REPORT No. 219

SOME ASPECTS OF THE COMPARISON OF MODEL AND FULL-SCALE TESTS

By D. W. TAYLOR

National Advisory Committee for Aeronautics

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Secretary National Advisory Committee for Aeronautics

Aeronautics now covers a large field. The bibliography alone, compiled and published annually by the United States National Advisory Committee for Aeronautics, requires something like 200 pages of a book 7 inches by 10 inches. Needless to say, I am not undertaking to review the whole field.

Owing to the difficulties of conducting free flight tests of performance and the fact that we can not afford to make many mistakes in an appliance whose operation involves the risk of human life, it is peculiarly desirable that we may be able to predict the performance of the completed airplane from small-scale experiments; and probably in no other branch of mechanical science have we at present so many research laboratories.

In view, then, of the universal use of models and wind tunnel tests to obtain results upon which are based predictions of performance of full-sized airplanes, it appears worth while to give some consideration to the foundation, as it were, of such methods. The mathematical basis of the law of mechanical similitude has been traced back as far as Sir Isaac Newton, but it is believed that the first serious practical application was that made by Mr. William Froude, when, some 65 years ago, with the aid of the Admiralty, he built in his garden at Torquay a long tank filled with water, in which he tested models of vessels. Froude's methods have been universally accepted by naval architects as of great value, and they are able to predict performances of full-sized ships with accuracy adequate to the purposes of the engineer. Nevertheless, they are not exact, and in the last analysis their justification is due to the fact that the results they predict for the full-sized ship are substantially verified in practice. However, Froude separated the frictional resistance of the model from its wave-making resistance, or the resistance absorbed in the production of waves, and it is to the latter only that Froude's law of comparison applies. Frictional resistance is calculated from coefficients originally determined by Froude upon the basis of tests with comparatively small plane surfaces at low speeds, and it is generally recognized now by naval architects that large-scale experiments would be desirable to give us greater assurance of accuracy when dealing with present-day ships.

The most fundamental and instructive method of covering this whole question of the value of model experiments is based upon the principle of dimensional homogeneity first fully enunciated, I believe, some 15 years ago by a Russian, Riabouchinski. In the United States, Doctor Buckingham has taken up the matter and done much work to amplify, clarify, and apply the principle. In a paper in 1915 before the American Society of Mechanical Engineers he gave a number of illuminating applications. In the mathematical treatment below I follow essentially Buckingham's methods.

Instead of considering the general formula, which may be of a beautiful simplicity to the mathematical physicist but is not too easy to follow for us who are not mathematical physicists, I will consider only the general case applying to motion of objects in a fluid medium. The

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[1] The thirteenth annual Wilbur Wright memorial lecture, read in London before the Royal Aeronautical Society of Great Britain Apr. 30, 1925, by Commander J. C. Hunsaker for Dr. D. W. Taylor and published by permission of that society.

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first thing to establish is the quantities involved; that is, the physical quantities present which can affect the case. Let us denote by R the resistance of the object; by V its speed; by L its size, or some linear dimension; by ρ the density of the fluid; by μ the viscosity of the fluid; by C the compressibility of the fluid; and by g the acceleration of gravity. In addition there are certain ratios present which I will denote by r_1, r_2, etc. These ratios express certain physical facts, such as aspect ratio of an airfoil, its angle of attack, etc. They are all independent of size.

It may be that the quantities enumerated above do not comprise all of the quantities we should consider, but they do comprise the most obvious ones and are sufficient in number to illustrate the point desired to be made. Now all these quantities can be expressed in terms of three units, and we will choose the simplest and most commonly used units, namely, those of mass, m; length, l; and time, t. Each of these physical quantities also has well-known dimensions. The table below gives the quantities enumerated above and their dimensions in m, l, and t.

	Quantities involved	Dimensions in m, l, and t
R	Resistance	$m\,l\,t^{-2}$
V	Speed	$l\,t^{-1}$
L	Size	l
ρ	Density of fluid	$m\,l^{-3}$
μ	Viscosity of fluid	$m\,l^{-1}\,t^{-1}$
C	Compressibility of fluid	$m^{-1}\,l^{-1}\,t^{2}$
g	Acceleration of gravity	$l\,t^{-2}$
r_1, r_2, etc.	Ratios	Dimensionless.

Now if the quantities above have a relation connecting them it may be written as follows:

$$F\,(L,\, \rho,\, V,\, R,\, \mu,\, C,\, g,\, r_1,\, r_2,\, -\!-\!-) = 0 \tag{1}$$

This equation, of course, teaches us nothing except that there is some relation between the seven physical quantities entering the case. Now let us choose three of the above quantities (this is because we have three units to express them all), and, instead of writing the relation symbolically between the seven simple quantities, let us use the following, involving the seven quantities in four compound quantities or variables:

$$F\,(L^a\,\rho^b\,V^c\,R,\; L^d\,\rho^e\,V^f\,\mu,\; L^g\,\rho^h\,V^k\,C,\; L^j\,\rho^m\,V^n\,g,\, r_1,\, r_2,\, -\!-\!-) = 0 \tag{2}$$

By the principle of dimensional homogeneity, since the physical relations or facts expressed by the above do not change with change of units, the compound variables or quantities above must be dimensionless; that is, of zero dimensions. By expressing their dimensions in terms of the dimensions of the three fundamental units, we have for each quantity three equations to determine the exponents a, b, c, etc. Let us take the first quantity. Our dimensional equation is

$$L^a\,\rho^b\,V^c\,R = l^a\,m^b\,l^{-3b}\,l^c\,t^{-c}\,m\,l\,t^{-2}$$
$$= l^{a-3b+c+1}\,m^{b+1}\,t^{-c-2}$$

In order that the expression may be dimensionless, we must have the index of l, for instance, equal to zero; that is, $a-3b+c+1$ equal to zero; similarly, the indices of m and t must equal zero. This gives us the three equations below, whose solution is obvious:

$$\left\{ \begin{array}{l} a-3b+c+1=0 \\ b+1=0 \\ -\,c-2=0 \end{array} \right\} \text{whence} \begin{array}{l} a=-2 \\ b=-1 \\ c=-2 \end{array}$$

So, our first quantity is $\dfrac{R}{\rho L^2 V^2}$

Proceeding to the second quantity and treating it in exactly the same manner, we have the following:

$$L^d\ \rho^e\ V^f\ \mu = l^d\ m^e\ l^{-3e}\ l^f\ t^{-f}\ m\ l^{-1}\ t^{-1}$$
$$= l^{d-3e+f-1}\ m^{e+1}\ t^{-f-1}$$

Whence—

$$d - 3e + f - 1 = 0$$
$$e + 1 = 0$$
$$f - 1 = 0$$
$$d = -1$$
$$e = -1$$
$$f = -1$$

Then our second expression is $\dfrac{\mu}{L\rho V}$. Of course in practice it makes no difference whether

we use the above expression or its reciprocal. The ratio $\dfrac{\mu}{\rho}$ is a quantity a good deal used in

physical parlance, called the kinematic viscosity, and denoted by ν. So our second vari-

able will be $\dfrac{\nu}{L V}$.

Proceeding in just the same manner for the third and fourth quantities we finally reduce the general equation to

$$F_1\left(\frac{R}{\rho L^2 V^2}, \frac{\nu}{L V}, \rho L^2 V^2 C \frac{Lg}{V^2},\ r_1, r_2 - - \right) = 0 \tag{3}$$

Now $\rho L^2 C$ has of course the dimensions of $\dfrac{1}{V^2}$, and since the velocity of sound in air is propor-

tional to $\dfrac{1}{\sqrt{\rho C}}$, if we denote by V_s the velocity of sound in air we may use instead of the variable

$\rho L^2 V^2 C$ the variable $\dfrac{V^2}{V_s^2}$.

Now we can solve the above symbolically for any one of the compound variables. Solving for the first, we have

$$\frac{R}{\rho L^2 V^2} = F_2\left(\frac{\nu}{L V}, \frac{V^2}{V_s^2}, \frac{Lg}{V^2},\ r_1, r_2\right)$$

Since we are interested primarily in the resistance, R, let us transform the above as below:

$$R = \rho L^2 V^2 \times F_2\left(\frac{\nu}{L V}, \frac{V^2}{V_s^2}, \frac{Lg}{V^2},\ r_1, r_2 - - \right) \tag{4}$$

The ratios r_1, r_2, etc., express such things as aspect ratio, angle of attack, etc., and hence are obviously the same for the model as for the full-sized object, so that for purposes of comparison between model and full-sized object they affect the case only as constants or fixed coefficients, to be determined by experiment or some other independent method, and can be eliminated from the equation above. This reduces us finally to the general equation:

$$R = \rho L^2 V^2 F_3\left(\frac{\nu}{L V}, \frac{V^2}{V_s^2}, \frac{Lg}{V^2}\right) \tag{5}$$

This, then, is a relation which follows if all of the factors which we originally assumed enter into the case and affect our results. We do not know whether, as a matter of fact, all these factors do affect them as indicated by the general expression for R above. But, obviously, if all of the factors enumerated materially affect our results, producing the preceding equation (5), model experiments are of no value for predicting the performance of the full-sized object. In the model experiment, we make an object differing in scale from the full-sized object, and test it at a speed different from that of the full-sized object. If this method is to be of value in

practice, as a general thing models should be smaller and tested at a lower speed than for the full-sized object. From equation (5) above, however, we see, considering the first combined variable, which we must have constant in proceeding from model to full-sized object, that LV must be constant, since we know that ν is practically constant for standard air. Considering the second term, however, since the velocity of sound in air is constant, if $\frac{V^2}{V_s^2}$ is to be constant V must be constant. Considering the third term, g is constant, and if $\frac{Lg}{V^2}$ is to be constant we must have $\frac{L}{V^2}$ constant. These three requirements evidently reduce to the single one, namely, that neither L nor V can change. In other words, we can not use the model and obtain results for the full-sized object.

