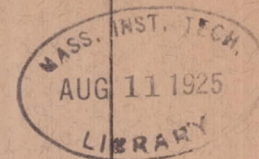


NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 206

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NONMETALLIC DIAPHRAGMS FOR INSTRUMENTS

By H. N. EATON and C. T. BUCKINGHAM



WASHINGTON
GOVERNMENT PRINTING OFFICE
1925

AERONAUTICAL SYMBOLS.

1. FUNDAMENTAL AND DERIVED UNITS.

	Symbol.	Metric.		English.	
		Unit.	Symbol.	Unit.	Symbol.
Length.....	l	meter.....	m.	foot (or mile).....	ft. (or mi.).
Time.....	t	second.....	sec.	second (or hour).....	sec. (or hr.).
Force.....	F	weight of one kilogram.....	kg.	weight of one pound.....	lb.
Power.....	P	kg.m/sec.....		horsepower.....	HP
Speed.....		m/sec.....	m. p. s.	mi/hr.....	M. P. H.

2. GENERAL SYMBOLS, ETC.

Weight, $W = mg$.

Standard acceleration of gravity,

$$g = 9.806 \text{ m/sec.}^2 = 32.172 \text{ ft/sec.}^2$$

Mass, $m = \frac{W}{g}$

Density (mass per unit volume), ρ

Standard density of dry air, 0.1247 (kg.-m.-sec.) at 15.6°C. and 760 mm. = 0.00237 (lb.-ft.-sec.)

Specific weight of "standard" air, 1.223 kg/m.³ = 0.07635 lb/ft.³

Moment of inertia, mk^2 (indicate axis of the radius of gyration, k , by proper subscript).

Area, S ; wing area, S_w , etc.

Gap, G

Span, b ; chord length, c .

Aspect ratio = b/c

Distance from $c. g.$ to elevator hinge, f .

Coefficient of viscosity, μ .

3. AERODYNAMICAL SYMBOLS.

True airspeed, V

Dynamic (or impact) pressure, $q = \frac{1}{2} \rho V^2$

Lift, L ; absolute coefficient $C_L = \frac{L}{qS}$

Drag, D ; absolute coefficient $C_D = \frac{D}{qS}$.

Cross-wind force, C ; absolute coefficient

$$C_c = \frac{C}{qS}$$

Resultant force, R

(Note that these coefficients are twice as large as the old coefficients L_c , D_c .)

Angle of setting of wings (relative to thrust line), i_w

Angle of stabilizer setting with reference to thrust line i_t

Dihedral angle, γ

Reynolds Number = $\rho \frac{Vl}{\mu}$, where l is a linear dimension.

e. g., for a model airfoil 3 in. chord, 100 mi/hr., normal pressure, 0°C: 255,000 and at 15.6°C, 230,000;

or for a model of 10 cm. chord, 40 m/sec., corresponding numbers are 299,000 and 270,000.

Center of pressure coefficient (ratio of distance of C. P. from leading edge to chord length), C_p .

Angle of stabilizer setting with reference to lower wing. $(i_t - i_w) = \beta$

Angle of attack, α

Angle of downwash, ϵ

SYNOPSIS OF REPORT NO. 206.
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

NONMETALLIC DIAPHRAGMS FOR INSTRUMENTS

By

H. N. Eaton and C. T. Buckingham

This report, the second of a series of reports relating to the general subject of instrument diaphragms, was prepared by the Bureau of Standards at the request of the National Advisory Committee for Aeronautics. The first report of the series was published as Technical Report No. 185, "Diaphragms for Aeronautic Instruments," and comprised an outline of historical developments and theoretical principles with a discussion of expedients for making the best use of existing diaphragms.

The present report relates entirely to nonmetallic diaphragms, the use of which in certain types of pressure elements has been increasing for some time. Little, if any, information has been available, however, to aid the designer of instruments using this form of pressure element. It was to attempt to meet the need for such information that the investigation reported in this paper was undertaken.

The report describes the various materials which have been used as nonmetallic diaphragms, discusses the factors which affect the performance of the diaphragms and gives the results of tests made for the purpose of investigating the effect produced by these factors. A theoretical discussion is given in which it is shown that the effective area of a nonmetallic diaphragm can be computed for specified conditions and hence the pressure-deflection curve can be predicted. Curves are given to facilitate the computation of effective areas under any given conditions. The theory was tested experimentally and was found accurate within about 5 per cent. Finally pressure-deflection curves are given to illustrate the control which the designer has over the shape of the curve by varying the different parts of the pressure element.

Report No. 206 is available for reference in the library of this institution.

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By H. N. EATON and C. T. BUCKINGHAM
Bureau of Standards

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NOTATION

- Let p =hydrostatic pressure applied to diaphragm (acting upward in figs. 16 and 18).
 F =external resisting force acting downward on rigid center.
 F_r =reaction of rim.
 A =horizontal projection of area of diaphragm.
 A_e =effective area of diaphragm $=\frac{F}{p}$.
 s =length of diametral cross-section of diaphragm from rim to rigid center.
 y or y_a =deflection of rigid center from horizontal plane at level of rim, being positive when measured upward and negative when measured downward. (See fig. 18.)
 y_s =deflection of end of spring from its normal unstressed position.
 c =distance from rim to edge of rigid center when $y=0$.
 D =diameter of diaphragm chamber.
 R =radius of rigid center.
 H =highest point on the arc s (the plane of the rim being horizontal).
 x =horizontal distance from rim to vertical line through H .
 ρ =horizontal distance from edge of rigid center to H , considered positive when measured toward the rim and negative when measured toward the center of the diaphragm.
 O =center of circle of which the arc s is a portion.
 r =radius of circle of which the arc s is a portion.
 a =distance between end of spring when the latter is in its unstressed state and the top of the rigid center.
 (See figs. 18 E and F.)
 α =angle at O subtended by arc s in Case I or $s+\rho$ in Case II.
 β =angle at rim subtended by deflection y at point of contact of diaphragm and rigid center.
 K =a constant.
 Subscript zero indicates that $y=0$.
 s/c is defined as the "slackness" of the diaphragm.

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SUMMARY

This report, the second of a series of reports relating to the general subject of instrument diaphragms, was prepared by the Bureau of Standards at the request of the National Advisory Committee for Aeronautics. The first report of the series was published as Technical Report No. 165, "Diaphragms for Aeronautic Instruments," and comprised an outline of historical developments and theoretical principles with a discussion of expedients for making the best use of existing diaphragms.

The present report relates entirely to nonmetallic diaphragms, the use of which in certain types of pressure elements has been increasing for some time. Little, if any, information has been available, however, to aid the designer of instruments using this form of pressure element. It was to attempt to meet the need for such information that the investigation reported in this paper was undertaken.

The report describes the various materials which have been used as nonmetallic diaphragms, discusses the factors which affect the performance of the diaphragms and gives the results of tests made for the purpose of investigating the effect produced by these factors. A theoretical discussion is given in which it is shown that the effective area of a nonmetallic diaphragm can be computed for specified conditions and hence the pressure-deflection curve can be predicted. Curves are given to facilitate the computation of effective areas under any given conditions. The theory was tested experimentally and was found accurate within about 5 per cent. Finally pressure-deflection curves are given to illustrate the control which the designer has over the shape of the curve by varying the different parts of the pressure element.

I. INTRODUCTION

The general use of nonmetallic diaphragms is a comparatively recent development. For many years metallic diaphragms appear to have been used exclusively, as a perusal of the first portion of Part I¹ of this series of reports will show. Nonmetallic diaphragms were first introduced in all probability in order to obtain a much more flexible pressure element than could be made using a metallic diaphragm of the same size.

Nonmetallic diaphragms can be used in two distinct types of pressure elements. In the first the elasticity of the diaphragm itself is used to resist the hydrostatic pressure applied, as in the case of a taut rubber diaphragm. In the second, a slack, pliable, but relatively inelastic diaphragm is used, its motion restrained by a suitable spring. In the latter case the elastic properties of the diaphragm would not be used at all, the diaphragm acting as a scale pan, so to speak, to transfer the pressure to the spring which would be made with excellent elastic properties, better than those of any metallic diaphragm. It might be thought that in this way the effect of the poor elastic properties of the diaphragm on the performance of the instrument could be made negligible, but this has not proved to be the case owing to other properties characteristic of these diaphragms.

¹ National Advisory Committee for Aeronautics Technical Report No. 165, 1923, "Diaphragms for Aeronautic Instruments," M. D. Hersey.

Nonmetallic diaphragms have been used in a number of aeronautic instruments, including air-speed indicators and recorders,² katanoscopes, oxygen apparatus,³ balloon manometers,⁴ ballonnet volume indicators, side-slip indicators, yaw meters,⁵ and turn indicators.⁶ They have also been used in a few laboratory manometers. Figure 1 shows a number of different types of nonmetallic diaphragm pressure elements.⁷ A taut rubber diaphragm is shown in Figure 1A, the instrument being a Smith air-speed indicator (British). It is a characteristic of a taut rubber diaphragm when used in this way that the deflection of the center of the diaphragm can be made approximately proportional to the square root of the differential pressure applied, provided the diaphragm is not too taut. (See fig. 14.) Owing to the fact that the differential pressure produced by an air-speed nozzle is proportional to the square of the speed, it is obvious that a diaphragm of this type will give very nearly a uniformly graduated scale under these circumstances with the use of an extremely simple multiplying mechanism.

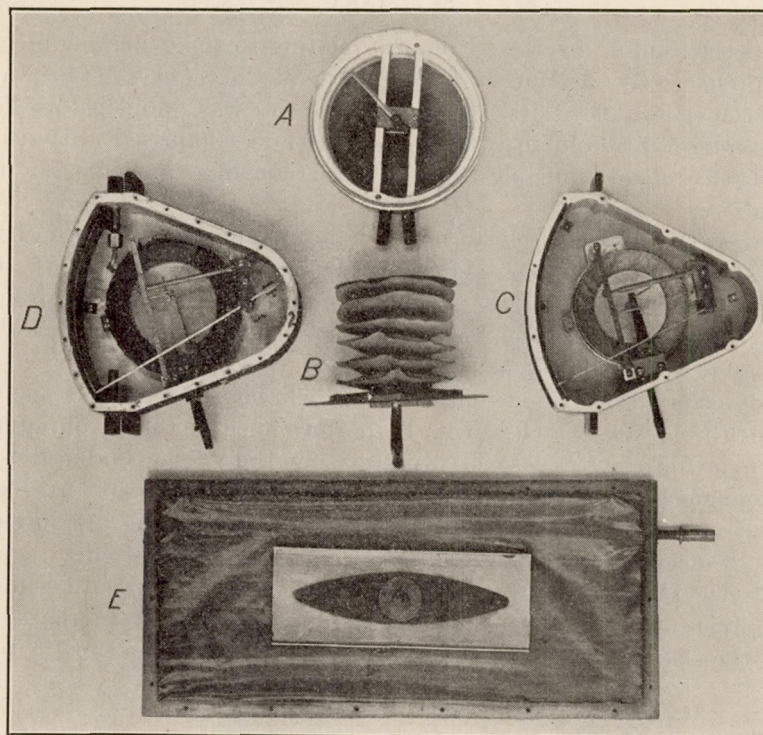


FIG. 1.—Types of nonmetallic diaphragm pressure elements

Figure 1, B, shows a nonmetallic bellows quite similar to those used in the Toussaint-Lepère air-speed recorder.² The bellows is built up of sections of rubberized fabric cemented alternately at the center and at the outer edge. A large deflection is obtained for a comparatively small differential pressure.

Figures 1, C, D, and E, are examples of slack diaphragm pressure elements. Figure 1, C, shows the slack zephir leather diaphragm used in the Pioneer balloon manometer. Here again the diaphragm deflects in opposition to a steel spring. Figure 1, D, shows a static pressure type ballonnet volume indicator constructed at the Bureau of Standards. Since the differential pressure producing full-scale deflection of this instrument is less than 1 centimeter of water, it was necessary to use an extremely flexible diaphragm material. Alum-tanned colon leather

² National Advisory Committee for Aeronautics Technical Report No. 127, "Aircraft Speed Instruments," by F. L. Hunt and H. O. Stearns, 1922.

³ National Advisory Committee for Aeronautics Technical Report No. 130, "Oxygen Instruments," by F. L. Hunt, 1922.

⁴ Technologic Paper No. 237, Bureau of Standards, "Aeronautic Instruments," by F. L. Hunt.

⁵ National Advisory Committee for Aeronautics Technical Report No. 112, "Control in Circling Flight," by F. H. Norton and E. T. Allen, 1921.

⁶ National Advisory Committee for Aeronautics Technical Report No. 128, "Direction Instruments," by W. S. Franklin and M. H. Stillman, 1922.

⁷ The term "pressure element" includes both the spring and the diaphragm.

was found to be ideal for this purpose since its flexibility was greater than that of any other material examined. Figure 1, E, shows the rather unusual element used in the Smith air-speed recorder. The diaphragm is made large in order to give a sufficiently great working force for the comparatively small differential pressure developed by a Pitot nozzle. The motion of the diaphragm is resisted by a flat steel spring. It should be observed that each of the last three instruments discussed has a large metal center on which is mounted the arm that transmits the force of the diaphragm to the spring. The size of the rigid center plays an important part in the design of a pressure element of this type, since the amount of force transmitted to the spring for a given pressure can be changed by altering the size of this center.⁸

II. MATERIALS USED FOR NONMETALLIC DIAPHRAGMS

The materials which have been used in the construction of nonmetallic diaphragms can be divided into four general classes:

1. Rubber.
2. Treated fabrics.
 - (a) Rubberized.
 - (b) Oiled.
 - (c) Doped.
3. Leathers and skins.
 - (a) Untreated.
 - (b) Doped.
 - (c) Oiled.
4. Paper.

In most instances when rubber has been used, its own elasticity has been utilized to furnish the resisting force. In the H \ddot{u} e automatic katanoscope, however, a rubber diaphragm is used with a rigid central disk and an opposing spring.

The other three classes of materials are always used in combination with a metal spring, and so it is advantageous to have them as pliable as possible. They should never be so used that the deflection is to an appreciable degree dependent on the stretching of the diaphragm, since under these circumstances the comparatively poor elastic properties of the diaphragm are brought into play.

III. NATURE AND PROPERTIES OF MATERIALS USED

RUBBER

The physical and chemical properties of rubber vary according to its source and preparation. Vulcanized rubber is very elastic and pliable at ordinary room temperatures, but these properties are temporarily destroyed when the rubber is subjected to the low temperatures which aeronautic instruments sometimes experience. The effect is transient, however, hence when the rubber returns to room temperature, its elasticity and pliability are completely restored.

Rubbers and rubberized fabrics to be suitable for use as diaphragms must be vulcanized; that is, a certain amount of sulphur must be combined with the rubber by some process so as to transform the original product into one which has much better properties. A complete description of these processes whereby rubber can be vulcanized will not be given here as descriptions are available elsewhere.⁹

Among the changes which are produced by vulcanization may be noted an increase in the strength and elasticity of the rubber and loss of adhesiveness. It becomes insoluble in ordinary rubber solvents and it is affected to a less degree by changes in temperature. The most serious defect of thin rubber sheets suitable for use as diaphragms lies in the rapid deterioration which takes place in the material, the elastic properties becoming markedly poorer within from six months to two years, depending on the method of preparation. It has been shown that relatively undervulcanized rubber compounds deteriorate less than those which are overvulcanized.

⁸ See discussion of "effective area," Section VII.

⁹ Bureau of Standards Circular No. 38, "The Testing of Rubber Goods."

FABRICS

*Untreated fabrics.*¹⁰—An elementary knowledge of the structure of the fibers comprising different fabrics will make much clearer the reasons for some of the effects exhibited by non-metallic diaphragms made of these materials. For example, wool fiber is built up of layers of animal cells of various forms which are capable of absorbing moisture. As a result the wool fiber is far more hygroscopic than any other fibers used and so is wholly unsuitable for use as a diaphragm material.

The silk fiber is apparently a continuous thread but probably is made up of a number of exceedingly minute elements. Hygroscopically it ranks next to wool, but in spite of this fact it has been used in diaphragms because it possesses other desirable properties.

The cotton fiber can be easily recognized from its ribbonlike unequally twisted appearance. It has the desirable property of being much less hygroscopic than either wool or silk.

Two forms of moisture are contained in the fibers: (1) That in chemical combination with the fibers, called water of hydration, more or less fixed in amount, and (2) that in the pores or cells of the fiber, variable in amount and depending on the changes in relative humidity of the air to which the fiber is exposed. Only this second form, the hygroscopic moisture, had to be taken into account in this investigation, since the water of hydration is not driven off until the fiber has been partially destroyed by exposure to high temperatures.

Rubberized fabrics.—A rubberized fabric consists of a fabric backing on which is spread or calendered one or more coats of thin rubber. In this way the flexibility of the fabric is retained to a high degree while the rubber adds the necessary impermeability. The rubber can be vulcanized by either the cold or the heat cure.

Rubberized fabric has been used extensively in the construction of bellows for the Toussaint-Lepère air-speed recorder. (See fig. 1, B.) An excellent grade of silk fabric is used, having a very thin coating of rubber calendered on one side. The resultant product possesses many desirable characteristics for use as nonmetallic diaphragms, but possesses the usual defect of rubber that it deteriorates after one or two years, rapidly becoming so stiff as to be useless.

The desirable qualities of this material have led to an attempt in this country to produce a rubberized fabric for the construction of instrument bellows, but the product thus far has not been equal in quality to the Toussaint-Lepère.

The behavior of rubberized fabric under varying conditions of temperature and humidity is more complex than the performance of either the fabric or the rubber when used alone, but the changes are comparatively small.

Oiled fabrics.—A number of oiled materials were tested during this investigation. It was found that both drying and nondrying oils served the purpose of filling the pores so as to remain in the material indefinitely, thus rendering the material relatively impermeable to gases and moisture. It has been found that the majority of the materials so treated are affected by changes in temperature to such an extent that they are nearly worthless as diaphragm materials for use in aeronautics.

