REPORT No. 376

SOME APPROXIMATE EQUATIONS FOR THE STANDARD ATMOSPHERE

By Walter S. Diehl

SUMMARY

This report, which was prepared for publication by the National Advisory Committee for Aeronautics, contains the derivation of a series of simple approximate equations for density ratios $\frac{\rho}{\rho_o}$, $\frac{\rho_o}{\rho}$, $\sqrt{\frac{\rho_o}{\rho}}$ *and for the pressure ratio* $\frac{p}{p_o}$, *in the standard atmosphere. The accuracy of the various equations is discussed and the limits of applications are given. Several of these equations are in excellent agreement with the standard values.*

INTRODUCTION

A number of factors were considered before the adoption of an aeronautical standard atmosphere for use in the United States. A large majority of aeronautical requirements are met in any standard atmosphere which simply specifies the pressures, temperatures, and densities at various altitudes. For this reason it was decided that the standard atmosphere should be based on some simple but definite relation which gave pressures, temperatures, and densities in close agreement with the average observed values. The Weather Bureau, after comparing various formulas with observed data, concluded that Toussaint's formula for linear decrease in temperature with altitude up to the lower level of the isothermal atmosphere at a temperature of −55° C. not only gave the most satisfactory results, but it also had the great advantage of extended official use abroad.[1] For these reasons it was adopted for official use in this country.

In extended use the standard atmosphere has proved quite satisfactory, but there are certain special cases in which simple mathematical relations are essential in order that integrations may be performed. This condition has led to some criticism of the present standard atmosphere, chiefly by those who prefer a simple exponential expression for the density ratio. It is possible to write approximate equations of various forms which will give fair to excellent agreement with the specified values in the standard atmosphere. Such approximate equations are sufficiently accurate to meet all ordinary requirements. The need for a systematic series of these equations is apparent.

[1] "Standard Atmosphere." W. R. Gregg, N. A. C. A. Technical Report No. 147 (1922).

In this report no attempt has been made to assemble more than a representative series of simple approximate equations of the forms most frequently required. It should be understood that many other types may be used and that greater accuracy can be obtained by the introduction of additional terms or constants. The complex forms, however, have such limited application that their inclusion here is hardly justified.

In all of the equations given in this report the values of the constants are based on the altitude h expressed in feet. Since the values of nondimensional pressure and density ratios at a given point are independent of the altitude units, a conversion to the metric system may readily be made if required by the substitution of $(3.28083h_m)$ for h in the equation, h_m being the altitude in meters.

EXACT EQUATIONS

By definition, the variation of absolute temperature in the standard atmosphere is given by

$$T = T_o - ah, \text{ or } \frac{T}{T_o} = \left(1 - \frac{a}{T_o}h\right)$$

For T in ° C. and h in meters, the value of a is 0.0065, or for T in ° F. and h in feet the value of a is 0.00356617. The value of T_o is 288° C. or 518.4° F.

In N. A. C. A. Technical Note, No. 99[2] it is shown that

$$\frac{p}{p_o} = \left(\frac{T}{T_o}\right)^{\frac{g}{aR}} = \left(1 - \frac{a}{T_o}h\right)^{\frac{g}{aR}}$$

where R is the gas constant for air. Substituting the standard numerical values of the constants gives for h in feet,

$$\frac{p}{p_o} = \left(1 - \frac{h}{145366}\right)^{5.255} \tag{1}$$

and

$$\frac{\rho}{\rho_o} = \left(1 - \frac{h}{145366}\right)^{4.255} \tag{1a}$$

Equations (1) and (1a) are exact up to the lower level of the isothermal atmosphere ($h = 35332$ feet). Other equations[3] apply in the isothermal atmosphere.

[2] "Notes on the Standard Atmosphere," by Walter S. Diehl, 1922.

[3] For a compilation of constants, exact equations and other data, see "Standard Atmosphere—Tables and Data," by Walter S. Diehl, N. A. C. A. Technical Report No. 218 (1925).