However, the equation (5) does not necessarily apply to the case unless it is confirmed by theoretical demonstration, experience, or practical tests. We do not know that as a matter of fact, the compound quantities which we originally considered as possibly affecting the case do all affect it. Suppose none of these quantities has any effect. We then have the exceedingly simple formula

$$R = \rho L^2 V^2 \text{ times a coefficient}$$

If this expresses the facts, a single experiment at a single speed of a model gives us complete information on the resistance at all speeds for all sizes of similar objects.

There is a theoretical basis for regarding this as a basic expression for resistance, the departures from it being, if appreciable, of secondary importance. If the disturbance of a fluid by an object moving through it, or, what is simpler to grasp, if the lines of flow of a fluid past a submerged object do not change with speed, then all forces vary as the square of the speed. For any force is measured by the momentum generated in unit time in the opposite direction, and, taking momentum at such a distance that pressure is not affected, the momentum generated in unit time is proportional to the square of the velocity. Similarly, if the lines of flow are similar as we change size, the momentum generated must vary as L^2. From consideration of a perfect nonviscous fluid we reach similar conclusions, but, as it happens, in a theoretical perfect fluid objects have theoretically no resistance. Concluding, then, that the expression

$$R = \rho L^2 V^2 \text{ times a coefficient}$$

is a correct first approximation, let us see what we can do to reach a closer approximation.

Suppose that only one of the terms of F_s in (5) is significant, the rest having no bearing. If the first term is the only significant one, we have

$$R = \rho L^2 V^2 F_s \left(\frac{\nu}{LV} \right)$$

Similarly, if the second and third terms are the only significant ones, we have

$$R = \rho L^2 V^2 F_s \left(\frac{V^2}{V_s^2} \right)$$

$$R = \rho L^2 V^2 F_s \left(\frac{Lg}{V^2} \right)$$

Now, it is obvious that if only one term is significant, we may or may not have a possible basis for model experiment, depending upon the nature of the term. Consider first the expression

$$R = \rho L^2 V^2 F_s \left(\frac{\nu}{LV} \right)$$

Here the requirement is that $\frac{\nu}{LV}$ should be constant, or, what is the same thing, that $\frac{LV}{\nu}$

should be constant. This results in the undesirable condition that if ν is constant LV must be constant as we pass from model to full-sized object, so that the smaller the model the higher the speed at which it must be tested. This is not a very desirable condition.

Consider now the second requirement:

$$R = \rho L^2 V^2 F_2 \left(\frac{V^2}{V_s^2} \right)$$

If $\frac{V^2}{V_s^2}$ is constant, since V_s itself is constant, V must be constant, and we can not use a low-speed model.

Consider now the third condition. Here our requirement is that $\frac{Lg}{V^2}$ is constant. This is the form found so useful in testing ships' models. The relation that speeds shall be as the square root of linear dimensions results in the test speed for the model being low, so that tests can be easily made. Evidently, however, for a body completely submerged in and surrounded by a fluid, the action of gravity can have practically no effect until the proposed speeds approach the point where vacuua are formed in the fluid. Hence we can confidently eliminate from our general equation above the variable $\frac{Lg}{V^2}$ as the one to govern our second approximation.

Consider next the variable $\frac{V^2}{V_s^2}$. Practically all the speeds with which we are concerned in airplane work, except some propeller speeds, are far below the velocity of sound through air, and there is little reason to believe that the compressibility of the air has a material effect, because the compression is so small. Also, experiments with projectiles indicate the same conclusion. Hence we can eliminate the second term as the one that we must keep when we seek a second approximation. When we come to the first term, however, the case is different. We know that air has viscosity, and we know that the viscosity must have some action at all speeds. Hence the first term can not be argued away on general principles. Also, it may be remarked in passing that the expression

$$R = \rho L^2 V^2 F_3 \left(\frac{\nu}{LV} \right)$$

can be derived independently of considerations of dimensional homogeneity from the equations of motion of a viscous fluid. However, these equations of motion necessarily assume in the first place that there is no other factor, such as compressibility, gravity, etc., involved. We might have originally assumed some more physical quantities present and affecting matters such as nature of surface, or sizes of turbulent vortices in the wind tunnel, but we seem to be restricted to one variable in our F_3 function if we are to profit by model experiments and the viscosity variable seems the one we should choose. The wisdom, or otherwise, of the choice will be shown if model experiments in accordance with the formula do or do not predict full scale performance.

Having then reduced our original broad formula to

$$R = \rho L^2 V^2 F_3 \left(\frac{LV}{\nu} \right) \tag{6}$$

involving the density, the size, the speed, and some unknown function of $\frac{LV}{\nu}$, the well-known Reynolds Number, we need to form some conception of the effect of Reynolds Number, commonly called the scale effect. While we do not known the form of the function, we do know for the flow of water, oil, and air in pipes the relative experimental values. The original wonderful experiments by Reynolds have been repeated and amplified by others since 1880, and it seems established that at low speeds where the fluid flows smoothly F_3 has one set of alues, and at high speeds when the motion is completely turbulent there is another well-defined set of

values, while for intermediate speeds values are rather indeterminate. Wind tunnel investigations on such objects as cylindrical wires, struts, and streamline wires show that the resistance departs appreciably from the law of the square with variation of Reynolds Number.

When we come to such objects as an airplane, however, we have difficulty with the ordinary wind tunnel. For constant Reynolds Number to test a model, say, one-twentieth scale, would require wind tunnel speed 20 times the actual flying speed, and there are no wind tunnels that can come in sight of this performance. Such speeds would be greater than the velocity of sound. There appears to be only one practicable solution of the difficulty, namely, the use of a testing tunnel where we vary the density of the air and hence the value of ν.

The kinematic viscosity coefficient ν for air varies inversely as the pressure, and decreases with temperature according to a somewhat complicated relation. Table I below gives numerical values when the unit of length is the centimeter and the unit of time the second.

TABLE I

KINEMATIC VISCOSITY COEFFICIENTS ν IN cm²/sec.

Temperature, centigrade	Pressure in atmospheres				
	1/10	1	5	10	20
50	1.284	0.1284	0.02568	0.01284	0.00642
40	1.292	.1292	.02584	.01292	.00646
30	1.300	.1300	.02600	.01300	.00650
20	1.308	.1308	.02616	.01308	.00654
10	1.318	.1318	.02636	.01318	.00659
0	1.329	.1329	.02658	.01329	.00665
−10	1.340	.1340	.02680	.01340	.00670
−20	1.351	.1351	.02702	.01351	.00676
−30	1.364	.1364	.02728	.01364	.00682
−40	1.378	.1378	.02756	.01378	.00689
−50	1.392	.1392	.02784	.01392	.00696

The variable-density wind tunnel of the National Advisory Committee for Aeronautics, as originally suggested by Doctor Munk of our staff, was described to the society two years ago, and a few sample results given. A good deal of experience has been had since then with the appliance. One lesson of experience has been that when we are working under a pressure of 20 atmospheres it takes but a small electrical spark to kindle a substantial fire. However, these little practical difficulties have been overcome, and experience in testing a number of different airfoils, etc., indicates that this apparatus, or the equivalent, is essential if we are to make a thoroughly reliable second approximation to the performance of an airplane from model tests.

Reynolds Number $\dfrac{LV}{\nu}$ is a compound ratio whose numerical value in the case of any given object depends upon the ratios between the actual values of LV and ν and their unit values. Unfortunately, each type of object has its own series of Reynolds Numbers because as a rule the values of L are not comparable for dissimilar objects. Thus for an airplane wing we naturally use for L in Reynolds Number the length of the chord. For an airship we would use the length or the diameter or any linear function of the two. But L for the airship would not be comparable with L for the airplane.

Considering airplanes as they are, using the chord of the wing in inches as L, and speeds in statute miles per hour, the Reynolds Numbers come out fairly large. Thus for an airplane of 5-foot chord, at 100 miles per hour in a normal atmosphere, the Reynolds Number will be some 4,800,000. For its model of 6-inch chord, in a wind tunnel at 100 miles per hour with normal air, the Reynolds Number will be 480,000.

