NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 356

STRENGTH OF RECTANGULAR FLAT PLATES UNDER EDGE COMPRESSION

By LOUIS SCHUMAN and GOLDIE BACK

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AERONAUTICAL SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Symbol	Unit	Symbol
Length	l	meter	m	foot (or mile)	ft. (or mi.)
Time	t	second	s	second (or hour)	sec. (or hr.)
Force	F	weight of one kilogram	kg	weight of one pound	lb.
Power	P	kg/m/s		horsepower	hp
Speed		km/hr	k. p. h.	mi./hr.	m. p. h.
		m/s	m. p. s.	ft./sec.	f. p. s.

2. GENERAL SYMBOLS, ETC.

W, Weight, $=mg$

g, Standard acceleration of gravity $=9.80665$ m/s^2 $=32.1740$ ft./sec.^{2}

m, Mass, $=\frac{W}{g}$

ρ, Density (mass per unit volume).

Standard density of dry air, 0.12497 (kg-m^{-4} s^2) at 15° C and 760 mm $=0.002378$ (lb.-ft.$^{-4}$ sec.^{2}).

Specific weight of "standard" air, 1.2255 kg/m^3 $=0.07651$ lb./ft.^{3}

mk^2, Moment of inertia (indicate axis of the radius of gyration, k, by proper subscript).

S, Area.

S_w, Wing area, etc.

G, Gap.

b, Span.

c, Chord length.

b/c, Aspect ratio.

f, Distance from C. G. to elevator hinge.

μ, Coefficient of viscosity.

3. AERODYNAMICAL SYMBOLS

V, True air speed.

q, Dynamic (or impact) pressure $=\frac{1}{2}\rho V^2$

L, Lift, absolute coefficient $C_L=\frac{L}{qS}$

D, Drag, absolute coefficient $C_D=\frac{D}{qS}$

C, Cross-wind force, absolute coefficient $C_C=\frac{C}{qS}$

R, Resultant force. (Note that these coefficients are twice as large as the old coefficients L_C, D_C.)

i_w, Angle of setting of wings (relative to thrust line).

i_t, Angle of stabilizer setting with reference to thrust line.

γ, Dihedral angle.

$\rho\frac{Vl}{\mu}$, Reynolds Number, where l is a linear dimension.

e. g., for a model airfoil 3 in. chord, 100 mi./hr. normal pressure, 0° C: 255,000 and at 15° C., 230,000;

or for a model of 10 cm chord 40 m/s, corresponding numbers are 299,000 and 270,000.

C_p, Center of pressure coefficient (ratio of distance of C. P. from leading edge to chord length).

β, Angle of stabilizer setting with reference to lower wing, $=(i_t-i_w)$.

α, Angle of attack.

ϵ, Angle of downwash.

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Bureau of Standards

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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STRENGTH OF RECTANGULAR FLAT PLATES UNDER EDGE COMPRESSION

By Louis Schuman [1] and Goldie Back [2]

SUMMARY

Flat rectangular plates of duralumin, stainless iron, Monel metal, and nickel were tested under loads applied at two opposite edges and acting in the plane of the plate. The edges parallel to the direction of loading were supported in V grooves. The plates were all 24 inches long and varied in width from 4 to 24 inches by steps of 4 inches, and in thickness from 0.015 to 0.095 inch by steps of approximately 0.015 inch. There were also a few 1, 2, 3, and 6 inch wide specimens. The loads were applied in the testing machine at the center of a bar which rested along the top of the plate. Load was applied until the plate failed to take any more load.

The tests show that the loads carried by the plates generally reached a maximum for the 8 or 12 inch width and that there was relatively small drop in load for the greater widths. This is explained by the fact that when the plate buckles, since the greatest deflection occurs at the center, its vertical chords will shorten more there than at the ends. In consequence there will be less load on the plate at the center and more toward the ends where it is better supported to resist bending and can continue to take load after buckling has occurred. In this way, the load carried by plates of a given thickness would tend to be constant for all plates wider than that at which the maximum load is reached.

Deflection and set measurements perpendicular to the plane of the plate were taken and the form of the buckle determined. The number of buckles was found to correspond in general to that predicted by the theory of buckling of a plate uniformly loaded at two opposite edges and simply supported at the edges.

The tests were made by the Bureau of Standards in cooperation with the Bureau of Aeronautics of the Navy Department, and submitted to the National Advisory Committee for Aeronautics for publication. The materials chosen were those suitable for aircraft construction. The data obtained will be of use in the design of floats, pontoons, wings, etc., of aircraft when the plating is subjected to pressure against the edges. It is desired to make this as light as possible, yet strong enough to take the required loads without permanent deformation.

[1] Junior physicist, Bureau of Standards.

[2] Research Associate, Bureau of Standards.

I. INTRODUCTION

Plates are used in large beams, in columns, in fuselages of aircraft, in pontoons and floats of seaplanes, etc. In many of these structures the plates carry compressive loads applied perpendicularly to two opposite edges and acting in the plane of the plate. The present investigation was undertaken by the Bureau of Standards in cooperation with the Bureau of Aeronautics, Navy Department, for the purpose of determining the strength of plates loaded in this way. The plates tested were loaded in the direction of rolling. Under ideal conditions all four edges of the plate would be supported so that they remain in the original plane. The unsupported portion of the plate may then buckle under the load.

The test procedure was determined by H. L. Whittemore and L. Schuman, and the tests were carried out by L. Schuman.

Acknowledgments are due William R. Osgood, of the Bureau of Standards, for suggestions in analyzing the data, particularly for the explanation of why the load could be increased beyond the value at which buckling began. Acknowledgments are due Messrs. R. G. Sturm and E. C. Hartmann, of the Aluminum Co. of America, for pointing out that the maximum load carried by the plate might be affected by the flexibility of the loading bar which was used.

II. ACKNOWLEDGMENTS FOR MATERIAL

The following firms donated the materials:

The Allegheny Steel Co. (stainless iron).
The Aluminum Co. of America (duralumin).
The International Nickel Co. (nickel and Monel metal).
The Universal Steel Co. (stainless iron).

III. MATERIALS

1. SPECIFICATIONS

Four materials suitable for aircraft construction were used in the tests, viz, duralumin, stainless iron, Monel metal, and nickel. Six thicknesses, varying from 0.015 in. to 0.095 in. were used. As the materials are for use in naval aircraft construction, Navy specifications were followed wherever possible in obtaining materials.

The principal requirements of the Navy specifications for physical properties of duralumin and Monel metal are given in Table I. The materials received conformed in general to these specifications. For the other materials (stainless iron and nickel), no Navy specifications were available.

TABLE I.—SPECIFICATIONS FOR DURALUMIN AND MONEL METAL

Material	Navy specification no.	Condition	Thickness	Tensile strength	Yield	Elongation in 2 inches (minimum)
			Inch	*Lb./in.²*	*Lb./in.²*	*Per cent*
Duralumin	47-A-3 (Sept. 1, 1926).	Sheet, heat treated.	0.013-0.020	55,000	30,000	15
			.020-.128	55,000	30,000	18
Monel metal	46-M-7c (Jan. 3, 1922).	Sheets and plates.		65,000	30,000	15

2. DETERMINATION OF PROPERTIES

(a) CHEMICAL ANALYSES

Two broken tensile specimens, one about 0.03 inch and the other about 0.08 inch thick, of each of the four materials were analyzed by the Chemical Division of the Bureau of Standards—the nonferrous metals by J. P. Hancock, the stainless iron by C. P. Larrabee. From the results of the analyses of these samples, representing the thin and the thick material, it appeared that the composition did not vary greatly. It was therefore not considered necessary to analyze samples of the other four thicknesses of the materials. In Table II are given the results of the analyses.

TABLE II.—RESULTS OF CHEMICAL ANALYSES

DURALUMIN

Thickness (inch)	Cu	Mn	Fe	Si	Mg	Zn	Sn	Al (by diff.)
	Per cent							
0.030	4.2	0.71	0.75	0.32	0.59	Not detected	Not detected	93.43
0.075	4.2	.72	.77	.32	.60	do	do	93.39

STAINLESS IRON

Thickness (inch)	C	Cr
	Per cent	
0.034	0.16	14.3
0.076	.12	14.7

MONEL METAL

Thickness (inch)	Ni	Cu	Fe	Mn	Si	Zn
	Per cent					
0.033	65.5	32.4	1.6	0.31	0.01	Not detected.
0.079	67.0	30.9	1.6	.26	.01	Do.

TABLE II.—RESULTS OF CHEMICAL ANALYSES—Con.

NICKEL

Thickness (inch)	Ni	Fe	Cu	Si	Mn	Zn
	Per cent					
0.032	99.1	0.54	0.13	0.06	Not detected	Not detected.
0.080	99.2	.50	.14	.09	do	Do.

(b) TENSILE TESTS

The tensile properties of the materials, in the direction of rolling, were determined by tests on two specimens [3] (see fig. 1) of each thickness of each of the four materials. The tests were made in a 20,000-pound Olsen machine, the 2,000-pound poise being used for the thinner specimens. Templin grips [4] were used for holding the specimens during the test. (Fig. 2.) Deformations were measured by means of Huggenberger tensometers. Two of these instruments were used, one on each flat side of the specimen, placed on a center line. (Fig. 2.) One of the two edges of contact of the instrument is on the end of a lever arm which is pivoted at a short distance from the specimen. The motion of this lever is magnified by a mechanical lever system and the corresponding deformation is indicated by a pointer moving over a graduated scale The gage length is 1 inch.

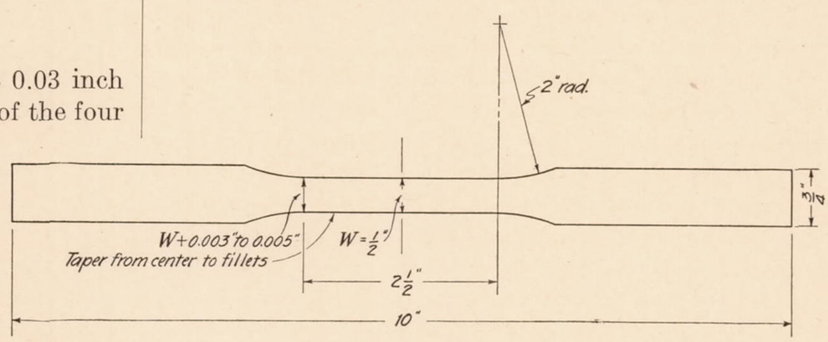

FIGURE 1.—Tensile specimen

The average deformation corresponding to a scale division was 0.000315 inch for one of the instruments and 0.000333 inch for the other. Readings for any part of the scale could be estimated to less than one-tenth of a scale division. Deformations could thus be estimated to about 0.00003 inch.

From the data thus obtained, stress-strain curves were drawn for each thickness of each of the four materials. (Tables III, IV, V, VI; figs. 3, 4, 5, 6.) These curves showed that while the duralumin was fairly uniform for different thicknesses, the tensile properties of the other materials varied considerably with the thickness.

In a few cases the results for the tensile tests showed large variations. For these, check specimens were

[3] See Proc. A. S. T. M., Tentative Standards, vol. 27, Pt. I, 1927, p. 1069.

[4] Special grips designed by Mr. R. L. Templin. See Proc. A. S. T. M., vol. 27, Pt. II, 1927, p. 242.

prepared and tested with Huggenberger tensometers having scale divisions reading to 0.0001 inch. These check data, marked (1) in Tables IV, V and VI, are probably more accurate than those obtained with the extensometers reading to 0.0003 inch.

Since the materials showed no definite yield points, the stress at which the slope of the stress-strain curve was one-third that of the modulus line was designated as the yield point. In addition, the yield point defined by the 1929 issue of the Army-Navy Specification AN 9092, issued since these tests were made, is given in Table III for the duralumin specimens.

Elongations in a 2-inch gage length were determined by means of dividers.

Young's modulus was obtained directly from the stress-strain curves.

(*c*) Brinell and Rockwell Tests

Brinell numbers were obtained in a Baby Brinell machine, with a 1/16-inch ball and a 6.4-kilogram load applied for 30 seconds. The Rockwell B-scale numbers were determined with a 1/16-inch ball and a 100-kilogram load. On the thinner specimens (below 0.04 inch) the Rockwell numbers were probably not so accurate as those for the thicker specimens since the indentation of the ball made a mark on the reverse side of the specimen.

(*d*) Erichsen Tests

In the Erichsen sheet-metal tester, the diameter of the opening over which the specimen was clamped was 27 millimeters and the indenting tool had a radius of 10 millimeters.

Erichsen values were obtained for each of the six thicknesses of duralumin. For the other materials only the three thinnest specimens were tested, as it was found that the force required to rupture the thicker specimens could not be applied by hand.

(*e*) Summary of Mechanical Properties

The preceding mechanical properties of the materials are summarized in Tables III, IV, V, and VI. The stress-strain curves are shown in Figures 3, 4, 5, and 6.

