REPORT No. 247

PRESSURE OF AIR ON COMING TO REST FROM VARIOUS SPEEDS

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SUMMARY

The text gives theoretical formulas from which is computed a table for the pressure of air on coming to rest from various speeds, such as those of aircraft and propeller blades. Pressure graphs are given for speeds from 1 cm. sec. up to those of swift projectiles.

The present treatment, slightly modified, was prepared for the Bureau of Aeronautics, Navy Department, February 17, 1926, and by it was submitted for publication to the National Advisory Committee for Aeronautics.

PRESSURE-SPEED FORMULAS FOR MODERATE SPEEDS

A solid surface in uniform translation through a frictionless incompressible fluid, otherwise quiescent, can thereby receive at one point or more a maximum pressure increase $\rho_0 V_0^2/2$, where ρ_0 is the fluid density, V_0 the body's speed. One calls $\rho_0 V_0^2/2$ the "full impact" or "stop" pressure; and any point where it occurs a "stagnation point" or "stop point." Likewise if the body is fixed in a uniform stream, of speed V_0, the incompressible fluid comes to rest at a stop point with the pressure increase $\rho_0 V_0^2/2$. Here the whole stop pressure above vacuo is

$$p_1 = p_0 + \rho_0 V_0^2/2 \tag{1}$$

if p_0 is the pressure in the unchecked part of the stream. In every case here treated p_0 is assumed void of gravity effect.

When a gas for which $p/p_0 = (\rho/\rho_0)^\gamma$ comes to rest adiabatically the stop pressure is

$$p_2 = p_0\left[1 + \frac{(\gamma-1)\rho_0 V_0^2}{2\gamma p_0}\right]^{\frac{\gamma}{\gamma-1}} \tag{2}$$

as shown in hydrodynamics, $\gamma = C_p/C_v$ being the ratio of the specific heat at constant pressure to that at constant volume. This formula is valid for engineering speeds below that of sound in the fluid; for higher and for extremely low speeds other formulas will be given presently.

Expanding (2) gives

$$p_2 = p_0 + \rho_0 V_0^2/2 + p_0\left(\frac{\rho_0 V_0^2}{8\gamma p_0^2} + \quad - \quad - \quad -\right)$$

which exceeds (1) by the parenthetical factor. This excess is negligible at sufficiently low speeds; but not at the speed of a fast airplane or propeller blade, as presently will be shown by some examples.

To furnish the aeronautical engineer with ready numerical values of (1), (2) for air on coming to rest, Table I has been computed for the standard values specified below it.[1] Taking $\gamma = 1.40$, one first writes (1), (2) in the convenient working forms

$$p_1/p_0 = 1 + 0.60471\ V_0^2 \times 10^{-9}, \qquad p_2/p_0 = (1 + 1.7277\ V_0^2 \times 10^{-10})^{3.50}$$

For speeds below that of sound no material error ensues from taking $\gamma = 1.40$ instead of the slightly different values given in physics.

[1] A like table was computed for C. & R. Report No. 129, dated May 13, 1919, and one giving five speeds was published by Finzi and Soldati in 1903, in their pamphlet "Esperimenti Sulla Dinamica dei Fluidi."

The computations were made by various members of the aerodynamics staff in the Construction and Repair Aerodynamical Laboratory of the United States Navy, and checked by the aeronautics staff at the Bureau of Standards. The diagrams were made by Mr. F. A. Louden.

The importance of the pressure excess due to compression may be judged from the tenth column of the table. For speeds under 70 miles an hour the excess is less than $\frac{1}{6}$ per cent of the impact pressure computed for air without compression. At 100 miles an hour, it is 0.41 per cent; at 150 miles, 0.96 per cent; at 300 miles, 4 per cent; at 800 miles, 31 per cent. The last is about the speed of sound and of some propeller tips, while 300 miles is attained by fast airplanes in diving.

