REPORT No. 293

TWO PRACTICAL METHODS FOR THE CALCULATION OF THE HORIZONTAL TAIL AREA NECESSARY FOR A STATICALLY STABLE AIRPLANE

By WALTER S. DIEHL
Bureau of Aeronautics

REPORT No. 293

TWO PRACTICAL METHODS FOR THE CALCULATION OF THE HORIZONTAL TAIL AREA NECESSARY FOR A STATICALLY STABLE AIRPLANE

By Walter S. Diehl

SUMMARY

This report is concerned with the problem of calculation of the horizontal tail area necessary to give a statically stable airplane. Two entirely different methods are developed, and reduced to simple formulas easily applied to any design combination. Detailed instructions are given for use of the formulas, and all calculations are illustrated by examples. The relative importance of the factors influencing stability is also shown.

INTRODUCTION

In 1925 the author began a study of the problem of horizontal tail-surface design. A preliminary survey disclosed that several of the published methods appeared to give good results but were too complicated for general use. No method was found to combine the qualities of simplicity and accuracy, necessary to give it wide use. Many designers were using empirical methods based largely on average values of a coefficient such as Hunsaker's "th."[1] These methods were obviously incorrect and leading to serious deficiency of tail area in some cases. There was an evident need for a logical design method which could be reduced to a practical form easily and quickly applied to any design combination. With these requirements in view, two methods were finally developed and thoroughly tested by application to a number of designs for which wind-tunnel data were then available. The very encouraging results which were obtained have been fully verified by subsequent use over a period of about two years. It is believed that these methods will prove of considerable interest and value to all airplane designers.

THE FIRST EQUATION FOR HORIZONTAL TAIL AREA

A general equation for horizontal tail area may be derived by writing the equation for pitching moment either about the leading edge of the mean wing chord or about the center of gravity. From a theoretical standpoint the leading edge of the mean wing chord has certain advantages, but these appear to be offset by the fact that most of the available data are referred to the center of gravity. In either case the final results are substantially the same. The following derivation will therefore be based on moments about the center of gravity, with the degree instead of the radian as the unit for angular measure.

Assuming that the resultant force vector is normal to the wind chord and equal to the lift, the equation for wing pitching moment about the *c. g.* is

$$M_w = qS_wc\left[C_L\left(\frac{a-x}{c}\right)\right] \qquad (1)$$

where q is the dynamic pressure $\frac{1}{2}\rho V^2$, S_w the total wing area, c the mean aerodynamic wing chord, C_L the absolute lift coefficient, x the center of pressure location, and a the fore and aft *c. g.* location on the mean wing chord.

[1] $\text{th} = \frac{\text{horizontal tail area}}{\text{total wing area}} \times \frac{\text{tail length}}{\text{mean chord}} = \frac{S_t.l}{S_w.c}$

Differentiating M_w with respect to α gives

$$\frac{\mathrm{d}M_w}{\mathrm{d}\alpha}=qS_wc\left[\frac{\mathrm{d}C_L}{\mathrm{d}\alpha}\left(\frac{a}{c}-\frac{x}{c}\right)-C_L\frac{\mathrm{d}\left(\frac{x}{c}\right)}{\mathrm{d}\alpha}\right]$$

$$=qS_wc\frac{\mathrm{d}C_L}{\mathrm{d}\alpha}\left[\frac{a}{c}-\left(C_P+\alpha_a\frac{\mathrm{d}C_P}{\mathrm{d}\alpha}\right)\right] \qquad (2)$$

since

$$\frac{x}{c}=C_P \text{ and } C_L=\alpha_a\frac{\mathrm{d}C_L}{\mathrm{d}\alpha}$$

The pitching moment due to the lift on the horizontal tail surface is

$$M_t=-qC_{Lt}\cdot S_t\cdot l \qquad (3)$$

Where C_{Lt} is the absolute lift coefficient for the tail surfaces, S_t the total horizontal tail area, and l the distance from the center of pressure of tail lift to the center of gravity. Without appreciable error, l may be taken as the distance from the center of gravity to the elevator hinge axis, and considered constant. The negative sign is required since a positive lift causes a diving, or negative moment.

The slope of the curve of tail pitching moment against angle of attack is

$$\frac{dM_t}{\mathrm{d}\alpha}=\frac{\mathrm{d}M_t}{\mathrm{d}\alpha_t}\frac{\mathrm{d}\alpha_t}{\mathrm{d}\alpha}=-\frac{\mathrm{d}C_{Lt}}{\mathrm{d}\alpha_t}\frac{\mathrm{d}\alpha_t}{\mathrm{d}\alpha}qS_tl \qquad (4)$$

α_t being the effective angle of attack of the tail surfaces.

The resultant moment on the entire airplane may be divided into three components due, respectively, to the wings, the tail surfaces, and the remaining parts such as fuselage, landing gear, etc. Denoting the residual moment by M_r, the total moment is

$$M=M_w+M_t+M_r \qquad (5)$$

The variation of M_r with α is usually small in comparison with that of M_w and M_t, so that

$$\frac{\mathrm{d}M}{\mathrm{d}\alpha}=\frac{\mathrm{d}M_w}{\mathrm{d}\alpha}+\frac{\mathrm{d}M_t}{\mathrm{d}\alpha} \qquad (5a)$$

It has been customary to base the horizontal tail area on the geometrical proportions of the airplane. This results in a restoring moment proportional to the product of the wing area by the mean wing chord, while for constant effectiveness the restoring moment should vary as the product of the weight by the mean chord. Wind-tunnel tests on models of airplanes having satisfactory static stability show that the slope of the curve of pitching moment against angle of attack is substantially constant over a considerable angular range. Changing the stabilizer setting merely shifts the curve without changing the slope, as shown by Figure 1. Since the wind-tunnel tests are made at a constant dynamic pressure q, the equation for the slope of the moment curve is either

$$\frac{\mathrm{d}M}{\mathrm{d}\alpha}=KqWc \qquad (6)$$

or

$$\frac{\mathrm{d}M}{\mathrm{d}\alpha}=K_1qSc \qquad (6a)$$

Table I contains values of K and K_1 obtained from wind-tunnel test data on various airplanes. It will be noted that K_1 is more nearly constant than K, owing to the former arbitrary design methods. An inspection of the values of K, however, shows definitely that it should be greater than -0.0005 to insure stability at all speeds, while values greater than -0.0010, probably indicate excessive stability.

