, as in the case of a screw with narrow thread which does not turn its nut if a force in the direction of its axis is applied, as it would do without friction. The deformation of structural members follow a certain law only up to a certain limit; then another law suddenly replaces the first one. The behavior of the air around a biplane also can be investigated independently of the viscosity under certain conditions only, and it is not yet possible to express these conditions. If the viscosity can be neglected at first, its small influence can be taken into account afterwards by making use of empirical results. This case alone is the subject of the following report. It is the most important one. But this paper also refers to the more difficult part of the problem. This can not be solved without systematic series of tests, but for the interpretation of these tests, to be made in the future, the following results are hoped to be useful. For the influence of friction is always associated with the influence of other variables, and it can not be separated from them unless the original and ideal phenomenon without friction is known.

The phenomenon in a nonviscous fluid is still three dimensional and complicated enough, and we are far from being able to describe even this completely. Consider a single aerofoil. In the middle section the direction of the air indeed is parallel to the plane of symmetry. At some distance from it it is no longer so, and so far as it can be described approximately by a two-dimensional flow, this flow is different at different sections. Near the ends the flow is distinctly three dimensional. On the upper side the direction of the air flow near the surface is inclined toward the center, on the lower side it is inclined toward the ends and finally flows around the ends. It is a fortunate circumstance however that along the greatest part of the span the flow is almost two dimensional. Moreover, most of the variables are linearly connected with each other, and hence the effect can easily be summed up to an average. Hence, the consideration of the two-dimensional problem is a very useful method to clear up all questions which refer to the variables given in the two-dimensional section; these are not only the dimensions of the wing section but also chord, gap, stagger, and decalage. The truth of this procedure is felt intuitively by everybody who considers the wing section separately. This problem will be discussed in the first part of this paper. The results are useful however only by combining them with the effect of the dimensions in the direction of the span. This effect is discussed in the second part. The third part will contain the consideration of the viscosity and the final results for the use of the designer, developed not only from the preceding theory but also by taking into consideration the results of experience. The fourth and last part contains a list of the important formulas and the necessary tables.

TWO-DIMENSIONAL FLOW NEGLECTING VISCOSITY.

2. GENERAL METHOD.

In order to investigate the influence of two aerofoils on each other, I take into account the fact that the dimensions of the wings at right angles to the chord are generally small when compared with either the chord or the gap. It can not be assumed, however, that the chord is small when compared with the gap. On the contrary, it is often greater than the gap. The first assumption reduces the problem to the consideration of the influence of two flat plates on each other, or, as I will generally express myself throughout this part, the mutual influence of two limited straight lines. This does not mean, however, that I intend to confine myself to considering the effect of this particular section only, as for one particular case has been done by Dr. W. M. Kutta (ref. 5). The flow around a straight line is by no means determined by the general conditions governing potential flows, but in addition to these the character of the flow near the rear edge is to be taken into account. I do not intend to choose this last additional condition indiscriminately, and the same for any wing section; besides, the decision as to the direction of the straight line to be substituted for the wing section must be made. The effect of the direction of this wing section—that is, of the angle of attack—is expressed by the moment of the air force produced about the center of the wing. If the angle of attack of a section shaped like a straight line is zero, this moment of course is zero. The most suc-

FIG. 1.—Section flow without circulation.

FIG. 2.

cessful proceeding is therefore to choose the direction of the substituted straight line so as to give always the same moment around the center as the replaced section does. An easy method for the calculation of this moment is discussed by me in a former paper. (Ref. 3.) For the present discussion it is not essential whether the moment is determined in the way described there or by any other theoretical or empirical method. The direction of the straight line determined according to this precept always becomes nearly parallel to the chord of the section. This is particularly true if the section is not S-shaped; but even then the angle between the chord of the section and the substituted straight line will seldom exceed 2°. This angle is $2/\pi C_{mo}$ where C_{mo} denotes the coefficient of the moment about the center of the section at zero angle of attack. It is always small. The assumption of a straight line not exactly parallel to the chord is thus justified, as it will always run near the points of the chord. (Fig. 2.) One such isolated substituted straight line at the angle of attack, zero, thus experiences no moment, but the air force due to the physical straight line in that position would still be different from that of the replaced wing section, for the lift of the straight line is zero, too, but this is not so in general for the actual wing section, in consequence of its curvature.

Consider the theoretical flow of smallest kinetic energy around the wing section instead of the flow actually occurring. (Fig. 1.) The former flow has no circulation around the wing;

that is to say, the velocity integral is not increased if a closed path around the section is taken. Hence the lift is zero and a straight line at the angle of attack, zero, can be taken as the most perfect substitution among all straight lines, for the air produces neither lift nor moment in either case. The effect of the wing section on the flow at some distance is very small in the case of this flow without circulation. It can be assumed, therefore, that two such wings, producing individually neither moment nor lift, have the smallest influence possible on each other at the usual distance and continue to experience no air forces when arranged in pairs. The influence, indeed, can be entirely described by sources and sinks, and I have shown in a former paper (ref. 4) that such influence is always exceedingly small. I have thus arrived at two straight lines replacing two sections in the particular case that the moment is zero in consequence of the particular angle of attack, and the lift is zero in consequence of the flow artificially chosen without circulation. Now it is easier to fix the thoughts if the different things occurring are designated by particular names. I will call this particular flow around the section without lift and moment the "section flow." (Fig. 1.) It differs from the flow around the two straight lines only in the neighborhood of the section, but there it differs very much, for at the rear edge the velocity of the section flow (which we remember is only imaginary) is infinite. This infinite velocity near the rear edge, which I will call "edge velocity" for sake of brevity, is the reason why the pure section flow generally does not really occur but has superposed on it a second type of flow with circulation (Fig. 3) in such a way that the edge velocity becomes finite. The "circulation flow," as I will call the second type, possesses an

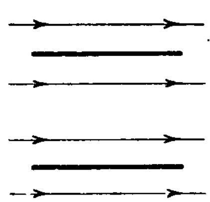

FIG. 3.—Longitudinal flow.

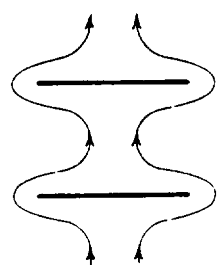

FIG. 4.—Vertical flow.

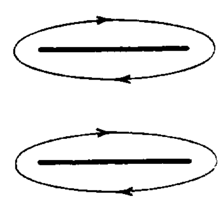

FIG. 5.—Circulation flow.

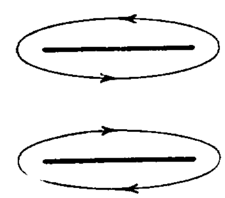

FIG. 6.—Counter-circulation flow.

infinite velocity also at the rear edge, but opposite to the previous one, and the superposition of section flow and circulation flow makes the infinite velocity vanish.

The magnitude of the infinity of the edge velocity can still be different in different cases, for it is infinite only directly at the edge. Near the edge, in this assumed case of an angle of 360° of the edge, it is proportional to $1/\sqrt{\epsilon}$, where ϵ denotes a small distance from the edge. The magnitude of the edge velocity at each point is given by an expression $m/\sqrt{\epsilon}$ where m is a constant near the edge; and for two different conditions the edge velocities, though infinite both times, can differ from each other by different value of the factor m. The superposed circulation flow is determined by the condition that its edge velocity is opposite and equal to the edge velocity of the original flow, which means that its $m_1 = -m_2$ of the original velocity. More generally, the sum of all the factors "m" occurring is zero. The circulation flow around the section differs in the same way from the circulation flow around the straight line as did the section flow from a flow with constant velocity parallel to the two lines; it differs only near the section and practically does not differ at some distance.

The idea is now to change the edge condition of the straight line so as to take into account the curvature of the section. The true section flow around the straight line no longer shall be considered as determining the infinite edge velocity. On the contrary, it is now supposed that the straight line is provided originally with the same edge velocity as the replaced section surrounded by the section flow alone. In consequence of this assumption, the same circulation flow is produced as by the replaced section if we prescribe the condition that the sum of the edge velocities of all the different types of superposing flows occurring, including the added original edge velocity, becomes zero. But then the air forces of the straight lines agree with those of the replaced section and so does the mutual influence of the two wings.

I proceed now to discuss the different types of flow. I suppose the position of the wings to be fixed and the direction of the velocity at infinity to be changing. Consider first the component in the mean direction of the two straight lines. The most important case is when two lines are parallel. If V is the velocity of flow at a great distance, and if β denotes the angle between the lines and direction of flow the component in the direction parallel to these lines, at infinity, is $V \cos \beta$. I call this type "longitudinal flow." (Fig. 3.) The other component is at right angles to it at infinity and is here practically vertical, although not exactly. It may be called "vertical flow." (Fig. 4.) At a great distance its velocity is $V \sin \beta$; near the wings it is variable and almost parallel to the wings. These two types have no circulation around either of the wings. There remain still two types with circulation, for the circulation around the two individual wings can be different. It would be possible to take two flows each having a circulation around one wing only. It is more convenient, however, to choose one flow with an equal circulation around each of the two wings, which may be called "circulation flow" (fig. 5), and a second flow (Fig. 6), with equal and opposite circulations around the two wings, the "countercirculation flow" (Fig. 6). These four types of flow will be sufficient for the development of the theory.

The longitudinal and vertical flows are fully determined by the velocity at infinity and by the angle of attack. The remaining circulation and countercirculation flows are to be determined so as to have such magnitudes as to make the two edge velocities vanish. This done, the air forces produced by the combined flow are to be calculated. This computation is much simplified by the relation between the forces and the types of flow. I have shown in a former paper (ref. 4) that the forces can always be represented by mutual forces between the singularities of the flow. The longitudinal flow has only a singularity at infinity, namely, a double source. The velocity of this flow exceeds in magnitude the average velocity of the other types. The longitudinal flow by itself, however, is unable to produce any air force. The vertical flow has an infinity, a double source of infinite strength, too, and besides, a system of vortexes along the two straight lines. Hence the vertical flow by itself produces a force, namely, a repulsion between the two wings. The circulation and countercirculation flows also produce forces, the latter giving rise to an attraction, for these two types of flow contain vortexes along the two wings also. These forces occur in pairs opposite to each other and may be called secondary. The main forces acting on the entire biplane are produced by the combination of the different types of flow in pairs. The entire lift of the pair of wings is produced by the combination of the flow due to the velocity at infinity with the circulation flow; the "counter lift," in the same way, by this velocity and countercirculation. This sum of lift and counterlift may be called primary lift. It is not the sum of the lifts of each individual wing, as there are in addition the repulsions mentioned between the wings. The entire moment of the pair of wings results from the combination of the velocity at a great distance with the vortexes of the vertical flow. The lift and counterlift generally contribute to the moment, too. The combination between the vortexes of the vertical flow and those of the circulation and countercirculation flow gives rise to a second mutual action between the two wings, namely, a secondary moment between them. This is of smaller importance and will not be discussed in this paper.

This seems to indicate a rather laborious calculation, but often it is much simplified in consequence of some symmetry, as I shall proceed to show.

3. THE BIPLANE HAVING EQUAL AND PARALLEL WINGS WITHOUT STAGGER.

As a preparation for the following development, the magnitude of the edge velocity of a single wing produced by the curvature must be calculated. The lift coefficient for $\beta = 0$, that is, for the angle of attack at which the moment of the air force around the center of the wing is zero, may be called C_{Lo}. A simple method for its calculation is given in a former paper (ref. 3), but it is not essential how this lift coefficient is determined. The velocity of the air with reference to the wing, at a great distance, may be V. The angle of attack of a straight line experiencing the same lift coefficient is theoretically $\beta_0 = 1/2\ \pi C_{Lo}$. The potential function of the vertical flow corresponding to this angle of attack is $W = -iV \sin \beta_0 \sqrt{(T/2)^2 - z^2}$ where z is the variable, T denotes the length of the chord, and where the origin of z is taken at the center of the wing. The magnitude of the velocity can be calculated from the length of the vector

$$\frac{dW}{dz}=iV\sin\beta_0\frac{z}{\sqrt{(T/2)^2-z^2}}$$

This is infinite near the rear edge. Let ϵ be the distance from the rear edge and accordingly let $z=1/2T+\epsilon$. Then the magnitude of the velocity becomes, for a small value of ϵ,

$$\frac{dW}{dz}=iV\sin\beta_0\frac{\sqrt{T/2}}{\sqrt{2\epsilon}}$$

Hence the factor m, mentioned before, is

$$m=iV\sin\beta_0 1/2\sqrt{T}$$

After this preparation I proceed now to the consideration of the biplane. The investigation is much simplified by a transformation of the biplane into a kind of "tandem," a method used by Kutta (ref. 5). The two straight lines of the biplane may be considered situated in the z-*plane*, the ends having the coordinates $z=\pm i\frac{G}{2}\pm\frac{T}{2}$, where G denotes the gap. The two horizontal straight lines may be transformed into two pieces of the same vertical straight line in the t-*plane*, running between the points $t=1$ and $t=k'$, and respectively $t=-1$, and $t=-k'$. The parts of the two planes at infinity are to correspond to each other without any change except for a constant factor. The expression "tandem" for the vertical pair of straight lines in the t-*plane* refers only to their mutual position, but not to their position with respect to the direction of the flow, for the tandem extends at right angles to the main velocity.

The upper wing of the tandem corresponds to the upper biplane wing and its lower wing to the lower biplane wing. However the edges do not correspond to each other. The ends of the biplane wings are transformed into points situated on the tandem wings at some distance from the end. It is not difficult to form the expression for the differential coefficient of the transformation $z=f(t)$. The transformation is performed if, following Kutta, we write

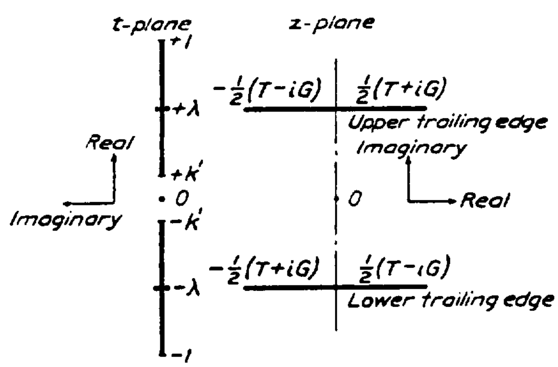

FIG. 7.—Transformation of the biplane. The biplane edges correspond to the points.

$$(1)\qquad \frac{dz}{dt}=C\frac{T}{2}\frac{t^2-\lambda^2}{\sqrt{(1-t^2)(t^2-k'^2)}}$$

The three constants λ, k', and C are to be determined so as to give the desired transformation.

If we take the integral of (1) around a closed path inclosing one of the tandem wings, z can not be increased, and hence this integral must be zero. Now it follows from the consideration of the entire flow that the integrand dz/dt has equal and opposite values on the two sides of the tandem wing, and so has the differential dt. Hence the entire integral is twice the integral between the two ends of the wing and this integral also must be zero. That means

$$(2)\qquad \int_k^1\frac{t^2-\lambda^2}{\sqrt{(1-t^2)(t^2-k'^2)}}dt=0$$

Substitute $t=\sqrt{1-k^2u^2}$, where $k=\sqrt{1-k'^2}$. Then substituting and replacing u by t the integral changes into

$$(3)\qquad \int_0^1\sqrt{\frac{1-k^2t^2}{1-t^2}}dt-\lambda^2\int_0^1\frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}}=0$$

These two definite integrals are known and their values are contained in most mathematical tables. They are called "complete elliptic integrals," complete because the limits are 0 and 1.

and they are always denoted by F (or K) and E. The number k which determines their value is called the modulus. It appears thus: $\lambda^2=\frac{E(k)}{F(k)}$. For $t=\pm\lambda$ the expression (1) changes its sign. These points therefore are the transformation of the biplane ends. Each point of the tandem in the *t-plane* corresponds to two points of the biplane. Thus $t=+\lambda$ corresponds to the front and rear ends of the upper wing; and $t=-\lambda$ to the two ends of the lower wing. k or k' can be chosen arbitrarily so as to obtain different ratios of gap/chord. C is to be determined so as to give the right scale in order that the integral of dz between $t=k'$ and $t=\lambda$ gives $T/2$, since by symmetry $t=k'$ corresponds to the middle of the chord.

$$\frac{T}{2}=C\frac{T}{2}\int_{k'}^{\lambda}\frac{\lambda^2-t^2}{\sqrt{(1-t^2)(t^2-k'^2)}}dt$$

Apply the same substitution as before,

$$\frac{1}{C}=\int_0^{\frac{\sqrt{1-\lambda^2}}{k}}\sqrt{\frac{1-k^2t^2}{1-t^2}}dt-\lambda^2\int_0^{\frac{\sqrt{1-\lambda^2}}{k}}\frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}}$$

$$\frac{1}{C}=E_{\left(k,\frac{\sqrt{1-\lambda^2}}{k}\right)}-\lambda^2F_{\left(k,\frac{\sqrt{1-\lambda^2}}{k}\right)}$$

These are no longer complete integrals but elliptic integrals for the modulus k and the argument $\frac{\sqrt{1-\lambda^2}}{k}$.

The gap G is given by the condition

$$\frac{G}{2}=C\frac{T}{2}\int_0^{k'}\frac{\lambda^2-t^2}{\sqrt{(1-t^2)(k'^2-t^2)}}dt$$

Substitute here $t=wk'$. It appears then that

$$\frac{G}{T}=C\left\{E_{(k')}-(1+\lambda^2)F_{(k')}\right\}$$

E and F are complete integrals again, but with the modulus k'.

In the case that $G=T$, $k'=0.14$ *and* $\lambda=0.55$. Each tandem wing thus has the length $1-k'$, or 0.86. The point λ is situated near the center of the wing, but not exactly, being nearer the other wing. If the gap of the biplane is increased more and more, the tandem wings become smaller and smaller, and the scale C increases accordingly. λ approaches the center of the tandem wings more and more, and at last the tandem wings are so small that they no longer influence each other, but each produces a flow like a single wing. C always gives the scale at a great distance from the wings, for at infinity dz/dt becomes $iC\ T/2$.

The transformation is thus completely given, and I proceed to the discussion of the different types of flow, as mentioned in the preceding section. The longitudinal flow is given more simply in the z-plane; the velocity is

$$\frac{dW}{dz}=V\cos\beta$$

Hence

$$\frac{dW}{dt}=V\cos\beta\frac{dz}{dt}$$

The vertical flow is easily given in the t-plane and is seen to be

$$\frac{dW}{dt}=-C\frac{T}{2}V\sin\beta$$

This expression assumes the desired value at infinity and fulfills the condition of flow near the two tandem wings, including the condition that the circulation around each of the wings be zero. For we remember that the circulation remains unchanged by a transformation. The velocity of the vertical flow in the z-plane is given by

$$\frac{dW}{dz}=\frac{-V\sin\beta C\ T/2}{dz/dt}$$

Now the relation between the velocity dW/dt at the points $t=\pm\lambda$ and the corresponding edge velocities dW/dz has to be established. For purposes of calculation, the velocity dW/dt may be taken

$$\frac{dW}{dt}=V\sin\beta\,C\frac{T}{2}$$

that is, the same as produced by the vertical flow at the angle of attack β. The transformation must be made from a point $\lambda+\zeta$ to $T/2+\epsilon$, where ζ and ϵ are small quantities, but not infinitesimal, dz/dt becomes zero at the exact ends of the wings and the second term in the expansion gives

$$\epsilon=-\frac{1}{2}\zeta^2C\frac{T}{2}\frac{2\lambda}{\sqrt{(1-\lambda^2)(\lambda^2-k'^2)}}$$

Introducing the abbreviation

$$B=\frac{1}{2}\frac{C}{\lambda}\sqrt{(1-\lambda^2)(\lambda^2-k'^2)}$$

$$\epsilon=-\frac{T/2\;C^2}{2B}\zeta^2\ i.\ e.,\ \zeta=i\frac{\sqrt{2B}\sqrt{\epsilon}}{C\sqrt{T/2}}$$

hence

$$\frac{dt}{dz}=\frac{i\sqrt{2B}}{2C\sqrt{T/2}\sqrt{\epsilon}}$$

and

$$\frac{dW}{dz}=\frac{dt}{dz}V\sin\beta\,C\frac{T}{2}=\frac{iV\sin\beta\sqrt{T/2}\sqrt{B/2}}{\sqrt{\epsilon}}$$

Therefore

$$m=iV\sin\beta\sqrt{T/2}\sqrt{B/2}$$

The comparison of this expression with the corresponding expression for the single wing at the beginning of this section shows that B must become 1 for an infinite gap. For other values of the ratio of gap/chord the value of B can be seen in Table I. It is always smaller than 1 and for very small values of gap/chord it is $1/2$.

