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REPORT No. 177

THE EFFECT OF SLIPSTREAM OBSTRUCTIONS ON AIR PROPELLERS

By E. P. LESLEY and B. M. WOODS

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AERONAUTICAL SYMBOLS.

1. FUNDAMENTAL AND DERIVED UNITS.

	Symbol.	Metric.		English.	
		Unit.	Symbol.	Unit.	Symbol.
Length	l	meter	m.	foot (or mile)	ft. (or mi.).
Time	t	second	sec.	second (or hour)	sec. (or hr.).
Force	F	weight of one kilogram	kg.	weight of one pound	lb.
Power	P	kg.m/sec		horsepower	IP
Speed		m/sec	m. p. s.	mi/hr	M. P. H.

2. GENERAL SYMBOLS, ETC.

Weight, $W=mg$.

Standard acceleration of gravity, $g=9.806\text{m/sec.}^2=32.172\text{ft/sec.}^2$

Mass, $m=\frac{W}{g}$

Density (mass per unit volume), ρ

Standard density of dry air, 0.1247 (kg.-m.-sec.) at 15.6°C. and 760 mm. = 0.00237 (lb.-ft.-sec.)

Specific weight of "standard" air, 1.223 kg/m.^{3} = 0.07635 lb/ft.^{3}

Moment of inertia, mk^2 (indicate axis of the radius of gyration, k, by proper subscript).

Area, S; wing area, S_w, etc.

Gap, G

Span, b; chord length, c.

Aspect ratio $=b/c$

Distance from $c.\ g.$ to elevator hinge, f.

Coefficient of viscosity, μ.

3. AERODYNAMICAL SYMBOLS.

True airspeed, V

Dynamic (or impact) pressure, $q=\frac{1}{2}\rho V^2$

Lift, L; absolute coefficient $C_L=\frac{L}{qS}$

Drag, D; absolute coefficient $C_D=\frac{D}{qS}$.

Cross-wind force, C; absolute coefficient $C_c=\frac{C}{qS}$.

Resultant force, R

(Note that these coefficients are twice as large as the old coefficients L_c, D_c.)

Angle of setting of wings (relative to thrust line), i_w

Angle of stabilizer setting with reference to thrust line i_t

Dihedral angle, γ

Reynolds Number $=\rho\frac{Vl}{\mu}$, where l is a linear dimension.

e. g., for a model airfoil 3 in. chord, 100 mi/hr., normal pressure, 0°C: 255,000 and at 15.6°C, 230,000;

or for a model of 10 cm. chord, 40 m/sec., corresponding numbers are 299,000 and 270,000.

Center of pressure coefficient (ratio of distance of C. P. from leading edge to chord length), C_p.

Angle of stabilizer setting with reference to lower wing. $(i_t-i_w)=\beta$

Angle of attack, α

Angle of downwash, ϵ

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REPORT No. 177.

THE EFFECT OF SLIPSTREAM OBSTRUCTIONS ON AIR PROPELLERS.

By E. P. Lesley and B. M. Woods.

This report was prepared by E. P. Lesley and B. M. Woods for publication by the National Advisory Committee for Aeronautics and describes an investigation to determine the effect of slipstream obstructions on air propellers.

PURPOSE OF INVESTIGATION.

The screw propeller on an airplane is usually placed near other objects, and hence its performance may be modified by them. Results of tests on propellers free from slipstream obstructions both fore and aft are therefore subject to correction for the effect of such obstructions, and the purpose of the investigation herein described was to determine the effect upon the thrust and torque coefficients and efficiency, for previously tested air propellers, of obstructions placed in the slipstream; it being realized that such previous tests had been conducted under somewhat ideal conditions that are impracticable of realization in flight.

At the start it was planned to use obstructions representative of the nose of the fuselage, of radiators, or of other parts of an airplane structure, but a consideration of the wide variety of forms thus defined led to the selection of simple geometrical forms for the initial investigation. Such forms offered the advantage of easy exact reproduction at another time, or in other laboratories, and it was believed that the effects of obstructions usually encountered might be deduced or surmised from those of the ones chosen.

APPARATUS AND PROGRAM.

Although the propeller testing dynamometer of the Stanford laboratory has been fully described in report No. 14, a brief statement of its peculiar features may be of value in this present report for ready reference.

The propeller shaft is carried in ring oiled bearings that are supported by a cast-iron standard which is securely attached to the experiment chamber floor of the wind tunnel. The shaft is free from longitudinal constraint except that afforded by the thrust balance and, when rotating, slides easily through the bearings. A ball-bearing collar communicates the thrust or pull to this balance, where it is weighed directly. The balance is sensitive to 0.005 pound, and readings are made to 0.01 pound. The shaft is driven through bevel gears from a motor that is placed at one side out of the wind stream. The torque or turning moment is determined by measuring the twist of a helical spring that constitutes a part of the drive shaft. The spring is calibrated by means of a Prony brake put in place of the propeller. The angular yield at 10 pound-feet moment is about 200°, so that, since the scale may be read to 0.1°, the turning moment may be determined within 0.005 pound-foot. A correction of measured torque is made for the frictional resistance of the bearings and gears of the dynamometer. The revolutions are counted by means of an accurate chronograph.

The wind velocity is determined from the reduction of pressure within the experiment chamber. Hundreds of calibrations have shown that for the range of velocities used (20 to 75 m. p. h.) the ratio of velocity head to reduction in experiment chamber pressure is practically constant. It was realized that a considerable obstruction placed in the wind stream might effect this ratio and careful tests were conducted to determine such effect. Although with the largest obstruction used an appreciable reduction in a wind velocity was noted for a given tunnel fan speed, there was a corresponding change in the experiment chamber pressure reduction, so that the ratio was not affected to an appreciable degree.

It was believed that the apparatus was well suited to the work in hand since the obstructions could be fastened to the dynamometer frame (see fig. 1 and 2) and the tests conducted as usual, resulting in the determination of the coefficients C_t (thrust), C_q (torque), and η (efficiency), which might be compared with the coefficients as derived from previous tests with unobstructed slipstreams.

The model propellers selected were Nos. 1, 3, 5, 7, 9, and 11. They are fully described in reports No. 14 and No. 141. It may be noted here that 1, 5, and 9 are of the straight type, having uniform width, while 3, 7, and 11 are of the curved, tapering or saber form. Nos. 1 and 3 have a nominal pitch-diameter ratio of 0.9, Nos. 5 and 7 one of 0.7, and Nos. 9 and 11 one of 0.5. All have a mean blade width of 0.15 of the radius, which is 18′′.

The obstructions used were as follows:

No. 1. Thin metal disk, 9′′ diameter.
2. Thin metal disk, 12′′ diameter.
3. Thin metal disk, 18′′ diameter.
4. Metal cylinder, 9′′ diameter, 30′′ long, end toward propeller closed, and other end faired to dynamometer.
5. Similar cylinder, 12′′ diameter.
6. Similar cylinder, 18′′ diameter.

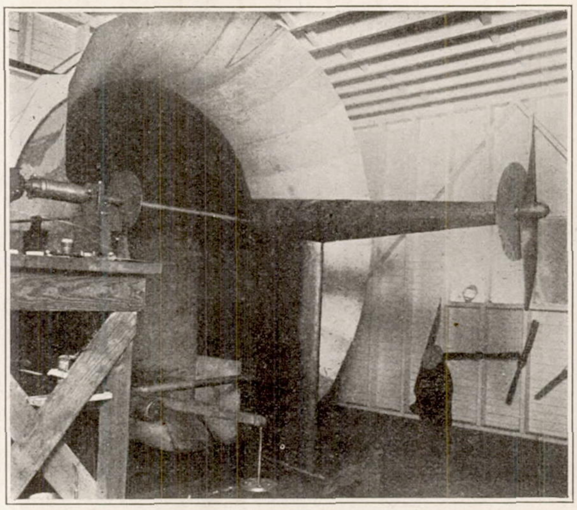

FIG. 1.—Showing obstruction No. 3 mounted on dynamometer frame.

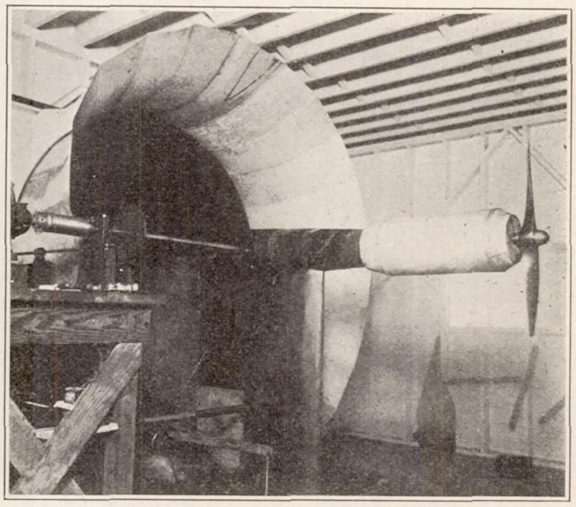

FIG. 2.—Showing obstruction No. 7 mounted on dynamometer frame

7. Metal cylinder, 12′′ diameter, with end toward propeller closed and tapered to 9′′ diameter, 60° taper. Other end faired to dynamometer.
8. Metal cylinder, 12′′ diameter, with end toward propeller closed and tapered to 6′′ diameter, 60° taper. Other end faired to dynamometer.
9. Metal cylinder, 12′′ diameter, with end toward propeller closed and tapered to 3′′ diameter, 60° taper. Other end faired to dynamometer.

It was originally planned to use a 6′′ diameter disk and a cylinder of the same size, but the early tests showed so slight an effect of these obstructions on a three ft. dia. model propeller that the 9′′ diameter was used instead.

The six propellers were tested each with the three disks at $\frac{1}{2}$′′ from the propeller hub, and propeller No. 3 was tested in addition with the remaining obstructions at the same distance and with the 12′′ and 18′′ disks at 6′′ and 12′′ from the hub.

It was at first contemplated only to measure the forces acting upon the propeller, with the obstructions mounted on the dynamometer frame as shown in Figures 1 and 2. The results of the first tests with propeller No. 1 and a 9′′ disk so mounted were as follows:

At low and moderate slips the thrust and torque were increased. At high slips the torque and thrust were decreased. At all slips the efficiency was apparently increased. The thrust was thus increased more or decreased less than torque.

These results were not altogether unexpected, since others [1] had given evidence of the same phenomena. With the 18″ disk, however, the apparent efficiency of propeller No. 1 reached a maximum of 115 per cent, and checks were made to insure that measurement of torque, thrust, revolutions, and velocity were correct. The measuring devices were carefully calibrated and the test was repeated. A pitot tube, placed 2 feet from the tips of the propeller blades and 1 foot within the line of the tunnel wall, was used to determine velocity. The results were practically identical with those of previous tests in which the reduction of pressure within the experiment chamber was used as an index of velocity.

In order to determine the total thrust reaction upon the obstruction, as well as that upon the propeller, additional tests were made with the obstruction mounted, by means of a ball bearing, on the propeller shaft and in the same space relation to the propeller as was used when it was mounted upon the dynamometer frame.

Letting T = pull exerted on the shaft by the propeller.
R = total reaction of the obstruction.

Then with the obstruction on the dynamometer T is measured, and with the obstruction on the shaft $T-R$ is measured. From these R may be determined.

In addition the resistance of each of the nine obstructions, without the propeller, was measured. This was done by mounting, with a ball bearing, the obstruction alone upon the shaft. The shaft was rotated to eliminate longitudinal shaft friction, and the resistance weighed by the thrust balance for wind velocities from 20 to 70 miles per hour.

RESULTS OF TESTS.

The results of the tests with the propellers and obstructions are given as tables of derived coefficients defined as follows:

$$C_t=\frac{gT}{\Delta\, v^2\, D^2}$$

$$C_q=\frac{gQ}{\Delta\, v^2\, D^3}$$

$$C_r=\frac{gR}{\Delta\, v^2\, D^2}$$

$$\eta=\text{Efficiency}=\frac{Tv}{2\pi\, nQ}=\frac{C_t\, v}{C_q\, nD\, 2\pi}.$$

In the above,

T = Thrust or pull on propeller shaft.
Q = Torque or turning moment of propeller shaft.
R = Total thrust reaction on obstruction.
v = Velocity of advance.
n = Revolutions of propeller per unit time.
D = Diameter of propeller.
g = Gravity acceleration constant.
Δ = Density of air in gravity units per cubic linear unit.

Any homogeneous system of units may be used. The letter M with subscript indicates the mounting of the obstruction as follows:

M_1, obstruction mounted on dynamometer frame.

M_2, obstruction mounted on a ball bearing on the shaft so that its total thrust reaction combined with that of the propeller is communicated to the shaft.

In addition to the tables, the results for propeller No. 3, on which the larger number of tests were made, are plotted as ordinates for the various coefficients with $\frac{v}{nD}$ as abscissae. See Figures 3 to 15.