*Doped fabrics.*¹¹—A number of special dopes were prepared through the courtesy of the Chemistry Division at the Bureau of Standards in hope that these might give the fabric the properties of being impervious to gases and of being unappreciably affected by changes of temperature and humidity. Some of the materials absorbed a large amount of the dope owing to their pronounced fibrous characteristics and hygroscopic properties and thus became stiff, heavy, and nonuniform, but certain other materials less fibrous and hygroscopic allowed the dope to form a smooth, thin, leatherlike finish on the surface, giving the material the desired properties.

*Leathers and skins.*¹²—Intestinal leathers and skins are the only materials belonging to this class which were found to have desirable characteristics for use as diaphragms. These leathers

¹⁰ Catalogue, Cramer System of Air Conditioning, 1909, Stuart W. Cramer, Howard Block, Providence, R. I. (or Courthouse Square, Charlotte, N. C., or Candler Building, Atlanta, Ga.).

¹¹ National Advisory Committee for Aeronautics Technical Report No. 38, "Airplane Dopes and Doping," by W. H. Smith, 1919.

¹² "Practical Tanning," by Allen Rogers, H. C. Baird & Co., New York City.

differ from kid skin and other similar leathers by the absence of large pores, thus rendering them comparatively impervious to gases. A few instruments have been constructed, using a treated sheepskin,¹³ but this material is too heavy and too porous for general use. The properties of the different intestinal leathers when used for diaphragms depend largely on the process of tanning. When using zephir leather or vegetable-tanned colon leather it has been found desirable to impregnate the material with oil or to coat it with some form of dope to render it more highly impervious to gases. On the other hand, the alum-tanned colon leather was highly impervious without any surface treatment. These intestinal leathers and skins are extremely thin and are very tough and strong. The leathers are distinguished from goldbeater's skin by their softness and pliability as compared with the relative stiffness of the goldbeater's skin.

Goldbeater's skin is wholly unsuitable for diaphragms, although it is highly impervious to gases. It expands considerably with increase in humidity and, as will be shown later in this report, this causes the stiffness of the pressure element to change appreciably.¹⁴

PAPER

Several specimens of parchment, Japanese papers, and the so-called nonporous papers¹⁵ were tested in this investigation but none were found to be of any value for use as diaphragms. Their lack of strength, their stiffness, and the high degree to which they were affected by changes in humidity all combined to cause their rejection.

IV. FACTORS INFLUENCING THE BEHAVIOR OF NONMETALLIC DIAPHRAGMS

Nonmetallic diaphragms possess characteristics which are peculiar to this type of diaphragm, as well as a number of characteristics which are also found in metallic diaphragms. It will probably be best to enumerate at this point the most important of the factors which affect their behavior. These may be classified as follows:

1. Seasoning.
2. Ageing.
3. Humidity.
4. Temperature.
 - (a) Transient effects.
 - (b) Permanent effects.
5. Stretching.
6. Permeability.
7. Indeterminate factors.

(1) and (2) *Seasoning and ageing*.¹⁶—The terms "seasoning" and "ageing" have been used interchangeably in connection with metallic diaphragms in the first report of this series¹ under "Irreversible effects." There a "seasoned" diaphragm is defined as "one which repeats its mechanical performance, including any irreversible effects¹⁷ * * * on successive occasions, separated by a sufficient interval of rest, provided of course, that the diaphragm is subjected to identical conditions on each occasion." The criterion for a seasoned diaphragm has been discussed in detail in the above-mentioned report to which the reader is referred for a more complete discussion. The subject is brought up here in order to explain the meaning of the terms "seasoning" and "ageing" as applied to nonmetallic diaphragms in the present report.

Here the term "seasoned" will be applied to any diaphragm which, when subjected to identical conditions on successive occasions, separated by a sufficient interval of rest, yields

¹ National Advisory Committee for Aeronautics Technical Report No. 165. "Diaphragms for Aeronautic Instruments," M. A. Hersey, 1923.

¹³ Hollocoombe-Clift air-speed indicator.

¹⁴ See Alum Tanned Colon Leather untreated—Flexibility of material.

¹⁵ Bureau of Standards Circular No. 107, "Testing of Paper."

¹⁶ National Advisory Committee for Aeronautics Technical Report No. 39, "The Testing of Balloon Fabrics," by J. D. Edwards and I. L. Moore, 1919.

¹⁷ Namely, load-deflection curve, drift, hysteresis, after-effect, etc. (the last three are irreversible effects).

the same pressure-deflection curve. This definition differs from the more precise definition given in the above-mentioned publication in that the irreversible effects are ignored.

A metallic diaphragm, when once seasoned, retains almost indefinitely the properties which characterize its seasoned state. This is not true, in general, of nonmetallic diaphragms. It will be found, for example, that most rubber diaphragms gradually grow more flexible in the course of six months or a year and then gradually become stiffer and stiffer until they become useless. In a similar way, rubberized fabrics grow appreciably stiffer in the course of several years. This gradual deterioration of the diaphragm over a long period of time will be called "ageing" in this paper.

(3) *Humidity*.¹⁰—All of the nonmetallic diaphragms tested in this investigation were affected to a greater or lesser degree by changes in humidity. Several materials were eliminated from further investigation because extreme changes in humidity altered by as much as 20 per cent. the pressure-deflection curves of pressure elements in which diaphragms of these materials were used. In all probability this effect is due to two causes, the first and more important of which is the expansion of the material¹⁸ with increase of humidity. This changes the effective area¹⁹ of the diaphragm and so alters the pressure-deflection curve. The second effect, which is undoubtedly very small, is the change in pliability of the diaphragm as the humidity changes.

The effect of humidity on nonmetallic diaphragms is even more important than the effect of temperature. It has led in some instances to the treatment of the surface of the material in an attempt to decrease the effect. These treatments, however, usually increase the temperature effect so greatly that nothing is gained.

(4) *Temperature*.—Two classes of temperature effects are found to exist in nonmetallic materials. In the first place a recoverable²⁰ change takes place in most materials whenever they are heated or cooled and are then brought back to their original temperature. This change in a nonmetallic diaphragm pressure element is comparable to the change in stiffness which takes place in a metallic diaphragm pressure element when its temperature is altered.²¹ The second effect is due to a permanent change which takes place in the material when it is heated or cooled sufficiently. Obviously a material which undergoes permanent changes when subjected to temperature variations such as may be experienced in use is totally unsuited for a diaphragm. Each material investigated was tested over a wide range of temperatures for permanent changes. Low temperatures affected permanently but few of the materials examined; the difficulties arose at high temperatures, from +40° to +60° centigrade.

Even though a material may prove to be free from permanent temperature effects over the desired range of temperatures, it may still be sufficiently subject to transient effects to be practically useless. For example, a number of materials, particularly when possessing a surface treatment, stiffen so greatly at temperatures of -10° to -20° centigrade as to be worthless for use in aeronautic instruments. The effect of temperature as well as the effect of humidity on nonmetallic diaphragms is probably due both to a change in the effective area and to a change in the pliability of the material itself. However, while the first of these is undoubtedly the important factor where humidity is concerned, sometimes one and sometimes the other of these two factors has the greater influence where changes in temperature are involved.

It was extremely difficult to investigate the effect of temperature alone on nonmetallic materials, since it was not practicable to control the atmospheric humidity during the tests except in a general way. This difficulty is due to the facts that it is the *relative* humidity which controls the moisture content of the material and that this effect is large. If the relative humidity could have been kept constant in the testing chamber at different temperatures, the difficulty

¹⁰ Catalogue, Cramer System of Air Conditioning, 1909, Stuart W. Cramer, Howard Block, Providence, R. I. (or Courthouse Square, Charlotte, N. C., or Candler Building, Atlanta, Ga.).

¹⁸ National Advisory Committee for Aeronautics Technical Report No. 36, "The Structure of Airplane Fabrics," by E. Dean Walen, 1919.

¹⁹ See Section VII of this paper.

²⁰ Namely, one which disappears when the diaphragm is allowed to remain under normal conditions for a long enough period.

²¹ The effects are comparable but the causes are quite different. In metallic diaphragm pressure elements the change is largely due to the effect of temperature on the elastic moduli of the material, while in nonmetallic diaphragm pressure elements it is mainly due to expansion or contraction to the material with the resultant change in the effective area of the diaphragm.

would have been eliminated, but no practicable method of doing this was devised during the investigation.

Instruments with nonmetallic diaphragms are, with few exceptions, unsuited for use under conditions where the instrument temperature will fall below -20° centigrade. It is obviously important to know in designing the diaphragm the range of temperature to which the instrument will be subjected in use.

(5) *Stretching.*²²—The comparatively low elastic moduli of the nonmetallic materials under discussion allow more or less stretching of the diaphragms to take place. If the stretch which occurs is perfectly elastic, the principal effect is to change the effective area of the diaphragm for any given deflection from what it would be if no stretching took place and thus to affect somewhat the shape of the pressure-deflection curve. However, if a certain amount of permanent set occurs each time the diaphragm is stretched, a progressive change takes place in the pressure-deflection curve of the pressure element. This permanent set often occurs, but fortunately the successive increments become smaller and smaller if the diaphragm material is properly chosen for the loads it is to carry, and so the change constitutes part of the seasoning process of the diaphragm.

As might be expected, the various materials undergo this stretching to a different degree for the same pressure, and each material is affected more and more with increasing pressure. The magnitude of the effect is, of course, dependent on the stresses existing in the diaphragm and so is affected by the slackness²³ of the diaphragm as well as the applied pressure. These stresses could be computed, at least approximately, for nonmetallic diaphragms, but no attempt has been made to do this in any of the instruments using nonmetallic diaphragm pressure elements designed at the Bureau of Standards. Each problem has been investigated individually and a diaphragm material chosen which experiment showed would give satisfactory performance under the stresses involved. Alum-tanned colon leather stretches the most of any of the materials which were tested in this investigation, but it has been found possible to season this leather and thus use it in many instances.

(6) *Permeability.*—Most nonmetallic diaphragms possess one defect which is not characteristic of metallic diaphragms and which often causes the rejection of a particular material. This defect is the permeability of the material; that is, its failure to prevent the passage of a gas. Under ordinary conditions, this is not a serious matter, provided the material is not so permeable as to produce a sufficient flow of air in the connecting tubes to cause an appreciable friction loss. When there are different gases on opposite sides of the diaphragm, however, the permeability of the material may be important. A good illustration of this may be found in the static pressure type ballonet volume indicator shown in Figure 1D. The case of this instrument is connected to the gas bag of the dirigible and contains hydrogen or helium, while the diaphragm chamber is connected to a ballonet and so is filled with air. Now when enough air leaks through the diaphragm to back up into the tube connecting the case to the gas bag, the differential pressure acting on the diaphragm will be altered, causing an error in the reading. Furthermore, if hydrogen is the inflating gas, an explosive mixture may be formed in the instrument. In this volume indicator provision has been made for flushing out the case and diaphragm chamber easily when desired, thus removing any mixture of gases which may have been formed.

As might be expected, the diaphragm materials were more highly permeable when subjected to high pressure than when subjected to low pressure. This is undoubtedly due to the material stretching and increasing the size of the pores. The permeability, therefore, is not a function of the applied pressure alone, but depends also on the amount of stretching, which in turn depends upon the stresses set up in the material. In other words, the size and slackness of the diaphragm must be considered as well as the applied pressure.

A test for permeability is the first which should be applied to a material which is under consideration for use as a nonmetallic diaphragm, for if this test gives unfavorable results, the

²² National Advisory Committee for Aeronautics Technical Report No. 16, "The Stretching of the Fabric and the Deformation of the Envelope in Nonrigid Balloons," by Rudolph Haas and Alexander Dietzius, 1917.

²³ See Sec. VII. Computation of effective area. Case 1.

material can usually be excluded from further investigation. Occasionally a highly permeable material may possess such excellent characteristics in other respects that it may be worth while to apply a surface treatment to close the pores, but this is not in general the case.

(7) *Indeterminate factors.*—Even when all the external conditions of the test are identical as far as can be determined the diaphragm may give slightly different results. The difficulty of duplicating the conditions exactly makes it practically impossible to study this point, but there is one obvious reason why the performance should be slightly erratic. In mounting a slack diaphragm, a number of wrinkles are usually produced, particularly at the edge. These wrinkles change from time to time and the resistance which they offer to deflection varies correspondingly. The slacker the diaphragm, the more trouble is caused from this effect. Consequently for diaphragms which are to be very slack, a thin, very flexible material is desirable. Dry goldbeater's skin is unsatisfactory in this respect, while alum-tanned colon leather gives excellent results.

V. METHOD OF TESTING MATERIALS AND DIAPHRAGMS

PERMEABILITY TESTS OF MATERIALS

The permeability of the materials used was determined by fastening a piece of the material between two metal plates in which had been drilled a hole 1 inch in diameter and subjecting it on one side to the air pressure existing in a closed system of known volume. A liquid manometer connected to the system enabled the observer to measure the rate of fall of pressure during tests and thus to measure the rate of flow of air through the material. The temperature was maintained constant within 2 degrees centigrade throughout the tests.

TEMPERATURE, HUMIDITY, AND SEASONING TESTS OF DIAPHRAGMS

In conducting tests to determine the effect of temperature and humidity upon the materials and to investigate seasoning, each material was tested in turn as a slack diaphragm in a Smith

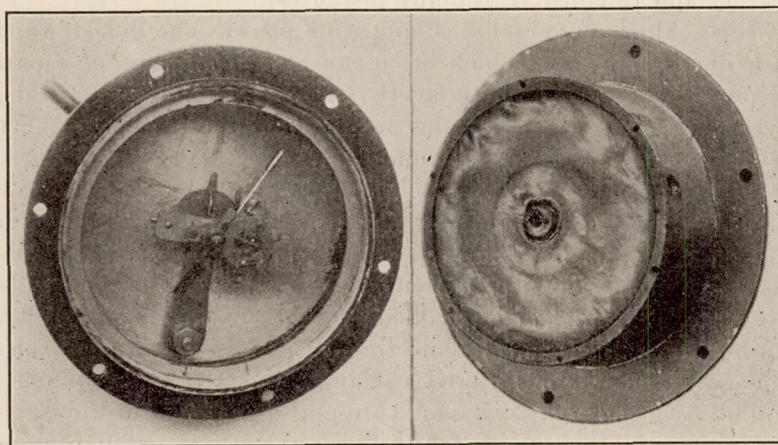


FIG. 2.—Manometer used for testing nonmetallic diaphragms

balloon manometer. (See fig. 2.) This type of instrument was chosen because the construction was such as to facilitate changing the diaphragms and because the mechanism was very satisfactory, exhibiting little friction and exceedingly small temperature errors. Before tests of the diaphragms were started, the effect on the mechanism of the instrument of temperature changes was investigated. This was done by mounting a micrometer head in place of the diaphragm and measuring over a wide range of temperatures the deflections of the spring (and therefore of the diaphragm deflections) corresponding to the different scale readings. The effect of temperature on the micrometer head was known and the necessary correction was applied. Tests at room temperature, at $+50^{\circ}\text{C}$. and -20°C . indicated the temperature effect

on the mechanism to be so small that it could be neglected in testing the diaphragms. From -20°C . to $+50^{\circ}\text{C}$., this effect averaged only 0.15 per cent for each 10-degree change from $+20^{\circ}\text{C}$.

The instrument was then used to determine the effects on the nonmetallic materials under investigation of seasoning, changes of temperature and changes of humidity assuming the performance of the mechanism to be constant under all conditions.

In testing the diaphragms it was considered desirable to have them sufficiently slack, so that when deflected enough to produce a full-scale reading, the material would not be put in direct tension. The deflection producing full-scale reading was found to be 1.85 millimeters. A minimum slackness of 1.003²⁴ was found sufficient to give this deflection without subjecting the diaphragm to direct tension.

An inclined U tube of large bore filled with benzol and calibrated by means of a vertical water column was used to measure the pressure required to deflect the diaphragm.

When the diaphragm had been mounted in the instrument it was rested for several hours, after which room-temperature tests were run. It was found in general that consecutive calibrations under the same conditions of temperature and humidity, as nearly as could be obtained, would not agree unless at least 20 minutes elapsed between the tests. Twenty-five deflections corresponding to full-scale reading were administered in some instances and several room-temperature tests were made, immediately after which the diaphragm appeared to be more flexible than at first, but after an hour's rest the further room-temperature tests agreed approximately with the original tests.

Difficulty was experienced in determining the effect on the diaphragm of temperature and humidity separately. With the object of obtaining the diaphragms in the driest possible condition, tubes containing calcium chloride were inserted on both sides of the diaphragm in the apparatus. The air in the system was forced back and forth on both sides of the diaphragm for several hours each day to facilitate the absorption of the moisture by the calcium chloride. It was assumed that this process of eliminating the moisture was practically complete although the humidity of the air in the system was not measured. Evidence as to this point will be given later in the paper under the detailed results for various materials.

In general, most of the pressure elements become stiffer upon the removal of moisture from the diaphragms, and since the rate at which the moisture was removed differed for each material and since no measure of the humidity was feasible, four or five days were allowed for drying, during which time several room-temperature tests were made each day in order to note the stiffening. When the process of stiffening appeared to be complete, the diaphragms were considered ready for temperature tests.

Following a period of 24 hours' rest the pressure elements were calibrated at approximately $+20^{\circ}$, $+10^{\circ}$, 0° , -10° , and -20°C . They were then allowed to return to room temperature, and after a further rest of 24 hours had been given them, room-temperature tests were made to see if the low temperatures had caused any permanent changes in the diaphragms. After another 24-hour period of rest the temperature was gradually increased from approximately $+20^{\circ}$, to $+30^{\circ}$, $+40^{\circ}$, and $+50^{\circ}\text{C}$., a calibration being made at each temperature. After the pressure element had been brought back to room temperature and had rested for 24 hours, a further series of room-temperature tests extending over several days was made. The drying tubes which were connected to the instrument throughout all the temperature tests were then removed and air under existing atmospheric conditions was allowed to come in contact with the diaphragm. After several days, room-temperature calibrations were run to determine how closely the diaphragms would agree with the original calibration, although owing to the usual change in humidity of the atmosphere since the beginning of the tests, it was not anticipated that exact agreement would be found.

²⁴ See Sec. VII. Computation of effective area. Case 1.