Attention is invited now to figures 1 to 3, giving in condensed form results of recent tests of three airfoils of well-known form in the variable-density wind tunnel. Necessary data as to the conditions and the airfoil section to which they apply are shown on each figure. Results are plotted as curves of lift and drag coefficients as ordinates over angles of attack—α—as abscissae, following the standard practice of the United States National Advisory Committee

for Aeronautics. Figure 1 shows results for an American section, U. S. A. 27; Figure 2 shows results for a British section, R. A. F. 15; Figure 3 shows results for a German section, Göttingen 387. It happens that these three typify the medium, the thin, and the thick sections.
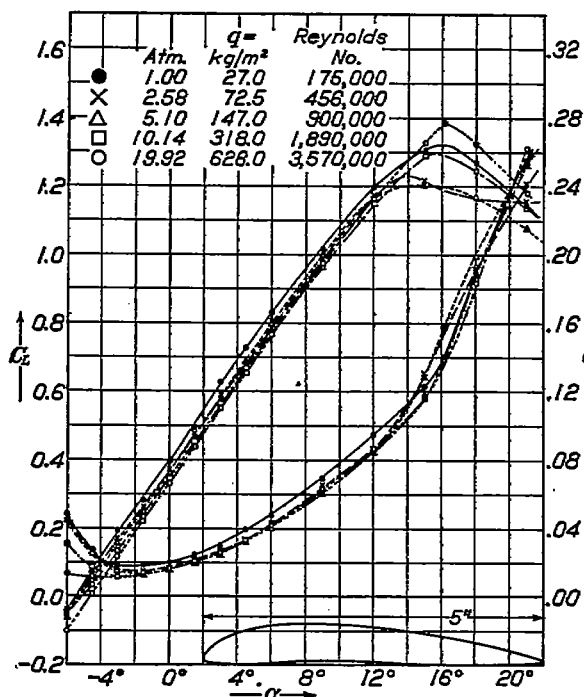

FIG. 1.—Lift and drag characteristics of USA 27 as tested in the Variable Density Wind Tunnel
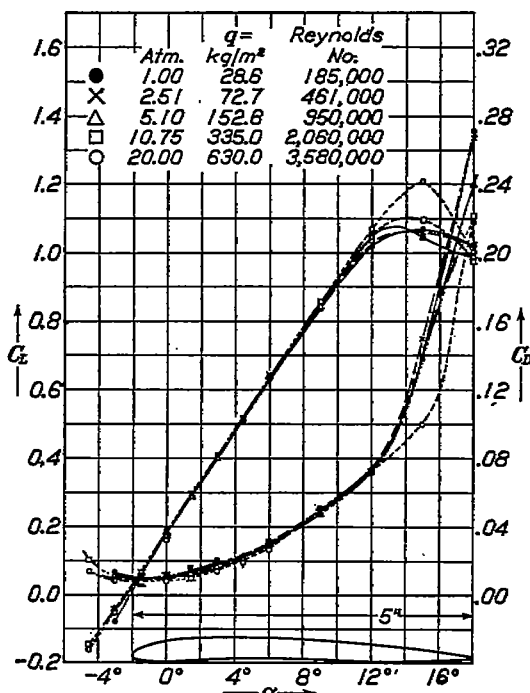

FIG. 2.—Lift and drag characteristics of RAF 15 as tested in the Variable Density Wind Tunnel
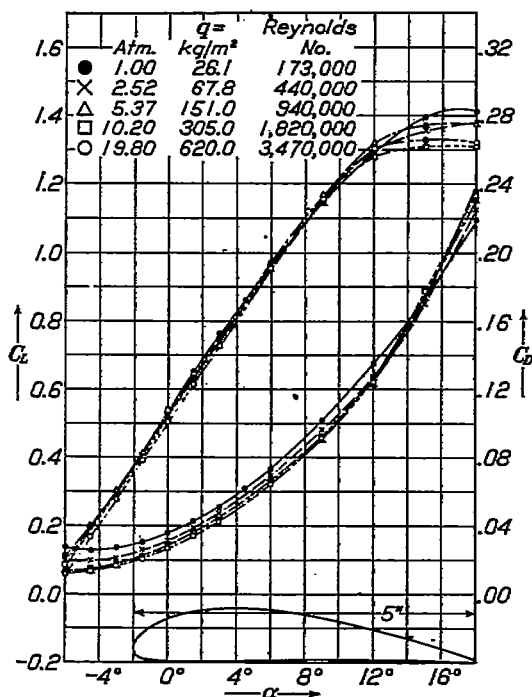

FIG. 3.—Lift and drag characteristics of Gottingen 387 as tested in the Variable Density Wind Tunnel

Ignoring minor eccentricities due to accidental causes, unavoidable experimental error, etc., these curves seem to warrant a few broad conclusions, which, by the way are in agreement with other results too numerous to include.

In the first place, the scale effect appears to have more influence upon the drag than upon the lift. This may be explained upon theoretical grounds.

In the second place, the scale effect increases more and more slowly as the Reynolds Number increases, so that conclusions drawn from experiments with airfoils within the Reynolds Number range of ordinary wind tunnels can not safely be extended to much larger Reynolds Numbers.

In third place, the consistency of the results gives us reason to think that for present-day airplanes we are justified in ignoring the effect of other factors than Reynolds Number in reaching our second approximation to aerodynamic properties of airfoils.

In the fourth place, the thin airfoil appears to show less scale effect than the thick airfoil.

In the fifth place, so far as airfoil action is concerned, the scale effect is, after all, secondary, though by no means negligible when we undertake to estimate closely.

The comments above apply only to airfoils. They do not necessarily apply to wires, struts, etc. Such appendages can be tested separately in the ordinary wind tunnel at Reynolds Numbers much closer to the numbers on the full-sized airplane than is possible with the airplane structure proper.

When we enter the somewhat vexed field of aircraft propellers, model experiment is unquestionably our surest guide. Here, as in all other cases where we utilize model experiment, we must finally assure ourselves by experience or full-scale experiment that we have a safe law of comparison, but the difficulty of accurate full-scale propeller tests in free flight renders it almost essential for the present that we investigate laws of propeller action by model experiment.

This has been done with good results in the marine field and air propellers are even more favorably circumstanced. For instance, for propellers in water we can not apply our law of comparison when cavitation is present. Cavitation does not trouble air propellers as yet. The propeller driving a vessel, assuming the atmospheric pressure as equivalent to 34 feet of water, is working in an inelastic fluid under a total head to the center of propeller of, say, from 35 to 60 feet. The airplane propeller is working in an elastic fluid under a total head, when near the ground, of something like 6 miles. This not only eliminates cavitation but enables us to adopt efficient blade sections that would be impossible in water. The airplane propeller designer is fortunate in this fact and in the further fact that he can use two-bladed propellers. These are capable of more efficiency than three or four bladed propellers, but can seldom be used for marine propellers because when working in an irregular stream, as they must at the stern of a ship, they are liable to cause excessive vibration of the ship.

Models in water act very much as in air, and experiments with thin, narrow two-bladed propellers in water show efficiencies fully as good as those of models of airplane propellers. Table II below gives maximum efficiencies in water of some two-bladed model propellers. They had ogival blade sections, straight faces of uniform pitch, and circular arcs for backs, the edges being sharp, not rounded.

TABLE II

MAXIMUM EFFICIENCIES OF 2-BLADED 16-INCH MODEL PROPELLERS IN WATER

Camber ratio at 0.75 radius	Mean width ratio	Pitch ratios					
		0.4	0.6	0.8	1.0	1.2	1.5
		Maximum efficiencies					
0.1241	0.075				0.85		
.0744	.125				.84		
.0465	.200	0.58	0.71	0.78	.82	0.83	0.84
.0338	.275				.79		

From "Some Results of Tests of Model Propellers," by A. V. Curtis and L. F. Hewins, Transactions Society of Naval Architects and Marine Engineers, Vol. 13, 1905.

The airplane propeller designer labors under one disadvantage. There is no doubt that as propeller tip speeds in air approach the velocity of sound, we may expect radical departures from the laws of action at lower speeds. That is a complication I shall not attempt to unravel.

Before taking up the model experiment end of propeller action, it may be as well to take up one general consideration.

Since thrust of a propeller is proportional to the sternward momentum per second generated by its action upon the fluid in which it works, it follows that there must be a certain energy carried off in the fluid to which velocity is communicated. Hence, no propeller can show an efficiency of 100 per cent, and the actual efficiency of an ideal propeller at a given speed of advance must always diminish as its thrust increases. Following the treatment proposed by McEntee in 1906 for propellers operating in water, we can gain some idea of limits in air.

Suppose we have an ideal frictionless propelling apparatus which takes hold of the air and discharges it directly aft without change of pressure and with uniform absolute velocity u feet per second, the velocity of advance of our ideal apparatus with reference to undisturbed air being v feet per second. Then, if A denotes the area in square feet of the slipstream, the mass of the air acted upon per second is

$$\left(\frac{w}{g}\right) A \; (v+u).$$

The thrust T in pounds from Newton's third law is equal to the sternward momentum generated per second, or

$$T = \left(\frac{w}{g}\right) A \; (v+u) \; u$$

Useful work equals

$$Tv = \left(\frac{w}{g}\right) A \; (v+u) \; uv$$

The lost work, or kinetic energy, of the air discharged equals

$$\left(\frac{w}{g}\right) A \; (v+u) \left(\frac{u^2}{2}\right)$$

Whence gross work equals

$$\left(\frac{w}{g}\right) A \; (v+u) \; uv + \left(\frac{w}{g}\right) A \; (v+u) \left(\frac{u^2}{2}\right)$$

Efficiency e equals

$$\frac{\text{Useful work}}{\text{Gross work}} = \left(\frac{v}{v+\frac{u}{2}}\right)$$

If we solve for u in the expression for thrust, we have

$$u = \sqrt{\left(\frac{v^2}{4} + \frac{gT}{wA}\right)} - \frac{v}{2}$$

Substituting in the expression for efficiency, we have

$$e = \frac{4}{\left\{ 3 + \sqrt{\left(\frac{4g}{w} \frac{T}{Av^2} + 1\right)} \right\}}$$

This, then is the general formula for the efficiency of an ideal frictionless propelling apparatus, discharging the fluid passing through it without increase of pressure and accompanying loss of efficiency.