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[1] Aeronautics in Theory and Experiment. Cowley and Levy 2d. ed.
British Advisory Committee for Aeronautics. Reports and Memoranda Nos. 305, 344, and 393. By A. Fage and H. E. Collins.
Design of Screw Propellers for Aircraft. Watts.

Table I shows the coefficients C_t, C_q, and η for the six propellers when operating with an unobstructed slipstream. These coefficients may be in some cases slightly different from those published in reports Nos. 14 and 141. This is due to the fact that the coefficients as here given are recent test results that have not been modified by cross fairing, in the interest of consistency, the curves as originally drawn.

Table II shows the coefficients C_t, C_q, C_r, and η as derived for the six propellers when operating with the obstructions as indicated. In this table it may be noted that one value only of C_q is given, and that is designated as $C_q\ M_{1-2}$. It was found that the torque was the same with the obstruction mounted upon the shaft as when it was placed upon the dynamometer. This was to be expected since the obstruction and propeller were for the two cases in the same space relation and no torque reaction of the obstruction was communicated to shaft except the almost negligible friction of the ball bearing to which the obstruction was secured in the case of shaft mounting; moreover, this was included in the correction of torque for friction of bearings and gears of the dynamometer.

$$C_r = C_t\ M_1 - C_t\ M_2.$$

This is apparent from the previous definitions.

DISCUSSION.

It is especially to be noted that there is no simple means of determining the propeller efficiency, per se, when the propeller is operated in front of an obstruction. If the usual quantities are measured or computed for the determination of the efficiency from the relation $\eta = \frac{Tv}{2\pi nQ}$, and if T, the thrust, is obtained by means of a balance on the shaft, it is apparent that the efficiency of the combination for the purpose of propelling an airplane will be obtained with the obstacle on the shaft—that is to say, with mounting No. 2, as previously described. With the obstruction on the dynamometer, mounting No. 1, the apparent efficiency resulting has little practical significance. The thrust measured in this case includes possibly a pressure reaction of the obstruction on the propeller as an external, unbalanced force, which is in reality balanced by the equal and opposite action on the obstruction, giving the effect of an internal force. Comparison of the thrust values obtained in this case, however, with those for obstruction mounting No. 2 exhibits the nature of the total reaction on the obstruction.

If it is desired to obtain the actual efficiency of the propeller, the *resistance* of the obstruction in the slipstream must be separated from the total reaction upon the obstruction and be credited to the propeller as thrust in the case of the mounting on the shaft. An approximation to this resistance was obtained by determining the resistance of the obstruction in a smooth, nonturbulent air stream having the velocity of the slipstream. The effect of turbulence of the stream was not taken into account and the numerical results of this approximate method are therefore sufficiently in question to justify their omission from the report. It suffices to say that no outstanding change in propeller efficiency was noted.

With the mounting of the obstruction on the dynamometer, it is important to observe the effect of distance between the obstruction and the propeller on the thrust, torque, and apparent efficiency. The velocity of the slip streamchanges little for a distance equal to one-half the radius of the propeller in its wake. Such change as occurs is, generally speaking, an increase in velocity, as evidenced by the converging of the stream lines. Hence, no material reduction in the resistance of an obstruction placed in the stream would be expected as it moved away from close proximity to the propeller. However, the effect of the pressure reaction, if any, in the space between the obstruction and the propeller should be less at greater distances. A lessening of pressure reaction would result in reducing the apparent thrust and efficiency with increasing distance, and would therefore make plausible the theory of a pressure reaction as above. The tests performed gave results supporting this point of view. For example, the maximum apparent efficiency with propeller No. 3 and the 12″ disk assumed the following values:

Propeller No. 3—12″ disk on Dynamometer.

Distance from propeller to obstruction.	Maximum apparent efficiency.
½″	0.91
6″	.89
12″	.86
No obstruction.	.81

Also for the same propeller with 18″ disk.

Propeller No. 3—18″ disk on Dynamometer.

Distance from propeller to obstruction.	Maximum apparent efficiency.
½″	1.19
6″	.96
12″	.88
No obstruction.	.81

At the same time no considerable change, with this increase of distance, was found in the efficiency of the combination with mounting No. 2 of the obstruction. Figures 5, 14, and 15 show the effect of distance with the 18″ disk and give the following:

For the working range of $\frac{v}{nD}$; i. e., from $\frac{v}{nD}=0.4$ to $\frac{v}{nD}=0.9$.

a. The apparent thrust decreases with increase of distance.

b. The torque increases slightly with distance at $\frac{v}{nD}=0.4$ and decreases slightly at $\frac{v}{nD}=0.9$.

c. The apparent efficiency decreases with distance for all values of $\frac{v}{nD}$. This is most marked, however, for large values of $\frac{v}{nD}$ (low slips).

PRACTICAL INFERENCES FROM THE TESTS.

The propeller exists as a mechanism for converting torque into thrust. The expression for its efficiency $\eta=\frac{Tv}{2\pi nQ}$ exhibits this fact fully. However, if this formula is to serve in the ordinary cases of the airplane, the numerator of the fraction must represent the useful work of the propulsion per unit of time in all cases and its denominator the power input. In performing tests of propellers with slipstream obstructions there is little difficulty in maintaining the analogy for the denominator. For the numerator it is necessary to decide what proportion of the thrust or thrust modified by resistance shall be used in determining efficiency.

It is at once apparent that a different definition of efficiency is necessary for each interpretation used. From the point of view of airplane propulsion it would seem logical to continue to interpret the numerator as the useful work per unit of time. Hence, the thrust becomes that which the airplane as a whole receives from the power plant and its accessories and the velocity is that of translation of the airplane as produced by this thrust. If the propeller, with the engine, the radiator, and the cowling, is thought of as producing the torque and the thrust, it is the net thrust of this assembly which is provided to pull the airplane. Let us call an efficiency derived from this thrust the *combined efficiency*. It corresponds to the efficiencies obtained with the obstructions mounted on the shaft (mounting No. 2). From the construction point of view, at least two possibilities appear: (*a*) The power plant assembly may be kept intact in one place with the propeller as a tractor or pusher screw in close proximity to the engine and radiator and with these latter in the slipstream; or (*b*) the propeller might be geared to the power plant and so separated from it at some distance, thus placing the latter out of the slipstream. With the obstructions used in this investigation, the former gives what has been called the *combined efficiency* and the latter what may be called the *parallel propulsive efficiency*. The former is obtained with the obstruction mounted on the shaft directly in the slipstream; the latter is derived by using as the net thrust the values obtained by subtracting the resistance of the obstruction in a free stream of the translation velocity assumed from the thrust of the propeller free and unobstructed. This would correspond roughly to the geared propellers of the early Wright machines with the radiators in the air stream but out of the slipstream, provided it is assumed that the engine is placed in the fuselage where it does not alter the existing resistances. In Table III the values of the parallel propulsive efficiency for the various propellers and obstructions are set forth. The tabulation of the combined efficiencies is included in Table II, giving the direct results of the tests. Table IV supplies the resistance coefficients K of the obstructions themselves as taken from the formula.

$$\text{Resistance} = \frac{K\Delta v^2}{g}$$

If the tabulated values of the combined and the parallel propulsive efficiencies for given propellers and obstructions are plotted, the resulting curves exhibit graphically the relative superiority of mounting a given obstruction in the slipstream or on the plane away from the slipstream.

Before attempting to state a general conclusion, let us examine the results given in the tables. With the disks of 9″, 12″, and 18″ diameter placed close to the propeller, the combined efficiency is generally less than the parallel propulsive efficiency throughout the working range of most propellers. This range may be taken as the middle third of the range of values of $\frac{v}{nD}$ for the propeller concerned. The difference is small for the 9″ disk, running in most cases from 0 to 2 points. For the 12″ disk it is slightly greater, and for the 18″ disk it is considerably greater, reaching values of as much as 10 points. The effect of low pitch ratio is to cause the combined efficiency and parallel propulsive efficiency curves to intersect in the working range; e. g., propellers Nos. 9 and 11. In every case both combined and parallel propulsion efficiencies are less than the efficiencies for propellers with unobstructed slipstreams.

With blunt-ended cylinders the results are similar except that the variations are smaller. Especially is the loss in efficiency from the unobstructed slipstream efficiency reduced. Hence, the fairing of the obstructions in the direction of a streamline form brings the curves nearer to those of the unobstructed slipstream, as might be anticipated.

Finally, the tests with obstructions 7, 8, and 9 (12″ cylinders with conical noses) show little difference among themselves, but all seem to indicate closer resemblance to the unobstructed slipstream curves than the tests of the blunt-ended 12″ cylinder. There is thus less and less variation from the unobstructed slipstream results as one considers successively disks, blunt-ended cylinders, and "nosed" cylinders.

General conclusions may be stated as follows:

1. The combined efficiency of a propeller with any obstruction in the slipstream is less than that of the propeller free and unobstructed.

2. For blunt obstructions, such as circular disks and flat-ended cylinders, placed close to the propeller in the slipstream, the difference between parallel propulsive efficiency and combined efficiency for obstructions of diameter up to one-third that of the propeller, is of little consequence. In no case is the advantage of either over the other such as to warrant a change from a simple and logical arrangement in order to effect a gain in efficiency.

TABLE I.

COEFFICIENTS FOR PROPELLERS WITH UNOBSTRUCTED SLIPSTREAMS.

$\frac{v}{nD}$	C_t	Cq	η	C_t	Cq	η
	Propeller No. 1.			Propeller No. 3.		
0.3	1.670	0.1720	0.463	1.660	0.1640	0.483
.4	.860	.0980	.558	.832	.0914	.580
.5	.500	.0626	.636	.485	.0580	.665
.6	.307	.0412	.711	.300	.0390	.734
.7	.187	.0273	.764	.184	.0262	.784
.8	.117	.0188	.792	.113	.0176	.808
.9	.069	.0125	.789	.069	.0122	.808
1.0	.038	.0082	.738	.038	.0078	.775
1.1	.015	.0049	.530	.017	.0049	.625
	Propeller No. 5.			Propeller No. 7.		
0.25	2.100	0.1900	0.439	2.020	0.1750	0.460
.3	1.360	.1280	.507	1.390	.1280	.519
.4	.715	.0735	.620	.670	.0690	.618
.5	.376	.0432	.694	.370	.0420	.701
.6	.212	.0272	.744	.210	.0265	.755
.7	.115	.0172	.745	.115	.0169	.759
.8	.059	.0110	.684	.060	.0110	.694
.9	.022	.0068	.464	.019	.0064	.425
	Propeller No. 9.			Propeller No. 11.		
0.25	1.465	0.1170	0.499	1.460	0.1150	0.505
.3	.925	.0783	.564	.925	.0770	.573
.4	.428	.0420	.650	.420	.0410	.653
.5	.204	.0242	.672	.200	.0231	.687
.6	.089	.0140	.607	.094	.0140	.641
.7	.027	.0098	.307	.028	.0080	.392

TABLE II.

COEFFICIENTS FOR PROPELLERS WITH OBSTRUCTED SLIPSTREAMS.

	$\frac{v}{nD}$	$C_t\ M_1$	Combined $C_t\ M_2$	$Cq\ M_{1-2}$	Apparent $\eta\ M_1$	Combined $\eta\ M_2$	C_r
Propeller No. 1 with obstruction No. 1 at ½″ from hub......	0.3	1.640	1.564	0.1690	0.463	0.442	0.076
	.4	.875	.820	.0995	.567	.531	.055
	.5	.506	.459	.0615	.655	.595	.047
	.6	.310	.273	.0408	.725	.637	.037
	.7	.195	.163	.0280	.779	.650	.032
	.8	.124	.095	.0195	.812	.623	.029
	.9	.076	.049	.0134	.815	.530	.027
	1.0	.046	.020	.0092	.795	.335	.026
Propeller No. 1 with obstruction No. 2 at ½″ from hub......	.3	1.620	1.466	.1650	.468	.425	.154
	.4	.895	.790	.0990	.581	.508	.105
	.5	.513	.434	.0608	.671	.568	.079
	.6	.321	.252	.0408	.751	.590	.069
	.7	.205	.145	.0282	.809	.573	.060
	.8	.132	.076	.0197	.854	.491	.056
	.9	.084	.031	.0138	.872	.322	.053
	1.0	.054	.003	.0099	.870	.049	.051
	1.1	.032	....	.0069	.815	....	.050
Propeller No. 1 with obstruction No. 3 at ½″ from hub......	.3	1.600	1.132	.1560	.490	.346	.468
	.4	.885	.585	.0900	.627	.413	.300
	.5	.534	.312	.0570	.739	.435	.222
	.6	.350	.168	.0395	.833	.408	.182
	.7	.240	.079	.0285	.938	.309	.161
	.8	.164	.019	.0205	1.016	.118	.145
	.9	.113	—.019	.0151	1.072	....	.132
	1.0	.077	—.047	.0110	1.113	....	.124
	1.1	.054	—.066	.0083	1.140	....	.120
	1.2	.038	—.081	.0063	1.150	....	.119
Propeller No. 3 with obstruction No. 1 at ½″ from hub......	.3	1.590	1.520	.1540	.493	.471	.070
	.4	.832	.780	.0880	.604	.565	.052
	5	.481	.435	.0555	.689	.624	.046
	.6	.297	.260	.0375	.755	.663	.037
	.7	.188	.157	.0260	.805	.673	.031
	.8	.119	.092	.0183	.829	.639	.027
	.9	.075	.048	.0128	.840	.533	.027
	1.0	.045	.016	.0088	.822	.290	.029

TABLE II—Continued.