It appears thus that the vertical flow of the same strength produces a smaller edge velocity with the biplane than with the monoplane having the same chord. This was to be expected, for each wing acts as if it produced a shadow in reference to the other wing and this stops the vertical flow. This is not so, however, with the longitudinal flow. If the edge velocity is produced by the longitudinal flow, it can not be materially influenced by the second wing. The edge velocity in this case remains unaltered, the transformed velocity in the t-plane is increased and has the magnitude

$$CV\sin\beta_0\frac{T}{2}\quad\frac{1}{\sqrt{B}}$$

From this discussion it follows that a finite velocity dW/dt at the point $t=\pm\lambda$ gives an infinite edge velocity. The condition of the vanishing edge velocity can therefore be expressed more conveniently by the prescription that the velocity dW/dt at the two points $t=\pm\lambda$ becomes zero. This velocity is the sum of the velocities of all single types of flow at this point and of the transformed edge velocity due to curvature, as just given.

The longitudinal flow does not give any velocity dW/dt at the transformed edge and the velocity of the vertical flow and section flow are already expressed. There remains only the circulation flow and the countercirculation flow. These two are to have the velocity zero at infinity and are to give two equal but opposite circulations. These conditions are fulfilled by the expressions:

$$\frac{dW}{dt}=P\frac{t}{\sqrt{(1-t^2)(t^2-k'^2)}}$$

for the circulation flow, and

$$\frac{dW}{dt}=Q\frac{1}{\sqrt{(1-t^2)(t^2-k'^2)}}$$

for the countercirculation flow, where P and Q are constants giving the intensity and are to be determined by the two-edge conditions. The circulation flow gives equal velocities at the two transformed edges, the countercirculation flow gives opposite and equal velocities. These velocities are respectively,

$$\frac{dW}{dt} = P \frac{\lambda}{\sqrt{(1-\lambda^2)(\lambda^2-k'^2)}} \text{ and } \frac{Q}{\sqrt{(1-\lambda^2)(\lambda^2-k'^2)}}$$

The determination of P and Q is easy now, for the edge velocities are equal at the upper and lower edge and so are the transformed edge velocities. Hence $Q=0$, and, to satisfy the zero conditions,

$$\frac{P\lambda}{\sqrt{(1-\lambda^2)(\lambda^2-k'^2)}} = \frac{PC}{2B} = C\frac{T}{2}V \sin\beta + \frac{C\;T/2\;V \sin\beta_0}{\sqrt{B}}$$

$$P = TV(B \sin\beta + \sqrt{B}\sin\beta_0)$$

The entire circulation is $2\pi P$, hence the entire lift is the product of the circulation, the velocity at infinity, and the density, that is

$$L = 2\pi VP\rho = 2\pi TV^2\rho(B\sin\beta + \sqrt{B}\sin\beta_0)$$

and the lift coefficient is

$$C_L = 2\pi \sin\beta\, B + 2\pi \sin\beta_0\sqrt{B}$$

B has a value somewhat less than 1. $(1-B)$, respectively $(1-\sqrt{B})$, gives the decrease of the lift when compared with that of the monoplane, whose lift coefficient is $2\pi \sin\beta$, as is well known. The former is due to β, the angle of attack; the latter, to β_0, the effect of curvature of the section. $1-\sqrt{B}$ is about $1/2(1-B)$. It can be stated therefore that:

The decrease of the lift due to the "biplane effect" is only half as great if the lift is produced by the curvature as if it is produced by the angle of attack.

The entire moment is the integral of the product of half the density, the square of velocity, the differential of the surface dz and the lever arm z, taken over both wings. The velocity is the sum of the velocities of the four types of flow; in the present case, only three types. The square of the velocity is accordingly the sum of the squares and the sum of twice the products of different velocities. In the preceding section it has been explained that the squares can not give a moment. For reasons of symmetry the product of the vertical velocity with the circulation does not give any moment either. There remains only the product of the longitudinal velocity, $V\cos\beta$, with the vertical flow. The entire moment is

$$M = \rho V\cos\beta \int \frac{dW}{dz}\,dz, \text{ over both wings}$$

$$= 4\rho V^2 \cos\beta \sin\beta\; C^2\frac{T^2}{4}\int_0^1 \frac{t(t^2-\lambda^2)}{\sqrt{(1-t^2)(t^2-k'^2)}}\,dt$$

$$= \pi\rho V^2 \cos\beta \sin\beta\; T^2C^2\frac{1-\lambda^2-k^2/2}{2}$$

as found also by Kutta in his particular case. The moment refers to the center of the biplane; that is, the intersection point of the two diagonals connecting one rear and one front edge. The position of the center of pressure on a line through this point parallel to the wings is found by dividing the moment by the component of the lift at right angle to the wings. For sections without curvature effect the distance of the center of pressure from the center of the biplane is constant and is

$$\frac{T}{4}\frac{C^2}{B}\,1-\lambda^2-\frac{k^2}{2}$$

The expression

$$x = \frac{C^2}{4B}\left(1-\lambda^2-\frac{k^2}{2}\right)$$

is calculated and contained in Table I. The distance from the center is xT. The distance from the leading point therefore is

$$\left(\frac{1}{2}-x\right)T$$

The factor x differs only slightly from $1/4$, hence $(1/2-x)T=T/4$, this being the same value as for a "monoplane without curvature." For gap = chord, the difference, $x-0.25$ is about .08, the center of pressure being nearer to the front. What has been said above applies to sections without curvature effect, as stated. For other sections the moment remains the same, but not so the lift, and hence a travel of the center of pressure takes place. The center of pressure of the entire lift is then

$$CP=\left(x-\frac{\sin\beta_0 2\pi x\sqrt{B}}{C_L}\right)T$$

For a single wing $x=1/4$ and $B=1$. For the practical range, the product $x\sqrt{B}$ is almost $1/4$ too. Hence the second term in the formula, which is the one giving rise to travel of the center of pressure, is almost equal to the corresponding term for a monoplane of the same section, indicating that there is a corresponding change in the lift. The change of the lift which gives rise to the travel is smaller, but the arm is increased; and so the total effect is almost neutralized. The position of the center of pressure is moved slightly to the front and the travel is almost the same as with the monoplane of the same section.

We remember that all results obtained in this section refer only to the two-dimensional problem. The influence of the lateral dimensions has still to be considered. It may be mentioned, however, that the fact of the travel of the center of pressure of both monoplane and biplane being the same does not mean that there is no difference between them with respect to the travel of the center of pressure. The biplane is superior, chiefly, of course, because the chord is only about half as great as the chord of the monoplane having the same wing section, and hence the absolute travel is only half as much too. But this is not all. The travel is equal only with reference to the change of the lift coefficient; it is smaller for the biplane with reference to the change of the angle of attack, and this is the determining factor for the calculation of the dimensions of the tail plane.

There remains finally the determination of the secondary repulsion between the two wings produced both by the circulation flow and by the vertical flow. For the circulation flow,

$$\frac{dW}{dt}=\frac{Pt}{\sqrt{(1-t^2)(t^2-k'^2)}}$$

$$\frac{dW}{dz}=\frac{Pt}{t^2-\lambda^2}\,\frac{1}{C\,T/2}$$

Repulsive force

$$=\frac{\rho}{2}\int\left(\frac{dW}{dz}\right)^2 dz$$

$$=\frac{\rho P^2}{C\,T/2}\int_1^{k_1}\frac{t^2}{(t^2-\lambda^2)}\,\frac{dt}{\sqrt{(1-t^2)(t^2-k'^2)}}$$

The same substitution as used before, $t^2=1-k^2w^2$, transforms the integral into

$$\int_0^1\frac{dw}{\sqrt{(1-w^2)(1-k^2w^2)}}+\frac{\lambda^2}{k^2}\int_0^1\frac{dw}{\left(\frac{1-\lambda^2}{k^2}-w^2\right)\sqrt{(1-w^2)(1-k^2w^2)}}$$

The first integral gives $F_{(k)}$ simply and the second one can be reduced to $-\frac{k^2}{2\lambda^2 B}F_{(k)}$. Hence the repulsive force is

$$\frac{\rho P^2}{C\,T/2}\,F_{(k)}\left(1-\frac{1}{2B}\right)$$

But

$$P=\frac{L}{2\pi\rho V}$$

and hence the repulsive force due to the lift is

$$\frac{L^2}{4\pi^2 \rho V^2 T/2} \frac{F_{(k)}(B-1/2)}{BC}$$

The repulsion due to the vertical flow is calculated by the same method.

$$\frac{dW}{dt} = -V \sin\beta \frac{T}{2} C$$

$$R = \frac{\rho}{2}\int\left(\frac{dW}{dz}\right)^2 dz = V^2 \sin^2\beta \frac{T}{2} \rho C \int_1^{k_1} \frac{\sqrt{(1-t^2)(t^2-k'^2)}}{t^2-\lambda^2} dt$$

$$= V^2 \sin^2\beta \frac{T}{2} \rho\, C\, F_{(k)} \left\{2\lambda^2 + k^2 - 2 + \frac{(1-\lambda^2)(\lambda^2+k^2-1)}{2B\lambda^2}\right\}$$

The second factor may be abbreviated again and denoted by v,

$$\text{i. e., } R = V^2 \frac{\rho}{2} \sin^2\beta\ Tv$$

v is contained in Table I.

It appears that the repulsion is proportional to the square of the lift, respectively to the square of the angle of attack; it is small, therefore, for small lift or angle and the ratio of the repulsive force to the lift is not constant. The entire repulsive force is the sum of the force due to the lift and that due to the angle of attack. For sections without curvature effect the two parts are proportional and can be expressed in terms of the angle of attack.

$$R = \pi \frac{\rho}{2} V^2 T \sin^2\beta \frac{F_{(k)}}{\pi}\left\{\frac{C(1-\lambda^2)(\lambda^2-k'^2)}{2B\lambda^2} + C(2\lambda^2+k^2-2) + \frac{4B(B-1/2)}{C}\right\}$$

Table I shows that the part due to the angle is much smaller than the part due to the lift. The lift produced by the curvature is accompanied by the one repulsive force only and therefore such biplanes have smaller repulsive forces and the upper and lower lifts are more equal, but the difference caused by considering curvature is very small.

4. BIPLANES WITH DIFFERENT WING SECTIONS, DIFFERENT CHORDS AND DECALAGE.

The method just employed can be used too for the investigation of varied arrangements.

The wing section of the upper and lower wing may be different, but the respective angles of attack for which the moments around the centers vanish may be taken by the two wings at the same time. It is assumed that the chords are still equal and the biplane unstaggered.

The two edge velocities are now different. It can easily be seen that the circulation flow and hence the entire lift is determined now by their arithmetic mean in the same way as before. Instead of β_0, the expression, $\frac{\beta_{01}+\beta_{02}}{2}$ enters in the equation for the entire lift. Besides, a countercirculation flow is now created by the difference of the two edge velocities from the mean value. This difference is in the *t-plane*,

$$C \frac{T}{2} V \frac{\sin\beta_{01} - \sin\beta_{02}}{2} \frac{1}{\sqrt{B}}$$

and must be neutralized by the velocity of the countercirculation flow

$$\frac{Q}{\sqrt{(1-\lambda^2)(\lambda^2-k'^2)}}$$

Hence

$$Q = C \frac{T}{2} V \frac{\sin\beta_{01} - \sin\beta_{02}}{2} \sqrt{\frac{(1-\lambda^2)(\lambda^2-k'^2)}{B}} = 2\lambda\sqrt{B} \frac{T}{2} V \frac{\sin\beta_{01} - \sin\beta_{02}}{2}$$

The lift of the wing with greater curvature is increased by the additional lift

$$2\rho V \cos\beta\ F_{(k)} Q = 4 \frac{\rho}{2} V^2 T \frac{\sin\beta_{01} - \sin\beta_{02}}{2} \lambda\sqrt{B} F_{(k)}$$

The other lift is decreased by the same amount. It is interesting, though not very important, that the upper and lower primary lifts have not the same ratio as if the two wings are isolated. The factor $\lambda\sqrt{B}F_{(k)}$ is somewhat greater than $\pi/2$ for the usual gap/chord ratio. The difference is not great, however.

The entire moment remains approximately unchanged, and for the calculation of the center of pressure the mean of the effective curvature may be taken. The difference comes in by the combined effect of countercirculation flow and the component of motion at right angles to the wings, so that the height of the center of pressure of the curvature lift is slightly changed. Besides the countercirculation flow produces an attraction which diminishes the differences of the upper and lower lift. The case of different section is not common, however, the differences of the effective curvature is small in these cases and hence the attraction which contains the square of the difference is very small. It is hardly worth while to discuss the magnitude of this.

The biplane with different lengths of the two chords can be treated according to the first development, by starting with a transformed tandem with different chords, so that the ends are k_1 and $-k_2$ and in the denominator two different λ's enter. The integrals occurring are somewhat complicated, although their solution can be performed systematically by well-known methods. But these are rather laborious. It does not seem proper to discuss them in this more general treatise, so much the more as the results are not expected to be very interesting for the following reason.

In the case of small differences of the two chords the effect can be discussed without any calculation. For the biplane behaves symmetrically whether the upper or lower wing has the smaller chord, and therefore all quantities referring to the entire biplane have a maximum or minimum for equal chords. Hence a small difference can not have a noticeable effect. From this follows that the entire lift and moment of a biplane with almost equal wings, without stagger and decalage, is equal to the biplane with two equal wings, which have the mean chord of the upper and lower wing. The lift of each individual wing was not equal before and the change of the primary lift is not proportional to the difference of size. It is to be expected, however, that this is at least approximately the case, and the question is not worth the while of a laborious investigation.

If the wings are very different, the arrangement approaches to a monoplane, and an ordinary interpolation seems to be justified and is likely to be exact enough for practical use. It must be remembered that the difference between the air forces of the monoplane and the biplane is not very great, anyhow, for the usual gap/chord ratio.

I proceed now to the biplane with equal, unstaggered wings, but with decalage. By decalage is meant the difference in the angles of attack of the two wings for which their individual moments around their centers are zero. Decalage is called positive if the angle for the lower wing is the greater. In the neutral position the angle of attack of the upper wing may be $-\delta$ and that of the lower wing δ. It is not possible to find a simple transformation in analogy to the former one, which transforms the tandem into two straight lines inclined toward each other. It is necessary to use a more elementary method for the calculation of the decalage effect, which, however, is likely to give as good results. It may be stated at once that the same consideration with respect to the entire lift and moment is valid as before. At small decalage, and a small decalage only is considered, the entire lift and entire moment remain practically unaltered. The lift of each individual wing however is changed considerably and in an interesting way, and it is well worth while to consider the reason of this phenomenon and to find a formula for it.

The solution of the problem of the biplane with decalage requires the knowledge of the flow around it in the neutral position. At first, the theoretical flow without circulation or countercirculation will be deduced. The edge velocity of this flow could be determined approximately by linear interpolation, if it were known for two positions of the upper wing while the lower wing retains its angle of attack δ. Now the edge velocity is known for parallel wings from the previous investigation, that is, for the angle of attack δ of the upper wing. As a second position, I try to find the particular position of the upper wing where it does not experience any influence at all from the lower wing, which continues to have the angle of attack δ. The influence does not vanish at the angle of attack zero of the upper wing. For the flow produced by the lower wing alone is almost straight in the space above and below the wing, but it is not parallel to the flow at infinity. Near the lower wing it is nearly parallel to it and hence has the angle δ. At some distance it gradually approaches zero. The disturbing velocity is given by the expression

$$\frac{dW}{dz}=\left(1-\frac{2z}{\sqrt{T^2-4z^2}}\right)V \sin \delta$$

At points above and below the center of the wing, z is purely imaginary and may be written iy. The angle of the flow at this point is

$$\tan \delta\left(1-\frac{2y}{\sqrt{T^2+4y^2}}\right)$$

Now this direction of the flow can be taken approximately as the direction of the wing in question. The bracket in the last expression may be denoted by d. The value of d is given in Table I, for different ratios gap/chord. The flow around the wing is parallel to the wing in its immediate neighborhood. At some distance it gradually assumes the direction of the undisturbed flow. Therefore, the second wing, when in the undisturbed position, has an angle of attack of the same sign as the other wing, but a smaller one. From Table I it can be seen that for equal chord and gap the angle of attack is only 1/10 of the other.

For parallel wings the edge velocity has the factor $m = V \sin \delta \sqrt{B/2}\sqrt{T/2}$. For the angle $d\,\delta$, there is no change in the edge velocity. For the angle of attack $-\delta$ the edge velocity therefore has the factor $m = -V\sqrt{T/2}\sqrt{B/2}\,\delta(1+2d)$. The sines of the angles are replaced by the angles themselves in this expression. The expression $\sqrt{B}\ (1+2d)$ is given in Table I also.

It is assumed that the decalage is small only and that therefore the former method can be applied for the remaining calculation. The entire lift remains unaltered, if the mean of the two angles of attack is considered as angle of attack. The entire moment is almost unaltered too. There is only a small contribution produced by the combined effect of vertical flow and countercirculation flow. This is

$$M=4\frac{\rho}{2}V^2T \sin \delta\ (1-2d)\ \sqrt{B}\lambda F_{(k)} \sin \beta$$

which is hardly considerable and is only mentioned for reason of completeness. The wing of greater angle of attack is turned forward by this moment. The additional primary opposite lift at each individual wing is $4\frac{\rho}{2}V^2\ T \sin \delta\ (1+2d)\sqrt{B}\lambda F_{(k)}$ and positive of course at the wing with the greater angle of attack.

In the neutral position the wing experiences the lift due to curvature, and the counterlift due to decalage as primary lift. The individual moments are opposite. Both additional influences tend to produce an attraction between the two wings and do actually produce one, if the curvature is small or the decalage great. For greater angle of attack the secondary force between the two wings changes its sign. The effect of this phenomenon is particularly conspicuous, if the lower wing has positive decalage. For then the lower lift is not only increased by the constant counterlift, but in the neutral position also by the attraction between the two wings. At greater angles, however, it is decreased by the repulsion and, therefore, it appears that the lift curve of the lower wing plotted against the angle of attack has an unusually low slope.

5. STAGGERED BIPLANES.

The calculation of the two-dimensional flow around staggered biplanes with equal wing chords is somewhat more complicated than the case without stagger. The same consideration with respect to symmetry is valid for staggered biplanes with small stagger as for the other variations. The influence of the small stagger on the entire lift and moment is given by an expression which does not contain the first power of the stagger, and therefore the lift and moment are almost constant at first. The difference of lift could be calculated to the first approximation alone. This approximation, however, is not likely to be a good one for somewhat greater stagger, nor is then the influence of these terms negligible which contain the powers

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of the stagger. The problem is one so important that it is worth while to perform the calculation in full for a series of different staggers and gaps.

In the following development two arbitrary constants occur for each of the two different ratios gap/chord and stagger/chord. Unfortunately the two ratios are functions of both the arbitrary constants, and it is not easy, therefore, to change only one of the two ratios.

The method of calculation is quite analogous to the previous one. First, a transformation is established, which transforms the same tandem in the *t-plane* as before into the staggered biplane in the *z-plane*. This transformation is

$$\frac{dz}{dt}=\frac{T}{2}C\left(\frac{t^2-\lambda^2}{\sqrt{(1-t^2)(t^2-k'^2)}}+a\right)$$

k or k' and a are arbitrarily chosen and λ has the same value as before. This follows from the condition that the line integral of dz around the tandem wing must be zero. $t=\lambda$ however, is no longer the transformation of the edge of the biplane wing. The corresponding points $t=\mu_1$, $t=\mu_2$ are found by the condition $dz/dt=0$. The length of the chords in the *z-plane* is $\int_{\mu_1}^{\mu_2} dz$ and by means of this integral the value of C is found. μ_1 and μ_2 are situated at different sides of the tandem wings. The integral gives

$$\Big|_1^2 E_{\left(k,\frac{\sqrt{1-\mu_1^2}}{k}\right)}-\lambda^2 F_{\left(k,\frac{\sqrt{1-\mu_1^2}}{k}\right)}+a\mu\Big|=\frac{2}{C}$$

C is twice the inverse value of this expression. The stagger is simply $T\,C\,a(\mu_1-\mu_2)$.

Now the different types of flow have to be considered. The vertical velocity is transformed into

$$\frac{dW}{dt}=i\;V\sin\beta\,\frac{T}{2}\,C\left(1+\frac{a(t^2-\lambda^2)}{\sqrt{(1-t^2)\,(t^2-k'^2)}}\right)$$

For infinity this expression assumes the value

$$V\sin\beta\,\frac{T}{2}\,C\;(1+ai)$$

and at the boundaries of the tandem the velocity is parallel to the boundaries. The substitution of μ_1 and μ_2 gives the transformed edge velocities due to the angle of attack $(1+a^2)$ as great as before. The transformed edge velocity, due to curvature, is again $T/2\;V\sin\beta_0$ multiplied by the factor of the second term, which gives the transformation of the two planes at $t=\mu$.

The circulation flow and countercirculation flow in the *t-plane* are the same as before. Their velocity at the transformed edges are obtained by substituting $t=\mu_1$, $t=\mu_2$.

All these velocities are different now in general at the upper and lower wing and P and Q have to be determined so as to make their sum vanish. This gives two linear equations for P and Q.

P and Q can be determined separately for the angle of attack and the curvature, and can be added afterwards. P and Q being known, the calculation is almost finished. P and P_0 give direct the factor of the lift, corresponding to B and B_0 in the previous development by dividing it by $T\;V\sin\beta$. Q has to be separated in the same way from $T\;V\sin\beta$ but then it does not yet give the counterlift. For the period of

$$\int\frac{dt}{\sqrt{(1-t^2)\;(t^2-k'^2)}}$$

is $4F$ and not 2π, therefore the value obtained has to be multiplied by $2F/\pi$.

