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REPORT No. 165

DIAPHRAGMS FOR AERONAUTIC INSTRUMENTS

By M. D. HERSEY

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AERONAUTICAL SYMBOLS.

1. FUNDAMENTAL AND DERIVED UNITS.

	Symbol.	Metric.		English.	
		Unit.	Symbol.	Unit.	Symbol.
Length	l	meter	m.	foot (or mile)	ft. (or mi.).
Time	t	second	sec.	second (or hour)	sec. (or hr.).
Force	F	weight of one kilogram	kg.	weight of one pound	lb.
Power	P	kg.m/sec		horsepower	HP
Speed		m/sec	m. p. s.	mi/hr	M. P. H.

2. GENERAL SYMBOLS, ETC.

Weight, $W = mg$.

Standard acceleration of gravity, $g = 9.806\text{m/sec.}^2 = 32.172\text{ft/sec.}^2$

Mass, $m = \frac{W}{g}$

Density (mass per unit volume), ρ

Standard density of dry air, 0.1247 (kg.-m.-sec.) at 15.6°C. and 760 mm. = 0.00237 (lb.-ft.-sec.)

Specific weight of "standard" air, 1.223 kg/m.^{3} = 0.07635 lb/ft.^{3}

Moment of inertia, mk^2 (indicate axis of the radius of gyration, k, by proper subscript).

Area, S; wing area, S_w, etc.

Gap, G

Span, b; chord length, c.

Aspect ratio $= b/c$

Distance from $c.\ g.$ to elevator hinge, f.

Coefficient of viscosity, μ.

3. AERODYNAMICAL SYMBOLS.

True airspeed, V

Dynamic (or impact) pressure, $q = \frac{1}{2}\rho V^2$

Lift, L; absolute coefficient $C_L = \frac{L}{qS}$

Drag, D; absolute coefficient $C_D = \frac{D}{qS}$.

Cross-wind force, C; absolute coefficient

$$C_c = \frac{C}{qS}.$$

Resultant force, R

(Note that these coefficients are twice as large as the old coefficients L_c, D_c.)

Angle of setting of wings (relative to thrust line), i_w

Angle of stabilizer setting with reference to thrust line i_t

Dihedral angle, γ

Reynolds Number $= \rho\frac{Vl}{\mu}$, where l is a linear dimension.

e. g., for a model airfoil 3 in. chord, 100 mi/hr., normal pressure, 0°C: 255,000 and at 15.6°C, 230,000;

or for a model of 10 cm. chord, 40 m/sec., corresponding numbers are 299,000 and 270,000.

Center of pressure coefficient (ratio of distance of C. P. from leading edge to chord length), C_p.

Angle of stabilizer setting with reference to lower wing. $(i_t - i_w) = \beta$

Angle of attack, α

Angle of downwash, ϵ

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Bureau of Standards

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TABLE OF CONTENTS.

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DIAPHRAGMS FOR AERONAUTIC INSTRUMENTS.

By Mayo D. Hersey.

RÉSUMÉ.

This report was prepared by the Bureau of Standards at the request of the National Advisory Committee for Aeronautics on the subject of diaphragms for aeronautic instruments. Flexible diaphragms actuated by hydrostatic pressure form an essential element of a great variety of instruments for aeronautic and other technical purposes. The various physical data needed as a foundation for rational methods of diaphragm design have not, however, been available hitherto except in the most fragmentary form.

The report comprises an outline of historical developments and theoretical principles, together with a discussion of expedients for making the most effective use of existing diaphragms, and a summary of experimental research problems. In connection with the material for this report the author is much indebted to Mr. H. N. Eaton and Mr. H. B. Henrickson of the Bureau of Standards.

INTRODUCTORY DETAILS.

The present use of diaphragms in the more familiar aeronautic instruments may be seen from Table I. This table shows that the closed capsule with corrugated top and bottom is the most frequent form of element; and that nickel brass(=nickel-silver=German Silver=neusilber=maillechort) has been by far the favorite material for metallic diaphragms. Further information concerning the form and action of diaphragm elements may be found in connection with the descriptions of individual instruments previously published.[1] Different sizes and types of corrugations are shown in Figure 1.

TABLE I.

TYPES OF DIAPHRAGMS IN AERONAUTIC INSTRUMENTS.

Instrument.	Diaphragm element.		
	Action.	Form.	Material.
Altimeter	Elastic with control spring	Corrugated capsule	Nickel-brass; rarely brass or steel.
Barograph	do	Same in multiple	Nickel-brass.
Venturi air-speed indicator	do	do	Do.
Oxygen regulator	do	do	Do.
Water ballast gauge	do	do	Do.
Statoscope	Elastic, self-acting	Single corrugated diaphragm	Nickel-brass, and steel.
Rate-of-climb indicator	do	do	Nickel-brass; phosphor bronze.
Pitot air-speed indicator	do	Corrugated capsule	Nickel-brass, also silver.
Gasoline depth gauge	do	do	Nickel-brass.
Gas bag manometer	do	do	Do.
Pitot air-speed indicator	do	Same in multiple	Do.
Statoscope	do	Flat disk	Rubber.
N. A. C. A. air-speed recorder	do	do	Steel.
Pitot air-speed indicator	do	do	Rubber.
Pitot air-speed indicator	Slack with spring control	Flat, annular	Doped fabric.
Toussaint-Lepère air-speed recorder.	do	Multiple like bellows	Rubberized silk.
N. A. C. A. yawmeter	Slack, balanced	Flat, annular	Fabric.

[1] General Report on Aeronautic Instruments by the Bureau of Standards, comprising Reports Nos. 125–132, inclusive, National Advisory Committee for Aeronautics. See in particular Reports Nos. 126, Parts I, II, and III; 127, Part I;, 129, Parts III, IV, and V; and 130.

The effective stiffness of individual diaphragms at full scale deflection (total force on the surface of the diaphragm due to hydrostatic pressure, divided by the corresponding deflection of the center of the diaphragm) varies from about 200 kg./cm. (over 1,000 lbs./in.) in the case of altimeters with corrugated metallic diaphragms, down to zero for slack fabric diaphragms.

The resultant stiffness of the complete elastic system comprising diaphragms and springs in combination has been measured at full scale deflection in a number of instances. The results are given in Table II. The numbers in the first column refer to the different types of elastic action previously noted in Table I in which the elastic diaphragm with spring control may be designated as class 1, the self-acting elastic diaphragm as class 2, the slack diaphragm with spring control as class 3, and the slack diaphragm with balanced action (the diaphragm being brought into equilibrium by the action of opposing forces without any elastic reaction) as class 4. The figures given for effective stiffness in the last column refer to the resultant stiffness of the complete spring and diaphragm combination, in the case of elastic systems of classes 1 and 3. The stiffness given for class 2 instruments likewise refers to the entire diaphragm element; in any such case the stiffness of an individual diaphragm may be computed by multiplying the observed stiffness by the number of diaphragms; for example, in the only such case occurring in the present table, that of the Sperry air-speed indicator, the stiffness of an individual diaphragm would be 5.9×2, or 11.8 kg./cm.

TABLE II.

DIAPHRAGM DATA FOR PARTICULAR INSTRUMENTS.

Elastic action.	Name and range.	Number of diaphragms.	Diaphragm diameter, cm.	Deflection, per cent diameter.	Stiffness, kg./cm.
2	Bristol water ballast indicator, 50 inches	4	5.1	2.2	235.0
1	Tycos altimeter, 20,000 feet	2	5.0	1.4	196.0
2	Bristol air-speed indicator, 160 m. p. h.	4	5.1	1.4	59.0
1	de Giglio barograph, 6,000 meters	4	4.5	9.1	28.0
2	B. S. rate-of-climb indicator, ±3,000 ft./min	1	14.0	.57	19.0
3	Atmos air-speed indicator (fabric), 300 km/hr	1	7.0	3.0	12.0
1	Dreyer oxygen regulator, 25,000 feet	14	5.0	23.4	10.0
2	Sperry air-speed indicator, 160 m. p. h.	2	7.0	3.0	5.9
1	Foxboro air-speed indicator, 160 m. p. h.	14	2.54	9.1	4.5
2	Statoscope, 200 feet	1	9.4	2.1	2.8
3	Smith gas bag manometer (fabric), 80 mm. water.	1	11.4	2.6	2.7
2	Ogilvie air-speed indicator (rubber), 160 m. p. h.	1	8.4	13.8	1.5
4	N. A. C. A. yawmeter	1	......	......	0

Table II is arranged with the instruments in descending order of stiffness at full scale deflection. The greater the stiffness for a given deflection, the greater will be the force available to overcome friction in the instrument; in fact, the total force actuating each instrument at full scale deflection is proportional to the product of the figures given in the last three columns. The effective stiffness of a given diaphragm is a maximum at full scale deflection and considerably less for small deflections, depending on the relative deflection of the diaphragm as a fraction of its diameter. Under otherwise similar conditions the effective stiffness approaches a constant value, which may be termed the initial stiffness, as the relative deflection approaches zero. Table II shows data on various representative instruments containing from 1 to 14 individual diaphragms, with diameters ranging from about 2.5 to 14 cms. and with full scale deflections between 0.6 per cent and 23 per cent of the diameter.

In addition to diaphragms properly so-called (see Fig. 1) Bourdon tubes and helical tubes have been employed in altimeters, pressure gauges, thermometers, and thermographs; and sylphons (a patented form) have been used in various experimental instruments. These elements present very much the same physical problems as do the ordinary diaphragms.

Diaphragms are constructed by spinning, by stamping, or by hydrostatic pressure. The spinning is done by hand in a lathe, by pressing the soft metal against a corrugated form. This results in an advantageous degree of hardening, and permits some control of the thickness of the finished metal. By suitable variation of thickness as a function of the radius it has also been possible to modify somewhat the form of the load-deflection curve, an important matter in connection with scale uniformity. Spinning is now generally preferred in spite of the simplicity of the stamping process for purposes of quantity production. Some manufacturers stamp the diaphragms first and finish by spinning. In constructing closed capsules or boxes the usual practice has been to turn up the edges of the two halves so they will overlap nearly the full depth of the box, one fitting inside the other, and then supply a liberal amount of solder. Other devices have been studied in search of improvement. Naturally the elastic performance of diaphragms depends very much on the manner of supporting the edges.

Such being the present day practice in the use and construction of diaphragms, it may be added that diaphragms are designed by trial and rejection following traditional patterns, and that all diaphragm instruments are subject to troublesome and obscure sources of error. Hence it comes about that a vast number of questions have presented themselves to the designer and investigator, of which the following are fair specimens:

1. What is the explanation for the failure of diaphragm instruments to repeat their indications under seemingly similar conditions? Are these inconsistencies truly erratic, or can they be attributed to a small number of definite performance characteristics, and how may these be so chosen as to reduce to a minimum number?

2. To what extent are the foregoing errors due to the diaphragm itself? How is the action of a diaphragm modified when coupled to a spring?

3. Is it true old diaphragms are better than new ones? If so, in what precise respect? Is there any way to accelerate aging artificially?

4. How do the qualities of spun diaphragms compare with stamped diaphragms? What are the results of different methods of fastening the edge?

5. What effect on the flexibility of a diaphragm is produced by corrugating the metal; also by varying the diameter of the solid central disk? How can one determine the best thickness and diameter of a metallic diaphragm?

6. Over how great a deflection range will the load-deflection curve of a diaphragm remain sensibly linear? What are the permissible stresses in diaphragm metals? At what point in the diaphragm will the greatest stress be found, and how may one hope to determine its actual magnitude?

7. Is there any connection between the two temperature coefficients of a diaphragm instrument, namely, the change of zero and change of sensitivity with temperature? Can the effect of temperature on a coupled system (spring plus diaphragm) be predicted from the separate temperature characteristics of the component elastic parts? Do all diaphragms become stiffer when chilled? Is there any prospect of discovering alloys which would make the stiffness of a diaphragm intrinsically independent of temperature—that is, without external mechanism? To what extent are the various elastic lag errors (irreversible effects due to imperfect elasticity) influenced by temperature?

8. To what extent are elastic lag errors mutually interdependent? What sort of theoretical or experimental researches would be necessary to establish such correlations if they do exist? To what extent are elastic lag errors determined solely by the properties of the diaphragm material, and to what extent are they influenced by the mechanical design of the diaphragm?

9. Why are steel diaphragms so rarely used? Is there any scientific evidence in support of the prevailing use of German silver? What can be said of the relative merits of numerous other alloys which may be suggested?

10. In regard to elastic lag errors, how does the variation produced in a given material by different methods of heat treatment compare in general magnitude with the variations due to the use of different materials treated in the same way? How may investigations of heat treatment, mechanical treatment, and chemical composition be planned so as to lead to some conclusive and useful information in a reasonable time?

11. Is there any way of designing an automatic mechanism to compensate for elastic lag effects? In general, how can the most effective use be made of existing diaphragms in the design of any given instrument? With a given instrument, is there any option as to the manner

in which the requisite observations may be made, or the surrounding conditions controlled so as to favor the instrument and secure a maximum degree of accuracy?

It is expected that some of these questions can be given a practical answer in the present series of reports, and that a reasonable method of attack can be indicated for the others.

HISTORICAL OUTLINE.