COEFFICIENTS FOR PROPELLERS WITH OBSTRUCTED SLIPSTREAMS—Continued.

	$\frac{v}{nD}$	$C_t\ M_1$	Combined $C_t\ M_2$	$Cq\ M_{1-2}$	Apparent $\eta\ M_1$	Combined $\eta\ M_2$	C_r
Propeller No. 3 with obstruction No. 2 at ½" from hub	0.3	1.560	1.340	0.1490	0.500	0.432	0.230
	.4	.840	.715	.0875	.614	.519	.125
	.5	.498	.407	.0567	.699	.571	.091
	.6	.311	.237	.0382	.776	.590	.074
	.7	.202	.137	.0269	.835	.565	.065
	.8	.133	.073	.0193	.880	.481	.060
	.9	.087	.031	.0140	.910	.320	.056
	1.0	.056	.003	.0100	.892	.048	.053
Propeller No. 3 with obstruction No. 3 at ½" from hub	.3	1.560	1.062	.1440	.518	.352	.502
	.4	.852	.520	.0830	.654	.402	.332
	.5	.519	.278	.0540	.765	.409	.241
	.6	.344	.150	.0380	.865	.377	.194
	.7	.238	.070	.0278	.952	.281	.168
	.8	.165	.015	.0204	1.030	.077	.150
	.9	.120	−.021	.0156	1.094	....	.141
	1.0	.085	−.050	.0119	1.150	....	.135
	1.1	.062	−.078	.0092	1.181	....	.140
Propeller No. 3 with obstruction No. 4 at ½" from hub	.3	1.555	1.530	.1590	.467	.459	.025
	.4	.813	.789	.0900	.575	.557	.024
	.5	.470	.448	.0560	.668	.638	.022
	.6	.292	.272	.0376	.740	.692	.020
	.7	.186	.166	.0258	.802	.717	.020
	.8	.120	.099	.0180	.851	.705	.021
	.9	.077	.057	.0127	.875	.645	.020
	1.0	.047	.027	.0088	.855	.485	.020
	1.1	.024	.005	.0057	.738	.160	.019
Propeller No. 3 with obtsruction No. 5 at ½" from hub	.3	1.579	1.490	.1540	.490	.460	.089
	.4	.822	.762	.0875	.598	.554	.060
	.5	.470	.425	.0550	.680	.614	.045
	.6	.297	.258	.0378	.750	.653	.039
	.7	.193	.158	.0266	.805	.661	.035
	.8	.125	.090	.0187	.847	.613	.035
	.9	.081	.047	.0132	.879	.510	.034
	1.0	.053	.021	.0097	.867	.344	.032
	1.1	.032	....	.0068	.815	....	....
Propeller No. 3 with obstructions No. 6 at ½" from hub	.3	1.645	1.280	.1510	.520	.382	.365
	.4	.861	.620	.0835	.642	.463	.241
	.5	.509	.346	.0540	.750	.507	.163
	.6	.327	.198	.0371	.848	.510	.129
	.7	.226	.108	.0269	.938	.447	.118
	.8	.160	.050	.0202	1.011	.316	.110
	.9	.112	.015	.0151	1.063	.142	.097
	1.0	.083	....	.0121	1.090	....	....
	1.1	.060	....	.0096	1.095	....	....
Propeller No. 3 with obstruction No. 7 at ½" from hub	.3	1.610	1.540	.1540	.500	.477	.070
	.4	.838	.795	.0890	.600	.570	.043
	.5	.487	.457	.0564	.688	.647	.030
	.6	.304	.283	.0382	.761	.708	.021
	.7	.190	.177	.0262	.817	.752	.013
	.8	.122	.110	.0182	.855	.770	.012
	.9	.077	.067	.0127	.868	.760	.010
	1.0	.044	.037	.0084	.835	.699	.007
	1.1	.023	.016	.0056	.735	.514	.007
Propeller No. 3 with obstruction No. 8 at ½" from hub	.3	1.630	1.550	.1570	.496	.472	.080
	.4	.840	.792	.0893	.600	.566	.048
	.5	.492	.467	.0572	.685	.650	.025
	.6	.302	.286	.0386	.750	.710	.016
	.7	.191	.179	.0266	.800	.750	.012
	.8	.121	.112	.0185	.835	.772	.009
	.9	.076	.069	.0130	.840	.761	.007
	1.0	.045	.039	.0090	.801	.690	.006
	1.1	.023	.016	.0060	.673	.470	.007
Propeller No. 3 with obstruction No. 9 at ½" from hub	.3	1.555	1.515	.1540	.483	.470	.040
	.4	.835	.805	.0900	.592	.570	.030
	.5	.478	.459	.0560	.680	.652	.019
	.6	.299	.284	.0380	.751	.715	.015
	.7	.190	.180	.0265	.800	.755	.010
	.8	.120	.111	.0184	.827	.770	.009
	.9	.072	.065	.0124	.830	.765	.007
	1.0	.043	.037	.0086	.800	.705	.006
	1.1	.022	.017	.0056	.700	.550	.005
Propeller No. 3 with obstruction No. 2 at 6" from hub	.3	1.570	1.345	.1510	.496	.425	.225
	.4	.835	.702	.0880	.604	.508	.133
	.5	.490	.394	.0565	.692	.556	.096
	.6	.302	.231	.0380	.760	.580	.071
	.7	.195	.137	.0265	.819	.575	.058
	.8	.127	.078	.0188	.860	.530	.049
	.9	.082	.037	.0133	.885	.395	.045
	1.0	.052	.006	.0093	.890	.103	.046
Propeller No. 3 with obstruction No. 2 at 12" from hub	.3	1.580	1.390	.1530	.493	.434	.190
	.4	.832	.715	.0883	.600	.516	.117
	.5	.483	.404	.0562	.684	.573	.079
	.6	.296	.233	.0377	.750	.590	.063
	.7	.188	.132	.0260	.805	.568	.056
	.8	.120	.067	.0181	.844	.470	.053
	.9	.076	.021	.0126	.865	.240	.055
	1.0	.045	−.006	.0085	.841	....	.051

TABLE II—Continued.

COEFFICIENTS FOR PROPELLERS WITH OBSTRUCTED SLIPSTREAMS—Continued.

	$\frac{v}{nD}$	C_t M_1	Combined C_t M_2	Cq M_{1-2}	Apparent η M_1	Combined η M_2	C_r
Propeller No. 3 with obstruction No. 3 at 6″ from hub	0.3	1.610	1.150	0.1480	0.520	0.370	0.460
	.4	.848	.547	.0850	.640	.410	.301
	.5	.508	.275	.0547	.739	.400	.233
	.6	.324	.135	.0380	.817	.340	.189
	.7	.213	.057	.0270	.879	.233	.156
	.8	.144	.007	.0198	.925	.048	.137
	.9	.098	−.024	.0147	.950	....	.122
	1.0	.065	−.046	.0108	.958	....	.111
	1.1	.043	−.064	.0081	.940	....	.107
Propeller No. 3 with obstruction No. 3 at 12″ from hub	.3	1.610	1.134	.1520	.507	.356	.476
	.4	.837	.555	.0861	.619	.410	.282
	.5	.490	.292	.0553	.705	.420	.198
	.6	.306	.150	.0376	.777	.371	.156
	.7	.198	.064	.0265	.832	.269	.134
	.8	.129	.010	.0189	.870	.067	.119
	.9	.082	−.030	.0133	.884	....	.112
	1.0	.051	−.055	.0094	.864	....	.106
	1.1	.031	−.070	.0067	.815	....	.101
Propeller No. 5 with obstruction No. 1 at ½″ from hub	.25	2.080	1.982	.1850	.448	.427	.098
	.3	1.423	1.360	.1330	.512	.489	.063
	.4	.712	.665	.0730	.621	.580	.047
	.5	.391	.349	.0435	.715	.638	.042
	.6	.221	.187	.0275	.770	.651	.034
	.7	.125	.097	.0178	.780	.605	.028
	.8	.069	.042	.0119	.740	.445	.027
	.9	.034	.003	.0078	.620	.055	.031
Propeller No. 5 with obstruction No. 2 at ½″ from hub	.3	1.440	1.275	.1330	.516	.458	.165
	.4	.715	.614	.0722	.633	.542	.101
	.5	.403	.322	.0440	.730	.582	.081
	.6	.237	.169	.0280	.809	.578	.068
	.7	.137	.082	.0182	.839	.502	.055
	.8	.079	.027	.0123	.817	.279	.052
	.9	.042	−.009	.0082	.734	....	.051
	1.0	.015	−.035	.0053	.450	....	.050
Propeller No. 5 with obstruction No. 3 at ½″ from hub	.25	2.160	1.560	.1740	.493	.357	.600
	.3	1.475	1.035	.1248	.565	.396	.440
	.4	.778	.488	.0708	.700	.439	.290
	.5	.445	.236	.0430	.824	.437	.209
	.6	.280	.107	.0290	.923	.353	.173
	.7	.183	.029	.0200	1.023	.162	.154
	.8	.120	−.021	.0141	1.082	....	.141
	.9	.078	−.054	.0100	1.115	....	.132
Propeller No. 7 with obstruction No. 1 at ½″ from hub	.25	2.000	1.900	.1760	.452	.430	.100
	.3	1.350	1.260	.1230	.523	.489	.090
	.4	.683	.630	.0690	.630	.582	.053
	.5	.377	.341	.0418	.718	.648	.036
	.6	.216	.183	.0264	.781	.662	.033
	.7	.125	.097	.0175	.796	.617	.028
	.8	.068	.039	.0110	.780	.452	.029
	.9	.032	.003	.0069	.663	.062	.029
Propeller No. 7 with obstruction No. 2 at ½″ from hub	.3	1.365	1.232	.1231	.530	.478	.133
	.4	.700	.593	.0685	.651	.558	.107
	.5	.399	.317	.0423	.751	.597	.082
	.6	.232	.167	.0272	.815	.587	.065
	.7	.140	.083	.0183	.853	.506	.057
	.8	.082	.027	.0123	.850	.280	.055
	.9	.044	.010	.0083	.760	....	.054
Propeller No. 7 with obstruction No. 3 at ½″ from hub	.3	1.475	.970	.1183	.596	.391	.505
	.4	.775	.456	.0673	.732	.431	.319
	.5	.461	.232	.0430	.852	.429	.229
	.6	.286	.104	.0285	.958	.349	.182
	.7	.186	.025	.0200	1.035	.139	.160
	.8	.125	.025	.0146	1.090	....	.150
	.9	.084	.057	.0108	1.112	....	.141
	1.0	.054	.079	.0078	1.089	....	.133
Propeller No. 9 with obstruction No. 1 at ½″ from hub	.25	1.480	1.410	.1140	.517	.491	.070
	.3	.965	.906	.0795	.580	.544	.059
	.4	.437	.398	.0416	.670	.609	.039
	.5	.210	.183	.0240	.695	.608	.028
	.6	.100	.074	.0147	.652	.480	.026
	.7	.043	.013	.0093	.520	.156	.030
	.8	.013	....	.0065	.255	....	....
Propeller No. 9 with obstruction No. 2 at ½″ from hub	.25	1.572	1.345	.1150	.544	.465	.227
	.3	.997	.838	.0770	.619	.520	.159
	.4	.470	.369	.0410	.729	.573	.101
	.5	.246	.170	.0245	.799	.551	.076
	.6	.128	.062	.0151	.810	.392	.066
	.7	.058	.002	.0090	.718	.028	.055
	.8	.016	....	.0050	.407	....	....

TABLE II—Continued.

COEFFICIENTS FOR PROPELLERS WITH OBSTRUCTED SLIPSTREAMS.