VI. TESTS OF NONMETALLIC MATERIALS AND DIAPHRAGMS

RESULTS OF PERMEABILITY TESTS

With a few exceptions the materials investigated proved to be sufficiently impermeable to air for general use. Tests were made up to pressures of 10 pounds per square inch. The following materials were found to be practically impermeable under this pressure:

1. Rubber.
2. Treated fabrics.
 - (a) Toussaint-Lepère rubberized silk.
 - (b) Goodyear rubberized silk.
 - (c) Balloon fabrics.
 - (d) Cotton treated with tung oil.
 - (e) Silk treated with tung oil.
 - (f) Airplane fabric.
 - (g) Oiled silk.
3. Leather and skins.
 - (a) Untreated.
 1. Alum-tanned colon leather.
 2. Goldbeater's skin.
 - (b) Treated.
 1. Zephir leather.
 2. Bung-gut leather.
 3. Vegetable-tanned colon leather.
 4. Kid skin.

The following materials were found to be so highly permeable as to be unsuited for use as nonmetallic diaphragms, the one possible exception to this statement being zephir leather. In spite of its fairly high permeability, this material appeared to be worthy of further investigation owing to other desirable characteristics.

1. Untreated fabrics.
 - (a) Silk.
 - (b) Cotton.
2. Untreated leathers and skins.
 - (a) Zephir leather.
 - (b) Bung-gut leather.
 - (c) Vegetable-tanned colon leather.
 - (d) Kid skin.

Since, as has been shown, the permeability of the material varies not only with pressure, but also with the size and the slackness of the diaphragm, the above results are not definitely conclusive. However, since the maximum pressure used in the tests was many times greater than any which would probably be used in practice, it will be found that these results are applicable to nearly any practical case which may arise.

RESULTS OF TESTS FOR EFFECTS OF TEMPERATURE, HUMIDITY, ETC.

A large number of materials²⁵ were tested in the Smith manometer, as already described, to determine their performance as slack diaphragms. It is not worth while to discuss in detail the results obtained for all of these materials. Instead, detailed results will be given for the most suitable materials tested, and in addition for several materials which, while not suitable for use, exhibited to a marked degree certain characteristics which many of the other materials showed to a slighter extent. A study of these outstanding characteristics sheds light on the performance of nonmetallic diaphragms. The results for all the materials tested are summarized in Table I.

²⁵ See Table I for the complete list.

ALUM-TANNED COLON LEATHER UNTREATED

Description.—Alum-tanned colon leather (fig. 3) when received from the manufacturer was in the form of creamy-white translucent skins, averaging about 24 inches in length, 10 inches in width, and 0.001 inch in thickness. It was very soft, pliable, smooth, limp, and light in weight, yet strong, tough, and uniform. The surface evidently had been powdered with a white talc, the excess of which was easily removed with a cotton duster.

*Slackness.*²⁶—After mounting a piece of the above-described material in a Smith manometer, the slackness was determined as 1.003, but after all tests had been completed the slackness was found to have increased to 1.005.

Seasoning.—The change in slackness in this instance constitutes, at least to a large extent, the seasoning process, and this effect can be recognized easily in Figure 3 by the gradual increase in flexibility of the diaphragm throughout the series of tests. This change in slackness produces a slight change in the load-deflection curve of the pressure element,²⁷ but is so small in this instance that it can be neglected for practical purposes. It is preferable, however, to season the diaphragm before it is used.

Repetition.—This material repeated its performance well both on the same and on different days, regardless of variations in weather conditions. This is probably due to the filling of

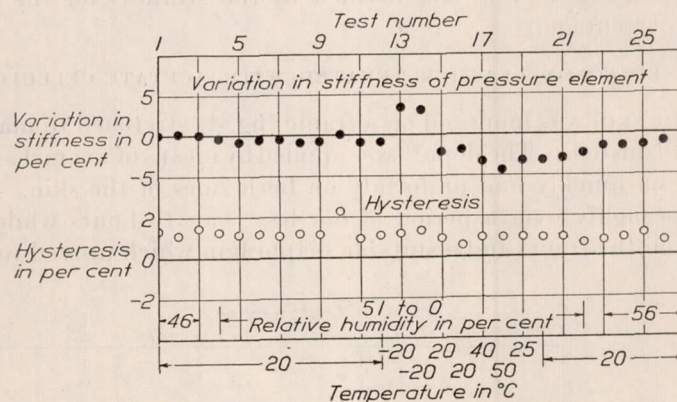


FIG. 3.—Untreated alum-tanned colon leather

the minute pores by the tanning process. Goldbeater's skin, the results for which are shown in Figure 8, is a good example of the performance of this same material²⁸ when the pores are not filled.

Temperature.—An increase in stiffness of about 3.3 per cent. at the lowest temperatures will be observed from Figure 3, while at high temperatures the maximum increase in flexibility was approximately 4.5 per cent. This maximum increase in flexibility was not caused entirely by increased temperature, but was aided by the seasoning produced during the series of tests, so bearing this in mind it can be seen that the effect of low temperature was practically the same numerically as that due to high temperature. No permanent effect due to high or low temperature was found.

*Hysteresis.*²⁹—The average hysteresis exhibited by this material was small, being about 1 per cent. of the maximum reading. The hysteresis was practically unaffected by changes in temperature.

Flexibility of material.—This leather was the most elastic and pliable material tested during the investigation.

The results of the tests of this material are shown in somewhat more detail in Figure 3. The data for each test are given on the vertical line through the number representing that test.

²⁶ See Sec. VII. Computation of effective area. Case 1.

²⁷ See Section VII of this report.

²⁸ Goldbeater's skin and the colon leather are made from the outer coat of the cæcum of the ox, the first being dried and the second tanned.

²⁹ Hysteresis is the excess of pressure required to produce a given instrument reading, with pressure increasing, over that required to produce the same reading with pressure decreasing.

Directly above the number of the test is given the temperature of the pressure element for that test. Above the temperature is indicated the approximate relative humidity existing in the testing system. Where the relative humidity is given over a number of tests as "51 to 0," as in Figure 3, for example, this is to be understood as meaning that the relative humidity was decreasing throughout the tests, tending toward zero, although, of course, it never reached this value. Above the values of the humidity are given the values of the hysteresis expressed as percentage of the total range. These are obtained by averaging the values of the hysteresis for all of the test points, dividing this result by the pressure required to produce full-scale deflection of the pointer, and expressing the result as a percentage.

Above the values of the hysteresis is given the variation in stiffness of the pressure element. The stiffness of the pressure element is defined as the ratio of the hydrostatic force acting on the diaphragm to the deflection of the spring which restrains the motion of the diaphragm; i. e.,

$$\text{Stiffness of pressure element}^{30} = \frac{Ap}{y_s}$$

An average value of the stiffness of the pressure element for a given test was obtained by computing the stiffness for each scale graduation at which readings were taken and averaging these results. The value plotted for each test is the difference between the average stiffness for the test in question and that for the first test divided by the stiffness for the first test, the result being expressed as a percentage.

ALUM-TANNED COLON LEATHER TREATED WITH ACETATE-CELLULOSE DOPE

Description.—The skin was mounted on a frame (fig. 4) stretched to make it as smooth and uniform as possible and dusted. The dope³¹ was applied by means of a camel's-hair brush. A thin coating was brushed on quickly and uniformly on both sides of the skin. The skin was then permitted to dry thoroughly over a period of not less than 10 hours while on the frame. It was then removed from the frame and a suitable diaphragm which proved to be 0.002 inch thick was cut out.

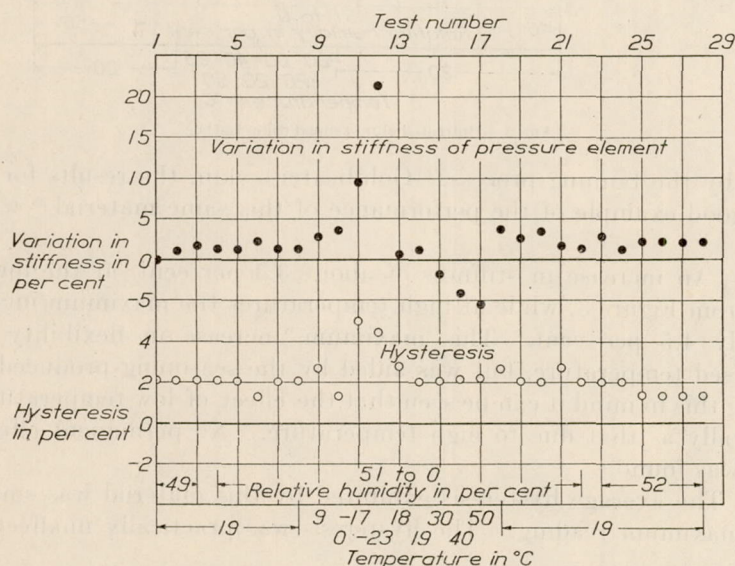


FIG. 4.—Alum-tanned colon leather treated with acetate-cellulose dope

Slackness.—The slackness of this diaphragm was not measured until all tests had been completed, when it was found to be 1.005.

Seasoning.—A small amount of seasoning appears to have taken place during the first two tests of the diaphragm. Figure 4 shows that the diaphragm stiffened slightly, the average change

³⁰ Compare with definition of stiffness of spring given in Section IX.

³¹ This dope consisted of 1.5 ounces of soluble gun cotton dissolved in two-thirds of a pint of amyl acetate and 1 ounce of Canada balsam mixed with 1 fluid ounce of castor oil dissolved in one-third pint of grain alcohol. The advice and cooperation of members of the Chemistry Division of the Bureau of Standards in connection with the doping of the materials are acknowledged.

for the first three tests being between 1.5 and 2 per cent. At the end of all the tests, the diaphragm appeared to be in a stable condition, repeating its performance and showing a stiffening of 2 per cent. from its original condition.

Repetition.—This material repeated its performance well, both on the same and on different days regardless of variations in weather conditions.

Temperature.—An increase in stiffness of about 21 per cent. took place at the lowest temperature while at the highest temperature the increase in flexibility amounted to about 5.5 per cent. These large temperature effects were mainly due to the surface treatment. No permanent effect due to high or low temperatures was indicated.

Hysteresis.—The average hysteresis exhibited by this diaphragm at room temperature was about 2.5 per cent. of the maximum pressure. The hysteresis increased to about 5 per cent., however, at temperatures of -17° centigrade and below. Obviously the surface treatment affects the hysteresis exhibited by the diaphragm, increasing it over the value obtained for the untreated colon leather, and causing it to be affected by low temperatures.

Flexibility of material.—This material was found to be slightly stiffer than the plain alum-tanned colon leather owing to the surface treatment.

ALUM-TANNED COLON LEATHER TREATED WITH COLLODION DOPE

Thickness.—This material averaged 0.003 of an inch in thickness.³² (See fig. 5.)

Slackness.—After this material had been prepared and mounted in a Smith manometer, its slackness was found to be 1.007.

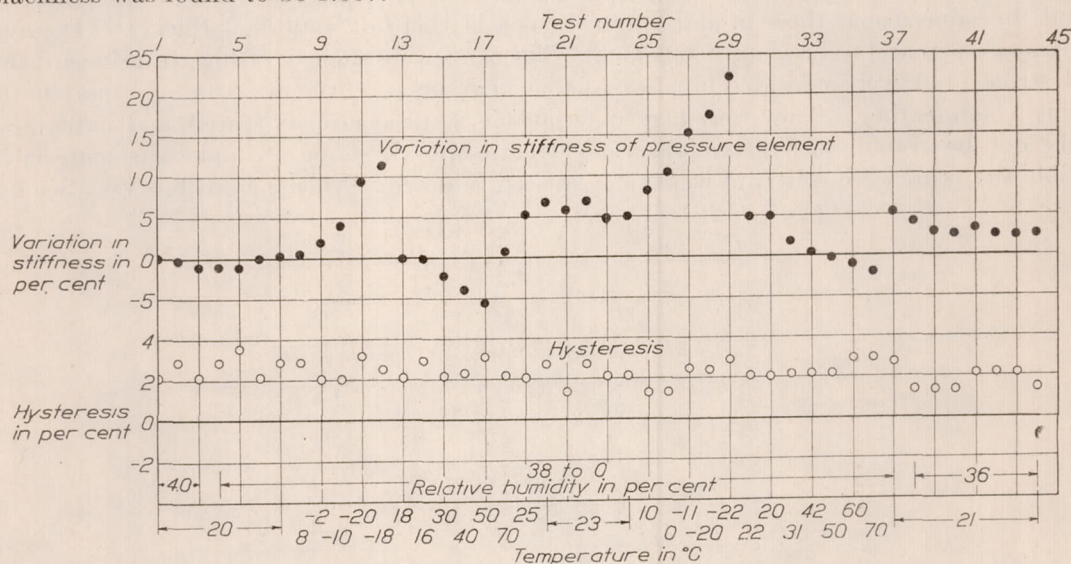


FIG. 5.—Alum-tanned colon leather treated with collodion dope

Seasoning.—Little, if any, effect of seasoning is shown in Figure 5.

Repetition.—The diaphragm repeated its performance very well both on same and on different days, regardless of weather conditions.

Temperature.—An increase in stiffness of about 11 per cent. at the lowest temperatures will be observed from the first group of tests on the plot while at the highest temperature the increase in flexibility amounted to about 6 per cent., these large temperature effects being accounted for by the surface treatment.

It was intended to stop the temperature tests here, but the instrument was accidentally allowed to remain at a temperature which increased from $+50^{\circ}$ C. to $+70^{\circ}$ C. over a period of 22 hours. At the end of this time a test was made at a temperature of $+70^{\circ}$ C. and the results showed considerable stiffening, as can be seen from Figure 5. It is believed that this stiffening

³² The dope consisted of a solution of 8 parts collodion, 1 part castor oil, and 1 part caoutchouc solution.

at increased temperatures was probably due to further drying of the diaphragm, since at the end of the series of tests, the diaphragm having been allowed to adjust itself to normal atmospheric humidity, there remained a residual stiffening of only 2 per cent. above the original stiffness of the element. This small value would seem to indicate that the high temperatures produced no appreciable permanent effect. Another series of tests was made at low and high temperatures. On this second series of tests, the diaphragm stiffened by approximately the same amount as it did for the same temperature decrease on the first series. On heating, however, the diaphragm continued to become more flexible up to the highest temperature reached, $+70^{\circ}\text{C}$. The fact that, on the second series of tests, the diaphragm continued to increase in flexibility up to $+70^{\circ}\text{C}$., while on the first series it grew stiffer from $+50^{\circ}\text{C}$. to $+70^{\circ}\text{C}$., is believed due to the time factor, the time for which the high temperature was maintained being 1 hour for the second series as against 22 hours for the first, the diaphragm thus having time to dry out much more during the longer time interval.

Hysteresis.—The average hysteresis exhibited by this material was small and fairly constant throughout these tests, being about 2.3 per cent. of the maximum-pressure range. No correlation with temperature or time was indicated.

Flexibility of material.—This material was found to be slightly stiffer than plain alum-tanned colon leather, owing to the surface treatment.

ZEPHIR LEATHER TREATED WITH ACETATE-CELLULOSE DOPE

Description.—Zephyr leather when received from the manufacturers was in the form of skins about the same size as those of alum-tanned colon leather but somewhat thicker. The average thickness was found to be about 0.004 inch. The skins were slightly yellow, translucent, smooth and strong, but were considerably less pliable and less uniform in structure than the alum-tanned colon leather. They were highly permeable to air as already stated, and on this account could not be tested until given a surface treatment. In order to make the material more flexible the skins were split and were then treated to decrease their permeability. (See fig. 6.)

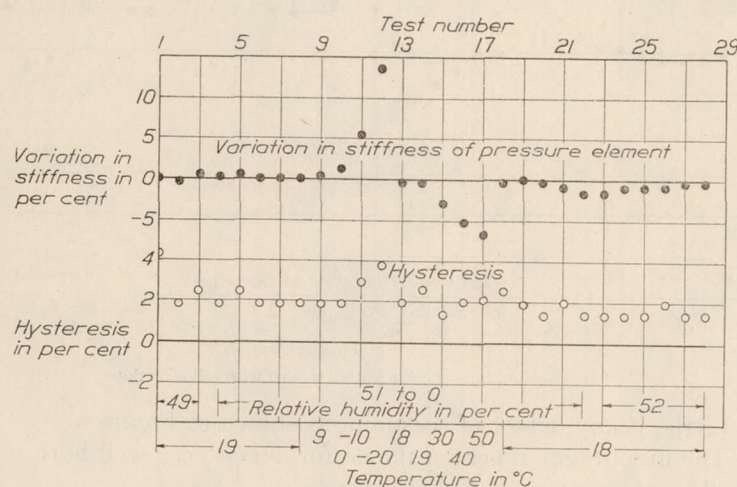


FIG. 6.—Zephyr leather treated with acetate-cellulose dope

Thickness.—This material measured 0.0025 inch.

Slackness.—The slackness of the diaphragm was not measured until the completion of the tests when it was found to be 1.005.

Seasoning.—No appreciable effect of seasoning can be detected from the results given in Figure 6.

Repetition.—This diaphragm repeated its performance very well regardless of weather conditions.

Temperature.—A temperature of -20°C . caused a stiffening of 13.5 per cent., while a temperature of $+50^{\circ}\text{C}$. caused an increase in flexibility of 7 per cent. No permanent effect

due to temperature changes could be detected, but the final room-temperature tests showed an increase in flexibility of about 0.5 per cent. over the original room-temperature tests. This effect may be due to seasoning or to a change in humidity. The effect appears to be permanent but is very small.

Hysteresis.—The average hysteresis exhibited by this material was small, about 2 per cent. of the maximum-pressure range. The hysteresis increased to 3.8 per cent. at the lowest temperature.

Flexibility of material.—The double thickness zephir leather was considerably stiffer than alum-tanned colon leather, but after it had been split and treated it was found to be much more flexible, although not equal to alum-tanned colon leather in this respect.