The complete equation for stability can now be written. Substituting equations (2), (4), and (6) into (5a) gives

$$KqWc = qS_w c \cdot \frac{dC_L}{d\alpha}\left[\frac{a}{c} - \left(C_p + \alpha_a \frac{dC_p}{d\alpha}\right)\right] + \frac{dC_{Lt}}{d\alpha_t} \cdot \frac{d\alpha_t}{d\alpha} qS_t l \quad (7)$$

Dividing by $\left(qS_w c \frac{dC_{Lt}}{d\alpha_t} \cdot \frac{d\alpha_t}{d\alpha}\right)$ and arranging terms, one obtains

$$\frac{S_t}{S_w} \cdot \frac{l}{c} = \frac{1}{\frac{dC_{Lt}}{d\alpha_t} \cdot \frac{d\alpha_t}{d\alpha}}\left[-K\left(\frac{W}{S_w}\right) + \left[\frac{a}{c} - \left(C_p + \alpha_a \frac{dC_p}{d\alpha}\right)\right]\frac{dC_L}{d\alpha}\right] \quad (8)$$

Letting

$$\frac{dC_{Lt}}{d\alpha_t} = F_1, \quad \frac{d\alpha_t}{d\alpha} = F_2, \quad \left(C_p + \alpha_a \frac{dC_p}{d\alpha}\right) = F_3$$

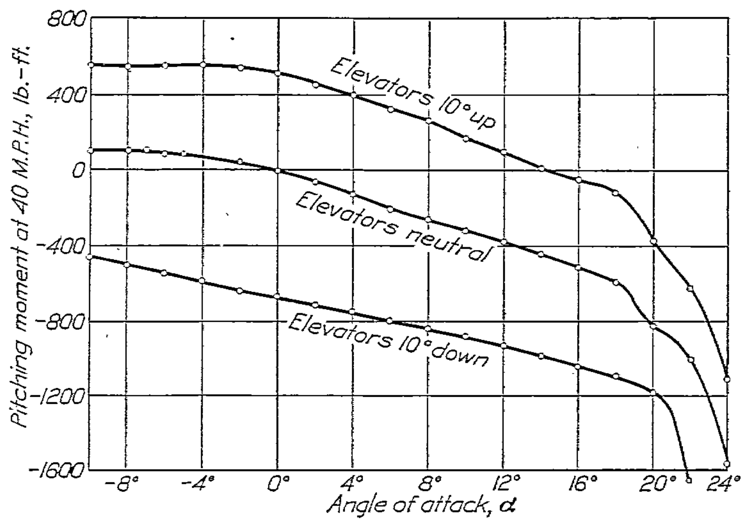

FIG. 1.—Pitching moments for a typical airplane. Wind tunnel test data, model scale 1:12

and

$\frac{dC_L}{d\alpha} = F_4$, equation (8) becomes

$$\frac{S_t}{S_w} \cdot \frac{l}{c} = \frac{1}{F_1 F_2}\left[-K\left(\frac{W}{S_w}\right) + \left(\frac{a}{c} - F_3\right)F_4\right] \quad (9)$$

An analysis of this equation shows that it is easily applied to the design of horizontal tail surfaces. The left-hand side is the well-known horizontal surface coefficient "th" used by Hunsaker.[1] F_1 is the slope of the lift curve of the tail surfaces, F_2 is a downwash factor, F_3 is a wing section stability factor, and F_4 is the slope of the lift curve of the wings. These factors can readily be determined for any particular design. Their derivation will be given briefly before the equation is analyzed further.

FACTORS F_1 AND F_4—SLOPE OF LIFT CURVES

The slope of the lift curve against angle of attack depends on the airfoil section and the effective aspect ratio. For any given section it will depend only on the aspect ratio. The variation with section must be determined experimentally, but the variation with aspect ratio

[1] See footnote, p. 291.

may be calculated by the method used in N. A. C. A. Technical Note No. 79 (Reference 2). Briefly, this method is as follows:

The difference between the induced angles of attack for two aspect ratios is

$$\Delta\alpha = (\alpha_1 - \alpha_2) = \frac{57.3 C_L}{\pi}\left[\frac{S_1}{(k_1 b_1)^2} - \frac{S_2}{(k_2 b_2)^2}\right] \qquad (10)$$

Where S_1 and S_2 are the total areas of the wings having respective maximum spans b_1 and b_2, and k_1 and k_2 are Munk's factors for equivalent monoplane span. Since $\Delta\alpha$ is the difference in angle of attack for the same lift coefficient at the two aspect ratios, the relation between the two slopes is

$$\frac{dC_L}{d\alpha_2} = \frac{\Delta C_L}{\left[\frac{\Delta C_L}{\left(\frac{dC_L}{d\alpha_1}\right)}\right] + \Delta\alpha} \qquad (11)$$

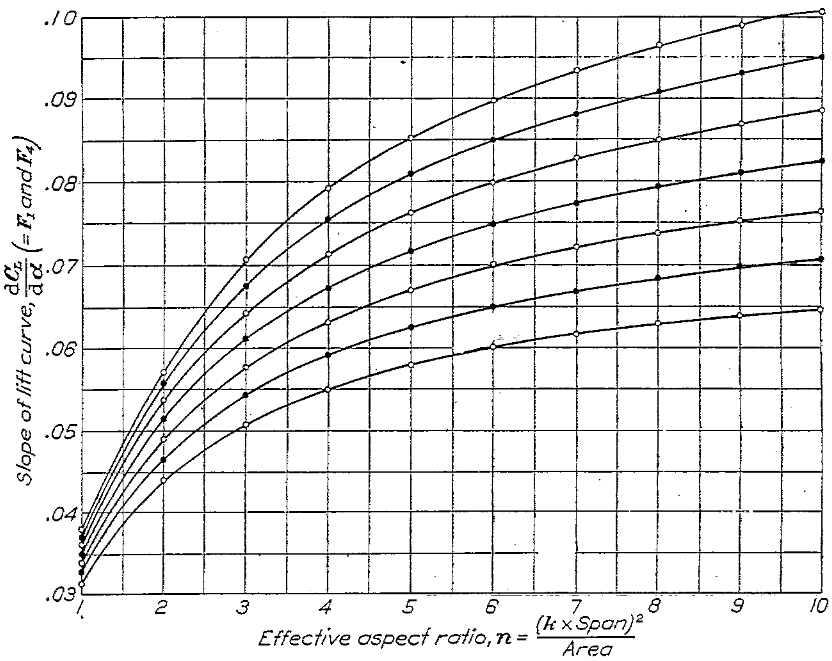

FIG. 2.—Slope of lift curve, variation with aspect ratio

ΔC_L being any convenient increment of lift. Equation (10) shows that $\Delta\alpha$ is positive or negative according as the effective aspect ratio is decreased or increased. $\frac{dC_L}{d\alpha}$ therefore increases with aspect ratio.

Figure 2 is a family of curves of $\frac{dC_L}{d\alpha}$ against effective aspect ratio, as calculated by equations (10) and (11). In order to use Figure 2, the value of $\frac{dC_L}{d\alpha}$ must be known at some given effective aspect ratio. Table II contains the values of $\frac{dC_L}{d\alpha}$ at aspect ratio 6 for a number of standard wing sections.

The first step in finding F_1 and F_4 is to find the effective aspect ratio of the horizontal tail surfaces and the wings. The effective aspect ratio n of any wing arrangenent is

$$n = \frac{(kb)^2}{S} \qquad (12)$$

where S is the total area, b the maximum span, and k Munk's factor for equivalent monoplane span. For a monoplane k=1.00, but for a biplane k varies with the ratios of gap to maximum span $\frac{G}{b_1}$ and shorter span to longer span $\frac{b_2}{b_1}$, and also with the area distribution. The value of k for any normal biplane may be obtained from either Figure 3 or Figure 4, representing