VALIDITY OF FORMULA

The validity of (2) is here assumed without proof; viz, the compression is assumed to occur without sensible heat transfer. At speeds above 150 miles an hour, for which the density increment is no longer negligible, the compression of the air filament from p_0 to p_2 may occur in very brief time. To illustrate, suppose air streaming at 200 feet a second across a rod 1 inch in diameter. From both theory and experiment one knows that the speed is sensibly unchecked at points 1 foot before the rod and 1 foot behind it. Hence a particle traversing this range must receive its maximum compression in about $\frac{1}{200}$ second. The dissipation of compression heat in this case may be assumed negligible, both because of suddenness and because the heating or cooling of any filament is paralleled by that of its immediate neighbors, thus lessening the temperature gradient.

It is commonly assumed also that for usual wind tunnel speeds the stop pressure on a large body equals that on a like small one in like conditions. In 1902 the writer found the impact pressure in the nozzle of a $\frac{1}{8}$-inch pitot the same as in a pipe 5 inches in diameter when both were pointed upstream in a 6-foot wind tunnel maintaining a steady 40-mile wind. (Reference 1.) In December, 1925, he found the impact pressure in a square-ended glass tube of $\frac{1}{8}$-inch bore, pointed into a 40-mile wind, equal to that in a like tested neatly pointed hypodermic tube 0.01 inch in diameter,[2] truly to $\frac{1}{5000}$ inch of water.

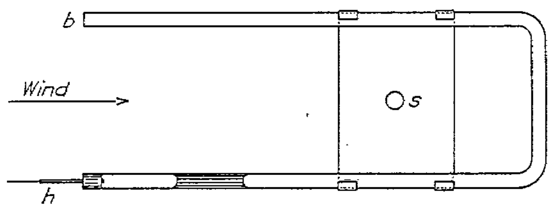

FIG. 1.—Glass U-tube pointing upwind and held by spindle s of aerodynamic balance. In a 40-mile wind, pressure in hypodermic nozzle h balances that at b truly to $\frac{1}{5000}$ inch of water. Bore of glass tube $\frac{1}{8}$ inch; bore of hypodermic tube 0.0097 inch

The arrangement for this latter test is shown in Figure 1. A glass U-tube of $\frac{1}{8}$-inch bore with arms in a horizontal plane and pointing upstream, is held by a sheet metal clamp mounted on the spindle of the aerodynamic balance in the 4-foot wind tunnel. With both ends wide open in a 40-mile wind the U-tube was adjusted, by canting the balance, first till the small piston of alcohol there shown just moved forward, then till it just moved backward. The amount of cant was indicated by an Ames dial gauge at the tip of the balance beam. Now, with the glass arms in their neutral position, and one plugged with a hypodermic needle of $\frac{1}{100}$-inch bore, as shown, the piston rested in the same equilibrium position as before in the same wind. Since the dial gauge showed that a cant of 1 in 10,000 is sufficient to move a piston 3 inches long, of alcohol of specific gravity 0.81, the differential pressure between the fine and coarse nozzle can not exceed about $\frac{1}{5000}$ inch of water.

For the medium speeds listed in Table I many experiments have shown that the pitot impact pressure equals the reservoir pressure; that is, the pressure of stagnant air from which the stream would issue with the speed V_o through a perfect nozzle. For such speeds therefore no corroborative data need be presented. For swifter flows Dr. Briggs furnishes some unpublished measurements made by himself and Dr. Buckingham showing that the static air pressure in a reservoir equals the pitot pressure in its fair discharge nozzle at exit speeds of 400 to 1,000 feet a second, but progressively exceeds it for higher speeds up to that of sound, though the exess is but a few per cent. Check measurements with improved apparatus will be made by them before publication.

[2] Inside diameter 0.0097; outside, 0.0111 inch.