	$\frac{v}{nD}$	C_t M_1	Combined C_t M_2	Cq M_{1-2}	Apparent η M_1	Combined η M_2	C_r
Propeller No. 9 with obstruction No. 3 at ½″ from hub	.2	2.500	1.900	.1710	.466	.354	.600
	.3	1.067	.685	.0762	.670	.430	.382
	.4	.550	.292	.0420	.833	.442	.258
	.5	.306	.113	.0256	.955	.350	.193
	.6	.181	.017	.0169	1.025	.096	.164
	.7	.112	.032	.0120	1.042	....	.144
Propeller No. 11 with obstruction No. 1 at ½″ from hub	.25	1.540	1.440	.1170	.525	.489	.100
	.3	1.000	.920	.0800	.598	.549	.080
	4.4	.452	.404	.0412	.699	.625	.048
	.5	.225	.194	.0242	.740	.640	.031
	.6	.107	.081	.0146	.700	.530	.026
	.7	.043	.013	.0092	.522	.157	.030
Propeller No. 11 with obstruction No. 2 at ½″ from hub	.25	1.560	1.310	.1110	.560	.470	.250
	.3	1.015	.827	.0760	.640	.520	.188
	.4	.492	.381	.1048	.750	.580	.111
	.5	.255	.177	.0247	.820	.570	.078
	.6	.135	.070	.0155	.835	.431	.065
	.7	.070	.009	.0102	.768	.100	.061
Propeller No. 11 with obstruction No. 3 at ½″ from hub	.25	1.660	1.078	.1100	.600	.390	.582
	.3	1.130	.681	.0770	.696	.423	.449
	.4	.571	.289	.0425	.855	.434	.282
	.5	.330	.116	.0265	.982	.350	.214
	.6	.204	.020	.0174	1.075	.110	.184
	.7	.119	.035	.0119	1.115	....	.154
	.8	.071	.070	.0082	1.090	....	.141

TABLE III.

DERIVED PARALLEL PROPULSIVE EFFICIENCY OF PROPELLERS AND OBSTRUCTIONS.

$\frac{v}{nd}$	Propeller No. 1.			Propeller No. 3.		
	Obs. No. 1.	Obs. No. 2.	Obs. No. 2.	Obs. No. 1.	Obs. No. 2.	Obs. No. 3.
0.3	0.457	0.451	0.433	0.476	0.470	0.451
.4	.543	.527	.487	.562	.547	.503
.5	.603	.580	.495	.631	.600	.513
.6	.653	.601	.454	.673	.618	.463
.7	.660	.568	.311	.675	.578	.311
.8	.622	.460	.041	.628	.465	.143
.9	.503	.240	....	.515	.247	....
1.0	.248	....	....	.261	....	....

	Propeller No. 3.			Propeller No. 3.		
	Obs. No. 4.	Obs. No. 5.	Obs. No. 6.	Obs. No. 7.	Obs. No. 8.	Obs. No. 9.
.3	0.479	0.474	0.462	0.482	0.482	0.482
.4	.570	.558	.526	.575	.576	.577
.5	.645	.623	.550	.658	.659	.660
.6	.698	.661	.548	.719	.723	.725
.7	.720	.655	.460	.758	.761	.765
.8	.705	.592	.265	.765	.772	.779
.9	.640	.454	....	.740	.750	.763
1.0	.479	.157	....	.653	.674	.694
1.1	.107	....	....	.407	.436	.482

	Propeller No. 5.			Propeller No. 7.		
	Obs. No. 1.	Obs. No. 2.	Obs. No. 3.	Obs. No. 1.	Obs. No. 2.	Obs. No. 3.
.25	0.433	.....	0.416	.....	.....	.....
.3	.497	0.488	.465	0.512	0.500	0.477
.4	.597	.577	.522	.594	.574	.515
.5	.645	.604	.488	.654	.610	.490
.6	.655	.575	.354	.665	.583	.357
.7	.581	.434	.026	.617	.441	.026
.8	.391	.127	....	.398	.139	....

	Propeller No. 9.			Propeller No. 11.		
	Obs. No. 1.	Obs. No. 2.	Obs. No. 3.	Obs. No. 1.	Obs. No. 2.	Obs. No. 3.
.25	0.489	0.482	....	0.496	0.488	0.467
.3	.548	.535	.496	.557	.544	.505
.4	.610	.575	.480	.612	.577	.480
.5	.587	.513	.306	.600	.524	.307
.6	.434	.279	....	.469	.314	....
.7	.020	....	....	.039	....	....

TABLE IV.

COEFFICIENTS K FOR VARIOUS OBSTRUCTIONS, FROM FORMULA RESISTANCE$=\frac{K\Delta v^2}{g}$

Obstruction No.	*K.*
1	0.2268
2	.4320
3	.9990
4	.1305
5	.2727
6	.6840
7	.0549
8	.0477
9	.0360

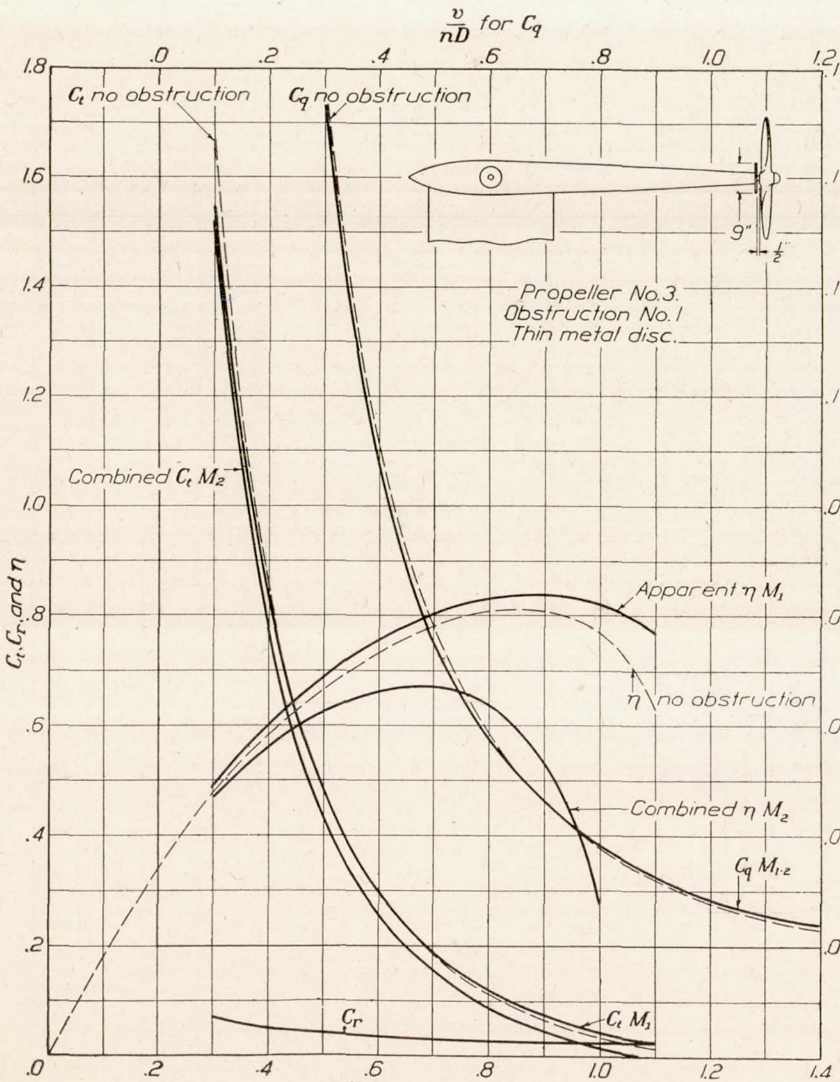

$\frac{v}{nD}$ for C_q
.0
.2
.4
.6
.8
1.0
1.2
C_t no obstruction
C_q no obstruction
9"
$\frac{1}{2}$"
Propeller No.3.
Obstruction No.1
Thin metal disc.
Combined C_t M_2
Apparent η M_1
η no obstruction
Combined η M_2
C_q $M_{1\cdot2}$
C_r
C_t M_1
C_t, C_r, and η
C_q
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
.0
.16
.14
.12
.10
.08
.06
.04
.02
.00
.2
.4
.6
.8
1.0
1.2
1.4
$\frac{v}{nD}$ for C_t, C_r, and η

Fig.3

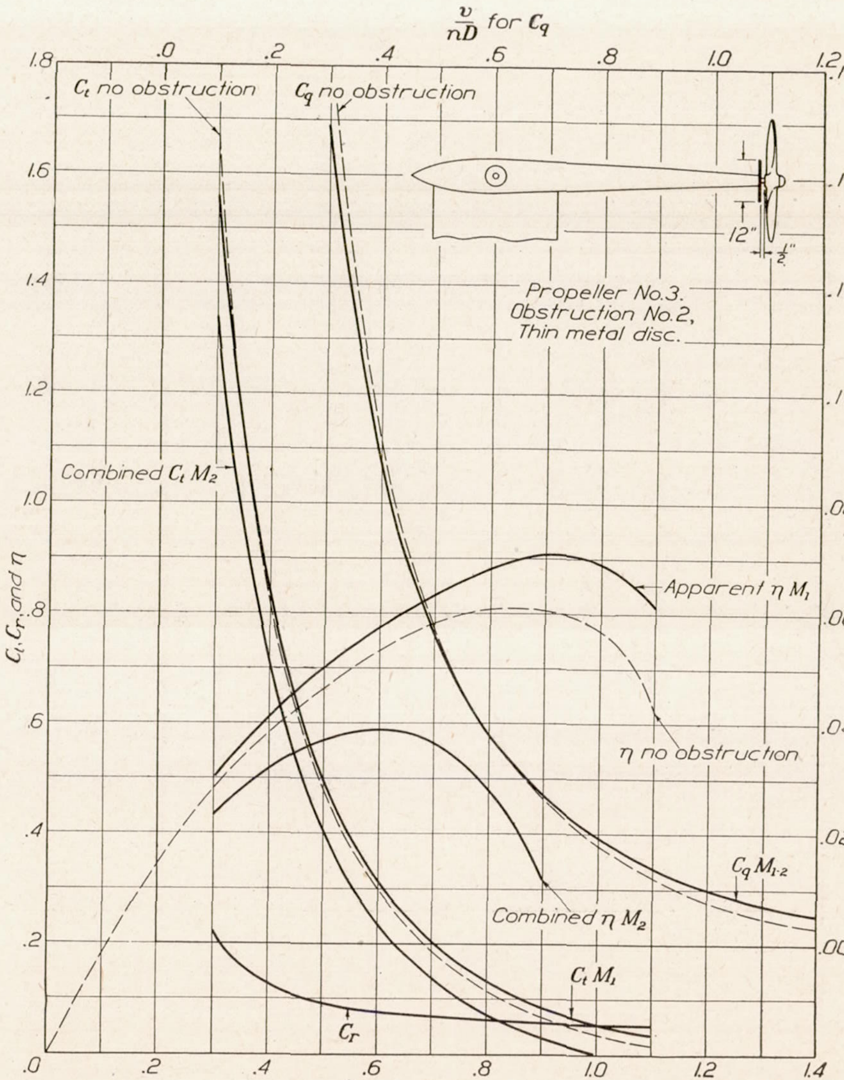

$\frac{v}{nD}$ for C_q
.0
.2
.4
.6
.8
1.0
1.2
C_t no obstruction
C_q no obstruction
12"
$\frac{1}{2}$"
Propeller No.3.
Obstruction No.2,
Thin metal disc.
Combined C_t M_2
Apparent η M_1
η no obstruction
C_q $M_{1\cdot2}$
Combined η M_2
C_t M_1
C_r
C_t, C_r, and η
C_q
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
.0
.16
.14
.12
.10
.08
.06
.04
.02
.00
.2
.4
.6
.8
1.0
1.2
1.4
$\frac{v}{nD}$ for C_t, C_r, and η

