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REPORT No. 185

THE RESISTANCE OF SPHERES IN WIND TUNNELS AND IN AIR

By D. L. BACON and E. G. REID

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AERONAUTICAL SYMBOLS.

1. FUNDAMENTAL AND DERIVED UNITS.

	Symbol.	Metric.		English.	
		Unit.	Symbol.	Unit.	Symbol.
Length	l	meter	m.	foot (or mile)	ft. (or mi.).
Time	t	second	sec.	second (or hour)	sec. (or hr.).
Force	F	weight of one kilogram	kg.	weight of one pound	lb.
Power	P	kg.m/sec		horsepower	HP
Speed		m/sec	m. p. s.	mi/hr	M. P. H.

2. GENERAL SYMBOLS, ETC.

Weight, $W = mg$.

Standard acceleration of gravity, $g = 9.806\text{m/sec.}^2 = 32.172\text{ft/sec.}^2$

Mass, $m = \frac{W}{g}$

Density (mass per unit volume), ρ

Standard density of dry air, 0.1247 (kg.-m.-sec.) at 15.6°C. and 760 mm. = 0.00237 (lb.-ft.-sec.)

Specific weight of "standard" air, 1.223 kg/m.^{3} = 0.07635 lb/ft.^{3}

Moment of inertia, mk^2 (indicate axis of the radius of gyration, k, by proper subscript).

Area, S; wing area, S_w, etc.

Gap, G

Span, b; chord length, c.

Aspect ratio = b/c

Distance from $c.\ g.$ to elevator hinge, f.

Coefficient of viscosity, μ.

3. AERODYNAMICAL SYMBOLS.

True airspeed, V

Dynamic (or impact) pressure, $q = \frac{1}{2} \rho V^2$

Lift, L; absolute coefficient $C_L = \frac{L}{qS}$

Drag, D; absolute coefficient $C_D = \frac{D}{qS}$.

Cross-wind force, C; absolute coefficient

$$C_c = \frac{C}{qS}.$$

Resultant force, R

(Note that these coefficients are twice as large as the old coefficients L_c, D_c.)

Angle of setting of wings (relative to thrust line), i_w

Angle of stabilizer setting with reference to thrust line i_t

Dihedral angle, γ

Reynolds Number $= \rho \frac{Vl}{\mu}$, where l is a linear dimension.

e. g., for a model airfoil 3 in. chord, 100 mi/hr., normal pressure, 0°C: 255,000 and at 15.6°C, 230,000;

or for a model of 10 cm. chord, 40 m/sec., corresponding numbers are 299,000 and 270,000.

Center of pressure coefficient (ratio of distance of C. P. from leading edge to chord length), C_p.

Angle of stabilizer setting with reference to lower wing. $(i_t - i_w) = \beta$

Angle of attack, α

Angle of downwash, ϵ

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REPORT No. 185.

THE RESISTANCE OF SPHERES IN WIND TUNNELS AND IN AIR.

By David L. Bacon and Elliott G. Reid.

SUMMARY.

To supplement the standardization tests now in progress at several laboratories, a broad investigation of the resistance of spheres in wind tunnels and free air has been carried out by the National Advisory Committee for Aeronautics.

The subject has been classic in aerodynamic research and, in consequence, there is available a great mass of data from previous investigations. This material was given careful consideration in laying out the research, and explanation of practically all the disagreement between former experiments has resulted. A satisfactory confirmation of Reynolds law has been accomplished, the effect of means of support determined, the range of experiment greatly extended by work in the new variable density tunnel, and the effects of turbulence investigated by work in the tunnels and by towing and dropping tests in free air.

It is concluded that the erratic nature of most of the previous work is due to support interference and differing turbulence conditions. While the question of support has been investigated thoroughly, a systematic and comprehensive study of the effects of scale and quality of turbulence will be necessary to complete the problem, as this phase was given only general treatment.

INTRODUCTION.

Rapid developments in both apparatus and technique have made the wind tunnel an accurate and sensitive experimental device but this development has also brought out one of its greatest shortcomings. This is found in the fact that the disagreement between data obtained from different wind tunnels is much greater than can be attributed to experimental errors.

It has been noted that the disagreement between the values of sphere resistance, as given by various investigators, is proportionally greater than that found for any other universally tested object. It has also been recognized that the air flow about a sphere is of very unstable character and all the existing data points to it as an extremely sensitive indicator of air-stream characteristics.

The tests of the standardization program confirmed this belief, and the present research was instituted with the purpose of separating the factors which control the resistance, ascertaining the magnitude and character of their effects, and formulating certain criteria for sphere testing and its interpretation when used as a means of standardizing wind tunnels.

RÉSUMÉ OF PREVIOUS RESEARCH.

A multitude of methods has been applied to the problem of sphere resistance. Although the greater part of the work has been done in wind tunnels, some experiments have been made in free air and in water. In the last two cases, towing as well as free ascent and descent have been applied, and at least one experimenter measured the resistance of a sphere in natural winds, using a specially constructed spring balance for the purpose. The collected data from previous work are amazing; the graphical representation of the results shows such large discrepancies that one really hesitates to consult the tabular records. The results of the more important researches are shown in Figure 1.

It is the purpose of this résumé to enumerate, briefly, the conditions of each of these tests, in so far as is possible, and to point out those features which seem to have the greatest bearing on the fundamental problem.

N. P. L. (Pannell).—Pannell's experiments were carried out in the 3, 4 and 7 foot (0.92, 1.22 and 2.13 meters) square wind channels of the National Physical Laboratory, and in free air (natural winds). The tunnels were all of the closed throat, open circuit type, and the N. P. L. balance was used throughout. While no specific mention of the method of support is made in the report covering the work,[1] Colonel Steadman, of the Canadian Air Board, who was associated with the N. P. L. at the time of this research, is authority for the information that a cross-wind spindle was used in nearly all cases, although some tests were made in which the spheres were supported by right-angle spindles which entered from downstream, and additional support was had by wires attached at the ends of a cross tunnel diameter or the extreme upstream point. Indications point toward an air stream of better than average turbulence characteristics. The curve of resistance coefficient (C_D) against Reynolds number (E) has no very unusual characteristics except that it has two points of inflection close to the minimum value of C_D. The minimum value is a little higher than average.

The tests made in natural winds show very little. The drag coefficients are not consistent among themselves although they are consistently lower than those obtained in the tunnel. Support was by a spindle perpendicular to the wind vector.

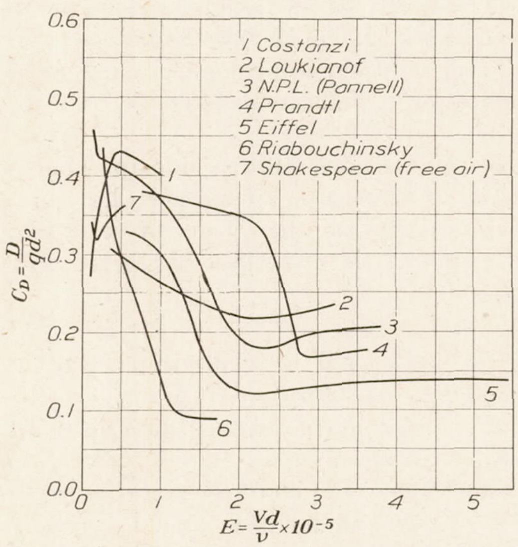


FIG. 1.—Collected wind tunnel data (taken from B. A. C. A. R. & M. No. 190).

Göttingen (Prandtl and Wieselsberger).—The investigations of Prandtl and Wieselsberger[2] are the most comprehensive on record. Attention was given to the factors governing the flow of air about the sphere rather than to the absolute values of C_D. The effects of artificially produced turbulence were studied, the movement of the circle of discontinuity was observed by the use of smoke filaments, and the effect of forcing the formation of the discontinuity was also ascertained. Surface roughness, as well, had some study. Some theories advanced by Prandtl will be mentioned later.

The tests were carried out in a tunnel of the continuous-circuit, Eiffel chamber type. The method of support is shown in Figure 2. As regards turbulence, the condition was exceptionally good. Figure 1 shows only one curve from these tests, and it is an average of the results obtained with a smooth sphere in the air stream when as free from turbulence as possible. The C_D curve shows a very sharp transition from one flow régime to another at the extraordinarily high Reynolds number 3.0×10^5 and its slope from the minimum point on is more steeply upward than found elsewhere.

Eiffel.—Eiffel's tests[3] were conducted in an open-circuit, Eiffel-chamber type tunnel. Two methods of support were used, pendulum and back spindle. Both are illustrated in Figure 2. The air stream was known to be very turbulent, and this is mentioned by Wieselsberger in his comment on the tests. The minimum value of the drag coefficient, as found with the spindle support, is lower than that obtained by any other experimenter with exception of Riabouchinsky. The transitional régime of flow occurs at the Reynolds number, 2.0×10^5, and beyond this point the curves from the different methods of support gradually approach each other. Eiffel succeeded in reaching a higher VL value than has been attained anywhere else in tunnel work, his maximum being 6.0×10^5.