IV. METHOD OF TESTING

1. TEST FIXTURE

The test fixture used (figs. 7 and 8) was designed and built after several forms of apparatus had been tried. This fixture consisted essentially of a base plate to which two channels were attached by means of angle irons and bolts. Spreading of the channels at the top was limited by a horizontal bar loosely bolted to them. Each channel was provided with two screws placed on the vertical center line of the web. On the screws, which were threaded into each channel, was mounted a straight bar in which a V-groove (45°) had been cut. The test specimen was set into these grooves, and rested on the base plate. By means of the screws the grooved bars were adjusted against the vertical edges of the specimen. The specimen could rotate about its edges and slide vertically in the grooves. The specimen extended about one-eighth of an inch beyond each end of the grooves, so that the loads could be applied without loading the fixtures. The load was applied through a bar 1 inch thick by 4 inches wide, which was free to rotate about an axis perpendicular to the plane of the plate at the middle of the upper edge, so that a fairly uniform distribution of load

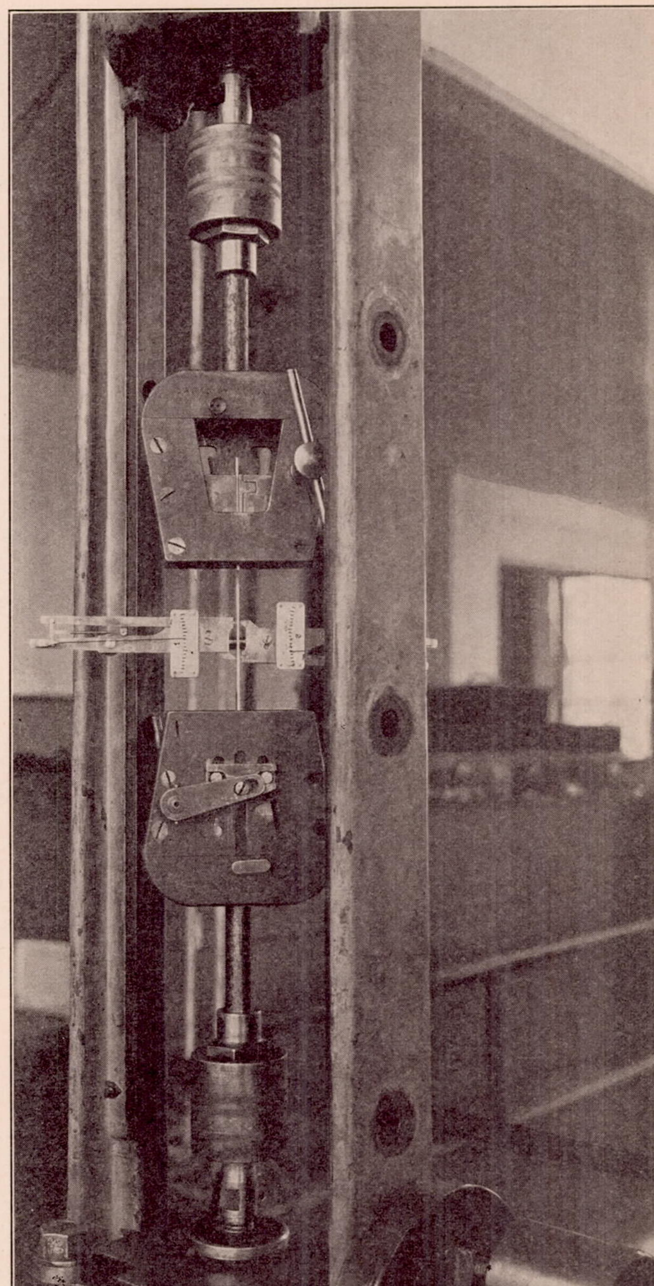

Figure 2.—Apparatus for tensile test, showing Templin grips and Huggenberger tensometers for measuring elongation

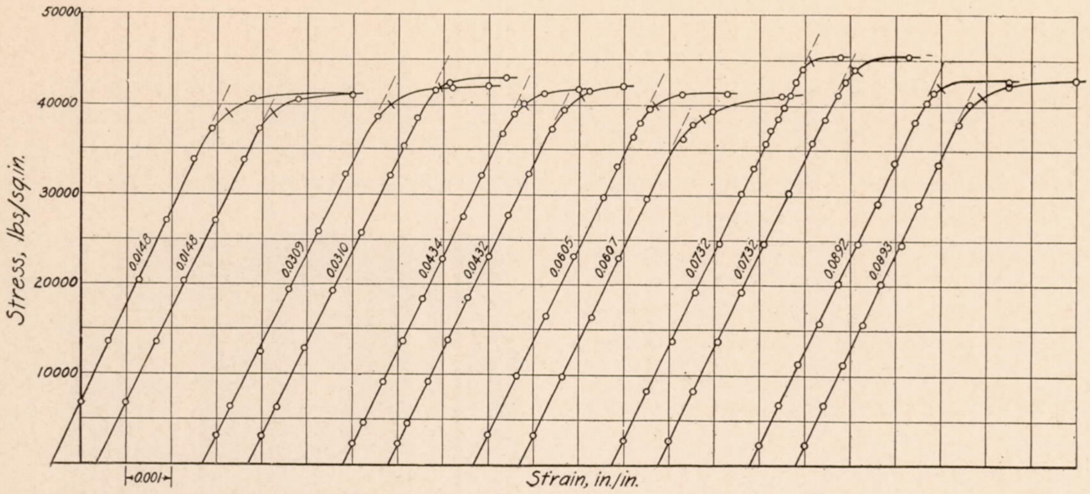

FIGURE 3.—Stress-strain curves for duralumin. The thickness of the specimen is given on each curve. The yield point which is here defined as the stress for which the slope is one-third that of the modulus line is indicated by a short line crossing each curve

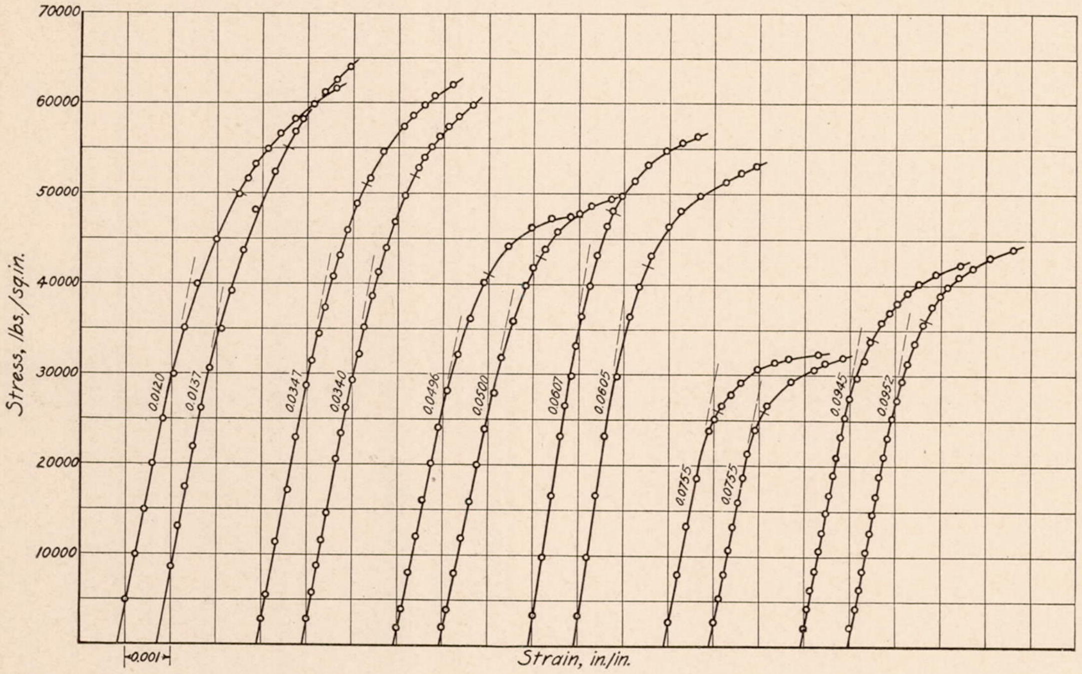

FIGURE 4.—Stress-strain curves for stainless iron. The thickness of the specimen is given on each curve. The yield point which is here defined as the stress for which the slope is one-third that of the modulus line is indicated by a short line crossing each curve

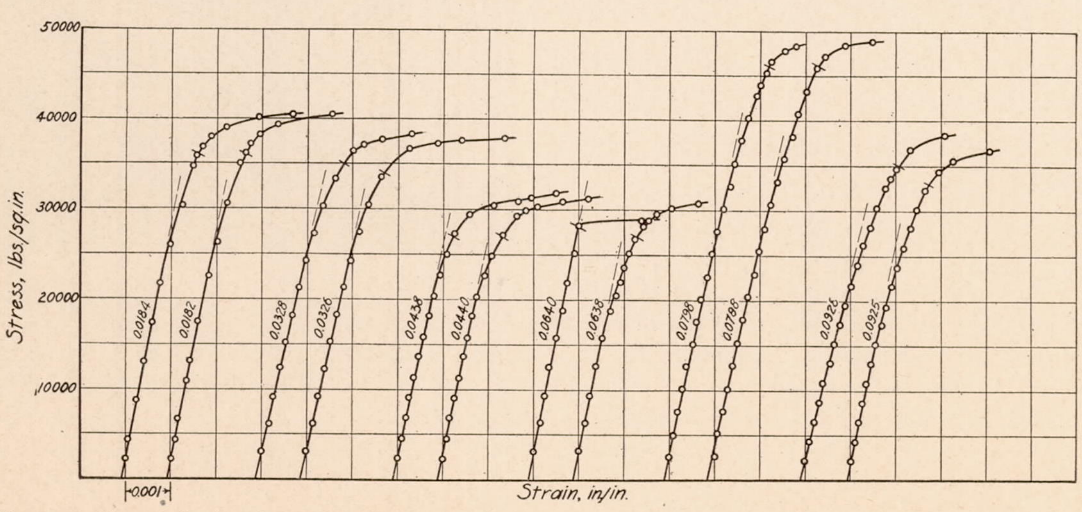

FIGURE 5.—Stress-strain curves for Monel metal. The thickness of the specimen is given on each curve. The yield point which is here defined as the stress for which the slope is one-third that of the modulus line is indicated by a short line crossing each curve

might be obtained until the buckling became appreciable. Holes were drilled and tapped in the base plate to permit varying the distance between the channels so that different widths of plate could be accommodated.

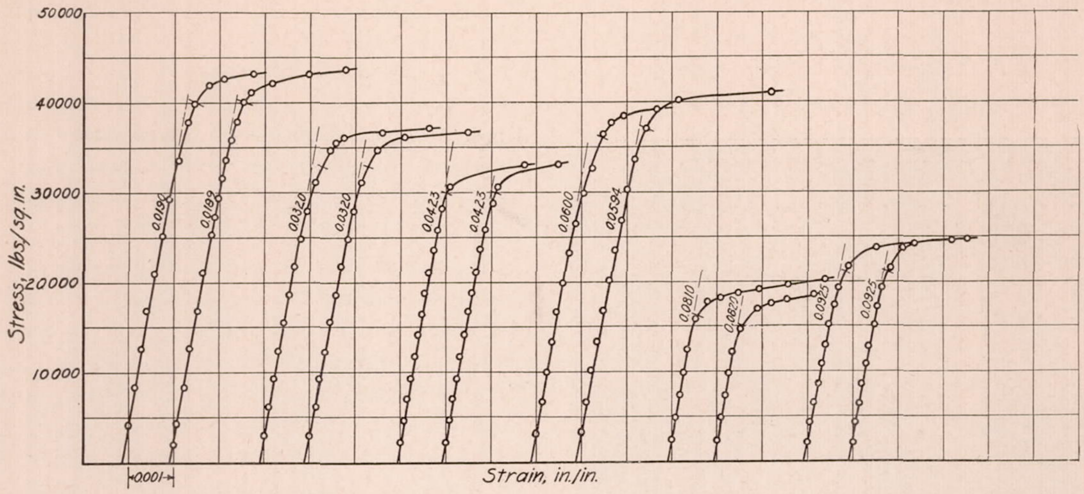

FIGURE 6.—Stress-strain curves for nickel. The thickness of the specimen is given on each curve. The yield point which is here defined as the stress for which the slope is one-third that of the modulus line is indicated by a short line crossing each curve

The deflection of the specimen was determined by measurements with a dial micrometer. (Figs. 7 and 8.) The micrometer was attached to a round bar five-eighths of an inch in diameter, through which, at one end, 24 holes were drilled 1 inch apart. These holes fitted over pins extending from the flanges of the channels. The pins were spaced 1 inch apart vertically and so arranged at the two flanges that the bar, when supported horizontally, would rest on a pin of one flange and fit over a pin of the other flange. By means of this apparatus the micrometer could be moved in steps of 1 inch, both vertically and horizontally. The dial reading was taken with the bar in contact with the flanges of the channel.

2. SIZE OF SPECIMENS

All the test specimens were about 24 inches long, parallel to the direction of rolling. Widths, transverse to the direction of rolling, of 1, 2, 3, 4, 6, 8, 12, 16, 20, and 24 inches were used, but only a few compression and no deflection tests were made on the 1, 2, 3, and 6 inch specimens, and, owing to their initial lack of flatness, no stainless iron specimens wider than 12 inches were tested. Six thicknesses of each material, varying from 0.015 to 0.095 inch, by steps of approximately 0.015 inch, were used.

The thinner specimens were sheared to the desired width; the thicker ones were sawed.

The edges to which the loads were applied were milled straight and parallel.

In addition to the 1, 2, 3, and 6 inch specimens, 18 specimens of stainless iron and 36 specimens of each of the other materials were tested.

3. LOADS AND DEFLECTIONS

The tests were made in a 50,000-pound Riehlé vertical screw testing machine. (Fig. 8.) For most of the plates, especially the narrow and thick ones, the maximum load was indicated by a distinct drop of the beam of the testing machine. In the case of some of the wide plates (over 12 inches) the load began to fall slowly after considerable buckling had taken place. This was especially noticeable in the wider Monel metal specimens.

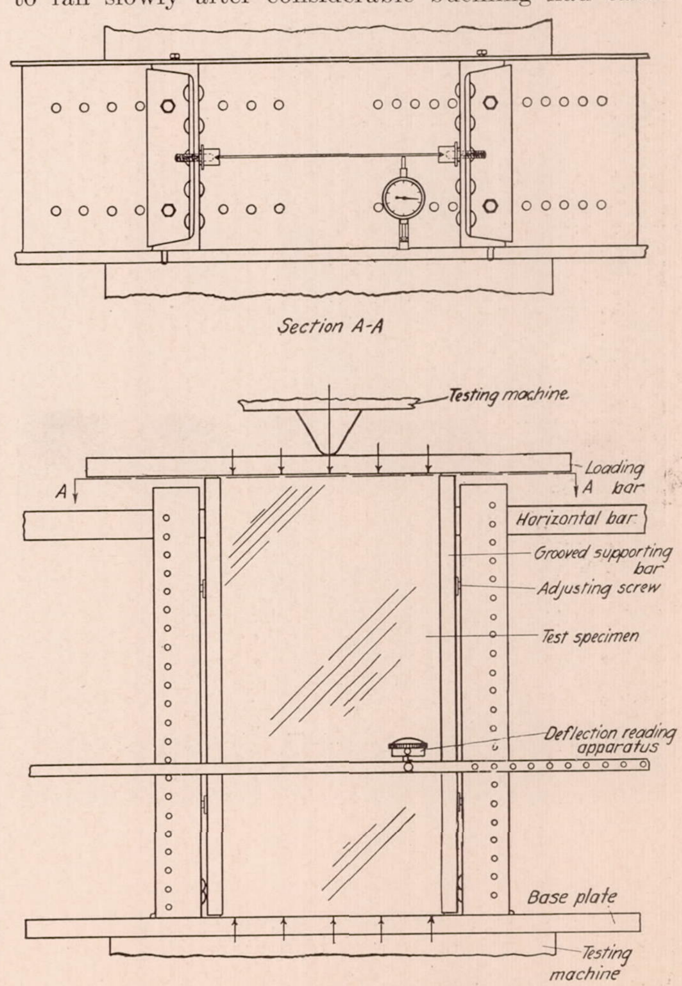

FIGURE 7.—Diagram of testing apparatus

After a drop of load the specimen was found to be deformed permanently. The load could not then be

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increased further, but continued to fall as the head of the machine came down on the specimen, increasing the permanent deformation.