The more significant items of progress in the study of instrument diaphragms may be briefly reviewed in four groups, namely, the earliest observations and developments (1798–1881); the first systematic investigations (1881–1905); experiments at the Bureau of Standards since 1911; and recent work abroad.

EARLIEST OBSERVATIONS AND DEVELOPMENTS.

M. Conté,[2] director of the aerostatical school at Meudon, constructed in 1798 a small pocket barometer with a very thin steel-foil diaphragm supported on numerous springs. This diaphragm was convex in form, and without corrugations; it extended clear across the heavy concave metal case, from which the air was pumped out; and its deformation under changing external pressure served to actuate the pointer in much the same manner as with present-day altimeters. The temperature errors of this ingenious instrument proved to be so great, however, that its use in balloon ascensions was later abandoned.

Vidi,[3] in 1847, independently invented a similar but more successful instrument, for which he coined the name "aneroid" (from the Greek meaning without fluid). A vacuum box or capsule with rigid base but flexible top was used, this latter surface consisting of a corrugated metallic diaphragm, supported externally on a helical spring. Temperature compensation depended on admitting a suitable amount of air into the capsule before sealing up. Vidi is commonly regarded as the inventor of the modern aneroid barometer, and indeed no radical change has been made in the commercial design of this instrument since, though its accuracy has been improved by minor modifications.

Dent,[4] in 1849, described what appears to have been the first experiment for determining the *effective area* of a corrugated diaphragm (equivalent piston area). A diaphragm 2½ inches in diameter would have a total load of 73 pounds under normal sea-level pressure. This, he found, was supported in a spring balance showing 44 pounds. Hence the effective area was about five-eighths of the diaphragm area.

Professor Lovering,[5] of Harvard University, in 1849, was the first to discover elastic lag in aneroids, as caused by large or rapid changes in pressure; for at the conclusion of his experiments on ordinary calibration errors he gives a table of results to show "with what fidelity and dispatch the index returned to its original position when the original pressure was restored."

Secular changes were first scientifically recorded by Kämtz [6] in 1861 in connection with his Goldschmid aneriod, a type in which any erratic errors due to the multiplying mechanism would be particularly small. He found a progressive change of the order of 20 mms. in two years, which was verified by laboratory tests at the factory, and which he attributed to the changing elastic quality of the metal. This change appeared to diminish exponentially. He also studied the temperature coefficient, and similar observations on both these subjects have been made by various writers since.

In 1867 a number of experiments were made with aneroids at Kew Observatory by Stewart,[7] in the course of which he supplemented Lovering's observations by discovering the difference in the calibration curve according as the pressure is changed rapidly or slowly. He concluded that large aneroids were more reliable than small ones, and that aneroids should be in a quiescent state before using. His observations showed a difference of about 2 per cent between rising and

[2] Bulletin des Sciences (Soc. Philomathique) Paris, vol. 1, Floreal, An. 6 (=1798-99), p. 106.

[3] Comptes Rendus d. l'Acad. d. Sci., Paris, v. 24, p. 975, 1847.

[4] E. J. Dent, A Treatise on the Aneroid: 34 pp.; published by the author, London, 1849.

[5] J. Lovering: Remarks on the Aneroid Barometer, Proc. Am. Acad. Arts and Sci., Vol. II, 1849; Am. Jour. Sci., Vol. IX; p. 249, 1850; Fortschritte d. Phys. Bd. VI, VII; 1850+.

[6] L. F. Kämtz, Ueber ein von Goldschmid in Zurich construirtes aneroid, Barometer, Repertorium f. Meteorologie Vol. II; 241-245, 1861.

[7] B. Stewart: Experiments on Aneroids at Kew Observatory, B. A. Report 1867; Phil. Mag. 37: 65-74, 1869; Proc. Roy. Soc. 16:472-480, 1869

falling readings. Similar observations with various types of aneroids have been published and compared by subsequent writers, but need not be enumerated here.

A new form of diaphragm element somewhat resembling the modern "sylphon" (See Fig. 1) was patented by Möller, constructed by Holstein, and described by Kleeman[8] in 1881. It differed from the sylphon in that the component units were separately stamped first and joined together later, instead of being pressed out of a single piece of metal; but it resembled the sylphon in its accordionlike action. The stiffness was sufficiently great, due to the thickness of the metal, to dispense with the complication of a steel spring. This is not ordinarily true for sylphons. The Möller-Holstein instrument is an ingenious solution of the problem of designing a diaphragm element of the self-acting class (i.e., without spring) which will be stiff enough and at the same time will give a sufficient range of deflection to serve in an altimeter. But in view of the difficulties experienced by diaphragm investigators of the present day in repeating observations on a given diaphragm when tested twice in succession, it may be suggested that the author of the above description ventured too far when he asserted that the instruments would not need individual calibration because they were all made alike.

FIRST SYSTEMATIC INVESTIGATIONS

Hartl[9] in 1881 published an investigation of the temperature coefficients of about 80 Naudet aneroids extending over a period of 12 years. These coefficients were in all cases negative, and approximately proportional to the barometric pressure; and in only two instances was there any appreciable secular change in the temperature coefficient.

Reinhertz[10] in 1887 published his investigation of the laws of elastic after-effect in aneroids, for which purpose he made laboratory tests on 7 instruments comprising the Naudet, Goldschmid, Bohne, and Reitz constructions. His work was chiefly concerned with the form of the curve which one obtains by plotting the change of reading at constant pressure as ordinate, against time as abscissa, when the experiment is conducted under the following conditions: (*a*) The aneroid is kept under a constant pressure (approximating normal atmospheric pressure for the locality) for a day or more before beginning the experiment so that the effects of all previous disturbances will have practically vanished; (*b*) the pressure is now diminished at a uniform rate until the desired range has been covered, after which it is held constant; and in plotting the curve above mentioned, time is reckoned from the instant the lowest pressure was reached, and the change in aneroid reading during this time is called positive if the pointer moves toward lower pressures.

This change of reading may be termed *drift* in order to distinguish it from other instances of elastic after-effect, such as, for example, the residual displacement upon removal of load observed by Lovering. Reinhertz, then, appears to have been the first investigator to make a systematic study of drift curves.

Reinhertz concluded that elastic after-effect was a regular and law-abiding phenomenon, and that the form of the drift curve was substantially in agreement with the general equation previously developed by Kohlrausch for other elastic bodies. Kohlrausch's equation, however, was only intended to represent the form of the recovery curve upon removal of load. In applying it to the drift curve, Reinhertz was obliged to assume that the drift would approach asymptotically some fixed numerical limit, and he took for this limit the value observed at the end of two or three days. After establishing the approximate mathematical form of these drift curves, Reinhertz proceeded to investigate experimentally the connection between the constants of his drift equation and the preceding pressure range and rate of change of pressure. The complete results of these experiments are presented in graphical form. He further deduced, qualitatively, the shape of the loop on calibration diagrams and its relation to the rate of change of pressure. Finally, he undertook to observe the influence of temperature on elastic after-effect phenomena,

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[8] R. Kleeman: Ein Neues Metallbarometer, Zs. f. Instrumentenkunde, V. 1: 266–267, 1881.

[9] H. Hartl: Ueber die Temperatur-Coefficienten Naudetschen Aneroide, Mitth. k. k. milit.-geog. Inst., vol. I. p. 1, 1881; Zeitschrift f. Instrumentenkunde vol. 21 p. 191, 1882.

[10] C. Reinhertz, Ueber die Elastische Nachwirkung beim Feder-barometer, Zs. f. Instrumentkunde VII: 153–170, 189–207, 1887

but on account of the small range of temperatures available the effect sought for was covered up by other variations.

Whymper,[11] the explorer, in 1891 gave an account of his experiences with aneroids in the Andes Mountains, where he discovered the remarkable fact that the drift curves do not approach any immediate asymptote, but keep on rising perceptibly for six weeks or more. This fact was well established with a number of different instruments (made by Hicks, of London) and later repeated in the laboratory, so that the conclusions reached by Reinhertz can only be valid for limited time intervals.

Barus[12] in 1896 described his "counter-twisted curl aneroid," an effort to apply "null methods" (familiar in electrical measurements) to the elastic type of instrument. A helical Bourdon tube served as the pressure element, but instead of taking the direct deflection of this element as a measure of the pressure, it was brought back to zero by the opposition of a steel spring. The measurement was recorded by observing the deflection of the steel spring, a more reliable factor than the tube.

Chree[1] published in 1898 the most complete collection of data on elastic lag errors yet available; these results were expressed by empirical equations and analyzed in great detail so as to bring to light all possible connections between the various quantities observed. Among these quantities may be mentioned (*a*) as independent variables, the pressure range; the rate of change of pressure; the time elapsed at constant pressure during any part of the cycle; and the temperature (small variations only); (*b*) as dependent variables, the error of the instrument relative to a standard, as observed under various stated conditions and represented by the usual calibration curve; the sum of the differences of the errors at each point of the scale with pressure ascending and descending; the drift (as defined above in connection with Reinhertz's investigation); and the residual error at any time after return to normal pressure. Chree's paper should be examined by anyone investigating the laws of elastic lag, but it may be remarked that his viewpoint was statistical rather than physical. Beyond drawing the inference that large instruments were better than small ones, he did not undertake to correlate his observations with the internal construction of the mechanism.

Professor Marvin[14] in 1898 called attention to the need for a physical study of diaphragms, in order to actually diminish the effects which previous investigators had been occupied in recording. He believed elastic lag in aneroids to be due to the heterogeneous elasticity and unstable molecular condition of the diaphragm resulting from crimping and rolling a sheet of metal originally annealed, and not to any elastic imperfection of the steel spring. This view was confirmed by hanging a variable weight of the order of 50 pounds from the spring, after removing the vacuum box. Not so much as 0.005 inch lag could be observed in the indications of the pointer. The diaphragms might behave better, he thought, if made of steel rather than of any anomalously elastic alloy like brass or German silver. In fact, at his request in 1896 meteorographs for kite observations had been constructed by Schneider Bros. with steel diaphragms. Their performance showed some improvement, but considerable lag still remained.

At the Physikalisch-Technischen Reichsanstalt[15] in 1897-8, aneroid temperature coefficients were determined for a series of diaphragms made of different materials. Constantan and "Waterbury metal" were found to yield smaller temperature coefficients than German silver, and the suggestion was made that nickel-steel should also be tried. The temperature coefficients in this investigation were determined both at normal atmospheric pressure and at lower pressures.

In 1905 de Bort[16] stated that recent advances in meteorological knowledge left outstanding instrumental faults of the order of 2 per cent as the major part of the error in barometric altitude

[11] E. Whymper: How to use the aneroid barometer, London (Murray), 1891.

[12] C. Barus: The counter-twisted curl aneroid, Am. Jour. Sci., v. 1: 114–129, 1896.

[13] Chree: Experiments with aneroid barometers at Kew Observatory, Phil. Trans. Roy. Soc. A, v. 191: 441–499, 1898; Zs. f. Instrk. v. 19: p. 284, 1899.

[14] C. F. Marvin: The Aneroid Barometer, Monthly Weather Review, v. 26: 410–412; 1898.

[15] Zs. f. Instrument enkunde v. 18: 183–184, 1898.

[16] L. T. de Bort: Verification des altitudes barometriques par la visée directe des ballons-sondes, Comptes Rendus d.l'Acad. d. Sci. v. 141: 153–155, 1905.

measurement. These he attributed to imperfect elasticity of the diaphragms; and he showed the effect of elastic lag on aeronautical observations in a striking manner by means of his data and diagrams. One of the latter consisted of a chart with the altitude of a balloon plotted vertically, and its distance of travel horizontally; this chart comprised two superimposed curves, one for the true height as observed from the ground (a sinuous curve showing the rise and fall and progress of the balloon in its flight); the other one, recording the height by aneroid, being a similar curve starting off from the same origin but continually lagging behind.

EXPERIMENTS AT THE BUREAU OF STANDARDS.

During the period from 1911 to 1916 a series of experiments on aneroid barometers, with special reference to diaphragm performance, was carried out by H. B. Henrickson and the author, which will be found summarized below under items 1 to 8, inclusive.[17]

During the interval from 1916 to 1920, experiments were also carried out under the author's direction by H. B. Henrickson on diaphragm metals in the form of flat disks and ribbons; by W. S. Nelms on the construction of testing apparatus for metallic and rubber diaphragms; by H. N. Eaton and J. R. Freeman jr., on seasoning processes; by H. N. Eaton and J. L. Wilson on the laws of deflection of diaphragms; and by J. B. Peterson and others on the design of a precision altimeter. These latter experiments are referred to below under items 9 to 13, inclusive.

An outline and brief description of the experimental work on diaphragms at the Bureau of Standards during the period 1911-1920 follows:

1. Elastic lag with repeated cycles.
2. Comparative test of commercial aneroids.
3. Investigation of temperature coefficients.
4. Study of secular changes.
5. Localization of elastic errors.
6. General laws of elastic lag.
7. Effect of temperature on elastic lag.
8. Progressive improvement of aneroid instruments.
9. Mechanical and thermal properties of flat disks.
10. Tension experiments on diaphragm metals.
11. Heat treatment and mechanical seasoning methods.
12. Load-deflection curves for corrugated diaphragms.
13. Precision altimeter design.