The curve shown in Figure 1 is merely a sample, for the C_D versus E curves from different spheres are not close to coincidence. The one shown arises from a test of a 33-centimeter (12.995 inches) sphere on a back spindle.

[1] British Advisory Committee, R. & M. No. 190.
[2] Z. F. M., 1914, p. 144. Physikalische Zeitschrift, 1921, V22, (N. A. C. A. Technical Note No. 84).
[3] Nouvelles Recherches sur la Resistance de l'Air et l'Aviation.

Koutchino (Riabouchinsky).—Riabouchinsky[4] has made a material contribution to testing technique in the form of his sphere support, which is illustrated in Figure 2. While his air flow was extremely turbulent, results show a lower minimum drag coefficient than has been found anywhere else, although he has established the transitional régime of flow at a lower Reynolds number than is usual without forcing the formation of a discontinuity.

Costanzi.—Costanzi's tunnel tests[5] were made at very low Reynolds numbers, and little weight is attached to them because they exhibit characteristics which are contradictory to those of all other experimenters.

His tests in the towing basin[6] are of little value. An unusual method of support was used (see N. A. C. A. Technical Note No. 44), and, as would be expected, the C_D versus E curves for different spheres do not follow Reynolds' law.

Université de Paris (Maurain).—Maurain's tests,[7] carried out under conditions quite similar to those of Eiffel, cover only a small range, and exhibit no unusual characteristics, agreeing well with Eiffel's values.

St. Cyr Laboratory (Toussaint and Hayer).—Toussaint and Hayer[8] made numerous tests of small spheres in a very high speed tunnel but obtained results in no way novel. Support was by a diametral wire, and the air flow was not particularly favorable, the tunnel having an entrance cone which terminated very abruptly a short distance ahead of the working section.

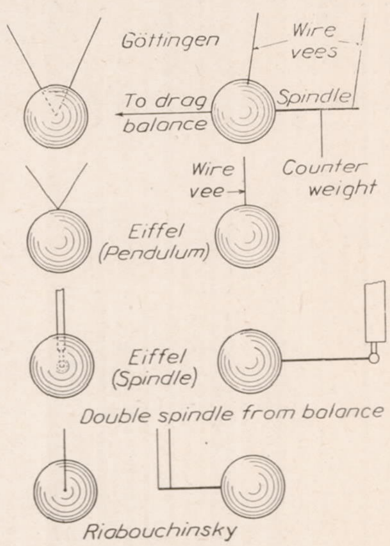


FIG. 2.—Methods of supporting sphere.

Shakespear.—The descent of celluloid spheres in free air was studied by Shakespear.[9] He worked in a very low Reynolds number range but obtained quite consistent results. His data indicates the existence of a bump in the C_D curve, a condition which Costanzi also found.

Imperial Technical School of Moscow (Loukianof).—Loukianof has made some tests on spheres,[10] but information concerning his methods is not available. His C_D curve resembles no other, having a very large minimum value and a shape somewhat similar to curves obtained at Göttingen in the work on artificial discontinuities.

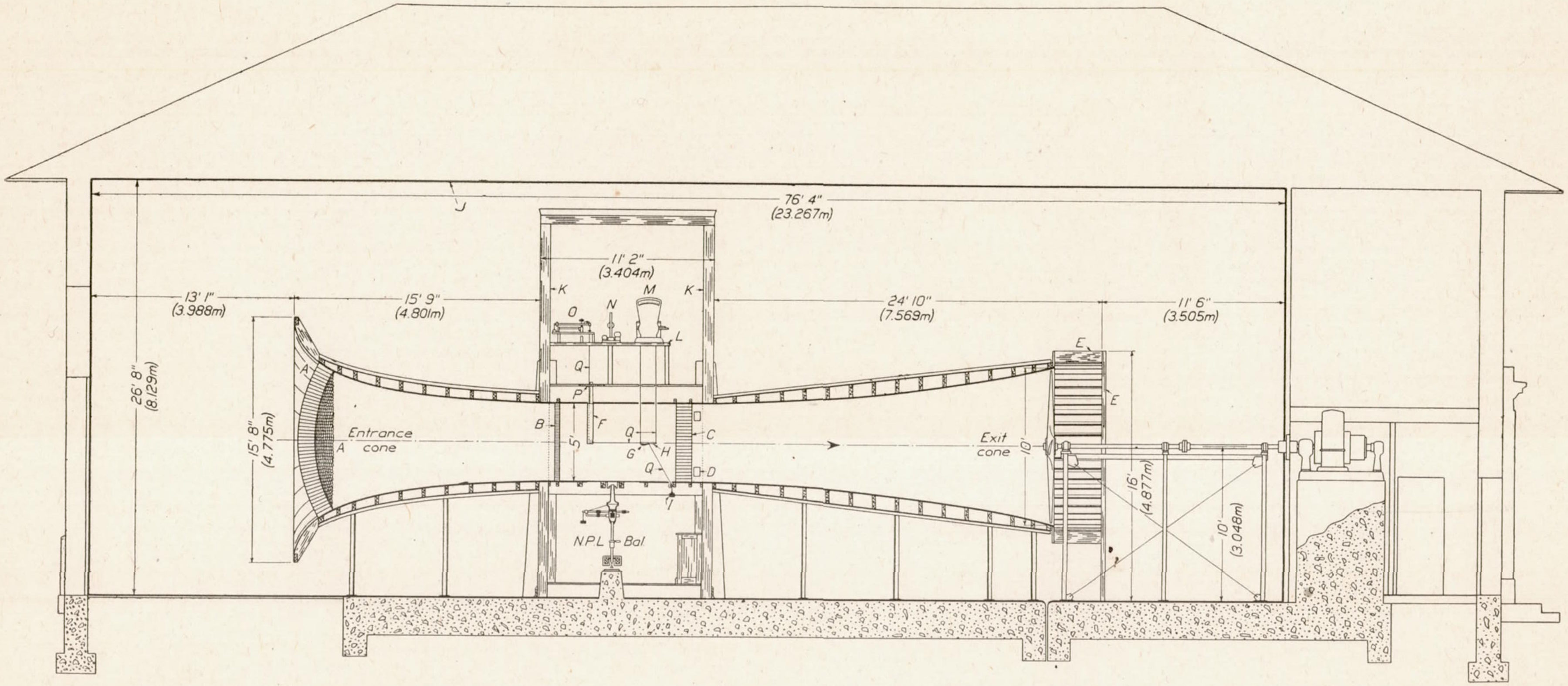
Hasselberg and Birkeland.—These experimenters worked with hydrogen-filled, rubber balloons. The time of ascent to a known height in very still air was measured and, buoyancy being known, resistance coefficients were calculated on the basis of a constant speed being attained at a height of 4 meters (13 feet).

The results of these tests are not as regular as those obtained in tunnels but a mean value of the resistance coefficients beyond the critical range is about 0.16 and the critical point occurs at approximately $E=2.75\times10^5$.

N. A. C. A. (1922).—The most recent research is that made last summer at Langley Field by Crowley and Brown of the National Advisory Committee for Aeronautics.[11] Their investigation was made by towing spheres of 7.5 to 38 centimeters (2.95 to 14.96 inches) diameter below an airplane in flight. The spheres were suspended by a single fine piano wire and the resistance calculated from the angle of trail. Wire drag was obtained by using different lengths of wire.

This research covers a very large range, reaching $E=9\times10^5$, and C_D has a minimum of 0.120. The existence of two points of inflection in the C_D curve, as in the N. P. L. tunnel tests, was found here. The Reynolds number for the critical range was very much higher than any tunnel value, occurring at $E=3.75\times10^5$.

[4] Bulletin de l'Institute Aerodynamique de Koutchino Fascicule V, (N. A. C. A. Technical Note No. 44).
[5] Rassegna aero—Marittima, April, 1914.
[6] Rendiconti delle Esperienze e deghli Studi, October, 1912.
[7] Aero de l'Université de Paris, Fascicule III, 1913. (N. A. C. A. Technical Note No. 45.)
[8] N. A. C. A. Technical Note No. 45.
[9] British Association Meeting, October, 1913.
[10] L'École Imperial Technique de Moscow, 1914.
[11] Unpublished report.



A-Spherical honeycomb (12"x2½"±conical tubes), bell mouth of beaver board
B-Honeycomb-fine (3"x⅜"-7.62x0.95 cm tubes)
C-Rear honeycomb (12"x2½"-30.48x6.35 cm tubes)
D-Bleeding holes-four (10"x5"-25.40x12.70 cm)
E-Squirrel cage of 48 radial vanes (3'x9"x⅞") and deflector of beaver board on 2x4s
F-Streamlined strut
G-Airfoil, 6" chord, inverted
H-Skids 12" long
I-Counterweight
J-Ceiling
K-Experiment chamber wall
L-Bench for instruments
M-Lift balance, angle of attack and center of pressure indicator
N-Micro-manometer
O-Drag balance
P-Cast-iron plate for supporting strut
Q-Wire

FIG. 3.—N. A. C. A. atmospheric No. 1 wind tunnel.