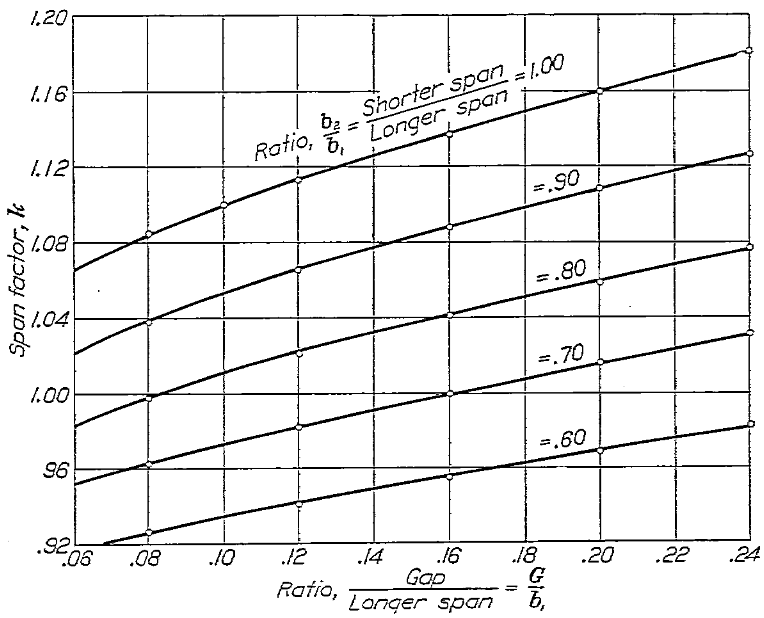

FIG. 3.—Span factors for biplanes with wings of equal chord

equal chords and equal aspect ratios, respectively. These data are based on the theoretical interference values given by Prandtl in N. A. C. A. Technical Report No. 116 (Reference 2). For a wing having raked tips the span should be taken slightly less than the extreme spread. This reduction is largely a matter of judgment and is usually unimportant.

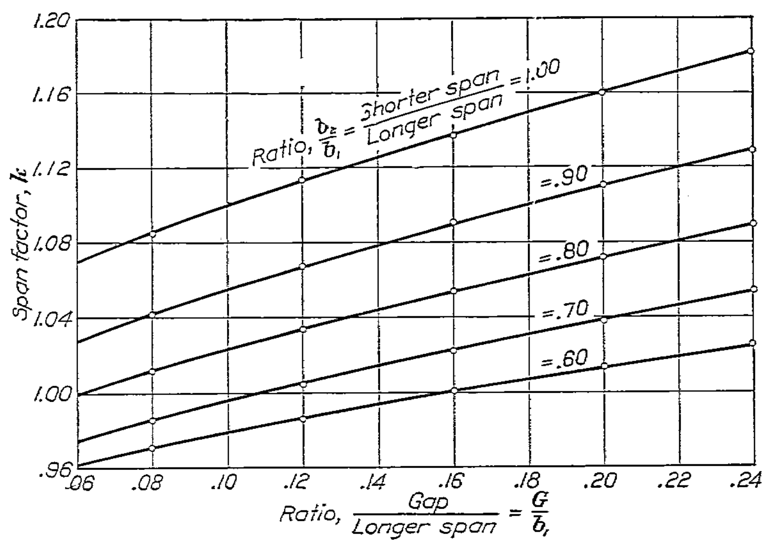

FIG. 4.—Span factors for biplanes with wings of equal aspect ratio

The effective aspect ratios of the wings and tail having been determined, the next step is to find the value of $\frac{dC_L}{d\alpha}$ at some given aspect ratio for the wing and tail sections. This value, if not given in Table II, may be obtained from wind-tunnel test data or it may be estimated. The average slope for the normal wing section is about 0.072 at aspect ratio 6. The average slope for the symmetrical cambered sections, commonly used in tail surfaces, runs slightly higher and may be taken as 0.075 at aspect ratio 6. At any other aspect ratio the value will lie on the curve in Figure 2 which passes through the given value of F_1 or F_4 at aspect ratio 6. For example, if $F_1=0.075$ at aspect ratio 6, Figure 2 shows that $F_1=0.061$ at aspect ratio 3; of if $F_1=0.072$ at aspect ratio 6, then $F_1=0.059$ at aspect ratio 3.

DOWNWASH FACTOR F_2

The angle of downwash at any given point depends on the lift coefficient, the effective aspect ratio of the wings and the location of the given point with respect to the wings. In N. A. C. A. Technical Note No. 42 (Reference 3) the writer has shown that the angle of downwash is given by

$$\epsilon=\frac{K}{n} F_x F_y C_L$$

$$=\frac{K}{n} F_x F_y \alpha_a \frac{\mathrm{d}C_L}{\mathrm{d}\alpha} \tag{13}$$

where F_x and F_y are empirical factors for the subsidence of the downwash angle in the horizontal and vertical planes respectively, n the effective aspect ratio, and K a constant. The value of K

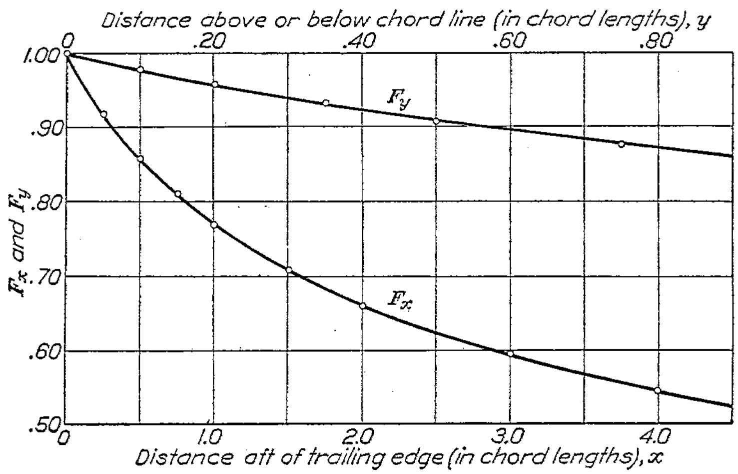

FIG. 5

has been calculated from a group of 10 tests on biplanes and monoplanes in which it varies from 45 to 54.6 with an average value of 52.

If the stabilizer is set at an angle β to the wing chord, the angle of attack of the tail surfaces is

$$\alpha_t=(\alpha_w+\beta)-\epsilon$$

$$=\alpha_w+\beta-\frac{52}{n} F_x F_y \alpha_a \frac{\mathrm{d}C_L}{\mathrm{d}\alpha} \tag{14}$$

Therefore

$$\frac{\mathrm{d}\alpha_t}{\mathrm{d}\alpha_w}=\left(1-\frac{52}{n} F_x F_y \frac{\mathrm{d}C_L}{\mathrm{d}\alpha}\right)=F_2 \tag{15}$$

since

$$\alpha_a=(\alpha_w+\alpha_0) \text{ and } \frac{\mathrm{d}^2C_L}{\mathrm{d}\alpha^2}=0.$$

F_2 is readily determined from equation (15), by the use of Figures 2 and 5, which give the values of $\frac{\mathrm{d}C_L}{\mathrm{d}\alpha}$, F_x and F_y. For the average case in which $\frac{\mathrm{d}C_L}{\mathrm{d}\alpha_w}=0.072$ and the tail plane is substantially in the plane of the wing of monoplane or midway between the wings of a biplane (F_y greater than 0.95) the value of F_2 may be read directly from Figure 6.

