REPORT No. 197

A NEW RELATION BETWEEN THE INDUCED YAWING MOMENT AND THE ROLLING MOMENT OF AN AIRFOIL IN STRAIGHT MOTION

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SUMMARY

In this paper, prepared for the National Advisory Committee for Aeronautics, the induced yawing moment, due to the rolling moment produced by the ailerons, is computed. This induced yawing moment is the greatest part of the entire yawing moment encountered by the wings. The following approximate formula results:

$$\frac{\text{Induced yawing moment}}{\text{Rolling moment}} \text{ is about } \frac{\text{Lift coefficient}}{\text{Aspect ratio.}}$$

I. DERIVATION OF THE FORMULA

We consider a displacement of the ailerons during flight. The lift is increased on one end of the airfoil and it is decreased on the other end. But the drag undergoes changes, too, and is usually increased and decreased on the same ends where the lift is increased or decreased. Thus, in addition to the rolling moment, which is desired, a yawing moment is set up by the displacement of the ailerons, and, according to what is said, it is an undesirable one, acting against the rudder. For instance, when turning from a straight flight path to the left, the right side of the airplane has to be banked up and has to be yawed ahead of the left side. But the yawing moment, just mentioned, tends to yaw the left side ahead of the right side. It is therefore desirable to have this yawing moment as small as possible. The following discussion deals with the magnitude of this yawing moment and leads to a numerical relation between it and the quantities which chiefly govern its size. This relation is thought to be of interest and use in all cases where the magnitude of the yawing moment occurs; that is, with all questions of stability and controllability.

The displacement of an aileron is equivalent to the change of the wing section, of which it forms a part. This change of section will generally cause a change of the frictional or section drag. The magnitude of this change depends upon many factors, and it is difficult to make a general statement concerning it. It can be said, however, that it is not so very large, except near the angle of attack of maximum lift, and that the changes are not necessarily of opposite sign on both wing ends. The changes of the induced drag will be much larger in most cases, and these changes are of opposite sign, giving rise to a yawing moment, directed as stated above. This induced yawing moment, forming probably the main part of the entire yawing moment encountered by the wings, lends itself readily to an analytical investigation. I will proceed, therefore, to compute the induced yawing moment of the wing, making assumptions which greatly simplify the mathematical treatment without essentially specializing the problem. On the contrary, the solution will be a good approximation for all practical cases.

I consider a single airfoil moving in an ideal fluid and assume the lift per unit length of the span to vary in proportion to the ordinates of an ellipse, with the span as a main axis. Let b denote the length of the span and introduce the angle δ by means of

$$\frac{b}{2}\cos\delta=x \tag{1}$$

where x denotes the distance of a wing element from the middle of the wing. Then the elliptical lift distribution can be expressed by

$$L'=\frac{dL}{dx}=2\ V\rho\ C\sin\delta \tag{2}$$

where V denotes the velocity of flight,
ρ denotes the density of the air, and
C is a constant of the dimension of a velocity potential.

The entire lift is then

$$L=\int_{-b/2}^{+b/2} L'\ dx=2\ C\ V\rho\ \frac{b}{2}\int_0^\pi \sin^2\delta\ d\delta$$

$$L=CV\frac{\rho}{2}b\pi,$$

whence

$$C=\frac{1}{\pi}\frac{L}{bV\frac{\rho}{2}} \tag{3}$$

The distribution of the lift produced by a displacement of the ailerons is assumed to be an odd function of x. Then it can be expanded in a Fourier's series with even multiples of δ only, giving the entire lift distribution,

$$L'=2\ V\rho\ (C\sin\delta+A_2\sin 2\,\delta+A_4\sin 4\,\delta+\ \cdot\ \cdot\ \cdot) \tag{4}$$

where the A's are any constants of the same kind as C.

The entire rolling moment [1] is then

$$M_r=\int_{-b/2}^{+b/2} L'\ x\ dx$$

$$2\ V\rho\left(\frac{b}{2}\right)^2\int_0^\pi (C\sin\delta+A_2\sin 2\,\delta+\ \cdot\ \cdot\ \cdot)\ \frac{\sin 2\delta\ d\delta}{2}$$

$$M_r=A_2V\rho\ b^2\frac{\pi}{8}$$

whence

$$A_2=\frac{4}{\pi}\frac{M_r}{b^2V\frac{\rho}{2}} \tag{5}$$

The induced angle of attack can be written down directly from reference (4) and is, by reference (1),

$$\alpha_i=\frac{1}{bV\sin\delta}\ (C\sin\delta+2\ A_2\sin 2\,\delta+4\ A_4\sin 4\,\delta+\ \cdot\ \cdot\ \cdot) \tag{6}$$

[1] The symbols M_r and M_y have been used in this paper instead of the standard symbols L and N, first, because L is also the standard symbol or lift, and in some of the equations both the lift and the rolling moment occur; and, second, because the axes of the moments do not move with the airplane but are orientated with respect to the relative motion between airplane and air.