Few of the experimenters have offered any explanations which would tend to clarify, materially, the muddled condition of the problem. Pannell and Prandtl agree that the critical or transitional régime may be shifted along the Reynolds number scale by a change of turbulence, the break coming at a smaller value for each increase in turbulence. Prandtl and Wieselsberger have shown that a ring of wire on the upstream surface of the sphere will make a hysteresis loop appear in the resistance coefficient curve, if the critical range is approached from both directions, and they have shown a small variation in minimum due to differing turbulence conditions. Riabouchinsky's unusual means of supporting the sphere seems the only possible explanation for his nonconforming results. Several investigators have cast aspersions on contemporary research, claiming that if the sphere diameter exceed a certain proportion of the tunnel throat diameter, a large boundary interference will appear and cause disagreement.

But beyond such generalities, practically no definite information has been obtained. This is the material which forms the groundwork for the present research. An outline of the work follows.

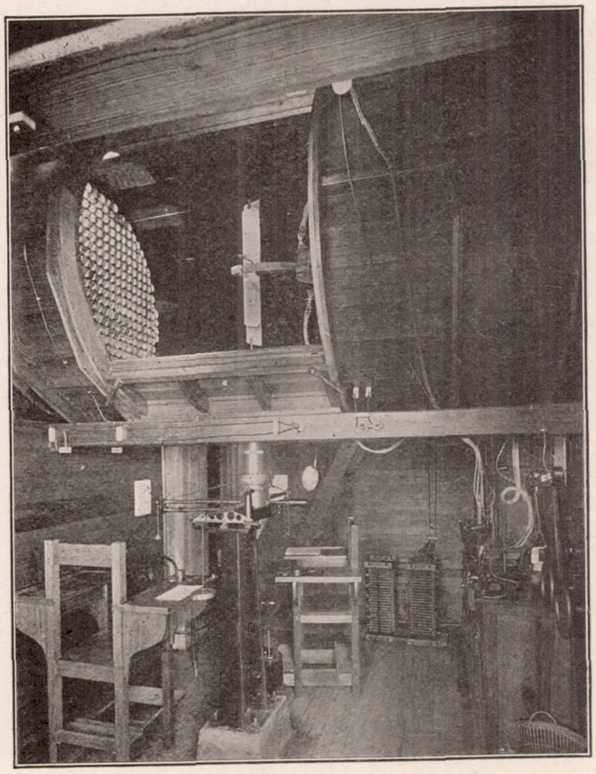


FIG. 4.—General view of balance room showing N. P. L. balance.

OUTLINE OF THE RESEARCH.

A thorough study of the information reviewed led to the conclusion that in this, as in most aerodynamic problems, the number of variables liable to become of major importance had been underestimated or not considered sufficiently. *With the object of separating the various factors and investigating the effect of each one upon the resistance of a sphere as measured experimentally* the research was outlined as follows:

Section I. Confirmation of Reynolds law by tests of two spheres under identical conditions of turbulence and support, the latter to have the least possible interference.

Section II. Investigation of the interference effects of various supports.

Section III. Investigation of the effects of turbulence.

Section IV. Tests in the variable density tunnel in an attempt to obtain confirmation of the results from the atmospheric tunnel, and to extend the experimental range to values of Reynolds number hitherto unexplored.

Section V. Tests in free air using falling spheres, with the objects of correlating, if possible, the turbulence condition there with that existing in wind tunnels, of checking the results obtained, at high values of E, in the variable density tunnel, and of determining, at least approximately, the absolute value of the resistance of spheres in free air.

DESCRIPTION OF TESTS; PRESENTATION OF DATA.

The tests required by Sections I, II, and III were carried out in the 5-foot (1.52 meters) atmospheric, No. 1, wind tunnel of the National Advisory Committee. This tunnel is of the open-circuit, closed-throat type, and is completely described in N. A. C. A. Technical Report No. 195. Figure 3 is a longitudinal section of the tunnel.

The balance used was of the N. P. L. type, specially constructed for this tunnel and a complete description of it is contained in Report No. 72 of the N. A. C. A. Figure 4 shows the balance as used. Only one change of importance has been incorporated in the balance since its installation. This consisted in replacing the original pivot with a ball-bearing which rests on three small balls supported in a spherical cup. The new arrangement has been found more sensitive and less liable to error and damage than the old one.

The balance is capable of measuring forces within the limits ± 0.0002 kilogram (0.0004 pound) and proper adjustment of stability and damping were so easily maintained that an error of more than ± 0.5 per cent seems improbable for the most adverse conditions encountered.

Details of the first three divisions of the research follow immediately.

SECTION I.

In attempting to confirm Reynolds law, it was thought best to test only two spheres, both small in comparison to the throat diameter. Because of the apparent effect of the comparative scale of turbulence in existing data, the fine honeycomb (tubes 3/8 x 3 inches) was kept in the throat to produce a turbulence of very small "grain."

The spheres used were of 15 (5.905) and 20 centimeters (7.874 inches) diameter, turned from laminated maple, accurately gauged for true sphericity and finished to a high gloss by varnishing and rubbing. A threaded brass plug was built into each sphere for spindle attachment. This plug was carefully finished flush with the surface.

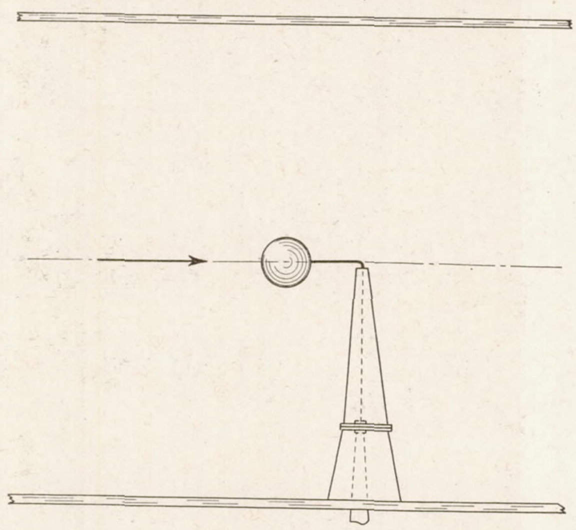


FIG. 5.—Set up for sphere tests.

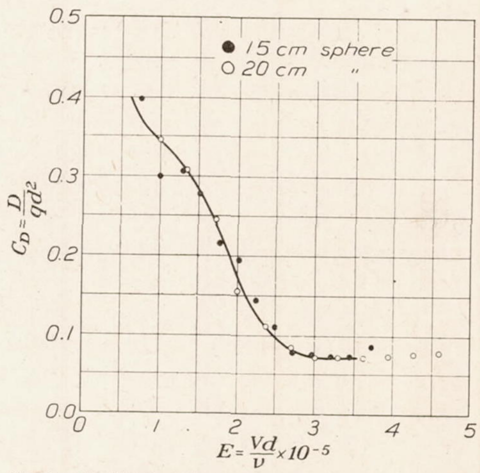


FIG. 6.—Similitude tests on spheres in No. 1 wind tunnel with fine honeycomb.

Watch th coefficients d^2

The drawing, Figure 5, shows the means of supporting the sphere for test. The bent spindle is screwed into a vertical spindle held in the balance chuck. Each sphere had its own bent spindle, the length of the horizontal portion being equal to the diameter of the sphere. The sphere is, of course, upstream from the fairing surrounding the vertical spindle and there was no auxiliary support used, the sphere being on a true cantilever spindle. The fairing was of conventional strut cross section, smooth and varnished, and was 5/8 by 1 3/8 inches (16 x 35 millimeters) at the top.

The actual taking of data for these tests was quite simple. Only the drag arm of the balance was used, the lift arm having been limited to the smallest possible motion which would give freedom of the pivot. Observations of the drag force were taken progressively in both directions throughout the operating range and as no hysteresis effects were observed, no comment is necessary.

The resistance coefficients were calculated from quantities expressed in units of the kilogram meter second system, as

$$C_D = \frac{D}{qd^2}$$

wherein

D is the measured drag,
q is the dynamic pressure, and
d is the diameter of the sphere.

It will be seen that the coefficient, C_D, is dimensionless and its unit is one-half that of the British system. The values of Reynolds number were calculated from the formula:

$$E=\frac{Vd\rho}{\mu}$$

wherein

V is the airspeed,
d is the sphere diameter,
μ is the coefficient of viscosity, and
ρ is the mass density of air, all in units of the kilogram meter second system.

Table I contains the data from these tests and they are graphically reproduced in Figure 6.

From these results, it would seem that Reynolds law is almost perfectly confirmed for the existing set of test conditions. Further discussion of the results will be reserved until the presentation of data is completed.

SECTION II.

The investigation of the interference effects of supporting devices was subdivided into two groups, one dealing with wires and one with spindles.