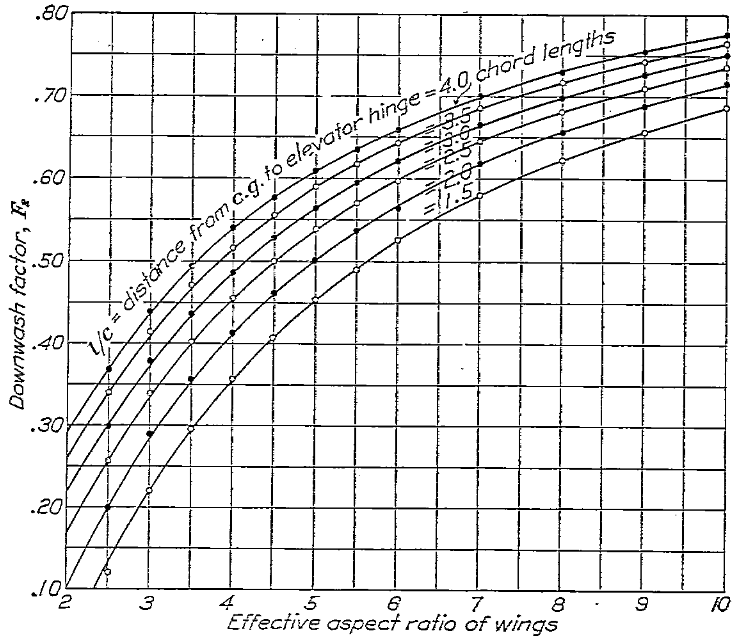

FIG. 6.—Downwash factor, F_2

NOTE.—This chart is based on $F=1.00$ (see eq. 15). If the tail location is either high or low a correction must be applied (see Fig. 5).

WING SECTION STABILITY FACTOR F_3

The wing section stability factor $F_3=\left(C_p+\alpha_a\dfrac{dC_p}{d\alpha}\right)$ is obtained by plotting C_p against α to a large scale so that the slope $\dfrac{dC_p}{d\alpha}$ may be determined with reasonable accuracy. Table III illustrates the method employed and Table IV contains values of F_3 obtained in a similar manner for a number of well-known wing sections. These values of F_3 are plotted against $\dfrac{V}{V_s}\left(=\sqrt{\dfrac{C_Lmax}{C_L}}\right)$ in Figure 7.

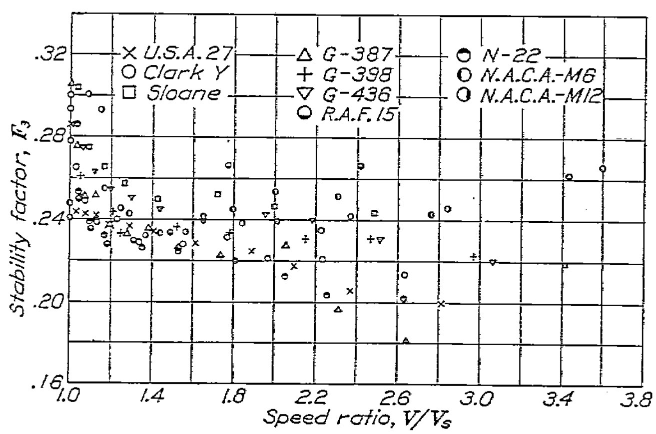

FIG. 7.—Wing section stability factor $F_3=\left(C_p+\alpha_a\dfrac{dC_p}{d\alpha}\right)$.

It will be noted (Table III) that $\dfrac{dC_p}{d\alpha}$ is negative under normal conditions where the center of pressure moves aft as α is decreased. However, the value of C_p is positive and greater than $\alpha_a\dfrac{dC_p}{d\alpha}$ so that the factor F_3 is positive although normally less than the usual values of the center of gravity location, $\frac{a}{c} \cdot \left(\frac{a}{c}-F_3\right)$ is positive under average conditions, and therefore the effect of moving the *c. g.* aft, i. e., increasing $\frac{a}{c}$, is to increase the horizontal tail area required. It is of considerable interest to note that a stable center of pressure movement does not necessarily mean a marked reduction in horizontal tail area required since the values of F_3 for the N. A. C. A.-M6 section do not differ greatly from those for the R. A. F. 15, owing to the change in sign of $\frac{dC_p}{d\alpha}$.

SECOND EQUATION FOR TAIL AREA

A very simple equation for horizontal tail area may be derived from a consideration of the conditions at zero lift. Neglecting the effects of slip stream and fuselage interference the pitching moment due to the horizontal tail surfaces is

$$M_t = C_{Lt} q S_t l = \alpha_v \frac{dC_{Lt}}{d\alpha_t} q S_t l \qquad (16)$$

where α_v is the effective longitudinal dihedral measured between the zero lift lines of the wings and tail surfaces $\frac{dC_{Lt}}{d\alpha_t}$ the slope of the lift curve for the tail surfaces, q the dynamic pressure, S_t the tail area, and l the distance from the center of gravity to the center of pressure of the tail surfaces.

When the wing lift is zero the downwash is zero and α_v is the aerodynamic angle of attack of the tail surfaces. Under these conditions the wing pitching moment about any lateral axis is

$$M_w = C_{M0} q S_w c \qquad (17)$$

where C_{M0} is the absolute moment coefficient about the leading edge of the wing chord, taken at zero lift, S_w the total wing area, and c the aerodynamic mean chord.

It has previously been shown (equation (6) and Table I) that the slope of the resultant pitching moment is

$$\frac{dM}{d\alpha} = KqWc \qquad (6)$$

If the airplane be balanced at an absolute angle of attack α_a', the resultant moment at zero lift should be

$$M_0 = \alpha_a' \frac{dM}{d\alpha} = K_0 q W c \qquad (18)$$

equating the moments

$$M_t + M_w = M$$

or

$$\alpha_v \cdot \frac{dC_{Lt}}{d\alpha_t} q S_t l + C_{M0} q S_w c = K_0 q W c \qquad (19)$$

from which

$$\frac{S_t}{S_w}\frac{l}{c} = \frac{1}{\alpha_v \frac{dC_{Lt}}{d\alpha_t}}\left[K_0\left(\frac{W}{S}\right) - C_{M0}\right] \qquad (20)$$

$$= \frac{1}{\alpha_v F_1}\left[K_o\left(\frac{W}{S}\right) - C_{M0}\right] \qquad (21)$$

Values of K_0 are determined for various airplanes in Table V, and these values are plotted against α_a' in Figure 8. An inspection of Figure 8 shows K_0 to vary linearly with α_a', that is,

$$K_0 = k\alpha_a' \qquad (22)$$

where k varies from 0.00040 to 0.0010, according to the stability.

If the tail setting α_s be plotted against the absolute angle of attack for balance α_a', as in Figure 9 where data from Table V are used, a linear relation is found. For the average airplane