One part of the moment is to be calculated in the same way as before; that is, the part created by the combination of the longitudinal and vertical flow. It results $(1+a^2)$ times the same value as before.

The moment with respect to the center of each individual wing due to the circulation has an opposite sign. The countercirculation, however, gives a moment. This can most conveniently be calculated by considering the change of the moment when compared with the biplane without stagger, for which this part of the moment was zero. The moment is expressed by

$$\int \frac{Q\ at\ L/2\ C}{\sqrt{(1-t^2)\ (t^2-k'^2)}}\, dt = Qa\, \frac{L}{2}\, C\, 2\, \pi$$

since the integral is taken around the two tandem wings. Besides, there is a small additional moment around the two tandem wings due to the countercirculation forces in the direction of the chord. This moment is

$$VQ \times 4F \times gap \times sin\ \beta$$

and has the effect that the height of the center of pressure is changed.

Besides the counterlift from the countercirculation flow, there are secondary repulsive forces which contain the squares of the angle of attack as before and which are small therefore for small angles. Their calculation is laborious and the result hardly interesting. This repulsive force is somewhat smaller than for the biplane without stagger, partly due to the increased distance and partly due to the difference of the upper and lower primary lift and changes in the flow. For small stagger the factor of course approaches *1*, and the difference is not great in practical cases.

This method is employed for the computation of the aerodynamical constants of 10 different staggered biplanes, and the results are given in Table II. It was necessary to perform the laborious calculation work with a slide rule, and as a consequence the results are not very exact. They are exact enough, however, for practical application, and this only is the standard of exactness in the present paper.

It appears, as was expected from consideration of symmetry, that the two kinds of lift remain almost unaltered at a small stagger. The change can be expressed, as a first approximation, as proportional to the square of the stagger. This holds true also for the quantities determining the entire moment and the travel of the center of pressure. The approximation is exact enough up to a stagger *1/3* of the chord, within the usual range of the ratio gap/chord and may even be employed up to $G/T = 1/2$ in order to obtain the range of magnitude. For very great stagger, equal to a multiple of the chord, the law is quite different of course, but such an arrangement is no longer a biplane but rather a tandem. It appears that with increasing stagger the lift produced by the angle of attack is increased and the lift produced by the curvature is diminished. At high lift, at which the coefficients are chiefly needed, both parts are positive. Under these circumstances the changes neutralize each other partly and the lift is even more independent of the stagger.

The change of primary upper and lower lift of each individual wing is directly proportional to the stagger, as long as the stagger is small. The front wing has a greater primary lift. For gap/chord *1* and stagger/chord *1/2* the difference of upper and lower primary lift is about 10 per cent of the entire lift. The difference of the primary lifts is a linear function of the entire lift, but by no means proportional to it. Hence the ratio of the difference to the entire lift is not constant, but even changes sign. The usual arrangement has a greater lift for the rear wing at small angles of attack and a greater lift for the front wing at greater angles of attack only.

The two centers of pressure move apart with increasing gap, at first only proportional to the square. Moreover, the ratio of the lift produced by the angle of attack to the lift due to curvature increases. The consequence is a greater travel of the center of pressure. For $G/T = 1$ and stagger/chord $= 1/2$ the two coefficients B and B_0 are almost equal and the distance of the two poles or centers of pressure of the two parts of the lift has increased by 10 per cent. Relative to the lift coefficient, the travel of the center of pressure is 10 per cent greater therefore when compared with the monoplane of the same section.

The method demonstrated could be employed for many other problems. The previous computations are sufficient for the present purpose. The benefit of the new method of calculation not only consists in the useful numerical results. The method shows also how two aerofoils situated near each other produce a common flow, the effect being that of one aerofoil, particularly if they move nearer and nearer together.

THE INFLUENCE OF THE LATERAL DIMENSIONS.

5. THE AERODYNAMICAL INDUCTION.

I proceed now to the discussion of the air forces with a biplane cellule as influenced by its lateral dimensions. The fact that the span of the wings is finite is not compatible with the conception of a two-dimensional flow. The variation of the flow in the lateral direction is particularly marked at the two ends. Near the middle the flow resembles the two-dimensional flow in so far as the lateral variations are small. But there are still important differences between this pseudo two-dimensional flow in the middle of the biplane cellule and the real two-dimensional flow; even in the middle, these two by no means agree.

The difference comes in owing to the fact that the flow behind the wing is not actually a real potential flow, for there is an unsteady layer which separates the air which has passed over the wing from the air which passed under it. At the rear edge, where the two airstreams flow together, they possess different lateral components of velocity and hence are unable to unite to a potential flow free from unsteadiness. The effect can be taken into account by assuming the direction of the airflow to be changed and turned by a certain angle. To be sure, the air near the wings flows parallel to the boundary whether the flow be two-dimensional or not. But the distribution of the velocity and the resulting pressure is changed as if the incident air originally had an additional downward component at right angle to the direction of flight. This imagined downwash can be calculated and is generally different from point to point. I have proved in a former paper (ref. 1) that under some admissible simplifying assumptions the entire resulting induced drag does not depend on the longitudinal coordinates of the points where the lift is produced. Only the front view is to be considered.

I have also given there the conditions under which the induced drag has its minimum value. These conditions are never exactly fulfilled, but the real induced drag will not be very much greater than the minimum value. Besides, it is interesting to know this smallest value possible, in order to have an idea as to whether or not an improvement is possible and promising. The induced drag can be conveniently calculated by means of the formula $D=\frac{L^2}{k^2b^2q\pi}$ where b is the greatest span of the biplane and $k\cdot b$ the span of the equivalent monoplane having the same induced drag under the same conditions. q denotes the dynamical pressure. The factor k depends on the front view of the biplane and not on the stagger. Its value for different gap/span ratio is given in Table III. For very small gap it assumes the value $k=1$, for very great gap it would finally become 1.41. It is chiefly a question of experience to decide how close the distribution of the lift comes to the most favorable one, so that the minimum induced drag expresses the real induced drag. This question is discussed in the last part of this paper. One remark concerning the distribution of lift, however, properly finds its place at this point. The investigation in the first part makes it possible to describe the most favorable distribution more exactly than is done in the original treatise. There the assumption was that the lift was small, and it was mentioned that for greater lift the description could be improved. That is simple now, for all deductions were drawn from the assumption that the lift at each point is proportional to the intensity of the transversal vortices at that point. But it is not the entire lift that is proportional, but only that part of the lift which I have called "primary lift." Only the primary lift is subject to the conditions for the minimum induced drag stated in the paper mentioned. The secondary lift, being a component of the mutual forces between parts of the whole arrangement—for instance, a repulsion between the two wings increasing the upper lift and decreasing the lower lift—must be omitted. This last makes no difference in the entire lift, for the sum of all secondary lifts is zero.

This is not without interest in the consideration of the most "regular" biplane, with two parallel and equal wings without stagger. It appeared that in the two-dimensional flow the upper and lower primary lift are equal, but not so the sum of primary and secondary lift. The condition of minimum drag for this biplane calls for equal induced downwash over both wings and, from reasons of symmetry, it follows that this is the case only if the lift which produces the downwash is equal, too, at both wings. But that is only the primary lift, and therefore the biplane in question fulfills the conditions as far as the entire upper and entire lower lift is concerned, although the two lifts including the secondary lifts are not equal.

The induced drag appears as a consequence of the total air force being no longer at right angles to the direction of motion but at right angles to the negative velocity of flight with induced downwash superposed on it. The entire surrounding and passing air appears to be turned, and with it the air force is turned and has now a component in the direction of flight. Hence the angle of turning, being small, has the magnitude $\frac{\text{induced drag}}{\text{lift}}$; that is, $\frac{L}{k^2 b^2 q \pi}$. But now the position of the section with respect to the incident airflow and hence the angle of attack has changed. It appears to be decreased by the same induced angle, and in order to create the same lift as in the case of the two-dismensional flow, the original angle of attack has to be increased by this induced angle. Considering the wing turned by this additional induced angle, the airflow around it is almost the same as in the two-dimensional case, and the distribution of pressure is the same, too. Therefore the moment and the center of pressure remain practically the same for the same lift coefficient, though not for the same angle of attack. For this reason and because all formulas become much more simple, it is recommended always to consider the lift coefficient instead of the angle of attack as the independent variable and to start with it. This is easier, too, because the lift coefficient can more easily be found for a certain condition of flight and a certain project than the angle of attack.

For an unstaggered biplane with equal and unstaggered wings, the induction at the upper and lower wing is almost equal, and therefore the change of the upper and lower lift is equal too. No additional difference of lift is induced. For a biplane with decalage or with different chords this is not exactly the case, but the differences are very small and it is not necessary to consider them. The staggered biplane, however, deserves a discussion at fuller length.

The staggered biplane in general has different upper and lower primary lift, and the ratio is variable in most cases for different angles of attack also. The distribution of lift is no longer the most favorable one, but in consequence of the induced drag the lift of the front wing is somewhat increased. This increase now, not very great anyhow, seems to be neutralized for the ordinary biplane with positive stagger (upper wing in front). The reason is the following: In Part I of this paper, dealing with the two-dimensional flow, the stagger had to be counted with respect to the direction of the wing chord. For the flow was resolved in components determined by this direction. But not so in the present case. Now, the stagger is no longer determined by the dimensions of the biplane only and is not constant, therefore, for all conditions of flight, but it is determined by the direction of flight, though not exactly parallel to it, and is therefore variable for different conditions of flight. So is the gap, which is to be measured at right angles to the stagger. For the effects of the aerodynamical induction are determined by the position of the layer of unsteadiness of the potential flow behind the wings, and the direction of this layer nearly coincides with the direction of flight. Hence, if the stagger and angle of attack are positive, the effective gap is increased, and in consequence the induced drag is decreased. This may neutralize the unfavorable effect of the differences of upper and lower primary lift. This is very convenient for practical applications, for it makes it possible to use the same coefficient k for both staggered and nonstaggered biplanes, as far as the induced drag is concerned.

A similar simplification for the angle of attack is possible in an important series of cases. It can be proved that the entire lift is only slightly changed by the effect of the aerodynamical induction if the coefficient of the primary lift was equal originally for all the individual wings. This includes the important case that the wings are parallel.

I have shown previously (ref. 1) that the entire induced drag remains constant if the lift remains constant on each longitudinal line. It does not, if the wing is moved longitudinally, for under ordinary conditions the downwash behind the front wing of the staggered pair diminishes the lift of the rear wing, and at the same time the lift of the front wings is increased in consequence of the diminished front down wash. Imagine, first, the two angles of attack α changed in such a way by the angles $\Delta\alpha_1$ and $\Delta\alpha_2$ that the lift of each individual wing is the same as before. $\Delta\alpha_1$ and $\Delta\alpha_2$ are the differences of the two induced directions of air before and after the change. It is known that the entire induced drag is the same as before; this gives the equation

$$\Delta\alpha_1\, L_1 + \Delta\alpha_2\, L_2 = 0.$$

If, now, the lift coefficients of the two wings are equal, the two sides of this equation can be divided by this lift coefficient, and it appears that the sum of the wing areas each multiplied by its change of downwash is zero too. If, now, the two wings are turned back into their original positions, the change of the entire lift takes place only so far as the induced drag is increased as a consequence of the less favorable new distribution of lift. But this is very little, if it was the minimum before, and hence the approximate constancy of the lift is demonstrated.

Drag and total lift remain almost constant. There is, however, the change of the effective gap already mentioned. The effective gap coincides with G only when $\beta = 0$, otherwise it is approximately $G\,(1 + \beta s/G)$. The effective gap is increased at positive stagger and angle of attack. The substitution of the usual dimensions shows that the influence amounts to from 1 to 2 per cent. By this much the lift may be increased at unusually great positive stagger. The interference effect of the two-dimensional flow was chiefly an increase of the lift within the same limits for either positive and negative stagger. The two influences have equal signs chiefly at positive stagger and opposite signs at negative stagger. The influence of the stagger is to be expected to be particularly small at negative stagger; at positive stagger, from this consideration, slight increase of the lift appears.

The moment and the difference of upper and lower lift is changed, however, by the aerodynamical induction to a considerable degree. It follows from the previous discussion that the effective angle of attack of the front wing is increased and that of the rear wing decreased by the same amount, and it remains to determine this quantity. The change of induced downwash takes place, of course, only with that part of the induced downwash which is produced by the second wing. If the wings are parallel and not staggered, the self-induced downwash can be assumed to be equal to the downwash of the corresponding monoplane—that is, $\frac{L}{2\pi b^2 q}$ where L denotes the entire lift of the two wings. The entire induced downwash of each wing is

$$\frac{L}{2\,k^2 b^2 \pi q}$$

There remains therefore

$$\frac{L}{b^2 \pi q}\left(\frac{1}{k^2} - \frac{1}{2}\right)$$

as downwash of each wing induced by the other wing.

This part of the induced drag can be considered as the effect of all the longitudinal vortices of the other wing, forming the layer where the flow is unsteady. In the plane at right angles at their ends, the downwash is exactly half of what it would be if the vortices were to extend infinitely in both directions. The change of downwash per unit of change of the longitudinal coordinate depends on the average distance of the investigated point from the longitudinal vortices. It will be sufficient to consider only the middle of one wing and to calculate how great the change is there. The differential of change produced by one infinitesimal longitudinal vortex is $1/R$ of its induced downwash, where R denotes the distance $\sqrt{x^2+G^2}$ between the middle of one wing and the origin of the longitudinal vortex in question.

G denotes the gap and x the lateral coordinate. The intensity of the vorticity can be taken according to an elliptical distribution of lift over each wing; that is, proportional to $x/\sqrt{1-x^2}$ for the span $b=2$. The downwash is then proportional to

$$const. \int_0^1 \frac{x}{\sqrt{1-x^2}} \frac{dx}{(x^2+G^2)}$$

The change is proportional to

$$\int_0^1 \frac{x^2}{\sqrt{(1-x^2)}} \frac{dx}{(x^2+G^2)^3}$$

and it follows that the average distance R must be taken as

$$R=\frac{b}{2}\frac{\int_0^1 \frac{x^2 dx}{\sqrt{(1-x^2)}\ (x^2+G^2)^2}}{\int_0^1 \frac{x^2 dx}{\sqrt{(1-x^2)}\ (x^2+G^2)^3}}$$

The upper integral is

$$\frac{\pi}{2}\left[1+\frac{G}{\sqrt{1-G^2}}\right]$$

The lower integral is $\frac{F\ (p)-E\ (p)}{\sqrt{1+G^2}}$ where $F\ (p)$ and $E\ (p)$ are the complete elliptic normal integrals for the modulus

$$p=\frac{1}{\sqrt{1+G^2}}$$

Hence

$$\frac{R}{b}=\frac{\pi}{4}\frac{\sqrt{1+G^2}-G}{F\ (p)-E\ (p)}$$

A staggered biplane of infinite span may have a lift coefficient $C_L=\frac{L}{qS}$ and a moment coefficient $C_{m_1}=moment/qST$. Hence the position of the center of pressure, $CP=T\ C_m/C_L$. S is the entire area, i. e., the sum of the areas of the upper and lower wings.

In order to deduce the moment coefficient and the CP for the same biplane, but with finite span b, define

the new moment coefficient $C_{m_2}=C_{m_1}+C_m{}'$
the new center of pressure $CP_2=CP_1+CP'$

The aerodynamical induction is equivalent to changing the effective angles of attack by equal and opposite amounts β', where

$$\beta'=\frac{C_L}{\pi}\frac{S}{b^2}\left(\frac{1}{k^2}-0.5\right)\frac{s}{R}$$

in which s denotes the stagger and R is explained above. Hence the individual upper and lower lift coefficients are changed by equal and opposite amounts $\pm 2\pi\beta'$, so that the total lift coefficient remains unchanged. The corresponding changes in the two lifts are $\pm 2\pi\beta'\ S/2\ q$; so that these two produce a moment, their distance apart being s. Therefore the additional moment is $2\pi\beta'\ S/2\ qs$, corresponding to the additional moment coefficient

$$C_m{}'=\frac{2\pi\beta'\ S/2\ qs}{SqT}=\frac{\pi\beta's}{T}$$

This additional moment coefficient divided by the total lift coefficient and multiplied by the chord T gives the change of the CP

$$CP'=\frac{C_m{}'}{C_L}T=\frac{\pi\beta's}{C_L}=\frac{s^2}{RT}\frac{s}{b^2}\left(\frac{1}{k^2}-0.5\right)T$$

which is constant.

The change of lift is produced by changes of the effective angle of attack; therefore the center of pressure which is moved is the center of pressure belonging to the lift due to the angle of attack. The other pole keeps its original position. An increase of the travel of center of pressure is the consequence, for the distance apart of the two poles is increased. The expression for the arm contains the square of the stagger as long as the stagger is small.

The induced difference of upper and lower lift depends on the stagger and is zero for unstaggered biplanes. It contains the angle of attack or the lift to the first power and the stagger directly. It may be called "primary" in analogy to the nomenclature of Part I, for there is still a secondary term of induced difference of upper and lower lift worth mentioning. This term comes in by the change of the effective stagger and therefore is always to be considered whether the biplane is staggered or not. The effective stagger of an unstaggered biplane is proportional to the effective angle of attack, for it results from the angle between the direction of the wings and the surrounding flow. The effect is proportional to the stagger and to the lift or angle of attack. Hence the square of the angle or of the lift occurs in the expression for the secondary induced difference of upper and lower lift, and the denomination "secondary" is fully justified. This secondary difference of lift has the opposite sign from the secondary lift resulting from the two-dimensional flow. For with increasing angle the upper wing moves backward and its lift decreases. Therefore the two secondary lifts have the opposite sign.

The effective stagger is

$$\frac{G C_L}{2\pi B}$$

The change of each induced angle of attack is

$$\beta_1 = \frac{C_L^2}{2\pi^2 B} \frac{S}{b^2} \left(\frac{1}{k^2} - 0.5\right) \frac{G}{R}$$

and hence the change of the induced upper and lower lift coefficient is

$$\frac{C_L^2}{\pi B} \frac{S}{b^2} \left(\frac{1}{k^2} - 0.5\right) \frac{G}{R}$$

The coefficient B is taken, assuming the lift to be produced by the angle of attack. Otherwise a coefficient between B and B_0 enters into the equation

THE DETERMINATION OF THE WING FORCES FOR PRACTICAL USE.

7. THE AERODYNAMICAL COEFFICIENTS.

The results of the theoretical investigation of the first two parts of this paper, together with experience from tests, make it possible to give simple rules for the determination of the wing forces. The application of these formulas is made more convenient by tables forming the fourth part of this paper and containing the results of the calculation to such an extent that there remains only some multiplication and addition work. The whole proceeding is restricted to the useful range of the angle of attack. The knowledge of the lift, drag, angle of attack, and center of pressure is important for the determination of the performance and stability of the airplane. These quantities can now be determined as exactly as other technical quantities and more easily and quickly than most of them.

As in other departments of technics, it is useful in aeronautics to use absolute coefficients in order to express the different quantities. The most important coefficient is the lift coefficient. It is derived from the load per unit of wing area and is formed by dividing this unit load by the dynamical pressure, as indicated by the Pitot tube. This dynamical pressure can be taken from Table VII for any velocity and altitude. Nor is it difficult to calculate it according to the equation

$$q = V^2\rho/2$$

where q denotes the dynamical pressure

V the velocity and

ρ the density of the air; that is, its specific weight divided by the acceleration of gravity g. The density decreases with the altitude and depends on the weather, so that Table VII gives only average values. At sea level, it can be assumed that

$$q\,\frac{lbs.}{sq.\,ft.} = \frac{1}{850}\left(V\,\frac{ft.}{sec.}\right)^2 = \frac{1}{390}\left(V\,\frac{mi.}{hr.}\right)^2.$$

With the use of Table VII, the lift coefficient can be quickly found for any altitude and velocity by dividing the load per unit of wing area by the values of this table

$$C_L = \frac{W/S}{q}.$$

There is some uncertainty as to what is to be considered as the entire wing area. The question is whether the tail plane and the space of the wing filled by the fuselage is to be considered as additional wing area. This is not quite a matter of definition, for the decision affects the value of the different coefficients. These coefficients are chiefly determined from wind tunnel tests with models without tail planes and the space for the fuselage filled. It seems the best definition therefore to add the space for the body and to omit the tail plane. The difference is not very great on the whole and for most practical calculations the designer may take that load per unit of wing area he is accustomed to use.

The drag coefficient is defined in the same way as the lift coefficient; that is, the drag per unit of wing area is divided by the dynamical pressure q. In the first place this refers to the entire drag of the airplane. But it is usual to divide the drag into several parts and it makes no difference whether the drag coefficient is divided into parts or the drag itself is divided and the coefficients of the parts formed afterwards.