The maximum loads were estimated from a few preliminary tests. Loads were then applied in increments equal to about one-fifth of the estimated maximum load, and readings of deflection of the specimen

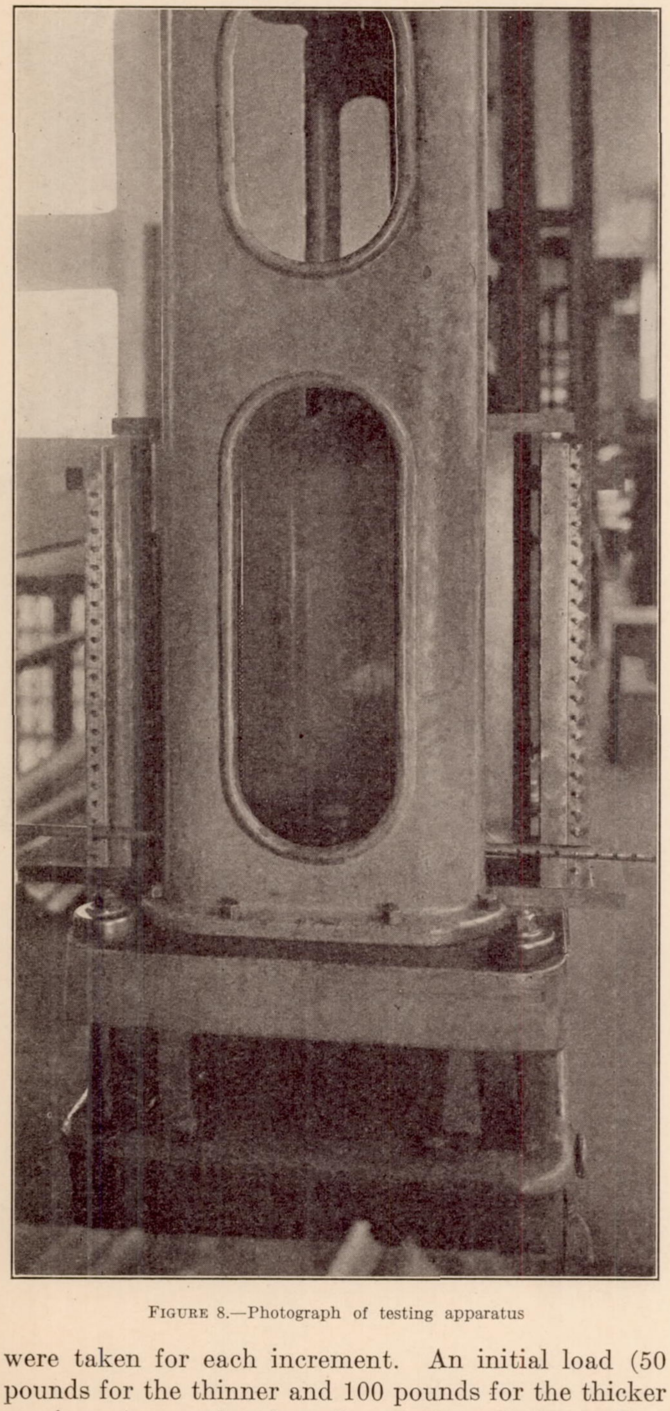

FIGURE 8.—Photograph of testing apparatus

were taken for each increment. An initial load (50 pounds for the thinner and 100 pounds for the thicker specimens) was placed on the specimen before taking the first set of dial readings. The intervals for the readings were so chosen as to give a sufficient number of readings from which to draw curves of deflection. For the 4-inch plates the intervals were 1 inch, both vertically and horizontally. For the wider plates the horizontal intervals were about one-fifth of the width. The vertical intervals were 2 inches in most cases. For the specimens which buckled in long waves the intervals could be taken larger without loss of accuracy.

In order to determine the amount of permanent deformation after definite loads had been placed on the specimen, the load was released and additional readings of deflection were taken under the initial load.

V. RESULTS

The results are shown in Figures 9 to 35, inclusive. Those for duralumin are in Figures 9 and 16 to 26; for stainless iron, in Figures 10, 27, 28, and 29; for Monel metal, in Figures 11, 30, 31, and 32; and for nickel, in Figures 12, 33, 34, and 35. The continuous portion of each curve of deflection has been drawn through the points representing the observed values; the broken portions are extrapolations over regions in which no measurements could be taken on account of proximity to an edge of the specimen.

VI. DISCUSSION OF RESULTS

1. CHARACTERISTICS OF THE CURVES OF MAXIMUM LOAD (FIGURES 9-12)

In a short thick ductile compression specimen we expect the average maximum stress to be at least equal to the yield point of the material. On the other hand, as soon as the dimensions of the specimen become such as to permit buckling, then a lower average maximum stress results.

Now consider Figure 9. First of all we note that the point of failure instead of being given in terms of stress—i. e., pounds per square inch, as is usual for tensile strength, yield point, etc.—is here given in terms of total load—i. e., pounds. The reason for this is obvious from an examination of the curves. Looking at Figure 9 (*b*), thickness 0.090 inch, we see that the load increases approximately proportionally to the width up to a 3-inch width and then the curve continues across approximately horizontally to the 24-inch width, the maximum width tested. The maximum load in this range, 8,000 pounds for the 8 and 12 inch widths, is $\frac{8{,}000}{6{,}500}=1.23$ times that of the minimum load (20-inch plate). In addition, the 24-inch plate, which is 8 times as wide as the 3-inch plate, carries a load $\frac{69}{68}=1.015$ as great as the 3-inch plate. The width, then, so far as failure to take load is concerned, is a minor factor in the range considered, since for large changes of width there are comparatively very small changes in the load carried. We see that a similar situation holds for all the other thicknesses. A compressive strength, then, in terms of average stress instead of total load would not show clearly the behavior of the plates.

In Figure 9 (*a*) are plotted loads against thickness for the range of widths considered useful, 4 to 24 inches. In this figure are also shown two dotted lines marked 4 and 24. These represent the buckling loads (derived from Bryan's theoretical formula) for 4 and 24 inch plates, respectively, which are uniformly loaded at two opposite edges. A discussion of this formula is given in the next section, where, also, these loads are designated as Bryan loads. It is seen that these do not give any measure of the maximum load. In particular, for the widest plate, the 24-inch, the maximum load varies from 6.2 times the Bryan buckling load for the 0.089-inch plate to 21.4 for the 0.031-inch plate. Even for the 4-inch plate the variation of this load ratio for the thickest to the thinnest plate is 1.05 to 7.7. For the wide thin plates, then, the Bryan load is very much lower than the maximum load. As the ratio of width over thickness of the plate decreases, the Bryan load approaches and may quite appreciably exceed the maximum load. Note the 4-inch plate in Figures 10, 11, 12. The character of the results, then, indicate that the maximum load is not the same as the Bryan load. It will be seen on page 14 (Sec. VI–3), that the Bryan loads could not be expected to apply to the test results because of the different methods of loading.

What has been said of the curves of maximum load for duralumin (fig. 9) is also true qualitatively for the corresponding curves of the other three materials, stainless iron, Monel metal, and nickel (figs. 10, 11, 12.) The Monel metal, in particular, shows greater variation of load with width. In the three greatest thicknesses there is a more marked dropping off of load, characteristic of buckling phenomena, when the plate width is increased from12 to 24 inches. However, the ratio of variation in load for the two extreme widths of the practical range, 4 and 24 inches, is still small compared to the ratio of variation in width.

To sum up, the (*b*) curves apparently present two different ranges of compression failure. In the thicker specimens we see that at first the loads increase approximately with the width, indicating a failure up toward the yield point of the material. (This region for the thinner specimens would be expected to occur with plates much narrower than those tested.) Then there is a rapid curving to the right, representing a combined buckling and direct compression failure. If it were purely buckling, then the wider plates would

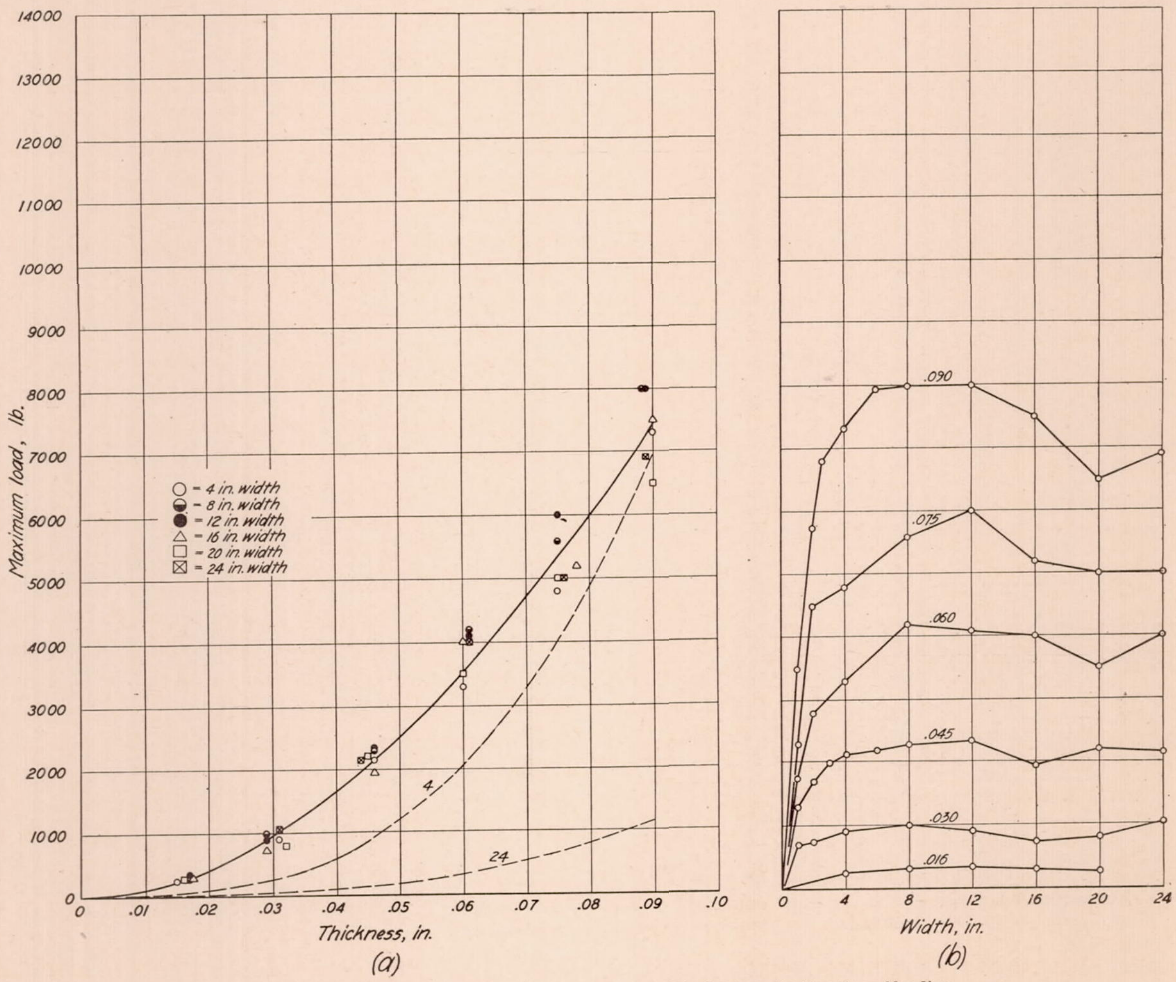

FIGURE 9.—Maximum loads for duralumin plates 24 inches long in direction of loading

(*a*) Load plotted against thickness; various widths. Broken lines show the Bryan loads for the widths (inches) given on the curves
(*b*) Load plotted against width; thicknesses (inches) are given on the curves

fail at a lower load than the narrower plates instead of failing, as they do, at higher loads for widths up to 8 or 12 inches. On the other hand, it can not be a pure compression failure across the entire plate because the average stress is well below the yield point. The comparatively minor change in load with width for specimens 4 inches and wider indicate that in some fashion the wider plates tend to act as though they were narrower. The explanation is to be found in the non-uniform distribution of the load after buckling begins. (See p.14, Sec. VI–3.)

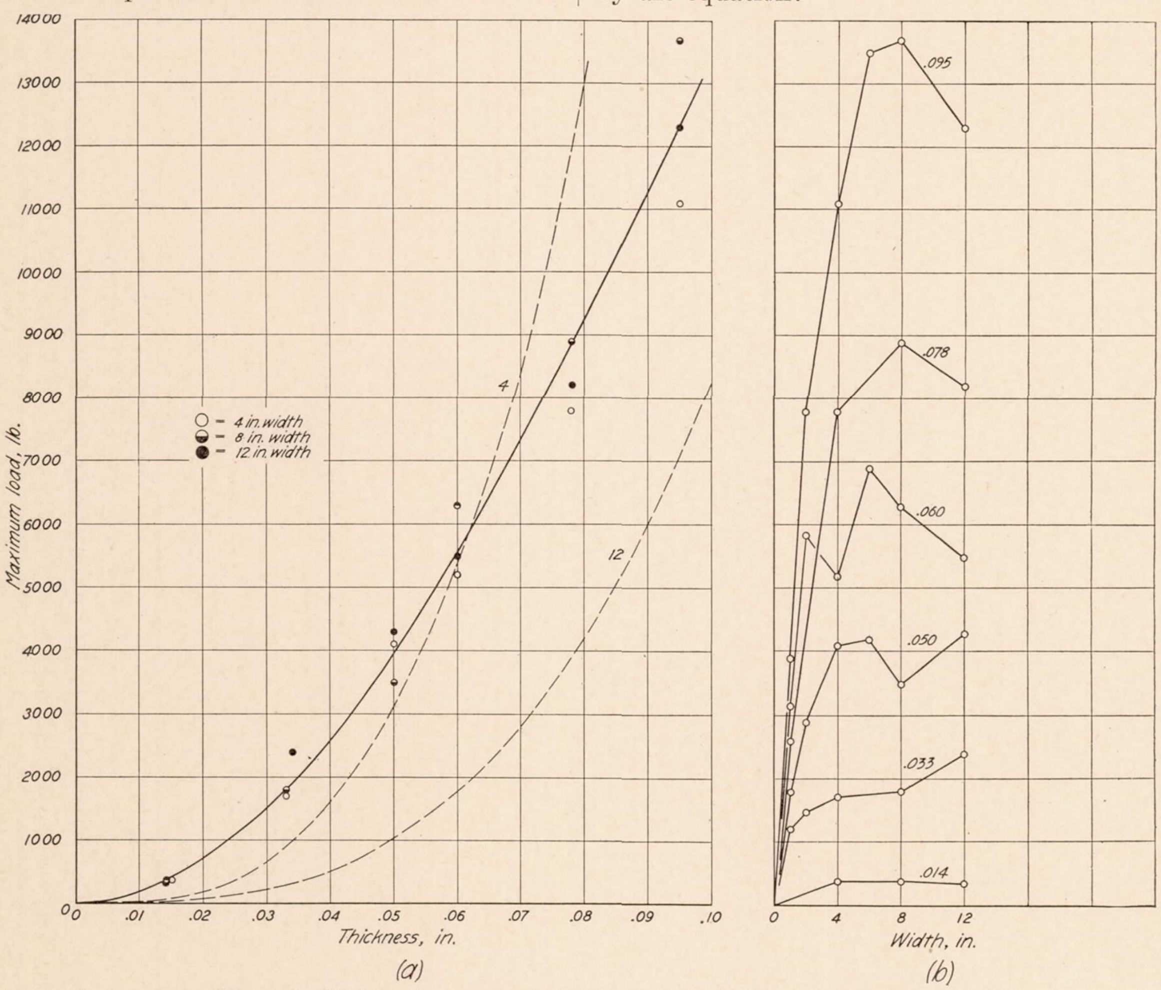

FIGURE 10.—Maximum loads for stainless iron plates 24 inches long in direction of loading

(a) Load plotted against thickness; various widths. Broken lines show the Bryan loads for the widths (inches) given on the curves.
(b) Load plotted against width; thicknesses (inches) are given on the curves.