Applying to an air propeller of diameter d feet, substitute $\frac{\Pi d^2}{4}$ for A. Also give g its standard value of 32.174 foot/seconds^2, w the value for standard air of 0.07651 pound/feet^3, and express v in miles per hour V instead of feet per second.

When this is done our formula for air becomes

$$e = \frac{4}{\left\{3 + \sqrt{\left[\frac{(16 \times 32.174)}{(0.07651 \times 2.15\text{II})} \frac{T}{d^2 V^2} + 1\right]\right\}}} = \frac{4}{\left\{3 + \sqrt{\left(\frac{99.56}{d^2 V^2} \frac{T}{} + 1\right)\right\}}}$$

Figure 4 shows contours of ideal efficiency derived from the above equations plotted upon values of d and $\frac{T}{V^2}$.

It is seen that curves of constant efficiency are parabolas, with values of $\frac{T}{V^2}$ as ordinates and values of diameter as abscissae; also, once we have fixed the diameter and the value of $\frac{T}{V^2}$ we fix the efficiency. It is obvious also that if, for a given diameter, we increase thrust without changing speed, or if, for a given diameter, we decrease speed without changing thrust, the efficiency necessarily falls off.
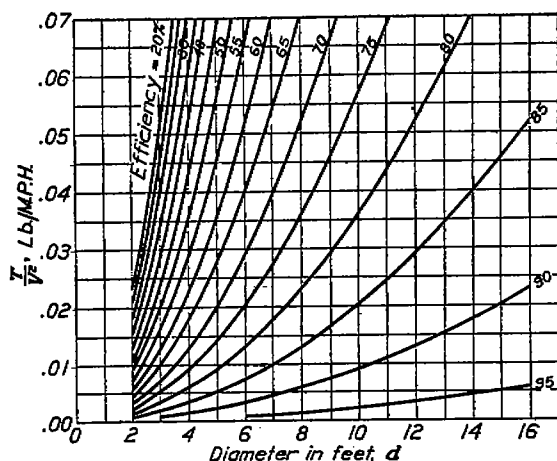

FIG. 4.—Ideal propeller efficiency in air as affected by thrust in
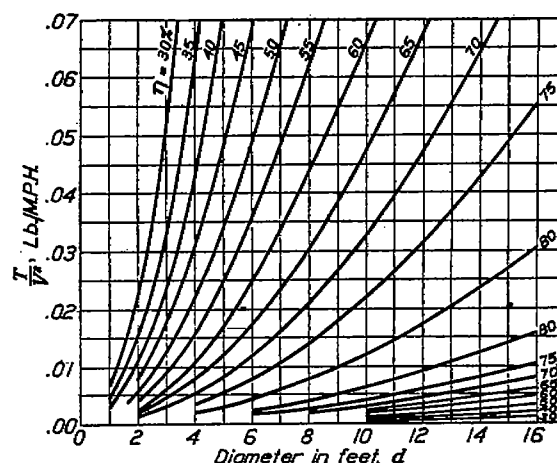pounds—T, speed in M. P. H.-V, and diameter in feet—d

FIG. 5.—Actual propeller, efficiency in air derived from model pro-
peller tests—propeller E Table III

While the above conclusions can be legitimately drawn from Figure 4, we must not forget that this refers to an ideal propeller of the best possible efficiency. Actual propellers in operation lose not only by the energy carried off in the wake, but by their friction and the energy due to transverse motion in the wake, both tending to reduce efficiency. If we assume a law of comparison, which will be discussed later, we can, from tests of a model propeller, draw a diagram similar to Figure 4, covering the performance of all propellers similar to the model. This is done in Figure 5 for a propeller of 0.9 pitch ratio, propeller "E" of Table III tested by Doctor Durand. It will be observed that in its general features Figure 5 corresponds fairly well with the ideal diagram of Figure 4. However, the efficiency contours, instead of increasing indefinitely as we increase the diameter and decrease the values of $\frac{T}{V^2}$, reach a maximum, and then for smaller values of $\frac{T}{V^2}$ the efficiency falls off very rapidly. Above the maximum line the agreement with the ideal diagram is better. We still have the feature that the efficiency of this family of propellers is dependent upon the diameter and the value of $\frac{T}{V^2}$. If in level flight we are operating above the parabola of maximum efficiency and undertake to climb, the value of $\frac{T}{V^2}$ necessarily increases and the efficiency necessarily falls off. If we are operating in level flight in the region below the contour of maximum efficiency and then

undertake to climb, the efficiency increases for a time and then falls off as before. In any case, however, the efficiency of a given propeller varies with the flight conditions.

A propeller is an object moving through air and our general equation (4) applies. Rewriting this with T to denote thrust instead of R to denote any resistance, and diameter D in place of L, we have

$$T = \rho V^2 D^2 F_3 \left(\frac{\nu}{DV}, \frac{V^2}{V_s^2}, \frac{Dg}{V^2}, r_1, r_2, --- \right) \tag{4}$$

As before, we can confidently eliminate the gravity variable $\frac{Dg}{V^2}$. When it comes to the compression variable $\frac{V^2}{V_s^2}$, we are not upon such sure ground as for the airplane, because tip speeds are much greater than airplane speeds. Probably some types of blade section can approach much more closely than others the velocity of sound in the air without it being necessary to take account of the compression variable. When that does become necessary the problem can be met by testing propeller models in high speed wind tunnels, V being airplane speed. It would seem from this point of view that even now model propellers should be tested at full speed. Eliminating the compression variable as not yet important, we come to the same general expression as for the airplane or

$$T = \rho V^2 D^2 F_3 \left(\frac{\nu}{DV} \right)$$

Here we meet the same Reynolds Number complication. Now, we have seen in dealing with airfoils that the scale effect correction for them was secondary. We know, too, that in propeller action, where a blade is attacking air already somewhat disturbed by its previous passage or the passage of another blade, we may expect great turbulence in the action and we know that the greater the turbulence the less the scale effect correction for viscosity.

So we conclude that if we drop the Reynolds Number variable we have left a first approximation sufficiently close to exactness for practical present-day engineering purposes.

This leads us to the simple expression

$$T = \rho V^2 D^2 F_3 \left(r_1, r_2, --- \right)$$

If this were exact, a model of any size at any speed would tell us all we need know, but bearing in mind that our expression is approximate and the nature of the quantities ignored, we should make our models as large as possible and test them at as high a speed as possible.

The quantities r_1, r_2, etc., are characteristic ratios and it is very important if we are to utilize model propeller tests to best advantage that we use ratios that are truly and adequately characteristic. Systematic treatment is necessary here. Much progress has been made in connection with aeronautic propellers by regarding their blades as composed of sections of airfoils, and this is essentially a fruitful method of procedure in our search for more efficient forms. It is interesting to note that the underlying idea is the same as that of Mr. William Froude, when for the marine propeller, in 1878, he put forward his blade theory, in which the propeller blades were regarded as made up of plane elements advancing through the water. Mr. Froude at that time said, with much justice:

No theoretical treatment of the action of an actual screw can be sound which does not incorporate and mainly rest on the principles embodied in the treatment of the problem of the plane, and, indeed, the character of the results must, in their most essential features, be the same in both cases.

The fundamental difficulty with the Froude blade theory was that propeller blades are not a plane of no thickness. This fact has been fully recognized by aeronautic designers in treating them as made up of airfoil elements, but after all the model propeller and the full-sized propeller must each be treated in the end as a whole, and our ratios must be based upon that fact.

It is obviously desirable to have diameter appear in all ratios if possible, since diameter is our basic dimension.

The first thing we need to consider is the most desirable ratio by which to express the obliquities of the blades, or the angles which they make with the axis of the propeller. This is sometimes done by stating the angles, but the best plan appears to be to adhere to the idea of pitch. This tends to become absurd in propellers with blades relatively as thick as those used on airplanes, but after all it is always possible at any section to establish a line making a definite angle with the axis. If the propeller has a working face with any material portion of it flat or straight in section on the driving face, this is naturally the line used to express pitch. However that may be, for the family of propellers derived from a given model, the ratio between pitch and diameter is always constant, and as a rule the whole family may be characterized by the extreme pitch ratio. This is about the simplest quantity we can use which gives an idea of the general features of the propeller with reference to the pitch, or blade obliquity if we prefer that expression. Then one of our ratios is the ratio between pitch and diameter, denoted by a.

We need something expressing relative blade width. Aspect ratio will do it, but there seems no necessity for departing from what is frequently called in marine propellers the mean width ratio, namely, the ratio between the mean or average width of the blade and the diameter. If we were always dealing with blades of the same developed outline, it would be simpler and better to use the ratio between the maximum width, a thing we need always to know and use, and the diameter. This would quite well characterize the propeller, but does not seem to be quite so good for universal use in view of variations in blade outline.