ZEPHIR LEATHER TREATED WITH SULPHONATED OILS

A skin of double thickness zephir leather was split, and was treated with oil³³ by the method already described for doping the skins (fig. 7). It was allowed to dry for five hours and

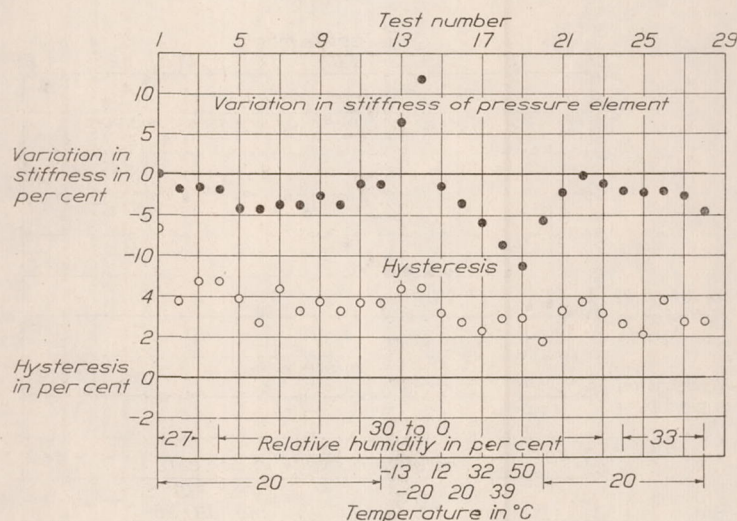


FIG. 7.—Oiled zephir leather

then a diaphragm was cut and mounted in a Smith manometer. The material was found to be 0.0025 inch in thickness.

Slackness.—The slackness as determined before any tests were conducted was found to be 1.003.

Seasoning.—The decrease in stiffness from test No. 4 to test No. 5 is probably a seasoning effect. Test No. 5 was made 30 minutes after 25 full-scale deflections had been given. It is believed that this was a permanent change. The gradual increase in stiffness with further tests is accounted for by the drying out of the material.

Repetition.—The repetition of performance was slightly more variable than that shown by the materials already discussed. The oil appeared to be comparatively sensitive to slight changes in humidity and temperature.

Temperature.—At the lowest temperature an increase in stiffness of 13 per cent. over that at room temperature was exhibited. A portion of this appeared to be permanent, for after resuming room temperature, the diaphragm appeared more flexible than before the low-temperature tests were made. This increased flexibility was undoubtedly caused by the absorption by the diaphragm of condensed moisture resulting from the change from low to room temperature and not by a permanent temperature effect.

An increase in flexibility of 8 per cent. from test No. 16 at +20° C. to test No. 19 at +50° C. is indicated in Figure 7. As the drying process was continued, the diaphragm continued to

³³ The composition of this mixture of oils was not definitely determined, but in all probability it was made up of cod and neat's foot oils.

stiffen, the effect shown in tests 20, 21, and 22 supporting the view that the diaphragm had absorbed condensed moisture in warming up after the low-temperature tests. In the final tests the performance agreed fairly well with that of the first room-temperature tests after preliminary seasoning had occurred. No permanent effects due to temperature changes could be detected. Tests showed, however, that from 48 to 72 hours was required for changes produced by abnormal temperature and humidity conditions to die out.

Hysteresis.—The average hysteresis exhibited by this material was slightly greater than shown by the materials previously discussed, being about 3.5 per cent. of the maximum-pressure range. The hysteresis was erratic throughout the series of tests and in addition increased noticeably at low temperatures and decreased at high temperatures.

Flexibility of material.—This material was not as flexible as alum-tanned colon leather.

GOLDBEATER'S SKIN

Slackness.—This diaphragm of ordinary goldbeater's skin (fig. 8) 0.0009 inch thick was mounted in the Smith manometer and found to have a slackness 1.003 before tests were made.

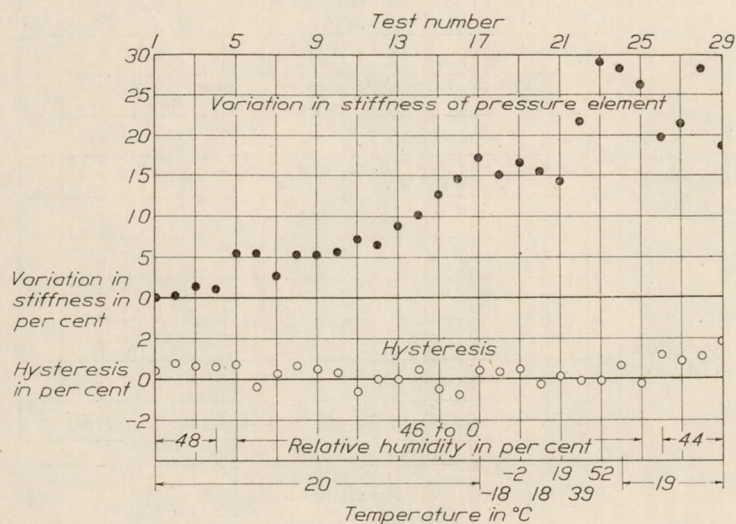


FIG. 8.—Untreated goldbeater's skin

Seasoning.—No effect of seasoning could be detected definitely because of other factors having a greater influence on the behavior of the material.

Repetition.—This material did not repeat its performance either on the same or on different days. This may be accounted for by its extreme sensitivity to changes in humidity. The difference in stiffness between the first and the final tests would probably have been greatly reduced if the diaphragm had been allowed to adjust itself to ordinary atmospheric humidity for a longer period of time and had then been tested.

Temperature.—This material illustrates the difficulties experienced in attempting to distinguish the effects due to temperature from those due to humidity. From Figure 8 it will be seen that as the drying process proceeded (after test No. 5) the diaphragm gradually became stiffer, the change being more or less erratic but in the same general direction. When the temperature was decreased the only apparent effect was that this progressive change practically ceased, and when the temperature was increased the progressive change continued much more rapidly than at room temperature. Now this effect may have been due to the fact that the relative humidity was greater at the low temperatures and less at the high temperatures than it would have been at room temperature, or it may have been due to temperature alone. Either explanation is plausible. It is probable, however, that change in humidity is the controlling factor.

Hysteresis.—The hysteresis exhibited by this material was very small. In some tests it was negative and others positive. The average hysteresis was about +0.25 per cent. of the

maximum pressure range. No entirely satisfactory theory has yet been advanced to explain this negative hysteresis, although a fairly reasonable explanation of negative drift in non-metallic diaphragm pressure elements is given later in this paper.

Flexibility of material.—This material was much stiffer than alum-tanned colon leather.

The fact that goldbeater's skin is so sensitive to changes in humidity and temperature makes it worthless for use as nonmetallic diaphragms.

COTTON TREATED WITH TUNG OIL

Description.—This material (fig. 9) when received was already prepared. The exact treatment is not known. It was found upon examination that the cotton cloth had a thread count of 130 per inch in one direction and 140 per inch at right angles. It contained approximately 3.4 ounces of oil per square yard and appeared as a dirty green glistening fabric with a fairly rough surface. It measured 0.007 inch in thickness.

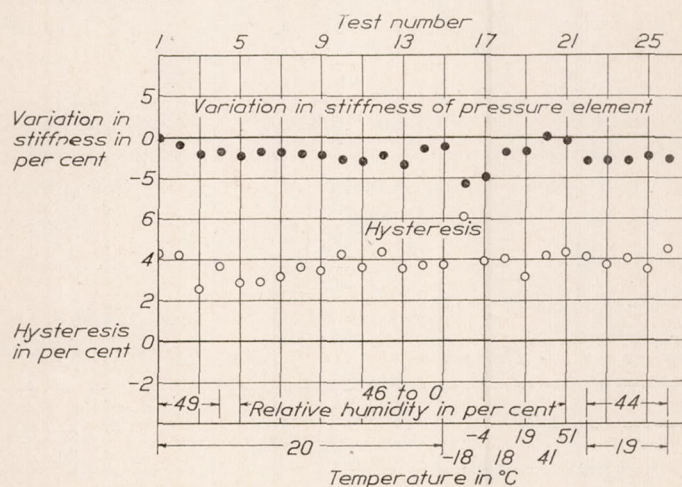


FIG. 9.—Cotton treated with tung oil

Slackness.—After mounting a diaphragm of the above-described material in a Smith manometer the slackness was determined and found to be 1.008. No measurement was made after the tests were completed.

Seasoning.—This material showed a small progressive increase in flexibility throughout the series of tests as can be seen from Figure 9. This change appeared to be permanent. The difference in average pressures between the first two room-temperature tests and the last five room-temperature tests was about 2.5 per cent. This effect does not appear to be due to the drying out of the material, since if the high and low temperature tests are left out of consideration the results of the room temperature tests appear to be nearly independent of humidity.

Repetition.—This was fairly good both on the same and on different days, regardless of variations in weather conditions.

Temperature.—If the results for tests 14 and 15 are neglected, the effect of lowering the temperature to -18°C . was to decrease the stiffness of the pressure element by about 3 per cent. Similarly, an increase in temperature to $+50^{\circ}\text{C}$. increased the stiffness of the diaphragm about 2.5 per cent. The fact that the diaphragm grew stiffer with increase in temperature and more flexible with decrease in temperature is worthy of note as this is unusual. It is doubtful if the effect is due to humidity, owing to the apparently small effect of changing the humidity while holding the temperature constant.

Hysteresis.—The average hysteresis exhibited by this material was about 3.8 per cent of the maximum pressure range. There was an increase in hysteresis to 6 per cent at the lowest temperatures.

Flexibility of material.—This material was considerably stiffer than alum-tanned colon leather.

RUBBERIZED COTTON

Description.—The exact preparation of this material (fig. 10) was not learned. It was determined that the material was cotton with a thin coating of rubber on one side. It averaged 0.0055 inch in thickness.

Slackness.—After mounting a diaphragm of this material in a Smith manometer the slackness was determined and found to be 1.008. No measurement was made after tests were completed.

Seasoning.—The reason for the abrupt increase in stiffness for test No. 3 is not clear. If it were due to the drying out of the material a progressive change would be expected, although it is possible that the diaphragm responded very rapidly to the sudden change in humidity. Evidence in favor of this change being due to humidity is found in the results of the last six tests. Here an equally abrupt decrease in stiffness is found when the diaphragm is again subjected to existing atmospheric humidity. In addition, it can be seen that the results of all tests at normal atmospheric humidity (1, 2, and 39 to 44) agree very well if we discard the results of test No. 42. On the whole, the evidence seems to indicate that no measurable seasoning has taken place.

Repetition.—This material repeated its performance very well under similar conditions.

Temperature.—This diaphragm proved to be surprisingly free from temperature effects. No appreciable change in stiffness occurred either at high or at low temperatures. Following

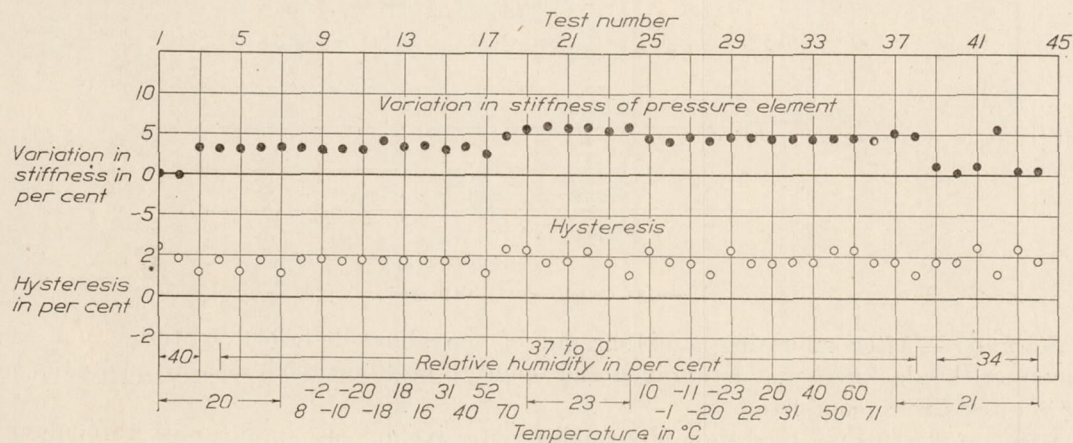


FIG. 10.—Rubberized cotton

tests at high temperatures, a series of room-temperature tests indicated a slight but temporary increase in stiffness. No reason for this could be assigned.

Hysteresis.—The hysteresis exhibited by this diaphragm was small and remained nearly constant in amount under all conditions encountered in the tests.

Flexibility of material.—The material was far stiffer than the alum-tanned colon leather.

TOUSSAINT-LEPÈRE RUBBERIZED SILK

Thickness.—This material (see fig. 11) was 0.0048 inch in thickness.

Slackness.—After mounting a diaphragm of this material in a Smith manometer the slackness was found to be 1.005.

Seasoning.—If any seasoning effects were produced they were too small to be detected. The final tests of the diaphragm agreed very well with the first.

Repetition.—This was very good both on same and on different days under varying weather conditions.

Temperature.—In order to understand the effect of temperature on this material, it will be necessary to consider in detail the conditions experienced during the tests.

After test No. 5 the drying tubes were connected to the apparatus and the gradual increase in stiffness resulting is obvious from the plot. The effect of humidity alone on the material is thus clear. Tests Nos. 18 and 19 were then run at temperatures of -12°C . and -20°C ., respectively, and showed little effect on the performance. The next test, however, at -25°C . caused further stiffening of the diaphragm. Upon bringing the diaphragm back to room temperature and testing, it was found to be in a more flexible condition (test No. 21) than just before the low-temperature runs. A further series of room-temperature tests showed a gradual stiffening of the diaphragm, approaching the condition which existed before the low-temperature runs were made.

The explanation of this performance appears to be somewhat as follows: The fabric would have stiffened at low temperatures had the relative humidity been constant, owing to the effect of low temperatures on the rubber coating. As the temperature was lowered, however, under the conditions of the tests, the relative humidity increased rapidly, thus tending to make the diaphragm more flexible. These two opposite tendencies appear to have balanced approximately between temperatures of -12°C . and -20°C ., but at -25°C . the stiffening of the rubber appears to have overbalanced the tendency toward increased flexibility owing to the

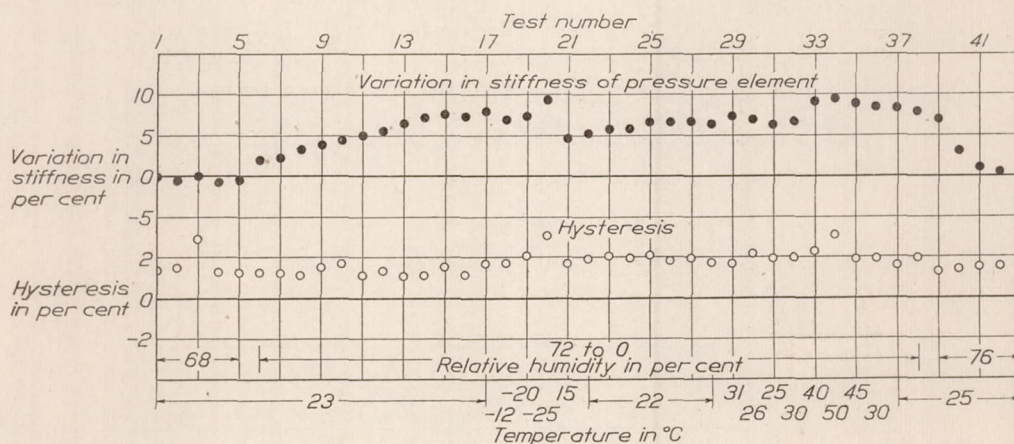


FIG. 11.—Toussaint-Lepere rubberized silk

increased relative humidity. Immediately after the diaphragm was brought back to room temperature, part of the effect of the high relative humidity remained and this may have been accentuated by the absorption of condensed moisture on the diaphragm. Following this the diaphragm continued to dry out and the pressure element became gradually stiffer. There appears to have been little, if any, permanent effect due to low temperatures.

Ordinarily the effect of increased temperatures is to increase the flexibility of a pressure element, but in this instance for temperatures above normal room temperature the pressure element exhibited an increase in stiffness amounting to about 2 per cent. at the highest temperature ($+50^{\circ}\text{C}$.). This effect may be accounted for by assuming that further drying due to the decreased relative humidity had a much greater influence on the performance than did the high temperature. After resuming room temperature again the diaphragm indicated by its increased stiffness that still further drying had taken place.

When the drying tubes were removed a few tests sufficed to show that the material had absorbed its normal amount of moisture, had returned to its original room-temperature performance, and showed no permanent effects due to temperature.

Hysteresis.—The average hysteresis exhibited by this material was small, being about 1.7 per cent. of the maximum pressure range.

Flexibility of material.—This material was only slightly stiffer than alum-tanned colon leather.

SINGLE-PLY RUBBERIZED BALLOON FABRIC

Description.—The exact nature of this material (fig. 12) was not known, but it was found to be a single-ply fabric treated with rubber in such a way that the latter was absorbed. It measured 0.0077 inch in thickness. The surface was smooth with a wax-like finish. The material was tested to obtain further evidence as to the performance of rubberized materials.

Slackness.—After a diaphragm of this material had been mounted in a Smith manometer the slackness before tests was found to be 1.007. No determination was made after completion of tests.

Seasoning.—No effect of seasoning could be detected positively. There appeared to be a slight increase in stiffness before the drying tubes were put on, yet at the conclusion of the tests the pressure element returned approximately to its original performance.

Repetition.—This was fairly good both on the same and on different days, regardless of changes in weather.

Temperature.—Decreasing the temperature caused the pressure element to become stiffer, while increasing the temperature produced the opposite effect. There is either a permanent effect caused by temperature changes or, what appears to be somewhat more probable, a long-

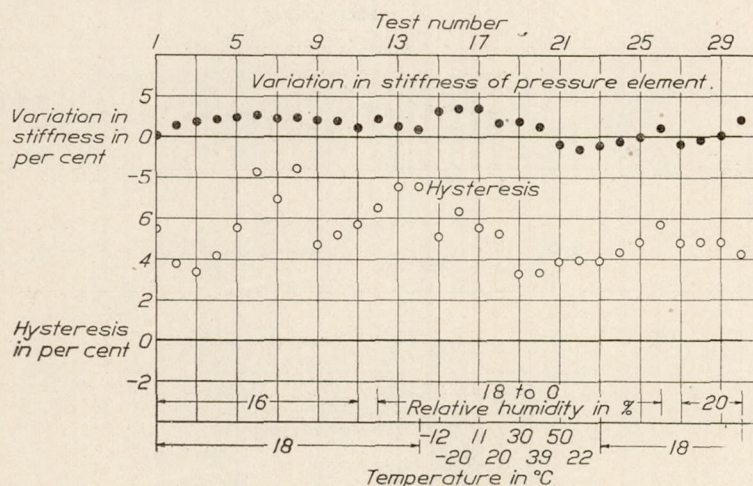


FIG. 12.—Rubberized balloon fabric (single ply)

continued transient effect. This is shown by the gradual stiffening of the pressure element following the increased flexibility caused by high temperatures.