2. ELASTIC STABILITY

The problem of the elastic stability of a plane rectangular plate has been discussed mathematically by Bryan, [5] Southwell, [6] Timoshenko, [7] Westergaard, [8] Love, [9] Nádai,[10] and others. Of the cases considered, the case of interest here is that of an ideally flat rectangular plate, *simply* supported at all four edges and *uniformly* loaded at two opposite edges, by a compressive load acting in the plane of the plate and perpendicular to these edges. As the load is increased from zero, a critical load is reached at which the plate becomes unstable and may buckle.

The critical value of the compressive stress is given by the equation:

$$\frac{P}{A}=k\frac{\pi^2 E}{12\ (1-\sigma^2)}\frac{t^2}{b^2}$$

in which P = total load, uniformly distributed.

A = area of section perpendicular to direction of loading.

E = Young's modulus of elasticity.

σ = Poisson's ratio.

t = thickness of plate.

b = width of plate, perpendicular to direction of loading.

a = length of plate, parallel to direction of loading.

$k=\left(\frac{a}{mb}+\frac{mb}{a}\right)^2$, where m is an integer which is chosen so as to make k a minimum.

Several values of k and m are given in Table VII.[11]

[5] Bryan, G. H. On the stability of a plane plate under thrust in its own plane. Proc. Lond. Math. Soc., vol. 22, 1890, pp. 54–67.

[6] Southwell, R. V. On the general theory of elastic stability. Phil. Trans. Royal Soc., series (a), vol. 213, 1913, pp. 187–244.

[7] Timoshenko, S. Einige Stabilitätsprobleme der Elastizitästheorie. Zeit. f. Math. u. Phys., vol. 58, 1910, pp. 337–385; and Über die Stabilität Versteifter Platten. Der Eisenbau, vol. 12, 1921, pp. 147–163.

[8] Westergaard, H. M. Buckling of Elastic Structures. Proc. Am. Soc. Civil Eng., vol. XLVII, No. 9, 1921, pp. 455–533.

[9] Love, A. E. H. Math. Theory of Elasticity, 4th edition, 1927.

[10] Nádai, A. Elastische Platten. Julius Springer, 1925.

[11] See Timoshenko and Lessels: Applied Elasticity, 1925, p. 292.

TABLE VII

$\frac{a}{b}=$	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.7	3
$m=$	1	1	1	1	1	1	2	2	2	2	2	3	3
$k=$	8.41	5.14	4.20	4.00	4.13	4.47	4.20	4.04	4.00	4.04	4.13	4.04	4.00

When buckling occurs, the vertical and the horizontal sections are sine curves. Corresponding to the minimum buckling stress there is but one half wave across the width, b, and the integral number, m, of half waves of equal length in the length, a.

from one to two half waves, from two to three half waves, etc., are obtained by substituting $m=1$, 2, 3, etc., in the expression,

$$\frac{a}{b}=\sqrt{m(m+1)}.$$

When $\frac{a}{b}$ is an integer, then m, or the number of half waves, is equal to this integer; in general this number, m, is determined by the nearest integer above or below the ratio $\frac{a}{b}$. The plate, in buckling, therefore tends to divide into square panels.

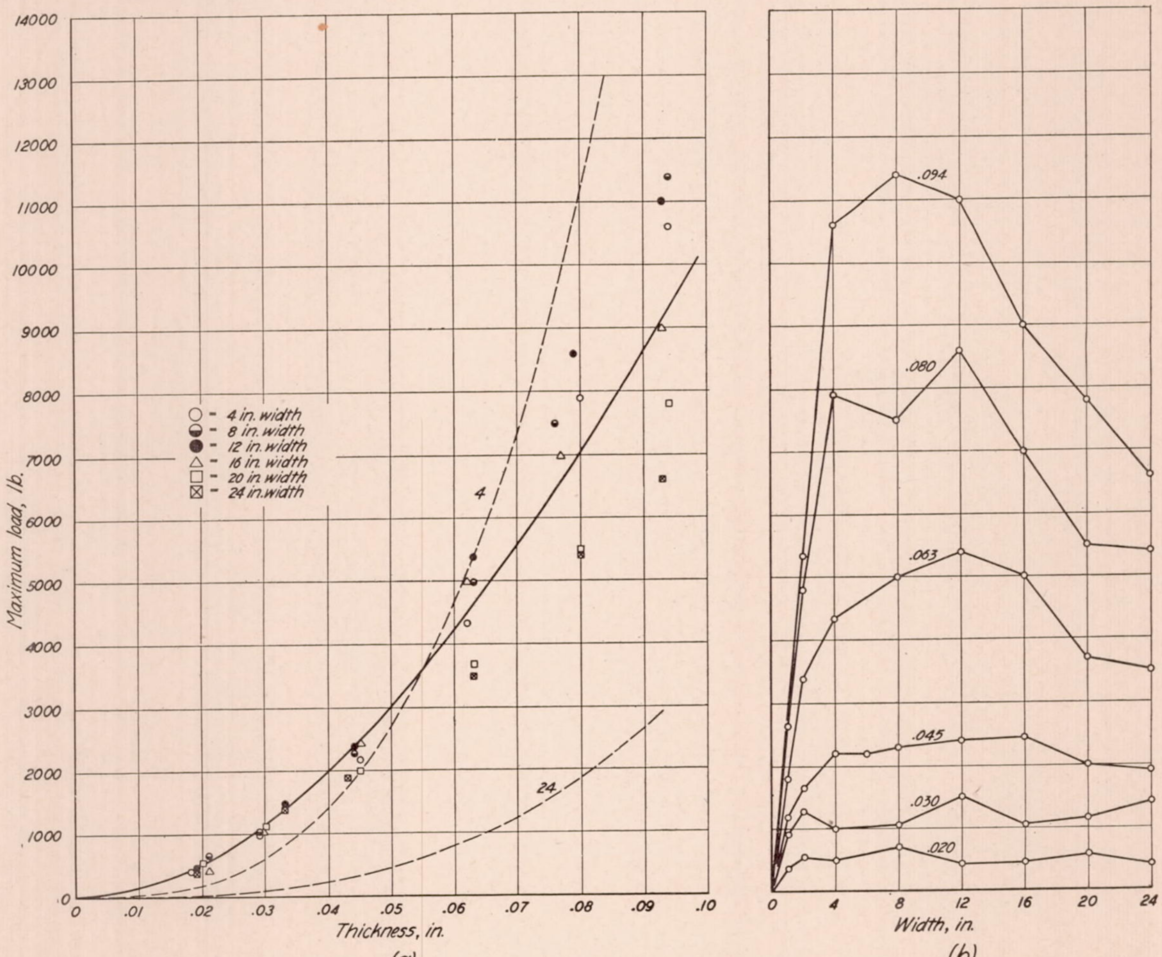

FIGURE 11.—Maximum loads for Monel metal plates 24 inches long in direction of loading

(a) Load plotted against thickness; various widths. Broken lines show the Bryan loads for the widths (inches) given on the curves.
(b) Load plotted against width; thicknesses (inches) are given on the curves.

For values of $\frac{a}{b}$ up to $\sqrt{2}$, $m=1$,

$\frac{a}{b}$ between $\sqrt{2}$ and $\sqrt{6}$, $m=2$,

$\frac{a}{b}$ between $\sqrt{6}$ and $\sqrt{12}$, $m=3$, etc.

The values of $\frac{a}{b}$ which mark the transition values

It should be observed that the minimum buckling stress depends only on the elastic constants (Young's modulus and Poisson's ratio) of the material and on the dimensions of the specimen. The equations for the buckling stress apply only so long as neither the proportional nor the elastic limit of the material is exceeded. For a thin plate the buckling stress may be very small compared to the proportional limit of the material.

G. H. Bryan was the first to give the above solution for the critical load, P, and in the discussion following, the load determined in this way will be called the Bryan load.

In Figures 9, 10, 11, and 12, the Bryan load is indicated by dotted lines [12] for 4 and 24 inch widths, except in the case of stainless iron, for which the greatest width tested was 12 inches, and the Bryan loads for this width are given instead of for 24 inches. The ordinate, or Bryan load, for the 4-inch plate is six times the corresponding ordinate for the 24-inch plate. If the ordinate for the 24-inch plate be taken as unity, the ordinates for 20, 16, 12, and 8 inch plates will be, respectively, 1¼, 1½, 2, and 3. It is seen that for only the very narrow and thick plates do the Bryan loads approach or exceed the maximum loads found in the tests. For the wide, thin plates, the Bryan load is as low as 1/30 of the maximum load and in general varies from 1/10 to 1/20 of the maximum.

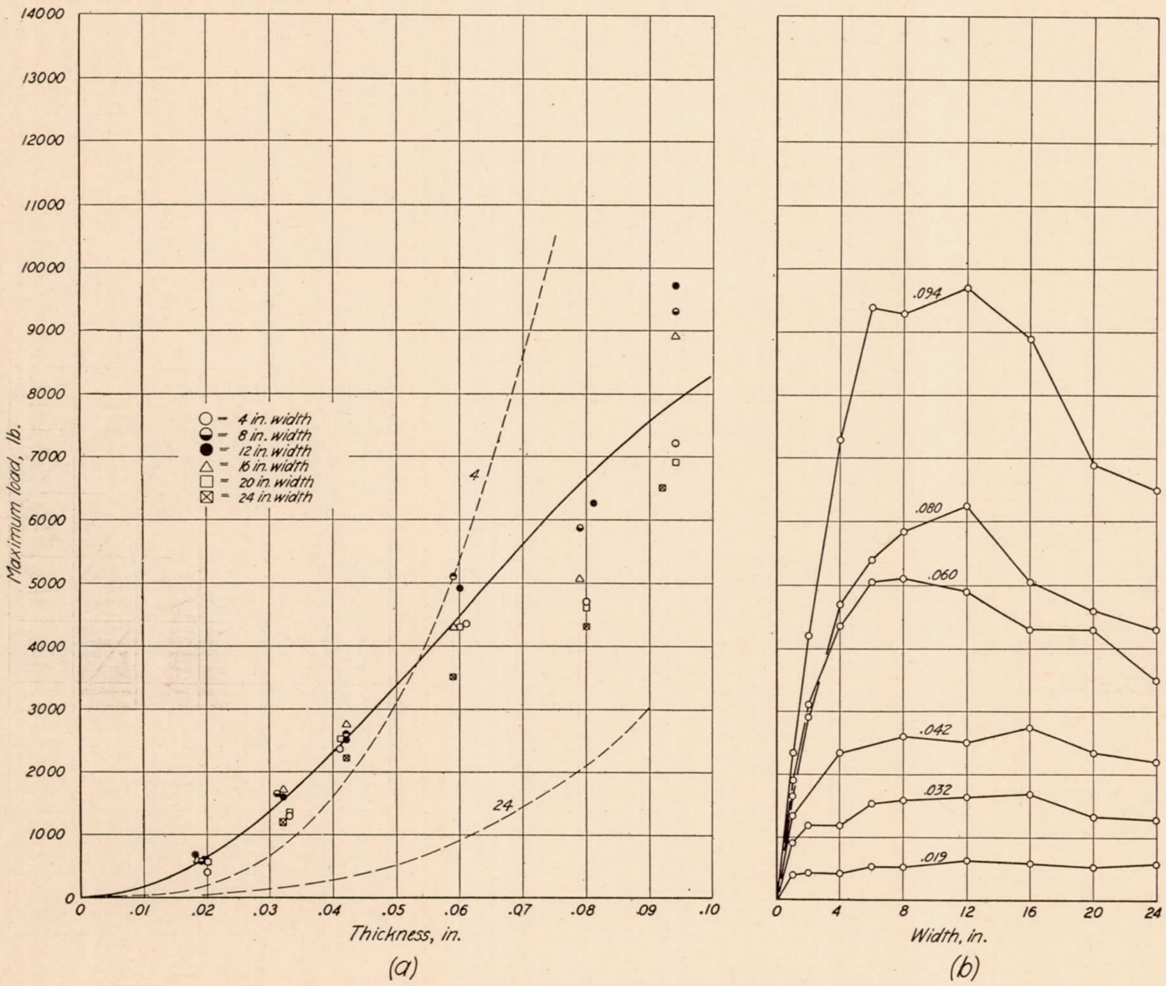

FIGURE 12.—Maximum loads for nickel plates 24 inches long in direction of loading

(*a*) Load plotted against thickness; various widths. Broken lines show the Bryan loads for the widths (inches) given on the curves. (*b*) Load plotted against width; thicknesses (inches) are given on the curves.