We come now to the most difficult ratio to express in practice. We need something to characterize the blade thickness. If propellers all had radially straight faces and straight backs, the simplest and obvious plan would be to extend the line of the face to the axis, the line of the back at maximum thickness to the axis, and express the characteristics of the propeller as regards blade thickness by the ratio between the intercept on the axis thus obtained and the diameter. This is a method which has come into a good deal of use for marine propellers of late years, but the backs of aeronautical propellers vary so much that it is doubtful if we are yet ready to adopt this as a standard ratio. I suggest tentatively for this last ratio the camber ratio at three-fourths of the radius.

It is now necessary to consider what to do with results of model tests.

These results, such as curves of thrust and torque or dimensionless coefficients derived from them, including curves of efficiency of propeller, are usually plotted initially upon the dimensionless quantity $\frac{V}{nD}$ when V is speed of advance of the propeller with reference to undisturbed air in feet per second, n denotes revolutions per second, and D is diameter of the propeller in feet. Now $\frac{V}{nD}$ is a natural coefficient and excellent as a basic variable when we are dealing with one propeller, but when dealing with systematic propeller research and making diagrams for design purposes it is somewhat lacking. For a single propeller, when we plot upon $\frac{V}{nD}$ we are virtually plotting upon the slip ratio s, since if a denote pitch ratio $\frac{V}{nD} = a(1-s)$.

The question of the basic variable to be used in plotting experimental data for design purposes is a very important one and worthy of a little examination. In the first place this basic variable must be dimensionless since we wish to use model results for dealing with full-sized propellers. There are any number of dimensionless functions available and they are readily converted one to another.

In the second place, looking at the matter from the design point of view, our basic variable should take account of or involve all the quantities known or assumed upon which a propeller design depends. Here we meet the fact that we do not necessarily base a propeller design always upon the same quantities. However, considering airplane, propeller, and motor separately, let us see what quantities we have.

For the airplane we have speed, drag or resistance, and effective or useful power used in overcoming the drag at the speed of the airplane. For the propeller we have speed, revolutions, torque, thrust, power absorbed by torque and power delivered by thrust. For the motor we have revolutions, torque, and power delivered to and absorbed by the propeller.

Now, the propeller is the middleman, as it were, and may be considered as driving the airplane or as absorbing the motor power. From the first point of view we need a basic variable involving speed, revolutions, and either thrust or power delivered by thrust. From the second point of view our basic variable should involve speed, revolutions, and either torque or power absorbed by the propeller.

It appears then that to meet all contingencies we really need two basic variables and obviously they should be readily convertible or connected by a simple relation. This indicates that our two basic variables should both involve speed and revolutions and then one should involve torque and the other thrust, or one should involve power absorbed by the propeller and the other power delivered by the propeller or useful power. Each set has its advantages but the set involving power seems preferable for two reasons. When we deal with motors we normally deal with power not torque, and the relation between power delivered to and delivered by the propeller is very simple, being the efficiency of the propeller with no intervening factor.

Having settled upon the quantities to enter into our basic variable, its form is readily determined by applying the principle of dimensional homogeneity. Consider the variable $P^x R^y V^z \rho$ where P denotes power; R, revolutions; and V, speed; and the exponents x, y, and z are to be determined. The dimensions of P are $m\, l^2\, t^{-3}$; of R, t^{-1}; and of V, lt^{-1}. Then dimensionally $P^x R^y V^z = m^x\, l^{2x}\, t^{-3x}\, t^{-y}\, l^z\, t^{-z}\, l^{-3}$.

$$X+1=0 \qquad 2X+Z-3=0 \qquad -3X-Y-Z=0$$
$$X=-1 \qquad Y=-2 \qquad Z=5$$

Our expression is $\dfrac{\rho V^5}{PR^2}$. This or the equivalent is well known and has been used more or less for many years. Of course we can use the reciprocal or any power. For marine propellers a very convenient expression is

$$1\Big/\sqrt{\left(\frac{\rho V^5}{PR^2}\right)}$$

or calling

$$\rho=1, \quad \frac{(R\sqrt{P})}{V^{2.5}}$$

For aeronautic work we need to keep ρ and from a practical point of view there seem to be some advantages, as in marine work, in using expressions where R appears in the numerator and in the first power.

These considerations lead us to the expressions below for basic variables.

Based on motor power

$$\frac{R\sqrt{P}}{\rho^5\, V_m^{2.5}}$$

Based on useful power

$$\frac{R\sqrt{U}}{\rho^5\, V_m^{2.5}}$$

Now we must select the essential things to be plotted upon our basic variable. Efficiency is one, of course. The other quantity that we need is some dimensionless function involving diameter—our primary dimension—and preferably it should involve diameter in the first power and in the numerator.

Such a function is $\dfrac{DR}{V}$, which we may call δ. When we come to plot efficiency and δ upon our basic variable we find it desirable to use logarithmic scales to keep curves within manageable limits.
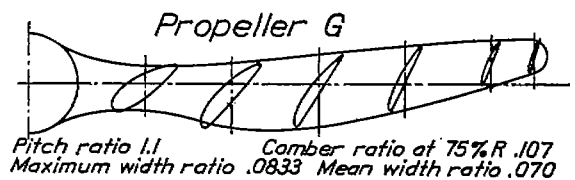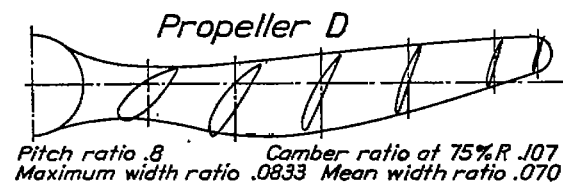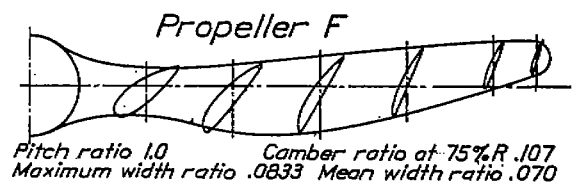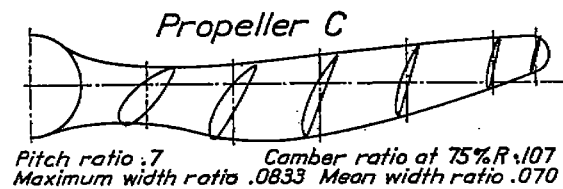
Propeller A

Pitch ratio .5   Camber ratio at 75%R .107
Maximum width ratio .0833 Mean width ratio .070

Propeller B

Pitch ratio .6   Camber ratio at 75%R .107
Maximum width ratio .0833 Mean width ratio .070

Propeller E

Pitch ratio .9   Camber ratio at 75%R .107
Maximum width ratio .0833 Mean width ratio .070

Propeller C

Pitch ratio .7   Camber ratio at 75%R .107
Maximum width ratio .0833 Mean width ratio .070

Propeller F

Pitch ratio 1.0   Camber ratio at 75%R .107
Maximum width ratio .0833 Mean width ratio .070

Propeller D

Pitch ratio .8   Camber ratio at 75%R .107
Maximum width ratio .0833 Mean width ratio .070

FIG. 6

Propeller G

Pitch ratio 1.1   Camber ratio at 75%R .107
Maximum width ratio .0833 Mean width ratio .070

FIG. 7

Propeller H

Pitch ratio .7   Camber ratio at 75%R .118
Maximum width ratio .0833 Mean width ratio .070

Propeller K

Pitch ratio .7   Camber ratio at 75%R .107
Maximum width ratio .0666 Mean width ratio .046

Propeller I

Pitch ratio .7   Camber ratio at 75%R .128
Maximum width ratio .0833 Mean width ratio .070

Propeller L

Pitch ratio .7   Camber ratio at 75%R .107
Maximum width ratio .1000 Mean width ratio .085

Propeller J

Pitch ratio .7   Camber ratio at 75%R .139
Maximum width ratio .0833 Mean width ratio .070

FIG. 8

Propeller M

Pitch ratio .7   Camber ratio at 75%R .107
Maximum width ratio .0769 Mean width ratio .066
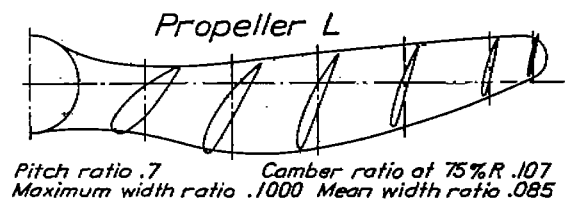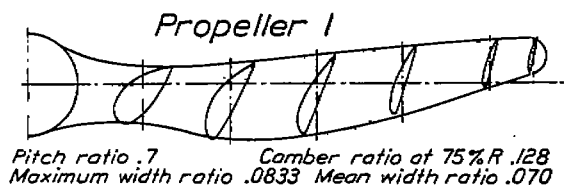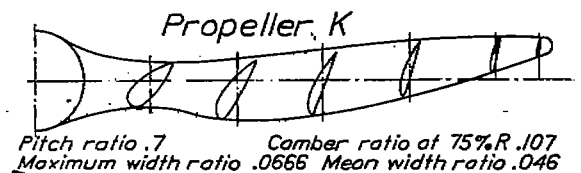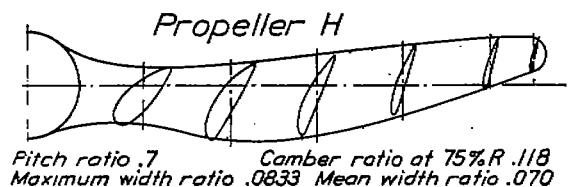
FIG. 9

The most fruitful model propeller experiments are those made with series of propellers changing one variable at a time. The field is too vast to cover fully in this way, but experiment soon shows that some of our variables are primary in their effects, while others are rather secondary, and may be mainly taken account of in a first approximation without special experiments. Experiments made with a group of propeller models varying systematically were recently completed by Doctor Durand for the United States National Advisory Committee for Aeronautics, and illustrate the points referred to above. Thirteen propellers, "A" to "M," were tested. They are shown by projection and sections in Figures 6 to 9 inclusive, and Table III below gives their essential characteristics.