Hysteresis.—The hysteresis exhibited by this material was excessive and was very erratic. The average value was 5.3 per cent. of the maximum pressure range.

Flexibility of material.—This material was very much stiffer than alum-tanned colon leather.

SMITH OILED SILK DIAPHRAGM

Description.—This diaphragm ³⁴ had a smooth, glossy surface, and appeared to be entirely free from air bubbles (fig. 13). It had a greenish color and appeared fairly uniform.

Slackness.—This was not determined for this diaphragm.

Seasoning.—No effect of seasoning is evident from the results plotted in Figure 13.

Repetition.—The diaphragm repeated its performance fairly well on the same and on different days, regardless of weather conditions.

Temperature.—The diaphragm showed a slight increase in flexibility under low temperatures and a slight increase in stiffness at high temperatures. There was no permanent effect evident after the diaphragm was finally brought back to room temperature.

³⁴ From Smith Balloon Manometer Identification No. 733.

Hysteresis.—This diaphragm exhibited negative hysteresis in a number of room and high-temperature tests. The average hysteresis for all tests was 0.75 per cent of the maximum pressure range.

Flexibility.—This material was somewhat stiffer than alum-tanned colon leather.

AGEING TEST OF A TAUT RUBBER DIAPHRAGM

A heavy rubber diaphragm (figs. 14 and 15) 1 millimeter in thickness was mounted in a very taut condition in an Ogilvie air-speed meter case. This rubber was prepared³⁵ by the Rubber Section of the Bureau of Standards, using the hot process of vulcanization. The object of

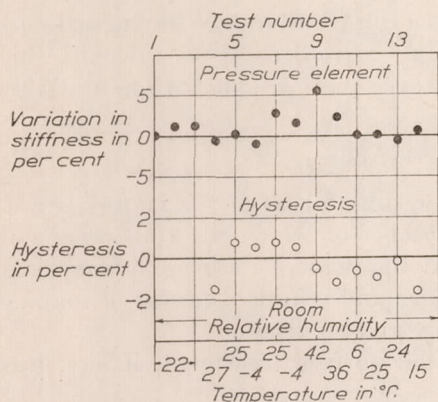


FIG. 13.—Oiled silk diaphragm in Smith balloon manometer

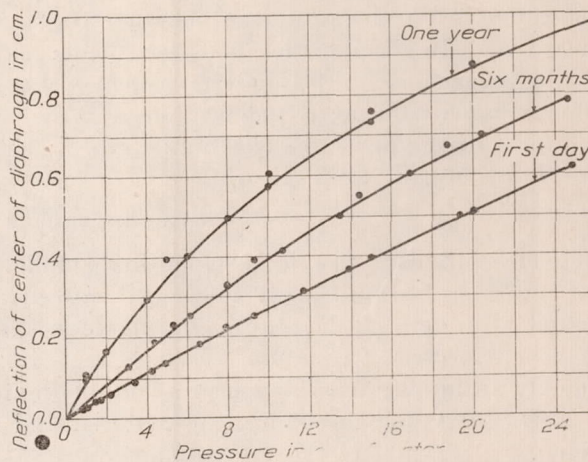


FIG. 14.—Load-deflection curves for taut rubber diaphragm showing effect of ageing

the tests was to determine the magnitude of the effect of ageing under ordinary conditions and the rapidity with which this effect took place.

Two days after the diaphragm was mounted the pressure-deflection curve obtained was found to be almost a straight line as shown in Figure 14. About six months later the diaphragm was found to have become much more flexible and the curvature of the pressure-deflection curve had become more pronounced. See Figure 14. At the end of one year the flexibility of the diaphragm had increased still further and the curvature of the pressure-deflection curve

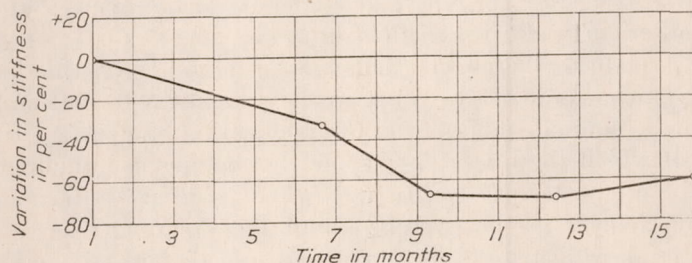


FIG. 15.—Change in flexibility of rubber diaphragm with time

had become still more pronounced. Following this the diaphragm began to stiffen slowly as shown in Figure 15. The maximum increase in flexibility exhibited by the diaphragm was about 70 per cent. In spite of the magnitude of this effect it is believed that no appreciable part of it was due to slipping of the diaphragm on the rim. The ageing was caused by the slow oxidation taking place in the rubber and the decrease in elasticity due to the initial tension.

This diaphragm is inferior to the rubber diaphragms used in the Ogilvie air-speed indicators, but it does illustrate the characteristic effect of ageing on rubber. Such results as are shown in Figures 14 and 15 indicate that the diaphragm would be useless owing to the great change

³⁵ By Mr. S. Collier.

in its elastic properties. Recent investigations in France³⁶ indicate, however, that, by adding certain substances to the rubber in the process of manufacture, the oxidation of the rubber can be greatly retarded and the elastic properties of the material retained over a period of years.

EXPANSION OF MATERIALS WITH INCREASE IN RELATIVE HUMIDITY

In order to study the expansion and contraction of nonmetallic diaphragm materials with changes in humidity, known lengths were marked on strips of these materials, which were then suspended from a frame in a bell jar in which the humidity could be controlled. Not only the magnitude of the changes in length of these strips with changes in humidity was thus determined, but the rate at which the strips responded to sudden changes in humidity was measured. In general, the strips lengthened as the relative humidity increased. The effect of this expansion of the material where a diaphragm is concerned would be to increase its effective area³⁷ for positive deflections and to decrease it for negative deflections, thus affecting the form of the pressure-deflection curve. The results of these tests are shown qualitatively in columns 6 and 7 of Table I.

SUMMARY OF RESULTS FOR ALL MATERIALS

This summary (Table I) includes a compilation of the results for all of the materials previously described and for a number of miscellaneous materials, for which the tests were not discussed in detail. The table is valuable in that it informs the reader in a general way what may be expected of each material with regard to performance and affords a means of making quickly an intelligent selection of a suitable material for any given purpose.

The first column names the material and specifies what surface treatment, if any, was given.

The second column gives the average thickness of the material in thousandths of an inch. The thickness of any one material of this nature is an extremely variable quantity as might be expected.

The third column, giving the flexibility or pliability of the materials relative to alum-tanned colon leather has no very scientific basis, but is the result of the judgment of several individuals who had handled these materials. The larger the number assigned to the relative flexibility the less flexible was the material.

Columns 4 and 5 are clear. All of the materials were satisfactory, as far as seasoning is concerned. All of the materials which age "very slowly" and most of those which age "slowly" are satisfactory in this respect. It should be understood, however, that any material containing rubber will be seriously affected in the course of time.

Columns 6 and 7 relate to the effect of changes in humidity on the diaphragm. Column 6 indicates the relative magnitude of this effect on the different materials and shows the marked improvement resulting in some instances from the application of a surface treatment. Column 7 indicates the rate at which the materials respond to changes in humidity. Those materials which respond slowly might be practically unaffected by daily variations in humidity but would be affected by changes in the average humidity extending over a period of weeks or months. Those materials which are said to respond quickly are affected appreciably by the ordinary changes in humidity which take place from hour to hour.

Columns 8, 9, 10, and 11 show the effect of temperature changes on the materials. Those materials which undergo permanent changes when subjected to high or low temperatures are obviously unsuited for use under conditions such that these temperatures may be experienced. All of the materials exhibited a measurable transient effect with the exception of the rubberized cotton. Columns 8 and 10 show the increased effect of temperature changes on colon leather after a surface treatment had been applied. In general, the effect of high temperatures is to make the pressure elements more flexible and of low temperatures to make them less flexible.

³⁶ The Action of Oxidation Inhibitors on Rubber, Comptes Rendus, vol. 177, pp. 204-6 C. A. 17, 3625 (1923), by André Helbronner and Gustave Bernstein.

³⁷ See Section VII of this report.

This is shown in columns 8 and 10. It is seen, however, that anomalous results were obtained for some of the materials. Cotton and silk treated with tung oil and Goodyear rubberized silk were found to give pressure elements which became more flexible at low temperatures and stiffer at high temperatures. The reason for this is not known positively, but dependence can be placed on the results, for in each case several diaphragms were carefully investigated. In addition untreated goldbeater's skin and Toussaint-Lepère rubberized silk stiffened with both increase and decrease in temperature. The probable reason for this peculiar behavior is discussed earlier in this report under the description of the tests of these materials.

Columns 12, 13, and 14 show the character of the hysteresis of pressure elements of the various materials. The statement that the hysteresis was "stable" means that the magnitude of the hysteresis was practically the same for successive tests under the same conditions. Figure 3 illustrates "stable" hysteresis, while Figure 12 illustrates erratic hysteresis.

Column 15 distinguishes between those materials which were too permeable for practical use as diaphragms at pressures less than about 10 pounds per square inch and those which were satisfactory.

DISCUSSION OF CERTAIN RESULTS

NEGATIVE DRIFT AND HYSTERESIS

The negative drift³⁸ and negative hysteresis³⁹ exhibited by some of the pressure elements studied in this investigation are worthy of comment. When metallic diaphragm pressure elements are used the drift is always positive, since it is due to the inelastic yielding of the material under stress. With nonmetallic diaphragm pressure elements, however, it has been found that a negative drift may often be obtained.

It is believed that in all probability this phenomenon is caused by abnormal diurnal changes in humidity. Since drift tests, as made at the Bureau of Standards, usually extend over a period of five hours, they are started in the morning and end late in the afternoon. In general the relative humidity decreases appreciably during this time.⁴⁰ Now tests which have already been described showed that decreasing the relative humidity caused most of the material investigated to shrink. This in turn would decrease the effective area of a nonmetallic diaphragm provided the deflection were positive; i. e., the rigid center was above the plane of the rim (see fig. 18), and would increase the effective area if the deflection were negative; i. e., the rigid center was below the plane of the rim, the pressure in the diaphragm chamber being in excess of that of the external air. Under these circumstances, assuming negative deflection, it would require less pressure to produce a given deflection with low relative humidity than with high relative humidity. This, of course, is the condition for positive drift. It is probable, particularly since the same pressure element gave positive drift on some occasions and negative drift on others, that the negative drift was due to an increase in the relative humidity during the test instead of the normal decrease. It is known that the rigid center of the diaphragm was below the plane of the rim for its total range of deflection in all cases.

This same explanation might be applied to account for negative hysteresis, but the time interval involved in a test for hysteresis is much shorter than that in a drift test, and there is hardly time for the relative humidity to change very greatly. This may, however, be the explanation of the negative hysteresis obtained for goldbeater's skin, since this material responds very rapidly to changes in humidity.

EFFECT OF STATIC ELECTRICITY

In the course of this investigation a peculiarity in the performance of a Toussaint-Lepère rubberized silk diaphragm when in a dry condition was found.

It was found that, if tests were run in quick succession while the diaphragm was slowly drying out and therefore stiffening slowly, the rate at which the diaphragm stiffened was

³⁸ Drift is defined as the increase in reading of the instrument when held for a prolonged period at a constant pressure.

³⁹ See reference No. 29.

⁴⁰ Monthly Weather Review, Supplement No. 6, "Relative Humidities and Vapor Pressures over the United States," Table 4, pages 35-43. By Preston C. Day.

greatly increased, but that this increased stiffness disappeared when the diaphragm was allowed to rest for 24 hours. The longer the interval between tests the less marked was this effect, until when 24 hours elapsed between the tests no such effect was exhibited. Now Toussaint-Lepère rubberized silk has a thin coating of rubber on one side only. It is believed that the process of deflecting the diaphragm with the consequent friction due to the changes taking place in the wrinkles about the rim may have sufficed to charge electrically the fibers of the silk. The fibers would then tend to repel each other and this would probably produce a slight stiffening of the material.

VII. PREDICTION OF THE PRESSURE-DEFLECTION CURVES OF NONMETALLIC DIAPHRAGM PRESSURE ELEMENTS

EFFECTIVE FORCE AND EFFECTIVE AREA

In order to discuss the theory of deflection of nonmetallic diaphragms of various types, it is desirable to introduce the conceptions of *effective force* and *effective area*. Slack diaphragms possess the advantage over metallic diaphragms, at least for purposes of design, that the force which such a diaphragm is capable of exerting at its center can be computed when the geometrical properties of the diaphragm, the differential pressure applied, and the magnitude of the deflection are known.

Suppose that the diaphragm (fig. 18, A) is subjected to a hydrostatic pressure p .⁴¹ The center of the diaphragm will deflect until it comes to a position such that the external resisting force F at the center (due to a spring, weights, etc.) just balances the upward force due to the pressure. The vertical component of the total hydrostatic force on the diaphragm is equal to $1/4 \pi D^2 p$, or to Ap if we represent by A the horizontal projection $1/4 \pi D^2$ of the area of the diaphragm. Not all of this hydrostatic force is available at the center of the diaphragm, however, since there is in general a reaction at the rim. A certain fraction of this force, equal in magnitude to the oppositely directed force F , is applied at the center. This is called the *effective force* for the given pressure p .⁴² It will be shown presently that this effective force depends also on the deflection of the center of the diaphragm.

Since a certain fraction of the total force is available at the center of the diaphragm we may consider that this force is due to the product of the pressure p and an area A_e , A_e being less than A . This area A_e may be called the *effective area*.⁴³ The effective area for a given diaphragm is not in general a constant, but is a function of the deflection of the center of the diaphragm.³

From the above considerations we can write

$$Ap = F + F_r, \quad (1)$$

where F_r is the vertical component of the reaction at the rim.

Also

$$A_e p = F, \quad (2)$$

whence

$$\frac{A_e}{A} = \frac{F}{F + F_r}. \quad (3)$$

COMPUTATION OF EFFECTIVE AREA

An expression will now be derived for the effective area of a slack diaphragm of circular form in terms of the dimensions of the diaphragm and the deflection of the rigid center. Figures 16, A, and 16, B, represent the geometrical conditions of the problem for the two cases which will be discussed. (The center of the diaphragm is assumed to have zero deflection initially.)

If a vertical diametral plane be passed through the diaphragm, the curve cut from the flexible portion of the diaphragm will be the arc of a circle, since the area under the curve tends

³ National Advisory Committee for Aeronautics Technical Report No. 130 (1922), "Oxygen Instruments," by F. L. Hunt. (Fig. 10 of this reference shows a rubber diaphragm whose effective area is independent of the deflection.)

⁴¹ It is assumed in what follows that the pressure p in the diaphragm chamber is greater than the external pressure.

⁴² Namely, for a given pressure element.

⁴³ This definition of "effective area" applies also to metallic diaphragms.

to become a maximum owing to the applied pressure, and this is the case when the cross section is the arc of a circle.

It is assumed that the flexible material does not stretch under the influence of the tension in it. This is true to a close degree of approximation for all the materials tested, except rubber. The following discussion would not apply to a rubber diaphragm.

Since the flexible material of the diaphragm can not transmit shear, no portion of the load inside of the horizontal circle of radius $R + \rho$ passing through the highest point H of the cross section and having its center on the axis can be transmitted to the rim, and likewise no portion of the load outside of the circle through H can pass to the rigid center. Consequently the effective force F is the vertical component of the total force due to hydrostatic pressure acting inside of the circle of radius $R + \rho$ having its center on the axis of the diaphragm and passing through H . The problem therefore resolves itself into finding the position of the point H , or the value of ρ , for different deflections.

In any specific problem there are given the values of R , the radius of the rigid center, c the distance from the rim to the edge of the rigid center when the deflection is zero, and s the length of the cross section of the diaphragm from the rim to the edge of the rigid center. If s is unknown, it can be determined for a given diaphragm by deflecting the diaphragm inward

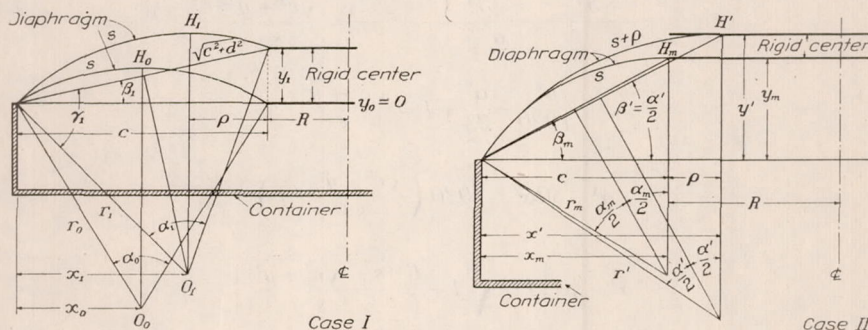


FIG. 16.—Cross sections of slack diaphragms

with a concentrated load applied at the center until the material of the diaphragm is in direct tension. The geometrical conditions then obtaining are represented in Figure 18, B. Since $p = 0$, the cross section is straight and the length s of the flexible portion can be obtained from

$$s = \sqrt{c^2 + y_m^2}, \quad (4)$$

where y_m is the maximum deflection of the diaphragm.

In the majority of instruments using slack diaphragms of this type the flexible material is not attached to the rigid center at the edge of the latter, but only at the center. Consequently two distinct cases may arise in practice. The first is represented in Figure 16, A. In this case the diaphragm is not tangent to the rigid center at its edge, but the highest point H of the diaphragm lies between the rim and the rigid center. The second case is shown in Figure 16, B. Here the flexible diaphragm is tangent to the rigid center at a point inside the radius of the rigid center. The conditions for the two cases differ so greatly that it is not convenient to treat them as special cases of a general solution, but it is preferable to derive the equations for each case separately. The solutions should give the same numerical result for the limiting case when $\rho = 0$.