1. *Elastic lag with repeated cycles.*—Preliminary to the establishment of standard methods of testing, a number of aneroids were calibrated over direct return cycles at a moderate rate of pressure change to determine how closely the curves would repeat on successive cycles. The discrepancies proved to be so large that it was found impracticable to verify the calibration in this matter. Experiments were next undertaken to determine whether aneroids could be put into a cyclic state, as has been done in magnetic testing; that is, a state, induced by a large number of repeated cycles, such that the hysteresis loop would subsequently maintain an invariable form. This experiment likewise led to a negative result, the hysteresis loop being hardly any more stable after carrying the instrument back and forth over its full pressure range all day long with fairly rapid cycles. It was concluded that such a cyclic state, if obtainable with aneroids at all, would require a larger number of cycles than could be made in one day; moreover, the time rate of pressure change during the preliminary cycles should be identical with that of the subsequent calibration cycles. These preliminary experiments revealed at first hand the magnitude and complexity of the elastic lag errors in diaphragm instruments; and led to the adoption of a single cycle, preceded by several days' freedom from disturbance, as a standard for routine testing. Such a cycle must be made at a constant rate of change of pressure, which must be specified. The errors will be more plainly brought out if there is a delay of several hours

[17] M. D. Hersey: Aneroid Barometers, Physical Review, Vol. VI: 75-77, July, 1915; Cf. also The Testing of Barometers, Bureau of Standards Circular No. 46 (12 pp.), issued Feb. 1, 1914. The author's work was continued at Harvard University during 1916.

at the lowest point of the range; this time interval likewise must be specified, and uniformly adhered to.

2. *Comparative test of commercial aneroids.*—Between 50 and 60 aneroid barometers, representing a considerable diversity of types of construction, were temporarily collected and put through a very complete series of tests. The information thus obtained regarding the numerical magnitude of the various sources of error in available instruments served as a basis for the performance specifications which were issued in 1914. This information also made it possible to select individual instruments for further investigations with some assurance that the results so obtained would be representative, and such results will be referred to below. A tabulation of constructive details for the entire collection of instruments was also preserved for subsequent use in correlating performance characteristics with constructional features; but this phase of the investigation has not yet been completed.

3. *Investigation of temperature coefficients.*—Temperature tests from +50° C. to −10° C., and in some cases from +60° C. to −40° C., were made in two ways: First, by varying the temperature at constant pressure; second, by repeating the calibration over the full range of the pressure scale at a succession of different constant temperatures. From the first set of data one can determine the ordinary temperature coefficient, i. e., the change of reading, in pressure units, at atmospheric pressure per degree rise of temperature. From the second set one can determine the temperature coefficient of the scale value, i. e., the per cent change of scale value per degree rise in temperature, where scale value is defined as the ratio of the true pressure change to the indicated pressure change. Some degree of correlation was found between these two coefficients, but not enough to warrant omitting either of the tests. Theoretical considerations which will be brought up later in this paper show that a closer degree of correlation might be expected between the temperature coefficient of scale value and a third factor, namely, the change of the temperature coefficient of the reading with pressure. This conclusion was verified by experiment in a very few tests which were made for that purpose, but it was found difficult to maintain a constant pressure, other than atmospheric, in a closed container simultaneously subjected to a wide range of temperature change. While it remains for future investigations to determine to what extent, if at all, the temperature coefficient of scale value varies with absolute temperature, it was readily evident that in a good proportion of the instruments tested the other coefficient, namely, the temperature coefficient of reading, did vary markedly with temperature. In all such cases the curve for reading (in pressure units) plotted as ordinate against temperature as abscissa proved to be a parabola, with vertex up. It is believed that this departure from linear form is due to an excessive amount of residual air in the vacuum box. Uncompensated instruments normally give straight lines on this diagram, sloping upward toward the higher temperatures, i. e., the temperature coefficient of the reading is normally positive. Overcompensation by bimetallic devices makes the coefficient negative. In either case with increasing pressures the temperature coefficient of the reading decreases. Hence, an aneroid which is found not properly compensated for change of temperature at sea-level pressure may be exactly compensated at some lower pressure; while an aneroid commercially described as compensated, will probably be found overcompensated at lower pressures.

4. *Study of secular changes.*—A number of aneroids from the comparative test were observed almost daily for more than a year in order to make a detailed study of progressive changes in the correction at normal atmospheric pressure. These results can not be gone into here beyond stating that the observed changes were small. The observations were supplemented by two experiments to trace the cause of the changes. First, a number of typical aneroids were packed in a box, hinged at one end and resting on a cam; this cam was rotated fast enough that the aneroids received several million bumps in the course of a few days. The instrumental corrections were determined at regular intervals and plotted as ordinate against the number of shocks as abscissa. The curves were distinctly systematic and progressive and so correlated with the constructive details of the respective instruments as to provide a complete explanation for the progressive changes observed without recourse to any assumption as to secular changes in the elastic quality of the diaphragms. To check this conclusion, several of the aneroids were placed

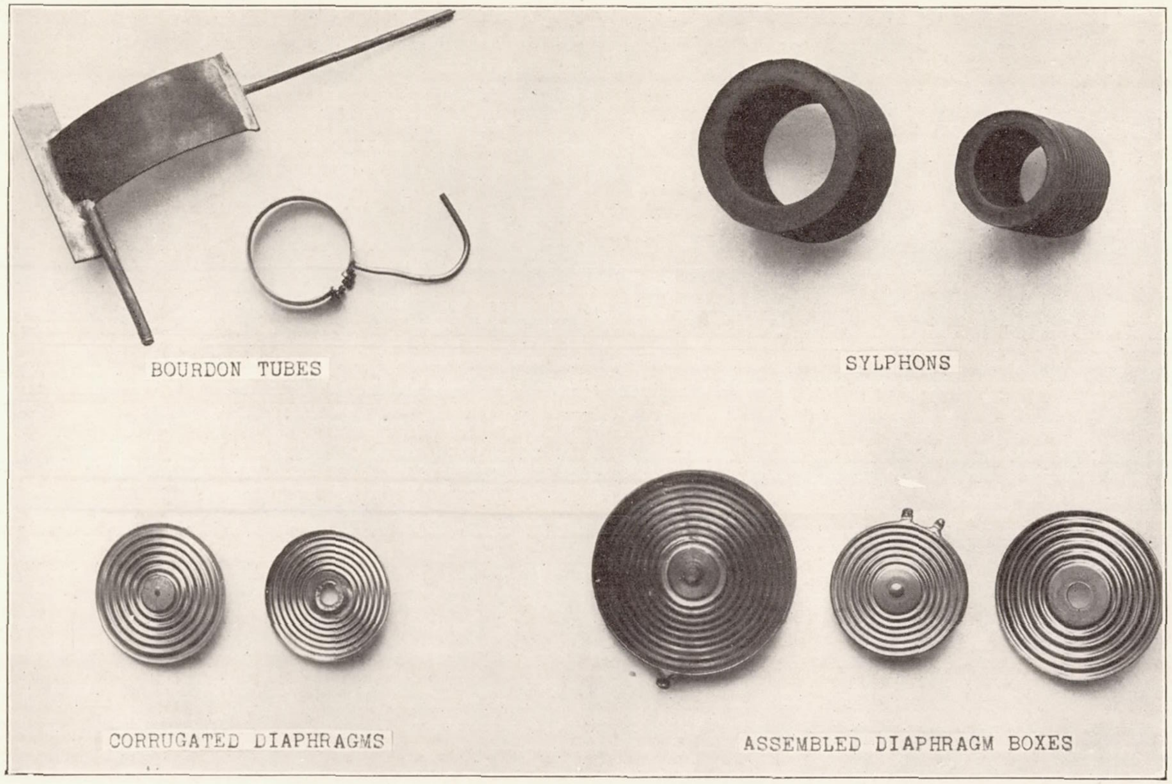
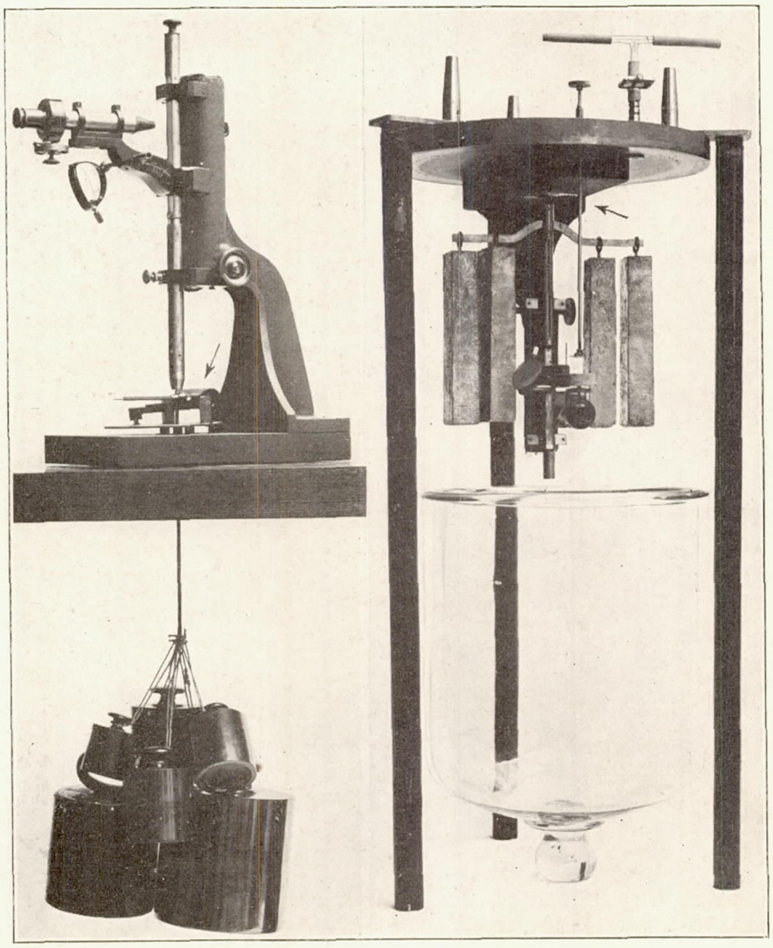


FIG. 1.—TYPES OF DIAPHRAGMS.



A. Spring alone under test. *B*. Diaphragm box under test.

FIG. 2.—EXPERIMENTS FOR LOCALIZING ELASTIC LAG.

under a partial vacuum and automatically subjected to rapid alternations of pressure for a long time, these alternations covering the working range of the dials. No perceptible change could be produced in this way, after allowing proper time for recovery from the usual transient elastic lag. These conclusions of course apply only to well-seasoned (e. g., old) aneroids. Newly made diaphragms, on the contrary, are considerably altered by such treatment, which may in fact be employed as an artificial seasoning process.

5. *Localization of elastic errors.*—As a confirmation and extension of Professor Marvin's experiments with an aneroid from which the vacuum box had been removed, tests were made on three representative aneroids, as follows. (*a*) The steel spring alone was tested over its working range of deflection, its motion being observed under the cross hairs of a microscope, so that it would be free from the disturbing influence of transmission mechanism. (*b*) The hair-spring, together with the transmission mechanism, was tested as a unit by applying a cycle of loads sufficient to run the pointer over the full scale and back, thus exhibiting directly any errors due to looseness or friction. (*c*) The vacuum box was isolated and distended by a constant load equal to the average spring tension, and its deflection observed microscopically under a cycle of varying air pressures comparable to those experienced in service. (*d*) As a check on the three foregoing tests or so-called vivisection experiments, each instrument was reassembled and operated over its full range by the variation of air pressure under a bell jar. During this operation the vacuum box deflection was continuously observed microscopically, while at the same time readings were taken of the position of the aneroid pointer and the connected mercurial standard. Thus the parts played by the spring, box, and mechanism, respectively, were quantitatively determined for each instrument, fully confirming Professor Marvin's opinion that the chief source of difficulty was in the vacuum box, i. e., in the corrugated metallic diaphragms. Experiments (*a*) and (*c*) are shown by Figure 2.

6. *General laws of elastic lag.*—As a result of experiments thus far made, the conclusion was reached that elastic lag errors in diaphragm instruments could be completely specified by measuring not more than four qualitatively different effects, namely, (*a*) variation of scale value; (*b*) drift; (*c*) hysteresis; (*d*) after-effect. By *variation of scale value* is meant the phenomenon of dependence of scale value (true pressure change per unit scale interval, sometimes known as sensibility reciprocal) upon the preceding rate of change pressure. The term *drift* was introduced at the bureau to distinguish specifically the change of reading at constant pressure from time-lag phenomena in general such as "creep" and "elastische nachwirkung"; it has been more exactly defined above in connection with the reference to the investigations of Reinhertz. *Hysteresis* will be understood to denote the observed difference between the instrument readings (or diaphragm deflections) with pressure increasing and decreasing; the hysteresis will be called positive if the deflection at a given pressure (or at a given reading) is greater for decreasing than for increasing pressures. By *after-effect* is meant the residual deflection at any time after complete removal of load, i. e., after return to the normal pressure for which the displacement was initially zero. To avoid confusion in regard to the use of the terms hysteresis and after-effect, it must be emphasized that these two terms are employed in the present report in the arbitrary special sense above defined, and are not synonymous with the corresponding terms frequently met in German publications. The present terminology has been in use at the Bureau of Standards since 1911, but may be open to improvement. Warburg and Heuse, for example, in connection with work published in 1915, which will be reviewed below, have used the same terms in a quite different sense. Hysteresis for them signifies only that hypothetical portion of the observed calibration loop which may be attributed to physical causes operating independently of the time elapsed; hysteresis in this report designates the total width of the calibration loop as actually observed, regardless of the physical explanation of the phenomena. Again, elastic after-effect (elastische nachwirkung) signifies for Warburg and Heuse any elastic lag effect of whatever nature which may be wholly regarded as a time phenomenon; but in this report the term "after-effect" is specifically restricted to the effect which may be observed after the cycle of pressure change has been completed.