12 For these curves Poisson's ratio for duralumin was taken as 0.32 and for the other materials as 0.30. The value used for Young's modulus was 10,400,000 lb./in.² for duralumin, 27,300,000 lb./in.² for stainless iron, 23,600,000 lb./in.² for Monel metal, and 27,500,000 lb./in.² for nickel. These moduli are average values obtained from Tables III, IV, V and VI. Only the first two values of Young's modulus in each group of thicknesses were used in computing the average.

3. DISTRIBUTION OF LOAD

Ideally, under the conditions of the test, if the plate were perfectly flat, the material perfectly uniform, and the load uniformly distributed, we should expect no buckling to appear until the Bryan load was reached, and then the buckling would appear all at once. Immediately there would be a redistribution of load, since the vertical central portion, after buckling, exerts less force upon the loading bar. Consequently the load would be thrown toward the vertical edges of the plate, which are better supported to resist bending, and the side portions would continue to support an increase of load until, possibly, they failed in direct compression.

An idea of the nature of the loading may be obtained from the following consideration. Figure 36 shows a diagram of a square plate with loading in the direction of the axis of x. Let the equation of the deflected surface be

$$w = A \sin \frac{\pi y}{b} \sin \frac{\pi x}{b},$$

where w is the deflection perpendicular to the plate. This expression assumes no deflection at the edges

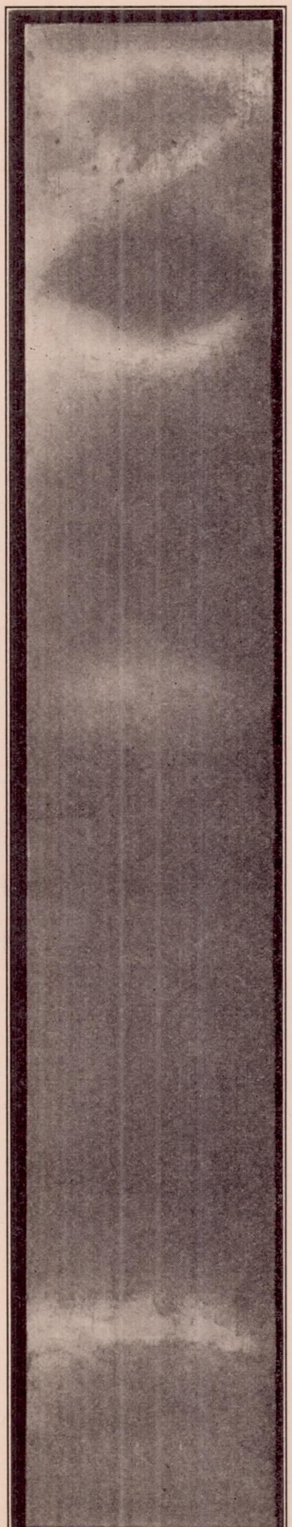

FIGURE 13.—Photograph of duralumin plate (0.090×4×24 inches) after test, showing wavelike deformation

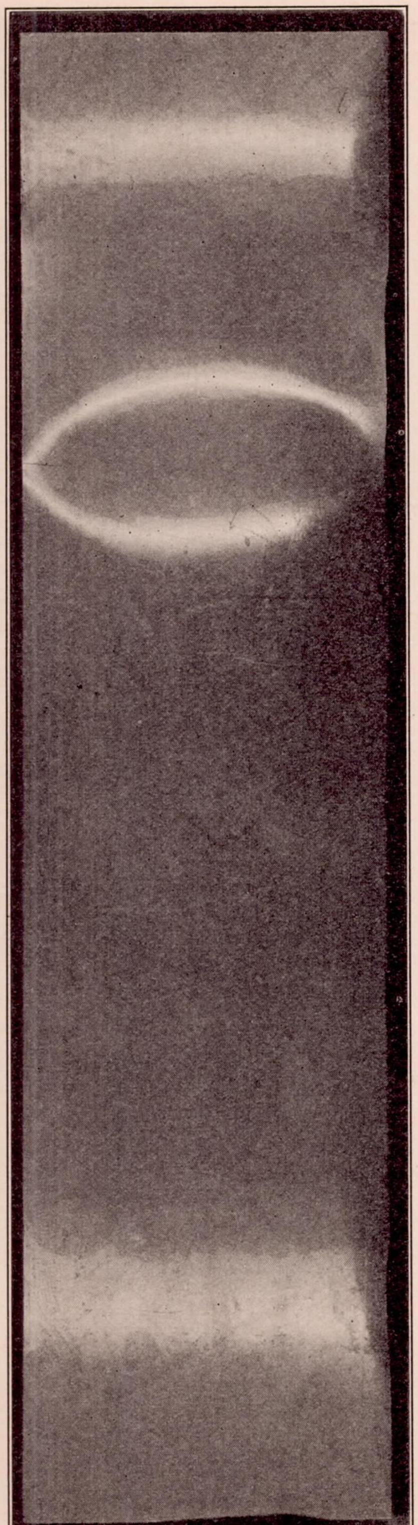

FIGURE 14.—Photograph of nickel plate (0.060×0.6×24 inches) after test, showing wavelike deformation

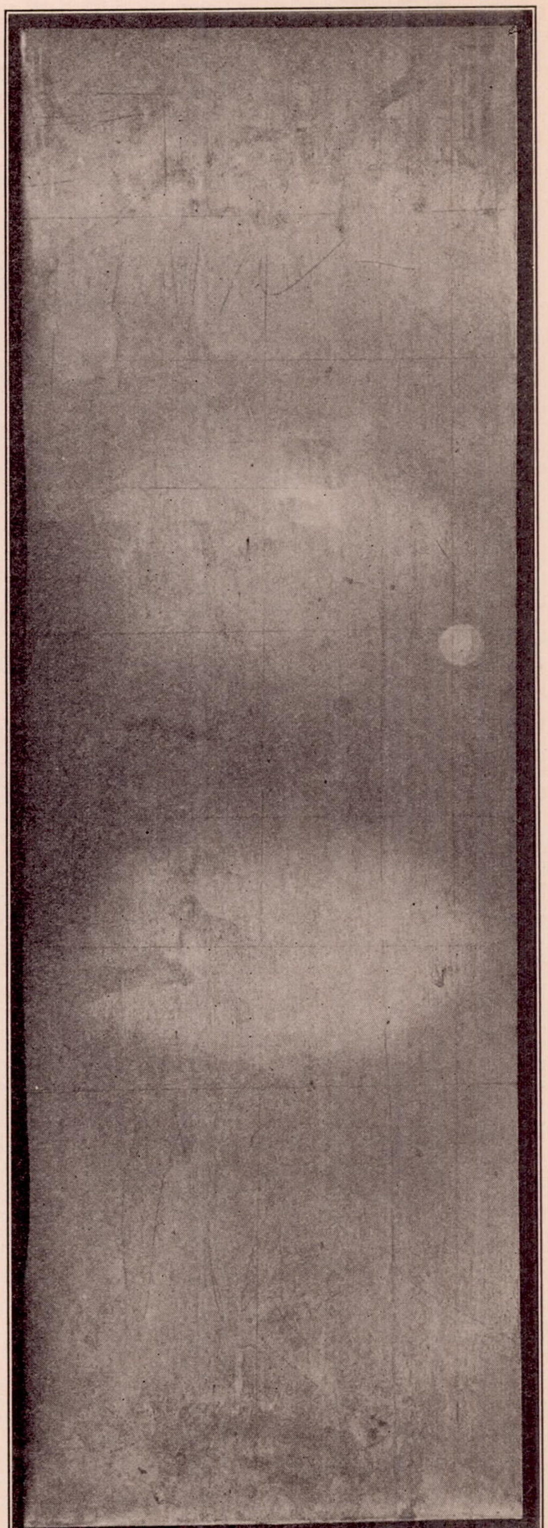

FIGURE 15.—Photograph of nickel plate (0.078×8×24 inches) after test, showing wavelike deformation

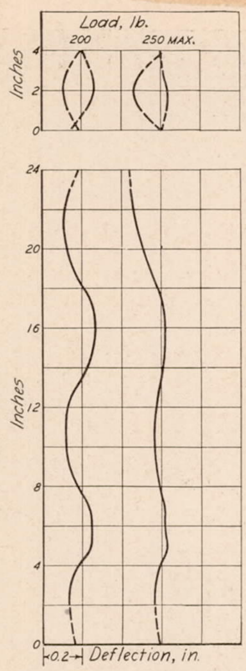

FIGURE 16.—Duralumin, 0.015×4×24 inches

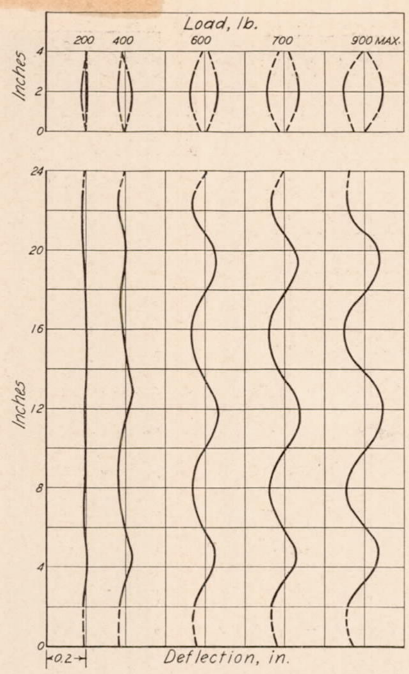

FIGURE 17.—Duralumin, 0.031×4×24 inches

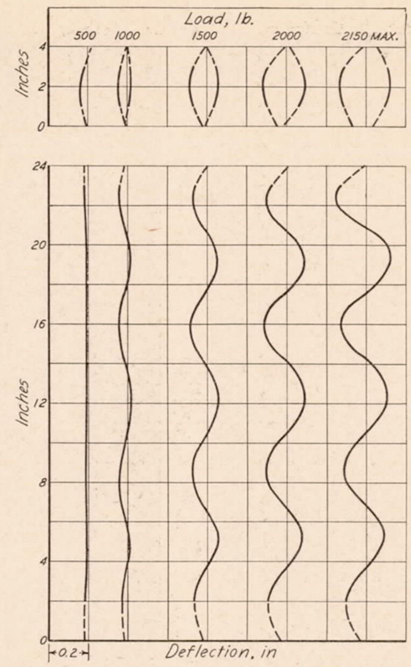

FIGURE 18.—Duralumin, 0.046×4×24 inches

Buckling of plates under the action of loads parallel to their lengths. Note that the horizontal scale is ten times the vertical scale. The lower curves show the shape of the central longitudinal section where the deflections are usually greatest. The upper curves show the shapes of the transverse sections at which the deflection in the indicated direction is a maximum. Dotted lines indicate extrapolation

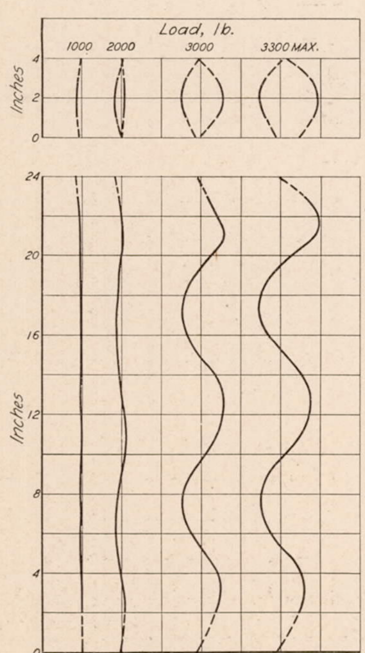

FIGURE 19.—Duralumin, 0.060×4×24 inches

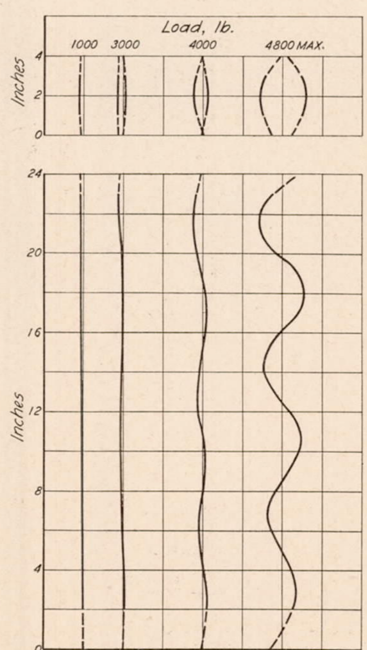

FIGURE 20.—Duralumin, 0.075×4×24 inches

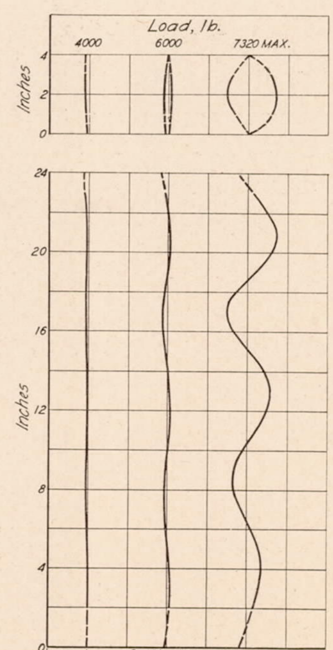

FIGURE 21.—Duralumin, 0.090×4×24 inches

Buckling of plates under the action of loads parallel to their lengths. Note that the horizontal scale is ten times the vertical scale. The lower curves show the shape of the central longitudinal section where the deflections are usually greatest. The upper curves show the shapes of the transverse sections at which the deflection in the indicated direction is a maximum. Dotted lines indicate extrapolation.

and maximum deflection at the center. Let Δ_y represent the difference between the lengths of chord and arc in an element at section AA, distant y from Ox. Let ϵ_y represent the direct compressive strain (assumed uniform) in this element, and let ϵ_o represent the same in the elements $y=0$ and $y=b$. Then, with the upper and lower edges of the plate, the loading bar, and the base plate parallel and true, the following relation should hold:

$$\epsilon_o = \frac{\Delta_y}{b} + \epsilon_y$$

Under the assumption that p_y, the load per unit of area, at any point is proportional to the compression in the strip under the load, $\epsilon_y = \frac{p_y}{E}$

deflect under the load and tend to give a more uniform distribution, and therefore probably produce failure in the plate at a lower load. This is evidently what happened in the case of the wider plates, where the loading bar is relatively very much more flexible (deflection varies as the cube of the length).