This also holds true for the horsepower, corresponding to the different parts of the drag. The necessary horsepower is the product of drag times velocity, and an old formula can be obtained by expressing the velocity from the equation of the lift coefficient and substituting it in the expression for the horsepower. It appears then

$$550\,\frac{HP}{W}=\frac{C_D}{C_L^{3/2}\sqrt{\rho/2}}\sqrt{\frac{W}{S}}.$$

These are net horsepower per unit of weight; the engine has to deliver more horsepower according to the efficiency of the propeller. A small table for $C_L^{3/2}\sqrt{\rho/2}$ is given as Table IV, where the expression can be taken directly for several lift coefficients and altitudes. $C_L^{3/2}\sqrt{\rho/2}$ is given in lbs.$^{1/2}$ sec. ft.$^{-2}$.

It is easily seen from the formula for the unit horsepower, that it can be divided into several parts corresponding to the parts of the drag. The additional horsepower per unit weight for climbing is simply equal to the vertical velocity of climbing.

The division of the drag ordinarily adopted is that into the drag of the wings and the drag other parts of the airplane. The coefficient of the latter part is generally assumed to be constant. This paper only deals with the wings. The drag coefficient of the wings is not constant but depends on the angle of attack. It is very useful now to divide the drag of the wings into two parts again, which are generally called section drag and induced drag. The section drag consists chiefly of the skin friction of the wings and other additional drag due to the viscosity of the air. It is analogous to the drag of the other parts of the airplane. It is essential to note that this drag coefficient depends practically on the wing section only, and that the coefficient, which is not very variable for different angle of attack within the useful range, is the same for different wing arrangements with the same wing section and the same lift coefficient. The induced drag coefficient behaves just the opposite way. It depends only on the arrangement of the wings and is equal for the same arrangement and different wing sections. It is very variable for different angles of attack. For a particular airplane the induced drag is inversely proportional to the dynamical pressure; the coefficient of induced drag is inversely proportional to the square of the dynamical pressure or directly proportional to the square of the lift coefficient. This quality makes the induced drag so useful for calculation, for, as a consequence, it can be easily calculated and laid down in tables. The general procedure for obtaining the drag of a particular airplane cellule is to take the drag coefficient from any test with the same wing section but not necessarily the same wing arrangement. This drag coefficient is divided into the two parts mentioned and the induced part is replaced by the induced drag coefficient of the new arrangement in question. This can be done simply, as will be shown now.

8. DETERMINATION OF THE DRAG COEFFICIENT.

The drag coefficient is obtained by splitting the known drag coefficient of an arrangement of wings not necessarily equal to the arrangement in question but with equal wing section into the drag coefficient of section and the induced drag coefficient, and by replacing the induced drag coefficient by the induced drag coefficient of the new arrangment. This is done by the use of the following equation:

$$C_{D_2}=C_{D_1}-\frac{C_L^2}{\pi}\left[\frac{S_1}{b_1^2k_1^2}-\frac{S_2}{b_2k_2^2}\right] \tag{1}$$

The lift coefficient occurs once only, for it is assumed that the two drag coefficients are compared with each other for the same lift coefficient. The designer who wishes to know the drag coefficient for any particular lift coefficient starts with the drag coefficient of the model at that same lift coefficient. The indices of the other symbols refer to the one or the other arrangement of wings. S is the entire area and b the greatest span. k_1 and k_2 are factors which depend merely on the gap/span ratio of the biplane and assume the value $k=1$ for monoplanes. If the two spans of a biplane are slightly different, an average span is to be substituted. The values of k are determined by the author empirically as described in a former paper (ref. 2). The theoretical values of k, which are its upper possible limits, are given in Table V and in Figure 3; both are plotted against the gap/span ratio. The differences are not very great. In view of the fact that the comparison has been made with one wing section only, and that it is difficult to obtain exact values of k, these values are not very reliable and an average curve must be taken until more comprehensive tests are made. The result of the calculation of the drag coefficient is practically unaffected by this small change of k. For rough calculation it is even sufficient to take once for all $k=1$ for monoplanes and $k=1.1$ for all biplanes used in practice.

It is not necessary now to calculate actually the two induced drag coefficients and to exchange them with each other. In equation (1) there occurs the expression S/b^2k^2. For monoplanes with rectangular plan view, for which k is 1, this is the inverse aspect ratio. It is helpful to introduce a name for S/b^2k^2, and since numerator and denominator both contain areas, it seems proper to call the expression "area ratio."

From equation (1) it can be seen now:

The difference of the induced drag coefficients of two wing arrangements with different area ratios is equal to the induced drag coefficient of an arrangement having an area ratio equal to the difference of the two area ratios.

The procedure is therefore this:

(a) Determine the two area ratios $S_1/b_1^2k_1^2$ and $S_2/b_2^2k_2^2$ and subtract one from the other.

(b) Take from Table VI the induced drag coefficient for this difference and subtract it from the original drag coefficient.

The drag coefficient must be taken for the particular lift coefficient in question. If the difference of the two area ratios is negative; that is, when the new arrangement has a greater area ratio, the figure from Table VI is to be added. If the difference of the two area ratios is so small that it is not contained in Table VI, take 10 times as great an area ratio and divide the result by 10.

Example.—A model test with a single rectangular wing gives for a particular section $C_D=0.040$ for the lift coefficient 0.50. The drag coefficient is to be determined for a biplane with a ratio of the chords, gap, and span 1:1:6, and the same lift coefficient. The area section of the model is $1/6=0.167$. Table VI gives $k=1.11$ for the biplane, hence its area ratio is $\frac{2\times 6}{36\times 1.11_2}=0.271$. The difference of the two area ratios is 0.104. Table VI gives for 0.104 (first column) and C_L 0.50 (on top) the answer 0.0083. This is to be added to 0.040, the area ratio of the model being smaller; and the final answer is $C_D=0.048$. For wings with any other plan form the greatest span is always to be taken. Stagger and decalage do not materially influence the value of k. If one of the wings is very much smaller than the other, the whole arrangement approaches a monoplane. In this case one must interpolate between the k for the complete biplane with that particular gap/span ratio and $k=1$ of a monoplane. The greatest of the spans is to be taken again.

9. DETERMINATION OF THE ANGLE OF ATTACK.

It is usual at present to ask what lift a certain biplane produces at a certain angle of attack, although it would be more natural to ask at which angle of attack the biplane produces a certain lift. For the weight of the airplane, and in consequence the lift, is the primary quantity known. In a wind-tunnel test, indeed, the angle of attack is the primary quantity and the lift is measured afterwards. This is probably the reason for always beginning with the angle of attack. But the design of the airplane is the main object and the wind-tunnel tests only an auxiliary procedure to foster it. It is obvious that both questions finally lead to the same answer, for if the angle of attack is known for a greater number of lift coefficients, the lift coefficient for any angle of attack can be taken therefrom. It is, however, much more easy to calculate the angle if the lift coefficient is given, than the lift coefficient if the angle is given; and chiefly for this reason the problem is always so stated in the following that the lift coefficient is chosen and the angle of attack belonging to it is calculated.

The connection between the lift and the angle of attack is more simple than that between the drag and the angle of attack, and can be calculated (ref. 3). Whether it be found by calculation or by tests, it may be supposed now that it is known for a particular arrangement of wings, monoplane or biplane, and it is asked how great the angle of attack belonging to a certain lift coefficient is for a second arrangement with the same wing section.

The difference of the two angles of attack for the same lift coefficient is due chiefly to two reasons: The induction and the interference between the upper and lower wing section. Hence the angle of attack necessary for producing a certain lift coefficient can be divided into three parts: (*a*) The original angle of attack belonging to the wing section in question and to the lift coefficient, (*b*) the additional induced angle of attack, and (*c*) the additional interference angle of attack. The procedure is now the same as before: The given angle of attack is split into the original angle of attack and the sum of the additional induced and interference angles of attack, and the second part is replaced by the corresponding sum of the two additional angles of attack for the new arrangement. The equation for this proceeding is the following:

$$\alpha_2 = \alpha_1 - \frac{C_L}{\pi}\left[\left(\frac{S_1}{k_1^2 b_1^2} + I_1\right) - \left(\frac{S_2}{k_2^2 b_2^2} + I_2\right)\right] \text{ in radians.} \tag{2}$$

In this equation the index 1 again refers to one of the two biplanes or monoplanes and the index 2 to the other. S/b^2k^2 is the same area ratio as before, k has the same value, which can be taken equal for all biplanes with the same gap/span ratio and is $k=1$ for monoplanes. I gives the interference effect and is approximately a function of the gap/chord ratio only. It is true that it varies somewhat with the stagger and with the section, being smaller for the lift produced by the curvature of the section than for the lift produced by the inclination of the section. But the curvature of all sections in actual use is not so very variable. Moreover, the interference angle is not great, so that the entire result is not very much affected if for each gap/chord ratio an average interference effect is taken. In Table I such an average value of the interference effect I is given as a function of the gap/chord ratio. c is always positive and is zero for the monoplane.

The expression S/b^2k^2+I can be considered as a kind of effective area ratio, being the area ratio which requires the same additional angle of attack as the real area ratio and interference together.

It is again seen that the difference of the two effective area ratios can be calculated first, and then the additional angle of attack can be taken from Table V for this difference. The figure of Table V has to be added again, if the effective area ratio is increased, otherwise subtracted.

Example.—The same monoplane as before may have the angle of attack 2.0° for $C_L=0.50$. Which angle has the biplane?

The effective area ratio of the monoplane is 1/6 or 0.167 as before, there being no biplane interference. The biplane has the real area ratio 0.271 as before. The coefficient J of interference is 0.060, as given by Table and Diagram I for the gap/chord ratio 1.0. The effective area ratio is $0.271+0.060=0.331$. The difference of the two effective area ratios is $0.331-0.167=0.164$. Table VIII gives for this value and $C_L=0.50$, $1.495°$ or approximately $1.5°$. Hence, the answer is $2.0°+1.5°=3.5°$.

10. DETERMINATION OF THE CENTER OF PRESSURE.

As is known, the exact determination of the center of pressure is one of the most difficult problems. The approximate determination is not so difficult, however.

The center of pressure of the unstaggered biplane is almost the same as that of a monoplane with the same section and the same lift coefficient. Compared with the monoplane, it is moved slightly toward the leading edge, about 2 per cent of the chord for the ratio gap/chord equal one. The center of pressure is moved more for staggered biplanes, and it can be calculated in the easiest way by introducing the moment coefficient with respect to the center of the biplane. This moment coefficient is increased for two reasons, from induction and from interference. The increase from induction is

$$\Delta C_m = 4\,\frac{s^2}{T^2}\,\frac{S}{b^2}\,\frac{T}{b}\left[\frac{b\,(1/k^2-0.5)}{R}\right] C_m \tag{3}$$

and the increase from interference can be approximated by the formula:

$$\Delta C_m{}'' = C_m\left(.08+\frac{.16\,s^2}{G^2}\right) + C_L\frac{.16s}{G^2} \tag{4}$$

where C_m refers to the monoplane.

These two additional moment coefficients are to be determined with the aid of Table III, which contains the last bracket of (3) as a function of gap/chord. If both arrangements are staggered biplanes, the one additional moment coefficient is to be subtracted and to be replaced by the new one. In most cases one of the two arrangements only is a staggered biplane, and then the additional moment coefficients are to be added.

The symbols in the expressions have the same meaning as before, that is, s denotes the stagger, T the chord, S the entire wing area, and b the greatest span.

11. CONCLUSION.

The investigation thus finished is not as exact as is desirable, chiefly in the first part. If the thickness of the section is finite, it is better to subtract from the length of the chord half of the radius of curvature of the leading edge, as explained in a former paper, before substituting in the formulas (ref. 3). The calculation of the two-dimensional flow around a staggered biplane ought to be continued for more values of the variables, and it is much to be regretted that the computation for this paper could not be made exact to four places, owing to technical difficulties.

The investigation of the biplane, chiefly of the staggered biplane, by model tests ought to be continued. The tests are likely to give more general and useful results if they are made with symmetrical sections, in order to separate the two different influences and if they are completed with different cambered sections at moderate angles of attack.

TABLES AND DIAGRAMS.

S area of both wings.
q dynamic pressure.
L entire lift of both wings.
α angle of attack, where $\alpha = 0$ means that the chord coincides with the direction of the air flow.
β angle of attack, where $\beta = 0$ means that the moment around the center of the wing is zero.
$\beta_0 = \frac{C_{L_0}}{2\pi}$ is the effect due to curvature, C_{L_0} being the lift coefficient for $\beta = 0$.
T chord.
s stagger.

I. TWO-DIMENSIONAL FLOW, UNSTAGGERED BIPLANE.

Lift produced by curvature $L_0 = 2\pi S q \sin \beta_0 B_0$.
Coordinates of C. P., $x_0 = 0$, $y_0 = 0$.
Lift produced by angle of attack $L = 2\pi S q \sin \beta B$.
Coordinates of C. P., $x = T$, $y = 0$.
Secondary repulsive force between the wings $\frac{S}{2} q \left[\sin^2 \beta v + \frac{C_L^2}{2\pi^2 B^2} C \right]$.

Additional angle of attack in order to compensate for loss of lift $\frac{C_L}{\pi} J$.

Additional lift coefficient for decalage $\pm\delta$, $\pm 2\pi \frac{S}{2} B_0 (1 + 2d)\delta$.

DIAGRAM FOR TABLE I.

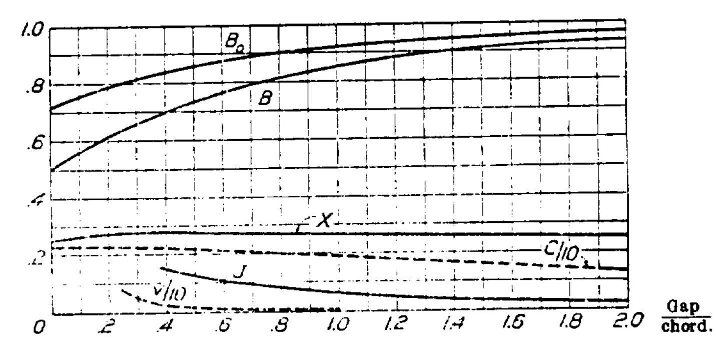

TABLE I.

Gap/chord	B	B_0	X	XB_0	C	v	J	d	$B_0(1+2d)$
∞	1	1	1/4	1/4	0	0	0	0	1
57.7	1	1	0.250	0.250	.053	.000	.000	.000	1.00
13.88	0.997	0.999	.250	.250	.238	.000	.000	.000	1.00
5.76	.992	.996	.252	.250	.530	.001	.002	.004	1.00
2.87	.972	.981	.254	.250	1.00	.003	.012	.012	1.01
2.02	.948	.974	.257	.250	1.32	.019	.024	.028	1.03
1.46	.912	.954	.262	.250	1.62	.047	.030	.055	1.06
1.11	.872	.934	.266	.248	1.82	.062	.055	.092	1.10
.98	.851	.923	.269	.248	1.91	.088	.060	.110	1.12
.79	.811	.902	.273	.246	2.01	.129	.082	.160	1.19
.64	.775	.880	.276	.242	2.07	.154	.104	.210	1.25
.56	.751	.868	.278	.241	2.11	.182	.116	.250	1.30
.46	.717	.846	.279	.236	2.13	.200	.135	.320	1.38
.39	.692	.833	.280	.233	2.14	.223	.151	.390	1.48

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II. TWO-DIMENSIONAL FLOW, STAGGERED BIPLANE.

Lift produced by curvature, $L_0 = 2\ \pi\ S\ q\ sin\ \beta_0\ B_0$.
Coordinates of C. P., $x_0 T$, $y_0 T$.
Lift produced by angle of attack, $L = 2\ \pi\ S\ q\ sin\ \beta\ B$.
Coordinates of C. P., $x T$, $y T$.
Difference of primary upper and lower lift:

Lift of curvature, $2\ \pi\ S\ q\ sin\ \beta_0\ C_0$.
Lift of angle of attack, $2\ \pi\ S\ q\ sin\ \beta\ C$.

TABLE II.

$\frac{\text{Gap}}{\text{chord}}$	$\frac{\text{Stagger}}{\text{chord}}$	B	B_0	C	C_0	x	x_0	y	y_0	$\frac{\text{Stagger}}{\text{gap}}$
1.51	0.32	0.91	0.96	0.046	0.041	0.28	0.01	0.025	0.03	0.21
1.44	.60	.915	.93	.092	.126	.35	.03	.05	.10	.42
1.32	.83	.925	.87	.125	.213	.41	.08	.07	.16	.63
1.10	.25	.88	.95	.054	.056	.28	.005	.03	.03	.23
1.04	.49	.89	.94	.095	.155	.31	.03	.05	.09	.47
1.00	.68	.895	.87	.145	.246	.36	.08	.08	.14	.68
.776	.20	.82	.92	.046	.034	.29	.005	.03	.015	.26
.742	.38	.83	.92	.100	.152	.33	.03	.06	.06	.51
.740	.53	.83	.89	.186	.302	.40	.08	.08	.125	.72
.600	.78	.83	.72	.232	.545	.56	.31	.14	.23	1.30

III. AERODYNAMICAL INDUCTION.

Minimum induced drag, $D = \frac{L^2}{k^2_{max}\ b^2\ \pi\ q}$

Induced drag, $D = \frac{L^2}{k^2\ b^2\ \pi\ q}$

Additional lift coefficient of individual staggered wings $\pm 2\ C_L \frac{S}{b^2}\left(\frac{1}{k^2} - 0.5\right)\frac{b}{R}\frac{s}{T}\frac{T}{b}$

Additional arm of moment as produced by stagger and induction, $T\frac{S}{b^2}\left(\frac{1}{k^2} - 0.5\right)\frac{b}{R}\left(\frac{s}{T}\right)^2\frac{T}{b}$

DIAGRAM FOR TABLE III.

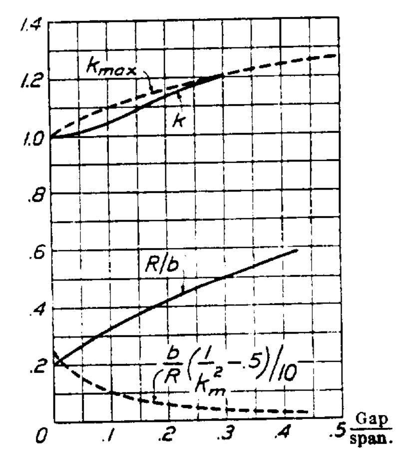

TABLE III.

$\frac{\text{Gap}}{\text{span}}$	k_{max}	$\frac{1}{k^2_{max}} - 0.5$	$\frac{R}{b}$	$\frac{b}{R}\left(\frac{1}{k^2_{max}} - 0.5\right)$	k
0	1	0.50	0.20	2.5	1
.05	1.06	.39	.27	1.5	1.02
.10	1.10	.32	.32	1.0	1.05
.15	1.13	.28	.37	.75	1.09
.20	1.16	.24	.42	.57	1.15
.30	1.21	.18	.50	.36	1.21
.40	1.24	.14	.57	.25	....
.50	1.27	.11	.65	.17	....

TABLE IV.—*Calculation of horsepower.*

$C_L{}^{3/2}\sqrt{\frac{\rho}{g}}$ *lbs.*$^{1/2}$ *sec. ft.*$^{-2}$

C_L	Attitude in feet.							C_L
	0	5000	10,000	20,000	30,000	40,000	50,000	
0.1	0.0011	0.0010	0.0009	0.0008	0.0007	0.0006	0.0005	0.1
.2	.0031	.0028	.0026	.0022	.0019	.0015	.0014	.2
.3	.0056	.0052	.0048	.0041	.0035	.0028	.0025	.3
.4	.0086	.0080	.0074	.0063	.0054	.0043	.0039	.4
.5	.0121	.0112	.0103	.0087	.0075	.0060	.0054	.5
.6	.0159	.0148	.0136	.0115	.0098	.0079	.0071	.6
.7	.0201	.0186	.0171	.0145	.0124	.0100	.0090	.7
.8	.0243	.0226	.0207	.0176	.0151	.0121	.0109	.8
.9	.0291	.0270	.0248	.0211	.0180	.0145	.0131	.9
1.0	.0343	.0318	.0292	.0248	.0212	.0171	.0154	1.0
1.1	.0395	.0367	.0337	.0286	.0245	.0197	.0178	1.1
1.2	.0451	.0418	.0384	.0326	.0279	.0225	.0203	1.2
1.3	.0507	.0471	.0432	.0367	.0314	.0253	.0228	1.3
1.4	.0565	.0525	.0482	.0409	.0350	.0282	.0254	1.4
1.5	.0624	.0579	.0531	.0451	.0386	.0311	.0280	1.5
1.6	.0696	.0645	.0593	.0503	.0430	.0347	.0313	1.6
1.7	.0761	.0706	.0648	.0550	.0471	.0380	.0342	1.7
1.8	.0830	.0769	.0707	.0600	.0513	.0414	.0373	1.8
1.9	.0902	.0836	.0768	.0652	.0557	.0450	.0405	1.9
2.0	.0974	.0903	.0829	.0704	.0602	.0486	.0437	2.0

TABLE V.

Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S.	Induced drag coefficient.										Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S.
		Lift coefficient C_L.											
		0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00		
10.0	0.1	0.0318	0.1273	0.2865	0.5093	0.7958	1.446	1.560	2.037	2.578	3.183	10.0	0.1
5.0	.2	.0159	.0637	.1433	.2547	.3980	.5731	.7801	1.019	1.290	1.592	5.0	.2
3.33	.3	.0106	.0424	.0955	.1698	.2652	.3820	.5199	.6790	.8594	1.061	3.33	.3
2.30	.4	.0080	.0318	.0716	.1274	.1990	.2866	.3900	.5094	.6448	.796	2.50	.4
2.00	.5	.0064	.0254	.0572	.1018	.1590	.2290	.3116	.4070	.5152	.636	2.00	.5
1.667	.6	.0053	.0212	.0478	.0849	.1326	.1910	.2600	.3396	.4298	.5306	1.667	.6
1.429	.7	.0046	.0182	.0409	.0728	.1137	.1627	.2228	.2911	.3684	.4548	1.429	.7
1.25	.8	.0040	.0159	.0358	.0636	.0995	.1034	.1949	.2546	.3222	.3978	1.25	.8
1.11	.9	.0035	.0141	.0318	.0566	.0884	.1273	.1733	.2263	.2864	.3536	1.11	.9
1.00	1.0	.0032	.0127	.0287	.0509	.0796	.1146	.1560	.2037	.2578	.3183	1.00	1.0
.909	1.1	.0029	.0116	.0261	.0463	.0724	.1042	.1418	.1852	.2344	.2894	.909	1.1
.833	1.2	.0027	.0106	.0239	.0424	.0663	.0955	.1209	.1697	.2148	.2652	.833	1.2
.769	1.3	.0025	.0098	.0220	.0392	.0612	.0881	.1110	.1567	.1983	.2448	.769	1.3
.714	1.4	.0023	.0091	.0205	.0364	.0569	.0810	.1114	.1455	.1842	.2274	.714	1.4
.667	1.5	.0021	.0085	.0191	.0340	.0531	.0764	.1010	.1358	.1719	.2122	.667	1.5
.625	1.6	.0020	.0080	.0179	.0318	.0497	.0716	.0975	.1273	.1611	.1989	.625	1.6
.588	1.7	.0019	.0075	.0169	.0300	.0468	.0674	.0917	.1198	.1516	.1872	.588	1.7
.556	1.8	.0018	.0071	.0159	.0283	.0442	.0637	.0866	.1132	.1432	.1768	.556	1.8
.526	1.9	.0017	.0067	.0151	.0268	.0419	.0603	.0821	.1072	.1357	.1675	.526	1.9
.509	2.0	.0016	.0064	.0143	.0255	.0398	.0573	.0780	.1019	.1290	.1592	.500	2.0
.476	2.1	.0015	.0061	.0136	.0243	.0379	.0546	.0743	.0970	.1228	.1516	.476	2.1
.455	2.2	.0015	.0058	.0130	.0232	.0368	.0521	.0709	.0926	.1172	.1447	.455	2.2
.435	2.3	.0014	.0055	.0125	.0221	.0346	.0498	.0678	.0886	.1121	.1384	.435	2.3
.417	2.4	.0013	.0053	.0119	.0212	.0332	.0477	.0650	.0849	.1074	.1326	.417	2.4
.400	2.5	.0013	.0051	.0115	.0204	.0318	.0458	.0624	.0815	.1031	.1273	.400	2.5
.385	2.6	.0012	.0049	.0110	.0196	.0306	.0441	.0600	.0783	.0991	.1224	.385	2.6
.371	2.7	.0012	.0047	.0106	.0189	.0295	.0424	.0578	.0755	.0955	.1179	.371	2.7
.357	2.8	.0011	.0045	.0102	.0182	.0284	.0409	.0557	.0728	.0921	.1137	.357	2.8
.316	2.9	.0011	.0044	.0099	.0176	.0275	.0395	.0538	.0703	.0889	.1098	.346	2.9
.333	3.0	.0011	.0012	.0096	.0170	.0265	.0382	.0520	.0679	.0859	.1061	.333	3.0
.323	3.1	.0010	.0041	.0092	.0164	.0257	.0370	.0503	.0657	.0832	.1027	.323	3.1
.313	3.2	.0010	.0040	.0090	.0159	.0249	.0358	.0488	.0637	.0806	.0995	.313	3.2
.303	3.3	.0010	.0039	.0087	.0154	.0241	.0347	.0472	.0617	.0781	.0934	.303	3.3
.294	3.4	.0009	.0037	.0084	.0150	.0234	.0337	.0459	.0599	.0758	.0936	.294	3.4
.286	3.5	.0009	.0036	.0082	.0145	.0227	.0327	.0445	.0582	.0736	.0909	.286	3.5
.278	3.6	.0009	.0035	.0080	.0141	.0221	.0318	.0433	.0566	.0716	.0884	.278	3.6
.270	3.7	.0009	.0034	.0077	.0138	.0215	.0310	.0421	.0550	.0697	.0860	.270	3.7
.263	3.8	.0008	.0031	.0075	.0134	.0210	.0302	.0411	.0536	.0679	.0838	.263	3.8
.256	3.9	.0008	.0033	.0073	.0131	.0204	.0294	.0400	.0522	.0661	.0816	.256	3.9
.250	4.0	.0008	.0032	.0072	.0127	.0199	.0287	.0390	.0509	.0645	.0796	.250	4.0
.244	4.1	.0008	.0031	.0070	.0124	.0194	.0279	.0380	.0497	.0629	.0776	.244	4.1
.238	4.2	.0008	.0030	.0068	.0121	.0190	.0273	.0371	.0485	.0614	.0758	.238	4.2
.233	4.3	.0007	.0030	.0067	.0118	.0185	.0266	.0363	.0476	.0994	.0740	.233	4.3
.227	4.4	.0007	.0030	.0065	.0116	.0181	.0261	.0355	.0463	.0586	.0724	.227	4.4
.222	4.5	.0007	.0028	.0064	.0113	.0177	.0255	.0346	.0453	.0573	.0707	.222	4.5
.217	4.6	.0007	.0028	.0062	.0111	.0173	.0249	.0339	.0443	.0561	.0692	.217	4.6
.213	4.7	.0007	.0027	.0061	.0108	.0169	.0244	.0332	.0433	.0518	.0677	.213	4.7
.208	4.8	.0007	.0027	.0060	.0106	.0166	.0239	.0325	.0424	.0537	.0663	.208	4.8
.204	4.9	.0007	.0026	.0059	.0104	.0163	.0234	.0319	.0416	.0527	.0650	.204	4.9
.200	5.0	.0006	.0025	.0057	.0102	.0159	.0229	0312	.0408	.0516	.0637	.200	5.0

TABLE V—Continued.

Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S.	Induced drag coefficient.										Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S.
		Lift coefficient C_L.											
		1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00		
10.0	0.1	3.852	4.584	5.379	6.239	7.162	8.149	9.199	10.31	11.49	12.73	10.0	0.1
5.0	.2	1.926	2.292	2.690	3.120	3.582	4.075	4.601	5.158	5.747	6.368	5.0	.2
3.33	.3	1.284	1.528	1.793	2.080	2.387	2.716	3.066	3.438	3.830	4.244	3.33	.3
2.50	.4	.9632	1.146	1.345	1.560	1.791	2.038	2.300	2.579	2.874	3.184	2.50	.4
2.00	.5	.7696	.9158	1.075	1.247	1.431	1.628	1.838	2.061	2.296	2.544	2.00	.5
1.667	.6	.6420	.7641	.8967	1.040	1.194	1.358	1.533	1.719	1.915	2.122	1.667	.6
1.429	.7	.5503	.6549	.7686	.8914	1.023	1.164	1.314	1.474	1.642	1.819	1.429	.7
1.25	.8	.4813	.5728	.6723	.7797	.8950	1.018	1.150	1.289	1.436	1.591	1.25	.8
1.11	.9	.4279	.5092	.5976	.6931	.7956	.9052	1.022	1.146	1.276	1.414	1.11	.9
1.00	1.0	.3852	.4584	.5379	.6239	.7162	.8149	.9199	1.031	1.149	1.273	1.00	2.0
.909	1.1	.3502	.4167	.4891	.5672	.6512	.7409	.8364	.9377	1.045	1.158	.909	1.1
.833	1.2	.3209	.3819	.4482	.5198	.5967	.6789	.7664	.8592	.9574	1.061	.833	1.2
.769	1.3	.2962	.3525	.4137	.4798	.5508	.6267	.7075	.7932	.8837	.9792	.769	1.3
.714	1.4	.2752	.3275	.3843	.4457	.5116	.5821	.6572	.7368	.8209	.9096	.714	1.4
.667	1.5	.2568	.3056	.3586	.4159	.4774	.5432	.6133	.6875	.7560	.8488	.667	1.5
.625	1.6	.2407	.2864	.3361	.3898	.4475	.5092	.6748	.6444	.7180	.7956	.625	1.6
.588	1.7	.2265	.2695	.3164	.3669	.4212	.4792	.5410	.6065	.6758	.7488	.588	1.7
.556	1.8	.2139	.2646	.2988	.3464	.3978	.4526	.5110	.5728	.6382	.7072	.556	1.8
.526	1.9	.2027	.2412	.2831	.3283	.3769	.4288	.4841	.5427	.6047	.6700	.526	1.9
.500	2.0	.1926	.2292	.2690	.3120	.3582	.4076	.4601	.5158	.5747	.6368	.500	2.4
.476	2.1	.1834	.2183	.2562	.2971	.3411	.3881	.4381	.4912	.5473	.6064	.476	2.1
.455	2.2	.1751	.2084	.2445	.2836	.3256	.3704	.4182	.4688	.5224	.5788	.455	2.2
.435	2.3	.1675	.1993	.2339	.2713	.3114	.3543	.4000	.4484	.4996	.5536	.435	2.3
.417	2.4	.1604	.1909	.2241	.2599	.2984	.3396	.3832	.4296	.4787	.5304	.417	2.4
.400	2.5	.1540	.1833	.2151	.2495	.2864	.3259	.3679	.4124	.4596	.5092	.400	2.5
.385	2.6	.1481	.1763	.2069	.2399	.2754	.3133	.3537	.3966	.4419	.4896	.385	2.6
.371	2.7	.1427	.1698	.1992	.2311	.2653	.3018	.3407	.3820	.4256	.4716	.371	2.7
.357	2.8	.1376	.1637	.1922	.2228	.2558	.2911	.3286	.3684	.4105	.4548	.357	2.8
.346	2.9	.1329	.1581	.1856	.2152	.2471	.2811	.3173	.3558	.3964	.4392	.346	2.9
.333	3.0	.1284	.1528	.1793	.2080	.2387	.2716	.3066	.3438	.3830	.4244	.333	3.0
.323	3.1	.1243	.1479	.1736	.2013	.2311	.2629	.2968	.3328	.3708	.4108	.323	3.1
.313	3.2	.1204	.1433	.1682	.1950	.2239	.2547	.2876	.3221	.3592	.3980	.313	3.2
.303	3.3	.1166	.1388	.1629	.1889	.2169	.2468	.2786	.3123	.3480	.3856	.303	3.3
.294	3.4	.1133	.1348	.1582	.1835	.2100	.2396	.2705	.3033	.3379	.3744	.294	3.4
.286	3.5	.1100	.1309	.1536	.1782	.2045	.2327	.2627	.2945	.3282	.3636	.286	3.5
.278	3.6	.1070	.1273	.1494	.1733	.1989	.2263	.2555	.2864	.3191	.3536	.278	3.6
.270	3.7	.1041	.1238	.1453	.1686	.1935	.2202	.2485	.2786	.3106	.3440	.270	3.7
.263	3.8	.1014	.1207	.1416	.1642	.1886	.2145	.2422	.2715	.3025	.3352	.263	3.8
.256	3.9	.0987	.1175	.1379	.1599	.1836	.2089	.2358	.2644	.2946	.3264	.256	3.9
.250	4.0	.0963	.1146	.1345	.1560	.1791	.2038	.2300	.2579	.2874	.3184	.250	4.0
.244	4.1	.0939	.1117	.1311	.1521	.1746	.1987	.2243	.2514	.2801	.3104	.244	4.1
.238	4.2	.0917	.1092	.1281	.1486	.1706	.1940	.2191	.2456	.2736	.3032	.238	4.2
.233	4.3	.0895	.1066	.1251	.1450	.1665	.1894	.2139	.2398	.2671	.2960	.233	4.3
.227	4.4	.0876	.1043	.1224	.1419	.1629	.1853	.2092	.2346	.2614	.2896	.227	4.4
.222	4.5	.0856	.1018	.1195	.1386	.1591	.1810	.2043	.2291	.2552	.2828	.222	4.5
.217	4.6	.0837	.0997	.1170	.1356	.1557	.1772	.2000	.2242	.2498	.2768	.217	4.6
.213	4.7	.0819	.0975	.1144	.1327	.1523	.1733	.1956	.2194	.2444	.2708	.213	4.7
.208	4.8	.0802	.0955	.1120	.1300	.1492	.1697	.1916	.2148	.2393	.2652	.208	4.8
.204	4.9	.0787	.0936	.1098	.1274	.1462	.1664	.1878	.2106	.2346	.2600	.204	4.9
.200	5.0	.0771	.0917	.1076	.1248	.1433	.1631	.1841	.2064	.2300	.2548	.200	5.0

TABLE V—Continued.

Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S.	Induced drag coefficient.										Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S.
		Lift coefficient C_L.											
		0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00		
0.196	5.1	0.0006	0.0025	0.0056	0.0100	0.0156	0.0225	0.0306	0.0399	0.0505	0.0624	0.196	5.1
.192	5.2	.0006	.0025	.0055	.0098	.0153	.0220	.0300	.0392	.0496	.0612	.192	5.2
.188	5.3	.0006	.0024	.0054	.0096	.0150	.0216	.0295	.0385	.0487	.0602	.188	5.3
.185	5.4	.0006	.0024	.0053	.0094	.0148	.0212	.0289	.0378	.0478	.0590	.185	5.4
.182	5.5	.0006	.0023	.0052	.0093	.0145	.0208	.0284	.0372	.0469	.0579	.182	5.5
.179	5.6	.0006	.0023	.0051	.0091	.0142	.0204	.0278	.0364	.0460	.0568	.179	5.6
.175	5.7	.0006	.0022	.0050	.0089	.0140	.0201	.0273	.0357	.0452	.0559	.175	5.7
.172	5.8	.0006	.0022	.0049	.0088	.0137	.0198	.0269	.0351	.0445	.0549	.172	5.8
.169	5.9	.0005	.0022	.0049	.0086	.0135	.0194	.0265	.0346	.0437	.0540	.169	5.9
.166	6.0	.0005	.0021	.0048	.0085	.0133	.0191	.0260	.0340	.0430	.0531	.166	6.0
.164	6.1	.0005	.0021	.0047	.0084	.0131	.0188	.0256	.0334	.0423	.0522	.164	6.1
.161	6.2	.0005	.0021	.0046	.0082	.0128	.0185	.0251	.0328	.0416	.0513	.161	6.2
.159	6.3	.0005	.0020	.0045	.0082	.0126	.0182	.0247	.0323	.0409	.0505	.159	6.3
.156	6.4	.0005	.0020	.0045	.0080	.0125	.0179	.0244	.0319	.0403	.0498	.156	6.4
.154	6.5	.0005	.0020	.0044	.0078	.0123	.0176	.0240	.0314	.0397	.0490	.154	6.5
.152	6.6	.0005	.0019	.0043	.0077	.0121	.0174	.0236	.0309	.0390	.0482	.152	6.6
.149	6.7	.0005	.0019	.0043	.0076	.0119	.0171	.0233	.0304	.0385	.0475	.149	6.7
.147	6.8	.0005	.0019	.0042	.0075	.0117	.0169	.0229	.0300	.0379	.0468	.147	6.8
.145	6.9	.0005	.0018	.0042	.0074	.0115	.0166	.0226	.0295	.0373	.0461	.145	6.9
.143	7.0	.0005	.0018	.0041	.0073	.0114	.0164	.0223	.0291	.0369	.0455	.143	7.0
.140	7.1	.0005	.0018	.0040	.0072	.0112	.0161	.0220	.0287	.0363	.0448	.140	7.1
.139	7.2	.0004	.0018	.0040	.0071	.0111	.0159	.0217	.0283	.0358	.0442	.139	7.2
.137	7.3	.0004	.0017	.0039	.0070	.0109	.0157	.0214	.0279	.0353	.0436	.137	7.3
.135	7.4	.0004	.0017	.0039	.0069	.0108	.0155	.0211	.0275	.0348	.0430	.135	7.4
.133	7.5	.0004	.0017	.0038	.0068	.0106	.0153	.0208	.0271	.0343	.0424	.133	7.5
.132	7.6	.0004	.0017	.0038	.0067	.0105	.0151	.0204	.0268	.0339	.0419	.132	7.6
.130	7.7	.0004	.0017	.0037	.0066	.0104	.0149	.0203	.0265	.0335	.0414	.130	7.7
.128	7.8	.0004	.0017	.0037	.0065	.0102	.0147	.0200	.0261	.0331	.0408	.128	7.8
.127	7.9	.0004	.0016	.0036	.0065	.0102	.0145	.0198	.0258	.0326	.0403	.127	7.9
.125	8.0	.0004	.0016	.0036	.0064	.0100	.0143	.0195	.0255	.0322	.0398	.125	8.0
.124	8.1	.0004	.0016	.0035	.0063	.0098	.0142	.0193	.0252	.0318	.0393	.124	8.1
.122	8.2	.0004	.0016	.0035	.0062	.0097	.0140	.0190	.0248	.0314	.0388	.122	8.2
.121	8.3	.0004	.0016	.0035	.0061	.0096	.0138	.0188	.0246	.0311	.0384	.121	8.3
.119	8.4	.0004	.0015	.0034	.0061	.0095	.0136	.0186	.0243	.0307	.0379	.119	8.4
.118	8.5	.0004	.0015	.0034	.0060	.0094	.0135	.0183	.0239	.0303	.0374	.118	8.5
.116	8.6	.0004	.0015	.0033	.0059	.0093	.0133	.0181	.0237	.0300	.0370	.116	8.6
.115	8.7	.0004	.0015	.0033	.0059	.0092	.0132	.0179	.0234	.0297	.0366	.115	8.7
.114	8.8	.0004	.0015	.0033	.0058	.0091	.0130	.0177	.0232	.0293	.0362	.114	8.8
.112	8.9	.0004	.0014	.0032	.0057	.0090	.0129	.0175	.0229	.0290	.0358	.112	8.9
.111	9.0	.0004	.0014	.0032	.0057	.0089	.0127	.0174	.0227	.0287	.0354	.111	9.0
.110	9.1	.0004	.0014	.0032	.0056	.0088	.0126	.0172	.0224	.0284	.0350	.110	9.1
.109	9.2	.0004	.0014	.0031	.0055	.0087	.0125	.0170	.0221	.0280	.0346	.109	9.2
.107	9.3	.0003	.0014	.0031	.0055	.0086	.0123	.0168	.0219	.0277	.0342	.107	9.3
.106	9.4	.0003	.0014	.0031	.0054	.0085	.0122	.0166	.0217	.0275	.0339	.106	9.4
.105	9.5	.0003	.0013	.0030	.0054	.0084	.0121	.0164	.0214	.0271	.0335	.105	9.5
.104	9.6	.0003	.0013	.0030	.0053	.0083	.0120	.0163	.0213	.0270	.0332	.104	9.6
.103	9.7	.0003	.0013	.0030	.0053	.0082	.0118	.0161	.0210	.0266	.0328	.103	9.7
.102	9.8	.0003	.0013	.0029	.0052	.0081	.0117	.0159	.0208	.0263	.0325	.102	9.8
.101	9.9	.0003	.0013	.0029	.0052	.0081	.0116	.0158	.0206	.0261	.0322	.101	9.9
.100	10.0	.0003	.0013	.0029	.0052	.0080	.0115	.0156	.0204	.0258	.0318	.100	10.0

TABLE V—Continued.

Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S.	Induced drag coefficient. Lift coefficient C_L. 1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00	Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S.
0.196	5.1	0.0755	0.0899	0.1055	0.1223	0.1404	0.1597	0.1803	0.2022	0.2253	0.2496	0.196	5.1
.192	5.2	.0741	.0881	.1034	.1200	.1377	.1567	.1769	.1983	.2209	.2448	.192	5.2
.188	5.3	.0727	.0865	.1016	.1178	.1352	.1539	.1737	.1917	.2170	.2404	.188	5.3
.185	5.4	.0714	.0849	.0997	.1156	.1328	.1510	.1705	.1912	.2130	.2360	.185	5.4
.182	5.5	.0701	.0834	.0979	.1135	.1303	.1482	.1673	.1876	.2090	.2316	.182	5.5
.179	5.6	.0687	.0818	.0960	.1113	.1278	.1454	.1642	.1840	.2050	.2272	.179	5.6
.175	5.7	.0675	.0804	.0943	.1094	.1256	.1428	.1616	.1811	.2018	.2232	.175	5.7
.172	5.8	.0664	.0791	.0928	.1076	.1235	.1405	.1587	.1777	.1982	.2196	.172	5.8
.169	5.9	.0653	.0778	.0913	.1058	.1215	.1382	.1561	.1750	.1949	.2160	.169	5.9
.166	6.0	.0643	.0765	.0897	.1041	.1195	.1359	.1535	.1720	.1917	.2124	.166	6.0
.164	6.1	.0632	.0752	.0882	.1023	.1174	.1336	.1509	.1691	.1884	.2088	.164	6.1
.161	6.2	.0621	.0739	.0867	.1006	.1154	.1313	.1483	.1662	.1852	.2052	.161	6.2
.159	6.3	.0611	.0727	.0853	.0990	.1136	.1293	.1460	.1636	.1823	.2020	.159	6.3
.156	6.4	.0603	.0717	.0842	.0976	.1120	.1275	.1439	.1614	.1798	.1992	.156	6.4
.154	6.5	.0593	.0706	.0828	.0960	.1102	.1254	.1416	.1588	.1769	.1960	.154	6.5
.152	6.6	.0583	.0694	.0815	.0945	.1084	.1234	.1393	.1562	.1740	.1928	.152	6.6
.149	6.7	.0575	.0684	.0803	.0931	.1069	.1216	.1373	.1539	.1715	.1900	.149	6.7
.147	6.8	.0566	.0674	.0791	.0917	.1053	.1198	.1352	.1516	.1690	.1872	.147	6.8
.145	6.9	.0558	.0664	.0779	.0904	.1037	.1180	.1332	.1494	.1664	.1844	.145	6.9
.143	7.0	.0551	.0655	.0769	.0892	.1024	.1165	.1315	.1474	.1642	.1820	.143	7.0
.140	7.1	.0542	.0645	.0757	.0878	.1008	.1147	.1295	.1452	.1617	.1792	.140	7.1
.139	7.2	.0535	.0637	.0747	.0866	.0994	.1132	.1277	.1432	.1596	.1768	.139	7.2
.137	7.3	.0528	.0628	.0737	.0855	.0981	.1116	.1260	.1413	.1574	.1744	.137	7.3
.135	7.4	.0520	.0619	.0727	.0843	.0968	.1101	.1243	.1393	.1552	.1720	.135	7.4
.133	7.5	.0513	.0611	.0717	.0831	.0954	.1085	.1225	.1374	.1531	.1696	.133	7.5
.132	7.6	.0507	.0303	.0708	.0821	.0943	.1073	.1211	.1358	.1513	.1676	.132	7.6
.130	7.7	.0501	.0596	.0700	.0811	.0932	.1060	.1196	.1341	.1494	.1656	.130	7.7
.128	7.8	.0494	.0588	.0690	.0800	.0918	.1044	.1179	.1322	.1473	.1632	.128	7.8
.127	7.9	.0488	.0580	.0681	.0790	.0907	.1032	.1165	.1306	.1455	.1612	.127	7.9
.125	8.0	.0482	.0573	.0673	.0780	.0896	.1019	.1150	.1290	.1437	.1592	.125	8.0
.124	8.1	.0476	.0566	.0664	.0770	.0884	.1006	.1136	.1273	.1419	.1572	.124	8.1
.122	8.2	.0470	.0559	.0656	.0761	.0873	.0993	.1121	.1257	.1401	.1552	.122	8.2
.121	8.3	.0465	.0553	.0647	.0753	.0864	.0983	.1110	.1244	.1386	.1536	.121	8.3
.119	8.4	.0459	.0546	.0641	.0743	.0853	.0970	.1095	.1228	.1368	.1516	.119	8.4
.118	8.5	.0453	.0539	.0632	.0733	.0842	.0957	.1081	.1212	.1350	.1496	.118	8.5
.116	8.6	.0448	.0533	.0025	.0725	.0833	.0947	.1069	.1199	.1336	.1480	.116	8.6
.115	8.7	.0443	.0527	.0619	.0717	.0824	.0937	.1058	.1186	.1321	.1464	.115	8.7
.114	8.8	.0438	.0521	.0612	.0710	.0815	.0927	.1046	.1173	.1307	.1448	.114	8.8
.112	8.9	.0433	.0516	.0605	.0702	.0806	.0917	.1035	.1160	.1292	.1432	.112	8.9
.111	9.0	.0428	.0510	.0598	.0694	.0797	.0906	.1023	.1147	.1278	.1416	.111	9.0
.110	9.1	.0424	.0504	.0592	.0686	.0788	.0896	.1012	.1134	.1264	.1400	.110	9.1
.109	9.2	.0419	.0498	.0585	.0678	.0779	.0886	.1004	.1121	.1249	.1384	.109	9.2
.107	9.3	.0414	.0493	.0578	.0670	.0770	.0876	.0988	.1108	.1235	.1368	.107	9.3
.106	9.4	.0410	.0488	.0573	.0664	.0763	.0868	.0980	.1098	.1224	.1356	.106	9.4
.105	9.5	.0405	.0482	.0566	.0657	.0754	.0858	.0968	.1085	.1209	.1340	.105	9.5
.104	9.6	.0402	.0478	.0561	.0651	.0747	.0850	.0960	.1076	.1198	.1328	.104	9.6
.103	9.7	.0397	.0472	.0554	.0643	.0738	.0840	.0948	.1063	.1184	.1312	.103	9.7
.102	9.8	.0393	.0468	.0549	.0637	.0731	.0832	.0939	.1053	.1173	.1300	.102	9.8
.101	9.9	.0390	.0464	.0544	.0631	.0725	.0824	.0931	.1043	.1162	.1288	.101	9.9
.100	10.0	.0385	.0458	.0537	.0623	.0716	.0814	.0919	.1030	.1148	.1272	.100	10.0

TABLE VI.

Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S.	Induced angle of attack in degrees. Lift coefficient C_L. 0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S.
10.0	0.1	18.238	36.476	54.713	72.951	91.189	109.427	127.665	145.902	164.140	182.378	10.0	0.1
5.0	.2	9.119	18.238	27.357	36.476	45.595	54.713	63.832	72.951	82.070	91.189	5.0	.2
3.33	.3	6.073	12.146	18.220	24.293	30.366	36.439	42.512	48.585	54.659	60.732	3.33	.3
2.50	.4	4.559	9.119	13.678	18.238	22.797	27.357	31.916	36.476	41.035	45.595	2.50	.4
2.00	.5	3.648	7.295	10.943	14.590	18.238	21.885	25.533	29.180	32.828	36.476	2.00	.5
1.667	.6	3.040	6.080	9.121	12.161	15.201	18.241	21.282	24.322	27.362	30.402	1.667	.6
1.429	.7	2.606	5.212	7.819	10.425	13.031	15.637	18.243	20.849	23.456	26.062	1.429	.7
1.25	.8	2.280	4.559	6.839	9.119	11.399	13.678	15.958	18.238	20.517	22.797	1.25	.8
1.11	.9	2.024	4.049	6.073	8.098	10.122	12.146	14.171	16.195	18.220	20.244	1.11	.9
1.00	1.0	1.824	3.648	5.471	7.295	9.119	10.943	12.766	14.590	16.414	18.238	1.00	1.0
.909	1.1	1.658	3.316	4.973	6.631	8.289	9.947	11.605	13.262	14.920	16.578	.909	1.1
.833	1.2	1.519	3.038	4.558	6.077	7.596	9.115	10.634	12.154	13.673	15.192	.833	1.2
.769	1.3	1.402	2.805	4.207	5.610	7.012	8.415	9.817	11.220	12.622	14.025	.769	1.3
.714	1.4	1.302	2.604	3.907	5.209	6.511	7.813	9.115	10.417	11.720	13.022	.714	1.4
.667	1.5	1.216	2.433	3.649	4.866	6.082	7.299	8.515	9.732	10.948	12.165	.667	1.5
.625	1.6	1.140	2.280	3.420	4.559	5.699	6.839	7.979	9.119	10.259	11.399	.625	1.6
.588	1.7	1.072	2.145	3.217	4.290	5.362	6.434	7.507	8.579	9.651	10.724	.588	1.7
.556	1.8	1.014	2.028	3.042	4.056	5.070	6.084	7.098	8.112	9.126	10.140	.556	1.8
.526	1.9	.959	1.919	2.878	3.837	4.797	5.756	6.715	7.674	8.634	9.593	.526	1.9
.500	2.0	.912	1.824	2.736	3.648	4.559	5.471	6.383	7.295	8.207	9.119	.500	2.0
.476	2.1	.868	1.736	2.604	3.472	4.341	5.209	6.077	6.945	7.813	8.681	.476	2.1
.455	2.2	.830	1.660	2.489	3.319	4.149	4.979	5.809	6.638	7.468	8.298	.455	2.2
.435	2.3	.793	1.587	2.380	3.173	3.967	4.760	5.553	6.347	7.140	7.933	.435	2.3
.417	2.4	.761	1.521	2.282	3.042	3.803	4.563	5.324	6.084	6.845	7.605	.417	2.4
.400	2.5	.730	1.459	2.189	2.918	3.648	4.377	5.107	5.836	6.566	7.295	.400	2.5
.385	2.6	.702	1.404	2.106	2.809	3.511	4.213	4.915	5.617	6.319	7.022	.385	2.6
.371	2.7	.677	1.353	2.030	2.706	3.383	4.060	4.736	5.413	6.090	6.766	.371	2.7
.357	2.8	.651	1.302	1.953	2.604	3.255	3.906	4.558	5.209	5.860	6.511	.357	2.8
.346	2.9	.631	1.262	1.893	2.524	3.155	3.786	4.417	5.048	5.679	6.310	.346	2.9
.333	3.0	.607	1.215	1.822	2.429	3.037	3.644	4.251	4.858	5.466	6.073	.333	3.0
.323	3.1	.589	1.178	1.767	2.356	2.945	3.534	4.124	4.713	5.302	5.891	.323	3.1
.313	3.2	.571	1.142	1.713	2.283	2.854	3.425	3.996	4.567	5.138	5.708	.313	3.2
.303	3.3	.553	1.105	1.658	2.210	2.763	3.316	3.868	4.421	4.973	5.526	.303	3.3
.294	3.4	.536	1.072	1.609	2.145	2.681	3.217	3.753	4.290	4.826	5.362	.294	3.4
.286	3.5	.522	1.043	1.565	2.086	2.608	3.130	3.651	4.173	4.794	5.216	.286	3.5
.278	3.6	.507	1.014	1.521	2.028	2.535	3.042	3.549	4.056	4.563	5.070	.278	3.6
.270	3.7	.492	.985	1.477	1.970	2.462	2.955	3.447	3.939	4.432	4.924	.270	3.7
.263	3.8	.480	.959	1.439	1.919	2.398	2.878	3.358	3.837	4.317	4.797	.263	3.8
.256	3.9	.467	.934	1.401	1.868	2.334	2.801	3.268	3.735	4.202	4.669	.256	3.9
.250	4.0	.456	.912	1.368	1.824	2.280	2.736	3.192	3.648	4.103	4.559	.250	4.0
.244	4.1	.445	.890	1.335	1.780	2.225	2.670	3.115	3.560	4.005	4.450	.244	4.1
.238	4.2	.434	.868	1.302	1.736	2.170	2.604	3.038	3.472	3.906	4.341	.238	4.2
.233	4.3	.425	.850	1.275	1.700	2.125	2.550	2.975	3.400	3.824	4.249	.233	4.3
.227	4.4	.414	.828	1.242	1.656	2.070	2.484	2.898	3.312	3.726	4.140	.227	4.4
.222	4.5	.405	.810	1.215	1.619	2.024	2.429	2.834	3.239	3.644	4.049	.222	4.5
.217	4.6	.396	.792	1.187	1.583	1.979	2.375	2.770	3.166	3.562	3.958	.217	4.6
.213	4.7	.388	.777	1.165	1.554	1.942	2.331	2.719	3.108	3.496	3.885	.213	4.7
.208	4.8	.379	.759	1.138	1.517	1.897	2.276	2.655	3.035	3.414	3.793	.208	4.8
.204	4.9	.372	.744	1.116	1.488	1.860	2.232	2.604	2.976	3.348	3.721	.204	4.9
.200	5.0	.365	.730	1.094	1.459	1.824	2.189	2.553	2.918	3.283	3.648	.200	5.0

TABLE VI—Continued.

Area ratio S/l^2b^2.	Aspect ratio k^2b^2/S.	Induced angle of attack in degrees.										Area ratio S/l^2b^2.	Aspect ratio k^2b^2/S.
		Lift coefficient C_L.											
		1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00		
10.0	0.1	200.616	218.854	237.091	255.329	273.567	291.805	310.043	328.280	346.518	364.756	10.0	0.1
5.0	.2	100.308	109.427	118.546	127.665	136.784	145.902	155.021	164.140	173.259	182.378	5.0	.2
3.33	.3	66.805	72.878	78.951	85.025	91.098	97.171	103.244	109.317	115.391	121.464	3.33	.3
2.50	.4	50.154	54.713	59.273	63.832	68.392	72.951	77.511	82.070	86.630	91.189	2.50	.4
2.00	.5	40.123	43.771	47.418	51.066	54.713	58.361	62.009	65.656	69.304	72.951	2.00	.5
1.667	.6	33.443	36.483	39.523	42.563	45.604	48.644	51.684	54.724	57.765	60.805	1.667	.6
1.429	.7	28.668	31.274	33.880	36.487	39.093	41.699	44.305	46.911	49.517	52.124	1.429	.7
1.25	.8	25.077	27.357	29.636	31.916	34.196	36.476	38.755	41.033	43.315	45.594	1.25	.8
1.11	.9	22.268	24.203	26.317	28.341	30.366	32.390	34.415	36.439	38.463	40.488	1.11	.9
1.00	1.0	20.062	21.885	23.709	25.533	27.357	29.180	31.004	32.828	34.652	36.476	1.00	1.0
.909	1.1	18.236	19.894	21.552	23.209	24.867	26.525	28.183	29.841	31.498	33.156	.909	1.1
.833	1.2	16.711	18.231	19.750	21.269	22.788	24.307	25.827	27.346	28.865	30.384	.833	1.2
.769	1.3	15.427	16.830	18.232	19.635	21.037	22.440	23.842	25.245	26.647	28.050	.769	1.3
.714	1.4	14.324	15.626	16.928	18.230	19.533	20.835	22.137	23.439	24.741	26.043	.714	1.4
.667	1.5	13.381	14.598	15.814	17.030	18.247	19.463	20.680	21.896	23.113	24.329	.667	1.5
.625	1.6	12.538	13.678	14.818	15.958	17.098	18.238	19.378	20.517	21.657	22.797	.625	1.6
.588	1.7	11.796	12.869	13.941	15.013	16.086	17.158	18.230	19.303	20.375	21.448	.588	1.7
.556	1.8	11.154	12.168	13.182	14.196	15.210	16.224	17.238	18.252	19.266	20.280	.556	1.8
.526	1.9	10.552	11.512	12.471	13.430	14.390	15.349	16.308	17.267	18.227	19.186	.526	1.9
.500	2.0	10.031	10.943	11.855	12.766	13.678	14.590	15.502	16.414	17.326	18.238	.500	2.0
.476	2.1	9.549	10.417	11.285	12.154	13.022	13.890	14.758	15.626	16.494	17.362	.476	2.1
.455	2.2	9.128	9.958	10.788	11.617	12.447	13.277	14.107	14.937	15.766	16.596	.455	2.2
.435	2.3	8.727	9.520	10.313	11.107	11.900	12.693	13.487	14.280	15.073	15.867	.435	2.3
.417	2.4	8.366	9.126	9.887	10.647	11.408	12.168	12.929	13.689	14.450	15.210	.417	2.4
.400	2.5	8.025	8.754	9.484	10.213	10.943	11.672	12.402	13.131	13.861	14.590	.400	2.5
.385	2.6	7.724	8.426	9.128	9.830	10.532	11.234	11.937	12.639	13.341	14.043	.385	2.6
.371	2.7	7.443	8.119	8.796	9.473	10.149	10.823	11.503	12.179	12.856	13.532	.371	2.7
.357	2.8	7.162	7.813	8.464	9.115	9.766	10.417	11.068	11.719	12.371	13.022	.357	2.8
.346	2.9	6.941	7.572	8.203	8.834	9.465	10.096	10.727	11.358	11.989	12.620	.346	2.9
.333	3.0	6.680	7.288	7.895	8.502	9.110	9.717	10.325	10.932	11.539	12.146	.333	3.0
.323	3.1	6.480	7.069	7.658	8.247	8.836	9.425	10.014	10.603	11.193	11.782	.323	3.1
.313	3.2	6.279	6.850	7.421	7.992	8.563	9.133	9.704	10.275	10.846	11.417	.313	3.2
.303	3.3	6.079	6.631	7.184	7.736	8.289	8.842	9.394	9.947	10.499	11.052	.303	3.3
.294	3.4	5.898	6.434	6.970	7.507	8.043	8.579	9.115	9.651	10.188	10.724	.294	3.4
.286	3.5	5.738	6.259	6.781	7.302	7.824	8.346	8.867	9.389	9.910	10.432	.286	3.5
.278	3.6	5.577	6.084	6.591	7.098	7.605	8.112	8.619	9.126	9.633	10.140	.278	3.6
.270	3.7	5.417	5.909	6.401	6.894	7.386	7.879	8.371	8.864	9.356	9.848	.270	3.7
.263	3.8	5.276	5.756	6.235	6.715	7.195	7.674	8.154	8.634	9.113	9.593	.263	3.8
.256	3.9	5.136	5.603	6.069	6.536	7.003	7.470	7.937	8.404	8.871	9.338	.256	3.9
.250	4.0	5.015	5.471	5.927	6.383	6.839	7.295	7.751	8.207	8.663	9.119	.250	4.0
.244	4.1	4.895	5.340	5.785	6.230	6.675	7.120	7.565	8.010	8.455	8.900	.244	4.1
.238	4.2	4.775	5.209	5.643	6.077	6.511	6.945	7.379	7.813	8.247	8.681	.238	4.2
.233	4.3	4.674	5.099	5.524	5.949	6.374	6.799	7.224	7.649	8.074	8.499	.233	4.3
.227	4.4	4.554	4.968	5.382	5.796	6.210	6.624	7.038	7.452	7.866	8.280	.227	4.4
.222	4.5	4.454	4.858	5.263	5.668	6.073	6.478	6.883	7.288	7.693	8.097	.222	4.5
.217	4.6	4.353	4.749	5.145	5.541	5.936	6.332	6.728	7.124	7.519	7.915	.217	4.6
.213	4.7	4.273	4.662	5.050	5.438	5.827	6.215	6.604	6.992	7.381	7.769	.213	4.7
.208	4.8	4.173	4.552	4.931	5.311	5.690	6.069	6.449	6.828	7.207	7.587	.208	4.8
.204	4.9	4.093	4.465	4.837	5.209	5.581	5.953	6.325	6.697	7.069	7.441	.204	4.9
.200	5.0	4.012	4.377	4.742	5.107	5.471	5.836	6.201	6.566	6.930	7.295	.200	5.0

TABLE VI—Continued.

Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S.	Induced angle of attack in degrees. Lift coefficient C_L. 0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S.
0.196	5.1	0.357	0.715	1.072	1.430	1.787	2.145	2.502	2.860	3.217	3.575	0.196	5.1
.192	5.2	.350	.700	1.050	1.401	1.751	2.101	2.451	2.801	3.151	3.502	.192	5.2
.188	5.3	.343	.686	1.029	1.371	1.714	2.057	2.400	2.743	3.086	3.429	.188	5.3
.185	5.4	.337	.675	1.012	1.350	1.687	2.024	2.362	2.699	3.037	3.374	.185	5.4
.182	5.5	.332	.664	.996	1.328	1.660	1.992	2.323	2.655	2.987	3.319	.182	5.5
.179	5.6	.326	.653	.979	1.306	1.632	1.959	2.285	2.612	2.938	3.265	.179	5.6
.175	5.7	.319	.638	.957	1.277	1.596	1.915	2.234	2.553	2.872	3.192	.175	5.7
.172	5.8	.314	.627	.941	1.255	1.568	1.882	2.196	2.510	2.823	3.137	.172	5.8
.169	5.9	.308	.616	.925	1.233	1.541	1.849	2.157	2.466	2.774	3.082	.169	5.9
.166	6.0	.303	.605	.908	1.211	1.514	1.816	2.119	2.422	2.725	3.027	.166	6.0
.164	6.1	.299	.598	.897	1.196	1.495	1.795	2.094	2.393	2.692	2.991	.164	6.1
.161	6.2	.294	.587	.881	1.174	1.468	1.762	2.055	2.349	2.643	2.936	.161	6.2
.159	6.3	.290	.580	.870	1.160	1.450	1.740	2.030	2.320	2.610	2.900	.159	6.3
.156	6.4	.285	.569	.854	1.138	1.423	1.707	1.992	2.276	2.561	2.845	.156	6.4
.154	6.5	.281	.562	.843	1.123	1.404	1.685	1.966	2.247	2.528	2.809	.154	6.5
.152	6.6	.277	.554	.832	1.109	1.386	1.663	1.940	2.218	2.495	2.772	.152	6.6
.149	6.7	.272	.543	.815	1.087	1.359	1.630	1.902	2.174	2.446	2.717	.149	6.7
.147	6.8	.268	.536	.804	1.072	1.340	1.609	1.877	2.145	2.413	2.681	.147	6.8
.145	6.9	.264	.529	.793	1.058	1.322	1.587	1.851	2.116	2.380	2.644	.145	6.9
.143	7.0	.261	.522	.782	1.043	1.304	1.565	1.826	2.086	2.347	2.608	.143	7.0
.140	7.1	.255	.511	.766	1.021	1.277	1.532	1.787	2.043	2.298	2.553	.140	7.1
.139	7.2	.254	.507	.761	1.014	1.268	1.521	1.775	2.028	2.282	2.535	.139	7.2
.137	7.3	.250	.500	.750	.999	1.249	1.499	1.749	1.999	2.249	2.499	.137	7.3
.135	7.4	.246	.492	.739	.985	1.231	1.477	1.723	1.970	2.216	2.462	.135	7.4
.133	7.5	.243	.485	.728	.970	1.213	1.455	1.698	1.940	2.183	2.426	.133	7.5
.132	7.6	.241	.481	.722	.963	1.204	1.444	1.685	1.926	2.167	2.407	.132	7.6
.130	7.7	.237	.474	.711	.948	1.185	1.423	1.660	1.897	2.134	2.371	.130	7.7
.128	7.8	.233	.467	.700	.934	1.167	1.401	1.634	1.868	2.101	2.334	.128	7.8
.127	7.9	.232	.463	.695	.926	1.158	1.390	1.621	1.853	2.085	2.316	.127	7.9
.125	8.0	.228	.456	.684	.912	1.140	1.368	1.596	1.824	2.052	2.280	.125	8.0
.124	8.1	.226	.452	.678	.905	1.131	1.357	1.583	1.809	2.035	2.261	.124	8.1
.122	8.2	.223	.445	.668	.890	1.113	1.335	1.558	1.780	2.003	2.225	.122	8.2
.121	8.3	.221	.441	.662	.883	1.103	1.324	1.545	1.765	1.986	2.207	.121	8.3
.119	8.4	.217	.434	.651	.868	1.085	1.302	1.519	1.736	1.953	2.170	.119	8.4
.118	8.5	.215	.430	.646	.861	1.076	1.291	1.506	1.722	1.937	2.152	.118	8.5
.116	8.6	.212	.423	.635	.846	1.058	1.269	1.481	1.692	1.904	2.116	.116	8.6
.115	8.7	.210	.419	.629	.839	1.049	1.258	1.468	1.678	1.888	2.097	.115	8.7
.114	8.8	.208	.416	.624	.832	1.040	1.247	1.455	1.663	1.871	2.079	.114	8.8
.112	8.9	.204	.409	.613	.817	1.021	1.226	1.430	1.634	1.838	2.043	.112	8.9
.111	9.0	.202	.405	.607	.810	1.012	1.215	1.417	1.619	1.822	2.024	.111	9.0
.110	9.1	.201	.401	.602	.802	1.003	1.204	1.404	1.605	1.805	2.006	.110	9.1
.109	9.2	.199	.398	.596	.795	.994	1.193	1.392	1.590	1.789	1.988	.109	9.2
.107	9.3	.195	.390	.585	.781	.976	1.171	1.366	1.561	1.756	1.951	.107	9.3
.106	9.4	.193	.387	.580	.773	.967	1.160	1.353	1.547	1.740	1.933	.106	9.4
.105	9.5	.191	.383	.574	.766	.957	1.149	1.340	1.532	1.723	1.915	.105	9.5
.104	9.6	.190	.379	.569	.759	.948	1.138	1.328	1.517	1.707	1.897	.104	9.6
.103	9.7	.188	.376	.564	.751	.939	1.127	1.315	1.503	1.691	1.878	.103	9.7
.102	9.8	.186	.372	.558	.744	.930	1.116	1.302	1.488	1.674	1.860	.102	9.8
.101	9.9	.184	.368	.553	.737	.921	1.105	1.289	1.474	1.658	1.842	.101	9.9
.100	10.0	.182	.365	.547	.729	.912	1.094	1.277	1.459	1.641	1.824	.100	10.0

TABLE VI—Continued.

Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S.	Induced angle of attack in degrees.										Area ratio S/k^2b^2.	Aspect ratio k^2b^2/S
		Lift coefficient C_L.											
		1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00		
0.196	5.1	3.932	4.290	4.647	5.004	5.362	5.719	6.077	6.434	6.792	7.149	0.196	5.1
.192	5.2	3.852	4.202	4.552	4.902	5.252	5.603	5.953	6.303	6.653	7.003	.192	5.2
.188	5.3	3.772	4.114	4.457	4.800	5.143	5.486	5.829	6.172	6.515	6.857	.188	5.3
.185	5.4	3.711	4.049	4.386	4.723	5.061	5.398	5.736	6.073	6.410	6.748	.185	5.4
.182	5.5	3.651	3.983	4.315	4.647	4.979	5.311	5.643	5.975	6.306	6.638	.182	5.5
.179	5.6	3.591	3.917	4.244	4.570	4.897	5.223	5.550	5.876	6.203	6.529	.179	5.6
.175	5.7	3.511	3.830	4.149	4.468	4.787	5.107	5.426	5.745	6.064	6.383	.175	5.7
.172	5.8	3.451	3.764	4.078	4.392	4.705	5.019	5.333	5.646	5.960	6.274	.172	5.8
.169	5.9	3.390	3.699	4.007	4.315	4.623	4.931	5.240	5.548	5.856	6.164	.169	5.9
.166	6.0	3.330	3.633	3.936	4.238	4.541	4.844	5.147	5.449	5.752	6.055	.166	6.0
.164	6.1	3.290	3.589	3.888	4.187	4.486	4.785	5.085	5.384	5.683	5.982	.164	6.1
.161	6.2	3.230	3.523	3.817	4.111	4.404	4.698	4.992	5.285	5.579	5.872	.161	6.2
.159	6.3	3.190	3.480	3.770	4.060	4.350	4.640	4.930	5.220	5.510	5.800	.159	6.3
.156	6.4	3.130	3.414	3.699	3.983	4.268	4.552	4.837	5.121	5.406	5.690	.156	6.4
.154	6.5	3.089	3.370	3.651	3.932	4.213	4.494	4.775	5.055	5.336	5.617	.154	6.5
.152	6.6	3.049	3.327	3.604	3.881	4.158	4.435	4.713	4.990	5.267	5.544	.152	6.6
.149	6.7	2.989	3.261	3.533	3.804	4.076	4.348	4.620	4.891	5.163	5.435	.149	6.7
.147	6.8	2.949	3.217	3.485	3.753	4.021	4.289	4.558	4.826	5.094	5.362	.147	6.8
.145	6.9	2.909	3.173	3.438	3.702	3.967	4.231	4.495	4.760	5.024	5.289	.145	6.9
.143	7.0	2.869	3.130	3.390	3.651	3.912	4.173	4.434	4.694	4.955	5.216	.143	7.0
.140	7.1	2.809	3.064	3.319	3.574	3.830	4.085	4.340	4.596	4.851	5.106	.140	7.1
.139	7.2	2.789	3.042	3.296	3.549	3.803	4.056	4.310	4.563	4.817	5.070	.139	7.2
.137	7.3	2.748	2.998	3.248	3.498	3.748	3.998	4.247	4.497	4.747	4.997	.137	7.3
.135	7.4	2.708	2.955	3.201	3.447	3.693	3.939	4.186	4.432	4.678	4.924	.135	7.4
.133	7.5	2.668	2.911	3.153	3.396	3.638	3.881	4.124	4.366	4.609	4.851	.133	7.5
.132	7.6	2.648	2.889	3.129	3.370	3.611	3.852	4.092	4.333	4.574	4.815	.132	7.6
.130	7.7	2.608	2.845	3.082	3.319	3.556	3.793	4.031	4.268	4.505	4.742	.130	7.7
.128	7.8	2.568	2.801	3.035	3.268	3.502	3.735	3.968	4.202	4.435	4.669	.128	7.8
.127	7.9	2.548	2.779	3.011	3.243	3.474	3.706	3.938	4.169	4.401	4.632	.127	7.9
.125	8.0	2.508	2.736	2.964	3.192	3.420	3.648	3.875	4.103	4.331	4.559	.125	8.0
.124	8.1	2.488	2.714	2.940	3.166	3.392	3.618	3.844	4.071	4.297	4.523	.124	8.1
.122	8.2	2.448	2.670	2.903	3.115	3.338	3.560	3.783	4.005	4.228	4.450	.122	8.2
.121	8.3	2.427	2.648	2.869	3.089	3.310	3.531	3.751	3.972	4.193	4.413	.121	8.3
.119	8.4	2.387	2.604	2.821	3.038	3.255	3.472	3.689	3.906	4.123	4.340	.119	8.4
.118	8.5	2.367	2.582	2.798	3.013	3.228	3.443	3.658	3.874	4.089	4.304	.118	8.5
.116	8.6	2.327	2.539	2.750	2.962	3.173	3.385	3.596	3.808	4.019	4.237	.116	8.6
.115	8.7	2.307	2.517	2.726	2.936	3.146	3.356	3.565	3.775	3.985	4.195	.115	8.7
.114	8.8	2.287	2.494	2.703	2.911	3.119	3.327	3.534	3.742	3.950	4.158	.114	8.8
.112	8.9	2.247	2.451	2.655	2.860	3.064	3.268	3.472	3.677	3.881	4.085	.112	8.9
.111	9.0	2.227	2.429	2.632	2.834	3.036	3.239	3.441	3.644	3.846	4.049	.111	9.0
.110	9.1	2.207	2.407	2.608	2.809	3.009	3.210	3.410	3.611	3.812	4.012	.110	9.1
.109	9.2	2.187	2.385	2.584	2.783	2.982	3.181	3.379	3.578	3.777	3.976	.109	9.2
.107	9.3	2.147	2.342	2.537	2.732	2.927	3.122	3.317	3.513	3.708	3.903	.107	9.3
.106	9.4	2.127	2.320	2.513	2.706	2.900	3.093	3.286	3.480	3.673	3.866	.106	9.4
.105	9.5	2.106	2.298	2.489	2.681	2.872	3.064	3.255	3.447	3.638	3.830	.105	9.5
.104	9.6	2.086	2.276	2.466	2.655	2.845	3.035	3.224	3.414	3.604	3.793	.104	9.6
.103	9.7	2.066	2.254	2.442	2.630	2.818	3.005	3.193	3.381	3.569	3.757	.103	9.7
.102	9.8	2.046	2.232	2.418	2.604	2.790	2.976	3.162	3.348	3.534	3.720	.102	9.8
.101	9.9	2.026	2.210	2.395	2.579	2.763	2.947	3.131	3.316	3.500	3.684	.101	9.9
.100	10.0	2.006	2.188	2.371	2.553	2.736	2.918	3.100	3.283	3.465	3.647	.100	10.0

TABLE VII.

Speed m.p.h.	Dynamic pressure in lbs./sq. ft. — Altitude in feet.											Speed m.p.h.
	0	5,000	10,000	15,000	20,000	25,000	30,000	35,000	40,000	45,000	50,000	
1	0.003	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1
2	.010	.009	.007	.006	.005	.005	.004	.003	.003	.002	.002	2
3	.023	.020	.017	.014	.012	.010	.009	.007	.006	.005	.005	3
4	.041	.035	.030	.025	.022	.018	.016	.013	.011	.010	.008	4
5	.064	.055	.047	.040	.034	.029	.024	.021	.018	.015	.013	5
6	.092	.079	.067	.057	.049	.041	.035	.030	.026	.022	.019	6
7	.126	.107	.091	.078	.066	.056	.048	.041	.035	.030	.025	7
8	.164	.140	.119	.101	.086	.074	.063	.053	.045	.039	.033	8
9	.208	.177	.151	.128	.109	.093	.079	.067	.058	.049	.042	9
10	.256	.219	.186	.159	.135	.115	.098	.084	.071	.061	.052	10
11	.310	.264	.225	.192	.164	.139	.119	.101	.086	.073	.063	11
12	.369	.315	.268	.228	.195	.166	.141	.120	.103	.087	.074	12
13	.433	.369	.315	.268	.228	.195	.166	.141	.120	.103	.087	13
14	.503	.428	.365	.311	.265	.226	.192	.164	.140	.119	.101	14
15	.577	.492	.419	.357	.304	.259	.221	.188	.160	.136	.116	15
16	.656	.559	.476	.406	.346	.295	.251	.214	.182	.155	.132	16
17	.741	.631	.538	.458	.390	.333	.283	.242	.206	.175	.149	17
18	.831	.708	.603	.514	.438	.373	.318	.271	.231	.197	.167	18
19	.926	.789	.672	.572	.488	.416	.354	.302	.257	.219	.187	19
20	1.03	.874	.745	.634	.540	.461	.392	.334	.285	.243	.207	20
21	1.13	.963	.821	.699	.596	.508	.433	.369	.314	.268	.228	21
22	1.24	1.06	.901	.768	.654	.557	.475	.404	.345	.294	.250	22
23	1.35	1.16	.985	.839	.715	.609	.519	.442	.377	.321	.273	23
24	1.48	1.26	1.07	.913	.778	.663	.565	.481	.410	.349	.298	24
25	1.60	1.37	1.16	.991	.844	.719	.613	.522	.445	.379	.323	25
26	1.73	1.48	1.26	1.07	.913	.778	.663	.565	.481	.410	.349	26
27	1.87	1.59	1.36	1.16	.985	.839	.715	.609	.519	.442	.377	27
28	2.01	1.71	1.46	1.24	1.06	.903	.769	.655	.558	.476	.405	28
29	2.16	1.84	1.57	1.33	1.14	.968	.825	.703	.599	.510	.435	29
30	2.31	1.97	1.68	1.43	1.22	1.04	.883	.752	.641	.546	.465	30
31	2.46	2.10	1.79	1.52	1.30	1.11	.943	.803	.684	.583	.497	31
32	2.63	2.24	1.91	1.62	1.38	1.18	1.00	.855	.729	.621	.529	32
33	2.79	2.38	2.03	1.73	1.47	1.25	1.07	.910	.775	.660	.563	33
34	2.96	2.53	2.15	1.83	1.56	1.33	1.13	.966	.823	.701	.597	34
35	3.14	2.68	2.28	1.94	1.66	1.41	1.20	1.02	.872	.743	.633	35
36	3.32	2.83	2.41	2.06	1.75	1.49	1.27	1.08	.922	.786	.670	36
37	3.51	2.99	2.55	2.17	1.85	1.58	1.34	1.14	.974	.830	.708	37
38	3.70	3.15	2.69	2.29	1.95	1.66	1.42	1.21	1.03	.876	.716	38
39	3.90	3.32	2.83	2.41	2.06	1.75	1.49	1.27	1.08	.923	.786	39
40	4.10	3.50	2.98	2.54	2.16	1.84	1.57	1.34	1.14	.971	.827	40
41	4.31	3.67	3.13	2.67	2.27	1.94	1.65	1.40	1.20	1.02	.869	41
42	4.52	3.85	3.28	2.80	2.38	2.03	1.73	1.47	1.26	1.07	.911	42
43	4.74	4.04	3.44	2.93	2.50	2.13	1.81	1.55	1.32	1.12	.956	43
44	4.96	4.23	3.60	3.07	2.62	2.23	1.90	1.62	1.38	1.17	1.00	44
45	5.19	4.42	3.77	3.21	2.74	2.33	1.99	1.69	1.44	1.23	1.05	45
46	5.43	4.62	3.94	3.36	2.86	2.44	2.08	1.77	1.51	1.28	1.09	46
47	5.66	4.83	4.11	3.50	2.98	2.54	2.17	1.85	1.57	1.34	1.14	47
48	5.91	5.03	4.29	3.65	3.11	2.65	2.26	1.93	1.64	1.40	1.19	48
49	6.16	5.25	4.47	3.81	3.24	2.76	2.35	2.01	1.71	1.46	1.24	49
50	6.41	5.46	4.65	3.96	3.38	2.88	2.45	2.09	1.78	1.52	1.29	50

TABLE VII—Continued.

Speed m.p.h.	Dynamic Pressure in lbs./sq. ft. — Altitude in feet. 0	5,000	10,000	15,000	20,000	25,000	30,000	35,000	40,000	45,000	50,000	Speed m.p.h.
51	6.67	5.68	4.84	4.12	3.51	2.99	2.55	2.17	1.85	1.58	1.34	51
52	6.93	5.91	5.03	4.29	3.65	3.11	2.65	2.26	1.93	1.64	1.40	52
53	7.20	6.14	5.23	4.45	3.80	3.23	2.76	2.35	2.00	1.70	1.45	53
54	7.48	6.37	5.43	4.62	3.94	3.36	2.86	2.44	2.08	1.77	1.51	54
55	7.76	6.61	5.63	4.80	4.09	3.48	2.97	2.53	2.15	1.83	1.56	55
56	8.04	6.85	5.84	4.97	4.24	3.61	3.08	2.62	2.23	1.90	1.62	56
57	8.33	7.10	6.05	5.15	4.39	3.74	3.19	2.71	2.31	1.97	1.68	57
58	8.63	7.35	6.26	5.33	4.55	3.87	3.30	2.81	2.40	2.04	1.74	58
59	8.93	7.60	6.48	5.52	4.70	4.01	3.41	2.91	2.48	2.11	1.80	59
60	9.23	7.86	6.70	5.71	4.86	4.14	3.53	3.01	2.56	2.18	1.86	60
61	9.54	8.13	6.93	5.90	5.03	4.28	3.65	3.11	2.65	2.26	1.92	61
62	9.86	8.40	7.15	6.10	5.19	4.43	3.77	3.21	2.74	2.33	1.99	62
63	10.17	8.67	7.39	6.29	5.36	4.57	3.89	3.32	2.83	2.41	2.05	63
64	10.50	8.95	7.62	6.50	5.53	4.72	4.02	3.42	2.92	2.48	2.12	64
65	10.83	9.23	7.86	6.70	5.71	4.86	4.14	3.53	3.01	2.56	2.18	65
66	11.17	9.52	8.11	6.91	5.89	5.01	4.27	3.61	3.10	2.64	2.25	66
67	11.51	9.81	8.36	7.12	6.07	5.17	4.40	3.75	3.20	2.72	2.32	67
68	11.85	10.10	8.61	7.33	6.25	5.32	4.54	3.86	3.29	2.80	2.39	68
69	12.21	10.40	8.86	7.55	6.43	5.48	4.67	3.98	3.39	2.89	2.46	69
70	12.56	10.70	9.12	7.77	6.62	5.64	4.81	4.09	3.49	2.97	2.53	70
71	12.93	11.01	9.38	7.99	6.81	5.80	4.94	4.21	3.59	3.06	2.61	71
72	13.29	11.33	9.65	8.22	7.00	5.97	5.08	4.33	3.69	3.14	2.68	72
73	13.65	11.64	9.92	8.45	7.20	6.13	5.23	4.45	3.79	3.23	2.75	73
74	14.04	11.96	10.19	8.68	7.40	6.30	5.37	4.58	3.90	3.32	2.83	74
75	14.42	12.29	10.47	8.92	7.60	6.48	5.52	4.70	4.00	3.41	2.91	75
76	14.81	12.62	10.75	9.16	7.80	6.65	5.66	4.83	4.11	3.50	2.99	76
77	15.20	12.95	11.04	9.40	8.01	6.83	5.82	4.95	4.22	3.60	3.06	77
78	15.60	13.29	11.32	9.65	8.22	7.00	5.97	5.08	4.33	3.69	3.14	78
79	16.00	13.63	11.62	9.90	8.43	7.18	6.12	5.22	4.44	3.79	3.23	79
80	16.41	13.98	11.91	10.15	8.65	7.37	6.28	5.35	4.56	3.88	3.31	80
81	16.82	14.33	12.21	10.40	8.86	7.55	6.43	5.48	4.67	3.98	3.39	81
82	17.24	14.69	12.52	10.66	9.08	7.74	6.59	5.62	4.79	4.08	3.47	82
83	17.66	15.05	12.82	10.92	9.31	7.93	6.76	5.76	4.90	4.18	3.56	83
84	18.09	15.41	13.13	11.19	9.53	8.12	6.92	5.90	5.02	4.28	3.65	84
85	18.53	15.78	13.45	11.46	9.76	8.32	7.09	6.04	5.14	4.38	3.73	85
86	18.96	16.16	13.77	11.73	9.99	8.51	7.25	6.18	5.27	4.49	3.82	86
87	19.41	16.54	14.09	12.00	10.23	8.71	7.42	6.32	5.39	4.59	3.91	87
88	19.86	16.92	14.41	12.28	10.46	8.91	7.60	6.47	5.51	4.70	4.00	88
89	20.31	17.30	11.74	12.56	10.70	9.12	7.77	6.62	5.64	4.80	4.09	89
90	20.77	17.70	15.08	12.85	10.94	9.32	7.96	6.77	5.77	4.91	4.19	90
91	21.23	18.09	15.41	13.13	11.19	9.53	8.12	6.92	5.90	5.02	4.28	91
92	21.70	18.49	15.75	13.42	11.44	9.74	8.30	7.07	6.03	5.13	4.37	92
93	22.18	18.89	16.10	13.72	11.69	9.96	8.48	7.23	6.16	5.25	4.47	93
94	22.66	19.30	16.45	14.01	11.94	10.17	8.67	7.38	6.29	5.36	4.57	94
95	23.14	19.72	16.80	14.31	12.19	10.39	8.85	7.54	6.43	5.47	4.66	95
96	23.63	20.13	17.15	14.61	12.45	10.61	9.01	7.70	6.56	5.59	4.76	96
97	24.13	20.56	17.51	14.92	12.71	10.83	9.23	7.86	6.70	5.71	4.86	97
98	24.63	20.98	17.88	15.23	12.98	11.06	9.42	8.03	6.84	5.83	4.96	98
99	25.13	21.41	18.24	15.51	13.24	11.28	9.61	8.19	6.98	5.94	5.07	99
100	25.64	21.85	18.61	15.86	13.51	11.51	9.81	8.36	7.12	6.07	5.17	100

TABLE VII—Continued.