To recapitulate; as a result of experiment, the conclusion was reached that all elastic-lag phenomena could be qualitatively described as belonging to one or another of four distinct types—namely, variation of scale value, drift, hysteresis, or after-effect; provided, of course, that oscillations and other inertia effects are excluded, such cases being outside the scope of this report.

A second series of experiments was undertaken to determine the quantitative laws governing the influence of the prevailing or preceding circumstances on the magnitude of each effect. For this purpose curves were plotted showing the form of the calibration curve, the drift curve, the hysteresis loop and the after-effect or so-called recovery curve, for a series of different conditions relative to the amplitude of pressure change, the rate of pressure change, etc. To some extent, particularly in the case of the drift curve, these results have been expressed by empirical equations, and the constants thereof taken as a measure of the quality of the diaphragm metal. Finally, a third stage of experimental work was entered upon, on which, however, relatively less progress has been made up to the present time; namely, the determination of possible interconnections and correlations between the four principal elastic lag effects. For it is believed that the four principal effects are not wholly independent. For example, it is very evident from the experimental data that the instruments which have the greatest hysteresis likewise have the greatest drift. Beside direct relations among the four principal effects, it is likewise possible that relations may be found connecting the respective coefficients which express the influence of various conditions on the magnitude of the principal effects, just as relations have already been found connecting the pressure coefficient of the ordinary temperature coefficient, with the temperature coefficient of scale value. The various experiments above described leading to the determination of characteristic curves for the principal elastic lag effects were for the most part carried out on aneroids selected from the comparative test; to a limited extent also such curves were obtained by direct observation of isolated vacuum boxes, viewed through a microscope (Fig. 2, *B*). The procedure for experimenting on the laws of elastic lag has to be very carefully planned, or it may lead to results which can not be definitely interpreted; theoretical considerations which have been found useful as a guide in planning and interpreting such experiments are briefly discussed below under the heading of irreversible effects, in the section on theoretical principles.

7. *Effect of temperature on elastic lag.*—A number of representative aneroids were put through the usual tests for calibration, drift, and hysteresis at a succession of widely different temperatures. In all cases the elastic lag effects were more pronounced at the higher temperatures. In general, rise of temperature and lapse of time were found to be qualitatively equivalent—that is, a rise of temperature changes all the elastic-lag effects in the same manner that they would be changed by allowing a longer time to elapse during the cycle of operations.

8. *Progressive improvement of aneroid instruments.*—The results of the diaphragm investigations at the bureau have been used to improve the quality of aneroid barometers commercially available in the United States. Suggestions and data were placed at the disposal of manufacturers and dealers, while definite performance specifications were rigidly enforced in which freedom from elastic lag was emphasized. To secure a Bureau of Standards certificate, the drift in 5 hours must not exceed 1½ per cent of the range, while the two temperature coefficients and other items must likewise be kept within stated limits. These requirements were followed by all Government departments in purchasing aneroids, and to a considerable extent by the general public. Statistics based on test results at the Bureau of Standards from year to year show that the average drift in the aneroids submitted for test has progressively decreased from 2½ per cent in 1914 to three-fourths of 1 per cent in 1918, since which date there has been a slight rise due to temporary conditions. The foregoing statements apply to commercial instruments.

9. *Mechanical and thermal properties of flat disks.*—Preliminary to a contemplated study of corrugated diaphragms, experiments were made on flat circular disks about 4 inches in diameter and from 0.01 to 0.04 inch thick. These disks were freely supported near the edge by a sharp brass ring, and loaded at the center. Readings were taken by an optical method which permitted

the apparatus to be kept in an air bath at any desired temperature. Two points were chiefly in view during these experiments: First, determination of the form of the load-deflection curves, in order to see over how great a range the various textbook formulas, all of which lead to a linear relation, could be applied in practice; second, determination of the change of stiffness with temperature, in order to see if any alloys could readily be found for which the temperature coefficient of the stiffness of the diaphragm is zero or positive. It was found that the linear relation between deflection and load held good, in most cases within 5 per cent, up to a deflection of the order of one-half of 1 per cent of the disk diameter; beyond this range the curve falls off very rapidly. Figure 3 shows the general character of the deflection curve for a typical copper disk, the scale of abscissas having been somewhat magnified near the origin so as to reveal the departure from proportionality more clearly. At a relative deflection of 1 per cent, only twice the limiting value above specified, the departure from direct proportionality is 60 per cent. Failure occurred by buckling at a deflection of about 5 per cent (10 times the proportional limit) and this deflection was produced by a load of about 150 pounds (just 100 times the amount of load corresponding to the above limit). Thus the stiffness (ratio of force to deflection) was 10 times as great at the buckling point as it was at the approximate proportional limit. This departure from proportionality, leading to an increased stiffness at the larger deflections, is of course not due to the failure of Hooke's law, but only to the geometrical necessities of the case. When Hooke's law does fail, the curvature of the deflection curve changes in quite the opposite direction, viz., from convex to concave, as is seen from Figure 3. Similar relations differing only in numerical amount are to be expected for corrugated diaphragms.

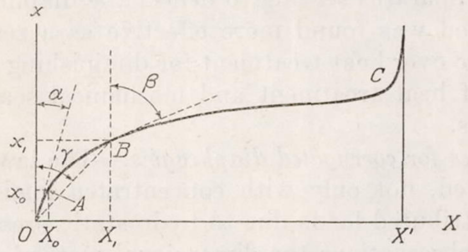


FIG. 3.—Typical diaphragm deflection curve.

Initial stiffness $= \left(\dfrac{dX}{dx}\right)_{x=0} = tan\ \alpha.$

Instantaneous stiffness at x_1 $= \left(\dfrac{dX}{dx}\right)_{x=x_1} = tan\ \beta.$

Effective stiffness at x_1 $= \dfrac{X_1}{x_1} = tan\ \gamma.$

Proportional limit (5% dev.) $= X_0$

Breaking load $= X'.$

Turning now to the second point, the question of temperature, the results were no less interesting. The temperature coefficient of stiffness at moderate deflections was greatest for soft sheet iron, and least of all for German silver. Tempered steel, phosphor-bronze, zinc, aluminum, copper, and brass fall in between, all having very small and nearly equal negative coefficients. The coefficient for the soft iron disk was excessively large, namely, -6 per cent per °C.; for the German silver it was zero at small deflections, and just perceptibly positive for large deflections. (This fact that the temperature coefficient of stiffness may be a function of the relative deflection will be further discussed below and is in agreement with the dimensional theory of diaphragms, equations (9) and (13)). Verification and continuation of these observations was interrupted by the war; they tend, however, to confirm the theoretical possibility that alloys with the usual negative temperature coefficients of elasticity may be found such that the temperature coefficient of stiffness of the diaphragm as a whole is zero or positive; as well as to suggest a possible scientific explanation for the traditional use of German silver diaphragms.

10. *Tension experiments on diaphragm metals.*—Direct observations by H. B. Henrickson on the drift of diaphragm metals under pure tension seem to provide still further evidence in favor of German silver. An interferometer method was employed owing to the minuteness of the effect to be measured, the tensile strains being of a smaller order of magnitude than diaphragm deflections. This work was done at the laboratory of the U. S. Geological Survey.

The characteristic drift curves (showing increase of strain under constant tension, as a function of time elapsed) were determined for a number of specimens in the forms of ribbons cut from the original sheets of diaphragm metal. Among four alloys thus far tested, the smallest amount of drift appears to be found in nickel and German silver, with distinctly inferior results for copper and brass.

11. *Heat treatment and mechanical seasoning methods.*—Observations were made on aluminum bronze diaphragms of different compositions; brass, bronze with different percentages of tin, monel metal, nickel (hard and soft), phosphor bronze, and German silver to determine the effect of heat treatment on the first few deflections of the diaphragm. The treatment consisted of annealing each group of diaphragms at different temperatures for certain periods of time, while the testing consisted of after-effect observations following a comparatively short application of a concentrated load at the center. This method was adopted to expedite observations, although for the most conclusive results a much longer period of strain would be preferable. The results demonstrated that seasoning in the strict sense of the word could not be secured by heat treatment alone, although such treatment does serve to diminish the magnitude of elastic lag effects. Experiments on mechanical seasoning were conducted by means of an automatic apparatus serving to deflect the diaphragms periodically by hydrostatic pressure. This method was found more effective as a seasoning process, although it may not have any advantage over heat treatment for diminishing the magnitude of the effects. Evidently a combination of heat treatment and mechanical seasoning would be of interest for subsequent investigations.

12. *Load-deflection curves for corrugated diaphragms.*—The laws of deflection of corrugated diaphragms were investigated, not only with concentrated loads as had been done for the flat disks, but also with distributed loads due to hydrostatic pressure. Both for the flat disk and corrugated diaphragm observations the dimensional method of planning and interpreting the experimental work is of advantage, and was followed throughout. In this way the deflection curves for different materials and different diameters can be made to coalesce into a single curve. This method of correlating scattered observations leads to the most reliable determination of the form of the curve, and when such a curve is represented by an empirical equation, all of the variables are automatically taken account of and the law of deflection becomes available in its most general form for actual use in design. This method is discussed below in connection with the dimensional theory of diaphragms.

13. *Precision altimeter design.*—The principle adopted at the Bureau of Standards to eliminate elastic errors in altimeters consists in the use of a relatively stiff main spring. If the spring is sufficiently stiff compared to the diaphragm, the quality of the latter is without appreciable influence. It has been found possible in this way to make the elastic errors of an aneroid negligible, even when using a diaphragm of common brass strained well beyond the elastic limit. Precision instruments with open scale dials have been constructed on this principle for the use of the Air Service and the National Advisory Committee for Aeronautics in aircraft testing. [18]

RECENT WORK ABROAD.

Warburg and Heuse [19] (Physikalisch-Technischen Reichsanstalt) in 1915 published a brief account of experiments on a series of metallic and nonmetallic specimens, though not in the form of diaphragms, tending to prove that about half of the observed loop on a stress-strain diagram is due to a purely directional effect independent of the period of the cycle. This part, only, the authors term elastic hysteresis, restricting the term elastic after-effect to the

[18] "Precision altimeter design," by J. B. Peterson and J. R. Freeman, jr., Report No. 126, N. A. C. A., Pt. II.

[19] E. Warburg u. W. Heuse: Elastische Hysteresis und Elastiche Nachwirkung; Verh. D. Deutsch, Physik. Ges. 17; 206–213: 1915

theoretical loop calculated by Boltzmann's method.[20] Their paper represents an important attempt to separate directional hysteresis from that part of the stress-strain loop which may be accounted for by the superposition of time effects.

The same authors in 1919 reported[21] an extended investigation of aneroid theory and practical design, from which it was concluded that elastic lag could be diminished by the following three methods: (I) Use of hard German silver for the diaphragms, no other material having been found by the authors to be superior to this. It is important to reduce the thickness of the metal, which they have carried with advantage from the usual 0.2 mm. down to 0.05 mm. The diaphragms should be pressed into shape by hand and thereby hardened, for softening was found to make the after-effect excessive. (II) Use of a stiff spring.[22] If this diminishes the sensitivity of the aneroid too much, it can be restored by increasing the multiplying power, while friction may be reduced by the vibration device of Goeppel. This consists of a steel plate attached to the aneroid and caused to vibrate by means of a toothed wheel operated by hand. (III) Use of vacuum boxes having flexible diaphragms for both top and bottom surfaces, in which event it appears from the theoretical discussion that the after-effect should only be half as much as with a single diaphragm. Experimental data are given in support of the foregoing three principles, by means of which aneroids were constructed for which the greatest width of the hysteresis loop was scarcely more than one-half of 1 per cent. In the course of this investigation empirical equations were established for the deflection of corrugated diaphragms under the combined action of hydrostatic pressure and an oppositely directed tension at the center.

Bennewitz[23] recently published a simplified version of Boltzmann's theory and showed how it might be applied to the development of aneroids automatically compensated for elastic lag. Two sets of diaphragm elements are required, which have to be selected with reference to a certain similarity in their lag characteristics. Such devices will be further discussed below.

Recent investigations in France[24] have included the study of elinvar in place of German silver as a means of temperature compensation without external devices.

The importance of elastic lag has been fully recognized by British investigators[25] but very little information appears to have been published since the work of Chree.

In Canada the Associate Air Research Committee[26] has an investigation under way with the object of developing an improved type of diaphragm for barographs. This work is being carried on by Stanley Smith, at the University of Alberta, and will include the study of new materials for diaphragms. The testing and general improvement of aneroid barometers are constantly in progress at the Surveys Laboratory[27] of the Interior Department, under the direction of W. C. Way, whose work has already led to some distinct improvements in the mechanism of such instruments.