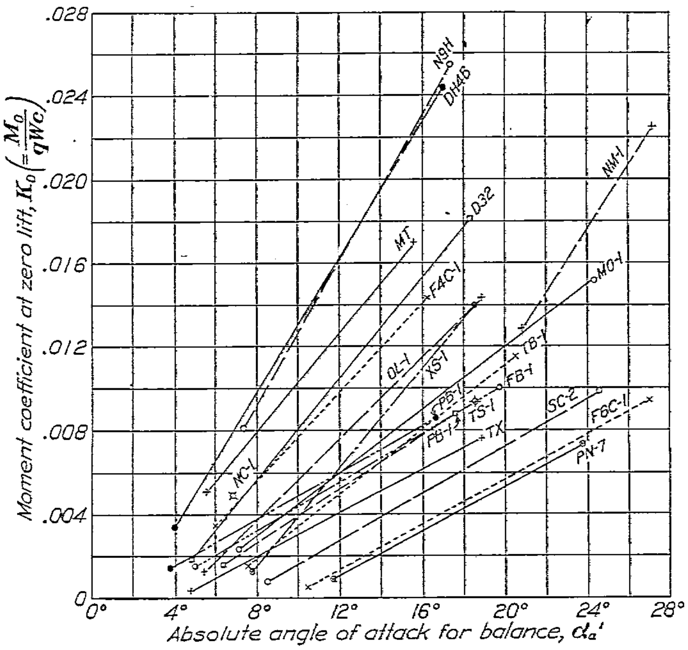

FIG. 8.—Pitching moment coefficient at zero lift

taking into consideration the stability characteristics desired, it appears that

$$\alpha_s = (3.0^0 + 0.25\ \alpha_a') \qquad (23)$$

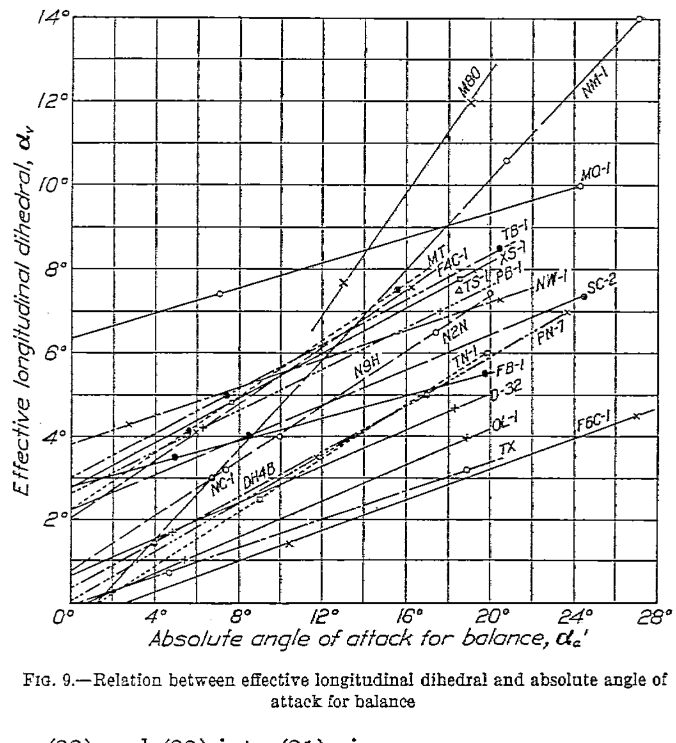

FIG. 9.—Relation between effective longitudinal dihedral and absolute angle of attack for balance

Substituting equations (22) and (23) into (21) gives

$$\frac{S_t}{S_w}\frac{l}{c} = \frac{1}{F_1\ (3+0.25\ \alpha_a')}\left[k\alpha_a'\left(\frac{W}{S}\right) - C_{M0}\right] \qquad (24)$$

As noted before k varies from 0.0004 to 0.0010 according to the stability desired. The average values of k for various types of airplanes are

Pursuit, racers	$k=0.0004$ to 0.0006
Observation, light bombers	$k=0.0005$ to 0.0008
Training, heavy bombers, boats	$k=0.0006$ to 0.0010

The value of α_a' is determined from the angular range between zero lift and maximum lift for the wing section used, and from the speed at which balance is desired. For example, a heavy bomber, or a flying boat might be balanced at its normal cruising speed which is about 1.5 times the stalling speed. Since $\frac{dC_L}{d\alpha}$ is substantially constant the corresponding absolute angle of attack is

$$\alpha_a'=\frac{\alpha_r}{\left(\frac{V}{V_s}\right)^2}=\frac{\alpha_r}{(1.5)^2} \tag{25}$$

where α_r is the angular range between zero and maximum lifts. The effect of α_a' on area required is very small and any convenient angle, say $\alpha_a'=6°$ may be used.

In order to simplify the application of equation (24), values of C_{M0} for various standard airfoils are given in Table VI.

DISCUSSION OF EQUATIONS

The first equation

$$\frac{S_t}{S_w}\frac{l}{c}=\frac{1}{F_1F_2}\left[-K\left(\frac{W}{S_w}\right)+\left(\frac{a}{c}-F_3\right)F_4\right] \tag{9}$$

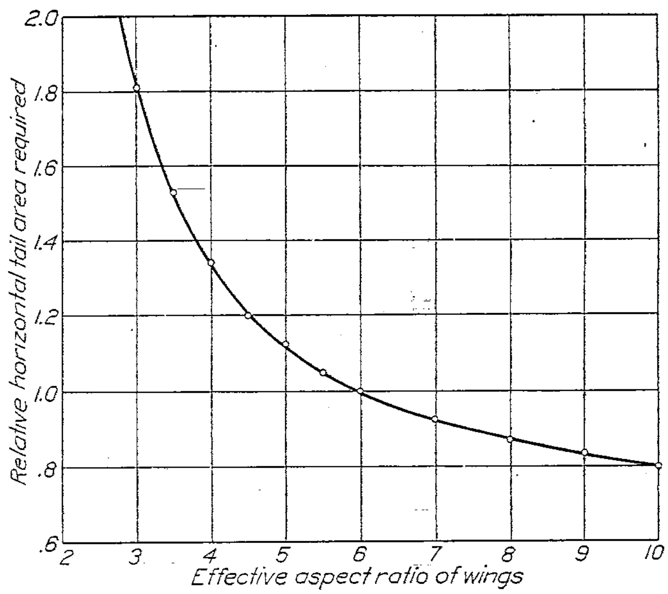

FIG. 10.—Effect of wing horizontal tail area required for constant static stability

is based on considerations affecting the slopes of the moment curves. It accounts for the effect of wing section, wing aspect ratio, tail aspect ratio, tail length, downwash, and fore and aft *c. g.* location. It is an approximation in so far as (1) the resultant force is not normal to the wing chord, (2) the residual moment (due to parts other than wing or tail) is not negligible, and (3) certain effects of vertical *c. g.* location are concerned. If the resultant force were always normal to the wing chord, then the vertical *c. g.* location would not affect the stability. The values of the constant K are based on normal *c. g.* locations between 0.20 c and 0.40 c below the mean chord. Lowering the *c. g.* improves stability; raising the *c. g.* decreases stability.

The second equation

$$\frac{S_t}{S_w}\frac{l}{c}=\frac{1}{F_1(3+0.25\,\alpha_a')}\left[k\alpha_a'\left(\frac{W}{S_w}\right)-C_{M0}\right]\tag{24}$$

is based on considerations at zero lift, and it merely insures an adequate positive moment for this condition. Experience indicates, however, that when this adequate restoring moment at zero lift is obtained with a normal *c. g.* location the moments at other lifts will be satisfactory.