APPROXIMATE EQUATIONS FOR DENSITY RATIO $\frac{\rho}{\rho_0}$

Expanding equation (1a) and neglecting all but the first two terms of the series gives

$$\frac{\rho}{\rho_0}=1-\frac{4.255h}{145366}=\left(1-\frac{h}{34160}\right) \qquad (2)$$

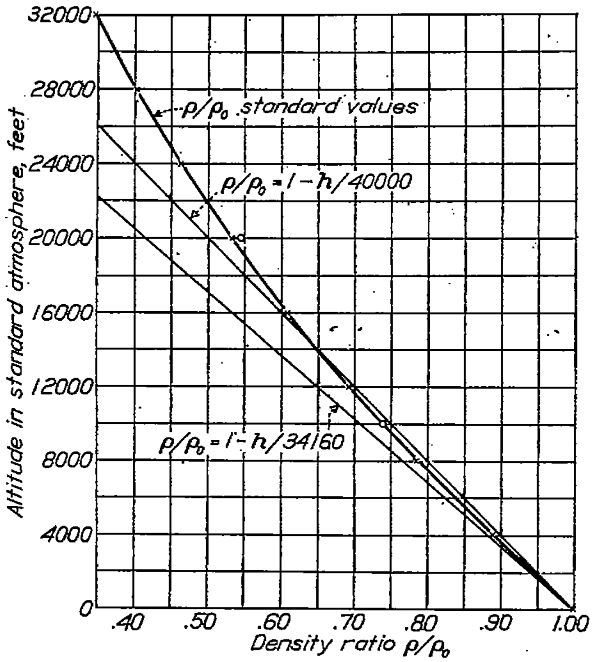

FIGURE 1

On Figure 1 there is given a plot of equation (2) compared with the standard values of $\frac{\rho}{\rho_0}$. The agreement is not satisfactory except for very low altitudes, not greater than about 4,000 feet. As shown on Figure 1, much better average agreement can be secured up to about 16,000 feet if the slope of the line is changed by using $\frac{h}{40000}$ instead of $\frac{h}{34160}$ in equation (2). The comparative values of $\frac{\rho}{\rho_0}$ are as follows:

	Altitude						
	2,000 feet	4,000 feet	6,000 feet	8,000 feet	10,000 feet	12,000 feet	14,000 feet
$\frac{\rho}{\rho_0}$ standard	0.9428	0.8881	0.8358	0.7859	0.7384	0.6931	0.6499
$\left(1-\frac{h}{34160}\right)$	.9414	.8829	.8244	.7658	.7073	.6487	.5902
$\left(1-\frac{h}{40000}\right)$	.9500	.9000	.8500	.8000	.7500	.7000	.6500

The second approximation is probably the best that can be obtained from a simple linear equation. It is sufficiently accurate at low altitudes for most purposes where an approximation can be used.

If the third term in the expansion of equation (1a) is retained, we have

$$\frac{\rho}{\rho_0}=1-\frac{h}{34160}+\left(\frac{h}{55236}\right)^2 \qquad (3)$$

Values from this equation are plotted on Figure 2 for comparison with the standard curve. The agreement is very good up to about 16,000 feet, above which the values from the equation are too high. A marked improvement can be obtained by the use of a larger constant in the denominator of the third term of the expansion. For example, if 59000 is used instead of 55236, the equation is

$$\frac{\rho}{\rho_0}=1-\frac{h}{34160}+\left(\frac{h}{59000}\right)^2 \qquad (3a)$$

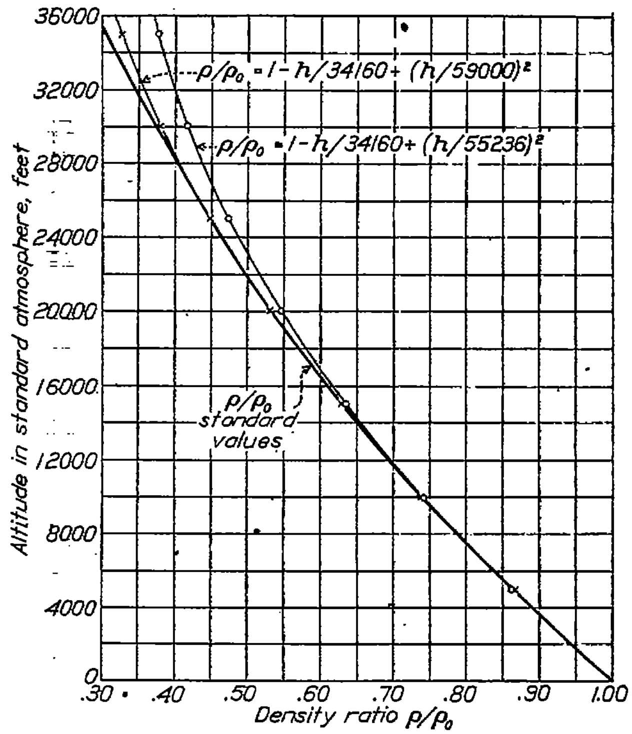

FIGURE 2

and the resulting agreement is excellent up to 25,000 feet. Comparative values are as follows:

	Altitude					
	5,000 feet	10,000 feet	15,000 feet	20,000 feet	25,000 feet	30,000 feet
$\frac{\rho}{\rho_0}$ standard	0.8616	0.7384	0.6291	0.5327	0.4490	0.3740
$1-\frac{h}{34160}+\left(\frac{h}{55236}\right)^2$	.8618	.7400	.6347	.5456	.4730	.4163
$1-\frac{h}{34160}+\left(\frac{h}{59000}\right)^2$	.8608	.7360	.6255	.5292	.4475	.3800

The exponential approximate equation for the density ratio is

$$\frac{\rho}{\rho_0}=e^{-ch} \qquad (4)$$

At one time this equation was used extensively to designate a standard atmosphere and it is still used in some lines of work on account of convenience in certain mathematical operations. In the N. A. C. A. standard atmosphere, c varies with altitude. A plot of calculated values gives $\frac{1}{c}=34160-0.12h$, so that below 20,000 feet the average value of c is $\frac{1}{33000}$. Hence

$$\frac{\rho}{\rho_o}=e^{-\frac{h}{33000}} \tag{4a}$$

and an excellent approximation is obtained by the use of

$$\frac{\rho}{\rho_o}=e^{-\left(\frac{h}{34160-0.12h}\right)} \tag{5}$$

which allows for the variation of c. Comparative values for equations (4a) and (5) are as follows:

	Altitude					
	5,000 feet	10,000 feet	15,000 feet	20,000 feet	25,000 feet	30,000 feet
$\frac{\rho}{\rho_o}$ standard	0.8616	0.7384	0.6291	0.5327	0.4480	0.3740
$e^{-\frac{h}{33000}}$	.8600	.7365	.6408	.5450	.4683	.4025
$e^{-\frac{h}{34160-0.12h}}$	.8615	.7385	.6290	.5325	.4483	.3745

An approximate equation of the form

$$\frac{\rho}{\rho_o}=\frac{1}{1+ch} \tag{6}$$

can not be used owing to the rapid variation of c. The variation in $\frac{1}{c}$ is linear with h, however, and given by

$$\frac{1}{c}=33600-0.53h \tag{7}$$

substituting this in equation (6) gives

$$\frac{\rho}{\rho_o}=\frac{33600-0.53h}{33600+0.47h} \tag{8}$$

This approximate equation gives excellent agreement with the standard values as shown by the following comparison:

	Altitude					
	5,000 feet	10,000 feet	15,000 feet	20,000 feet	25,000 feet	30,000 feet
$\frac{\rho}{\rho_o}$ standard	0.8616	0.7384	0.6291	0.5327	0.4480	0.3740
$\frac{33600-0.53h}{33600+0.47h}$	.8612	.7388	.6308	.5348	.4486	.3712

A logarithmic form of the approximate equation is

$$\frac{\rho}{\rho_o}=\log\frac{e}{1+ch} \tag{9}$$

where c has an average value of $\frac{1}{33600}$. Substituting this in equation (9) gives

$$\frac{\rho}{\rho_o}=\log\left(\frac{91300}{33600+h}\right) \tag{10}$$

Up to 25,000 feet this equation gives excellent agreement as shown by the following comparison:

	Altitude					
	5,000 feet	10,000 feet	15,000 feet	20,000 feet	25,000 feet	30,000 feet
$\frac{\rho}{\rho_o}$ standard	0.8616	0.7384	0.6291	0.5327	0.4480	0.3740
$\log\left(\frac{91300}{33600+h}\right)$	.8616	.7390	.6304	.5330	.4440	.3612

The foregoing equations represent the types most frequently required. Many others may be found and those given may be modified as necessary for particular purposes. Table I lists the various equations as given and shows the deviations from standard values.