For the work on spindle interference, the 20-centimeter (7.87 inches) sphere was set up as in Section I and the balance head rotated through a series of angles, drag being measured at

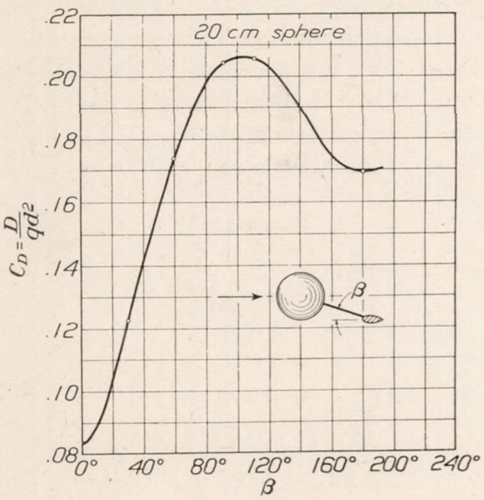


FIG. 7.—Effect of supporting spindle on sphere resistance at 25 m. p. s. in No. 1 tunnel with fine honeycomb.

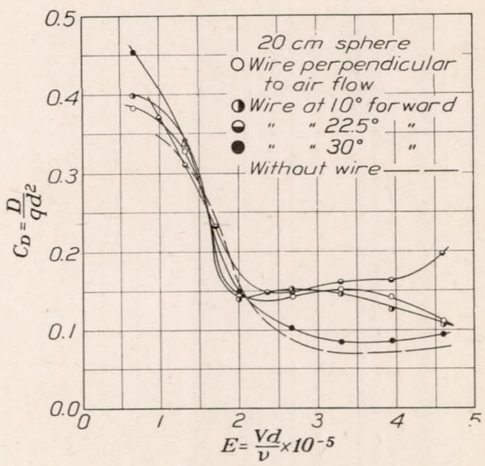


FIG. 8.—Effect of wire support on sphere resistance in No. 1 tunnel with fine honeycomb.

each setting. The entire run was made at 25 m. p. s. (82 ft. p. sec.). Spindle drag was then taken, without the sphere attached, at the same settings. While the spindle drags thus obtained can not be truly correct, it will be seen from the data that their magnitude is such that an error of 100 per cent would have little, if any, effect on the general nature of the conclusions to be drawn from this work. The data are tabulated in Table III and plotted in Figure 7.

To attempt to check up on all the methods of wire support previously used would have been an immense task and, without considerable coordinating work, quite useless alone. So it was decided to begin by studying the effects of radial wires and the results proved so pregnant with explanation of a large number of the discrepancies among previous researches that no further work on wires was done.

The tests which were made consisted in setting up the 20-centimeter sphere as in Section I, attaching an 0.018-inch (0.46 millimeter) wire radially to the sphere and measuring the drag forces throughout the speed range. One end of the wire was twisted to a tiny wire brad which was driven flush in the sphere and the other end was taken through an opening in the tunnel wall and attached directly to the drag arm so that the wire in no way restrained the balance. Several positions of the wire were used, varying from perpendicular to the air stream to an angle of 30° forward of the cross-tunnel plane.

These data will be found in Table IV and are graphically represented in Figure 8.

SECTION III.

The effects of turbulence on the resistance of spheres have been more firmly established by previous investigations than have most other factors, so the work in this line was largely one of confirmation.

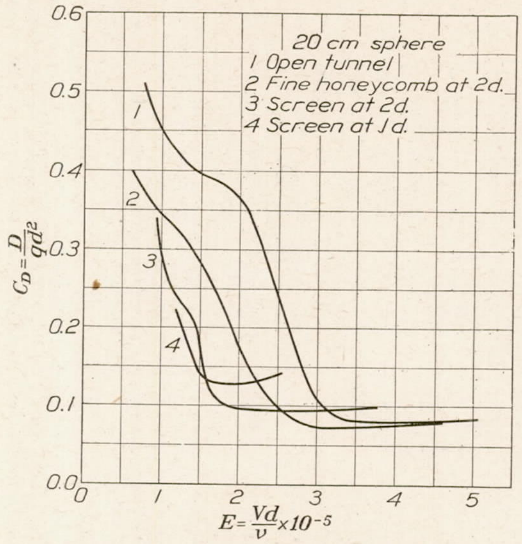


FIG. 9.—Effect of turbulence on sphere resistance.

The spheres were supported on the bent spindles as previously and all conditions maintained except for the presence of the honeycomb at the entrance of the experimental section. It was replaced by a wire screen of $\frac{1}{4}$-inch mesh (6.3 millimeters) and the drag forces measured as formerly. For the next run, the screen was moved up to close proximity with the sphere and a new set of data taken. Finally, the screen was removed, leaving the tunnel clear from entrance honeycomb to model and a third set of readings made. This last condition is characterized as "open tunnel."

The data on the 20-centimeter (7.874 inches) sphere, taken under these conditions, are contained in Table V, and are plotted with those from Table II in Figure 9. Although data were taken on the 15-centimeter (5.905 inches) sphere as well, they are not included because they are so very similar to those for the larger one that it would be somewhat confusing. This will be referred to again in the discussion.

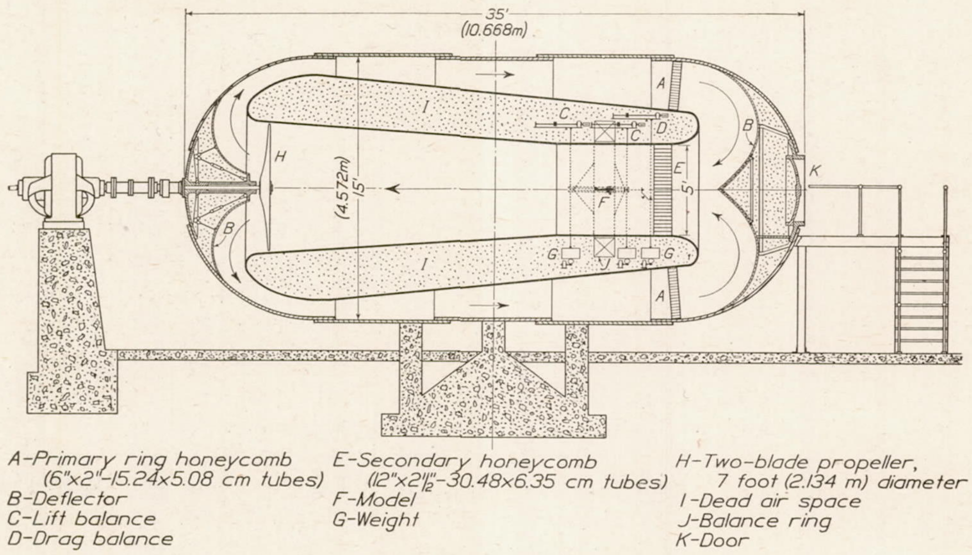


FIG. 10.—The N. A. C. A. variable density No. 2 wind tunnel.

SECTION IV.

The tests of this division were carried out in the new variable density No. 2 wind tunnel at Langley Memorial Aeronautical Laboratory. The unique feature distinguishing this tunnel is the use of air differing in density from that of the atmosphere. By this means, the kinematic viscosity, of the test medium and, consequently, the Reynolds number of the experiment, are variable, although the air speed is constant. The general arrangement and proportions may be seen in the sectional drawing, Figure 10, and the photograph, Figure 11.

Figure 10 indicates the use of a second honeycomb of $2\frac{1}{2}$ by 12 inches (6.3 by 30 centimeters) tubes, for sphere tests, installed at the front of the throat section. This measure became necessary when it was found that the ring honeycomb, 2 by 6 inches (5 by 15 centimeters) tubes, was insufficient to counteract the torque of the propeller. The result was

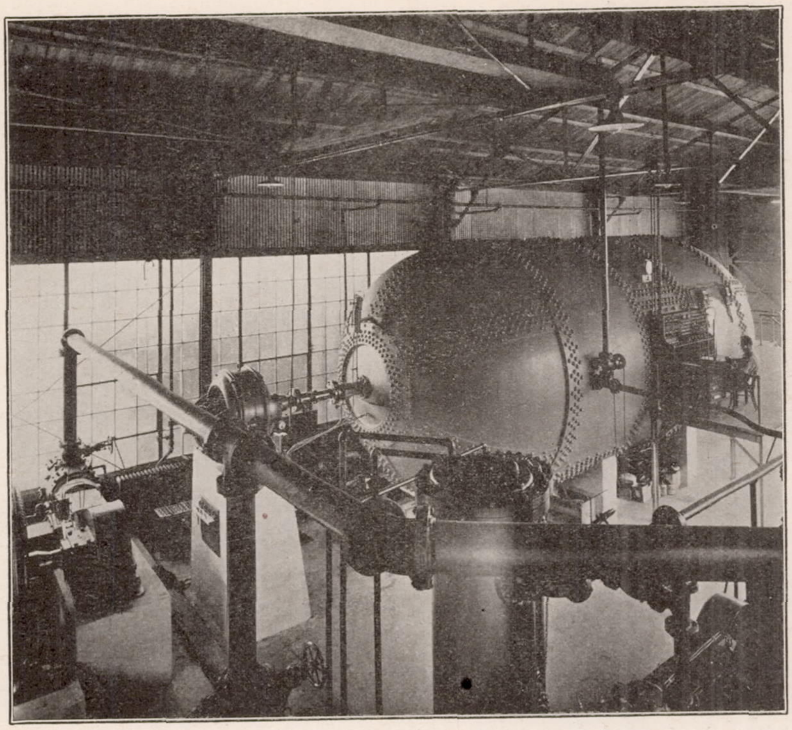


FIG. 11.—Variable density No. 2 wind tunnel.

naturally a coarse scale of turbulence at the test section, but, as this was only a temporary arrangement, the results need not be taken to indicate that a turbulent condition will exist in future testing.