Case 1: $x < c$ (See fig. 16, A), where x is the horizontal distance from the rim to the vertical line through H .

This case is characterized by the fact that the length of arc s is constant and that $x < c$.

From Figure 16, A, we obtain the equations

$$r\alpha = s, \quad (5)$$

$$2r \sin 1/2\alpha = \sqrt{c^2 + y^2}, \quad (6)$$

$$y = c \tan \beta, \quad (7)$$

where

r = radius of circle of which the arc s is a portion,

α = angle at O subtended by arc s ,

y = deflection of rigid center from horizontal plane at the rim,

and

β = angle at rim subtended by deflection y at point of contact of diaphragm and rigid center.

Dividing (6) by (5)

$$\frac{2 \sin \frac{1}{2}\alpha}{\alpha} = \frac{\sqrt{c^2 + y^2}}{s}.$$

Now expanding $\sin 1/2\alpha$,

$$\sin 1/2\alpha = 1/2\alpha - \frac{(1/2\alpha)^3}{3!} + \frac{(1/2\alpha)^5}{5!} - \frac{(1/2\alpha)^7}{7!} +$$

This series is convergent for all finite values of $1/2\alpha$. When $\alpha \leq \frac{\pi}{2}$ the first three terms of the series give the value of $\sin 1/2\alpha$ with an error of less than 1 per cent. Consequently only the first three terms of the series will be used.

$$\frac{2 \left\{ \frac{(1/2\alpha)^5}{5!} - \frac{(1/2\alpha)^3}{3!} + 1/2\alpha \right\}}{\alpha} = \frac{\sqrt{c^2 + y^2}}{s}$$

$$\frac{\alpha^4}{1920} - \frac{\alpha^2}{24} + 1 = \frac{\sqrt{c^2 + y^2}}{s}$$

$$\alpha^4 - 80\alpha^2 = 1920 \left(\frac{\sqrt{c^2 + y^2}}{s} - 1 \right)$$

$$\alpha^2 = 40 \left\{ 1 - \sqrt{1 - \frac{6}{5} \frac{(s - \sqrt{c^2 + y^2})}{s}} \right\} \quad (8)$$

The negative sign should be used before the radical since α must be less than π .

When α has been determined, r can be computed from (5). x is then found from

$$x = r \cos (\gamma - \beta),$$

where

$$\gamma = 90^\circ - 1/2\alpha \quad (9)$$

and

β is given by (7).

Finally

$$\rho = c - x. \quad (10)$$

Equations (8), (9), and (10) can be put in a more useful form by expressing the quantities involved as dimensionless ratios.

Thus:

$$\alpha^2 = 40 \left\{ 1 - \sqrt{1 - \frac{6}{5} \frac{\left(\frac{s}{c} - \sqrt{1 + \left(\frac{y}{c} \right)^2} \right)}{\frac{s}{c}}} \right\} \quad (8a)$$

$$\frac{x}{c} = \frac{r}{c} \cos (\gamma - \beta), \quad (9a)$$

and

$$\frac{\rho}{c} = 1 - \frac{x}{c}. \quad (10a)$$

$\frac{s}{c}$ is called the "slackness" of the diaphragm.

In Figure 17 are plotted values of $\frac{\rho}{c}$ against $\frac{y}{c}$ for different assumed values of $\frac{s}{c}$. By plotting these dimensionless quantities instead of the simple quantities ρ , y , and s , the plot can be used for diaphragms of all sizes.

Case 2. $x > c$ (See fig. 16, B). This case is characterized by the facts that the diaphragm is tangent to the rigid center at some point along the radius R and that consequently the deflection of the highest point of the diaphragm is now equal to the deflection of the rigid center.

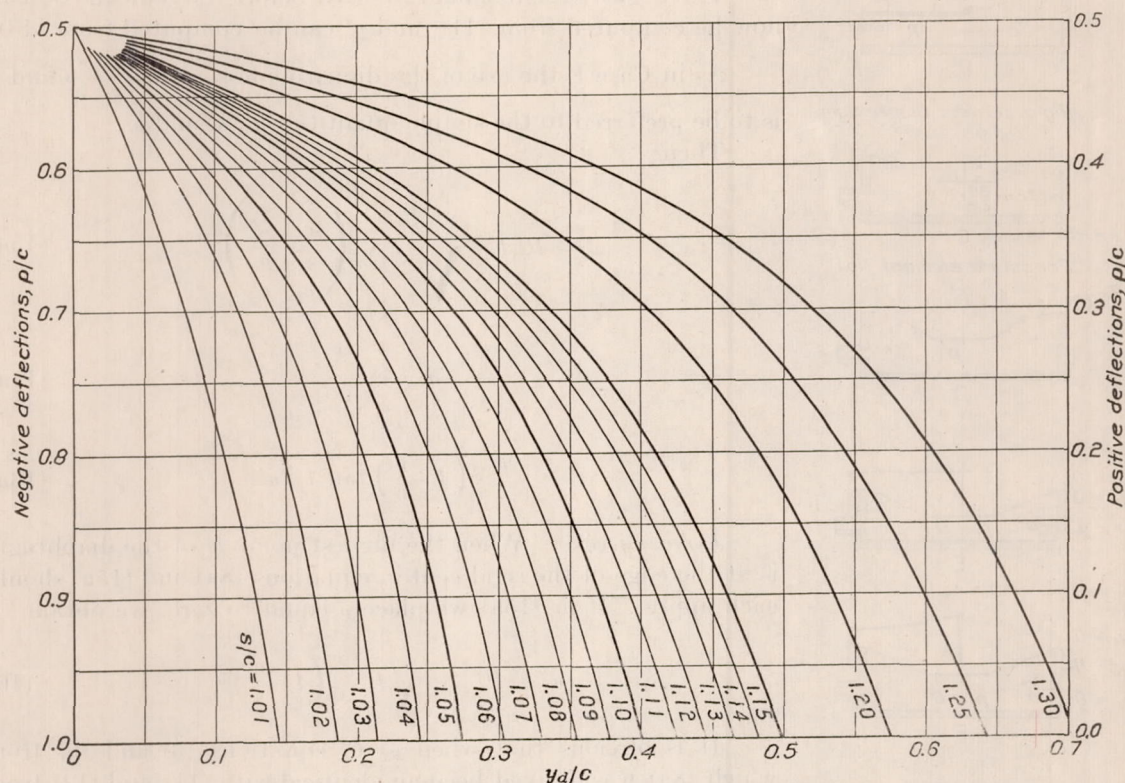


FIG. 17.—Curves for use in computing effective areas of slack diaphragms. Case I

From Figure 18 the following equations are obtained:

$$r'\alpha' = s + \rho \quad (11)$$

$$2r' \sin 1/2\alpha' = \sqrt{(c+\rho)^2 + y'^2} \quad (12)$$

$$y' = (c+\rho) \tan \beta' = (c+\rho) \tan 1/2\alpha' \quad (13)$$

The primes are used to signify Case II.

Dividing (11) by (12)

$$\frac{\alpha'}{2 \sin 1/2\alpha'} = \frac{s + \rho}{\sqrt{(c+\rho)^2 + y'^2}}.$$

Substituting for y' from (13)

$$\frac{\alpha'}{2 \sin 1/2\alpha'} = \frac{s + \rho}{\sqrt{(c+\rho)^2 (1 + \tan^2 1/2\alpha')}} = \frac{(s + \rho) \cos 1/2\alpha'}{c + \rho},$$

$$\alpha' = 2 \frac{s + \rho}{c + \rho} \sin 1/2\alpha' \cos 1/2\alpha'$$

$$\alpha' = \frac{s + \rho}{c + \rho} \sin \alpha'. \quad (14)$$

Expanding $\sin \alpha'$ in terms of α' , dividing by α' , and retaining only the first three terms,

$$\frac{\alpha'^4}{5} - \frac{\alpha'^2}{3} + 1 = \frac{c + \rho}{s + \rho},$$

whence

$$\alpha'^2 = 10 \left\{ 1 - \sqrt{\frac{6}{5} \left(\frac{c + \rho}{s + \rho} \right) - \frac{1}{5}} \right\} = 10 \left\{ 1 - \sqrt{1 - \frac{6}{5} \left(\frac{s - c}{s + \rho} \right)} \right\}. \quad (15)$$

The negative sign should be used before the radical. r' can now be computed from (11), and y' can be computed from (13).

As in Case I, the use of the dimensionless ratios $\frac{y'}{c}$, $\frac{s}{c}$, and $\frac{\rho}{c}$ is to be preferred to the simple quantities y' , s , and ρ .

Then:

$$\alpha'^2 = 10 \left\{ 1 - \sqrt{1 - \frac{6}{5} \left(\frac{\frac{s}{c} - 1}{\frac{s}{c} + \frac{\rho}{c}} \right)} \right\} \quad (15a)$$

$$\frac{r'}{c} = \frac{\frac{s}{c} + \frac{\rho}{c}}{\alpha'} \quad (11a)$$

$$\frac{y'}{c} = \left(1 + \frac{\rho}{c} \right) \tan \frac{1}{2} \alpha'. \quad (13a)$$

Limiting case.—When the highest point H of the diaphragm is at the edge of the rigid center, equations (8a) and (15a) should each apply. If in (15a) we place ρ equal to zero, we obtain

$$\alpha'^2 = 10 \left\{ 1 - \sqrt{1 - \frac{6}{5} \left(1 - \frac{c}{s} \right)} \right\}. \quad (16)$$

It is obvious that when $\rho = 0$, equations (5) and (6) from which (8a) was derived become identical with (11) and (12) from which (15a) was derived. Hence (8a) must be equivalent to (15a) when $\rho = 0$.

A consideration of the geometry of the problem shows that the method of solution given above for positive deflections holds equally well for negative deflections so long as the tangent to the diaphragm at the rim does not slope downward toward the axis of the diaphragm. See Figure 18D. Figure 17⁴⁴ gives curves for constant values of $\frac{s}{c}$ giving values of $\frac{\rho}{c}$ corresponding to different values of $\frac{y}{c}$. In this figure the curves for negative deflections are rotated through 180° to coincide with the corresponding curves for positive deflections, thus giving a more compact plot. It should be carefully noted that these curves apply only for Case I.

Once the value of $\frac{\rho}{c}$ is determined, ρ is known and the numerical value of the effective area can be computed. In terms of the notation used in Figure 16A,

$$A_e = \pi (R + \rho)^2, \quad (17)$$

care being taken to use the proper sign for ρ ⁴⁵ and the effective force can be determined from (2).

This furnishes a means of computing the pressure-deflection curve for any slack diaphragm and spring when used in combination. The method of doing this will be illustrated in the following section.

⁴⁴ The assistance of Mr. W. A. MacNair in working out some of the details of the theory and in making the computations for these curve is acknowledged.

⁴⁵ ρ is always positive for Case I.

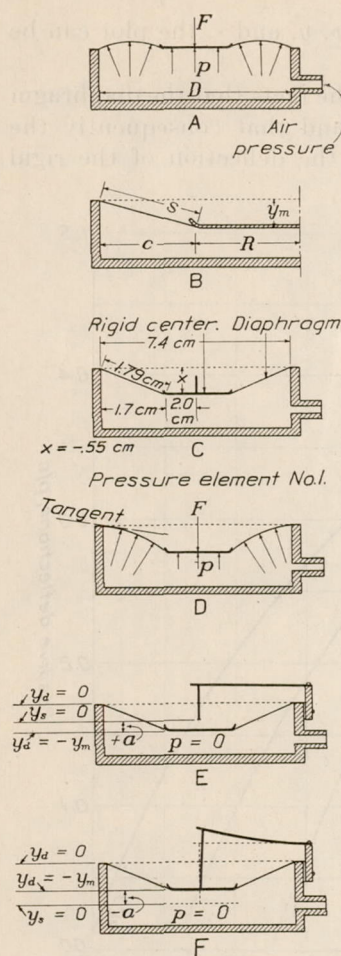


FIG. 18.

VIII. EXPERIMENTAL VERIFICATION OF THEORY OF DEFLECTION

The load-deflection curves for a number of nonmetallic diaphragms of widely different design were determined experimentally and were compared with the curves computed on the basis of the above theory. The detailed computations are given for the first case to illustrate the method. Only Case I is illustrated below, as Case II will rarely ever occur.

PRESSURE ELEMENT No. 1

A diaphragm of alum-tanned colon leather having the following geometrical characteristics was mounted (see fig. 18, C):

Radius of rigid center, $R=2.0$ centimeters.

Distance from rim to edge of rigid center, $c=1.70$ centimeters.

Slackness, $\frac{s}{c} = 1.05$.

A helical spring to be used with this diaphragm was designed and constructed of 0.08 inch steel wire. It was 1.4 inches long, 1 inch in diameter and had 10 turns. The load-deflection curve for this spring was determined experimentally. The deflections of the spring are given in column 2 for the loads in column 9 of Table II. The spring was mounted above the diaphragm so that when the rigid center had assumed its maximum negative deflection without placing the diaphragm in direct tension, the spring just touched the rigid center but exerted no force on it.

A brief statement of the considerations involved in computing the load-deflection curve for the pressure element might be helpful at this point. The hydrostatic pressure required to produce a given deflection of the diaphragm is dependent only on the resisting force of the spring and the effective area for that deflection. The effective area depends only on the geometry and the deflection of the diaphragm and can be computed from the formulas already developed in this paper. The resisting force of the spring is given by its load-deflection curve for the spring deflection corresponding to the assumed diaphragm deflection. The required pressure is then obtained by dividing the resisting force of the spring by the effective area of the diaphragm. These computed pressures can then be used with the deflections for which they were computed to plot the pressure-deflection curve for the pressure element.

TABLE II

1	2	3	4	5	6	7	8	9	10
y_d cm	y_s cm	$\frac{y_d}{c}$	$\frac{\rho}{c}$	ρ cm.	$R+\rho$ cm.	$(R+\rho)^2$ cm. ²	$\pi(R+\rho)^2$ $=A_s$ cm. ²	Load F gms.	$p=\frac{F}{A_s}$ cm. water
-0.55	0.0	-0.323	Beyond limit of family of curves.						
-0.50	.05	-.294	Beyond limit of family of curves.						
-0.45	.10	-.265	0.915	1.56	3.56	12.7	39.8	95	2.9
-0.35	.20	-.206	.740	1.26	3.26	10.6	33.4	190	5.6
-0.25	.30	-.147	.645	1.10	3.10	9.61	30.2	285	9.5
-0.15	.40	-.088	.575	.98	2.98	8.88	27.9	380	13.6
-0.05	.50	-.029	.525	.90	2.90	8.41	26.4	480	18.2
0	.55	.00	.500	.85	2.85	8.12	25.5	525	20.5
.05	.60	.029	.475	.81	2.81	7.90	24.8	575	23.2
.15	.70	.088	.425	.72	2.72	7.40	23.2	675	29.0
.25	.80	.147	.355	.60	2.60	6.76	21.2	775	36.5

In column 1 are given assumed values of the deflection of the diaphragm taking the plane of the rim as the zero position of the diaphragm and starting with the diaphragm in the lowest position it could assume without stretching the material. As already stated, the spring was so placed that the zero position of its end coincided with this lowest position of the diaphragm. Hence the spring deflections in column 2 are obtained by adding the amount 0.55 centimeters to the diaphragm deflection. The method of obtaining the values in column 3 is obvious. The values in column 4 are obtained from Figure 17, using the proper curve for the slackness of the diaphragm, in this case, 1.05. It will be observed that the first two values of $\frac{y_d}{c}$ give values of $\frac{\rho}{c}$ which lie outside of the plot. The geometrical meaning of this is that under these conditions ρ

is greater than c and no horizontal plane can be drawn tangent to the diaphragm. (See Figure 18, D.) For any diametral section, the lines tangent to the diaphragm at the rim slope downward toward the axis of the diaphragm. Obviously the above theory does not apply under these conditions and so, for a small range of values of $\frac{y_d}{c}$, the pressure-deflection curve can not be obtained by this method. Practically this will rarely if ever cause any difficulty since it is known that the curve passes through the origin.

The method of obtaining the values in columns 5, 6, 7, and 8 is obvious. The values of the load given in column 9 are obtained from the load-deflection curve for the spring and correspond to the spring deflections in column 2. The pressures required to give the assumed deflections can then be computed by dividing the central load due to the spring by the effective area of the diaphragm for the corresponding deflection.

Two of the assumptions involved in the above computations are worthy of notice. It was tacitly assumed that the weight of the rigid center and of the diaphragm itself could be neglected

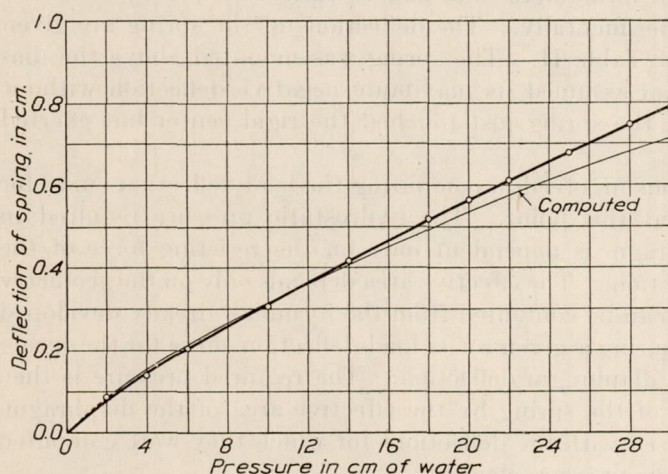


FIG. 19.—Alum-tanned colon leather diaphragm with helical spring. Pressure element No. 1

Comparison of computed curve with experimental data. Spring stiffness 0.97 kg/cm (5.4 lb./in.) $R=2$ cm. $c=11.7$ cm. $s/c=1.05$

for small deflections and slightly flatter for large deflections than is the experimental curve. The agreement, however, is excellent when the uncertainties involved in the computation are considered.