LOW-SPEED FORMULA

Table I therefore is valid for bodies of all but microscopic size. For small bodies at low speeds a viscous pressure may have to be added to the inertia pressure. As shown in hydrodynamics, the nose impact pressure on a sphere of radius a, fixed in a boundless uniform stream of liquid, of viscosity μ, is

$$p_3=\frac{1}{2}\rho_o V_o^2+\frac{3}{2}\frac{\mu V_o}{a} \tag{3}$$

The ratio of this to $\rho_o V_o^2/2$, found for an inviscid liquid is

$$y=p_3/.5\rho_o V_o^2=1+\frac{3}{R} \tag{4}$$

where $R=aV_o/\nu$ is Reynolds Number. On plain section paper y plots against R as an hyperbola asymptotic to the lines $y=1$, $R=o$.

For R quite large, as assumed for Table I, the viscous term is inappreciable; for R small, as when a mist particle falls in air, that term is predominant.

Assuming p_3 to express the impact pressure of a fine pitot at small speeds, Muriel Barker (Reference 2), of Cambridge University, plotted it against the impact pressure p_e determined by her for the point of a pitot 1 millimeter in diameter held at the center of a long brass pipe 11 millimeters in diameter[3] conducting water in steady stream-line flow at approximately 15° C. Fixed speeds of unchecked flow from 0.82 to 11.76 centimeter-seconds at the pipe's center were used. For $V_o<6$ centimeter-seconds, or for $R<30$, faired values of p_e plot as a straight line

$$p_e=p_3/1.1; \tag{5}$$

for $R>30$, $p_e=.5\ \rho_o V_o^2$.

No correction of p_e was made for the ratio of the diameters of the pipe and pitot. Until this has been done (5) may be regarded as but an approximate expression for the differential pressure of said pitot in a boundless stream.

HIGH-SPEED FORMULA

For speeds well above that of sound the value of (2) is doubtful; first because γ is then quite variable, secondly, because the pressures given by (2) are much higher than the nose pressures found in high-speed projectiles. For such speeds Rayleigh (Reference 3) derives a special formula which, with $\gamma=1.40$ reduces to

$$\frac{p_2}{p_o}=\frac{166.7\ V_o^7}{C^2(7V_o^2-C^2)^{2.5}} \tag{6}$$

where C is the speed of sound in the unchecked stream. For $V_o=C$, (6) and (2) give the same value of p_2/p_o. Rayleigh's formula is indorsed in Reference 4.

PRESSURE-SPEED GRAPHS

Fig. 2 shows graphically the absolute pressures given in Columns 4, 5 of Table I; also the impact pressures got from them by subtracting unity.

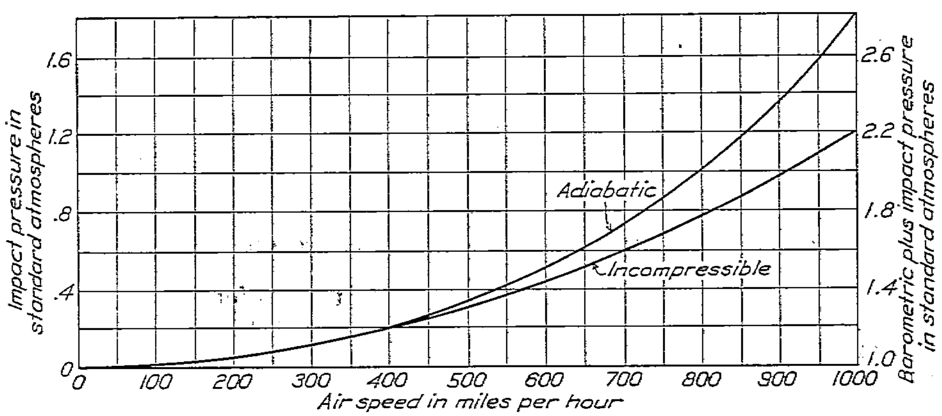

FIG. 2.—Pressure of air on coming to rest from various speeds

[3] The inner and outer diameters of the pipe and pitot can not be gleaned from the paper cited.