TABLE III

MODEL PROPELLERS TESTED BY DURAND

Designation	Pitch ratio	Maximum width ratio	Mean width ratio	Camber ratio at 0.75 radius
A	0.5	0.0833	0.070	0.107
B	.6	.0833	.070	.107
C	.7	.0833	.070	.107
D	.8	.0833	.070	.107
E	.9	.0833	.070	.107
F	1.0	.0833	.070	.107
G	1.1	.0833	.070	.107
H	.7	.0833	.070	.118
I	.7	.0833	.070	.128
J	.7	.0833	.070	.139
K	.7	.0666	.046	.107
L	.7	.1000	.065	.107
M	.7	.0769	.066	.107

It will be observed that propellers A to G, inclusive, have essentially the same blade sections similarly distributed radially, but differ in pitch, the pitch ratios running from 0.5 to 1.1. This makes seven propellers with variation in pitch as the primary characteristic. Propellers H to M, inclusive, and also C, all have the same pitch ratio, the differences being in mean width ratio and thickness, expressed by camber ratio at 0.75 radius.

Figures 10 and 11 show the results for the propellers of varying pitch plotted as nondimensional coefficients upon the basic variable deduced above. Figure 12 shows the seven propellers of uniform pitch but varying blade sections plotted in the same manner upon the basic characteristic or power only. We see from Figures 10 and 11 that the possible efficiency of an airplane propeller is essentially a question not of propeller design but of the requirements to be met by the propeller. Given the power to be absorbed or delivered, the speed, and the revolutions per minute for this family of propellers, and the maximum efficiency attainable is fixed, and it may well happen that it will fall below the 80 per cent efficiency, which is sometimes regarded as normal. Of course Figures 10 and 11 refer to only one family of propellers, but it will be found that almost any family will plot in the same general way. The efficiencies may be a little higher or a little lower, but the variations of efficiency will follow closely the variations of Figures 10 and 11.

Studying these figures, it will be found that for a given combination of power, speed, and revolutions a definite pitch ratio shows the maximum efficiency, but there is a relatively wide range of pitch ratio on each side of that for maximum efficiency where the falling off is slight. Keeping revolutions, power, and speed the same, we may use a smaller propeller of coarser pitch or a larger propeller of finer pitch without a reduction in efficiency of more than a point or so, an amount which could hardly be detected in service.

In view of Figures 10 and 11, inspection of the propellers of the original Wright plane produces admiration of the engineering genius of the pioneers of the air. This low-speed plane has two relatively very large propellers of coarse pitch. These characteristics are essential to the best efficiency under the conditions to be met. Fast planes of the present day may obtain good efficiency with single propellers of high revolutions and fine pitch, but if the Wrights had fitted such a propeller their plane probably would not have flown at all.

Coming now to Figure 12 it will be observed that the variation of efficiency is remarkably small for the variations of blade section of all seven propellers, C and H to M inclusive. The differences are almost within the limits of error to be expected in such experiments. As regards diameter, the variation resulting from change of section is normal, the thicker blades requiring smaller diameter because their virtual pitch ratio is greater. Thin blades and narrow blades, similarly, act as normal blades of slightly greater diameter. The efficiency curves in all three

Fig. 10

$$\delta = \frac{RD}{V}$$

ρ = Density of air.
R = Rev. per minute.
D = Diameter of pro-
         peller in feet.
P = Horsepower absorbed
         by propeller.
V_m = Speed of advance of pro-
         peller in statute miles per hour.

Values of efficiency and of δ in air
for model
propellers A to G,   plotted upon $\dfrac{R\sqrt{P}}{\rho^{.5}V_m^{2.5}}$

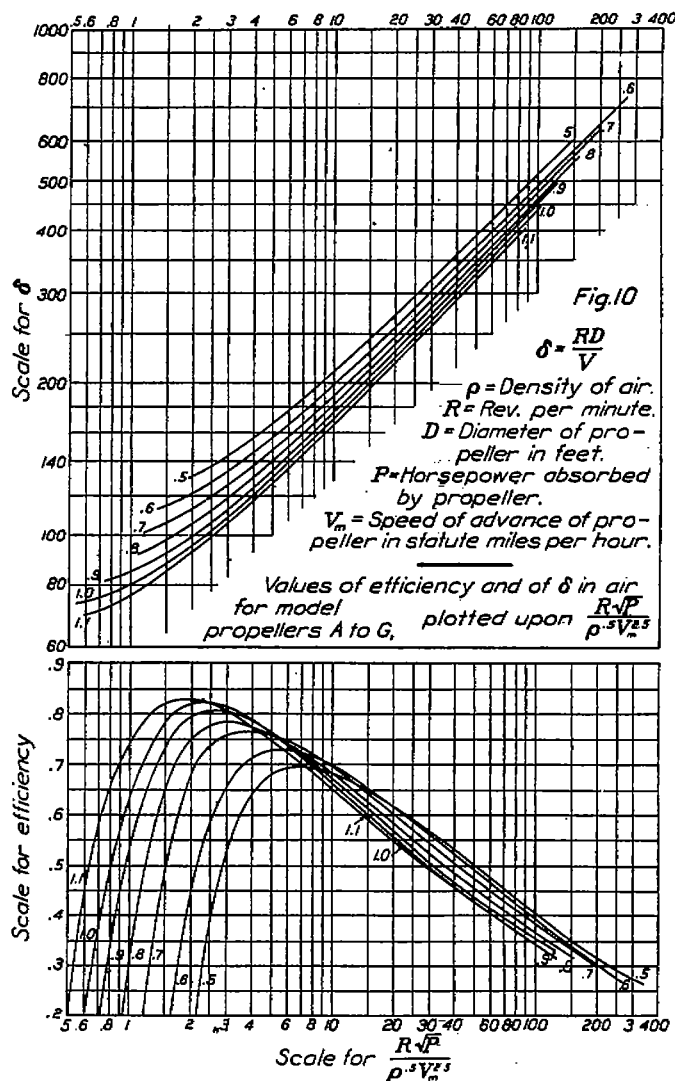Scale for $\dfrac{R\sqrt{P}}{\rho^{.5}V_m^{2.5}}$

FIG. 10

figures show minor inconsistencies which could readily be faired out. They were taken from curves plotted on entirely different variables.

In concluding this part of my subject, it might be pointed out that systematic diagrams such as Figures 10 and 11, from one family of propellers, may be used to extrapolate with a good deal of accuracy the results to be expected from propellers of another blade type when but one of the type has been tested. If, for instance, the one tested has a pitch ratio of 0.7, we will say, and its δ line falls 3 per cent above or below the δ line for the 0.7 pitch ratio in Figures 10 and 11 we may conclude with good approximation that the same relation will hold for pitch ratios of 0.6 and 0.8. It is rather remarkable at first sight to see how δ lines for propellers of quite different

blade sections tend to parallel one another when plotted upon the basic variables used and how little efficiency is affected by variations of blade sections, etc.

But after all when we go back to first principles these results are perfectly natural.

In air the pressure in the slipstream can not differ much from the undisturbed pressure of the air. Then the thrust is proportional to the sternward momentum, and as shown in Figure 4, there is a certain unavoidable loss or waste of power associated with it, even if we had a perfect propelling instrument. With actual propellers, that are not perfect, we have two further losses

Fig. 11

$$\delta = \frac{RD}{V}$$

ρ = Density of air.
R = Rev. per minute.
D = Diameter of propeller in feet.
U = Horsepower delivered by propeller.
V_m = Speed of advance of propeller in statute miles per hour.