The alum-tanned colon leather of which this diaphragm is constructed was the most elastic of any of the materials studied in this investigation. Hence stretching of the diaphragm might be expected to cause a greater discrepancy between the experimental and computed curves for this material than for any of the other materials. It will be interesting to investigate the discrepancy in the two curves shown in Figure 19 on the assumption that stretching has taken place. The stretching is dependent on the stresses existing in the diaphragm and so increases with the hydrostatic pressure applied and changes with alterations in the geometrical configuration of the diaphragm.

An attempt was made to reverse the method of computation used in Table II and, using the experimentally determined values of p in column 10, to compute the slackness. Computations for a few of the highest points on the curve, however, showed that values of ρ greater than $0.5c$ resulted for positive deflections and this was inconsistent with the theory. (See fig. 16, A.)

It can be seen from an examination of Figure 16, A⁴⁶ that increasing the slackness of the diaphragm would deviate the load-deflection curve in the direction of the experimentally determined curve. However, it was shown that the deviation found is much greater than can be explained by the stretching of the diaphragm. Nevertheless, it was decided to test next a diaphragm

in comparison with the force exerted by the spring and specifically assumed that no stretching of the diaphragm took place. Since the diaphragm and rigid center in this instance weighed only 10 grams, it is clear from a consideration of columns 2 and 9 in Table II that the first assumption is true except for exceedingly small deflections. The justification for the assumption that the stretching of the diaphragm is negligible will be considered in the light of the experimental results which follow.

The computed curve for pressure element No. 1 is plotted in Figure 19 together with the curve obtained experimentally. It will be observed that the computed curve is slightly steeper

⁴⁶ See also discussion of certain results, negative drift and hysteresis.

which was much less elastic than alum-tanned colon leather in order to reduce the effect of stretching. The material chosen for this purpose was the rubberized cotton fabric described earlier in this report.

PRESSURE ELEMENT No. 2

The diameter of the diaphragm chamber of this pressure element was 6 inches, and the diameter of the rigid center was 2.5 inches. When mounted the diaphragm was found to have a slackness of 1.03. The opposing helical spring was made of 0.08 inch steel wire. It was 2 inches long, 1 inch in diameter and had 18 turns.

The theoretical pressure-deflection curve for this pressure element was computed in the same manner as was done for the first pressure element. The actual curve was then determined experimentally. The two curves are plotted in Figure 20. The agreement between the two curves is excellent, the greatest discrepancy being approximately 2 per cent. of the maximum deflection. The excellent results shown by this diaphragm indicate that, in all probability, stretching of the material was an important factor in causing the discrepancy shown in Figure 19, although no wholly satisfactory explanation can be offered.

TESTS OF OTHER PRESSURE ELEMENTS

In order to get a more complete verification of the theory, further tests were run on several slack diaphragm pressure elements, the only differences in these elements being in the degree

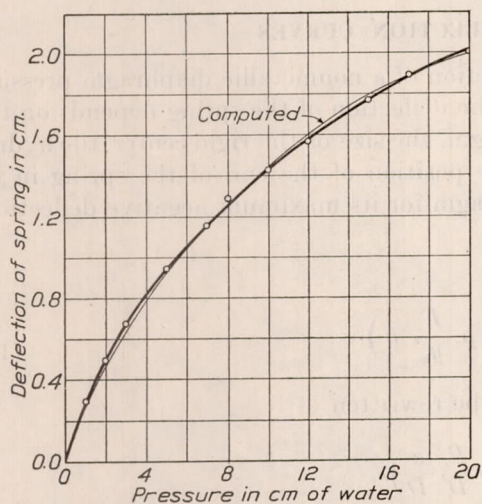


FIG. 20.—Rubberized cotton diaphragm with helical spring. Pressure element No. 2

Comparison of computed curve with experimental data.
Spring stiffness 0.60 kg/cm. (3.35 lb./in.) $R=2.5$ cm. $c=0.5$ cm. $s/c=1.03$

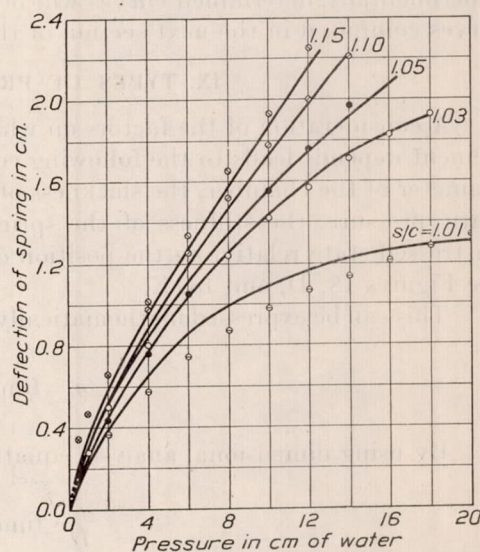


FIG. 21.—Comparison of computed curves with experimental data. Points experimental, curves computations

Spring stiffness 0.60 kg/cm. (3.35 lb./in.) $R=3.2$ cm. $c=4.7$ cm.

of slackness of the diaphragm and the position of the spring. Alum-tanned colon leather was used for most of the tests, as a pliable material is more satisfactory than a stiff material for mounting when a high degree of slackness is desired. The same diaphragm box, rigid center and spring used in pressure element No. 2 were used also for these tests. The spring was adjusted for each test so that it was just in contact with the rigid center when the diaphragm was in its lowest position. The computed and the experimentally determined curves are given in Figure 21 for values of $\frac{s}{c}$ of 1.01, 1.03, 1.05, 1.10, and 1.15. The results obtained with alum-

tanned colon leather agree well with the computed curves for values of $\frac{s}{c}$ of 1.05, 1.10, and 1.15, but it was found very difficult to obtain agreement for the values 1.01 and 1.03 since with such a comparatively taut diaphragm, slipping from the edge of the diaphragm box occurred and, of course, the stretching was increased.

In order to verify the computed curves for values of 1.01 and 1.03, Toussaint-Lepère rubberized silk was used for the diaphragm material, thus reducing greatly the stretching. After cementing the diaphragm to the box, good results were obtained for values of $\frac{s}{c}$ of 1.03 and 1.05 and fair results for 1.01. It is difficult to determine the slackness accurately for such a low value of $\frac{s}{c}$ as this last, and stretching is comparatively large.

For values of slackness greater than about 1.03, however, it would appear reasonable to assume that stretching of the material can be ignored in computing the load-deflection curve for a slack diaphragm pressure element. The error made in adopting this assumption will be greater for alum-tanned colon leather than for less elastic materials, and it may be quite appreciable if a fairly stiff diaphragm having a considerable degree of slackness is mounted on a diaphragm chamber of small diameter. Under these latter conditions wrinkling of the diaphragm would undoubtedly cause trouble.

The experimental verification of the theory of deflection of nonmetallic diaphragms is considered sufficiently good to warrant the assumption that this theory can be relied on to predict the pressure-deflection curves of this type of pressure element for purposes of design. The discrepancy between the computed and the actual curves should not exceed about 10 per cent. of the maximum range. The agreement thus found to exist between the computed and the experimentally determined curves will be used as justification for assuming the accuracy of the curves computed in the next section of the report.

IX. TYPES OF PRESSURE-DEFLECTION CURVES

A consideration of the factors on which the deflection of a nonmetallic diaphragm pressure element depends leads to the following conclusion: The deflection of the spring depends on the diameter of the chamber, the slackness of the diaphragm, the size of the rigid center, the hydrostatic pressure, the stiffness of the spring,⁴⁷ and the position of the end of the spring in its unstressed state relative to the position of the diaphragm for its maximum negative deflection. See Figures 18, D, and 18, E.

This can be expressed mathematically by writing

$$y_s = \text{funct} \left(D, \frac{s}{c}, R, p, \frac{F}{y_s}, a \right). \quad (18)$$

By using dimensional analysis equation (18) can be rewritten

$$\frac{y_s}{D} = \text{funct} \left(\frac{p D}{F}, \frac{s}{c}, \frac{R}{D}, \frac{a}{D} \right),$$

or

$$\frac{y_s}{D} = \text{funct} \left(\frac{p D^2}{F}, \frac{s}{c}, \frac{R}{D}, \frac{a}{D} \right). \quad (19)$$

Owing to the fact that a direct analytical solution of the problem was possible, it was not necessary to use this equation in developing the theory, but it is given here as a convenient means of showing at a glance what factors affect the shape of the pressure-deflection curve. A number of computed pressure-deflection curves will now be given, showing the effect of varying the dimensionless variables in equation (19).

Figures 22, A, and 22, B, show the effect of varying the slackness $\frac{s}{c}$, keeping all other factors, including the position of the end of the spring, constant. This, of course, necessitates varying $\frac{a}{D}$ at the same time. A little consideration will show that according to the theory, the pressure-deflection curves for pressure elements having diaphragms of the same dimensions

⁴⁷ The stiffness of the spring is defined as the ratio of its resisting force to its deflection, i. e., $\frac{F}{y_s}$ (or $\frac{A \cdot p}{y_s}$).

but of different degrees of slackness must all pass through a common point in addition to zero, provided the position of the spring is kept unchanged. This point is given by the zero position of the diaphragm and the corresponding pressure, for under these conditions $\rho = 1/2 c$ and obviously is independent of the slackness.

In Figure 22, A, the zero position of the spring is the same for the four slacknesses assumed and is 0.7 centimeter below the plane of the rim, this position corresponding to the maximum negative deflection of the diaphragm whose slackness is 1.01. In cases (2), (3), and (4) the diaphragm deflects some distance before coming in contact with the spring. Increasing the slackness makes it possible to obtain a greater deflection with the diaphragm and causes the pressure-deflection curves to become more nearly linear, as might be expected from a consideration of the geometry of the pressure element. It is interesting to note that changing the slackness under these conditions hardly alters the lower portion of the curves, but produces very great changes in the curves for the larger pressures.

The curves in Figure 22, B, are computed for a zero position of the spring corresponding to the maximum negative deflection of the diaphragm whose slackness is 1.03; i. e., for -1.2

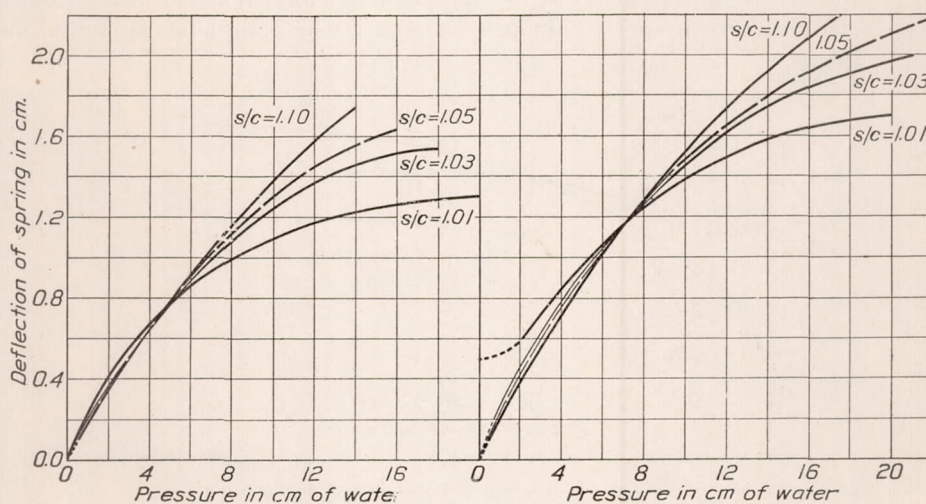


FIG. 22.—A. Zero position of spring 0.7 cm. below rim. B. Zero position of spring 1.2 cm. below rim
Effect of varying slackness of diaphragm keeping all other factors constant

When $s/c = 1.01$, maximum negative deflection = 0.70 cm.
When $s/c = 1.03$, maximum negative deflection = 1.20 cm.
When $s/c = 1.05$, maximum negative deflection = 1.50 cm.
When $s/c = 1.10$, maximum negative deflection = 2.15 cm.

centimeters. This results in the spring being initially in compression when used with the diaphragm whose slackness is 1.01.

Lowering the zero position of the spring increases the flexibility of the pressure element as long as the spring is not in compression initially, since the effective areas corresponding to given spring deflections are greater under these conditions.

The case just illustrated is not very practical, however, since it is desirable to have the spring attached to the diaphragm. The more practical case, which involves changing the slackness of the diaphragm $\frac{s}{c}$, and altering the zero position of the spring to coincide with the

maximum negative deflection of the diaphragm (i. e., keeping $\frac{a}{D}$ constant and equal to zero), is shown in Figure 21. The increased flexibility of the pressure element resulting from increasing the slackness of the diaphragm is clearly illustrated there. The pressure-deflection curves are more nearly linear for the relatively high slacknesses than for the low slacknesses.

The effect of varying the size of the rigid center is shown in Figure 23. The principal feature of interest in these curves is the extension of the computations for the rigid center of

large diameter to include its deflection under the conditions of Case II.⁴⁸ As no curves giving values of $\frac{p}{c}$ for given values of $\frac{y_d}{c}$ had been plotted for this case, it was necessary to substitute directly in formulas (13a) and (15a). The deflection of the diaphragm for the limiting case was determined and the pressure required to produce this deflection was computed independently by the formulas both of Case I and of Case II. The two results agreed very closely.

The three curves shown in Figure 23 were plotted on logarithmic cross-section paper to determine whether they approximated the relation

$$y_s = K(p)^{0.5}, \quad (20a)$$

which applies to air-speed indicators of the differential pressure type. When plotted in this way the curve for the pressure element with the small rigid center gave up to 60 centimeters of water very nearly a straight line whose average slope was 0.57, thus giving the relation

$$y_s = K(p)^{0.57}. \quad (20b)$$

By using this pressure element, therefore, or one geometrically and physically similar to it, it is possible to construct an air-speed indicator having a very simple mechanism, such as the

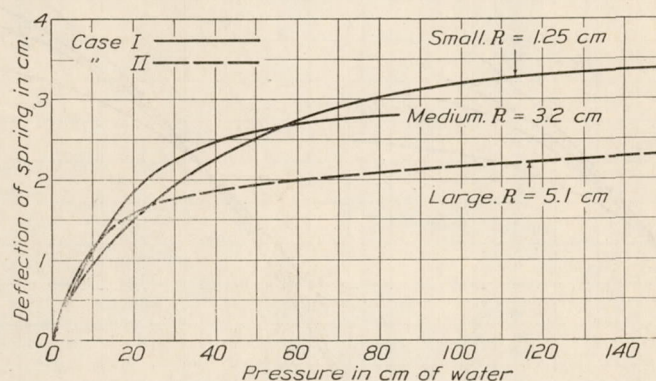


FIG. 23.—Effect of varying size of rigid center. Slackness constant. $s/c=1.05$.

Diameter of chamber 15.2 cm. Spring stiffness 0.97 kg/cm. (5.4 lb./in.)

silk thread used in the Ogilvie air-speed indicator (See fig. 1, A), at the same time obtaining a scale with practically uniform graduations. Furthermore, it is probable that by making slight adjustments in the stiffness of the spring, the slackness of the diaphragm, or other factors which affect the pressure-deflection curve, still better agreement can be obtained between the desired relation given by equation (20a) and the curve obtained from the pressure element.

The effect of changing the spring stiffness is obvious from equation (18). If the stiffness $\frac{F}{y_s}$ is increased, the dimensions of the element being left unchanged, it is necessary to increase the pressure p to keep $\frac{y_s}{D}$ constant. That is, the abscissas of all points on the pressure-deflection curve are multiplied by a constant factor to take account of this change in stiffness.

X. ADVANTAGES AND DISADVANTAGES OF NONMETALLIC DIAPHRAGM PRESSURE ELEMENTS

ADVANTAGES

The nonmetallic diaphragm pressure element possesses a number of advantages which, when generally realized, should make its use more common in a number of types of pressure-measuring instruments. Among its advantages may be mentioned (1) possibility of predetermining the pressure-deflection curve, thus making scientific design possible; (2) wide control of the shape of the pressure-deflection curve, thus making it possible to simplify the indicating mechanism of the instrument; (3) large deflection combined with small size, enabling the multi-

⁴⁸ See Section VII.

plying ratio of the mechanism to be decreased, the instrument to be made small and compact and the measurement of small differential pressures facilitated; (4) small inertia, which gives comparative freedom from position errors and the inertia effects caused by vibrations and accelerations and (5) means of accomplishing temperature compensation.⁴⁹ In addition to this, taut rubber diaphragms give a very good zero point.

DISADVANTAGES

The three principal disadvantages of this type of pressure element are peculiar to non-metallic diaphragms. They are (1) permeability of material, (2) effect of humidity, and (3) ageing of material. The first two can be overcome by the use of surface treatments, but these introduce at the same time large temperature effects in most cases. The last eliminates one of the materials which would be most useful in this connection—rubber. If a method of preparing or treating rubber can be devised so that it will not oxidize when in the form of thin sheets, a material advance will have been made in this field.³⁶

XI. DESIGN OF NONMETALLIC DIAPHRAGM PRESSURE ELEMENTS

CHOICE OF MATERIAL FOR DIAPHRAGM

The data in Table I will greatly facilitate the selection of the proper material for a specific purpose. Only the most general considerations will be discussed here, as it is impossible to take into account all of the factors which might be involved in the selection of the best available material. The first considerations are the permissible diameter of the element, the degree of permeability which is allowable, and the magnitude of the differential pressures involved. If the element must be small in size and if the differential pressures involved are small, alum-tanned colon leather, the most pliable material of those investigated, may be considered. With out surface treatment this material is ideal in all respects except two: (1) It is comparatively elastic and hence can not be used under conditions where the stresses in the material are excessive, and (2) it will grow stiffer in time if subjected to wide variations of humidity, particularly if the conditions are such that moisture condenses on the diaphragm. The ageing can be prevented, or at least materially delayed, by the use of a surface treatment on both sides of the material. This, however, has been found to introduce fairly large temperature effects for all of the dopes and oils tried.

If a less elastic material is required, surface-treated zephir leather, the cotton treated with tung oil, rubberized cotton, or the Toussaint-Lepère rubberized silk, may be considered. The fact that rubber ages rapidly should be borne in mind. The Toussaint-Lepère rubberized silk is excellent if not used for more than a year or so, but it stiffens in time and becomes worthless. This material can not be obtained at the present time, but an attempt has been made by an American firm to duplicate it. Tests have shown, however, that the American product is not satisfactory for use as a slack diaphragm material.