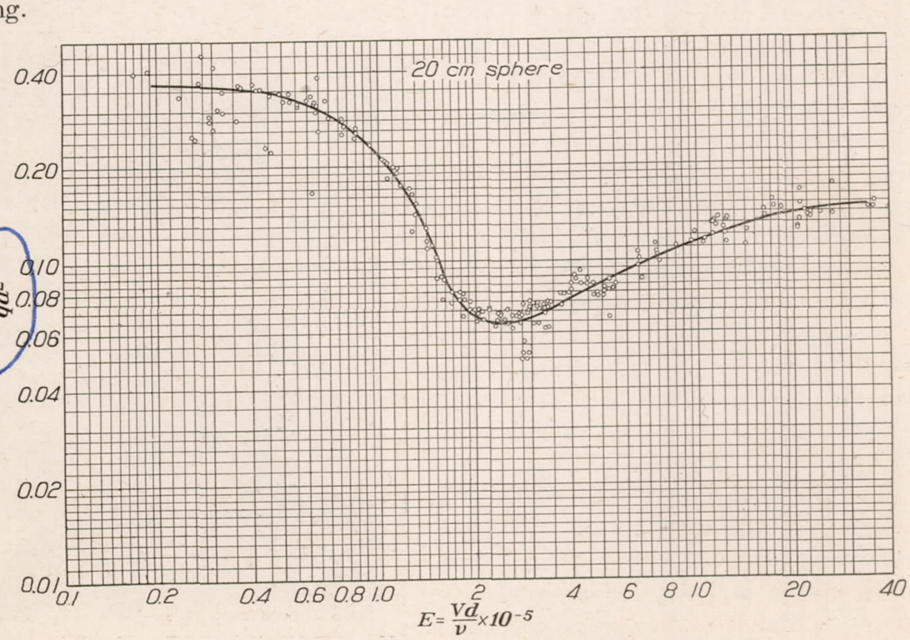


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FIG. 12.—Resistance of sphere in variable density tunnel.

Only the 20-centimeter (7.874 inches) sphere was tested in the variable density tunnel. A heavy steel bar was made fast to the main balance ring so that it was coincident with the horizontal diameter of the tunnel throat. Around it was fitted a hollow wood strut of 2½ inches (6.35 centimeters) maximum thickness and fineness ratio 3 (approximate). Through a small hole at the center of the strut, a straight spindle was screwed into the steel bar, its free end projecting axially upstream. The sphere was attached to the end of the spindle, the latter extending clear through it, and threading into the original brass plug in the upstream surface of the sphere. This method was deemed necessary in view of the long spindle and large forces. The set up, then, consisted of the sphere on a spindle entering its downstream side, and being 25 centimeters (9.8 inches) ahead of the cross tunnel strut and 65 centimeters (25 inches) behind the honeycomb.

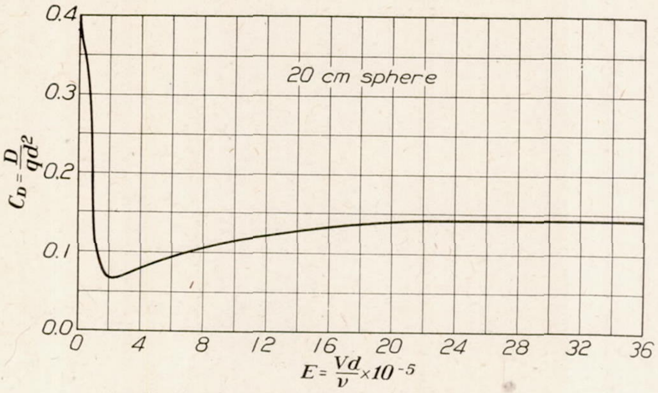

FIG. 13.—Resistance of sphere in variable density tunnel.

The actual testing consisted in measuring the drag forces under conditions of varying density and a constant speed of about 22 m. p. s. (72 ft. p. sec.). An enormous range of Reynolds number was covered and much new information discovered. It will be seen from the curve, Figure 12, that several sets of data were taken over the entire testing range, and, because of their bulk, tabular data are omitted. Figure 13 (nonlogarithmic) will give a better idea of the large *VL* range covered.

SECTION V.

The problem of procuring data on the resistance of spheres in free air presented many difficulties at the outset. However, following the suggestion of recording the rates of descent of spheres of known weight, while falling freely, a little preliminary work with a meteorological type theodolite paved the way for a very easy solution. A pair of identical recording theodolites, one of which is shown in Figure 14, were built and found to function very satisfactorily. The operation of the instrument consists in keeping the horizontal cross hair of a pair of 6-power artillery binoculars on the falling object and, by so doing, making a time elevation-angle record of the path.

Reference to the photograph will show that the glasses are attached to a frame which is free to rotate in the vertical plane about the pivots (*a*) and to which is attached the quadrant carrying the record blank of sensitized indicator paper. The emulsion used gives a black line when scratched with brass. The fixed arm (*b*) carries the recording assembly which consists of a pair of electromagnets which act upon a pivoted armature in which a brass stylus (*c*) is mounted. The motion of the stylus is about 1/32 inch (0.7 millimeter), radial as referred to the quadrant, and return is brought about by an elastic cord attached to the armature below the pivot. A chronometric contact makes and breaks the electromagnet circuit. The vertical post, upon which both assemblies are mounted, is free to traverse.

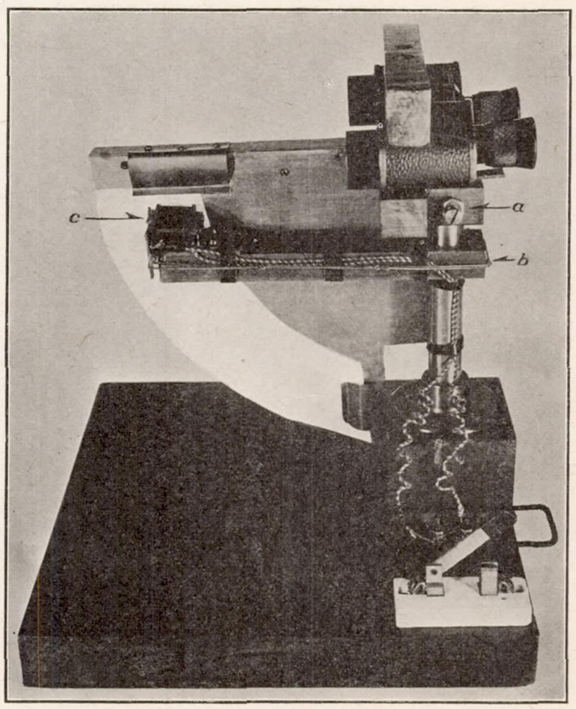

FIG. 14.—Recording theodolite.

Thus, with the interruption of the circuit at regular intervals, the angular motion of the binoculars and quadrant is recorded in the form of a notched arc. The observations being made on a vertical drop, these records are easily rectified into space-time curves which show whether or not a steady speed of descent has been attained and, if so, its magnitude. To locate the starting point of the rectification, the altitude, as read by an experienced test pilot from a carefully calibrated altimeter, was assumed correct.

Simultaneous records, taken with two such instruments, have shown that rather surprising accuracy may be obtained. It was found that with practiced observers and favorable visibility conditions the position of a 12-inch (30 centimeters) sphere, as determined from separate rectifications of the instrument records, would have a maximum departure of not more than $\pm$ 30 feet (9 meters) from a mean curve, although in a drop from 2,000 feet (610 meters) altitude a terminal speed of over 160 ft. p. sec. (48 m. p. s.) was attained. Figure 15 is a sample of the curves obtained.

The actual tests were carried out with a marked respect for weather conditions. Good visibility was necessary, and no drops were made unless there was practically no wind. Examination of the records of the Signal Corps meteorological station at Langley Field showed that it was unusual to have very much difference in wind velocity between ground level and 2,000 feet (610 meters) altitude, and so spheres were dropped under well-determined conditions. No account was taken of either the possible effect of wind or the initial horizontal speed of the sphere on leaving the airplane in the rectification of records. Conditions were so chosen that the existing windage would be of negligible consequence, and it was found by trial that the spheres fell almost truly vertically because the slipstream seemed to completely destroy the initial forward speed.