THEORETICAL PRINCIPLES.

The following discussion will be restricted to general principles avoiding, as far as possible, all special assumptions which might preclude the application of the results to practical research work. Certain special theories proposed by various authorities at different times may however be very briefly reviewed first, because of their historical or suggestive value. Such theories are limited in their usefulness because of stated mathematical restrictions or because of some physical hypothesis not readily capable of demonstration. Among them mention may be made of the conventional theory of elasticity as applied to thin flat disks undergoing only

[20] See below under the theory of irreversible effects.

[21] E. Warburg u. W. Heuse: Über Aneroide; Zs. f. Instrumentenkunde 39: 41–55, 1919; abstracted in Technical Note No. 72, National Advisory Committee for Aeronautics, 1921.

[22] The stiff spring principle was independently adopted at an earlier date at the Bureau of Standards.

[23] K. Bennewitz: Über die elastische Nachwirkung, Physik. Zs. 21: 7–3–705, 1920; Verfahren zur Kompensation der elastischen Nachwirkung, Ibid. 22: 329–332, 1921.

[24] Bull. du Section Technique de l'Aeronautique Militaire, Fasic. 4: p. 4, 1918.

[25] T. G. Hull: Creep Errors in Altimeters due to Hysteresis; (British) Advisory Com. for Aeronautics Report, 1916–17: 668–670.

[26] Report of the Air Board, Ottawa, for the year 1920, p. 14.

[27] The testing of Aneroid Barometers at the Laboratory of the Dominion Lands Survey, Bull. 42: 1–9, 1921.

infinitesimal deflections, and the theories of Maxwell, of Ewing and Rosenhain, and of Guillaume for the explanation of elastic lag.

The conventional textbook analysis of thin flat plates as presented by Grashof, Föppl, Lanza, Love, Morley and others is limited not only to perfectly elastic action but, also, to the following conditions:

(*a*) Infinitely small deflections.
(*b*) Geometrically simple shapes.
(*c*) Isotropic material.
(*d*) Homogeneous material.
(*e*) Strains following Hooke's law.

Nevertheless, these conventional formulas do provide definite numerical checks, or limiting values, of interest for comparison with the experimental data wherever the experimental conditions approach the conditions assumed in the formulas; for example, in the determination of the initial stiffness of flat disks or corrugated diaphragms—that is, the slope of their load-deflection curves at the origin (Fig. 3).

The theory of Maxwell [28] represents some molecular complexes as being more easily broken up than others, so that the entire substance of a body does not respond simultaneously to a change of stress but requires a certain relaxation time. Extensive experiments by Barus [29] on iron carbides are considered to support this view.

The theory of Ewing and Rosenhain [30] is based on the slippage of crystals, and is supported by recent metallographic observations of J. R. Freeman, jr., at the Bureau of Standards, but no such theory can easily explain the close resemblance of lag phenomena in metals to those in nonmetallic substances like glass, rubber, and silk.

Guillaume [31] found from his work on nickel steel that the older viscosity hypothesis was inconsistent with the facts, and he has personally described to the author his conception of elastic lag as being simply a gradual approach to equilibrium among the numerous solid phases present in an alloy, the relative proportions of which are altered by small variations of stress just as they would obviously be altered by change of temperature.

Turning now to the more general principles, the following subjects may be considered:

I. Dimensional theory of diaphragms:
   (*a*) Deflection curves.
   (*b*) Stresses in diaphragms.
   (*c*) Stiffness of diaphragms.
   (*d*) Temperature compensation.

II. General theory of coupled systems:
   (*a*) Stiffness.
   (*b*) Change of stiffness with temperature.
   (*c*) Elastic lag.
   (*d*) The two temperature coefficients.

III. Irreversible effects:
   (*a*) Phenomena and definitions.
   (*b*) Boltzmann's theory.
   (*c*) Drift superposition theory.
   (*d*) Dimensional theory.

In all cases except II (*c*) and III perfectly elastic materials are presupposed.

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[28] J. C. Maxwell: Constitution of Bodies, Encyc. Britt., 9th ed., 1874.
[29] C. Barus: Certain Physical Properties of Iron Carbides, Clark Univ. Lectures, 128–161, 1912.
[30] Cf. Poynting and Thomson: Properties of Matter, pp. 58–60; 8th Ed. London, Chas. Griffin, 1920; Ewing and Rosenhain, Proc. Roy. Soc. 45: 85.
[31] Ch. Ed. Guillaume: Applications of Nickel Steel: Paris, Gauthier-Villars, 1–215, 1904.

I. DIMENSIONAL THEORY [32] OF DIAPHRAGMS.

(*a*) *Deflection curves.*—It is required to determine the most general form which any correct and complete equation must take, whether discovered mathematically or experimentally, which expresses the laws of deflection of a diaphragm—that is, which expresses the relation between the deflection of the diaphragm x and the load or any other quantities which can influence the deflection.

Calling the total concentrated force or tension which acts on the diaphragm in an outward direction T, and the uniform hydrostatic pressure acting inward on the same surface p, while D denotes the diameter or any other designated linear dimension of the diaphragm, it is evident that x will be some function of p, T, D and of the geometrical shape of the diaphragm and of the elastic properties of the material. The shape can be specified by the relative thickness t/D together with such other ratios as may be needed for indicating the relative size of the rigid central disk, the relative width and depth of corrugations, etc., or by t/D alone if the resulting equation is restricted to any one series of diaphragms, differing in thickness, in size, or in material, but having geometrically similar surface contours. In what follows geometrically similar contours will be presupposed, but this supposition does not mean that any diaphragm however complicated is excluded from the argument.

Now, as to the effective properties of the material, Young's modulus E together with the shear modulus μ would suffice to fix its behavior in purely static experiments if the material were homogeneous, isotropic, and subject to Hooke's law. More generally the relative elastic moduli E'/E, E''/E, μ'/μ, μ''/μ, etc., in different parts of the diaphragm or in different directions at a given point must be included to take account of departure from homogeneity and isotropy, while departure from Hooke's law can be provided for, if desired, by introducing additional coefficients obtained from the empirical equations for elastic moduli in terms of strain. Such coefficients are dimensionless because strain is dimensionless; they are conspicuously large for cast iron and for soft rubber, but comparatively small for ordinary diaphragm materials and will therefore be neglected when treating diaphragms as perfectly elastic. Dimensional theory does not require the neglect of these coefficients, but their retention would be superfluous in calculations concerned only with the major characteristics of the laws of deflection, which at the moment are wholly unknown, and for lack of which diaphragm stiffnesses can not be predicted within several hundred per cent. The assumption of Hooke's law for diaphragm materials does not imply that the resultant deflection of the diaphragm itself should be proportional to load; on the contrary, as seen from Figure 3, any departure of the material from Hooke's law would but diminish the departure of the diaphragm from proportionality between load and deflection.

From the foregoing considerations it appears that all metallic diaphragms which are geometrically similar in surface contour and which, if not isotropic and homogeneous, possess the same relative distribution of elastic constants, must be governed by some general equation which, in qualitative form, may be expressed by the formula

$$x=f\left(p,\ T, D, E, \mu,\ \frac{t}{D}\right) \tag{1}$$

in which the form of the function f depends only on the shape of the die, or template, which fixes the corrugated contour of the diaphragm; e. g., it is the same for two diaphragms of different thickness, etc., struck from the same die. For rubber diaphragms, dimensionless coefficients representing departure from Hooke's law might be introduced, and regarded as additional elastic constants. But on account of the fact that such coefficients are dimensionless, they will not combine with other quantities and can be dropped out of all calculations

[32] Readers not familiar with dimensional analysis may consult E. Buckingham, Model Experiments and the Forms of Empirical Equations, Trans. Am. Soc. Mech. Engs. 37; 263-296, 1915; Notes on the Method of Dimensions, Phil. Mag., October, 1921. However, it is unnecessary to be versed in the details of the dimensional method in order to follow the present discussion provided the fact is appreciated that the dimensional argument in itself is absolutely rigorous and automatic; the only chance for any difference of opinion would lie in the selection of the original list of physical quantities concerned, the list which has to be written down in the form of a qualitative equation, like eq. (1), at the beginning.

involving diaphragms made of the same or substantially the same material. To recapitulate: In equation (1) x represents the observed deflection of the diaphragm, say at its center, under the simultaneous action of a hydrostatic pressure p (measured in force units per unit area) opposed by a total tension T (measured in force units alone), the diaphragm having a diameter D and thickness t, and being made out of some material whose Young's modulus is E and whose shear modulus is μ.

Routine dimensional reasoning serves to convert equation (1) into the more specific form

$$\frac{x}{D}=\varphi\left(\frac{p}{E}, \frac{T}{ED^2}, \frac{t}{D}, \sigma\right) \tag{2}$$

in which the symbol φ merely represents some unknown function of the four arguments or independent variables p/E, T/ED^2, t/D, and σ. The last of these, σ, stands for μ/E or any arbitrary function thereof and may therefore conveniently be recognized as denoting Poisson's ratio, $\frac{E}{2\mu}-1$, a definite property of the material.

The value of equation (2) in contrast with equation (1) as a framework for correlating experimental results lies in the fact that it involves only four independent variables, instead of six, thus greatly diminishing the labor and expense of exploring the form of the relation by direct experiment.

The practical use of equation (2) can, perhaps, best be suggested by examining special cases. For example, in the case of the flat disk and corrugated diaphragm experiments previously referred to, if the specimens are limited to materials having roughly the same Poisson's ratio while concentrated loads only are employed, (2) reduces to

$$\frac{x}{D}=\psi\left(\frac{T}{ED^2}, \frac{t}{D}\right) \tag{3}$$

or

$$u=\psi\,(v, w) \tag{4}$$

in which u has been written for x/D, v for T/ED^2, and w for t/D. Simply tabulating the observed values of u, v, and w in three columns enables one to plot a family of curves or to construct a space model which will completely represent the law of deflection for the conditions stated. A different surface would exist, of course, for materials have essentially different values of σ, or for contours of different shape; but a single surface suffices to represent all effects which can be realized by variations of load T, elastic modulus E, thickness t or diameter D. In the absence of dimensional analysis a doubly infinite family of surfaces instead of a single surface would have been required; moreover, with a given number of observed points available, the precision of determination of the surfaces would be far less if distributed over numerous surfaces than if concentrated on one. Having constructed this characteristic surface or plotted an equivalent system of plane curves, one has only to represent the same algebraically by a suitable empirical formula connecting u, v, and w, in order to deduce a complete mathematical expression for the general law of deflection by substituting in once more the original variables symbolized by u, v, and w. Such equations are needed for rational design. From equation (2) the usual inferences concerning dynamical similarity may also be drawn, and all the above reasoning can be applied to hydrostatic pressure.

(*b*) *Stresses in diaphragms.*—The same mode of reasoning which has been outlined above, together with the same physical assumptions, leads to the conclusion that any element of stress, f, such as the tensile or the shearing stress on a given plane at any point, or any stated component of such stress will be given, for geometrically similar diaphragms, by the general equation

$$\frac{f}{E}=\text{funct.}\left(\frac{p}{E}, \frac{T}{ED^2}, \sigma\right) \tag{5}$$

This equation can be discussed and applied in various ways analogous to the treatment given above, but on account of the fact that it is not commonly practicable to make direct measurement of internal stresses in the laboratory, it is more interesting to throw equation (5) into a quite different form.

This can be accomplished by reference to equation (2). Write equation (2) three times over, first, for the displacement x, second, for the displacement of some other specified point, x', and finally for the displacement of some third point, x''. These three equations together with (5) constitute a sysem of four equations from which the three variables p/E, T/ED^2, and σ may all be eliminated, leaving

$$\frac{f}{E}=\text{funct.}\left(\frac{x}{D}, \frac{x'}{D}, \frac{x''}{D}\right) \tag{6}$$

provided, as before, that t/D is treated as a constant on account of geometrical similarity. Thus when the relative displacement of any three points has been given, the stress is fixed at all points. If attention is confined to diaphragms having the same Poisson's ratio and for which T/ED^2 is zero or constant, the x' and x'' terms in (6) drop out, leaving simply

$$\frac{f}{E}=\phi\left(\frac{x}{D}\right) \tag{7}$$

where ϕ is some unknown function, different of course for different values of the constant parameters. In this case the stress at all points is determined solely by the relative deflection at the center, together with Young's modulus, and is directly proportional to the latter. Two geometrically similar diaphragms made up in different sizes and from different materials will have stresses proportional to Young's modulus when deflected in proportion to their diameters. They will have equal stresses if made of the same material and similarly deflected, no matter how different in size; i. e., if their relative deflections at the center, x/D, are the same, the diaphragms will be similarly deformed throughout, and everywhere equally stressed.

These considerations are applicable in the design of diaphragms, even though stresses are not directly measured. For it may be desired to reproduce, in a larger diaphragm of different material, conditions known to lie within safe limits of stress for some particular diaphragm as judged by satisfactory practical performance. Further, in experimenting on elastic lag, one would expect the same behavior in diaphragms of different size, if the stress history at each point is the same, and the foregoing relations show how the same stress conditions can be realized in different diaphragms.