4. BUCKLING

In most of the plates the buckling was gradual, increasing in magnitude with the load and showing no sudden change. In some of the thick and narrow specimens, however, there was no appreciable buckling until the load approached the maximum. Owing

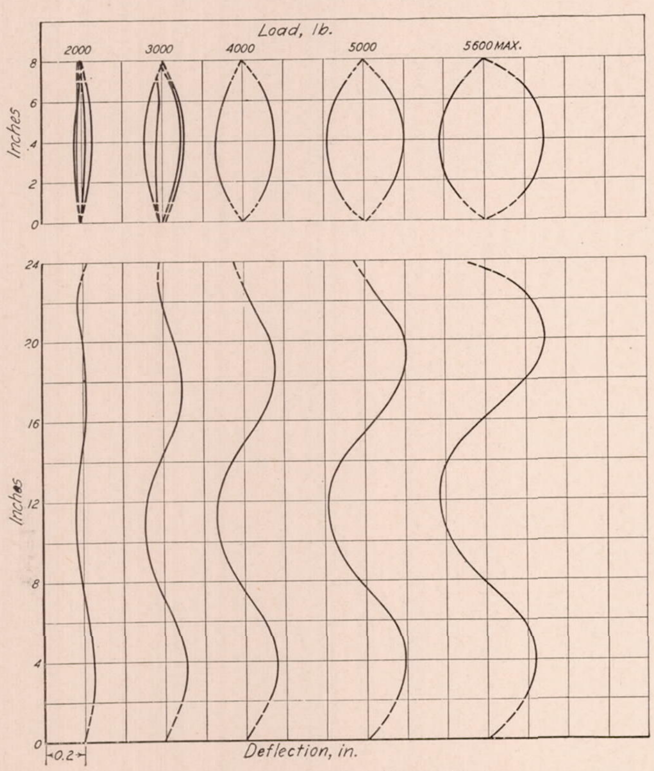

FIGURE 22.—Duralumin, 0.075×8×24 inches

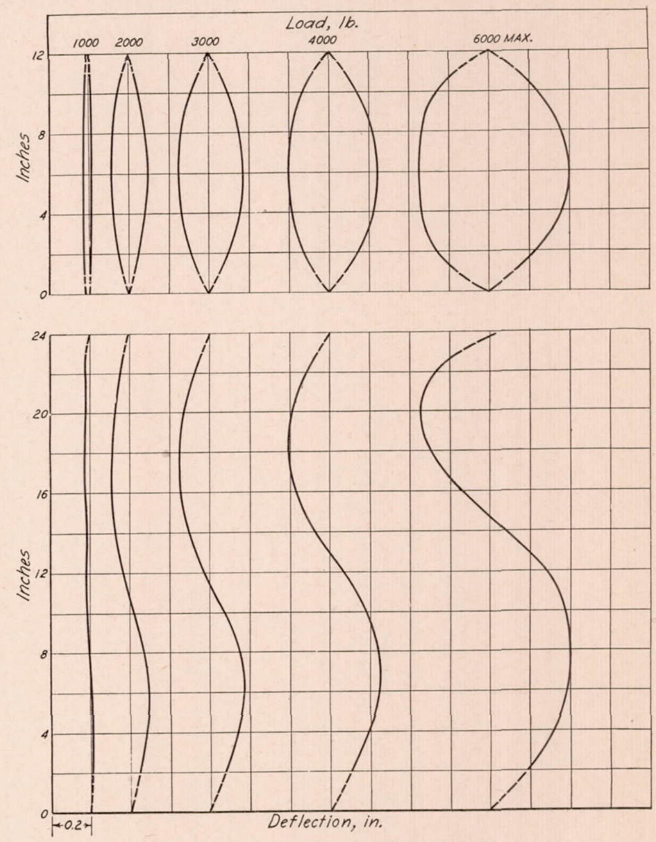

FIGURE 23.—Duralumin, 0.075×12×24 inches

Buckling of plates under the action of loads parallel to their lengths. Note that the horizontal scale is ten times the vertical scale. The lower curves show the shape of the central longitudinal section where the deflections are usually greatest. The upper curves show the shapes of the transverse sections at which the deflection in the indicated direction is a maximum. Dotted lines indicate extrapolation.

The difference between the lengths of chord and arc of a sine curve of small amplitude may be expressed by

$$\Delta_y = \frac{\pi^2 A^2_y}{4b}$$

where A_y = amplitude $= A \sin \frac{\pi y}{b}$

Substituting for ϵ_y and Δ_y and solving for p_y gives:

$$p_y = p_o - \frac{\pi^2 A^2}{4b^2} E \sin^2 \frac{\pi y}{b} \text{ (see fig. 37)}$$

where p_o equals the value of p_y for $y=0$, $(y=b)$.

If the loading bar is not rigid, however, as was assumed in the above calculation, then the bar will

to lack of ideal conditions, such as initial curvature, all plates buckled before the Bryan load was reached. Practically all of the measurements of deflection taken under load showed evidences of buckling of the plate.

The Bryan theory predicts the wave deformation of the plates quite satisfactorily. The number of half waves in the majority of the plates is given by the ratio of the length (24 inches) to the width. For instance, in the 4-inch plates there are six half waves, or six approximately equal square panels. In the 8-inch plates there are three panels; in the 12-inch plates, two. In the case of the 16-inch plates some specimens give two half waves, others one. The theory predicts two for this case, since $\sqrt{2}<\frac{24}{16}<\sqrt{6}$. Most of the 20 and 24 inch specimens give but one half wave. It should be observed that theoretically the plate may buckle into any whole number of half waves, and that the length-width ratio gives the number of half waves corresponding to a minimum value of the critical load. No other value is probable, however. It is believed that for some of the specimens the initial deviations from true planeness may have been large enough to contribute to the form of the buckling, especially in the case of the thinner specimens. For instance, many of the 4-inch plates buckled into 5 and 7 panels, some of the 12-inch widths into 3 panels, and some of the 24-inch widths into 2, while a few others gave still greater variation. On account of the comparatively large deviations from planeness in the stainless iron, no specimens of this material wider than 12 inches were tested. Inequalities in the set-up would also contribute to producing panels of unequal length and deflection.

5. CONDITIONS OF SUPPORT

The conditions of ideal support require the four edges of the initial mid-plane of the plate to remain in the same plane at all times. As actually supported, the transverse curvature assumed under load causes the vertical edges of the plate to move perpendicularly to the plate as soon as they leave their initial positions.

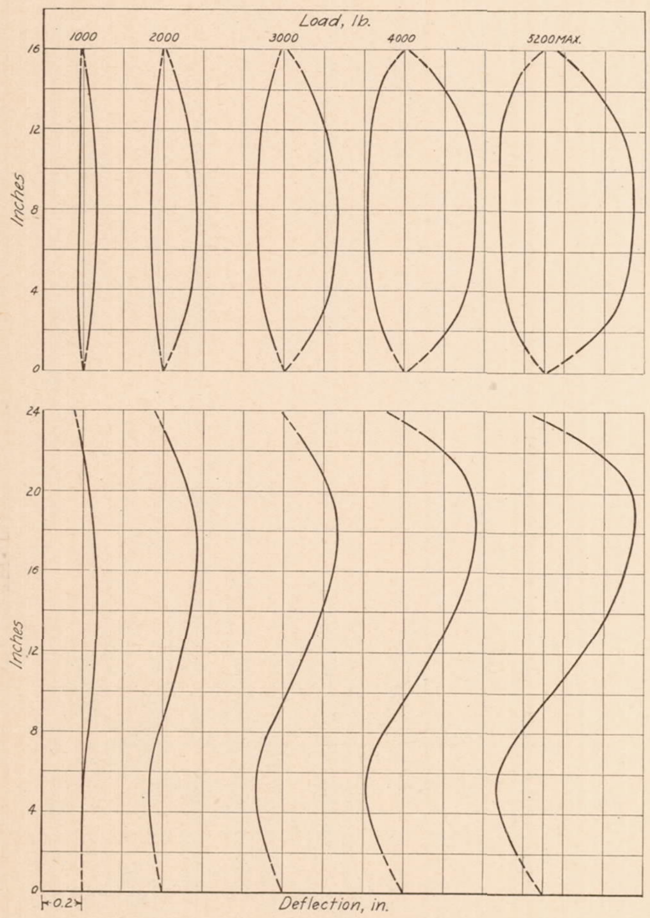

FIGURE 24.—Duralumin, 0.075×16×24 inches

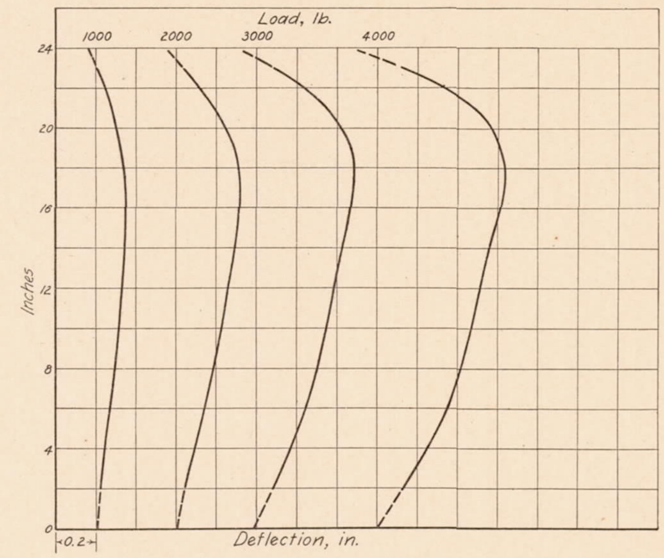

FIGURE 25.—Duralumin, 0.075×20×24 inches

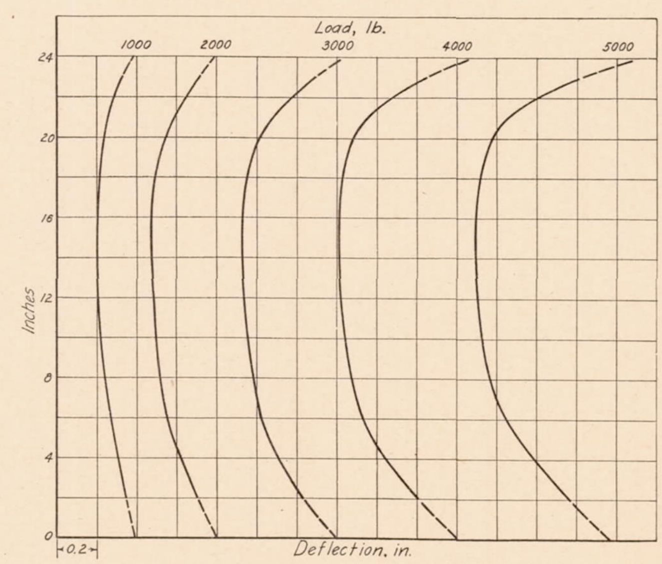

FIGURE 26.—Duralumin, 0.076×24×24 inches

Buckling of plates under the action of loads parallel to their lengths. Note that the horizontal scale is ten times the vertical scale. The lower curves show the shape of the central longitudinal section where the deflections are usually greatest. The upper curves show the shapes of the transverse sections at which the deflection in the indicated direction is a maximum. Dotted lines indicate extrapolation.

Figure 38, representing a horizontal cross section under load, illustrates the motion referred to. The point of the edge initially at M moves along the V-groove to N, and the point at M_o thus moves to N_o; the point at M_o has moved a distance P_oN_o perpendicular to the original position of the plate. Consequently, any initial curvature of the edges is increased, and this may be expected to cause failure at a lower load than otherwise.

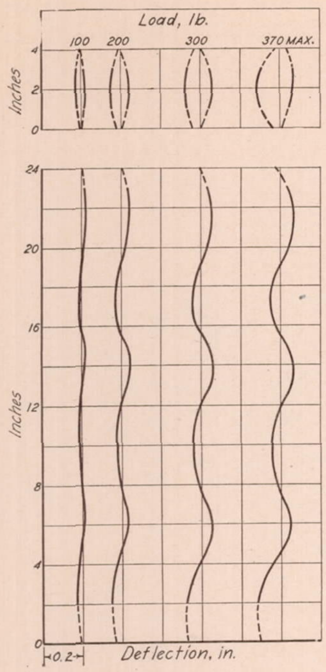

FIGURE 27.—Iron, 0.015×4×24 inches

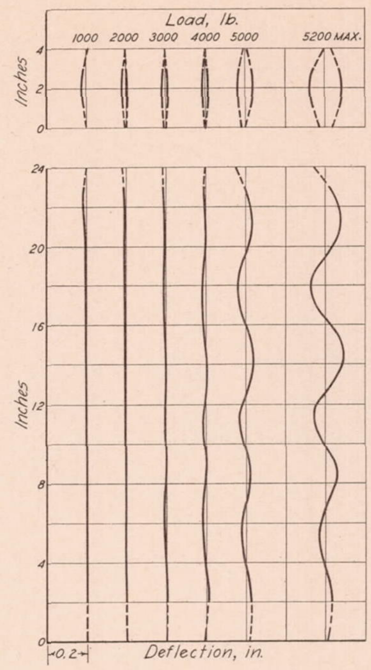

FIGURE 28.—Iron, 0.060×4×24 inches

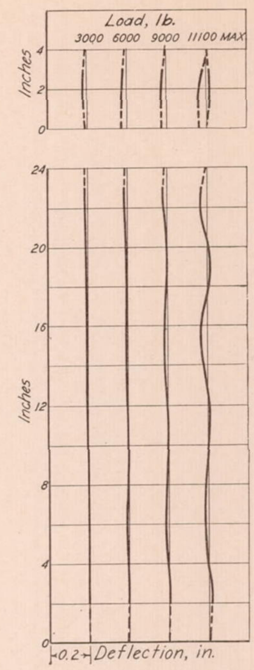

FIGURE 29.—Iron, 0.095×4×24 inches

Buckling of plates under the action of loads parallel to their lengths. Note that the horizontal scale is ten times the vertical scale. The lower curves show the shape of the central longitudinal section where the deflections are usually greatest. The upper curves show the shapes of the transverse sections at which the deflection in the indicated direction is a maximum. Dotted lines indicate extrapolation.