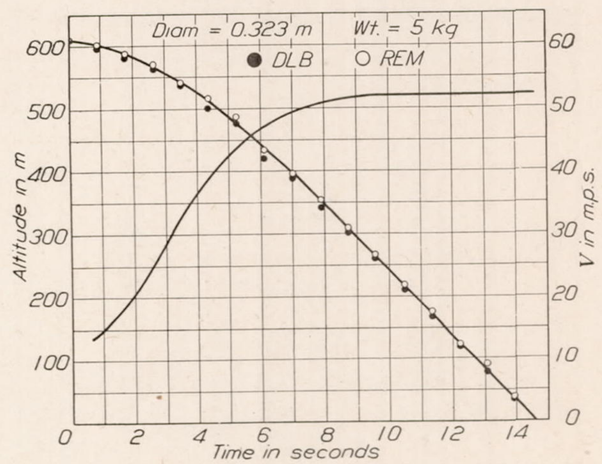

FIG. 15.—Sphere drop.

Spheres of three kinds were used. A split brass mold of 20 centimeters (7.8 inches) inside diameter was made in the shop and, in this, spheres of varying wall thickness were cast from Montana wax. They had a fine glazed surface when cast and were varnished with a mixture of varnish and chrome yellow to improve visibility. The varnish was rubbed with fine sandpaper and oil when dry.

In the attempts to cast thick walled wax spheres a good deal of difficulty was encountered because it seemed almost impossible to eliminate bubbles in the wax. About this time it was found that large rubber surf balls, which were very accurately made and had smooth, bright colored surfaces, could be purchased. Two of these spheres were obtained. They were about 1 foot (30 centimeters) in diameter and sufficiently heavy to attain high terminal speeds. One of them was used without alteration, but the other, denoted as "yellow" in the tabular results, was punctured, loaded with sand, reinflated, and patched. The patch applied was of the kind used on automobile inner tubes. It was found almost impossible to get a smooth juncture of patch against sphere surface. After several drops of each sphere, it was noted that the coefficients resulting from drops of the patched sphere were very erratic and higher than those from the other. An attempt to reach still higher terminal speeds by very heavy loading of the blue sphere resulted in equally suspicious data, and so the difficulty was attributed to the patched surface and the process, as such, abandoned. The patched spheres had been seen to twist and "corkscrew" when falling and this was not eliminated by fixing part of the contained load opposite the patch.

Spheres of the third variety used were made of wood. They were of 30 (11.81) and 38 centimeters (14.96 inches) diameter and of the same accuracy and fine finish as those used in the wind tunnel tests, as they had been made for and used in the towing tests. The drops of these spheres gave very regular results and, as a Reynolds number of 15×10^5 was attained, the research terminated with their destruction.

Listed in the summary, Table VI, will be found all the results of this work. The curve for C_D versus E appears in Figure 16. Figure 17 shows, graphically, the relation of the results from all the researches on spheres made by the N. A. C. A.

DISCUSSION OF RESULTS; COMPARISON WITH EXISTING DATA.

The similitude tests of Section I have shown that, under such conditions as existed there, the truth of Reynolds law is beyond question. The excellence of agreement between the two sets of data is attributed largely to the very fine "grain" or "texture" of turbulence in the airstream. This is said in view of the data obtained from the tests of Section II. In Table II will be found the results of tests of the 15 (5.9) and 20 centimeter (7.8 inches) spheres in the "open tunnel." It is not even necessary to plot the curves to see that they have noticeably different ordinates at the same values of Reynolds number. In the latter tests, such turbulence as existed must have been of larger scale than was possible with the fine honeycomb in the tunnel.

To obtain dynamic similarity between two systems of flow, it is ordinarily considered sufficient to establish equal Reynolds numbers for the cases compared, the bodies in the flow

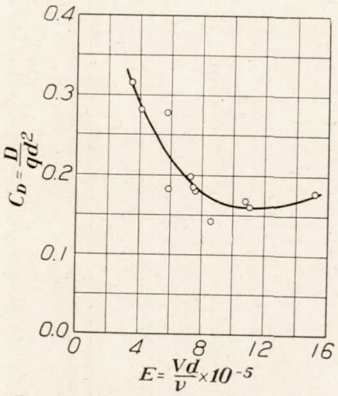

FIG. 16.—Resistance of spheres in free air.

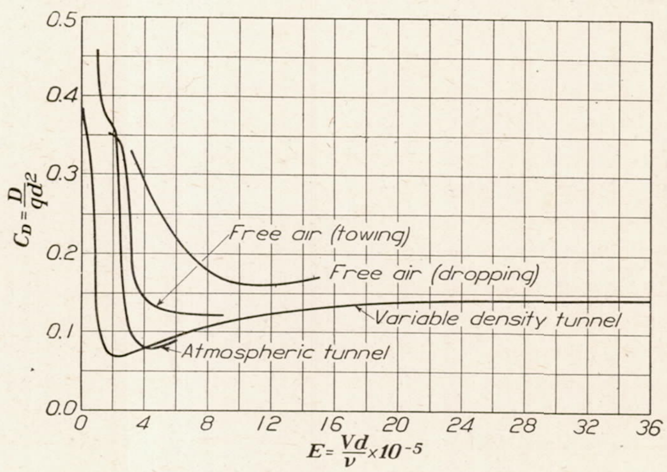

FIG. 17.—Resistance of spheres in air.

system being geometrically similar. However, in wind-tunnel testing, although we establish equal Reynolds numbers for two geometrically similar objects, if the scale of turbulence is fixed by such damping devices as honeycombs, etc., and we even neglect the fact that the Reynolds number of the tunnel itself (i. e., L being a tunnel dimension) must change with velocity, complete dynamic similarity is impossible because the airflow is not geometrically similar when referred to the dimensions of the objects tested.

Introducing this consideration, it will be seen at once that the ideal conditions were much more closely attained in the air stream of fine texture turbulence than the one existing with the honeycomb removed. This is a matter which will not be noticed in work with very stable systems of flow; but, for the fine accuracy necessary for the use of spheres in the standardization of wind tunnels, it assumes a more important rôle.

An explanation of a great part of the discrepancies noted is, in all probability, to be found in the novel results of the tests in Section II. It will be noticed, if the résumé be consulted, that in every former research for which details of the apparatus are known, excepting that of Riabouchinský and one section of Eiffel's work, the spheres were supported in one of the following ways:

(1) By a spindle perpendicular to the air stream.

(2) By a system entailing the use of wires of which at least one was attached to the sphere itself at or upstream from the equator.

The data in Table III show that the resistance of a sphere, supported as in (1), may be more than 2.5 times that when supported by a back spindle only. This value applies, of course, to a specific set of conditions and was observed at a value of E greater than the critical, but Figure 7 clearly demonstrates the nature of the influence.

The work on the effect of wire interference was inspired by the disagreement between the first wind-tunnel data on spheres and those Crowley and Brown obtained in their towing tests. Figure 8 is eloquent on this subject. It will be seen that the curves are reasonably well grouped until the transitional régime has been passed, but that, beyond this, the addition of a wire may more than double the resistance.

The effect of the wire is, of course, to force the formation of the closed curve of discontinuity at an unnatural position. Prandtl and Wieselsberger obtained a similar effect by putting a wire ring around their sphere upstream of the equator. Prandtl speaks at some length on the theory of this phenomenon, saying that a boundary air layer of very small thickness must exist at all points upstream of the normal circle of discontinuity and that, by adding a ring of wire whose thickness of 1 millimeter (0.039 inch) was said to be greater than that of the boundary layer, the effect of truncating this layer was obtained. Although destruction of the continuity of the layer at a single point seems superficially unimportant, it is regretted that, having obtained such unusual results by the use of a ring, and knowing that in the low resistance régime of flow the circle of discontinuity must be well back of the equator, the Göttingen experimenters did not investigate the effects of their equatorial supporting wires.

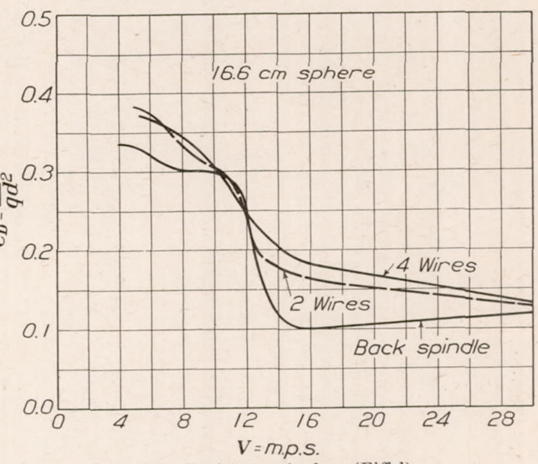

FIG. 18.—Resistance of sphere (Eiffel).

An interesting point in this connection came to light during the writing of this discussion. At the Bureau of Standards, an attempt to corroborate the above wire interference phenomena—by having the wire approach the surface of the sphere—failed because the wire was never brought closer than ½ inch (12.7 millimeters) from the surface. Any closer approach produced such violent instability that the work was abandoned for fear of damage to the apparatus.