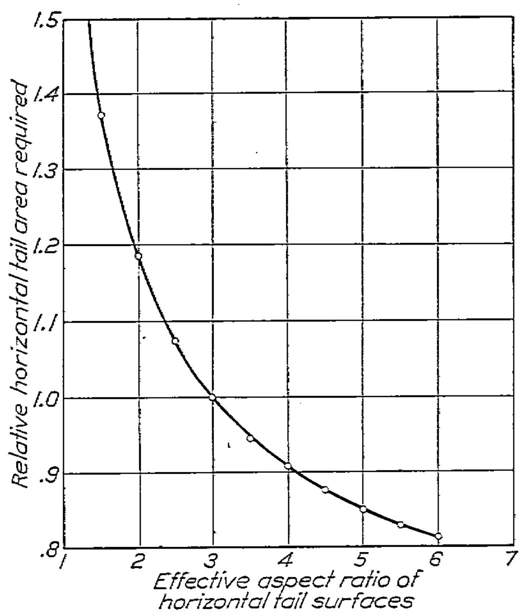

FIG. 11.—Effect of tail aspect ratio on horizontal tail area required for constant static stability

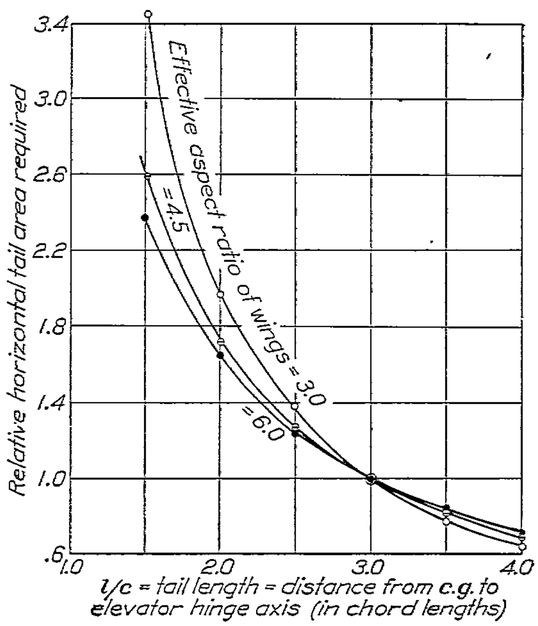

FIG. 12.—Effect of tail length on horizontal tail area required fo constant static stability

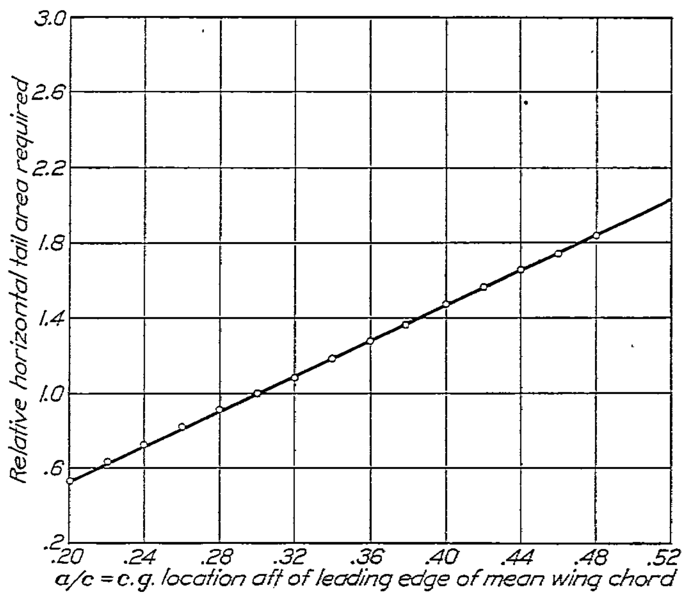

FIG. 13.—Effect of fore and aft c. g. location on horizontal tail area required for constant static stability

For a *c. g.* location at about 30 per cent of the mean chord the two equations give almost identical results, but the second method does not include the effect of fore and aft *c. g.* location. For this reason the first method should be used whenever the *c. g.* is forward of say, 0.28 *c*, or aft of 0.33 *c*.

From data now at hand it appears that in general a horizontal tail area less than about 90 per cent of the value indicated by the first method, will result in static instability. Three cases have been found in wind tunnel tests where the area indicated by the first method gave satisfactory static stability, while a 5 per cent reduction in area resulted in an unsatisfactory condition. In no case yet studied has the area indicated by the first method been found to give unsatisfactory stability.

It is of considerable interest to find the effect of varying the different factors in equation (9). Figures 10 to 13 show the effect of varying wing aspect ratio, tail aspect ratio, tail length and fore and aft *c. g.* location. The magnitude of some of these effects may appear surprising at first glance, but there seems to be little question as to the general correctness of these indications when they are compared with test data. There is one point, however, which demands qualification. For constant static stability the effect of fore and aft center of gravity location is as shown on Figure 13, but this does not consider the questions of control and loading on the tail surfaces. The effect of these factors is to offset to a great extent, the reduction in tail which would be possible with constant static stability for *c. g.* locations well forward.

INSTRUCTIONS FOR USING EQUATIONS

For the benefit of the aeronautical engineer who does not have the time to follow through the complete derivation of the equations and also to avoid any possible misunderstanding, an outline will be given of the steps necessary to calculate the "th" coefficient by the two methods.

I. *First method.*—Equation (9). This method may be used with any fore and aft *c. g.* location. The following steps are necessary:

1. Find effective aspect ratio $n=\frac{(kb)^2}{S}$ for wings and for tail surfaces. k may be obtained from Figure 3 or 4.
2. Find slope of lift curve at some aspect ratio for wing section and tail surface section and obtain slopes of lift curves at actual aspect ratios for wings and for tail, F_4 and F_1, from Figure 2. For average wing section $F_4=0.072$ at aspect ratio 6. For average tail section $F_1=0.075$ at aspect ratio 6.
3. Read downwash factor F_2 from Figure 6. For example, for effective wing aspect ratio of 5, tail length $\frac{l}{c}=3.0$, the value of F_2 is 0.564.
4. Find value of $F_3=\left(C_p+\alpha_a\frac{dC_p}{d\alpha}\right)$ for the wing section used; Tables III or IV, or Figure 7. Take value of F_3 at a high value of $\frac{V}{V_s}$, i. e., $\frac{V}{V_s}>2.0$.
5. Select value of stability constant K, according to type of airplane. The following limits may be used:

Type	$-K$
Pursuit	0. 0005 to 0. 0007
Observation, light bombers	0. 0006 to 0. 0008
Training, heavy bombers, boats	0. 0007 to 0. 0010

II. *Second method.*—Equation (24). This method should not be used unless the *c. g.* is between 0.28 and 0.34*c*. The following steps are necessary:

1. Find or assume effective aspect ratio of horizontal tail surfaces.
2. Find slope of lift curve of tail surfaces F_1, using Figure 2.
3. Assume value of absolute angle of attack for balance, say $\alpha_a'=6°$.
4. Find value of absolute moment coefficient at zero lift for wing section used. Table VI.
5. Select value of stability constant k, according to type of airplane. The following limits may be used:

Type	k
Pursuit	0. 0004 to 0. 0006
Observation, light bombers	0. 0005 to 0. 0008
Training, heavy bombers, boats	0. 0006 to 0. 0010

The calculations will be illustrated by the tabulation of data for a typical pursuit type airplane:

	First method	Second method
Gross weight W lb	2, 800	2, 800
Wing area S sq. ft	250	250
Wing loading $\frac{W}{S}$	11. 20	11. 20
Wing section	Clark Y.	Clark Y.
Span Upper b_1	31. 5	------------
Span Lower b_2	26. 0	------------
Span ratio $\frac{b_2}{b_1}$	. 83	------------
Average gap G	5. 44	------------
$\frac{\text{Gap}}{\text{Max. span}}$ $\frac{G}{b_1}$	. 173	------------
Span factor (equal aspect ratios fig. 4) k	1. 075	------------
Effective wing aspect ratio $\frac{(kb_1)^2}{S}$	4. 60	------------
Tail length l	13. 73′	13. 73′
Mean chord c	4. 83′	4. 83′
Tail aspect ratio	3. 35	3. 35
$\frac{dC_L}{d\alpha}$ for wings at aspect ratio 6	. 071	------------
$\frac{dC_L}{d\alpha}$ for wings F_4	. 0665	------------
$\frac{dC_L}{d\alpha_t}$ for tail at aspect ratio 6	. 075	. 075
$\frac{dC_L}{d\alpha_t}$ for tail F_1	. 0635	. 0635
Downwash factor $\left\{\begin{matrix} n=4.60 \\ \frac{l}{c}=2.84 \end{matrix}\right\}$ F_2	. 528	------------
Wing section stability factor F_3	. 22	------------
Moment coefficient at zero lift C_{M0}	------------	−. 080
Stability coefficient K and k	−. 0006	+. 0005
$c.\ g.$ location $\frac{a}{c}$	. 32	------------

Applying these data.