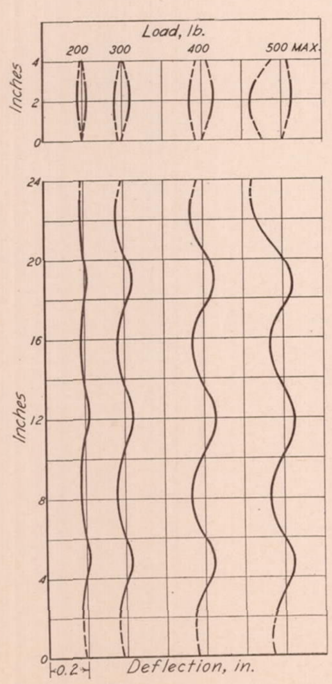

FIGURE 30.—Monel metal, 0.019×4×24 inches

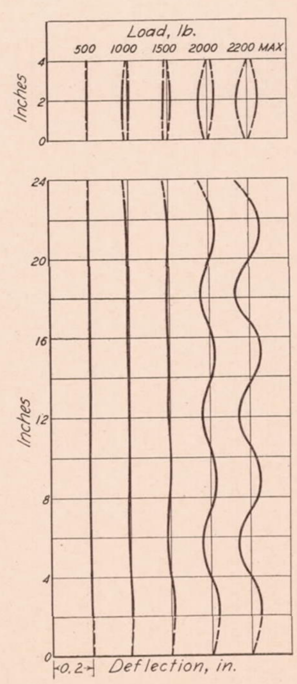

FIGURE 31.—Monel metal, 0.045×4×24 inches

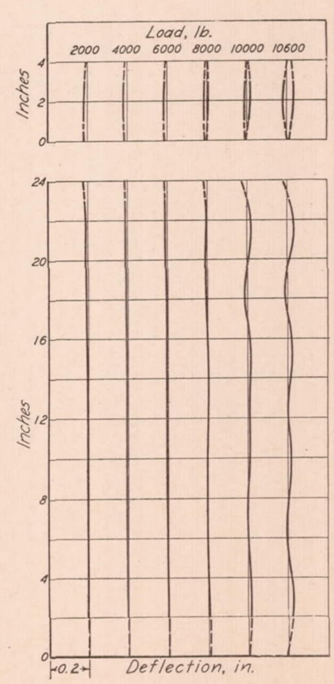

FIGURE 32.—Monel metal, 0.094×4×24 inches

Buckling of plates under the action of loads parallel to their lengths. Note that the horizontal scale is ten times the vertical scale. The lower curves show the shape of the central longitudinal section where the deflections are usually greatest. The upper curves show the shapes of the transverse sections at which the deflections in the indicated direction is a maximum. Dotted lines indicate extrapolation.

There is also a question of the rigidity of the supporting channels. Some of the plates snapped out of the grooves near the top, and it is probable that this action was due to a spreading of the channels near the top. If any spreading occurred, the effect in all cases

FIGURE 33.—Nickel, 0.020×4×24 inches

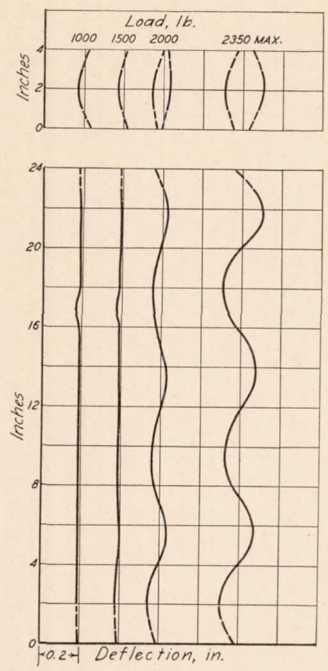

FIGURE 34.—Nickel, 0.041×4×24 inches

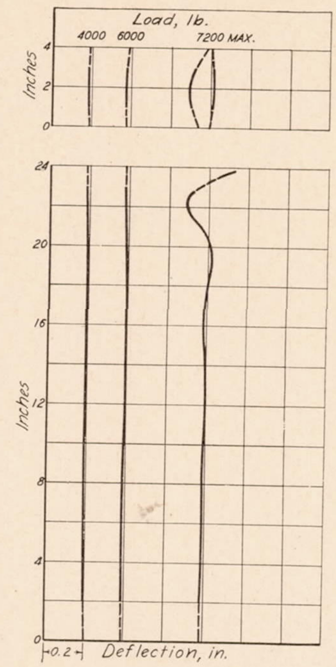

FIGURE 35.—Nickel, 0.094×4×24 inches

Buckling of plates under the action of loads parallel to their lengths. Note that the horizontal scale is ten times the vertical scale. The lower curves show the shape of the central longitudinal section where the deflections are usually greatest. The upper curves show the shapes of the transverse sections at which the deflection in the indicated direction is a maximum. Dotted lines indicate extrapolation.

would be similar to that noted in the preceding paragraph.

In future tests it might be well to arrange to equalize the pressure on the two screws (Fig. 7) holding the V-grooves against the plate.

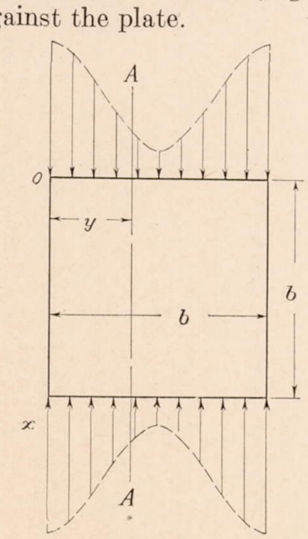

FIGURE 36.—Diagram of loaded plate

Such unsymmetrical curves of deflection as the lower curves in Figure 24 may possibly be explained by one or more of the conditions of support mentioned in this section.

6. VARIATION OF MAXIMUM LOAD WITH THE DIMENSIONS OF THE PLATE

The maximum loads plotted in Figures 9 to 12, inclusive, show a continuous increase with the thickness, which in each case may be approximated to by a curve, shown by the full black line, of the type:

$P = K_1t^2 - K_2t^3$ where P = total maximum load,
t = thickness of the plate,
K_1, K_2 are constants dependent on the properties of the material, the conditions of support, and the original condition of the plate—initial curvature, etc.

In the range of widths from 4 to 24 inches the Monel metal shows the largest variation of load with width.

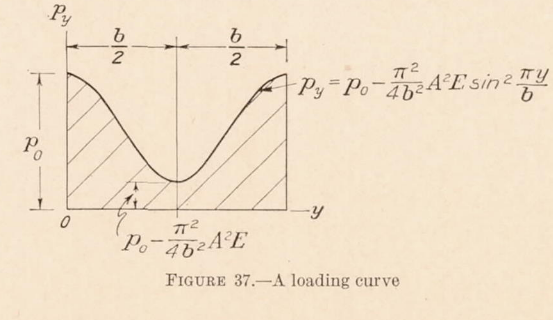

FIGURE 37.—A loading curve

(See figs. 9 (*b*), 10 (*b*), 11 (*b*), 12 (*b*).) The variation amounts at most to about ±40 per cent, though more generally to not more than ±20 per cent from the average value for a given thickness. The variation for duralumin is usually less than ±15 per cent and that for stainless iron is in general of the same order. The variation for nickel is somewhat larger.

The (*b*) curves (figs. 9–12) all have the same general form—peak load for the plate, 8 or 12 or, sometimes,

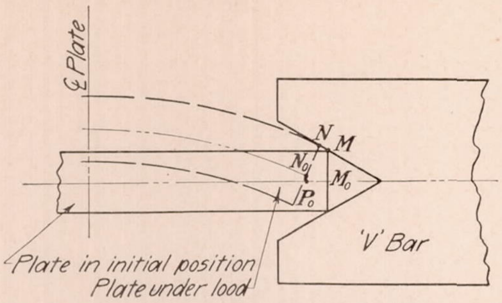

FIGURE 38.—Diagram showing displacement of the edge of the midplane of the plate when bending occurs (not to scale)

16 inches wide, dropping off for the 4-inch width and more considerably for the 20 and 24 inch widths. As width is so small that the effective load-carrying area extends across the plate.

7. VARIATION WITH THE TENSILE PROPERTIES OF THE MATERIAL

The loads carried by 4-inch plates of various thicknesses and materials are shown in Figure 39. It is seen that the two nickel plates of greatest thicknesses, which have low yield points, fall well out of the group of iron and Monel metal. In Figure 12 (*a*) it is seen that all the nickel plates 0.08 inch thick carry low maximum loads. Still other comparisons may be drawn from the results to show that for a given material, low tensile properties in general accompany low maximum loads.

Obviously a numerical formulation of the variation of plate strength with some property of the stress-strain curves will depend upon the specific property

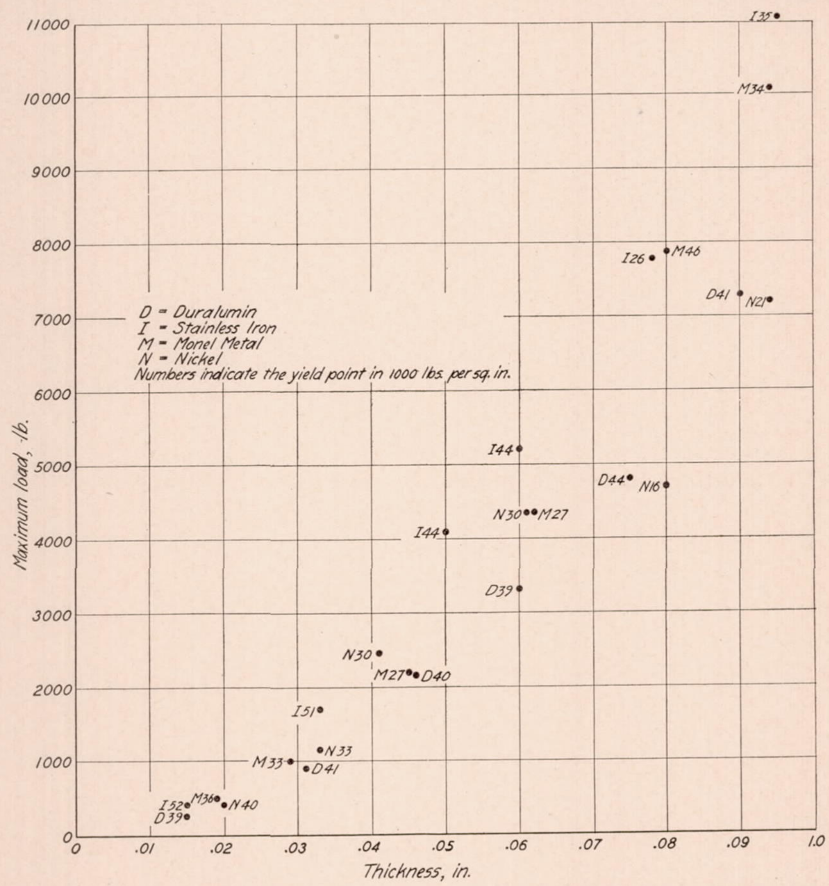

FIGURE 39.—Maximum loads for plates 4 inches wide and of various thicknesses and materials

mentioned previously, if a more rigid bar were used we should expect less dropping off of load for the wider plates.

It appears that after buckling, the wide plate acts as though it were a narrow plate of a width corresponding to that of the side portions which are taking most of the load. It might even be that the center portion is buckled sufficiently to lose contact with the loading bar. Consequently one would expect two plates of the same thickness, but of different widths, to fail at not very greatly different loads, unless the chosen. If the curves were affine, any set of homologous points would be a satisfactory measure of comparative strength, but with such variations in the stress-strain curves as are shown in Figures 3 to 6 one would not expect that such a blanket definition as that of the yield point used in this paper would specify points of the same significance in every case, even though the cases were limited to different thicknesses of the same material. A more highly specialized test than that described in this paper would be necessary to determine the best correlation between plate strength and the properties of the stress-strain diagram.

The duralumin plates generally showed larger deflections for a given load than those of the same dimensions of the other materials. This was to be expected, because this material has a lower modulus of elasticity than iron, monel metal, or nickel. Since a larger deflection with the same load produces larger bending stress, failure would be produced at a lower load, other things being equal, in the case where the deflection is larger. The maximum loads carried by 0.06-inch plates of various widths and materials are shown in Figure 40. It will be noticed (figs. 39 and 40) that, in general, the maximum load for duralumin is less than that for other materials, the dimensions of the plates being the same.

8. PERMANENT SET

In the case of the duralumin, no permanent deflection was measurable at the observed load next preceding the maximum. For the other materials this is not the case. The Monel-metal plates show permanent set at loads approximately three-fifths of the maximum loads. The nickel and stainless iron plates show a slight set at the loads next preceding the maximum loads; i. e., at loads equal to about four-fifths of the maximum loads.

This may perhaps be explained from the stress-strain diagrams for the different materials. If the maximum load is that at which the portions of the plate supporting the greatest stress are yielding rapidly as compared with the increase in load (that is, these portions are undergoing marked permanent deformation; their stresses are near the yield point), then the load at which a plate will exhibit a permanent set will be near or far from the maximum according as the tensile stress-strain graph does or does not curve sharply as the limit of proportionality between stress and strain is passed. The duralumin graphs (fig. 3) do curve sharply, and permanent set occurs near the maximum load. Those for the other materials (figs. 4, 5, 6) curve less sharply, and permanent set occurs farther from the maximum.

VII. CONCLUSIONS

1. For the plates of this investigation, plates loaded in the direction of rolling, buckling occurred at loads less than the Bryan load. This was found whenever observations at such smaller loads were taken.

2. Except for the cases noted below, the plates carried loads above the Bryan load. The wide and thin specimens in particular carried much greater loads. The stainless iron, Monel metal, and nickel plates 4 inches wide carried less than the Bryan load when their thicknesses equaled or exceeded 0.06 inch.

3. The maximum load carried by a plate depended far more upon the thickness than upon the width of

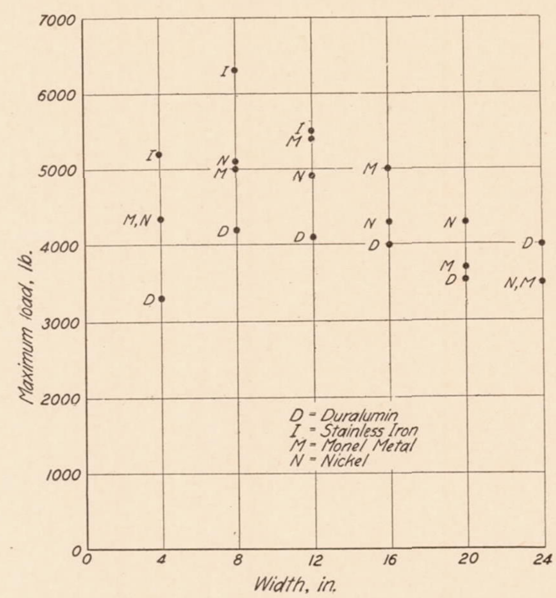

FIGURE 40.—Maximum loads for plates 0.06 inch thick and of various widths and materials

the plate unless the plate was narrow (in this work, less than 4 inches).