APPROXIMATE EQUATIONS FOR $\frac{\rho_o}{\rho}$

The linear approximation for $\frac{\rho_o}{\rho}$ is

$$\frac{\rho_o}{\rho}=1+ch \tag{11}$$

When $c=\frac{1}{30000}$ this equation deviates less than 1 per cent up to about 10,000 feet. When $c=\frac{1}{25000}$ it is within 5 per cent up to 20,000 feet. The value of $\frac{1}{c}$ varies almost linearly with altitude and a direct plotting gives

$$\frac{1}{c}=34160-0.567h \tag{12}$$

The equation obtained by substituting equation (12) into (11) is practically identical with that obtained by inverting equation (8). Hence, using the latter,

$$\frac{\rho_o}{\rho}=\frac{33600+0.47h}{33600-0.53h} \tag{13}$$

This is quite accurate up to 30,000 feet.

Introducing an h^2 term in equation (11) gives

$$\frac{\rho_o}{\rho}=1+\frac{h}{34160}+bh^2 \tag{14}$$

This form is not very satisfactory, since $\sqrt{b}$ varies from $\frac{1}{45000}$ at sea level to $\frac{1}{37000}$ at 20,000 feet. The average value is $\sqrt{b}=\frac{1}{40500}$ or $bh^2=\left(\frac{h}{40500}\right)^2$.

The exponential form is obtained by changing the sign of the exponent in equations (4) and (5).

$$\frac{\rho_o}{\rho}=e^{ch}=e^{\frac{h}{33000}} \qquad (15)$$

and

$$\frac{\rho_o}{\rho}=e^{\left(\frac{h}{34160-0.12h}\right)} \qquad (16)$$

Equation (15) is within 1 per cent up to 15,000 feet and within 2.5 per cent up to 20,000 feet, which is about the limit of its application. Equation (16) like equation (5) is very accurate up to 30,000 feet.

In the logarithmic form of the approximate equation

$$\frac{\rho_o}{\rho}=\log\ (e+ch) \qquad (17)$$

the coefficient c varies rapidly with h and below 25,000 feet has an average value of

$$c=\frac{1}{12000-0.33h} \qquad (18)$$

Substituting this into equation (17) gives

$$\frac{\rho_o}{\rho}=\log\left(\frac{32600+0.10h}{12000-0.33h}\right) \qquad (19)$$

This equation is fairly accurate up to 25,000 feet, above which it deviates rapidly from the standard values.

The accuracy of these equations at various altitudes is indicated in Table II.

APPROXIMATE EQUATIONS FOR $\sqrt{\frac{\rho_o}{\rho}}$

The linear approximation for $\sqrt{\frac{\rho_o}{\rho}}$, from equation (11), is

$$\sqrt{\frac{\rho_o}{\rho}}=1+\tfrac{1}{2}ch \qquad (20)$$

When $\frac{1}{2}c=\frac{1}{60000}$ this equation is within 1 per cent up to 15,000 feet and within 5 per cent up to 25,000 feet. When $\frac{1}{2}c=\frac{1}{50000}$ the equation deviates more at low altitudes, but it is within about 3 per cent up to 30,000 feet.

Introducting an h^2 term in equation (20) as in equation (14) gives

$$\sqrt{\frac{\rho_o}{\rho}}=1+\frac{h}{68320}+bh^2 \qquad (21)$$

When $bh^2=\left(\frac{h}{68320}\right)^2$ this equation is within 1 per cent up to 30,000 feet.

The approximate equation corresponding to (8) and (13) is

$$\sqrt{\frac{\rho_o}{\rho}}=\frac{K+ch}{K-(1-c)h} \qquad (22)$$

This is formed from equation (20) by determining the variation of ($\frac{1}{2}c$) with h, which on plotting is found to be

$$\tfrac{1}{2}c=\frac{1}{68320-0.707h}$$

Substituting this into (20) gives

$$\sqrt{\frac{\rho_o}{\rho}}=\frac{68320+0.293h}{68320-0.707h} \qquad (23)$$

This equation is within $\frac{1}{10}$ of 1 per cent up to 30,000 feet.