With p = impact pressure plus viscous pressure, the upper curve of Fig. 3 delineates $p/.5\rho_0 V_0^2$ versus V_0 for air on coming to rest against the nose of a sphere 1 millimeter in diameter. It shows the effect of viscosity at low speed and adiabatic compression at high speed. The lower curve gives $p/.5\rho_0 V_0^2$ for water, in comparison with Miss Barker's readings with the one millimeter pitot. Semilog paper is used to lengthen the low-speed scale.

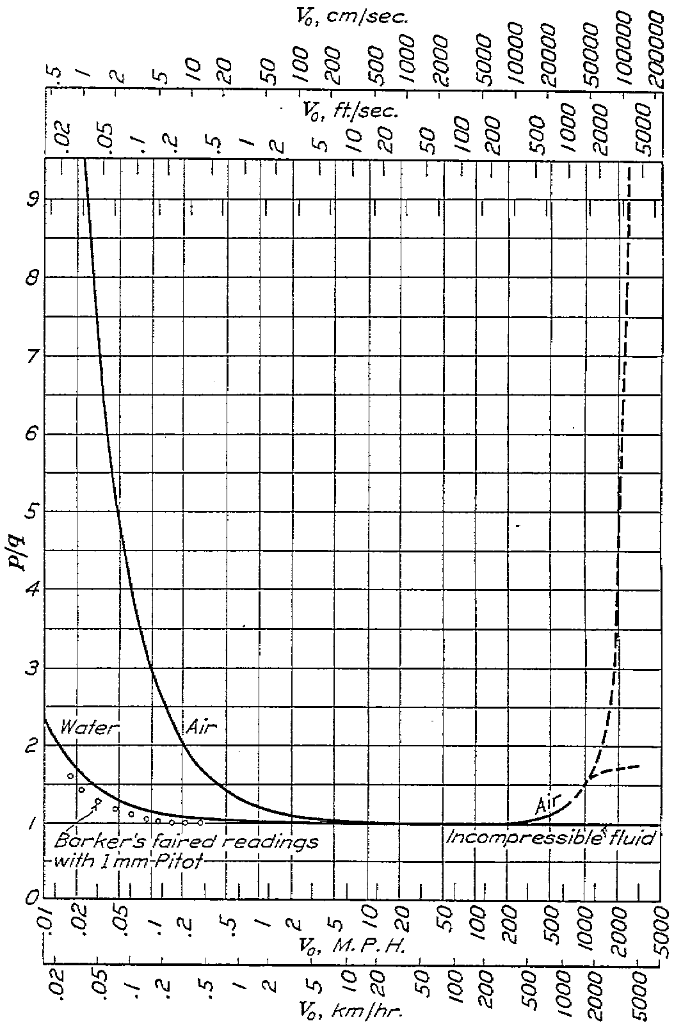

FIG. 3.—Pressure of fluid on coming to rest from various speeds. p/q vs. V_0; p=impact pressure plus viscous pressure, sphere 1 millimeter diameter; $q=\rho_0 V_0^2/2$.

For ultra sound speeds two graphs are given in Figure 3; the higher derived from (2), the lower from (6). The true impact pressure at these speeds is found by Stanton to be given by Rayleigh's formula within ½% at 2.3 times the speed of sound (Reference 4).

REFERENCES

1. A. F. Zahm, "Measurement of air velocity and pressure," Phys. Review, December, 1903.

2. BARKER, MURIEL, "On the use of very small pitot-tubes for measuring wind velocity," Proc. Roy. Soc. Lon., Series A, Vol. Cl, 1922. (This reference was furnished the writer by Dr. H. L. Dryden, United States Bureau of Standards.)