In general, the several maximum loads carried by duralumin plates of a given thickness and ranging in width from 4 to 24 inches did not individually depart from their average by more than 15 per cent, whatever the thickness within the range studied. For Monel metal the corresponding departures from the average are in general not more than 20 per cent.

4. Permanent deflections generally occurred between the loads mentioned below, M indicating the maximum load.

Material	Permanent deflection generally occurred between—
Duralumin	0.8 M and M.
Stainless iron [1]	0.6 M and 0.8 M.
Monel metal [1]	0.4 M and 0.6 M.
Nickel [1]	0.6 M and 0.8 M.

[1] Except 4-inch plates 0.06 inch or more thick, on which permanent deflection generally occurred between 0.8 M and M.

TABLE III.—MECHANICAL PROPERTIES OF DURALUMIN (FIG. 3)

TENSILE PROPERTIES

Thickness of tensile specimen	Yield point (stress at slope equal to ⅓ E)	Stress by Army-Navy Spec. AN 9092 (1929 issue)	Tensile strength	Young's modulus E	Elongation in 2 inches
Inch	*Lbs./in.²*	*Lbs./in.²*	*Lbs./in.²*	*Lbs./in.²*	*Per cent*
0.0148	39,000	41,100	60,700	10,500,000	18.5
.0148	39,000	41,100	60,700	10,400,000	18.5
.0309	40,000	42,000	63,650	10,200,000	19.5
.0310	42,000	43,000	64,300	10,300,000	18
.0434	40,000	42,000	61,700	10,400,000	19.5
.0432	41,000	42,300	61,300	10,400,000	19.5
.0605	40,000	41,500	61.750	10,400,000	20
.0607	38,500	41,100	61,700	10,400,000	20
.0732	45,000	45,500	64,000	10,500,000	19
.0732	44,000	45,500	64,000	10,400,000	21
.0892	42,000	42,600	62,300	10,400,000	20
.0893	41,000	42,800	62,000	10,600,000	20

OTHER PROPERTIES

Thickness of material	Brinell number (1/16-in. ball; 6.4-kg. load)	Rockwell B-scale (1/16-in. ball; 100-kg. load)	Erichsen value (opening, 27 mm. diameter; ball, 10 mm. diameter)
Inch			*Mm.*
0.015	103	67.4	7.32
	103	68.3	6.74
	103	67.7	6.97
		68.3	----------
	----------	67.7	----------
	Av. 103	Av. 67.9	Av. 7.01
.031	114	71.2	6.98
	114	71.3	6.92
	106	70.8	7.06
	----------	70.7	----------
	----------	70.8	----------
	Av. 111	Av. 71.0	Av. 6.99
.043	110	75.5	7.23
	117	75.8	7.25
	118	75.7	7.24
	----------	75.3	----------
	----------	75.0	----------
	Av. 115	Av. 75.5	Av. 7.24
.061	117	77.6	6.75
	114	77.6	6.63
	----------	77.4	6.82
	----------	77.7	----------
	----------	77.8	----------
	Av. 115	Av. 77.6	Av. 6.78
.073	100	77.3	5.51
	112	77.3	5.60
	110	77.4	----------
	----------	78.0	----------
	----------	77.0	----------
	Av. 111	Av. 77.4	Av. 5.55
.089	112	78.3	5.80
	109	77.8	5.53
	108	77.8	5.49
	----------	78.4	----------
	----------	78.3	----------
	Av. 110	Av. 78.1	Av. 5.61

TABLE IV.—MECHANICAL PROPERTIES OF STAINLESS IRON (FIG. 4)

TENSILE PROPERTIES

Thickness of tensile specimen	Yield point (stress at slope equal to ⅓ E)	Tensile strength	Young's modulus E	Elongation in 2 inches
Inch	*Lbs./in.²*	*Lbs./in.²*	*Lbs./in.²*	*Per cent*
0.0120	50,000	87,600	26,000,000	7
.0137	55,000	91,600	26,300,000	9.5
.0347	51,000	86,600	27,000,000	11.5
.0340	52,000	85,900	27,700,000	12
.0496	41,000	66,400	24,300,000	19
.0500	43,000	67,900	24,000,000	20
[1] .0500	44,000	72,800	25,000,000	18
.0607	48,000	83,000	32,700,000	18.5
.0605	42,000	82,300	34,000,000	19
[1] .0607	41,000	81,700	31,800,000	19
[1] .0610	46,000	81,100	32,800.000	20
.0755	26,000	68,000	26,000,000	25.5
.0755	26,000	67,500	25,700,000	25.5
.0945	34,000	72,000	26,800,000	26.5
.0952	36,000	72,800	27,400,000	23.5

[1] Check test (Sec. III-2*b*.).

OTHER PROPERTIES

Thickness of material	Brinell number (1/16-in. ball; 6.4-kg. load)	Rockwell B-scale (1/16-in. ball; 100-kg. load)	Erichsen value (opening, 27 mm. diameter; ball, 10 mm. diameter
Inch			*Mm.*
0.014	203	----------	6.43
	194	----------	6.50
	Av. 198	----------	Av. 6.47
.034	185	95.3	5.57
	203	93.2	5.40
	191	95.2	----------
	----------	93.3	----------
	----------	93.2	----------
	Av. 193	Av. 94.0	Av. 5.48
.050	173	90.3	----------
	185	90.3	9.38
	185	89.8	9.22
	----------	90.8	----------
	----------	89.5	----------
	Av. 181	Av. 90.1	Av. 9.31
.060	185	90.0	----------
	185	89.2	----------
	----------	89.5	----------
	----------	90.8	----------
	----------	91.2	----------
	Av. 185	Av. 90.1	----------
.076	134	89.3	----------
	134	88.5	----------
	139	89.2	----------
	----------	89.2	----------
	----------	88.7	----------
	Av. 136	Av. 89.2	----------
.095	150	89.3	----------
	139	89.0	----------
	----------	88.7	----------
	----------	88.5	----------
	----------	90.0	----------
	Av. 144	Av. 89.1	----------

The stress-strain curves of stainless iron, monel metal, and nickel showed a decrease of modulus with increasing stress at very low stresses. The values reported for Young's modulus are secant moduli corresponding to the dashed lines in Figures 4 to 6, inclusive. Values determined at stresses below 5,000 pounds per square inch were considerably higher. For the stainless iron these values varied from 28,000,000 to 35,000,000 pounds per square inch, and may be associated with the markedly differing grain structure of different specimens.

TABLE V.—MECHANICAL PROPERTIES OF MONEL METAL (FIG. 5)

TENSILE PROPERTIES

Thickness of tensile specimen	Yield point (stress at slope equal to ⅓ E)	Tensile strength	Young's modulus (E)	Elongation in 2 inches
Inch	*Lbs./in.*[2]	*Lbs./in.*[2]	*Lbs./in.*[2]	*Per cent*
0.0184	36,000	85,200	25,700,000	40
.0182	36,000	85,600	24,700,000	36.5
.0328	35,000	76,100	22,400,000	37
.0326	34,000	75,700	22,600,000	37.5
[1] .0328	33,000	75,400	24,200,000	37.5
[1] .0331	33,000	75,150	24,200,000	38
.0438	27,000	75,300	24,000,000	37
.0440	27,000	74,800	24,000,000	37
.0640	28,000	79,700	24,600,000	37.5
.0638	27,000	80,250	24,400,000	37.5
[1] .0638	26,000	79,000	23,800,000	37.5
[1] .0637	27,000	79,100	23,800,000	37.5
.0798	46,000	79,250	24,000,000	32
.0788	46,000	79,300	23,000,000	32.5
.0928	35,000	71,600	20,200,000	36.5
.0925	33,000	71,800	24,000,000	37
[1] .0929	35,000	71,000	23,800,000	36.5
[1] .0930	33,000	71,150	23,800,000	38.5

[1] Check test (Sec. III-2*b*).

OTHER PROPERTIES

Thickness of material	Brinell number (1/16-in. ball; 6.4-kg. load)	Rockwell B-scale (1/16-in. ball; 100-kg. load)	Erichsen value (opening, 27 mm. diameter; ball, 10 mm. diameter)
Inch			*Mm.*
0.019	118	89.7	10.80
	124	87.3	10.49
	118	87.8	10.50
	----	87.7	----
	----	87.9	----
	Av. 120	Av. 87.7	Av. 10.60
.033	118	71.2	11.15
	118	71.0	11.13
	120	71.4	----
	----	71.5	----
	----	70.9	----
	Av. 119	Av. 71.2	Av. 11.14
.044	114	76.8	11.93
	110	77.8	12.10
	110	77.5	12.10
	----	77.3	----
	----	77.2	----
	Av. 111	Av. 77.3	Av. 12.04
.064	101	83.4	----
	102.5	81.8	----
	102.5	84.3	----
	----	84.2	----
	----	82.3	----
	Av. 102	Av. 83.2	----
.079	129.5	88.7	----
	127	88.8	----
	129.5	87.5	----
	----	87.7	----
	----	88.7	----
	Av. 129	Av. 88.3	----
.093	110	89.7	----
	110	89.3	----
	----	89.9	----
	----	89.9	----
	----	88.7	----
	Av. 110	Av. 89.5	----

TABLE VI.—MECHANICAL PROPERTIES OF NICKEL (FIG. 6)

TENSILE PROPERTIES

Thickness of tensile specimen	Yield point (stress at slope equal to ⅓ E)	Tensile strength	Young's modulus (E)	Elongation in 2 inches
Inch	*Lbs./in.*[2]	*Lbs./in.*[2]	*Lbs./in.*[2]	*Per cent*
0.0190	40,000	73,600	28,000,000	34.5
.0189	40,000	74,200	28,000,000	33
.0320	33,000	71,200	28,300,000	36
.0320	33,000	71,200	27,400,000	36.5
[1] .0319	34,000	70,400	28,000,000	36
.0423	30,000	76,600	28,500,000	40
.0423	30,000	76,700	28,100,000	43
.0600	36,000	67,100	27,500,000	35.5
.0594	37,000	67,200	27,600,000	34.5
[1] .0593	30,000	62,900	27,800,000	39
[1] .0589	30,000	62,550	27,800,000	38
.0810	16,000	74,200	27,600,000	42
.0820	15,000	74,400	26,100,000	42
.0925	21,000	65,800	26,700,000	44
.0925	21,000	66,800	26,600,000	43.5

[1] Check test (Sec. III-2*b*).

OTHER PROPERTIES

Thickness of material	Brinell number (11/16-in. ball; 6.4-kg. load)	Rockwell B-scale (11/16-in. ball; 100-kg. load)	Erichsen value (opening, 27 mm. diameter; ball, 10 mm. diameter)
Inch			*Mm.*
0.019	124	82.2	9.56
	118	81.9	9.42
	----	81.4	9.30
	----	82.3	----
	----	83.2	----
	Av. 121	Av. 82.2	Av. 9.43
.032	117	77.2	10.89
	114	77.6	10.55
	118	77.7	10.28
	----	77.3	10.60
	----	77.8	----
	Av. 116	Av. 77.5	Av. 10.58
.042	112	77.8	----
	110	78.0	12.34
	110	77.7	12.32
	----	78.1	----
	----	78.2	----
	Av. 111	Av. 78.0	Av. 12.33
.060	116	75.3	----
	110	75.2	----
	112	75.9	----
	----	75.2	----
	----	75.3	----
	Av. 113	Av. 75.4	----
.080	77.5	64.2	----
	68.5	63.7	----
	81.0	64.2	----
	----	65.3	----
	----	65.2	----
	Av. 76	Av. 64.5	----
.093	102.5	71.7	----
	102.5	67.5	----
	102.5	69.3	----
	----	67.1	----
	----	69.7	----
	Av. 102.5	Av. 69.1	----

The stress-strain curves of stainless iron, monel metal, and nickel showed a decrease of modulus with increasing stress at very low stresses. The values reported for Young's modulus are secant moduli corresponding to the dashed lines in Figures 4 to 6, inclusive. Values determined at stresses below 5,000 pounds per square inch were considerably higher. For the stainless iron these values varied from 28,000,000 to 35,000,000 pounds per square inch, and may be associated with the markedly differing grain structure of different specimens.

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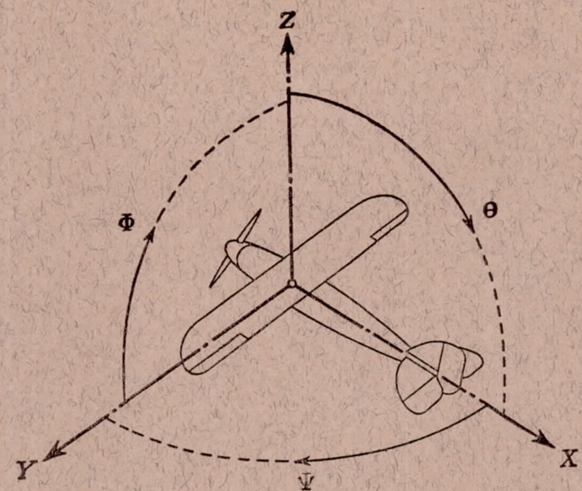

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	rolling	L	$Y \longrightarrow Z$	roll	Φ	u	p
Lateral	Y	Y	pitching	M	$Z \longrightarrow X$	pitch	Θ	v	q
Normal	Z	Z	yawing	N	$X \longrightarrow Y$	yaw	Ψ	w	r

Absolute coefficients of moment

$$C_L=\frac{L}{qbS} \qquad C_M=\frac{M}{qcS} \qquad C_N=\frac{N}{qfS}$$

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D, Diameter.
p_e, Effective pitch.
p_g, Mean geometric pitch.
p_s, Standard pitch.
p_v, Zero thrust.
p_a, Zero torque.
p/D, Pitch ratio.
V', Inflow velocity.
V_s, Slip stream velocity.

T, Thrust.
Q, Torque.
P, Power.
(If "coefficients" are introduced all units used must be consistent.)
η, Efficiency $=T\,V/P$.
n, Revolutions per sec., r. p. s.
N, Revolutions per minute, r. p. m.
Φ, Effective helix angle $=\tan^{-1}\left(\frac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS

1 hp = 76.04 kg/m/s = 550 lb./ft./sec.
1 kg/m/s = 0.01315 hp
1 mi./hr. = 0.44704 m/s
1 m/s = 2.23693 mi./hr.

1 lb. = 0.4535924277 kg
1 kg = 2.2046224 lb.
1 mi. = 1609.35 m = 5280 ft
1 m = 3.2808333 ft.