Data on the support used by Loukianof were not available, so his extraordinarily high minimum value of C_D can not have comment here, but it is significant that the values obtained at the N. P. L., which are next highest—if we consider only those beyond the transitional régime, are below the maximum indicated in Figure 7.

This leaves the work of Eiffel as the only remaining member of a one-time large group of nonconforming results. Three curves, obtained from the data given in Nouvelles Récherches are shown in Figure 18. The influence of the pendulum method of support is clearly shown, and it is significant that the differences between the results from two and four wire support are not as large as those between back spindle and two-wire methods. To account for the comparatively high minimum value of C_D obtained by the spindle method is difficult, but the following point seems important: Tests of three spheres do not indicate the validity of Reynolds' law and the minima are successively higher with increasing sphere diameters. In the light of the work on turbulence, this condition would be interpreted as the result of turbulence of very coarse scale and strongly defined pattern, both of which seem plausible on reference to drawings showing the size and location of honeycombs in the Eiffel tunnel.

Section III is almost entirely of confirmatory nature. The effect of increasing the degree of turbulence was found to agree with the behavior described in the works of Pannell and of Prandtl. Although the first step, from "open tunnel" to fine honeycomb is contradictory, the results show a systematic increase of minimum value with increasing turbulence. While no positive proof of the fact exists, there is a suspicion that the air-speed measurement during the tests with no honeycomb was somewhat in error and that the actual air speeds were slightly higher than those recorded. The doubt is founded on subsequent Pitot calibrations, but as the tunnel has been changed slightly since these tests, an absolute verification is out of the question. However, if this were to be found to be the case, the sequence of minima would be complete, for the coefficients for this condition would be reduced.

The data obtained from the work in the variable density tunnel form a valuable addition to the existing information on sphere resistance. As these tests were made before the installation was actually completed, i. e., adequate honeycombs not yet provided and the balance not completely free from those inaccuracies always present in new apparatus of such complicated nature, the small discrepancies between different sets of data were not at all unexpected.

The outstanding features of the results are, of course, the large range covered, the very low minimum coefficient obtained, and the flattening of the coefficient curve in the high VL range. Coincidence with the results from the atmospheric tunnel would be remarkably close if the curve were to be shifted slightly out on the E scale, and this is the effect which would be expected with finer texture of turbulence. It is also pleasing to note that the results from the variable density tunnel approach those of the towing and dropping tests at high values of E.

The minimum coefficient obtained in these tests is the lowest ever attained, it is believed, but that need cause no alarm. There were several factors present, all of which might tend to bring this about. The method of support must have had some effect for, with a small dead-air region behind the sphere—which must exist when C_D is very small—the large strut containing the balance bar would certainly tend to increase the "fineness ratio of the whole flow system," if such a conception is not too far fetched. The proximity of model and honeycomb and the size of the tubes in the latter, would probably work at cross purposes so that consideration has little explanatory value. A point not hitherto mentioned is the magnitude of directional fluctuations in the air flow. This is known to be several times greater as well as more rapid than that for the atmospheric tunnel; this has been found by taking photographic records, using a very sensitive yaw head connected to a special high-speed recording air-speed meter. The fluctuations in the variable density tunnel are small—merely fractions of a degree—but the condition in the atmospheric tunnel is so unusually fine that there is quite a difference. This fact, when considered in the light of the work of Katzmeyer, of Vienna, concerning the effect of directional variations on airfoil drag, would certainly admit the possibility of an explanation of the low minimum.

The experiments with falling spheres are probably of even greater value than those in the variable density tunnel, although the former were conducted with apparatus and under conditions not entirely ideal. The shape of the curve of C_D versus E is intensely interesting from a theoretical standpoint. Lanchester, in his Aerodynamics, advanced the theory that sphere resistance would have three phases if referred to a velocity base: First, the Stokes régime in which $D \propto V$; second, the Allen phase in which $D \propto V^{1.5}$ and finally the true Newtonian resistance wherein $D \propto V^2$. This sequence would result in a C_D versus E curve composed of the vertical branch of a rectangular hyperbola and a horizontal straight line, connected by a curve of exponent 1.5. Or, if resistance were plotted against velocity on logarithmic paper, the "curve" would be made up of segments of the straight lines having slopes of 1, 1.5, and 2, respectively. The curve in Figure 16 bears considerable resemblance to the predicted shape.

An interesting extrapolation of this curve may be had by using data from Humphrey's "Physics of the Air." His information, the fruit of countless observations by meteorologists, gives, for the fall of a rain drop (approximately spherical) of 3 millimeters (0.118 inch) diameter, a velocity of 7 m. p. s. (22.9 ft. p. sec.). Calculated for the system of coefficients used here, the result would be $C_D = 0.515$ at a value of $E = 0.0014 \times 10^5$.

The only other experiments with free spheres in air, those of Hasselberg and Birkeland, form an interesting comparison. It would be very hard to determine whether or not a critical point was found but, in that range in which C_D is sensibly constant, its value is within 5 per cent of that obtained from the dropping tests of the same phase.

CONCLUSIONS.

With the completion of the research, several facts stand out as new information.

The method of support, in sphere testing, is very important; to obtain reliable data, the support should be such that it can not interfere with the formation or natural movement of the boundary of discontinuous flow.

The effect of increasing turbulence is to cause the transition of flow to occur at smaller values of the Reynolds number, but its effect on the resistance beyond the critical phase is not so determinate. The results of these tests indicate that increasing turbulence will cause a rise in the minimum value of the resistance coefficient, but the scale, or "grain," of turbulence seems to be interlocked with that quality which might be termed "intensity" in such an involved way that conclusions regarding the minima are not justified. The one determined effect of scale of turbulence is to control the degree with which true dynamic similarity may be maintained throughout a series of tests with spheres of different sizes. If the scale is fine, as compared with the diameter of the smallest sphere, a good approximation may be had throughout; if it is coarse, Reynolds law no longer serves even as an indicator.

The absence of information concerning the specific nature of the various forms of turbulence, and the consequent nonexistance of terms definitive of its characteristics, preclude, for the present at least, a complete analysis of this phase of the subject, but it is interesting to note that at large values of E the effects of turbulence seem to become relatively unimportant, as the resistance coefficients obtained under greatly differing conditions are moving toward coincidence there.

The tests in free air have demonstrated the fact that no existing wind tunnel can even approximate the nonturbulent condition prevailing in the atmosphere.

In the presence of little turbulence, the resistance of spheres conforms well to Lanchester's predictions, increasing directly with velocity at small Reynolds numbers, then at a slightly faster rate and finally conforming almost perfectly with the V^2 law.

It is recommended that an extensive study be made of the effects of scale and quality, or "intensity," of turbulence, for, with this problem solved, the comparison of air flows in general, and the standardization of those in wind tunnels in particular, will be facilitated by this very powerful tool.

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APPENDIX.

ATMOSPHERIC WIND TUNNEL DATA.

No account of spindle drag has been taken except in Table III—and there only roughly—because, with the method of support used, the sphere so masked the spindle that its drag, with the sphere supported in the position of test, was found to be less than 2 per cent of the total. This proportion exists only at low speed; at high speeds spindle drag is entirely negligible.

The average temperature existing during these tests was 32° C., and all computations were based on this condition.

VARIABLE DENSITY WIND TUNNEL DATA (NOT INCLUDED).

Spindle drag was neglected here, as in the atmospheric tunnel, support being similar. Temperature corrections were applied, individually, to each set of readings.

SPHERE DROPPING DATA.

All computations were made on the basis of standard temperature and pressure, i. e., 15.6° C. and 760 mm. Hg.

GENERAL.

Coefficient of viscosity (absolute) $\mu = 0.0001824$ at 23° C. and 760 mm. Corrected by the formula:

$$\mu_t = 0.0001824 - 0.000000493\ (23 - t).$$

Air density in metric (kg–m–s) gravitational units:

$$\rho = 0.1247 \text{ at } 15.6° \text{ C. and } 760 \text{ mm.}$$

TABLE I.

q (kg./m.^{2})	D (kg.)	C_D	E ($\times 10^{-5}$)
20-CM. SPHERE.			
3.53	0.0488	0.346	0.98
6.38	.0784	.309	1.32
10.68	.1042	.244	1.71
14.73	.0926	.157	2.00
20.40	.0918	.112	2.36
26.45	.0904	.0853	2.68
32.75	.0962	.0735	2.99
39.9	.1145	.0718	3.30
48.1	.1399	.0725	3.62
57.0	.1693	.0743	3.94
66.8	.2045	.0766	4.26
77.3	.2464	.0798	4.59
15-CM. SPHERE.			
3.53	0.0316	0.398	0.735
6.38	.0430	.300	.99
10.68	.0735	.306	1.28
14.73	.0921	.278	1.50
20.40	.0994	.216	1.77
26.45	.1169	.196	2.01
32.75	.1070	.145	2.24
39.9	.1008	.112	2.48
48.1	.0858	.0793	2.72
57.0	.0988	.0770	2.96
66.8	.1122	.0745	3.20
77.3	.1295	.0745	3.44
82.4	.1495	.0807	3.68

Both spheres tested at 0.4 m. downstream from fine honeycomb (tubes ⅜ by 3 inches) (9.5 by 76.2 millimeters).