Fig.4

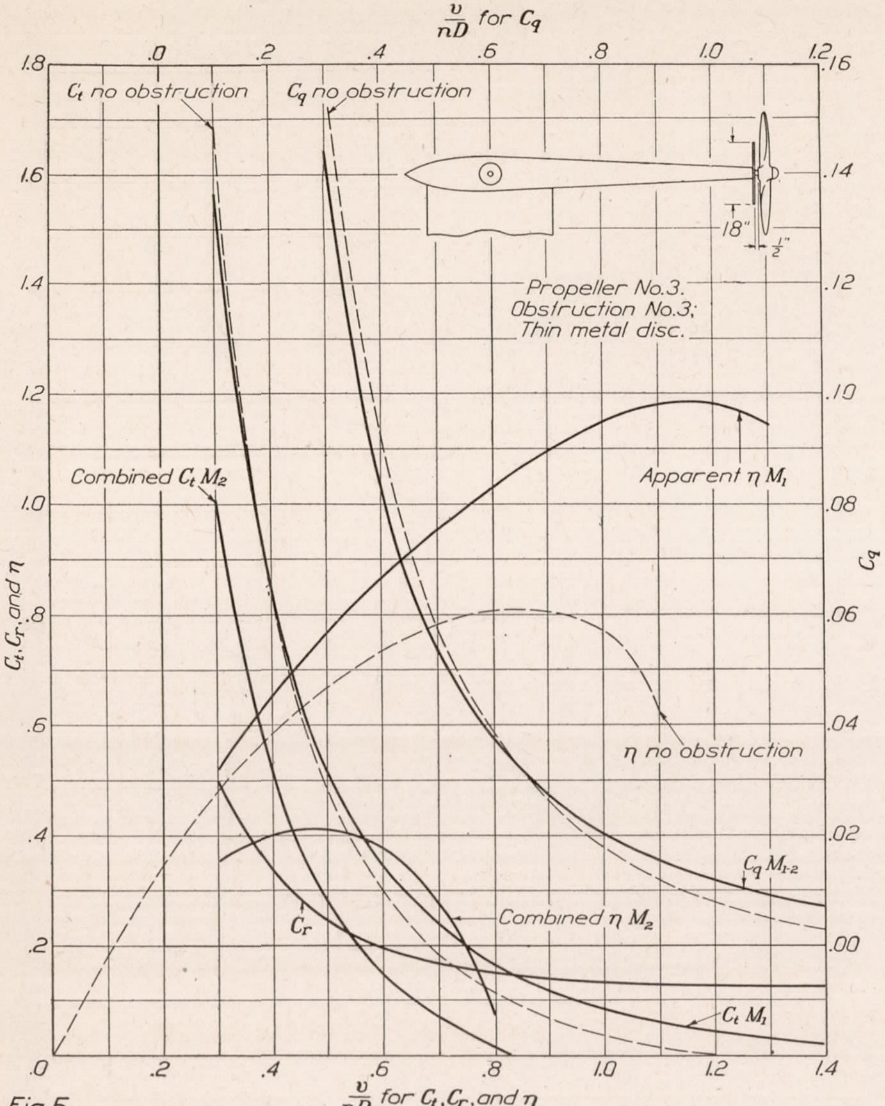

$\frac{v}{nD}$ for C_q
.0
.2
.4
.6
.8
1.0
1.2
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
.0
.16
.14
.12
.10
.08
.06
.04
.02
.00
C_t no obstruction
C_q no obstruction
18"
$\frac{1}{2}$"
Propeller No.3.
Obstruction No.3;
Thin metal disc.
Combined C_t M_2
Apparent η M_1
C_t, C_r, and η
C_q
η no obstruction
C_q $M_{1\text{-}2}$
C_r
Combined η M_2
C_t M_1
.0
.2
.4
.6
.8
1.0
1.2
1.4
$\frac{v}{nD}$ for C_t, C_r, and η

Fig.5

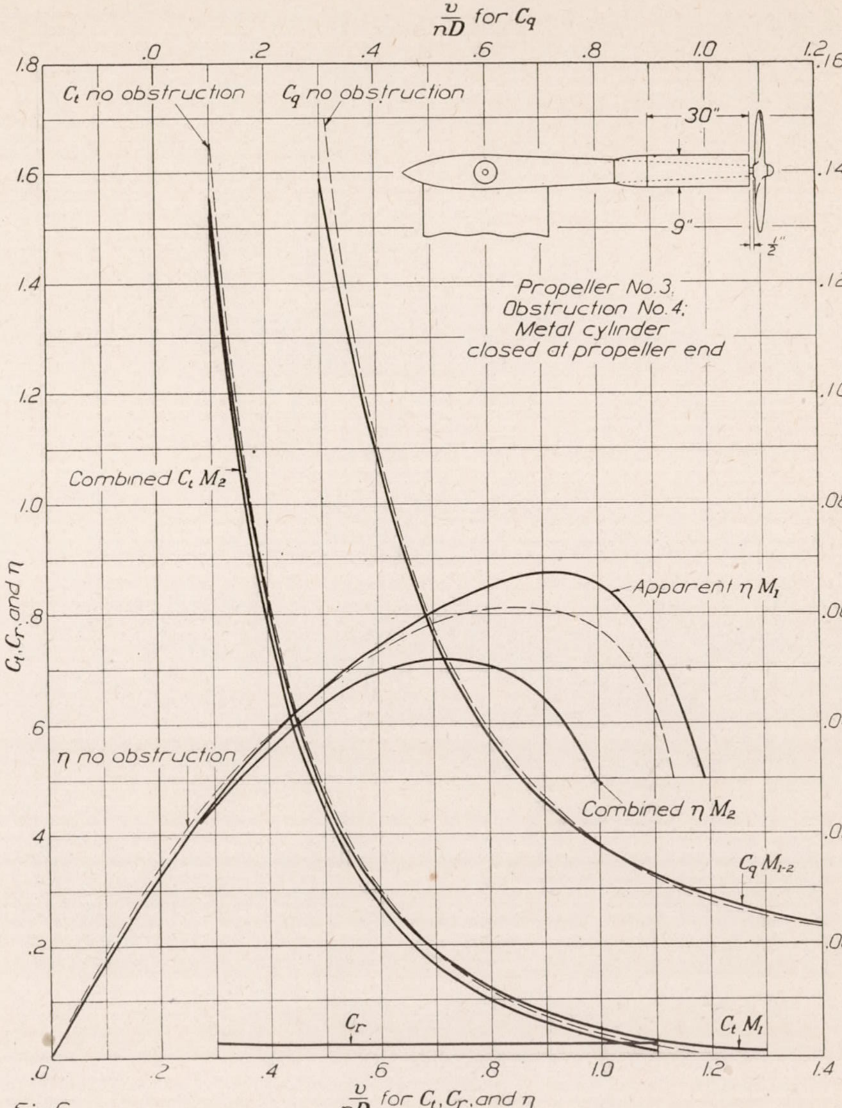

$\frac{v}{nD}$ for C_q
.0
.2
.4
.6
.8
1.0
1.2
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
.16
.14
.12
.10
.08
.06
.04
.02
.00
C_t no obstruction
C_q no obstruction
30"
9"
$\frac{1}{2}$"
Propeller No.3.
Obstruction No.4;
Metal cylinder
closed at propeller end
Combined C_t M_2
Apparent η M_1
C_t, C_r, and η
C_q
η no obstruction
Combined η M_2
C_q $M_{1\text{-}2}$
C_r
C_t M_1
.0
.2
.4
.6
.8
1.0
1.2
1.4
$\frac{v}{nD}$ for C_t, C_r, and η

Fig 6

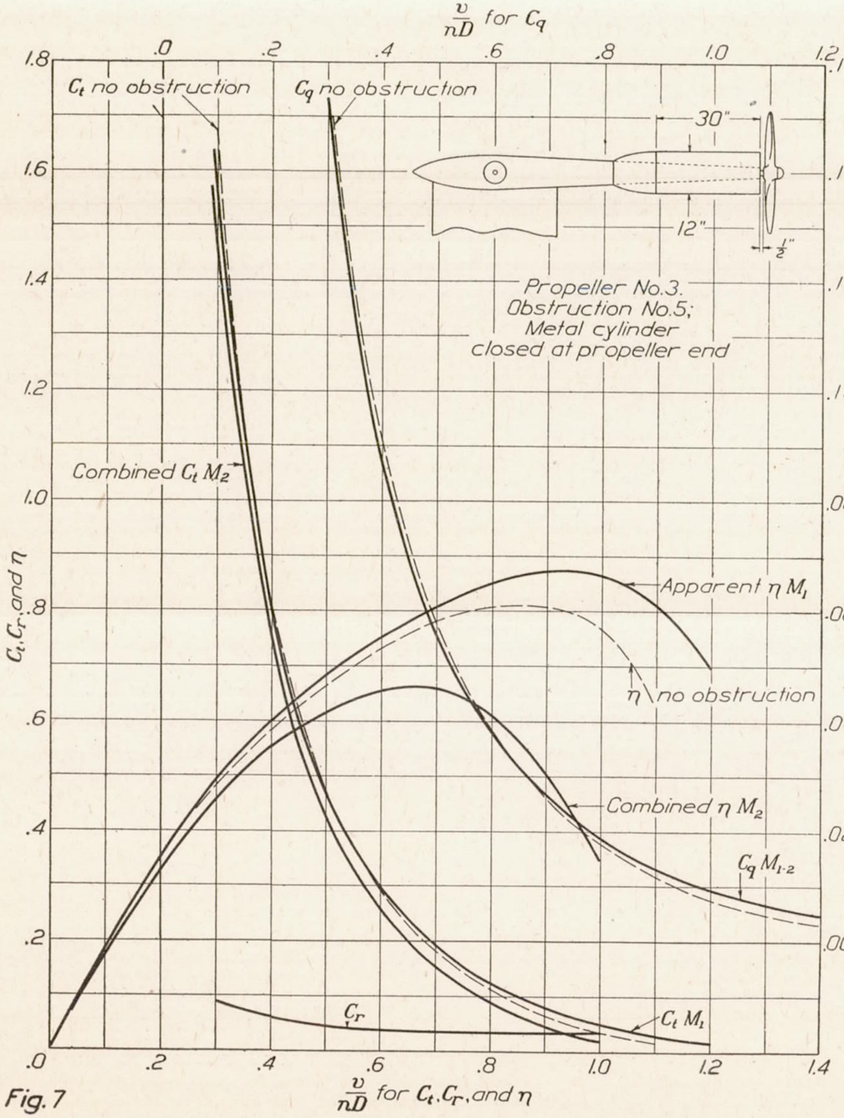

v/nD for Cq
.0
.2
.4
.6
.8
1.0
1.2
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
.0
.16
.14
.12
.10
.08
.06
.04
.02
.00
Ct no obstruction
Cq no obstruction
30"
12"
1/2"
Propeller No.3.
Obstruction No.5;
Metal cylinder
closed at propeller end
Combined Ct M2
Apparent η M1
η no obstruction
Combined η M2
Cq M1-2
Cr
Ct M1
Ct, Cr, and η
Cq
.0
.2
.4
.6
.8
1.0
1.2
1.4
v/nD for Ct, Cr, and η