The exponential approximate equations are obtained from equations (15) and (16) by dividing the exponent by 2 in each case. That is

$$\sqrt{\frac{\rho_o}{\rho}}=e^{\frac{h}{66000}} \qquad (24)$$

and

$$\sqrt{\frac{\rho_o}{\rho}}=e^{\left(\frac{h}{68320-0.24h}\right)} \qquad (25)$$

Equation (24) is within 1 per cent up to about 20,000 feet, but above this altitude it deviates rapidly. Better average agreement can be obtained by increasing the exponent slightly. For example

$$\sqrt{\frac{\rho_o}{\rho}}=e^{\frac{h}{63000}} \qquad (24a)$$

is within 0.75 per cent up to 25,000 feet and it is out by about 1.5 per cent at 30,000 feet.

Equation (25) is exact for all practical purposes, the maximum deviation being about two one-hundredths of 1 per cent.

In the logarithmic form of the approximate equation, similar to (17)

$$\sqrt{\frac{\rho_o}{\rho}}=\log\ (e+ch) \qquad (26)$$

it is found that

$$\frac{1}{c}=25000-0.42h \qquad (27)$$

Substituting this into (26) gives

$$\sqrt{\frac{\rho_o}{\rho}}=\log\left(\frac{68000-0.14h}{25000-0.42h}\right) \qquad (28)$$

This equation is within two-tenths of 1 per cent up to 30,000 feet.

APPROXIMATE EQUATIONS FOR PRESSURE RATIO $\frac{p}{p_o}$

The equation for pressure ratio, corresponding to equation (1), is

$$\frac{p}{p_o}=\left(1-\frac{a}{T_o}h\right)^{\frac{g}{aR}}$$

$$=\left(1-\frac{h}{145366}\right)^{5.255} \qquad (1)$$

Expanding this equation and neglecting all but the first two terms gives

$$\frac{p}{p_o}=1-\frac{h}{27660} \qquad (29)$$

Equation (29), like equation (2), does not give satisfactory agreement for altitudes greater than about 4,000 feet. Using $\frac{h}{33000}$ instead of $\frac{h}{27660}$ gives reasonable agreement up to about 14,000 feet.

Retaining the third term in the expansion of equation (1) gives

$$\frac{p}{p_o}=1-\frac{h}{27660}+\left(\frac{h}{43455}\right)^2 \qquad (30)$$

Equation (30) gives good agreement up to 15,000 feet, but it diverges rapidly at higher altitudes. Increasing the value of the constant in the denominator of the third term from 43455 to 48000, that is

$$\frac{p}{p_o}=1-\frac{h}{27660}+\left(\frac{h}{48000}\right)^2 \qquad (31)$$

gives much better general agreement up to 30,000 feet. The comparative values from these approximations are as follows:

	Altitude (h)					
	5,000 feet	10,000 feet	15,000 feet	20,000 feet	25,000 feet	30,000 feet
$\frac{p}{p_o}$ standard	0.8320	0.6876	0.5642	0.4594	0.3709	0.2968
$\left(1-\frac{h}{27660}\right)$	.8193	.6385	.4577			
$\left(1-\frac{h}{33000}\right)$	.8485	.6970	.5455	.3940	.2425	
$\left(1-\frac{h}{27660}+\left(\frac{h}{43455}\right)^2\right)$	.8325	.6914	.5759	.4858	.4273	.3925
$\left(1-\frac{h}{27660}+\left(\frac{h}{48000}\right)^2\right)$	.8302	.6819	.5554	.4507	.3677	.3063

These values are plotted on Figure 3.

The exponential form of the approximate equation for pressure ratio is

$$\frac{p}{p_o}=e^{-ch} \qquad (32)$$

c varies almost linearly with h, from $c=\frac{1}{27660}$ at sea level to $c=\frac{1}{24700}$ at 30,000 feet. Fair agreement is obtained up to 25,000 feet by the use of $c=\frac{1}{26000}$, that is

$$\frac{p}{p_o}=e^{-\frac{h}{26000}} \qquad (32a)$$

Plotting the calculated values of c against altitude gives $\frac{1}{c}=27660-.097h$. Substituting this into equation 32 gives