(c) *Stiffness of diaphragms.*—Defining the stiffness S of a diaphragm for any stated system of loading—hydrostatic pressure or concentrated tension, for example—as the ratio of force to displacement, it can readily be shown by dimensional reasoning that for geometrically similar diaphragms

$$\frac{S}{ED}=\text{funct}\left(\frac{x}{D}, \sigma\right) \tag{8}$$

For sufficiently small displacements, such as occur frequently in instruments, the stiffness will be practically independent of displacement, so in this limiting case

$$S=ED\,\phi\,(\sigma) \tag{9}$$

That is, two geometrically similar diaphragms with the same Poisson's ratio have stiffnesses directly proportional to Young's modulus and to their absolute sizes. Equation (9) can evidently be applied also to large deflections by simply specifying the magnitude of the relative deflection to which the stiffness in question corresponds; the stiffness S and the numerical value of ϕ being greater for large values of x/D than for small, in accordance with Figure 3.

(d) *Temperature compensation.*—Differentiating (9) logarithmically with respect to temperature θ gives

$$\frac{1}{S}\frac{dS}{d\theta}=\frac{1}{E}\frac{dE}{d\theta}+\frac{1}{D}\frac{dD}{d\theta}+\frac{1}{\phi}\frac{d\phi}{d\sigma}\frac{d\sigma}{d\theta} \tag{10}$$

On account of the dimensional derivation of (9) the quantity σ represents any function of μ/E and can be written

$$\sigma=\frac{1}{2}\frac{E}{\mu}-1 \tag{11}$$

which, for homogeneous isotropic materials, is the usual expression for Poisson's ratio. This can be differentiated in order to supply the last term of (10) above. Thus letting α denote the temperature coefficient of Young's modulus, β that of the shear modulus, and γ the ordinary linear thermal expansivity of the material of the diaphragm, gives

$$\frac{d\sigma}{d\theta}=\frac{1}{2}\frac{E}{\mu}(\alpha-\beta)=(1+\sigma)(\alpha-\beta) \tag{12}$$

hence, replacing ϕ by its equivalent S/ED, (10) becomes

$$\frac{1}{S}\frac{dS}{d\theta}=(1+C)\alpha-C\beta+\gamma \tag{13}$$

in which the dimensionless, isothermal shape-factor C is given by

$$C\equiv(1+\sigma)\frac{d}{d\sigma}\log\frac{S}{ED} \tag{14}$$

This factor may be numerically different for different relative deflections $\frac{x}{D}$, if the deflection is great enough to cause variations of stiffness, as in the flat disc experiments above cited. According to equation (13) it should be possible to determine the effect of temperature on the stiffness of a diaphragm or spring of any shape by purely mechanical (i. e., isothermal) experiments. Only one quantity has to be found by direct experiment on the complete diaphragm, namely C, which is simply $(1+\sigma)$ times the slope of a plot connecting log S/ED with σ. Further, it is not necessary to make these observations on the actual diaphragm, if geometrically similar models having any convenient values of E and D are available. The remaining coefficients α, β, and γ do not depend on the diaphragm at all, but are familiar thermal properties of the material from which the diaphragm is made.

For intrinsic temperature compensation the second member of (13) must equal zero, giving as the general criterion for compensation

$$\frac{\alpha}{\beta}=\frac{C}{1+C}-\frac{\gamma}{\beta}\frac{1}{1+C} \tag{15}$$

and, with a sufficient approximation for numerous materials, the term involving thermal expansion can be neglected.

The question whether intrinsic compensation is practicable remains as an important one for future study, but the conditions to be satisfied have been analyzed above. It is conceivable that compensation might be secured either by discovering alloys which satisfy equation (15) for a given value of C; or conversely by developing a suitable value of C through some radical departure in mechanical design, such as to satisfy equation (15) for any given values of α, β, and γ.

When C is positive, equation (13) shows that the α and β terms influence the stiffness in opposite directions. Increasing Young's modulus makes the diaphragm stiffer; increasing the shear modulus makes it more flexible. This paradox is equally evident in the textbook formulas for thin, flat discs, when written out explicitly as functions of both E and μ. If it were not for this fact, intrinsic compensation for temperature might require α and β to have opposite signs.

II. THEORY OF COUPLED SYSTEMS.

(*a*) *Stiffness.*—Let it be required to compute the stiffness S of a system consisting of a diaphragm element (i. e., either a single diaphragm or diaphragm capsule or a series of such) of stiffness S_2 coupled to a spring of stiffness S_1. Let the stiffnesses S_1 and S_2 relate to concentrated loads applied at the coupling, while S relates to the more general case in which the diaphragm is loaded by hydrostatic pressure. For this type of loading let S'_2 be the stiffness of the diaphragm by itself and let λ represent the ratio S'_2/S_2. Then it can be shown from the conditions for statical equilibrium, that for moderate deflections (so that all stiffnesses are constant and therefore λ also constant)

$$S=\lambda\ (S_1+S_2) \tag{16}$$

This relation has applications in instrument design where diaphragms and springs are employed in combination, because a knowledge of S is needed for computing the scale value, while λ, S_1 and S_2 are available as empirical characteristics of the component parts. The question how to modify (13) when λ is not strictly constant remains for further study; but the case presented above, which is sufficiently general for numerous problems of instrument work including barograph design and sylphon computations, may be proved as follows:

Let Δx be the resultant downward deflection of the top of the diaphragm element due to the load increment ΔF caused by an increase of hydrostatic pressure Δp; this deflection incidentally increases the tension of the spring by some unknown amount ΔT. The downward deflection of the diaphragm element due to ΔF alone if the tension remained constant would evidently be equal to $\Delta F/S'_2$, while its upward deflection due to ΔT alone if the pressure remained constant would similarly be given by $\Delta T/S_2$; hence by superposition

$$\Delta x=\frac{\Delta F}{S'_2}-\frac{\Delta T}{S_2} \tag{17}$$

Now by definition of the spring stiffness S_1 the increment of tension must be equal to $S_1\Delta x$. Making this substitution in (17) and solving for the ratio of force to deflection gives directly

$$\frac{\Delta F}{\Delta x}=\frac{S'_2}{S_2}\ (S_1+S_2) \tag{18}$$

which is identical with equation (16), as required.

The conception of effective area, or equivalent piston area investigated by Dent in 1849 (see above) may be defined as a_0 in the equation $\Delta T=a_0\ \Delta p$; while the actual area is given by a in the relation $\Delta F=a\ \Delta p$; in which ΔT now represents whatever increase of tension would be necessary to neutralize a downward deflection Δy caused by ΔF. Hence S'_2/S_2, which is equal by definition to $\Delta F/\Delta y\div\Delta T/\Delta y$, may be written $a\ \Delta p\div a_0\ \Delta p$ or simply a/a_0, therefore (18) becomes

$$S=\frac{a}{a_0}\ (S_1+S_2) \tag{19}$$

Thus λ, the ratio of the diaphragm stiffness under distributed load to its stiffness under concentrated load, may also be interpreted as the ratio of the actual area to the effective area. Finally if, instead of the ordinary stiffness S, it is found more convenient to employ the stiffness for hydrostatic pressure $S_p=\Delta p/\Delta x$ (a quantity which has the dimensions of stiffness divided by area), equation (19) can be rewritten

$$S_p=\frac{1}{a_0}\ (S_1+S_2) \tag{20}$$

A similar modification of equations (8) to (10), and (13) to (15), could also be made if desired, for which purpose only those terms involving D and γ need be altered.

(*b*) *Change of stiffness with temperature.*—Differentiating (16) with respect to any independent variable θ gives

$$\frac{1}{S}\frac{dS}{d\theta}=\eta\frac{1}{S_1}\frac{dS_1}{d\theta}+(1-\eta)\frac{1}{S_2}\frac{dS_2}{d\theta} \tag{21}$$

in which η represents the relative spring stiffness, $S_1/(S_1+S_2)$. Thus, if θ stands for temperature, (21) shows how the temperature coefficient of stiffness for the system operating as a whole can be computed very simply from the separate temperature coefficients of the component members, when the factor η is given and λ is constant.

(*c*) *Elastic lag.*—In equation (21) the increment $d\theta$ may be regarded as an interval of time or interpreted in any arbitrary way: the fact being that any observed change of stiffness dS caused by the component changes dS_1 and dS_2 is related to them by the equation

$$\frac{dS}{S}=\eta\frac{dS_1}{S_1}+(1-\eta)\frac{dS_2}{S_2} \tag{22}$$

This equation therefore makes it possible to compute the resultant effect due to elastic lag of amount dS_1 in the spring and dS_2 in the diaphragm element. In particular (22) confirms the suggestion that η should be chosen as nearly equal to unity as possible, i. e., for minimum elastic lag in the system as a whole, the spring should be made as stiff as possible compared to the diaphragm, in the usual case where elastic lag is intrinsically greater in the diaphragms than in the spring (i. e., $dS_2/S_2>dS_1/S_1$).

(*d*) *Equation for temperature coefficients.*—If the reading of an instrument r is some definite function

$$r=f\ (p,\theta) \tag{23}$$

of the pressure p and temperature θ, i. e., if irreversible effects are excluded, a certain relation must subsist between the various coefficients. Let the scale value v, the temperature coefficient of the reading a, and the temperature coefficient of scale value b be defined by the expressions

$$\frac{\partial r}{\partial p}=\frac{1}{v},\ \frac{\partial r}{\partial\theta}=a,\ \frac{\partial v}{\partial\theta}=b; \tag{24}$$

then since $\partial/\partial\theta$ of $\partial r/\partial p$ is identically the same as $\partial/\partial p$ of $\partial r/\partial\theta$, it follows that

$$\frac{\partial a}{\partial p}=-\frac{b}{v^2} \tag{25}$$

(As an approximation in the case of properly adjusted instruments, v can be set equal to unity). This result is applicable to any indicating mechanism, whether a coupled system or single diaphragm, and it should be more and more nearly realized in actual instruments, as their operation approaches the condition of perfect reversibility assumed in equation (23). The application of (25) to aneroid barometers has already been considered in connection with the investigation of temperature coefficients, item 3 above, under the head of experiments at the Bureau of Standards.

III. IRREVERSIBLE EFFECTS.

(*a*) *Phenomena and definitions.*—Freshly made diaphragms are likely to show progressive secular changes and fail to repeat their mechanical performance on successive occasions under identical conditions, even though sufficient time has been allowed for transient effects to disappear. Following the phraseology adopted in thermometer practice, where these phenomena are equally familiar, such diaphragms may be termed green or unseasoned. Conversely, diaphragms which do repeat their mechanical performance when experiencing the same load history on successive occasions separated by a sufficient time interval may be termed aged or seasoned. That these irregularities can be overcome by allowing the material to age itself naturally over a long period of time seems fairly established by the earlier experiments of the Bureau of Standards on aneroid barometers referred to above. Those observations were made chiefly on imported instruments several years old at least, and in nearly all cases showed excellent repetition of the hysteresis loops when determined under identical conditions with several days' rest between tests. However, the necessity for rapid production during the war naturally brought forward the question whether artificial seasoning methods were possible. Although evidence is available that seasoning can be partially accomplished by heat treatment, and more completely by repeated mechanical stress, the nature of the problem is not such as to promise a quantitative solution at any early date.

Since all diaphragms will ultimately become seasoned, and since it is only for such diaphragms that the results of laboratory tests will be of any quantitative value to the owner of the instrument, the discussion from this point on may be restricted to the consideration of seasoned diaphragms. To recapitulate, a seasoned diaphragm is one which repeats its mechanical performance, including any irreversible effects whatever that may be of interest, notably the hysteresis loop, on successive occasions, separated by a sufficient interval of rest, provided of course that the diaphragm is subjected to identical conditions on each occasion. The complete history of the external forces acting on the diaphragm, together with temperature variations, if any, forms an essential part of these conditions, and it may be represented by curves plotted against time as abscissa.

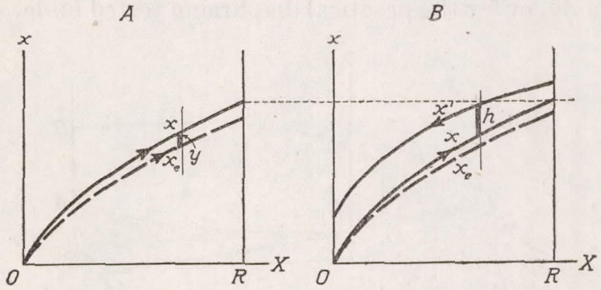

FIG. 4.—Deflection curves showing inelastic yield.

Ideal elastic deflection	$=x_e$	Rising deflection	$=x$
Actual deflection	$=x$	Falling deflection	$=x'$
Inelastic yield $x-x_e$	$=y$	Hysteresis $x'-x$	$=h$
Range of load	$=R$	Range of load	$=R$

The foregoing definition of a seasoned diaphragm must not be taken to mean that two successive hysteresis loops will be the same; on the contrary, successive loops taken one immediately after the other are just as likely to be progressively different for a seasoned diaphragm as for an unseasoned one, but if this entire series of loops be repeated after a long interval of rest, the time curve representing the force history for each series being the same, then, in the case of the seasoned diaphragm, one entire series of loops should be identical with the next entire series of loops; whereas, in the case of an unseasoned diaphragm, the second series of loops will differ notably from the first series. In practice it is not necessary to observe more than one loop in each series in order to distinguish between seasoned and unseasoned diaphragms, because the foregoing remarks apply equally well to the first loop in each series, and this may be sufficient.