3. RAYLEIGH, "Scientific papers," Vol. V, p. 608.

4. T. E. Stanton, "On the Flow of Gases at High Speeds," Proc. Royal Soc. IIIA, 1926, No. A758.

TABLE I

PRESSURE OF AIR ON COMING TO REST FROM VARIOUS SPEEDS

(Symbols defined below)

Air speed			Barometric plus impact pressure in standard atmospheres. 1 std. atmo. = 1.0133×10^6 dynes/cm^2 = p_o		Impact pressure in pounds per square foot 1 std. atmo. = 2116.8 lb. per sq. ft.		Impact pressure in inches of water 1 std. atmo. = 407.2 in. of water		Percentage difference
Miles per hour	Knots per hour	Kilometers per hour	Incompressible $p_1/p_o = 1+.60471\times10^{-9} V_o^2$	Adiabatic $p_2/p_o = (1+1.7277\times10^{-16} V_o^2)^{3.50}$	Incompressible	Adiabatic	Incompressible	Adiabatic	
0	0	0	1.00000	1.00000	0.000	0.000	0.000	0.000	0.00
10	8.7	16.1	1.00012	1.00012	.254	.254	.049	.049	.00
20	17.4	32.2	1.00048	1.00048	1.016	1.016	.196	.196	.00
30	26.1	48.3	1.00109	1.00109	2.307	2.307	.444	.444	.00
40	34.7	64.4	1.00193	1.00193	4.085	4.085	.786	.786	.00
50	43.4	80.5	1.00302	1.00302	6.393	6.393	1.230	1.230	.00
60	52.1	96.5	1.00435	1.00436	9.208	9.229	1.771	1.774	.17
70	60.8	112.6	1.00592	1.00593	12.532	12.553	2.411	2.415	.17
80	69.5	128.7	1.00773	1.00775	16.363	16.405	3.148	3.156	.26
90	78.2	144.8	1.00979	1.00982	20.724	20.787	3.987	3.999	.31
100	86.8	160.9	1.01208	1.01213	25.571	25.677	4.919	4.939	.41
110	95.5	177.0	1.01462	1.01470	30.948	31.117	5.953	5.986	.55
120	104.2	193.1	1.01740	1.01751	36.832	37.065	7.085	7.130	.63
130	112.9	209.2	1.02042	1.02057	43.225	43.543	8.315	8.376	.73
140	121.6	225.3	1.02368	1.02389	50.126	50.570	9.643	9.728	.89
150	130.3	241.4	1.02719	1.02745	57.556	58.106	11.072	11.178	.96
160	138.9	257.4	1.03094	1.03128	65.494	66.214	12.599	12.737	1.10
170	147.6	273.5	1.03492	1.03536	73.919	74.840	14.219	14.399	1.26
180	156.3	289.6	1.03915	1.03970	82.873	84.037	15.942	16.166	1.41
190	165.0	305.7	1.04362	1.04430	92.335	93.774	17.762	18.039	1.56
200	173.7	321.8	1.04834	1.04918	102.326	104.104	19.684	20.026	1.74
210	182.4	337.9	1.05329	1.05431	112.804	114.963	21.700	22.115	1.91
220	191.0	354.0	1.05849	1.05972	123.812	126.415	23.817	24.318	2.10
230	199.7	370.1	1.06392	1.06540	135.306	138.439	26.028	26.631	2.32
240	208.4	386.2	1.06960	1.07135	147.329	151.034	28.341	29.054	2.52
250	217.1	402.3	1.07553	1.07758	159.882	164.221	30.756	31.591	2.72
260	225.8	418.3	1.08169	1.08410	172.921	178.023	33.264	34.246	2.95
270	234.5	434.4	1.08809	1.09090	186.469	192.417	35.870	37.015	3.19
280	243.2	450.5	1.09474	1.09799	200.546	207.425	38.578	39.902	3.43
290	251.8	466.6	1.10163	1.10537	215.130	223.047	41.384	42.907	3.68
300	260.5	482.7	1.10876	1.11305	230.223	239.304	44.287	46.034	3.95