The distribution of the induced drag follows then from

$$\frac{dD}{dx}=D_i'=\alpha_i L'$$

$$D'_i=\rho\,\frac{2}{b}\,\frac{1}{\sin\delta}\,(C\sin\delta+A_2\sin 2\,\delta+A_4\sin 4\,\delta+\;\cdot\;\cdot\;\cdot)$$
$$(C\sin\delta+2\,A_2\sin 2\,\delta+4\,A_4\sin 4\,\delta+\;\cdot\;\cdot\;\cdot) \tag{7}$$

The resulting induced yawing moment is

$$M_y=\int_{-b/2}^{+b/2}D'_i x\,dx=\frac{b^2}{4}\int_0^{\pi}D'_i\,\frac{\sin 2\delta}{2}\,d\delta \tag{8}$$

or substituting (7)

$$M_y=\frac{\rho b}{2}\int_0^{\pi}\cos\delta\;(C\sin\delta+A_2\sin 2\delta+A_4\sin 4\delta)\;(C\sin\delta+2\,A_2\sin 2\delta+4\,A_4\sin 4\delta)\;d\delta$$

This integral can be split into the three following ones:

(I) $\quad M_y=\dfrac{\rho b}{2}\displaystyle\int^{\pi}\cos\delta\;C^2\sin\delta\;d\delta$

(II) $\quad +\dfrac{\rho b}{2}\displaystyle\int_0^{\pi}\cos\delta\;C\sin\delta\;(3\,A_2\sin 2\delta+5A_4\sin 4\delta+\;\cdot\;\cdot\;\cdot)\;d\delta$

(III) $\quad +\dfrac{\rho b}{2}\displaystyle\int^{\pi}\cos\delta\;(A_2\sin 2\delta+A_4\sin 4\delta+\;\cdot\;\cdot\;\cdot)\;(2\,A_2\sin 2\delta+4\,A_4\sin 4\delta+\;\cdot\;\cdot\;\cdot)d\delta$

The integrands I and III are the products of a symmetric function and cos δ, and hence the integrals are zero. The integral II can be written

$$M_y=\frac{\rho b}{2}\int^{\pi}\frac{C}{2}\sin 2\delta\;(3\,A_2\sin 2\delta+5\,A_4\sin 4\delta+\;\cdot\;\cdot\;\cdot)\;d\delta$$

which gives

$$M_y=\frac{\rho}{2}\,bCA_2\frac{3}{4}\,\pi$$

Substituting (3) and (5) gives the final result

$$M_y=M_r\,\frac{3}{\pi}\,\frac{L}{b^2V^2\frac{\rho}{2}}$$

or otherwise written

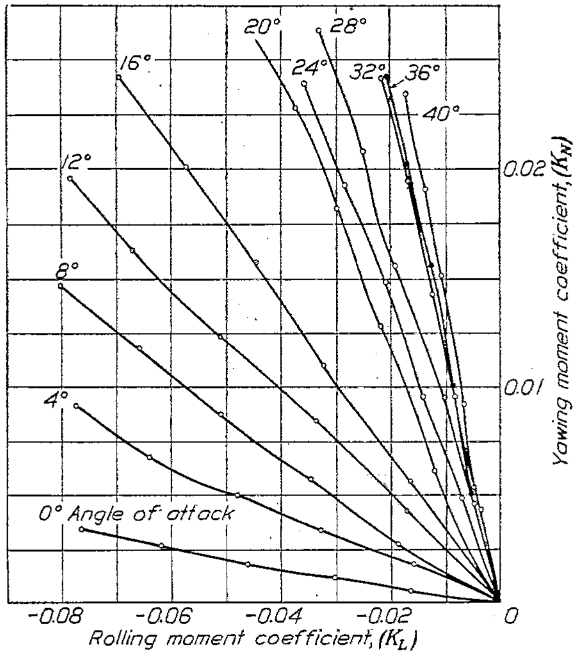
$$\frac{M_y}{M_r}=\frac{3}{\pi}\,C_L\,\frac{S}{b^2}$$

or

$$\frac{\text{Induced yawing moment}}{\text{Rolling moment}}\text{ is about }\frac{\text{Lift coefficient}}{\text{Aspect ratio}}$$