Values of efficiency and of δ in air for model propellers A to G, plotted upon $\frac{R\sqrt{U}}{\rho^{.5}V_m^{2.5}}$
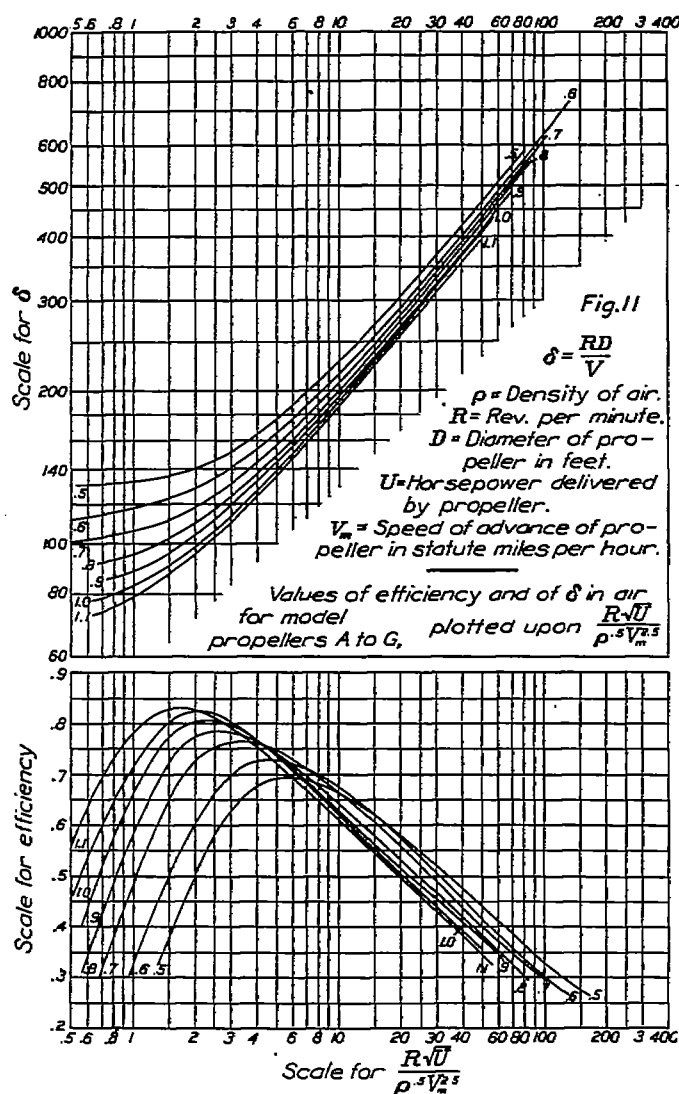
Scale for $\frac{R\sqrt{U}}{\rho^{.5}V_m^{2.5}}$

Fig. 11

due to edge or frictional resistance of the blades and to the transverse momentum communicated to the air involving energy. Neither of these two further losses can be eliminated, and from the nature of the case it does not seem that there is a large field for reducing them, though there is plenty of opportunity to increase them.

As tip speeds increase it will be more and more important to develop types of blade section to avod the quasi cavitation that must be guarded against.

It may be recalled that several times reference has been made to the difficulties of satisfactory full-scale trials. However, we can never rely absolutely upon model experiments until they have been checked by corresponding full-scale trials. During the last year the National

Advisory Committee for Aeronautics has attempted such a comparison, the model experiments and the full-scale tests being both carried out by Prof. E. P. Lesley. The full-scale experiments with five airplane propellers on a VE–7 airplane with a Wright E–4 engine, were conducted at the Langley Memorial Aeronautical Laboratory between May 1 and August 30, 1924. The model experiments were carried out subsequently with models of the same propellers and also a partial model of that part of the airplane exposed to the slip stream, the model being on the scale of 0.3674.

Fig.12

$$\delta = \frac{RD}{V}$$

ρ = Density of air.
R = Rev. per minute.
D = Diameter of propeller in feet.
P = Horsepower absorbed by propeller.
V_m = Speed of advance of propeller in statute miles per hour.

Values of efficiency and of δ in air for model props. C & H to M, plotted upon $\dfrac{R\sqrt{P}}{\rho^{.5}V_m^{2.5}}$

Scale for $\dfrac{R\sqrt{P}}{\rho^{.5}V_m^{2.5}}$
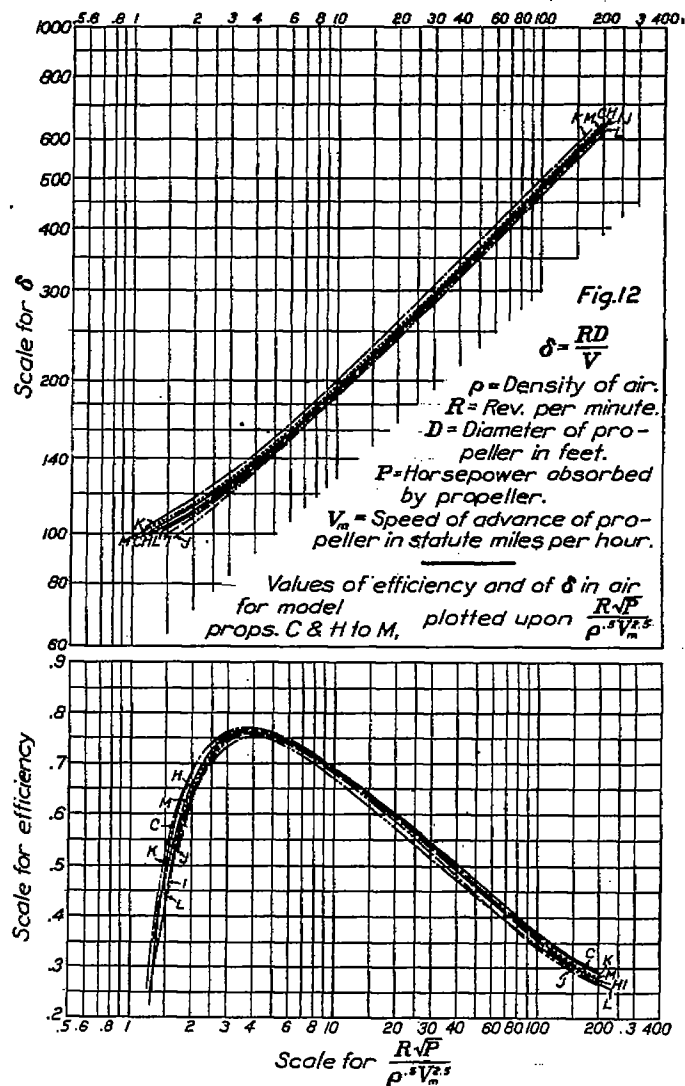
FIG. 12

Heretofore we have considered propeller performance alone. When we come to combine the propeller and its airplane, we meet the complication that each reacts upon and affects the performance of the other. The slip stream from the propeller affects its airplane for tractor propellers, increasing the resistance and somewhat disturbing the balance. This can usually be expressed as regards propulsion matters as an augment of the drag. Furthermore, the disturbance set up in the air by the airplane extends to the air around the propeller, the net result being that the propeller, instead of moving uniformly through the air at the speed of the airplane, moves through air variously disturbed. For the tractor propeller the net result is that the air acted upon by the propeller has already had its relative velocity more or less checked by the reaction from the airplane.

About the only practical way to deal with this matter is to regard this disturbance as equivalent to a uniform slowing up of the air, so that a propeller, instead of behaving as if it were passing through still air with velocity V of the airplane, behaves as if it were passing through still air with a velocity V_1, less than that of the airplane.

When we come to consider the efficiency of the combination, it is unfortunately necessary to make a clear distinction between the efficiency of the propeller and the "efficiency of propulsion." The efficiency of propulsion is best regarded as the ratio between the power delivered to the propeller and the power necessary to propel the airplane under the circumstances if there were no propeller acting. The efficiency of the propeller, however, is the ratio between the useful power which it delivers and the power delivered to it. The power which it delivers depends upon its actual thrust and its speed, V_1, through the air upon which it acts. The thrust is normally greater than the drag of the airplane without the propeller, and V_1 is normally less than V. These two factors affect efficiency in opposite directions, and the result is that the efficiency of propulsion may be greater or less than that of the propeller, according to circumstances. In practice we may usually expect to find it somewhat less.

In the free flight experiments at Langley Field it was necessary first to determine the drag of the airplane. This was done over a range of speed from 50 to 135 miles per hour by making steady glides at various steady angles, the propeller being throttled until the thrust was very close to zero, correction being subsequently made for its departure from zero. This being done, it was possible, from the angle of glide and velocity through the air during the glide, both of which were carefully measured, to determine the drag and the ratio between lift and drag. The power could be determined only indirectly, by means of careful calibration runs of the airplane engine on the testing stand with full throttle. The power flights which were used for reduction were made at full throttle, consisting of runs at air speeds from 50 to 135 miles per hour in level flight, climb, or power dive, as determined by the speed. It being impracticable in the full-scale trials to determine the effect of the slip stream upon the airplane or the airplane upon the propeller, the efficiency in the air was regarded as the efficiency of propulsion, not the efficiency of the propeller.

The model tests were made with models of the same five propellers used in the air, and, in order to make them comparable with the free flight test, they also were reduced to an efficiency of propulsion; that is to say, a thrust coefficient was obtained by using the net thrust, which was the actual thrust less the difference between the drag of the airplane model with the propeller working and its drag without the propeller, all being plotted upon the basis of the speed through the air. The five propellers used had the dimensions and coefficients given in Table IV.
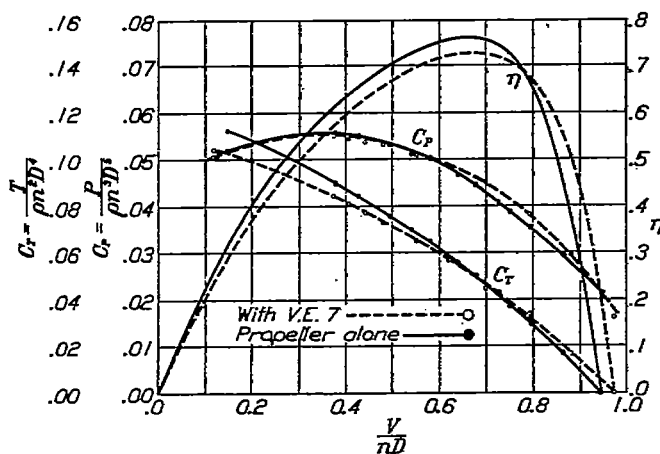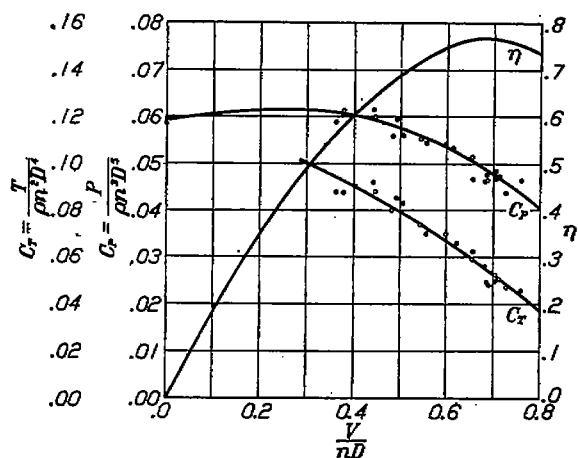