Fig. 7

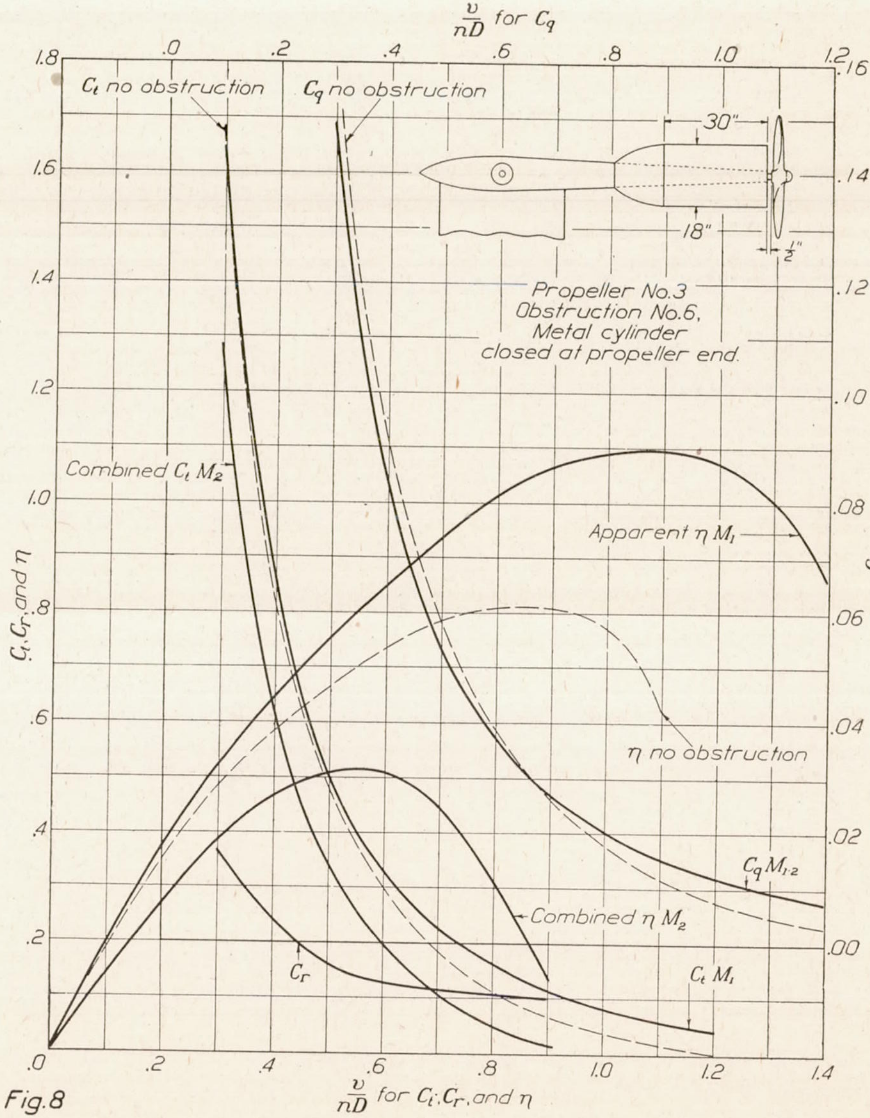
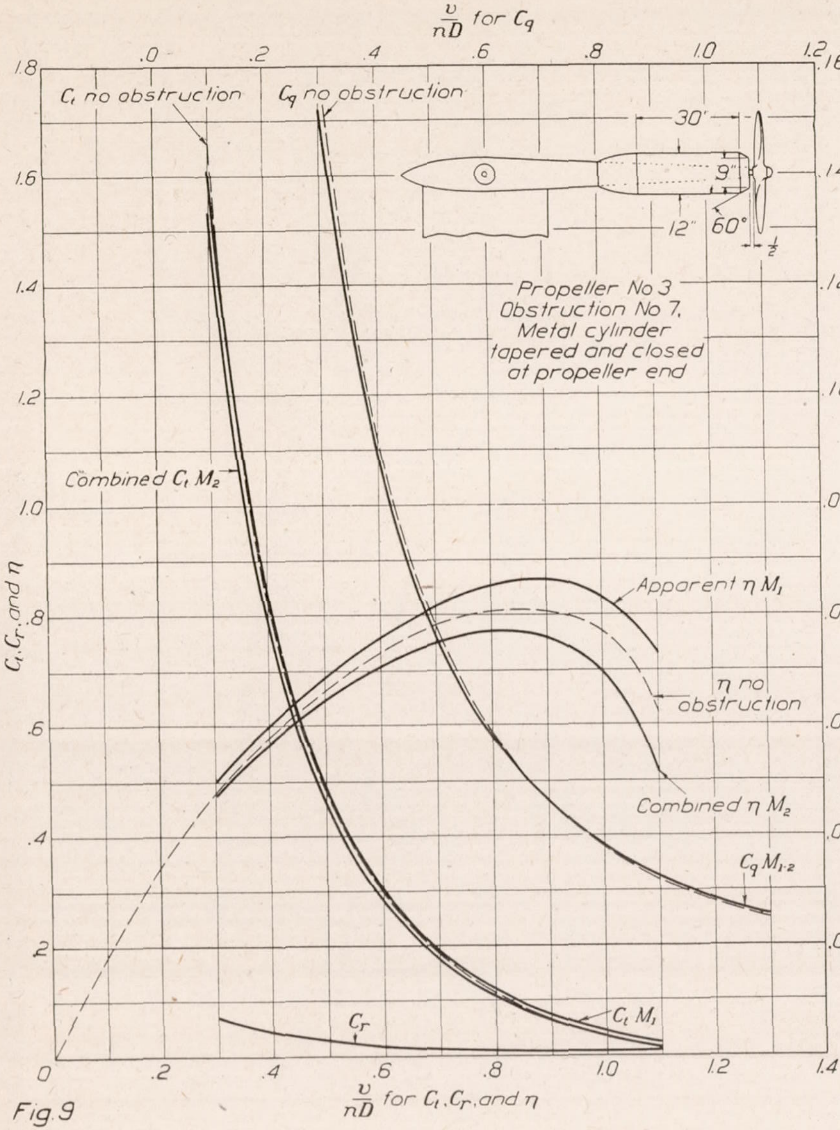
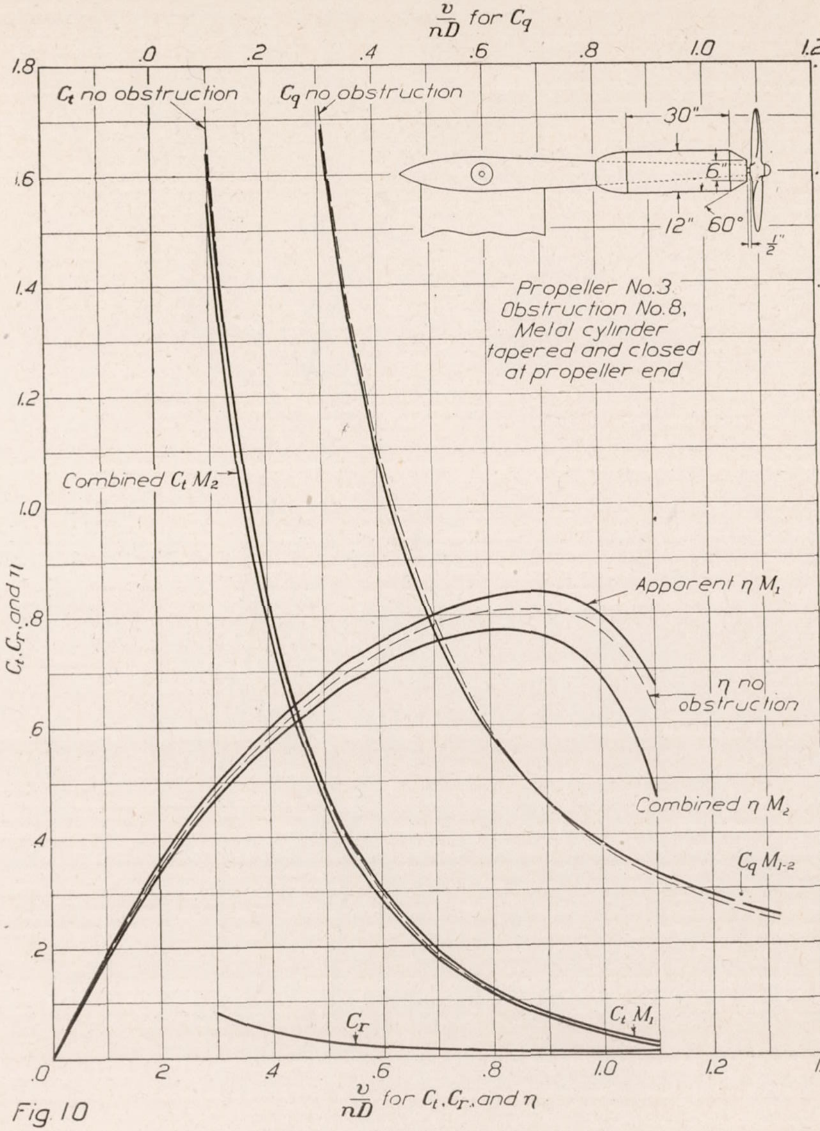

v/nD for Cq
.0
.2
.4
.6
.8
1.0
1.2
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
.0
.16
.14
.12
.10
.08
.06
.04
.02
.00
Ct no obstruction
Cq no obstruction
30"
18"
1/2"
Propeller No.3
Obstruction No.6,
Metal cylinder
closed at propeller end.
Combined Ct M2
Apparent η M1
η no obstruction
Cq M1-2
Combined η M2
Cr
Ct M1
Ct, Cr, and η
Cq
.0
.2
.4
.6
.8
1.0
1.2
1.4
v/nD for Ct, Cr, and η

Fig. 8



$\frac{v}{nD}$ for C_q
.0 .2 .4 .6 .8 1.0 1.2
1.8 1.6 1.4 1.2 1.0 .8 .6 .4 .2 0
.16 .14 .12 .10 .08 .06 .04 .02 .00
C_t no obstruction
C_q no obstruction
30"
9"
12"
60°
$\frac{1}{2}$
Propeller No 3
Obstruction No 7,
Metal cylinder
tapered and closed
at propeller end
Combined C_t M_2
Apparent η M_1
η no obstruction
Combined η M_2
C_q M_{1-2}
C_t M_1
C_r
C_t, C_r, and η
C_q
0 .2 .4 .6 .8 1.0 1.2 1.4
$\frac{v}{nD}$ for C_t, C_r, and η

Fig. 9



$\frac{v}{nD}$ for C_q
.0 .2 .4 .6 .8 1.0 1.2
1.8 1.6 1.4 1.2 1.0 .8 .6 .4 .2
.16 .14 .12 .10 .08 .06 .04 .02 .00
C_t no obstruction
C_q no obstruction
30"
6"
12"
60°
$\frac{1}{2}$"
Propeller No. 3
Obstruction No. 8,
Metal cylinder
tapered and closed
at propeller end
Combined C_t M_2
Apparent η M_1
η no obstruction
Combined η M_2
C_q M_{1-2}
C_t M_1
C_r
C_t, C_r, and η
C_q
.0 .2 .4 .6 .8 1.0 1.2 1.4
$\frac{v}{nD}$ for C_t, C_r, and η

Fig. 10

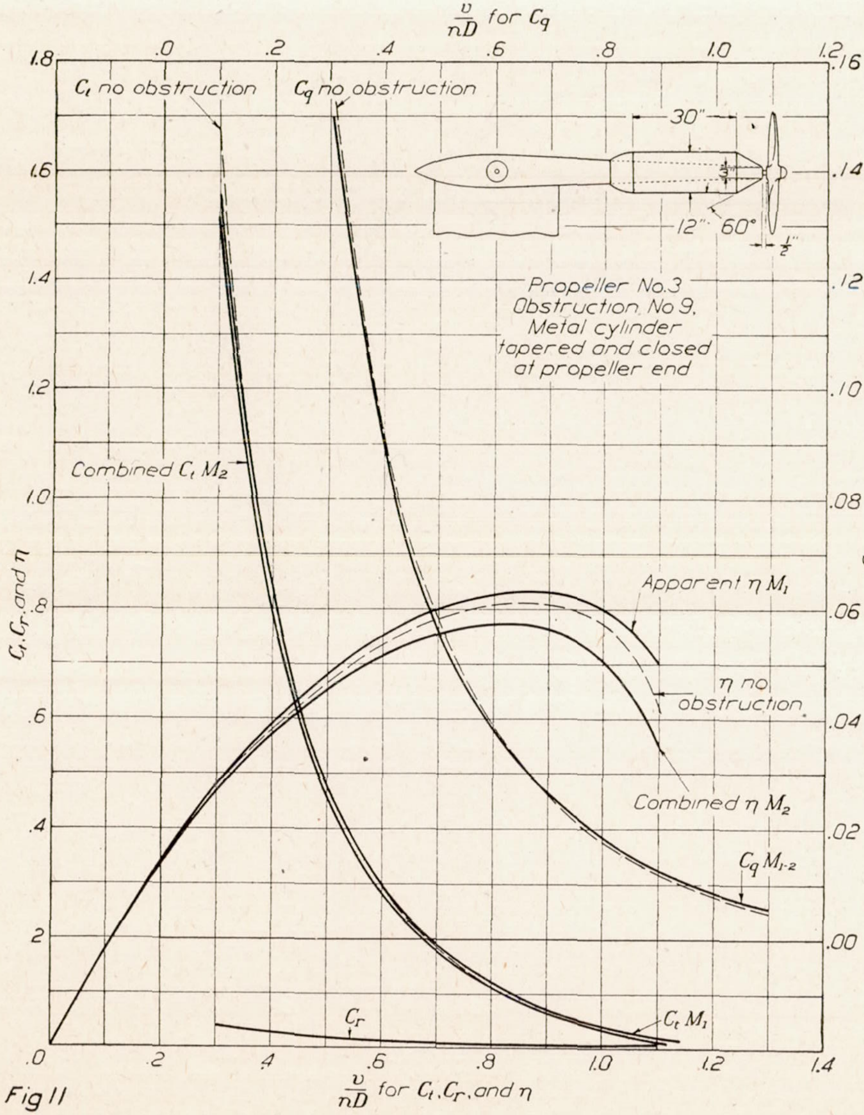

$\frac{v}{nD}$ for C_q
.0
.2
.4
.6
.8
1.0
1.2
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
2
.0
.16
.14
.12
.10
.08
.06
.04
.02
.00
C_t no obstruction
C_q no obstruction
30"
12"
60°
$\frac{1}{2}$"
Propeller No.3
Obstruction No 9.
Metal cylinder
tapered and closed
at propeller end
Combined C_t M_2
C_t, C_r, and η
C_q
Apparent η M_1
η no obstruction
Combined η M_2
C_q $M_{1\text{-}2}$
C_r
C_t M_1
.0
.2
.4
.6
.8
1.0
1.2
1.4
$\frac{v}{nD}$ for C_t, C_r, and η