I.
$$\frac{S_t}{S_w}\frac{l}{c}=\frac{1}{F_1F_2}\left[-K\left(\frac{W}{S_w}\right)+\left(\frac{a}{c}-F_3\right)F_4\right]$$

$$=\frac{1}{0.0635\times 0.528}\,[0.0006\times 11.20+(0.32-0.22)\times 0.0665]$$

$$=0.400$$

$$S_t=0.400\,\frac{S_w}{\left(\frac{l}{c}\right)}=0.400\left(\frac{250}{2.84}\right)=35.2\text{ sq. ft.}$$

II.
$$\frac{S_t}{S_w}\frac{l}{c}=\frac{1}{F_1\,(3+0.25\,\alpha_a')}\left[k\alpha_a'\left(\frac{W}{S_w}\right)-C_{M0}\right]$$

$$=\frac{1}{0.0635\,(3+0.25\times 6)}\,[0.0005\times 6\times 11.20-(-0.08)]$$

$$=0.397$$

from which, $S_t=35.0$ sq. ft.

The agreement obtained in this example is exceptional, but for normal $c.\ g.$ locations it is usually within 5 per cent.

REFERENCES

Reference 1. Hunsaker, J. C.: Naval Architecture in Aeronautics. Royal Aeronautical Journal, July, 1920.

Reference 2. Diehl, W. S.: Effect of Airfoil Aspect Ratio on the Slope of the Lift Curve. N. A. C. A. Technical Note No. 79 (1922).

Reference 3. Prandtl, L.: Applications of Modern Hydrodynamics to Aeronautics. N. A. C. A. Technical Report No. 116 (1921).

Reference 4. Diehl, W. S.: The Determination of Downwash. N. A. C. A. Technical Note No. 42 (1921).

Reference 5. Warner, E. P.: Statical Longitudinal Stability of Airplanes. N. A. C. A. Technical Report No. 96 (1920).

Reference 6. Hamburger, H.: Practical Method for Balancing Airplane Moments. N. A. C. A. Technical Note No. 179 (1924).

Reference 7. Bienen, Theodor: Approximate Calculation of the Static Longitudinal Stability of Airplanes. N. A. C. A. Technical Memorandum No. 387 (1926).

TABLE I

SLOPE OF STABLE PITCHING MOMENT CURVES FROM WASHINGTON NAVY YARD WIND-TUNNEL TEST DATA

Airplane	Class	Weight W (pounds)	Wing area S (square feet)	Mean wing chord c (feet)	Slope of pitching moment curve $\frac{dM}{d\alpha}$	K $=\frac{\frac{dM}{d\alpha}}{qWc}$	K_1 $=\frac{\frac{dM}{d\alpha}}{qSc}$	Remarks
N9H	Training	2, 765	467	4. 92	−100	−0. 00180	−0. 0107	Very stable.
N2N	do	2, 405	285	4. 63	−38	−. 000835	−. 00705	Do.
NY-1	do	2, 818	320	4. 50	−61	−. 00118	−. 0104	Do.
NB-1	do	2, 570	344	5. 00	−15	−. 000286	−. 00214	Unstable at high speed.
F4C-1	Pursuit	1, 700	185	3. 67	−21	−. 000825	−. 00759	Excellent.
F6C-1	do	2, 808	250	4. 60	−18	−. 000340	−. 00382	Slightly unstable at high speed.
FB-1	do	2, 945	242	4. 68	−32	−. 000568	−. 00696	Satisfactory.
D-38	do	2, 450	245	4. 83	−23. 5	−. 000488	−. 00488	Stable at all speeds.
TS-1	do	2, 025	227	4. 75	−18	−. 000460	−. 00410	Unstable at high speeds.
XS-1	Single seater	1, 000	99	3. 00	−7	−. 000490	−. 00495	Stable at all speeds.
NW-1	Racer	3, 000	180	4. 92	−20	−. 000333	−. 00553	Just stable at high speeds.
DH4B	Observation	3, 876	440	5. 50	−82	−. 000943	−. 00832	Very stable.
D-32	do	3, 876	400	6. 00	−50	−. 000530	−. 00514	Excellent.
OL-1	do	4, 800	504	6. 00	−50	−. 000426	−. 00405	Neutral at high speeds.
UO-1	do	2, 230	289	4. 63	−30	−. 000712	−. 00552	Very satisfactory.
VE-7	do	2, 125	289	4. 63	−40	−. 00100	−. 00736	Very stable.
MO-1	do	4, 885	488	9. 57	−97	−. 000510	−. 00510	Stable at all speeds.
T3M-1	Torpedo	9, 863	856	8. 25	−120	−. 000362	−. 00416	Just stable at high speed.
TN-1	do	10, 535	882	8. 5	−100	−. 000274	−. 00328	Neutral at high speeds.
TB-1	do	10, 550	882	8. 5	−220	−. 00060	−. 00718	Stable at all speeds.
PN-7	Boat	14, 236	1, 220	9. 0	−220	−. 000425	−. 00496	Just stable at high speeds.
PB-1	do	25, 000	1, 810	11. 0	−590	−. 000520	−. 00727	Excellent.
F5L	do	14, 000	1, 387	8. 0	−160	−. 000350	−. 00353	Neutral at high speeds.

TABLE II

SLOPE OF LIFT CURVE FOR WELL-KNOWN AIRFOIL SECTIONS—ASPECT RATIO=6

Section	$\frac{dC_L}{d\alpha}$	Section	$\frac{dC_L}{d\alpha}$
RAF-6	0.075	Navy N-9	0.072
RAF-15	.077	Navy N-10	.080
RAF-19	.094	Navy N-14	.081
USA-5	.082	Navy N-22	.074
USA-16	.082	Göttingen 387	.072
USA-27	.071	Göttingen 398	.072
USA-35A	.073	Göttingen 413	.078
USA-35B	.075	Göttingen 429	.072
USA-45	.076	Göttingen 430	.077
USA-TS-5	.075	Göttingen 436	.072
Sloane	.080	Eiffel 32	.075
Albatross	.075	Eiffel 36	.076
Clark Y	.071	NACA-81	.070
Loening M-80	.073	NACA M-6	.072