$$\frac{p}{p_o}=e^{\frac{-h}{27660-.097h}} \qquad (33)$$

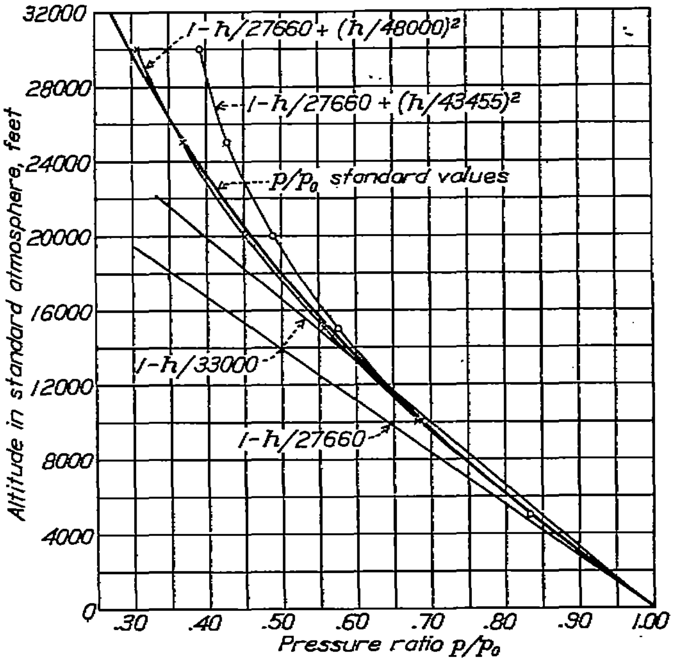

FIGURE 3

which is very accurate. Comparative values from equations (32a) and (33) are as follows:

	Altitude					
	5,000 feet	10,000 feet	15,000 feet	20,000 feet	25,000 feet	30,000 feet
$\frac{p}{p_o}$, standard	0.8320	0.6876	0.5642	0.4594	0.3709	0.2968
$e^{-\frac{h}{26000}}$	.8250	.6805	.5615	.4632	.3824	.3153
$e^{\frac{-h}{27660-.097h}}$	.8320	.6875	.5640	.4593	.3714	.2975

In the approximate equation of the form

$$\frac{p}{p_o}=\frac{1}{1+ch} \qquad (34)$$

it is found that

$$\frac{1}{c}=27000-0.48h \qquad (35)$$

Substituting equation (35) into equation (34) gives

$$\frac{p}{p_o}=\frac{27000-0.48h}{27000+0.52h} \tag{36}$$

which is in excellent agreement with the standard values up to 30,000 feet.

In the logarithmic form of the approximate equation

$$\frac{p}{p_o}=\log\left(\frac{e}{1+ch}\right) \tag{37}$$

c varies with h to a greater extent than was found for the density ratio in equation (10). However, good agreement can be secured up to 20,000 feet with $c=\frac{1}{27800}$, so that

$$\frac{p}{p_o}=\log\left(\frac{75500}{27800+h}\right) \tag{38}$$

Comparative values from equations (36) and (38) are as follows:

	Altitude					
	5,000 feet	10,000 feet	15,000 feet	20,000 feet	25,000 feet	30,000 feet
$\frac{p}{p_o}$ standard	0.8320	0.6876	0.5642	0.4594	0.3709	0.2968
$\frac{27000-0.48h}{27000+0.52h}$	.8310	.6890	.5690	.4650	.3750	.2958
$\log\left(\frac{75500}{27800+h}\right)$	.8335	.6920	.5672	.4572	.3576	--------

APPROXIMATE EQUATIONS FOR VERY HIGH ALTITUDES

The foregoing approximate equations have been derived for the altitude range ordinarily required in aeronautical work. In certain cases it may be desired to use approximate equations up to very high altitudes. For this purpose the constants in some of the equations may be so modified as to give good or reasonable agreement up to practically any required altitude. For example, equation (5) with modified constants

$$\frac{\rho}{\rho_o}=e^{-\left(\frac{h}{35000-0.16h}\right)} \tag{39}$$

gives good agreement up to altitudes greater than 60,000 feet. In a similar manner, equation (8) may be written

$$\frac{\rho}{\rho_o}=\frac{31000-0.43h}{31000+0.57h} \tag{40}$$

to give good agreement up to 60,000 feet. The range of equation (10) can not be extended greatly by a simple modification of the constants. However, the introduction of an h^2 term will result in satisfactory agreement, but the equation is too complicated to be of value.

In general, the linear and simple exponential equations can not be extended to high altitudes.