Fig 11

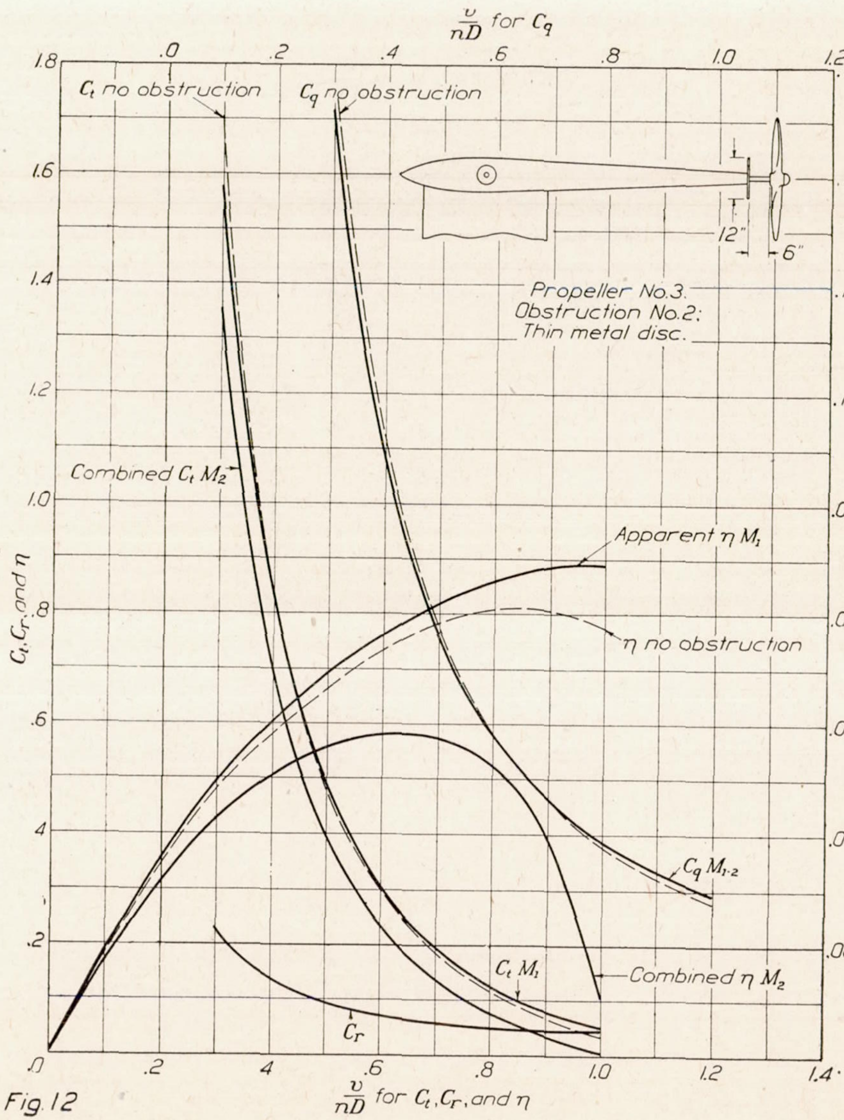

$\frac{v}{nD}$ for C_q
.0
.2
.4
.6
.8
1.0
1.2
1.8
1.6
1.4
1.2
1.0
.8
.6
4
.2
.0
.16
.14
.12
.10
.08
.06
.04
.02
.00
C_t no obstruction
C_q no obstruction
12"
6"
Propeller No.3.
Obstruction No.2:
Thin metal disc.
Combined C_t M_2
C_t, C_r, and η
C_q
Apparent η M_1
η no obstruction
C_q $M_{1\text{-}2}$
C_t M_1
Combined η M_2
C_r
.0
.2
.4
.6
.8
1.0
1.2
1.4
$\frac{v}{nD}$ for C_t, C_r, and η

Fig. 12

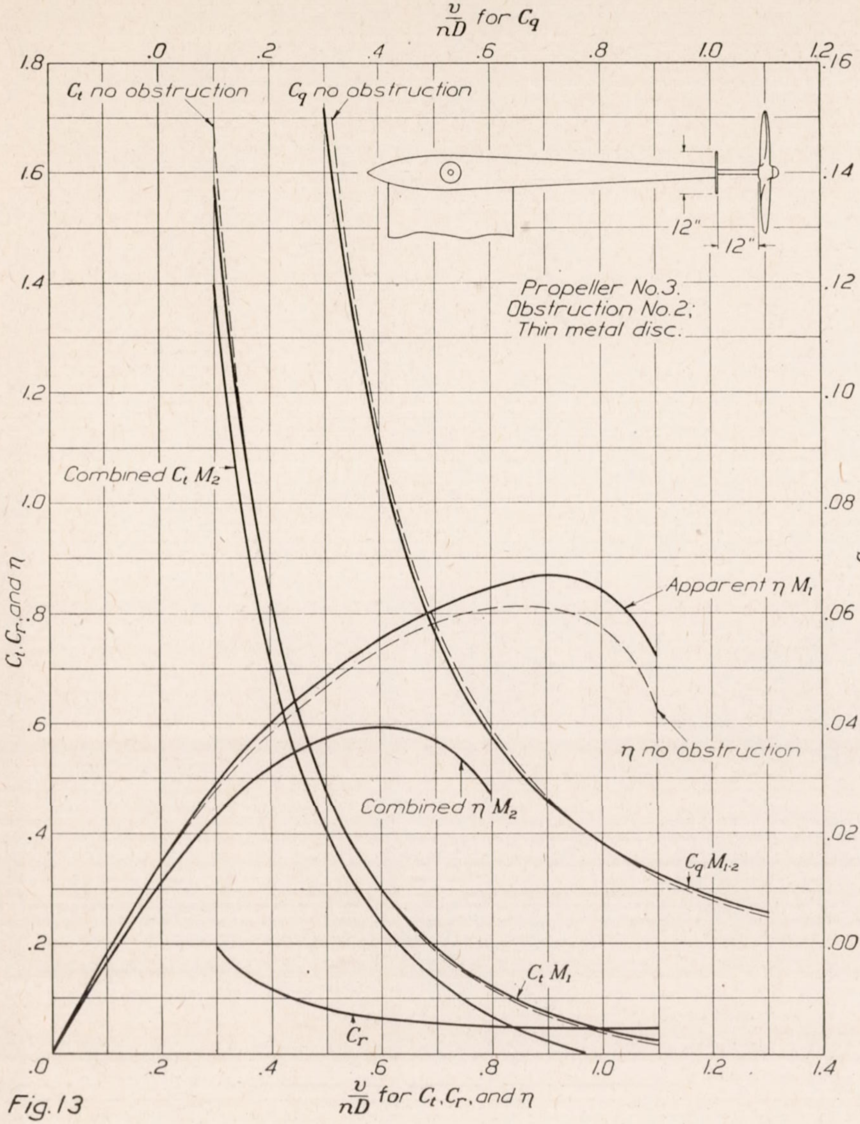

$\frac{v}{nD}$ for C_q
.0
.2
.4
.6
.8
1.0
1.2
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
.0
.16
.14
.12
.10
.08
.06
.04
.02
.00
C_q
C_t, C_r, and η
C_t no obstruction
C_q no obstruction
12"
12"
Propeller No.3.
Obstruction No.2;
Thin metal disc.
Combined C_t M_2
Apparent η M_1
η no obstruction
Combined η M_2
C_q $M_{1\cdot2}$
C_t M_1
C_r
.0
.2
.4
.6
.8
1.0
1.2
1.4
$\frac{v}{nD}$ for C_t, C_r, and η

Fig. 13

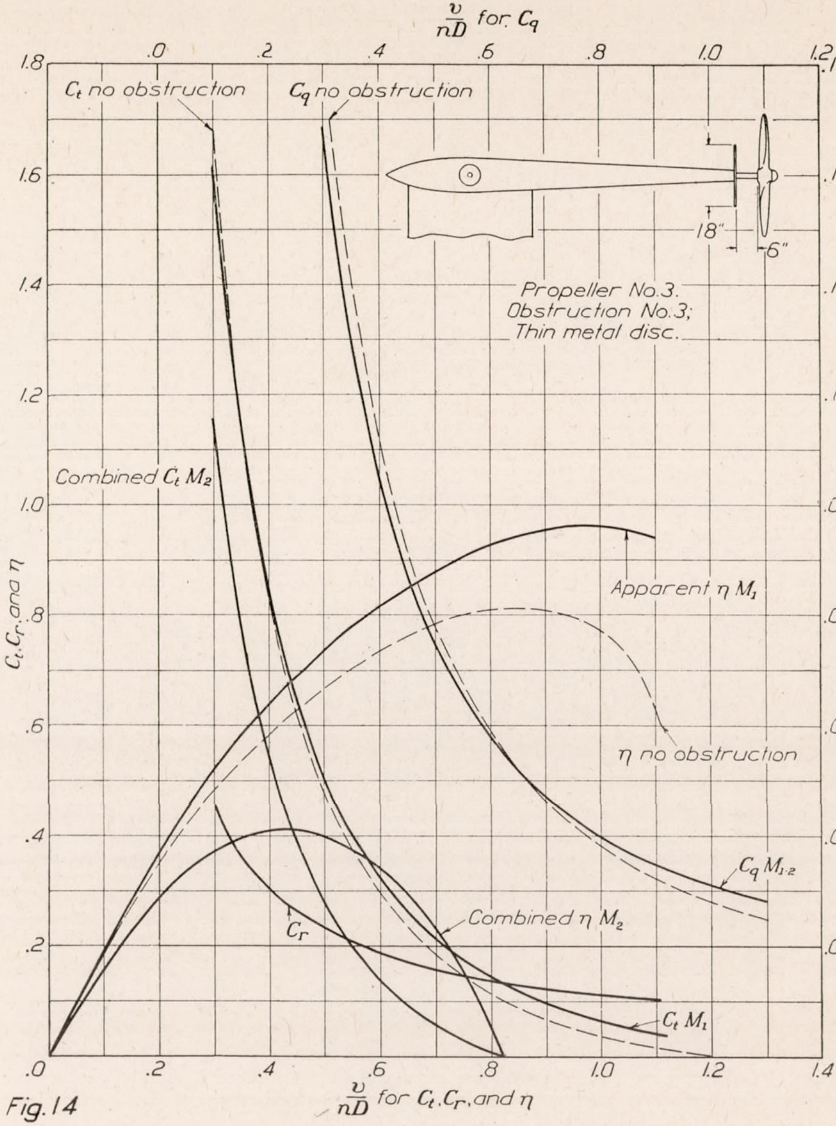

$\frac{v}{nD}$ for C_q
.0
.2
.4
.6
.8
1.0
1.2
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
.0
.16
.14
.12
.10
.08
.06
.04
.02
.00
C_q
C_t, C_r, and η
C_t no obstruction
C_q no obstruction
18"
6"
Propeller No.3.
Obstruction No.3;
Thin metal disc.
Combined C_t M_2
Apparent η M_1
η no obstruction
C_q $M_{1\cdot2}$
Combined η M_2
C_r
C_t M_1
.0
.2
.4
.6
.8
1.0
1.2
1.4
$\frac{v}{nD}$ for C_t, C_r, and η

Fig. 14

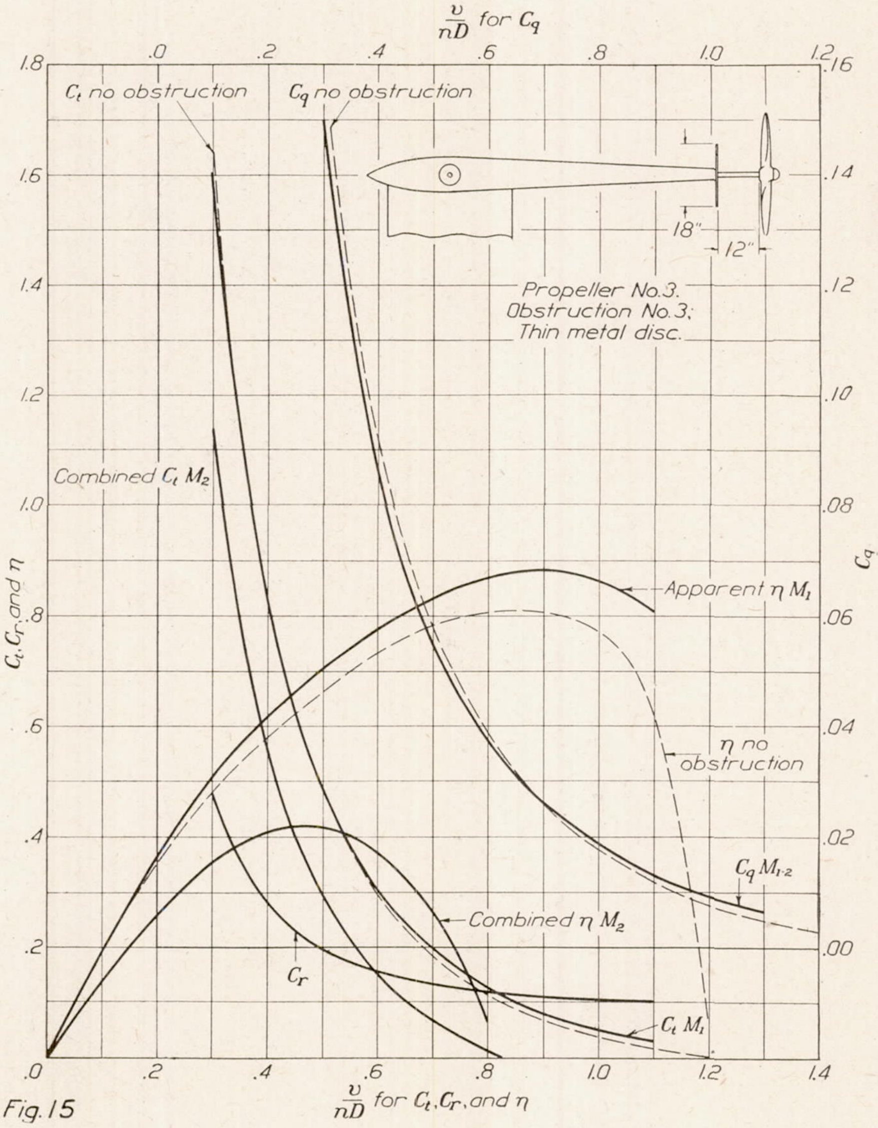

$\frac{v}{nD}$ for C_q
.0
.2
.4
.6
.8
1.0
1.2
C_t no obstruction
C_q no obstruction
18"
12"
Propeller No. 3.
Obstruction No. 3;
Thin metal disc.
Combined C_t M_2
Apparent η M_1
η no obstruction
C_q $M_{1\text{-}2}$
Combined η M_2
C_r
C_t M_1
C_t, C_r, and η
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
.0
C_q
.16
.14
.12
.10
.08
.06
.04
.02
.00
.0
.2
.4
.6
.8
1.0
1.2
1.4
$\frac{v}{nD}$ for C_t, C_r, and η

Fig. 15

APPENDIX.