TABLE III

WING SECTION STABILITY FACTOR F_3 FOR USA-27

$$F_3=\left[C_p+\alpha_a\frac{dC_p}{d\alpha}\right]$$

Angle of attack from chord line α	Absolute angle of attack α_a	C_L	$\sqrt{\frac{C_{L\max}}{C_L}}$ $\frac{V}{V_s}$	Center of pressure C_p	$\frac{dC_p}{d\alpha}$	$\alpha_a\frac{dC_p}{d\alpha}$	F_3
°	°						
−4	1.4	0.102	3.669				
−3	2.4	.174	2.817	0.728	−0.22	−0.528	+0.200
−2	3.4	.245	2.371	.580	−.11	−.374	.206
−1	4.4	.316	2.088	.500	−.064	−.282	.218
0	5.4	.387	1.887	.452	−.042	−.227	.225
2	7.4	.531	1.610	.388	−.0215	−.159	.229
4	9.4	.688	1.415	.360	−.0133	−.125	.235
6	11.4	.825	1.293	.336	−.0087	−.099	.237
8	13.4	.966	1.194	.320	−.0061	−.082	.238
10	15.4	1.086	1.125	.310	−.0044	−.068	.242
12	17.4	1.211	1.067	.304	−.0035	−.061	.243
14	19.4	1.289	1.031	.298	−.0028	−.054	.244
16	21.4	1.356	1.008	.288	−.0020	−.043	.245
18	23.4	1.378	1.000	.286	0	0	.286

TABLE IV

VALUES OF STABILITY FACTOR F_3 FOR WELL-KNOWN WING SECTIONS

Clark Y		Sloane		RAF-15		G-387		G-398	
$\frac{V}{V_s}$	F_3	$\frac{V}{V_s}$	F_3	$\frac{V}{V_s}$	F_3	$\frac{V}{V_s}$	F_3	$\frac{V}{V_s}$	F_3
1.00	0.294	1.000	0.310	1.000	0.280	1.000	0.306	1.000	0.300
1.028	.265	1.031	.304	1.041	.250	1.033	.276	1.052	.261
1.072	.249	1.093	.275	1.104	.235	1.066	.251	1.105	.244
1.137	.239	1.164	.266	1.184	.228	1.117	.252	1.175	.239
1.223	.240	1.262	.258	1.309	.230	1.186	.241	1.255	.233
1.336	.229	1.421	.250	1.487	.244	1.270	.241	1.372	.234
1.551	.228	1.719	.253	1.775	.266	1.377	.236	1.529	.237
1.774	.232	2.000	.247	2.413	.266	1.526	.227	1.780	.234
1.965	.221	2.498	.244	3.432	.262	1.736	.223	2.15	.231
2.226	.221	3.409	.219			2.057	.229	2.47	.232
2.635	.214					2.310	.197	2.963	.223
						2.64	.182		

TABLE IV—Continued

VALUES OF STABILITY FACTOR F_3 FOR WELL-KNOWN WING SECTIONS

G-436		N-22		M-6		M-12	
$\frac{V}{V_s}$	F_3	$\frac{V}{V_s}$	F_3	$\frac{V}{V_s}$	F_3	$\frac{V}{V_s}$	F_3
1.000	0.296	1.000	0.286	1.000	0.248	1.000	0.241
1.061	.276	1.047	.252	1.068	.256	1.083	.301
1.120	.264	1.098	.239	1.167	.255	1.150	.293
1.198	.255	1.164	.232	1.282	.243	1.250	.245
1.297	.251	1.255	.232	1.426	.234	1.375	.235
1.441	.246	1.367	.226	1.646	.241	1.565	.234
1.648	.241	1.532	.225	1.790	.246	1.835	.238
1.953	.244	1.803	.220	2.000	.254	2.010	.239
2.178	.241	2.050	.213	2.300	.252	2.370	.242
2.509	.233	2.243	.203	2.760	.248	2.84	.246
3.058	.220	2.630	.202			3.60	.266

TABLE V

MOMENT COEFFICIENT AT ZERO LIFT AND EFFECTIVE LONGITUDINAL DIHEDRAL

Airplane	Weight W (pounds)	Mean chord c (feet)	Balance at α_a'	Pitching moment at $C_L=0$ M_o (lb./ft. at 40 M. P. H.)	Coefficient $K_o=\frac{M_o}{qW_c}$	Effective longitudinal dihedral α_s
FB-1	2,945	4.68	5.0	85	0.00151	−3.5
			19.8	560	.0100	−5.5
F4C-1	1,700	3.68	6.0	90	.00353	−4.1
			16.2	365	.0143	−7.6
F6C-1	3,186	4.6	10.4	30	.00050	−1.4
			27.0	560	.0094	−4.5
MSO	2,780	8.0	12.5	280	.00308	−7.7
			19.0	1,060	.0117	−12.0
NW	3,000	4.92	2.8	60	.00099	−4.3
			20.5	730	.0121	−7.3
TX	2,900	4.33	4.7	18	.000352	−0.7
			18.9	390	.00762	−3.2
N9H	2,765	5.0	7.4	460	.00816	−3.2
			17.4	1,440	.0255	−6.5
N2N	2,405	4.62	10	310	.00682	−4.0
			20	1,040	.0228	−7.5
DH4B	3,876	5.5	4.0	300	.00344	−1.4
			17.0	2,130	.0244	−5.0
D-32	3,876	6.0	4.8	180	.00189	−1.7
			18.3	1,720	.0182	−4.7
OL-1	5,000	6.0	5.4	150	.00124	−1.0
			18.9	1,740	.0143	−4.0
MO-1	4,885	9.57	7.1	440	.00231	−7.4
			24.3	2,880	.0151	−10.0
XS-1	1,000	3.0	7.7	16	.00130	−4.8
			18.5	172	.0140	−7.8

TABLE V—Continued

MOMENT COEFFICIENT AT ZERO LIFT AND EFFECTIVE LONGITUDINAL DIHEDRAL

Airplane	Weight W (pounds)	Mean chord c (feet)	Balance at α_a'	Pitching moment at $C_L=0$ M_0 (lb./ft. at 40 M. P. H.)	Coefficient $K_0=\frac{M_0}{qWc}$	Effective longitudinal dihedral α_e
NM-1	4, 190	6. 5	20. 8	1, 400	0. 0128	−10. 6
			27. 1	2, 500	. 0225	−14. 0
MT	12, 098	7. 98	5. 6	2, 000	. 00506	−4. 1
			15. 6	6, 700	. 0170	−7. 5
PN-7	14, 250	9. 00	11. 7	500	. 00096	−3. 5
			23. 7	3, 800	. 00728	−7. 0
PB-1	25, 000	11. 00	6. 3	1, 850	. 00165	−4. 2
			17. 6	9, 900	. 00883	−7. 0
SC-2	9, 434	8. 24	8. 5	250	. 000793	−4. 0
			24. 5	3, 100	. 0098	−7. 3
TB-1	10, 550	8. 5	7. 5	550	. 00150	−5. 0
			20. 5	4, 200	. 0115	−8. 5
TN-1	10, 535	8. 5	8. 5	200	. 000547	−2. 5
			19. 9	3, 300	. 00904	−6. 0

TABLE VI

MOMENT COEFFICIENT AT ZERO LIFT FOR STANDARD WING SECTIONS

(Reference axis is at leading edge of wing chord)

Section	Moment coefficient at zero lift C_{M0}	Reference
G-398	−0. 079	McCook Field tests.
G-436	−. 078	Do.
G-387	−. 095	N. A. C. A. Technical Note No. 219.
RAF-15	−. 050	Do.
USA-27	−. 086	Do.
USA-35A	−. 120	Do.
USA-35B	−. 075	Do.
Clark Y	−. 080	Do.
NACA-M6	+. 010	Do.
NACA-M12	−. 005	Do.

BUREAU OF AERONAUTICS,
NAVY DEPARTMENT,
April 6, 1928.