BUREAU OF AERONAUTICS,
NAVY DEPARTMENT,
WASHINGTON, D. C., *September 19, 1930.*

TABLE I

ACCURACY OF APPROXIMATE EQUATIONS FOR DENSITY RATIO $\frac{\rho}{\rho_o}$

[Deviations from standard values are given as percentages in parentheses]

	Altitude and per cent of error					
	5,000 feet	10,000 feet	15,000 feet	20,000 feet	25,000 feet	30,000 feet
$\frac{\rho}{\rho_o}$ standard value	0.8616	0.7384	0.6291	0.5327	0.4480	0.3740
$1-\frac{h}{40000}$	.8750	.7500	.6250	.5000		
Per cent of error	(+1.55)	(+1.55)	(−0.66)	(−6.14)		
$1-\frac{h}{34160}+\left(\frac{h}{59000}\right)^2$	.8608	.7360	.6255	.5292	.4475	.3800
Per cent of error	(−0.09)	(−0.33)	(−0.57)	(−0.66)	(−0.11)	(+1.60)
$e^{-\frac{h}{33000}}$	.8600	.7365	.6408	.5450	.4683	.4025
Per cent of error	(−0.19)	(−0.26)	(+2.02)	(+2.31)	(+4.53)	(+7.62)
$e^{-\left(\frac{h}{34160-0.12h}\right)}$	.8615	.7385	.6290	.5325	.4483	.3745
Per cent of error	(−0.01)	(+0.01)	(−0.01)	(−0.04)	(+0.07)	(+0.13)
$\frac{33600-0.53h}{33600+0.47h}$	.8612	.7388	.6308	.5348	.4486	.3712
Per cent of error	(−0.05)	(+0.05)	(+0.27)	(+0.40)	(+0.13)	(−0.75)
$\log\left(\frac{91300}{33600+h}\right)$	.8616	.7390	.6304	.5330	.4440	.3612
Per cent of error	(0)	(+0.08)	(+0.21)	(+0.06)	(−0.89)	(−3.43)

TABLE II

ACCURACY OF APPROXIMATE FORMULAS FOR $\frac{\rho_0}{\rho}$

[Deviations from standard values are given as percentages in parentheses]

	Altitude					
	5,000 feet	10,000 feet	15,000 feet	20,000 feet	25,000 feet	30,000 feet
$\frac{\rho_0}{\rho}$ standard value	1. 1606	1. 3542	1. 5896	1. 8772	2. 2320	2. 6737
$1+\frac{h}{30000}$	1. 167	1. 333	1. 500	1. 667		
Per cent of error	(+0. 47)	(−1. 54)	(−5. 98)	(−10. 7)		
$1+\frac{h}{25000}$	1. 200	1. 400	1. 600	1. 800	2. 000	
Per cent of error	(+4. 25)	(+3. 38)	(+0. 65)	(−4. 11)	(−10. 4)	
$\frac{33600+0.47h}{33600-0.53h}$	1. 161	1. 354	1. 584	1. 870	2. 228	2. 695
Per cent of error	(0)	(0)	(−0. 35)	(−0. 38)	(−0. 02)	(+0. 78)
$1+\frac{h}{34160}+\left(\frac{h}{40500}\right)^2$	1. 161	1. 354	1. 576	1. 830	2. 113	
Per cent of error	(0)	(0)	(−0. 89)	(−2. 51)	(−5. 34)	
$e^{\frac{h}{33000}}$	1. 164	1. 358	1. 576	1. 834	2. 133	
Per cent of error	(+0. 29)	(+0. 28)	(−0. 89)	(−2. 29)	(−4. 03)	
$e^{\left(\frac{h}{34160-0.12h}\right)}$	1. 1608	1. 355	1. 590	1. 877	2. 230	2. 670
Per cent of error	(−0. 02)	(+0. 04)	(0)	(0)	(0)	(0)
$\log\left(\frac{32600+0.10h}{12000-0.33h}\right)$	1. 164	1. 355	1. 583	1. 870	2. 260	2. 880
Per cent of error	(+0. 29)	(+0. 07)	(−0. 41)	(−0. 38)	(+1. 25)	(+7. 71)

TABLE III

ACCURACY OF APPROXIMATE EQUATIONS FOR $\sqrt{\frac{\rho_0}{\rho}}$

[Deviations from standard values are given as percentages in parentheses]