The type of spring used with the diaphragm is important. Both flat and helical springs have been used with success. If the ratio of the spring deflection to the length of the spring is great, the end of a flat spring will be inclined considerably to the vertical and will tend to tilt the diaphragm. If this is the case, the helical spring may be preferable, as its reaction is always axial. Care should be taken that a helical spring is not sufficiently slender to buckle if compressed. The difficulty of making adjustments when a helical spring is used should be noted. The zero adjustment is not so difficult, but the stiffness of the spring can not be adjusted easily. A helical spring will give a greater position error than a flat one owing to its greater weight.

The central rigid disk should be made as light as possible and should be cemented to the diaphragm at the center. The edge of the disk should be turned up slightly so that any motion of the diaphragm relative to this edge will not damage the material.

The authors wish to express their appreciation of the advice and assistance of Mr. W. H. Smith and Mr. R. C. Bowker of the Bureau of Standards in connection with portions of this investigation.

³⁶ The Action of Oxidation Inhibitors on Rubber, *Comptes Rendus*, vol. 177, pp. 204-6 C. A. 17, 3625 (1923), by André Helbronner and Gustave Bernstein.

⁴⁹ By selecting the proper diaphragm material the pressure element can be made to increase in flexibility or decrease in flexibility with increase of temperature as may be desired. See Table I of this paper and accompanying discussion.

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TABLE I.—Summary—Properties of diaphragm materials

Material	Thick- ness thou- sandths inch	Rela- tive flexi- bility	Exhibits seasoning	Exhibits ageing	Affected by changes of humidity	Hygroscopic moisture content varies	Low temperature		High temperature		Average hysteresis			Highly per- meable
							Temporary effect	Per- ma- nent effect	Temporary effect	Per- ma- nent effect	Per cent of max- imum pres- sure	Stable	Affected by temperature	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Untreated alum-tanned colon leather.....	1.1 to 2.7	1	Yes.....	Very slowly..	Slightly.....	Slowly.....	Slight increase in stiffness.	No...	Very slight in- crease in flex- ibility.	No...	1.0	Yes..	No.....	No.
Alum-tanned colon leather treated with acetate-cellulose dope.	2.0	1	Yes.....	Very slowly..	Very slightly.	Very slowly..	Great increase in stiffness.	No...	Increase in flexibility.	No...	2.5	Yes..	Increased at low temperature only.	No.
Alum-tanned colon leather treated with collodion dope.	3.0	1	Very slightly.	Very slowly..	Very slightly.	Very slowly..	Great increase in stiffness.	No...	Increase in flexibility.	No...	2.3	Yes..	No.....	No.
Zephir leather treated with acetate-cel- lulose dope.	2.5	2	Very slightly.	Very slowly..	Very slightly.	Very slowly..	Great increase in stiffness.	No...	Increase in flexibility.	No...	2.0	Yes..	Increased at low temperature only.	No.
Oiled zephir leather.....	2.5	3	Yes.....	Slowly.....	Slightly.....	Fairly slowly.	Great increase in stiffness.	No..	Great increase in flexibility.	No...	3.5	No...	Increased at low and decreased at high tempera- tures.	No.
Untreated goldbeater's skin.....	.9	2	No.....	Slowly.....	Very greatly..	Very fast.....	Slight increase in stiffness.	Yes..	Great increase in stiffness.	Yes..	0.3	No...	No.....	No.
Cotton treated with tung oil.....	7.0	3	Very slightly.	Very slowly..	Very slightly.	Very slowly..	Increased flex- ibility.	No...	Small increase in stiffness.	No...	3.8	Yes..	Increased at low temperature only.	No.
Rubberized cotton.....	5.5	3	Very slightly.	Slowly.....	Very slightly.	Slowly.....	No effect.....	No...	No effect.....	No...	1.8	Yes..	No.....	No.
Toussaint-Lepère rubberized silk.....	4.8	3	Very slightly.	Slowly.....	Slightly.....	Fairly slowly.	Slight increase in stiffness.	No...	Slight increase in stiffness.	No...	1.7	Yes..	Slightly increased at high and low temperatures.	No.
Rubberized balloon fabric (single ply)...	7.7	4	Very slightly.	Slowly.....	Slightly.....	Slowly.....	Slight increase in stiffness.	No...	Slight increase in flexibility.	No...	5.3	No...	No.....	No.
Goodyear rubberized silk.....	6.7	3	Yes.....	Slowly.....	Slightly.....	Fairly slowly.	Increase in flexibility.	No...	Increase in stiffness.	No...	1.3	Yes..	No.....	No.
Rubber.....			Yes.....	Fast.....	Slightly.....	Slowly.....	Increase in stiffness.	No...	Increase in flexibility.	Yes..			No.....	No.
Silk treated with tung oil.....	4.7	5	Yes.....	Very slowly..	Slightly.....	Slowly.....	Increase in flexibility.	Yes..	Great increase in stiffness.	Yes..	0.7	Yes..	No.....	No.
Alum-tanned colon leather treated calolice oil.	3.0	1	Yes.....	Slowly.....	Slightly.....	Slowly.....	Great increase in stiffness.	No...	Great increase in flexibility.	No...		No...	No.....	No.
Oiled silk (Smith manometer diaphragm).....		2	Very slightly.	Slowly.....	Slightly.....	Slowly.....	Slight increase in stiffness.	No...	Increase in flexibility.	No...	0.8	Yes..	No.....	No.
Untreated bung gut leather.....	5.0			Slowly.....	Greatly.....	Fairly fast...								Yes.
Untreated vegetable-tanned colon leather	1.9	2	Very slightly.	Very slowly..	Slightly.....	Fairly slowly.	Great increase in stiffness.	No...	Increase in flexibility.	No...	1.3	Yes..	No.....	Yes.
Untreated kid skin.....			Slightly.....	Very slowly..	Greatly.....	Fairly fast...								Yes.
Untreated zephir leather.....	4.0	4	Very slightly.	Slowly.....	Greatly.....	Fast.....	Increase in stiffness.	No...	Increase in flexibility.	No...			No.....	Yes.
Silk treated with acetate-cellulose dope...	4.5	5		Very slowly..	Slightly.....	Slowly.....			Great increase in stiffness.	Yes..		No...	Yes.....	No.
Alum-tanned colon leather treated with rubber and spar varnish.	4.0	2	Yes.....	Fairly fast...	Slightly.....	Fairly slowly	Great increase in stiffness.	No...	Great increase in flexibility.	No...		No...	No.....	No.
Zephir leather treated with calolice oil...	2.0	3.	Yes.....	Slowly.....	Greatly.....	Fairly fast...	Great increase in stiffness.	No...	Great increase in flexibility.	No...		No...	No.....	No.

APPENDIX

METHODS OF MOUNTING NONMETALLIC DIAPHRAGMS

No particular emphasis was laid in the course of this investigation on methods of mounting nonmetallic diaphragms, but the experience gained during several years of experimenting may be worth setting down for the benefit of others. The materials to be considered in this connection may be divided into two groups: (1) Rubber, and (2) leathers and treated fabrics.

TAUT RUBBER DIAPHRAGMS¹

In some instruments which measure pressure, a thin vulcanized sheet of rubber is used as the diaphragm of the pressure element. When used in this manner the rubber is almost invariably mounted under tension and is used without a rigid center. The usual method employed in such cases is to stretch the rubber to the desired tautness and to attach it to a ring using a suitable adhesive, the ring and diaphragm fitting into the instrument case as a unit. This type of construction is used in the pressure element of the Ogilvie air-speed indicator as shown in Figure 1A. A very simple and dependable method of mounting the diaphragm on the ring has been employed at the Bureau of Standards.

The ring should be made of bakelite or hard rubber. In trials of the two materials bakelite was found superior to hard rubber, as the hard rubber had a tendency to soften if heated to a temperature higher than 50° Centigrade. Furthermore, the hard rubber warped badly under test conditions. However, the specimens of hard rubber used were of a very low grade and it is possible that if a good grade of hard rubber were used its performance might be as good as that of bakelite. Other materials such as aluminum and cast iron were tried but with only indifferent success. Cast-iron rings can be employed, but their weight is a serious disadvantage. Aluminum rings were used with fair success but it was found difficult to prevent an exceedingly taut diaphragm from slipping on an aluminum ring unless the joint was kept under a heavy pressure after the diaphragm had been mounted. The method of mounting described below should be suitable for all tensions which may be desired.

In the majority of cases, rings of bakelite will serve the purpose and the following method of mounting will be suitable for all tensions.

After the bakelite ring has been turned out, a large number of small holes about one-sixteenth inch in depth are drilled into one side of the ring near the outer edge. This surface is then sandpapered or filed so as to roughen it and is cleaned with benzol to receive the rubber diaphragm. A narrow strip of quick-vulcanizing material is then placed on the roughened surface. This strip should be about half as wide as the ring and should be placed on the outer edge. Ordinary rubber insulating tape is very good. The ring R is placed with the vulcanizing material up (see fig. 24) in the center of the table T, which has been dusted with talc. The table can be adjusted at A, to regulate to a certain extent the amount of tension which will result in the diaphragm. The sheet of rubber D, one side having been washed with benzol, is placed flat across the rim of case C with the washed side toward the ring. It is then secured to the case with rubber bands N which hold it in the groove G. The sheet should be flat, but not stretched, and the rubber bands should be very tight to prevent slipping or creeping of the sheet. The case C is then evacuated, drawing down the sheet of rubber and stretching it across the ring R and table T. In evacuating the case, the suction is gradually increased until the rubber is drawn sharply into the corners. The suction is then maintained constant at this value during the process of vulcanization. The rubber is thus kept in this position until the vulcanizing material has been cured. If leaks are apparent the suction may be left on, since the table T fits closely enough to the wall of the case C to prevent the sheet rubber from being drawn down through the opening.

¹ Mr. D. H. Strother assisted in the preparation of this section.

In order to cure the vulcanizing material and thus to attach the stretched rubber to the ring, the vulcanizing material is heated in the following manner. The surface of the stretched rubber is dusted with talc. The space *P* within the ring is filled with talc. A circular piece of cardboard *B*, or better, a thin sheet of asbestos, is cut out to fit inside the ring. This protects the portion of the rubber which will constitute the diaphragm from the heat of vulcanization. A thin sheet of aluminum *L* is placed over the entire ring extending over the edges. On top of this is placed a cast-iron disk or block *H* weighing about 3 pounds which has been heated to about 125° Centigrade. Immediately after this has been put in place and rests on the entire surface of the ring, the wooden insulator *I* and the weights *W* (200 pounds or more) are put on top of the heated block. This is permitted to stand for at least 10 or 15 minutes, the longer the better. The apparatus is then disassembled and the excess rubber trimmed from the outside edge of the ring. It is not necessary to place the ring under compression to prevent the rubber from slipping if the vulcanizing material has been properly cured. Even at a tem-

perature of +70° Centigrade the diaphragm will remain permanently secured. It should be allowed to age for several days before using in an instrument.

The vulcanizing material does not become a part of the bakelite as it does of the sheet rubber, but it adheres very well. Consequently the bakelite rings can be used over and over if desired. The pressure applied to the vulcanizing material when it is in a softened condition owing to the heat, forces it into the small holes drilled in the ring. After the rubber has cooled it continues to project down into these holes, thus effectually preventing any slipping between the ring and the vulcanizing material.

This method assures more uniform tension in the diaphragm than is at all necessary. No

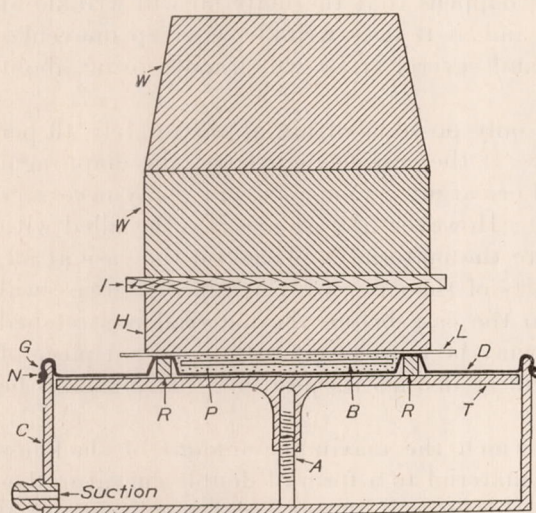


FIG. 24.—Section of apparatus for mounting rubber diaphragms

great amount of care need be exerted in constructing the apparatus. The adjustable table need not be elaborate, because no two diaphragms can be mounted exactly alike. Furthermore the amount which the rubber creeps over the rim varies for different diaphragms, the specimens vary in thickness and the elastic properties vary, not only for different mixes, but even in the same specimen.

MOUNTING LEATHERS, SKINS AND TREATED FABRICS

These materials are, or should be, always mounted in a slack condition. Difficulties in mounting often arise when the size of the pressure element is very small and the degree of slackness required is large. Some of the materials are rather stiff and very hard to handle, although most of the leathers and skins are comparatively flexible and elastic.

The following method has served well with various grades of rubberized, oiled, and doped silks and cottons, and with oiled and doped leathers, skins, and papers. The materials are often utilized without treatment and in most cases are much better and easier to mount when thus used.

A somewhat larger piece of the material than is necessary is cut out. If a rigid center is to be mounted it should be placed at the center of the material at this point in the procedure. It can be either cemented to the material or merely clamped with a suitable device. The rubber, oil, or dope on the fabric will serve as a sufficient air seal. A very small amount of stopcock grease² is also very effective as a seal here when the material used is untreated.

² Stopcock grease: Vaseline 16 parts, pure gum rubber 8 parts, paraffin 1 part. Melt all together. More paraffin may be added if the compound is not stiff enough.

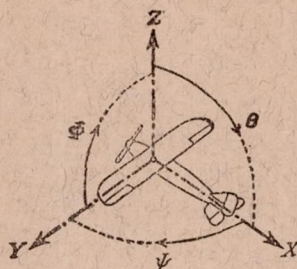
The material is then placed over the case on which it is to be mounted with the treated side placed toward the chamber. The rigid center is brought as near the center of the case as can be done conveniently by eye. Several rubber bands are put on to hold the material in place. It is preferable to have the walls of the case grooved on the outside (see fig. 24) when the diaphragm is to be tied permanently. Some instrument cases are separated like that of the Smith manometer (see fig. 2) and the diaphragm is clamped between the two parts. In these cases the same procedure may be followed.

Next by pulling the excess material on the outside, the rigid center is brought exactly to the center of the case and the material is pulled tightly across the case. More rubber bands may be put in place at this point to make more nearly uniform the creeping which is to be produced in order to secure the correct amount of slackness. In producing the slackness, either evacuate the case very slowly or else push evenly and gently on the rigid center until the right amount of slackness is obtained. The creeping about the rim is usually uniform and the rigid center will stay in the middle. It seldom happens that the material will wrinkle at the rim except with very small pressure elements and, if it does wrinkle or creep unevenly, the addition or removal of several rubber bands and several trials will usually bring about the desired results.

After obtaining the proper slackness (which is only possible within approximately 10 per cent.) the diaphragm may be permanently secured. If the case has a groove, the diaphragm is secured by tying it onto the rim with silk thread or catgut. Cement is not really necessary unless rather high pressures are to be encountered. However, the groove may be filled with a slow-drying cement, such as rubber cement, before the material is placed on the case at all. If the diaphragm is to be clamped between two parts of the case, a little stopcock grease will serve to seal the joint. This should be applied to the case before the material is stretched across it at all. If screw holes are necessary they may be burned out with a red-hot piece of wire. This procedure will not injure the diaphragm or disturb its position. This should be the last step.

In mounting very small slack diaphragms for which the maximum amount of slackness is to be obtained, it may be necessary to press the material to a form of double curvature between dies before mounting it in an instrument. However, if the material can be mounted according to the above procedure, it should assume its correct form when deflected during the process of adjusting the slackness. This part of the procedure must be investigated in each individual case, and no general statement can be made about it. Some materials with good elastic properties, a good example of which is alum-tanned colon leather, will give no trouble at all. Cotton fabric or goldbeater's skin will wrinkle badly if the slackness is adjusted too rapidly.





Positive directions of axes and angles (forces and moments) are shown by arrows.

Axis.		Force (parallel to axis) symbol.	Moment about axis.			Angle.		Velocities.	
Designation.	Sym- bol.		Designa- tion.	Sym- bol.	Positive direc- tion.	Designa- tion.	Sym- bol.	Linear (compo- nent along axis).	Angular.
Longitudinal.....	X	X	rolling.....	L	Y → Z	roll.....	Φ	u	p
Lateral.....	Y	Y	pitching....	M	Z → X	pitch.....	Θ	v	q
Normal.....	Z	Z	yawing.....	N	X → Y	yaw.....	Ψ	w	r

Absolute coefficients of moment

$$C_l = \frac{L}{q b S} \quad C_m = \frac{M}{q c S} \quad C_n = \frac{N}{q f S}$$

Angle of set of control surface (relative to neutral position), δ . (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS.

Diameter, D

Pitch (a) Aerodynamic pitch, p_a

(b) Effective pitch, p_e

(c) Mean geometric pitch, p_g

(d) Virtual pitch, p_v

(e) Standard pitch, p_s

Pitch ratio, p/D

Inflow velocity, V'

Slipstream velocity, V_s

Thrust, T

Torque, Q

Power, P

(If "coefficients" are introduced all units used must be consistent.)

Efficiency $\eta = T V / P$

Revolutions per sec., n ; per min., N

Effective helix angle $\Phi = \tan^{-1} \left(\frac{V}{2\pi r n} \right)$

5. NUMERICAL RELATIONS.

1 HP = 76.04 kg. m/sec. = 550 lb. ft/sec.

1 kg. m/sec. = 0.01315 HP

1 mi/hr. = 0.44704 m/sec.

1 m/sec. = 2.23693 mi/hr.

1 lb. = 0.45359 kg.

1 kg. = 2.20462 lb.

1 mi. = 1609.35 m. = 5280 ft.

1 m. = 3.28083 ft.