It has already been noted that the principal irreversible phenomena of a purely mechanical nature which are of interest for diaphragm work may be reduced to four distinct effects: First, variation of stiffness with rate of load application; second, drift, or change of displacement at constant load; third, hysteresis, or difference between rising and falling deflection at a given load; fourth, after effect, or residual displacement after removal of load.

These different effects may be exactly defined by reference to Figures 3, 4, and 5. The definition of stiffness is available from Figure 3; in this report, unless otherwise specified, the effective stiffness for a given deflection will be understood. In all three figures X represents the external load applied to the diaphragm, whether consisting actually of a concentrated force or a distributed pressure, while x represents the corresponding displacement or deflection of the center of the diaphragm. Thus Figure 4 shows an ordinary load-displacement diagram. The heavy curve x under A, Figure 4, shows the relation between displacement and load for increasing loads; the dotted curve shows x_e, the ideal or so-called elastic deflection which would be observed at some more rapid rate of load application—for example, an instantaneous load—excluding inertia effects. The excess of the actual displacement x over the elastic displacement x_e may be designated by the symbol y, and spoken of as the inelastic yield. This quantity y will be taken as a fundamental one in the discussion which follows, because it represents conveniently the departure of an actual diaphragm from the ideal performance of a perfectly elastic one, or from the standard performance of a practical diaphragm under stated conditions. Figure 4, B, consists of a reproduction of A with the return curve x' added. From this diagram the exact definition of the term hysteresis, as it will be used in the following text, will be apparent. Hysteresis, then, for the purpose of this report, signifies the excess of the falling deflection over and above the rising deflection at a given load, regardless of the cause of the phenomenon.

In Figure 5 the same effects recorded in Figure 4, B, are represented graphically in a more convenient manner by plotting the yield y as a function of the load X and the time t. Consider first the yX diagram. The shaded portion ORA shows the change of stiffness due to inelastic yield, which would vanish if the yield were zero throughout the entire range. In fact, whether for a perfectly elastic body, or for the practical diaphragm tested under a standard rate of change

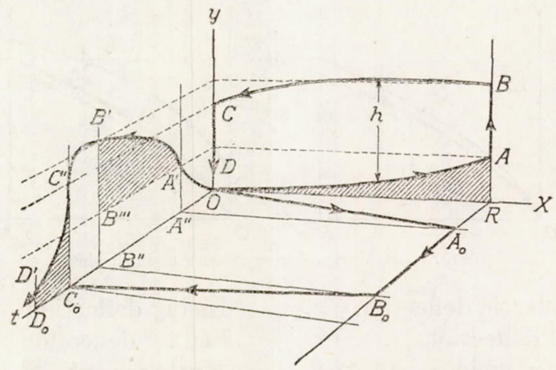

FIG. 5.—Graphical chart of elastic lag phenomena.

Variation of stiffness is shown by ORA.
Drift is shown by AB and by $A'B'B'''$.
Hysteresis, h, is shown by $OABCO$.
After-effect is shown by $C_0C'D'D_0$.

of pressure, all observations would lie flat on the Xt plane, and all ordinates would be zero. Returning to the yX plane, the curve OA shows the yield as a function of load for increasing loads; the displacement AB is the drift; the curve BC represents the yield for diminishing loads; while finally the displacement CD represents incomplete recovery, leaving a residual displacement DO which, for perfectly seasoned diaphragms will ultimately vanish. The lower plane shows merely the load X as a function of the time t; in this diagram, which is taken as a standard for practical testing, the load is applied at a uniform rate, during the time OA''; it is held constant during the time $A''\ B''$; it is removed at the same rate at which it was applied during the time interval $B''\ C_0$.

The time diagram in Figure 5 is equally important for the present discussion; the shaded part $A'\ B'\ B'''$ represents the drift curve in its usual form, while $C'\ D'$ shows the familiar recovery curve, according to which the aftereffect or residual displacement has diminished from an initial value C_0C' to a much smaller amount D_0D' at the instant represented by the diagram. This elastic after effect according to Figure 5 will approach zero asymptotically.

The performance characteristics of any diaphragm can be fairly completely specified by reference to Figure 5, which will be taken as an empirical starting point for the theoretical analysis below.

(*b*) *Boltzmann's theory.*—The treatment of elastic lag followed by recent German investigators, notably by Warburg and Heuse in their effort to separate directional hysteresis from time effects, is due to Boltzmann, whose general equation may, in the notation of this report, be written

$$x = \frac{X}{E} + \int_0^t X_\tau \psi(t-\tau)d\tau \tag{26}$$

Here E represents the stiffness of the body (X/x), for perfectly elastic displacements, or for displacements due to an instantaneous load. The integral, then, represents y, the excess of the actual displacement over and above the ideal displacement X/E. The equation is intended to express a relation between the load X and displacement x at any present time t; to accomplish this, Boltzmann found it necessary to take account of the load X_τ existing at all previous times τ between 0 and t. X, outside the integral, is to be regarded as a function of t; X_τ, inside, as a function of τ. The origin of time, $\tau=0$, has to be taken far enough back so that the body was then in a perfectly undisturbed state, all transient effects due to previous deformations having died out. Boltzmann indicated this by integrating back to $-\infty$, but the above convention is more simple, and equally satisfactory for seasoned diaphragms, though it would hardly do for unseasoned ones. Finally, the function ψ in equation (26), Boltzmann's characteristic function, is an arbitrary, perfectly unknown one, which has to be empirically determined in such a way that equation (26) will fit the facts. This function was later termed the "heredity function" by Volterra, and it has continued to be of interest to mathematicians engaged in the study of integral equations, none of whom, however, has succeeded in establishing the form of the function from theoretical considerations alone. Boltzmann's integral simply provides a certain inductive expression for the superposition of time effects generated by the previous action of external forces. It states that the total inelastic part of the displacement will be found by superposing a succession of small contributions, each of which is proportional to the product of three factors, viz, the magnitude X_τ of the force acting at some previous time τ; the length of time $d\tau$ during which that force was kept on; and some function or other, ψ, of the time elapsed from then until now, $t-\tau$. This was surely a reasonable conjecture, but it did not proceed from any distinct physical conception or observation, and must be subjected to experimental proof. Many scattered experiments have been published during the last 50 years which tend to show that Boltzmann's formula is roughly but not exactly correct.

In applying equation (26) it is understood that X_τ will be available as a function of τ from 0 to t; in other words that the load history has been given. In fact, the problem of irreversible effects in its most general form consists in the calculation of the displacement history (x as a function of t) corresponding to any given load history. When this has been done, the hysteresis loop and other features of the displacement-load curve (x as a function of X) can be derived by eliminating t between the (x, t) and (X, t) curves.

(*c*) *Drift superposition theory.*—Chree in 1898 appears to have been the first to suggest the interpretation of hysteresis and recovery curves as consisting merely of superposed drift effects. While it may not be possible to account for all the hysteresis of a body in this way, on account of the existence of purely directional effects, it is obvious that some hysteresis must ensue if the body continuously experiences drift, i. e., if curves of the type $A'B'$ (Fig. 5) are generated with each new increment of load. Evidently, then, it is important to be able to calculate how much hysteresis and how much of any other effect would necessarily result from the known drift characteristics of a body, i. e., from the known form of the curve $A'B'$ for the body in question, as represented by the function F discussed below. In this way it might be possible to make approximate predictions of the mechanical performance of a diaphragm, starting out with a knowledge only of its drift equation. In any event the calculated magnitudes can be taken as defining an ideal or standard degree of irreversibility, relative to which the observed peculiarities of a diaphragm made of any given alloy may be expressed, just as various fluids studied in thermodynamics are conveniently characterized by their slight departures from equations which define an imaginary ideal gas. One of the purposes which such calculations would immediately serve is the determination of the approximate amount of directional hysteresis present by the comparison of observed and computed values.

The analysis undertaken by Chree was not completely developed, but the same general point of view was later taken up by the present author and has led to a working formula (equation 31 below) curiously related both to Boltzmann's formula for elastic after effect, and to von Schweidler's formula for imperfect dielectrics.

For the inelastic yield write

$$x-\frac{X}{E}=y \tag{27}$$

and define the drift function F by the relation

$$\Delta y=F\ (t-\tau).\ \ \Delta X_\tau \tag{28}$$

in which Δy is the additional yield, or element of drift, existing at any time t, and which was generated at time τ by the load increment ΔX_τ.

The function F, then, can not be determined a priori, but has to be available as an experimental characteristic of the body in question (for example, a helical spring under tension, or a German silver diaphragm loaded by hydrostatic pressure); and it is taken as the starting point of our calculations. This seems logical because *drift* is the *first one* of the various irreversible phenomena which take place when a force acts on a body. Ideally, then, the drift function is to be determined in the laboratory by applying a moderate load ΔX to the body instantaneously, and plotting a curve, against time, for the excess of the observed displacement x over and above the initial instantaneous displacement x_e. The ordinates of this curve, when divided by ΔX, represent F (t $-\tau$), while the abscissas represent values of the elapsed time, $t-\tau$. In practice there are difficulties due to inertia, and to the indeterminateness of the point where the instantaneous displacement stops and drift begins. These can be avoided by applying the load at a finite uniform rate instead of instantaneously, and making suitable corrections; although the drift function thus determined may be taken as it stands as a satisfactory first approximation for the function F. In either case it has been found by experiments on aneroid barometers that the drift is roughly proportional to the cube root of the time elapsed, while a more exact expression is given by

$$F\ (T)=AT^{\frac{1}{3}}\ (1-e^{-mT}) \tag{29}$$

in which T stands for $t-\tau$ and in which A and m are empirical coefficients. Other forms of drift function have been discussed elsewhere [33] and the subject is still under investigation. For a good quality diaphragm the drift in 5 hours may be from 1 to 5 per cent of the deflection.

The coefficients of the drift equation, especially the constant of proportionality at the beginning, for example A of equation (29), will certainly be functions of temperature, and may also be functions of the total load X_τ (not ΔX_τ) on account of the influence of stress on the physical properties of bodies.

By summation, equation (28) leads to a working formula for graphical or arithmetical use, viz:

$$y=\Sigma\ F(t-\tau)\ \Delta X_\tau \tag{30}$$

When X_τ has been given as a continuous function of τ, it is more convenient to rewrite (30) in the equivalent form

$$y=\int_0^t \frac{dX_\tau}{d\tau}\ F\ (t-\tau)\ d\tau \tag{31}$$

In applying (31) suppose X_τ is given as some definite function of the time, $f\ (\tau)$; then $dX_\tau/d\tau$ is to be replaced by $f'(\tau)$ before integrating; and, if the coefficients of F involve X_τ, as, for example, in (33) below, they must be completely written out and X_τ replaced by $f\ (\tau)$ wherever it occurs.

[33] On the Theory of Irreversible Time Effects, Journ. Wash. Acad. Sci. 11: 149–155, 1921.

Since F is a known function, equation (31) can obviously be used for the calculation of y as a function of t in any desired problem, for example, in the case of the closed cycle in Figure 5. When this has been done, the relation between x and t follows from (27); suppose this relation turns out to be $x=\phi\ (t)$; then since the load at any time t is available from the equation $X=f\ (t)$, it only remains to eliminate t between these two equations either analytically or graphically in order to arrive at the final relation between x and X.

In conclusion it may be noted that (31) is mathematically identical with Boltzmann's integral in (26), *provided* the coefficients of the drift function are independent of the load. From a physical standpoint, this matter of the coefficients being influenced by stress constitutes an essential difference between Boltzmann's theory and the drift superposition theory, and should account for a considerable part of the discrepancies observed between Boltzmann's calculations and the corresponding experimental results.

The mathematical identity can be shown upon integrating (31) by parts, remembering that $X_\tau=0$ when $\tau=0$, and that $F\ (0)=0$. As a result of this identification of the two formulas, a physical interpretation has been found for Boltzmann's ψ, for upon carrying through the comparison in detail it appears that $\psi\ (T)=F'\ (T)$; i. e., the heredity function is simply the slope of the drift curve, or the actual mechanical velocity of drift at any time after the application of an instantaneous load.

Again, von Schweidler's formula (familiar in electrical research) is perfectly analogous to (31) in the special case where the stress coefficients entering F are considered negligible. For diaphragm investigations such coefficients are probably not negligible, so it is fortunate that (31) permits of taking proper account of them when desired.

(*d*) *Dimensional theory.*—Where complicated functions have to be dealt with the integration of (31) may become unwieldy but if so the dimensional method, reinforced by comparatively simple experimenting, may be called upon as an aid. In fact, the dimensional theory appears to be more general in its validity, because it does not involve the assumption of superposition.

Let the physical constants needed for specifying the irreversible properties of a body be represented by C_1, $C_2, \cdots C_\nu$ and let X_τ be the load at time τ. Suppose while τ varies from 0 to t the load passes through a maximum range R. Then the load history can be specified by R, t, and the geometrical shape of a diagram having X/R and τ/t for coordinates. Therefore

$$y=funct\ (R,\ t,\ C_1,\ C_2 \cdots C_\nu) \tag{32}$$

in which the form of the function is unknown, but the same for all processes with geometrically similar load diagrams. As (32) is a qualitatively complete physical equation, it is subject to the usual methods of dimensional reasoning.