TABLE II.

20-CM. SPHERE.			
q (kg./m.^{2})	D (kg.)	C_D.	E ($\times 10^{-5}$)
3.17	0.0626	0.494	0.925
5.77	.0906	.393	1.25
9.52	.1444	.380	1.60
13.01	.1979	.380	1.88
18.23	.2413	.331	2.22
23.76	.2077	.218	2.53
29.45	.1612	.137	2.82
36.0	.1277	.0866	3.12
42.7	.1418	.0830	3.40
51.5	.1676	.0814	3.74
60.6	.2000	.0814	4.05
70.5	.2395	.0827	4.36
81.0	.2615	.0806	4.68
91.1	.3110	.0854	4.96
15-CM. SPHERE.			
5.77	0.0574	0.442	0.93
9.52	.0822	.394	1.20
13.01	.1215	.415	1.41
18.23	.1561	.391	1.66
23.76	.1964	.368	1.90
29.45	.2313	.349	2.22
36.0	.2115	.261	2.34
42.7	.2201	.229	2.55
51.5	.1972	.170	2.80
60.6	.1446	.107	3.04
70.5	.1320	.0823	3.28
81.0	.1487	.0815	3.51
91.1	.1818	.0866	3.72
103.2	.2405	.105	3.96

Both spheres tested in the "open tunnel."

TABLE III.

20 CM. SPHERE.				
β (deg.).	D (gross).	D (spindle).	D (net).	C_D.
°	*kg.*	*kg.*	*kg.*	
0	0.0947	0.0193	0.0754	0.0838
30	.1300	.0198	.1102	.1224
60	.2074	.0508	.1566	.1740
90	.2614	.0769	.1845	.2048
110	.2774	.0818	.1856	.2060
180	.1646	.0118	.1528	.1696

Velocity 25 m. p. s. (approx.).
q=39.9 kg./m.^{2}
NOTE.—Spindle drags were obtained by simply removing the sphere.

TABLE IV.

20 CM. SPHERE.				
q (kg./m.^{2})	D (kg.)	C_D.	E ($\times 10^{-5}$).	
1.56	0.0241	0.386	0.65	Wire perpendicular to airflow.
6.38	.0839	.329	1.32	
14.73	.0852	.145	2.00	
26.45	.1514	.143	2.68	
39.9	.2421	.152	3.30	
57.0	.3223	.142	3.94	
77.3	.3364	.109	4.59	
1.56	.0250	.401	.98	Wire 10° forward.
6.38	.0841	.338	1.32	
14.73	.0845	.143	2.00	
26.45	.1628	.154	2.68	
39.9	.2409	.151	3.30	
57.0	.2871	.126	3.94	
77.3	.3402	.110	4.59	
3.53	.0525	.372	.98	Wire 22½° forward.
6.38	.0793	.311	1.32	
10.68	.0997	.233	1.71	
14.73	.1074	.182	2.00	
20.40	.1224	.149	2.36	
26.45	.1587	.150	2.68	
39.9	.2595	.163	3.30	
57.0	.3724	.163	3.94	
77.3	.6069	.197	4.59	
1.56	.0277	.455	.65	Wire 30° forward.
6.38	.0864	.339	1.32	
14.73	.0892	.151	2.00	
26.45	.1089	.103	2.68	
39.9	.1340	.0840	3.30	
57.0	.1994	.0875	3.94	
77.3	.2918	.0943	4.59	

Wire used was 0.018 inch (0.46 millimeter) diameter.
All results in this table apply to 20 centimeter sphere, supported on bent spindle at 0.4 meter behind fine honeycomb.
Drag of wire neglected in calculations because it was found to be 3 to 4 per cent of total for worst case.
Data on sphere alone, under same conditions, are contained in Table I.

TABLE V.

Data on resistance of 20 centimeter sphere behind fine honeycomb and in the "open tunnel" will be found in Tables I and II respectively.

Behind ¼-inch (6.3 millimeters) mesh screen.

20 CM. SPHERE AT 0.4 M. (APPROXIMATE).			
q (kg/m.^{2})	D (kg.)	C_D.	E ($\times 10^{-5}$).
5.77	0.0753	0.326	0.93
9.52	.0894	.235	1.20
13.01	.1125	.216	1.41
18.23	.0844	.116	1.66
23.76	.0941	.0991	1.90
29.45	.1129	.0958	2.12
36.0	.1387	.0961	2.34
42.7	.1645	.0965	2.55
51.5	.1969	.0955	2.80
60.6	.2359	.0971	3.04
70.5	.2780	.0985	3.28
81.0	.3509	.1082	3.51
20 CM. SPHERE AT 0.2 M. (APPROXIMATE).			
5.77	0.0511	0.222	0.93
13.01	.0676	.130	1.20
18.23	.0951	.130	1.66
23.76	.1225	.129	1.90
29.45	.1726	.106	2.12
36.0	.1939	.135	2.34

TABLE VI.

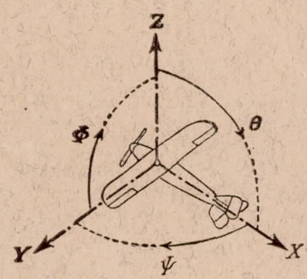
Summary of sphere dropping tests.

Altitude (ft.)	Weight (kg.)	Diameter (m.)	V (max.) (m. p. s.)	j (m.p.s.^{2})	Drag D (kg.)	C_D.	E ($\times 10^{-5}$)
WAX SPHERES.							
1,000	0.471	0.20	24.8	0.0	0.471	0.317	3.47
1,000	.610	.20	29.4	.0	.610	.282	4.10
1,000	.810	.20	42.0	.0	.810	.184	5.85
1,000	1.230	.20	41.0	.50	1.166	.278	5.71
RUBBER SPHERES. (BLUE.)							
1,000	1.330	0.324	32.0	0.0	1.330	0.197	7.23
1,000	1.330	.324	33.3	.0	1.330	.184	7.46
2,000	1.330	.324	37.8	.0	1.330	.142	8.54
3,000	1.330	.324	33.5	.0	1.330	.181	7.57
2,000	5.000	.324	52.0	.0	5.000	.284	11.70 (?)
(YELLOW.)							
2,000	2.500	0.308	41.0	0.0	2.500	0.252	8.82 (?)
2,000	2.500	.305	40.0	.0	2.500	.269	8.51 (?)
1,000	2.500	.308	50.0	.0	2.500	.169	10.75 (?)
WOODEN SPHERES.							
2,000	5.242	0.380	57.0	0.0	5.242	0.179	15.12
2,000	2.493	.299	53.0	.0	2.493	.159	11.05

j is the acceleration existing at the maximum velocity attained.

Data marked (?) is for spheres which had been loaded, re-inflated and patched. Data arising from these drops are not plotted on the curve sheet, Fig. 16.

O



Positive directions of axes and angles (forces and moments) are shown by arrows.

Axis.		Force (parallel to axis) symbol.	Moment about axis.			Angle.		Velocities.	
Designation.	Symbol.		Designation.	Symbol.	Positive direction.	Designation.	Symbol.	Linear (component along axis).	Angular.
Longitudinal....	X	X	rolling.....	L	$Y \longrightarrow Z$	roll.....	Φ	u	p
Lateral..........	Y	Y	pitching...	M	$Z \longrightarrow X$	pitch....	Θ	v	q
Normal..........	Z	Z	yawing.....	N	$X \longrightarrow Y$	yaw.....	Ψ	w	r

Absolute coefficients of moment

$$C_l=\frac{L}{q\,b\,S}\quad C_m=\frac{M}{q\,c\,S}\quad C_n=\frac{N}{q\,f\,S}$$

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS.

Diameter, D
Pitch (a) Aerodynamic pitch, p_a
(b) Effective pitch, p_e
(c) Mean geometric pitch, p_g
(d) Virtual pitch, p_v
(e) Standard pitch, p_s
Pitch ratio, p/D
Inflow velocity, V'
Slipstream velocity, V_s

Thrust, T
Torque, Q
Power, P
(If "coefficients" are introduced all units used must be consistent.)
Efficiency $\eta = T\,V/P$
Revolutions per sec., n; per min., N
Effective helix angle $\Phi=\tan^{-1}\left(\frac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS.

1 HP = 76.04 kg. m/sec. = 550 lb. ft/sec.
1 kg. m/sec. = 0.01315 HP
1 mi/hr. = 0.44704 m/sec.
1 m/sec. = 2.23693 mi/hr.

1 lb. = 0.45359 kg.
1 kg. = 2.20462 lb.
1 mi. = 1609.35 m. = 5280 ft.
1 m. = 3.28083 ft.