II. COMPARISON WITH EXPERIENCE

I found only one series of tests (reference 2) suitable for checking the formula obtained. That report contains a diagram with the coefficient of yawing moment plotted against the coefficient of rolling moment. Both coefficients are obtained by dividing the moments by the same quantity; which is, the product of the dynamic pressure, the square of the span, and the sum of the chords of the upper and lower wings. This diagram is reproduced in the accompanying figure.



A glance at this diagram makes it at once apparent that the type of relation suggested by the theory has actually been observed by these tests. The individual curves for particular angles of attack are substantially straight lines, passing through the point of origin, which shows that the ratio of the two moments is constant for each angle of attack. Furthermore, the tangents of the slopes of these straight lines are substantially a linear function of the angle of attack. The slope zero occurs at an angle of attack about $-2°$. Reference 3, Figure 1, shows this angle $-2°$, to be the angle of attack of zero lift for the airplane in question. The yawing moment appears to be proportional to the lift coefficient, in agreement with the formula. It remains only to examine how far the agreement includes the magnitude of the factor $\frac{3}{\pi}$. Since the type of relation has been demonstrated to agree, it is sufficient to check the factor for one condition only.

I choose for such check the angle of attack 12°, giving a lift coefficient $C_L = .96$ according to reference (3). Reference (2) gives $\frac{M_y}{M_r} = .25$. The wing area of the airplane is 188 sq. ft., and the span is 26.6 ft., giving an aspect ratio

$$\frac{b^2}{S} = \frac{26.6^2}{188} = 3.75$$

Hence my formula gives

$$\frac{M_y}{M_r} = \frac{3}{\pi} C_L = \frac{.96 \times 3}{3.75\pi} = .25$$

That happens to be a very good agreement. An error of some per cent should be expected, as, in my computation, the lift distribution is somewhat arbitrarily assumed. Substituting the effective aspect ratio of the biplane for the nominal aspect ratio $\frac{b^2}{S}$ would indeed decrease the result somewhat.

III. PRACTICAL CONCLUSIONS

The investigation shows that the yawing moment set up by the displacement of the ailerons is substantially the effect of the aerodynamic induction. It follows that no changes in the design of the aileron, nor substitution of equivalent devices for the ailerons, will do away with this yawing moment. The increase of drag on the side of increased lift can not be avoided. In order to get rid of the yawing moment, a parasite drag has to be produced on the side of decreased lift, which is bad, as the entire drag is then increased. It is probably more efficient to tolerate the induced yawing moment and to neutralize it by improved or increased rudder action. The induced yawing moment should be made as small as possible, however. According to the foregoing investigation, this can be done by distributing the lift somewhat differently from elliptic, and more concentrated near the middle. This is most effectively done by providing a proper washout of the angle of attack near the ends of the wings. Such a characteristic is good in other respects, too.

The question treated arises in connection with the controllability of airplanes, particularly at low speeds. It is then that the lift coefficient assumes large values, and hence the induced yawing moment becomes large. An even greater danger arises if the wing is near its angle of stall. Turning down the aileron may then suddenly decrease the lift and still greatly increase the drag, bringing about an effect opposite to the one desired. The discussion of this phenomenon is beyond the scope of this paper. It may be mentioned, however, that the choice of proper wing sections near the ends greatly helps to diminish this danger. Furthermore, a proper washout of the angle of attack near the ends will diminish the initial lift coefficient and hence the danger to overstall the wing ends by pulling the ailerons down.

This danger is diminished by the use of De Haviland's differential ailerons, where the up aileron is moved through twice the angle of the down aileron. The induced yawing moment is not substantially changed by this arrangement, for the computations in this paper show that it is not much affected by the distribution of lift being symmetrical or not. The De Haviland arrangement has the disadvantage of diminishing the entire lift during maneuvering in sharp turns and the like, a feature certainly unfavorable with regard to the flight characteristics of the airplane.

REFERENCES

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(3) GLAUERT & PEATFIELD. Lift and drag of S. E. 5 aeroplanes. British Reports and Memoranda No. 739.