FIG. 13.—Model Propeller I
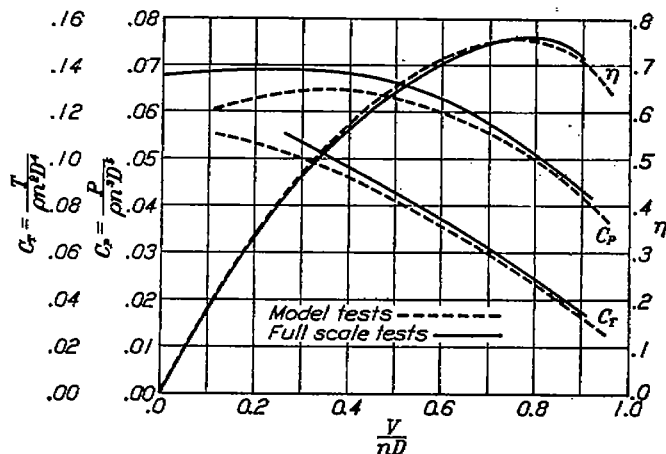
FIG. 14—Propeller I full scale with VE 7 airplane

FIG. 16.—Propeller D′
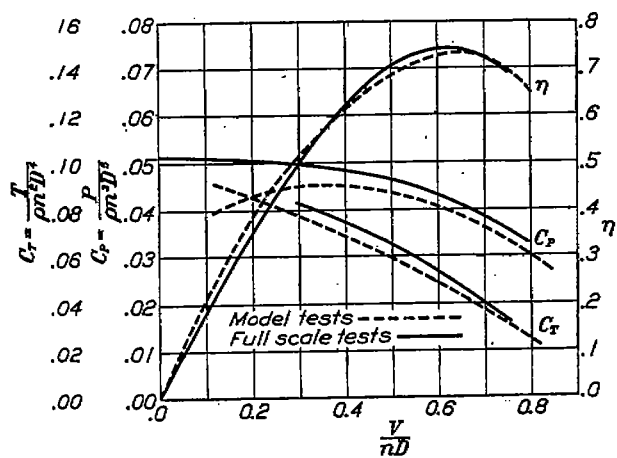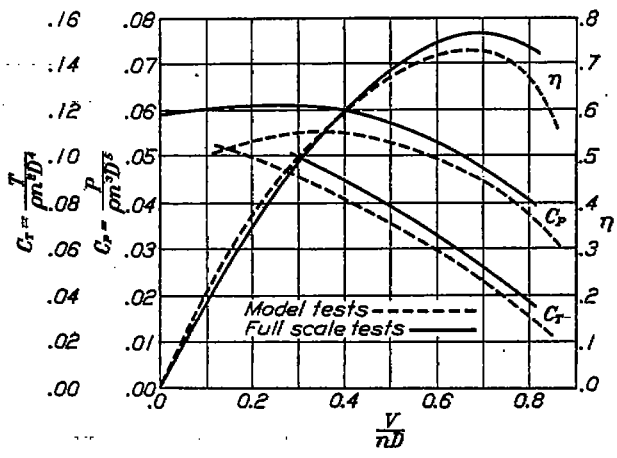

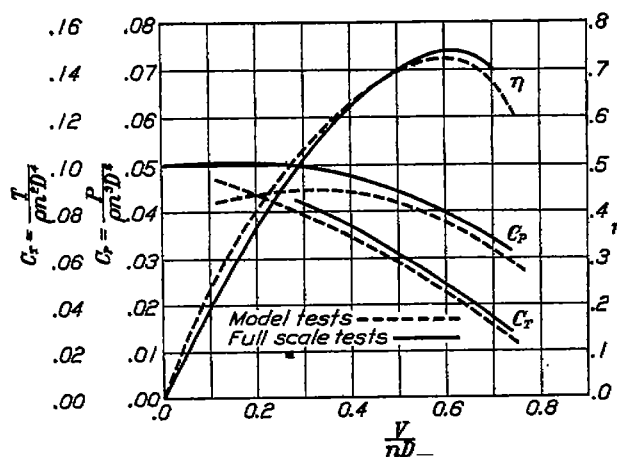FIG. 15.—Propeller B′

FIG. 17.—Propeller I

FIG. 18.—Propeller K′
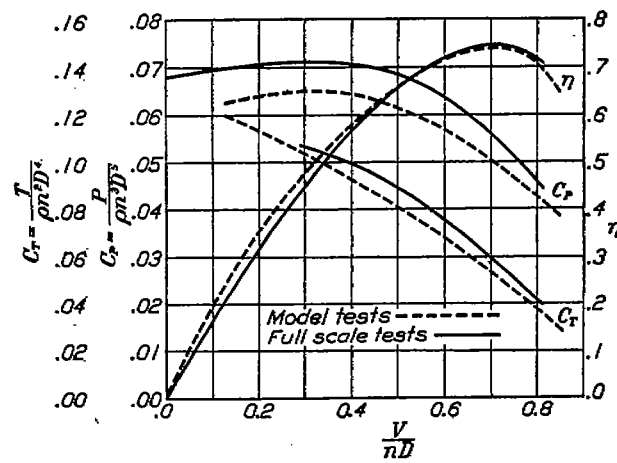

FIG. 19.—Propeller L′

TABLE IV

DATA OF FIVE PROPELLERS USED IN FREE FLIGHT TESTS

Designation	Diameter	Pitch	Pitch ratio	Maximum width ratio	Mean width ratio	Camber ratio at 0.75 radius
	′ ″	′ ″				
B′	8 6	5 1.5	0.6	0.0833	0.070	0.128
D′	7 10	6 3.2	.8	.0833	.070	.128
I	8 2	5 8.6	.7	.0833	.070	.128
K′	8 2	5 8.6	.7	.0666	.046	.128
L′	8 2	5 8.6	.7	.1000	.085	.128

The model results generally were quite consistent.

Figure 13 shows the results of model propeller I with the partial VE–7 model in place, compared with the model results of the propeller alone. In Figures 13 to 19, inclusive, P, upon which values of C_P depend, does not mean power, but foot-pounds per second. T denotes thrust in pounds. Of course, as regards the propeller, abscissae of $\frac{V}{nD}$ mean something slightly different according as the airplane model is or is not present. With the propeller alone it refers to the true slip; with the propeller and model it refers to the apparent slip, and the case is also affected by the use of the net thrust instead of the actual thrust. In the full-scale work we can not determine the actual drag and the air speed relative to the propeller.

Figure 14 shows for propeller I in free flight the curves of C_T, C_P, and corresponding efficiency of propulsion on the basis already explained. Finally, in Figures 15 to 19 there are brought together the results of the model tests with model of plane in place, and the free flight tests in the shape of curves of C_T, C_P, and efficiency. It will be observed that in each case the coefficients are larger in free flight than as estimated from the model results. While the differences vary, as is to be expected, they are consistently too great to be accidental, averaging somewhat on the order of 8 per cent, although for propeller D' they are very small. It is significant that the efficiency differences are very small indeed. Without entering too much into the realm of speculation, it may be pointed out that there are several more or less constant perturbing causes. One is the scale effect; another is the fact that in the model propeller tests the propeller shaft is always parallel to the direction of the flight, whereas in the flight tests the angle made by the propeller shaft with the flight path varied between zero and ten degrees. Inspection of Figures 15 to 19, however, indicates that the perturbation, broadly speaking, increases with the thrust; that is to say, it increases as $\frac{V}{nD}$ decreases. This points to a third cause of perturbation, namely, the elastic deformation of the blades of the propeller under stress, a deformation that would be much greater on the full-sized propeller than on the model tested at less than full speed. A moderate deformation of the full-sized propellers would account for all the discrepancies in Figures 15 to 19.

There are, however, too many uncertainties in such a complicated series of experiments to enable us to fix positively the causes of the discrepancies. In their general features the model and full-scale curves agree very well. It should be pointed out also that, although a difference of, say, 8 per cent of C_P looks large on a diagram, for practical purposes it is not of primary importance. For constant pitch ratio, revolutions, and speed the diameter of the proper propeller varies as the sixth root of C_P, so that a discrepancy of 8 per cent in the value of C_P means a discrepancy of only about 1 per cent in propeller diameter. This is an approximation adequate for the purposes of the engineer. Full-scale tests, where torque and thrust are determined by measurement instead of inference, are of course very desirable, but as far as they go, those to which I have invited your attention are encouraging to those of us who believe that model experiment properly interpreted is not only valuable but indispensable to aeronautical development.