To illustrate what are meant by the constants $C_1 \cdots C_\nu$ consider a diaphragm whose drift function can be approximately represented by

$$F\ (t-\tau)=B\ (1+\beta X_\tau)\ \ (t-\tau)^n \tag{33}$$

an expression equivalent to (29) when $m=\infty$, and $n=1/3$, in which case the drift constant A is seen to be a function of the load X_τ and equal to $B\ (1+\beta X_\tau)$. The drift coefficients B and n and the stress coefficient β here play the rôle of $C_1 \cdots C_\nu$, and $\nu=3$. The dimensions of the six physical quantities involved in the problem are therefore, taking x, X, and t as fundamental units,

$$\left.\begin{array}{ll} [y]=[x] & [B]=\left[\dfrac{x}{Xt^n}\right] \\ [R]=[X] & [\beta]=\left[\dfrac{1}{X}\right] \\ [t]=[t] & [n]=[1] \end{array}\right\} \tag{34}$$

Therefore (25) becomes

$$\frac{y}{BRt^n}=\phi\ (\beta R,\ n) \tag{35}$$

in which ϕ has to be determined experimentally, but in which there are now only two independent variables, as contrasted with five in equation (32).

Thus the complete equation for the yield of this diaphragm can be found by plotting y/BRt^n as ordinate against R and n for rectangular coordinates. As it is probable that n does not vary much for different diaphragms of the same material, a considerable part of the desired information can be obtained by varying R alone; that is, by altering the maximum load, while preserving the same shape of load-history curve. This experiment is to be repeated for each different shape of load-history curve which it is desired to investigate, as the form of the function ϕ may change in passing from one to another. It is not necessary to change any of the quantities B, β, or t unless desired as a check, yet the results appear to be applicable to diaphragms for which B and β are quite different, and to time intervals either shorter or longer.

The procedure for model experiments can also be illustrated by equation (35), letting primed symbols refer to the model, others to the original. The condition for similarity is that the model be made from a substance having the same value of n, and loaded over a range R' such that $R'/R=\beta/\beta'$. The yield y at any time t can now be computed from the yield y' observed at time t' by the relation

$$\frac{y}{y'} = \frac{B}{B'} \cdot \frac{R}{R'} \cdot \left(\frac{t}{t'}\right)^n \tag{36}$$

As before, the load history diagrams for the model and original are to be kept geometrically similar.

For diaphragms not following the drift function (33)—and the majority will not—one can apply the same mode of reasoning, using the appropriate physical constants in place of B, β, and n.

It is possible that dimensional theory may likewise be applicable to problems involving directional hysteresis, but the procedure for such cases has not yet been worked out. If, however, the amount of hysteresis due to drift be computed either by dimensional or superposition methods, and the result compared with experiment, the difference may be attributed to directional hysteresis provided a sufficiently accurate form of F has been employed. For the present this seems to be the best way to proceed when endeavoring to separate directional hysteresis from time hysteresis. Warburg and Heuse, as indicated above, have taken a significant step in this direction, using Boltzmann's principle; but the mathematical methods here outlined are thought to be more general and more accurate than Boltzmann's and are suggested for consideration by investigators having opportunity to pursue the subject further.

PRACTICAL EXPEDIENTS FOR THE USE OF DIAPHRAGMS.

While awaiting the development of actual improvements in the quality of diaphragms, various expedients are available for utilizing existing diaphragms in the most effective manner, among which the following may be briefly mentioned.

(*a*) *Automatic compensation for elastic lag.*—At least two methods have been proposed for solving this most elusive and interesting problem of instrument design. One method, applied by Peterson in his development of precision altimeters at the Bureau of Standards, consists in attaching the diaphragm—preferably an extra large and very thin diaphragm, free from corrugations—in such a way that a variable proportion of the total area will be supported externally. If so arranged that an increasing deflection of the diaphragm causes a diminution of unsupported area, then the tendency for drift under constant load is in some measure automatically suppressed; for if drift does take place to any extent, that is if the deflection under constant load does increase, the load itself will immediately be diminished. A second method consisting of a differential mechanism has been independently proposed by Bennewitz [34] and by the present author. This method requires the use of two separate diaphragm elements acting together in such a way that the instrument indication is proportional to the difference in the deflections of the two elements. The diaphragm elements may be selected with identical elastic lag characteristics, the resultant effect of which is eliminated upon taking the difference, but one element must be chosen stiffer than the other in order to leave a residual elastic deflection sufficient for operating the instrument.

[34] K. Bennewitz: Über die elastiche Nachwirkung, Physik. Zs. 21: 7-3-705, 1920; Verfahren zur Kompensation der elastichen Nachwirkung ibid. 22: 329-332, 1921. The principle has been proposed by the present author in a form which need not be restricted to elastic phenomena.

(*b*) *Null methods.*—A method proposed by Barus [35] has already been described in which the deflection of the pressure element is brought back to zero by the opposition of a steel spring. This method seems capable of further development and application to diaphragm instruments, since it shifts the responsibility for accurate performance from the uncertain diaphragm element to the more reliable steel spring. The N. A. C. A. balanced-diaphragm type of instrument is somewhat similar in principle. However, such methods usually require mechanical manipulation at the time of taking each observation, which would be likely to present difficulties in aeronautical use.

(*c*) *Stiff spring method.*—The theory of the stiffness of coupled systems, equation (22) above, led to the proposal for designing instruments with the steel spring relatively as stiff as possible compared to the diaphragm. In the case of an aneroid barometer, if the spring were made 100 times as stiff as the diaphragm element, irregularities of the order of 10 per cent in the action of the diaphragm element itself, due to elastic lag would be reduced to as little as one-tenth of 1 per cent in the final indication of the combined instrument, provided the errors of the spring were negligible; and it is a fact that the elastic lag of steel springs for commercial instruments, although capable of further improvement, is negligible for most purposes. The stiff spring principle was taken as the starting point in the development of precision altimeters at the Bureau of Standards, and has led to successful results.

(*d*) *Discontinuous operation.*—Two methods have been proposed for relieving the diaphragms from stress until the desired conditions have been reached for which final observations are desired, as, for example, near the maximum altitude in a flight. The first method has been commercially applied in the Watkins patent aneroid with vacuum box release. In this instrument the vacuum box unscrews from the bottom of the case and during the greater part of the time is left in a collapsed condition comparatively free from stress. When an observation is to be made, a large thumbscrew projecting through the bottom of the case is given several turns, which serves to distend the vacuum box and bring it up flush against a stop. It is now in the normal position for observations, which can be immediately made before any appreciable drift has accumulated. In this way the performance of the diaphragm element is made practically independent of the manner in which the pressure is varied during the flight previous to the moment of observation. A second method, suggested by the author in connection with altitude instruments, would consist in the use of an auxiliary altimeter in an airtight case. This case would be automatically closed at all altitudes greater than a certain minimum limit, say, 1,000 feet. Such an instrument would be protected from large stresses so that the diaphragms would be comparatively free from elastic lag, and therefore it could be constructed in a very sensitive open scale pattern to be reserved for observations near sea level, for example, when landing.

(*e*) *Alteration of scale value.*—It may at times be desired to extend the range of the scale on existing instruments, as had to be done, for example, with barographs during the war. It was found possible to do this without redesigning the diaphragms by simply adding an auxiliary external spring. The requisite stiffness of this spring can be calculated from the theory of coupled systems, equation (16), in such a manner as to provide approximately the desired scale value, final adjustments being made by trial.

(*f*) *Similarity of flight conditions with test conditions.*—Whatever instrument may be selected for use, one final opportunity for insuring the accuracy of the results consists in arranging similar physical conditions both for the flight and the laboratory calibration. To some extent this can be done by instructing the pilot to take observations preferably with increasing deflections, and for important work to use only an instrument which is in a rested condition before the flight. This is suggested on the understanding that the laboratory calibration is made for increasing deflections. Where the pilot is not free to regulate the conditions of flight and of observation, maximum accuracy in the final results of important observations may be secured by subjecting the instrument to a flight history test. This type of test, which has been found necessary in verifying barographs used for competitive altitude determination, consists in reproducing in the laboratory as closely as possible the same variation of temperature and pressure from moment to moment as was experienced during the flight.

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[35] C. Barus: The counter twisted curl aneroid, Am. Jour. Sci., v. 1: 114–129, 1896.

CLASSIFICATION OF DIAPHRAGM RESEARCH PROBLEMS.

As a result of the general survey presented in the previous sections of this report, it will be apparent that the more important problems of diaphragm investigation which remain for the future are essentially experimental and can be grouped together as follows:

1. Problems on elastic action of diaphragms.
   (*a*) Stiffness, and deflection curves, in relation to the entire geometrical design of the diaphragm, and to the elastic constants.
   (*b*) Determination of the mechanical factor C (equation 13), for diaphragms of different design; to be used in connection with the calculation of temperature effects.
   (*c*) Experimental measurement of internal stresses.
   (*d*) Effect of initial tension, for both metallic and rubber diaphragms.
   (*e*) Effective areas for different types of diaphragm.
2. Elastic properties of materials.
   (*a*) Determination of elastic constants, including coefficients for departure from isotropy, homogeneity, and Hooke's law.
   (*b*) Determination of temperature coefficients of elastic constants.
3. Elastic lag of materials.
   (*a*) General laws for seasoned samples as determined, for example, by simple tension and torsion experiments.
   (*b*) Correlation of the different phenomena and comparison with calculations.
   (*c*) Separation of directional from time hysteresis.
   (*d*) "Proportional limit," "elastic limit," and ultimate strength of seasoned samples; relation of "fatigue limit" to elastic lag phenomena.
   (*e*) Effect of temperature on the foregoing characteristics.
4. Elastic lag of seasoned diaphragms.
   (*a*) Relation between performance characteristics and mechanical design, for diaphragms constructed from any given material whose lag charactistics have been determined.
   (*b*) Influence on diaphragm performance of variation in the respective constants of the material.
   (*c*) Effect of different processes for rolling, stamping, spinning, or fastening the edges.
5. Comparison of seasoning processes.
   (*a*) Heat treatment.
   (*b*) Repeated stress.
   (*c*) Both together.
   (*d*) Natural aging.
6. Metallurgical investigations.
   (*a*) Correlation of above results with chemical constitution, mechanical and thermal treatment, and metallographic observations.
   (*b*) Development of new diaphragm alloys; pre-determination or control of physical constants investigated above; particularly lag constants, and the temperature coefficients of elastic constants.

Three general principles evident in the above outline which are often overlooked, but which should help to simplify any investigation of elasticity, are: First, the disentanglement of the properties of bodies (e. g., diaphragms) from the properties of substances (e. g. diaphragm materials); second, separate consideration of elastic action and irreversible (i. e., lag) effects; third, the distinction between recoverable and irrecoverable effects when dealing with irreversible phenomena, the former being experienced with seasoned diaphragms and the latter with unseasoned diaphragms.

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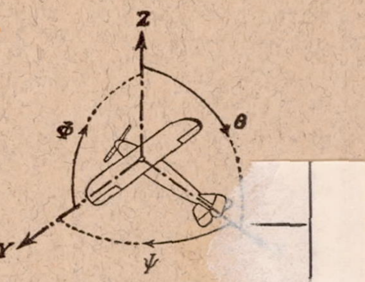

Positive directions of axes and angles (forces and moments) are shown by arrows.

Axis.		Force (parallel to axis) symbol.	Moment about axis.			Angle.		Velocities.	
Designation.	Symbol.		Designation.	Symbol.	Positive direction.	Designation.	Symbol.	Linear (component along axis).	Angular.
Longitudinal	X	X	rolling	L	$Y \longrightarrow Z$	roll	Φ	u	p
Lateral	Y	Y	pitching	M	$Z \longrightarrow X$	pitch	Θ	v	q
Normal	Z	Z	yawing	N	$X \longrightarrow Y$	yaw	Ψ	w	r

Absolute coefficients of moment

$$C_l = \frac{L}{q\,b\,S} \quad C_m = \frac{M}{q\,c\,S} \quad C_n = \frac{N}{q\,f\,S}$$

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS.

Diameter, D
Pitch (a) Aerodynamic pitch, p_a
(b) Effective pitch, p_e
(c) Mean geometric pitch, p_g
(d) Virtual pitch, p_v
(e) Standard pitch, p_s
Pitch ratio, p/D
Inflow velocity, V'
Slipstream velocity, V_s

Thrust, T
Torque, Q
Power, P
(If "coefficients" are introduced all units used must be consistent.)
Efficiency $\eta = T\,V/P$
Revolutions per sec., n; per min., N
Effective helix angle $\Phi = \tan^{-1}\left(\frac{V}{2\pi r n}\right)$

5. NUMERICAL RELATIONS.

1 HP = 76.04 kg. m/sec. = 550 lb. ft/sec.
1 kg. m/sec. = 0.01315 HP
1 mi/hr. = 0.44704 m/sec.
1 m/sec. = 2.23693 mi/hr.

1 lb. = 0.45359 kg.
1 kg. = 2.20462 lb.
1 mi. = 1609.35 m. = 5280 ft.
1 m. = 3.28083 ft.