Speed m.p.h.	Dynamic Pressure in lbs./sq. ft.											Speed m.p.h.
	Altitude in feet.											
	0	5,000	10,000	15,000	20,000	25,000	30,000	35,000	40,000	45,000	50,000	
101	26.16	22.29	18.99	16.18	13.78	11.74	10.00	8.52	7.26	6.19	5.27	101
102	26.68	22.73	19.36	16.50	14.06	11.97	10.20	8.69	7.41	6.31	5.38	102
103	27.20	23.18	19.75	16.82	14.33	12.21	10.40	8.86	7.55	6.43	5.48	103
104	27.73	23.63	20.13	17.15	14.61	12.45	10.61	9.04	7.70	6.56	5.59	104
105	28.27	24.09	20.52	17.48	14.89	12.69	10.81	9.21	7.85	6.69	5.70	105
106	28.81	24.55	20.91	17.82	15.18	12.93	11.02	9.39	8.00	6.82	5.81	106
107	29.36	25.01	21.31	18.16	15.46	13.18	11.23	9.57	8.15	6.94	5.92	107
108	29.91	25.48	21.71	18.50	15.76	13.43	11.44	9.75	8.30	7.07	6.03	108
109	30.46	25.96	22.11	18.84	16.05	13.68	11.65	9.93	8.46	7.21	6.14	109
110	31.03	26.43	22.52	19.19	16.35	13.93	11.87	10.11	8.61	7.34	6.25	110
111	31.59	26.92	22.93	19.54	16.65	14.18	12.08	10.30	8.77	7.47	6.37	111
112	32.16	27.40	23.35	19.89	16.95	14.44	12.30	10.48	8.93	7.61	6.48	112
113	32.74	27.89	23.77	20.25	17.25	14.70	12.52	10.67	9.09	7.74	6.60	113
114	33.32	28.39	24.19	20.61	17.56	14.96	12.75	10.86	9.25	7.88	6.72	114
115	33.91	28.89	24.62	20.97	17.87	15.22	12.97	11.05	9.42	8.02	6.83	115
116	34.50	29.40	25.05	21.34	18.18	15.49	13.20	11.24	9.58	8.16	6.95	116
117	35.10	29.90	25.48	21.71	18.49	15.76	13.43	11.44	9.75	8.30	7.07	117
118	35.70	30.42	25.92	22.08	18.81	16.03	13.65	11.63	9.91	8.45	7.20	118
119	36.31	30.94	26.36	22.46	19.13	16.30	13.89	11.83	10.08	8.59	7.32	119
120	36.92	31.46	26.80	22.84	19.45	16.57	14.12	12.03	10.25	8.73	7.44	120
121	37.54	31.98	27.25	23.22	19.78	16.85	14.36	12.23	10.42	8.88	7.57	121
122	38.16	32.52	27.70	23.60	20.11	17.13	14.60	12.44	10.60	9.03	7.69	122
123	38.79	33.05	28.16	23.99	20.44	17.41	14.84	12.64	10.77	9.17	7.82	123
124	39.43	33.59	28.62	24.38	20.77	17.70	15.08	12.84	10.94	9.32	7.94	124
125	40.06	34.13	29.08	24.78	21.11	17.98	15.32	13.05	11.12	9.48	8.07	125
126	40.71	34.68	29.55	25.17	21.45	18.27	15.57	13.26	11.30	9.63	8.20	126
127	41.36	35.24	30.02	25.57	21.79	18.56	15.82	13.47	11.48	9.78	8.33	127
128	42.01	35.79	30.50	25.98	22.14	18.86	16.07	13.67	11.66	9.94	8.47	128
129	42.67	36.35	30.97	26.39	22.48	19.15	16.32	13.90	11.84	10.09	8.60	129
130	43.33	36.92	31.46	26.80	22.83	19.45	16.57	14.12	12.03	10.25	8.73	130
131	44.00	37.49	31.94	27.21	23.18	19.75	16.83	14.34	12.22	10.41	8.87	131
132	44.68	38.06	32.43	27.63	23.54	20.06	17.09	14.56	12.40	10.57	9.00	132
133	45.36	38.64	32.92	28.05	23.90	20.36	17.35	14.78	12.59	10.73	9.14	133
134	46.04	39.23	33.42	28.47	24.26	20.67	17.61	15.00	12.78	10.89	9.28	134
135	46.73	39.81	33.92	28.90	24.62	20.98	17.87	15.22	12.97	11.05	9.42	135
136	47.43	40.41	34.43	29.33	24.99	21.29	18.14	15.45	13.17	11.22	9.56	136
137	48.13	41.00	34.93	29.76	25.36	21.60	18.41	15.68	13.36	11.38	9.70	137
138	48.83	41.60	35.45	30.20	25.73	21.92	18.68	15.91	13.56	11.55	9.84	138
139	49.54	42.21	35.96	30.64	26.10	22.24	18.95	16.14	13.75	11.72	9.98	139
140	50.26	42.82	36.48	31.08	26.48	22.56	19.22	16.37	13.95	11.89	10.13	140
141	50.98	43.43	37.00	31.52	26.86	22.88	19.50	16.61	14.15	12.06	10.27	141
142	51.70	44.05	37.53	31.97	27.24	23.21	19.78	16.84	14.35	12.23	10.42	142
143	52.43	44.67	38.06	32.42	27.63	23.54	20.06	17.08	14.56	12.40	10.57	143
144	53.17	45.30	38.60	32.88	28.01	23.87	20.34	17.32	14.76	12.57	10.71	144
145	53.91	45.93	39.13	33.34	28.41	23.77	20.62	17.56	14.97	12.75	10.86	145
146	54.66	46.57	39.68	33.80	28.80	24.10	20.91	17.81	15.17	12.93	11.01	146
147	55.41	47.21	40.22	34.26	29.19	24.43	21.19	18.05	15.38	13.10	11.16	147
148	56.16	47.85	40.77	34.73	29.59	24.76	21.48	18.30	15.59	13.28	11.32	148
149	56.93	48.50	41.32	35.20	29.99	25.10	21.77	18.55	15.80	13.46	11.47	149
150	57.69	49.15	41.88	35.68	30.40	25.44	22.07	18.80	16.02	13.64	11.62	150

TABLE VII—Continued.

Speed m.p.h.	Dynamic pressure in lbs./sq. ft. — Altitude in feet. 0	5,000	10,000	15,000	20,000	25,000	30,000	35,000	40,000	45,000	50,000	Speed m.p.h.
151	58.46	49.81	42.44	36.15	30.80	25.78	22.36	19.05	16.23	13.83	11.78	151
152	59.24	50.47	43.00	36.63	31.21	26.12	22.66	19.30	16.45	14.01	11.94	152
153	60.02	51.1[illegible]	43.57	37.12	31.63	26.46	22.96	19.56	16.66	14.20	12.09	153
154	60.81	51.82	44.14	37.60	32.04	26.81	23.26	19.81	16.88	14.38	12.25	154
155	61.[illegible]	52.49	44.72	38.09	32.46	27.16	23.56	20.07	17.11	14.57	12.41	155
156	62.40	53.16	45.30	38.59	32.88	27.51	23.87	20.33	17.32	14.76	12.57	156
157	63.20	53.85	45.88	39.08	33.30	27.87	24.17	20.59	17.55	14.95	12.74	157
158	64.01	54.54	46.46	39.58	33.73	28.22	24.48	20.85	17.77	15.14	12.90	158
159	64.82	55.23	47.06	40.09	34.16	28.58	24.79	21.12	17.99	15.33	13.06	159
160	65.64	55.93	47.65	40.59	34.59	28.94	25.11	21.39	18.22	15.52	13.23	160
161	66.46	56.63	48.25	41.10	35.02	29.30	25.42	21.65	18.45	15.72	13.39	161
162	67.29	57.33	48.85	41.61	35.46	29.67	25.74	21.92	18.68	15.91	13.56	162
163	68.13	58.04	49.45	42.13	35.90	30.04	26.06	22.20	18.91	16.11	13.73	163
164	68.96	58.76	50.06	42.65	36.34	30.41	26.38	22.47	19.14	16.31	13.90	164
165	69.81	59.48	50.67	43.17	36.78	30.78	26.70	22.74	19.38	16.51	14.07	165
166	70.66	60.20	51.29	43.69	37.23	31.15	27.03	23.02	19.61	16.71	14.24	166
167	71.51	60.93	51.91	44.22	37.68	32.10	27.35	23.30	19.85	16.91	14.41	167
168	72.37	61.66	52.53	44.75	38.13	32.49	27.68	23.58	20.09	17.12	14.58	168
169	73.23	62.39	53.16	45.29	38.59	32.87	28.01	23.86	20.33	17.32	14.76	169
170	74.10	63.14	53.79	45.82	39.04	33.26	28.34	24.14	20.57	17.53	14.93	170
171	74.98	63.88	54.43	46.37	39.51	33.66	28.68	24.43	20.81	17.73	15.11	171
172	75.86	64.63	55.06	46.91	39.97	34.05	29.02	24.71	21.06	17.94	15.28	172
173	76.74	65.38	55.71	47.46	40.43	34.45	29.35	25.00	21.30	18.15	15.46	173
174	77.63	66.14	56.35	48.00	40.90	34.85	29.69	25.29	21.55	18.36	15.64	174
175	78.53	66.90	57.00	48.56	41.38	35.25	30.04	25.58	21.80	18.57	15.82	175
176	79.43	67.67	57.65	49.12	41.85	35.65	30.38	25.91	22.05	18.78	16.00	176
177	80.33	68.44	58.31	49.68	42.33	36.06	30.73	26.17	22.30	19.00	16.19	177
178	81.24	69.22	58.97	50.24	42.81	36.47	31.07	26.47	22.55	19.21	16.37	178
179	82.16	70.00	59.64	50.81	43.29	36.88	31.42	26.77	22.81	19.43	16.55	179
180	83.08	70.78	60.31	51.37	43.77	37.29	31.78	27.07	23.06	19.65	16.74	180
181	84.00	71.57	60.98	51.95	44.26	37.71	32.13	27.37	23.32	19.87	16.93	181
182	84.93	72.36	61.65	52.52	44.75	38.13	32.49	27.67	23.58	20.09	17.11	182
183	85.87	73.16	62.33	53.10	45.24	38.55	32.84	27.98	23.84	20.31	17.30	183
184	86.81	73.96	63.02	53.68	45.74	38.97	33.20	28.28	24.10	20.53	17.49	184
185	87.76	74.77	63.70	54.27	46.24	39.39	33.57	28.59	24.36	20.75	17.68	185
186	88.71	75.58	64.39	54.86	46.74	39.82	33.93	28.90	24.63	20.98	17.87	186
187	89.66	76.39	65.09	55.45	47.24	40.25	34.30	29.21	24.89	21.21	18.07	187
188	90.63	77.21	65.79	56.04	47.75	40.68	34.67	29.53	25.16	21.43	18.26	188
189	91.59	78.04	66.49	56.64	48.26	41.12	35.03	29.84	25.43	21.66	18.46	189
190	92.56	78.86	67.19	57.24	48.77	41.55	35.41	30.16	25.70	21.89	18.65	190
191	93.54	79.70	67.90	57.85	49.29	41.99	35.78	30.48	25.97	22.12	18.85	191
192	94.52	80.53	68.61	58.45	49.80	42.43	36.16	30.80	26.24	22.35	19.05	192
193	95.51	81.38	69.33	59.06	50.32	42.87	36.53	31.12	26.51	22.59	19.25	193
194	96.50	82.22	70.05	59.68	50.85	43.32	36.91	31.44	26.79	22.82	19.45	194
195	97.50	83.07	70.78	60.29	51.37	43.77	37.29	31.77	27.07	23.06	19.65	195
196	98.50	83.92	71.50	60.91	51.90	44.22	37.68	32.09	27.34	23.30	19.85	196
197	99.51	84.78	72.23	61.54	52.43	44.67	38.06	32.42	27.62	23.53	20.05	197
198	100.52	85.64	72.97	62.16	52.97	45.12	38.45	32.75	27.90	23.77	20.26	198
199	101.54	86.51	73.71	62.79	53.50	45.58	38.84	33.08	28.19	24.01	20.46	199
200	102.56	87.38	74.45	63.43	54.04	46.04	39.23	33.42	28.47	24.25	20.66	200

TABLE VII—Continued.

Speed m.p.h.	Dynamic pressure in lbs./sq. ft.											Speed m.p.h.
	Altitude in feet.											
	0	5,000	10,000	15,000	20,000	25,000	30,000	35,000	40,000	45,000	50,000	
201	103.59	88.26	75.20	64.06	54.58	46.50	39.62	33.75	28.76	24.50	20.87	201
201	104.63	89.14	75.95	64.70	55.13	46.97	40.02	34.09	29.04	24.74	21.08	202
203	105.66	90.02	76.70	65.34	55.67	47.43	40.42	34.43	29.33	24.99	21.29	203
204	106.71	90.92	77.46	65.99	56.22	47.90	40.82	34.77	29.62	25.24	21.50	204
205	107.76	91.81	78.22	66.64	56.78	48.37	41.22	35.11	29.91	25.48	21.71	205
206	108.81	92.71	78.99	67.29	57.33	48.84	41.62	35.45	30.21	25.73	21.93	206
207	109.87	93.61	79.75	67.94	57.89	49.32	42.02	35.80	30.50	25.98	22.14	207
208	110.93	94.51	80.53	68.60	58.45	49.80	42.43	36.14	30.80	26.24	22.35	208
209	112.00	95.43	81.30	69.26	59.01	50.28	42.84	36.49	31.09	26.49	22.57	209
210	113.08	96.34	82.08	69.93	59.58	50.76	43.25	36.84	31.39	26.74	22.78	210
211	114.16	97.26	82.87	70.59	60.15	51.24	43.66	37.19	31.69	27.00	23.00	211
212	115.24	98.18	83.65	71.26	60.72	51.73	44.08	37.55	31.99	27.02	23.22	212
213	116.33	99.11	84.44	71.94	61.29	52.22	44.50	37.90	32.29	27.51	23.44	213
214	117.43	100.05	85.24	72.62	61.87	52.71	44.92	38.26	32.60	27.77	23.66	214
215	118.53	100.98	86.04	73.30	62.45	53.21	45.34	38.62	32.90	28.03	23.88	215
216	119.63	101.92	86.84	73.98	63.03	53.70	45.76	38.98	33.21	28.29	24.11	216
217	120.74	102.87	87.65	74.67	63.62	54.20	46.18	39.34	33.52	28.56	24.33	217
218	121.86	103.82	88.46	75.36	64.21	54.70	46.61	39.70	33.83	28.82	24.56	218
219	122.98	104.78	89.27	76.05	64.80	55.20	47.04	40.07	34.14	29.08	24.78	219
220	124.10	105.73	90.09	76.74	65.39	55.71	47.47	40.43	34.45	29.35	25.01	220
221	125.23	106.70	90.91	77.44	65.99	56.22	47.90	40.80	34.76	29.62	25.23	221
222	126.37	107.67	91.73	78.15	66.58	56.73	48.34	41.17	35.08	29.89	25.46	222
223	127.51	108.64	92.56	78.85	67.19	57.24	48.77	41.54	35.40	30.16	25.69	223
224	128.66	109.61	93.39	79.56	67.79	57.75	49.21	41.92	35.71	30.43	25.92	224
225	129.81	110.60	94.23	80.27	68.40	58.27	49.65	42.29	36.03	30.70	26.16	225
226	130.96	111.58	95.07	80.99	69.00	58.79	50.09	42.67	36.36	30.97	26.39	226
227	132.13	112.57	95.91	81.71	69.62	59.31	50.54	43.05	36.68	31.25	26.62	227
228	133.29	113.56	96.76	82.43	70.23	59.83	50.98	43.43	37.00	31.52	26.86	228
229	134.46	114.56	97.61	83.15	70.85	60.36	51.43	43.81	37.33	31.80	27.09	229
230	135.64	115.57	98.46	83.88	71.47	60.89	51.88	44.19	37.65	32.08	27.33	230
231	136.82	116.57	99.32	84.61	72.09	61.42	52.33	44.58	37.98	32.36	27.57	231
232	138.01	117.58	100.18	85.35	72.72	61.95	52.79	44.96	38.31	32.64	27.81	232
233	139.20	118.60	101.05	86.08	73.35	62.49	53.24	45.24	38.56	32.85	28.05	233
234	140.40	119.62	101.92	86.83	73.98	63.03	53.70	45.63	38.89	33.13	28.29	234
235	141.60	120.64	102.79	87.57	74.61	63.57	54.16	46.02	39.22	33.42	28.53	235
236	142.81	121.67	103.67	88.31	75.25	64.11	54.62	40.41	39.56	33.70	28.78	236
237	144.02	122.71	104.55	89.06	75.89	64.65	55.09	46.81	39.89	33.99	29.02	237
238	145.24	123.74	105.43	89.82	76.53	65.20	55.55	47.20	40.23	34.28	29.27	238
239	146.46	124.79	106.32	90.57	77.17	65.75	56.02	47.60	40.57	34.57	29.51	239
240	147.69	125.83	107.21	91.33	77.82	66.30	56.49	48.00	40.91	34.86	29.76	240
241	148.93	126.88	108.10	92.10	78.47	66.85	56.96	48.40	41.25	35.15	30.01	241
242	150.16	127.94	109.00	92.86	79.12	67.41	57.44	48.80	41.60	35.44	30.26	242
243	151.41	129.00	109.91	93.63	79.78	67.97	57.91	49.21	41.94	35.73	30.51	243
244	152.66	130.06	110.81	94.40	80.43	68.52	58.39	49.61	42.29	36.03	30.76	244
245	153.91	131.13	111.72	95.18	81.10	69.10	58.87	50.02	42.63	36.32	31.01	245
246	155.17	132.20	112.64	95.96	81.76	69.66	59.35	50.43	42.98	36.62	31.27	246
247	156.43	133.28	113.55	96.74	82.42	70.22	59.84	50.84	43.33	36.92	31.52	247
248	157.70	134.36	114.48	97.52	83.09	70.79	60.32	51.25	43.68	37.22	31.78	248
249	158.98	135.45	115.40	98.31	83.76	71.36	60.81	51.67	44.04	37.52	32.03	249
250	160.26	136.54	116.33	99.10	84.44	71.94	61.30	52.08	44.39	37.82	32.29	250

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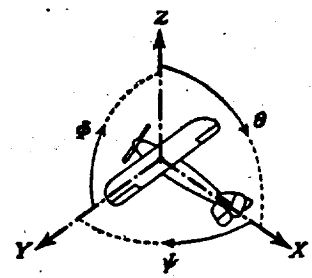

Positive directions of axes and angles (forces and moments) as shown by arrows.

Axis.		Force (parallel to axis) symbol.	Moment about axis.			Angle.		Velocities.	
Designation.	Symbol.		Designation.	Symbol.	Positive direction.	Designation.	Symbol.	Linear (component along axis).	Angular.
Longitudinal	X	X	rolling	L	Y ⟶ Z	roll	Φ	u	p
Lateral	Y	Y	pitching	M	Z ⟶ X	pitch	θ	v	q
Normal	Z	Z	yawing	N	X ⟶ Y	yaw	Ψ	w	r

Absolute coefficients of moment

$$C_l=\frac{L}{q\,b\,S},\ C_m=\frac{M}{q\,c\,S},\ C_n=\frac{N}{q\,f\,S}$$

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS.

Diameter, D.
Pitch (a) Aerodynamic pitch, p_a
(b) Effective pitch, p_e
(c) Geometric pitch, p_g
Pitch ratio, p/D
Inflow velocity, V′
Slip-stream velocity, V_s
Thrust, T

Torque, Q
Power, P
(If "coefficients" are introduced all units used must be consistent.)
Efficiency $\eta = T\,V/P$
Revolutions per sec., n; per min., N.
Effective helix angle $\Phi = \frac{V}{\pi D n}$

5. NUMERICAL RELATIONS.

1 HP = 76 kg. m/sec. = 550 lb. ft/sec.
1 kg. m/sec. = 0.01315 HP
1 mi/hr. = 0.4470 m/sec.
1 m/sec. = 2.237 mi/hr.

1 lb. = 0.4536 kg.
1 kg. = 2.204 lb.
1 mi. = 1609 m. = 5280 ft.
1 m. = 3.281 ft.