Subsequent to the preparation of the preceding report, contact with certain other aspects of this general problem has suggested a somewhat different form of analysis as presumably more useful in certain practical cases. This form of analysis is therefore outlined below, with corresponding results in tabular form.

The useful work of propulsion done, per unit time, by an airplane propeller may be defined as $D_0 v$; where D_0 is the drag or resistance of the airplane alone, without propeller, along the flight path, and v is the velocity of advance.

In a hypothetical case of an airplane, in steady flight, with the propeller so placed that there is (*a*), no obstruction offered to the slipstream and (*b*), no increase, due to slipstream, of drag, the shaft thrust of the propeller would be equal to D_0. The propeller efficiency, as determined for an unobstructed slipstream, would then be defined by $\eta=\frac{Tv}{2\pi nQ}$. What may be termed the *propulsive efficiency* and designated as η^1 would be defined by $\eta^1=\frac{D_0 v}{2\pi nQ}$. Since, in this case, T is equal to D_0, η would obviously be equal to η^1. Propulsive efficiency may also be defined as the ratio of tow line horsepower to brake horsepower.

In the actual case, however, the propeller is placed so that there is (*a*), a change in shaft thrust from that experienced with no slipstream obstructions, and (*b*), a change in drag from that obtaining with no slipstream. The propeller efficiency can no longer be defined as $\frac{Tv}{2\pi nQ}$, where T is the shaft thrust, since T may include an internal force that is not useful in propelling the airplane, and therefore, when multiplied by v, does not represent useful work per unit time. The useful work per unit time may nevertheless still be defined as $D_0 v$ and *propulsive efficiency* by $\eta^1=\frac{D_0 v}{2\pi nQ}$.

In the present tests then, to determine propulsive efficiency, the combination of propeller and obstruction on the shaft should be credited with the drag of the obstruction alone. This is obviously equivalent to crediting the propeller with all of the thrust apparently developed, where the obstruction is mounted on the dynamometer, and at the same time charging it with the apparent increase in drag of the obstruction.

The difference in point of view from that previously presented is readily seen. In the earlier discussion, particularly with reference to the terms, *combined efficiency* and *parallel propulsive efficiency*, the obstructions are regarded as wholly prejudicial, and whatever develops as a result of their presence on the airplane is considered as non-useful. In this later analysis, the obstruction is thought of as a useful or necessary part of the airplane, such as the radiator, the nose of the fuselage, or a part of the wing; and the work done in moving it through still air, at the velocity of advance, is therefore considered useful work and is credited to the propeller.

With the data in the form of coefficients as given in the tables, the equation, $\eta=\frac{Tv}{2\pi nQ}$, is transformed into $\eta=\frac{C_t}{C_q}\cdot\frac{v}{nD}\cdot\frac{1}{2\pi}$. For η^1 we may use either

$$\eta^1=\frac{C_t M_2+\frac{K}{D^2}}{C_q M_{1-2}}\cdot\frac{v}{nD}\cdot\frac{1}{2\pi}$$

or,

$$\eta^1=\frac{C_t M_1-\left(C_r-\frac{K}{D^2}\right)}{C_q M_{1-2}}\cdot\frac{v}{nD}\cdot\frac{1}{2\pi}$$

$C_t M_1$, $C_t M_2$, C_r and $C_q M_{1-2}$ are given in Table II, and K, for each obstruction, is given in Table IV. K is divided by D^2, where D is the diameter of the propeller, and in these tests equal to three feet, in order to derive a coefficient similar in form to C_t.

The values of propulsive efficiency, η^1, for the propellers and obstructions used are shown in Table V. For ready comparison the values of propeller efficiency, with unobstructed slipstream are given in the same table under the heading "Without obstruction."

Inspection of Table V leads to the following conclusions:

1. Moving a blunt obstruction, of diameter not exceeding one-third the diameter of the propeller, from a point outside the slipstream to one near the center, and close to the hub of the propeller, does not materially affect the propulsive efficiency.
2. The effect, at low slips, appears, in many cases, to be beneficial. This may be explained by fact that the hub of the propeller shields the obstruction to some extent, and consequently the obstruction offers less resistance to forward motion when in the slipstream than when out.
3. The distance of an obstruction from the propeller, while having marked effect upon the apparent propeller efficiency, has little effect upon propulsive efficiency, the advantage appearing to be with close spacing.
4. Blunt slipstream obstructions, having a diameter equal to half that of the propeller, materially reduce propulsive efficiency at high slips, but at low slips have little effect, and in some cases the effect is apparently beneficial.

It may be noted that, with the obstructions used, practically all cases of apparently beneficial effect occur with very small combined thrusts. In other words, the beneficial effect occurs when little or no thrust is available from the propeller other than that required to overcome the total drag of obstruction.

TABLE V.

PROPULSIVE EFFICIENCIES FOR PROPELLERS WITH OBSTRUCTED SLIPSTREAMS.

PROPELLER NO. 1.

$\frac{v}{nD}$	Obstruction No. 1 at ½″.	Obstruction No. 2 at ½″.	Obstruction No. 3 at ½″.	Without obstruction.
0.3	0.448	0.438	0.381	0.463
0.4	.540	.538	.492	.558
0.5	.627	.631	.592	.636
0.6	.698	.703	.675	.711
0.7	.749	.764	.743	.764
0.8	.785	.802	.807	.792
0.9	.792	.819	....	.789
1.0	.779	.821	....	.738

PROPELLER NO. 3.

$\frac{v}{nD}$	Obstruction No. 1 at ½″.	Obstruction No. 2 at ½″.	Obstruction No. 3 at ½″.	Without obstruction.
0.3	0.479	0.445	0.389	0.483
0.4	.582	.556	.484	.580
0.5	.659	.640	.574	.665
0.6	.726	.713	.656	.734
0.7	.780	.767	.725	.784
0.8	.814	.791	.785	.808
0.9	.816	.809	....	.808
1.0	.741	.812	....	.775

PROPELLER NO. 3.

$\frac{v}{nD}$	Obstruction No. 4 at ½″.	Obstruction No. 5 at ½″.	Obstruction No. 6 at ½″.	Without obstruction.
0.3	0.463	0.471	0.428	0.483
0.4	.568	.575	.531	.580
0.5	.657	.659	.622	.665
0.6	.732	.728	.706	.734
0.7	.778	.788	.762	.784
0.8	.800	.817	.795	.808
0.9	.801	.835	.862	.808
1.0	.741	.837	....	.775

PROPELLER NO. 3.

$\frac{v}{nD}$	Obstruction No. 7 at ½″.	Obstruction No. 8 at ½″.	Obstruction No. 9 at ½″.	Without obstruction.
0.3	0.479	0.472	0.471	0.483
0.4	.573	.567	.572	.580
0.5	.654	.657	.658	.665
0.6	.723	.720	.724	.734
0.7	.778	.770	.773	.784
0.8	.811	.805	.795	.808
0.9	.824	.818	.797	.808
1.0	.815	.785	.759	.775

PROPELLER NO. 3.

$\frac{v}{nD}$	Obstruction No. 2 at ½″.	Obstruction No. 2 at 6″.	Obstruction No. 2 at 12″.	Without obstruction.
0.3	0.445	0.440	0.448	0.483
0.4	.556	.542	.550	.580
0.5	.640	.622	.640	.665
0.6	.713	.701	.711	.734
0.7	.767	.778	.771	.784
0.8	.791	.852	.810	.808
0.9	.809	.915	.785	.808
1.0	.812	.925	....	.775

TABLE V—Continued.

PROPULSIVE EFFICIENCIES FOR PROPELLERS WITH OBSTRUCTED SLIPSTREAMS—continued.

PROPELLER NO. 3.

$\frac{v}{nD}$	Obstruction No. 3 at ½″.	Obstruction No. 3 at 6″.	Obstruction No. 3 at 12″.	Without obstruction.
0.3	0.389	0.407	0.390	0.483
0.4	.484	.493	.492	.580
0.5	.574	.561	.580	.665
0.6	.656	.618	.653	.734
0.7	.725	.692	.735	.784
0.8	.785	.759	.820	.808

PROPELLER NO. 5.

$\frac{v}{nD}$	Obstruction No. 1 at ½″.	Obstruction No. 2 at ½″.	Obstruction No. 3 at ½″.	Without obstruction.
0.25	0.433	....	0.382	0.439
0.3	.497	0.474	.439	.507
0.4	.602	.584	.538	.620
0.5	.685	.670	.642	.694
0.6	.736	.740	.718	.744
0.7	.763	.796	.780	.745
0.8	.715	.777	....	.684
0.9	.514	....	....	.464

PROPELLER NO. 7.

$\frac{v}{nD}$	Obstruction No. 1 at ½″.	Obstruction No. 2 at ½″.	Obstruction No. 3 at ½″.	Without obstruction.
0.25	0.435	....	....	0.460
0.3	.498	0.496	0.437	.519
0.4	.605	.595	.536	.618
0.5	.695	.686	.635	.701
0.6	.752	.757	.721	.755
0.7	.778	.798	.759	.759
0.8	.740	.777	....	.694
0.9	.581	....	....	.425

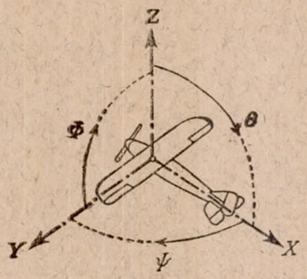
PROPELLER NO. 9.

$\frac{v}{nD}$	Obstruction No. 1 at ½″.	Obstruction No. 2 at ½″.	Obstruction No. 3 at ½″.	Without obstruction.
0.25	0.500	0.482	....	0.499
0.3	.558	.549	0.498	.564
0.4	.647	.648	.611	.650
0.5	.690	.708	.697	.672
0.6	.644	.695	.722	.607
0.7	.455	.620	....	.307

PROPELLER NO. 11.

$\frac{v}{nD}$	Obstruction No. 1 at ½″.	Obstruction No. 2 at ½″.	Obstruction No. 3 at ½″.	Without obstruction.
0.25	0.498	0.487	0.430	0.505
0.3	.565	.550	.491	.573
0.4	.663	.653	.599	.653
0.5	.721	.725	.682	.687
0.6	.693	.727	.718	.641
0.7	.460	.622	....	.392

○



Positive directions of axes and angles (forces and moments) are shown by arrows.

Axis.		Force (parallel to axis) symbol.	Moment about axis.			Angle.		Velocities.	
Designation.	Symbol.		Designation.	Symbol.	Positive direction.	Designation.	Symbol.	Linear (component along axis).	Angular.
Longitudinal....	X	X	rolling.....	L	$Y \longrightarrow Z$	roll.....	Φ	u	p
Lateral..........	Y	Y	pitching...	M	$Z \longrightarrow X$	pitch....	Θ	v	q
Normal..........	Z	Z	yawing.....	N	$X \longrightarrow Y$	yaw.....	Ψ	w	r

Absolute coefficients of moment

$$C_l=\frac{L}{q\,b\,S} \quad C_m=\frac{M}{q\,c\,S} \quad C_n=\frac{N}{q\,f\,S}$$

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS.

Diameter, D
Pitch (a) Aerodynamic pitch, p_a
(b) Effective pitch, p_e
(c) Mean geometric pitch, p_g
(d) Virtual pitch, p_v
(e) Standard pitch, p_s
Pitch ratio, p/D
Inflow velocity, V'
Slipstream velocity, V_s

Thrust, T
Torque, Q
Power, P
(If "coefficients" are introduced all units used must be consistent.)
Efficiency $\eta = T\,V/P$
Revolutions per sec., n; per min., N
Effective helix angle $\Phi=\tan^{-1}\left(\frac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS.

1 HP = 76.04 kg. m/sec. = 550 lb. ft/sec.
1 kg. m/sec. = 0.01315 HP
1 mi/hr. = 0.44704 m/sec.
1 m/sec. = 2.23693 mi/hr.

1 lb. = 0.45359 kg.
1 kg. = 2.20462 lb.
1 mi. = 1609.35 m. = 5280 ft.
1 m. = 3.28083 ft.