310	269.2	498.8	1.11613	1.12102	245.824	256.175	47.288	49.279	4.21
320	277.9	514.9	1.12374	1.12931	261.933	273.723	50.387	52.655	4.50
330	286.6	531.0	1.13160	1.13790	278.571	291.907	53.588	57.532	4.78
340	295.3	547.1	1.13969	1.14680	295.696	310.746	56.882	59.777	5.09
350	303.9	563.2	1.14803	1.15602	313.350	330.263	60.278	63.531	5.39
400	347.4	643.6	1.19335	1.20707	409.283	438.326	78.732	84.319	7.11
500	434.2	804.5	1.30210	1.33612	639.485	711.499	123.015	136.868	11.26
600	521.0	965.4	1.43503	1.50688	920.872	1,072.964	177.144	206.402	16.51
700	607.9	1,126.3	1.59212	1.72815	1,253.400	1,541.348	241.111	296.503	22.97
800	694.7	1,287.2	1.77338	2.01124	1,637.091	2,040.593	314.920	411.777	30.75
900	781.6	1,448.1	1.97881	2.35142	2,071.945	2,860.686	398.571	550.298	38.08
1,000	868.4	1,609.0	2.20841	2.82371	2,557.962	3,860.429	492.065	742.615	50.92
11.5	10	18.5	1.00016	1.00016	.339	.339	.065	.065	.00
23.0	20	37.1	1.00064	1.00064	1.355	1.355	.261	.261	.00
34.5	30	55.6	1.00144	1.00144	3.048	3.048	.586	.586	.00
46.1	40	74.1	1.00256	1.00256	5.419	5.419	1.042	1.042	.00
57.6	50	92.7	1.00401	1.00401	8.488	8.488	1.633	1.633	.00
69.1	60	111.2	1.00577	1.00578	12.214	12.235	2.350	2.354	.17
80.6	70	129.7	1.00785	1.00787	16.617	16.659	3.197	3.205	.25
92.1	80	148.3	1.01026	1.01029	21.718	21.782	4.178	4.190	.29
103.6	90	166.8	1.01298	1.01304	27.476	27.603	5.285	5.310	.47
115.2	100	185.3	1.01603	1.01612	33.932	34.123	6.527	6.564	.57
126.7	110	203.9	1.01939	1.01952	41.045	41.320	7.896	7.949	.67
138.2	120	222.4	1.02308	1.02327	48.856	49.258	9.398	9.476	.83
149.7	130	240.9	1.02708	1.02735	57.323	57.894	11.027	11.137	1.00
161.2	140	259.5	1.03141	1.03177	66.489	72.251	12.790	12.937	1.12
172.7	150	278.0	1.03606	1.03652	76.332	77.306	14.684	14.871	1.27
184.2	160	296.5	1.04103	1.04163	86.852	88.122	16.707	16.952	1.47
195.8	170	315.1	1.04632	1.04709	98.050	99.680	18.862	19.175	1.66
207.3	180	333.6	1.05192	1.05289	109.904	111.958	21.142	21.537	1.87
218.8	190	352.1	1.05785	1.05906	122.457	125.018	23.557	24.049	2.09
230.3	200	370.7	1.06410	1.06558	135.687	138.820	26.102	26.704	2.31

$p_o = 1.0133\times10^6$ dynes/cm^2 = 1 std. atmo } U. S. Std. values. (See N. A. C. A. Technical Report No. 218.)
$\rho_o = .0012255$ gm/cm^3 }
$\gamma = 1.40$.
V_o = Air speed in cm/sec.
p_1/p_o (incompress.) $= 1 + \rho_o V_o^2/2 p_o = 1 + .60471\times10^{-9} V_o^2$ atmo.
p_2/p_o (adiabatic) $= (1 + (\gamma-1) \rho_o V_o^2/2 \gamma p_o)^{\gamma/\gamma-1} = (1+1.7277\times10^{-16} V_o^2)^{3.50}$ atmo.
Using $\gamma = 1.404$ would lower the values in columns 7 and 9 less than 0.02 % for speeds less than 350 miles per hour.