	Altitude					
	5,000 feet	10,000 feet	15,000 feet	20,000 feet	25,000 feet	30,000 feet
$\sqrt{\frac{\rho_0}{\rho}}$ standard value	1.0773	1.1637	1.2608	1.3701	1.4940	1.6352
$1+\frac{h}{60000}$	1.0833	1.167	1.250	1.333	1.417	----------
Per cent error	(+0.56)	(+0.26)	(−0.86)	(−2.69)	(−5.17)	----------
$1+\frac{h}{50000}$	1.100	1.200	1.300	1.400	1.500	1.600
Per cent error	(+2.11)	(+2.86)	(+3.11)	(+2.18)	(+0.40)	(−2.15)
$1+\frac{h}{68320}+\left(\frac{h}{68320}\right)^2$	1.078	1.168	1.268	1.378	1.500	1.622
Per cent error	(+0.06)	(+0.37)	(+0.56)	(+0.60)	(+0.40)	(−0.81)
$\frac{68320+0.293h}{68320-0.707h}$	1.0772	1.1634	1.2599	1.3691	1.4936	1.6368
Per cent error	(−.009)	(−0.03)	(−0.07)	(−0.07)	(−0.03)	(+0.10)
$e^{\frac{h}{66000}}$	1.079	1.164	1.255	1.354	1.461	----------
Per cent error	(+0.13)	0	(−0.43)	(−1.17)	(−2.21)	----------
$e^{\frac{h}{63000}}$	1.083	1.172	1.269	1.374	1.487	1.610
Per cent error	(+0.49)	(+0.72)	(+0.64)	(+0.27)	(−0.47)	(−1.54)
$e^{\left(\frac{h}{68320-0.24h}\right)}$	1.0773	1.1639	1.2608	1.3704	1.494	1.635
Per cent error	(0)	(+0.02)	(0)	(+0.02)	(0)	(0)
$\log\left(\frac{68000-0.14h}{25000-0.42h}\right)$	1.078	1.164−	1.260	1.368	1.491	1.637
Per cent error	(+0.06)	0	(−0.07)	(−0.15)	(−0.20)	(+0.13)

89300—32——18

TABLE IV

ACCURACY OF APPROXIMATE EQUATIONS FOR PRESSURE RATIO $\frac{p}{p_0}$

[Deviations from standard values are given as percentages in parentheses]

	Altitude					
	5,000 feet	10,000 feet	15,000 feet	20,000 feet	25,000 feet	30,000 feet
$\frac{p}{p_0}$ standard value	0. 8320	0. 6876	0. 5642	0. 4594	0. 3709	0. 2968
$1-\frac{h}{27660}$	. 8193	. 6385	. 4577			
Per cent error	(−1. 53)	(−7. 14)	(−18. 9)			
$1-\frac{h}{33000}$	. 8485	. 6970	. 5455	. 3940	. 2425	
Per cent error	(+1. 98)	(+1. 37)	(−3. 31)	(−14. 23)	(−34. 60)	
$1-\frac{h}{27660}+\left(\frac{h}{43455}\right)^2$	. 8325	. 6914	. 5759	. 4888	. 4273	. 3925
Per cent error	(+0. 06)	(+0. 55)	(+2. 07)	(+6. 40)	(+15. 2)	(+32. 2)
$1-\frac{h}{27660}+\left(\frac{h}{48000}\right)^2$	. 8302	. 6819	. 5554	. 4507	. 3677	. 3063
Per cent error	(−0. 22)	(−0. 83)	(−1. 56)	(−1. 89)	(−0. 86)	(+3. 20)
$e^{\frac{-h}{26000}}$	. 8250	. 6805	. 5615	. 4632	. 3824	. 3153
Per cent error	(−0. 84)	(−1. 03)	(−0. 48)	(+0. 83)	(+3. 10)	(+6. 23)
$e^{\frac{-h}{27660-0.097h}}$	. 8320	. 6875	. 5640	. 4593	. 3714	. 2975
Per cent error	(0)	(−0. 01)	(−0. 04)	(−0. 02)	(+0. 13)	(+0. 24)
$\frac{27000-0.48h}{27000+0.52h}$	. 8310	. 6890	. 5690	. 4650	. 3750	. 2958
Per cent error	(0. 12)	(+0. 20)	(+0. 85)	(+1. 22)	(+1. 11)	(−0. 34)
$\log_e\left(\frac{75500}{27800+h}\right)$	. 8335	. 6920	. 5672	. 4572	. 3576	. 2678
Per cent error	(+0. 18)	(+0. 64)	(+0. 30)	(−0. 48)	(−3. 58)	(−9. 77)