REPORT No. 437

THE EFFECT OF AREA AND ASPECT RATIO ON THE YAWING MOMENTS OF RUDDERS AT LARGE ANGLES OF PITCH ON THREE FUSELAGES

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SUMMARY

In view of the paucity of data on the effect of systematic changes in the vertical tail surfaces, measurements have been made of the yawing moments produced by rudder displacement for seven rudders mounted on each of three fuselages at angles of pitch of 0°, 8°, 12°, 20°, 30° and 40°. The dimensions of the rudders were selected to cover the range of areas and aspect ratios commonly used, while the ratios of rudder area to fin area and of rudder chord to fin chord were kept approximately constant.

An important result of the measurements is to show that increased aspect ratio gives increased yawing moments for a given area, provided the maximum rudder displacement does not exceed, say, 25°. If large rudder displacements are used, the effect of aspect ratio is not so great.

The work was conducted at the Bureau of Standards in cooperation with the Aeronautics Branch of the Department of Commerce and the National Advisory Committee for Aeronautics.

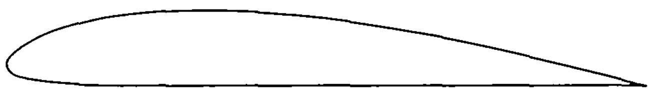
INTRODUCTION

Previous studies of the effectiveness of rudders have usually been made in connection with the design of a particular airplane. By the method of trial and error, an arrangement of the vertical surfaces is found which gives a satisfactory yawing moment for a given angular displacement of the rudder as judged by comparison with measurements on models of airplanes whose rudder control is known to be satisfactory. Such measurements do not readily lend themselves to analysis or to the determination of the influence of the several factors, such as the area and aspect ratio of the vertical tail surfaces, on the magnitude of the yawing moments. The present investigation represents a beginning at least of a systematic study of the effect of the area and the aspect ratio of the vertical surfaces, of the angle of pitch, and of the shape of the fuselage on the yawing moments produced by rudder displacement.

DESCRIPTION OF APPARATUS AND MODELS

Wing.—The model is a monoplane and is of wooden construction. The wing is rectangular in plan form, having a chord of 10 inches and a span of 60 inches. The profile and coordinates of the wing section (the Clark Y) are given in Figure 1.

Fuselages.—Three different fuselage shapes were used, designated as open flat deck, open round deck and cabin. The shapes and dimensions are shown in Figures 2, 3, and 4. The open flat deck and open round deck fuselages differ only in the shape of the upper part of the fuselage aft of the front cockpit. The length of the fuselage is 65 per cent of the wing span. The location of the wing was determined to satisfy the following relations: (1) the center of gravity to be 50 per cent of the wing span forward of the rear end of the fuselage (which was to be the rudder axis); (2) the center of gravity to be at a distance



Per cent of chord	Clark Y section Upper	Clark Y section Lower	Per cent of chord	Clark Y section Upper	Clark Y section Lower
0	3.62	3.58	40	11.37	0.00
1.25	5.38	1.86	50	10.49	0.00
2.5	6.43	1.42	60	9.13	0.00
5	7.83	0.91	70	7.34	0.00
7.5	8.79	0.59	80	5.21	0.00
10	9.56	0.39	90	2.79	0.00
15	10.63	0.12	95	1.50	0.00
20	11.32	0.01	100	0.12	0.00
30	11.68	0.00			

Source N.A.C.A. Report No. 233

FIGURE 1.—Dimensions of Clark Y wing section

equal to 25 per cent of the wing chord aft of the leading edge of the wing; (3) the leading edge of the wing to be 35 per cent of the chord above the top of the fuselage; (4) the angle of incidence of the wing to the axis of the fuselage to be 4°. The position of the center of gravity is indicated in Figures 2, 3, and 4 at C.

The cabin fuselage is a little shorter but all of the above conditions were satisfied except (3). Instead of relation (3), the chord of the wing was placed in coincidence with the upper deck of the cabin fuselage.

Horizontal tail surfaces.—The area of the horizontal tail surfaces was chosen equal to 15 per cent of the wing area, and the chord as 50 per cent of the wing chord. The span was adjusted to permit a cut-out to give clearance for rudder displacements of 44°. A rectangular plan form was used and the angle of incidence to the fuselage axis was made 0°. The dimensions and arrangement are shown in Figure 5.

Rudders.—To decrease the number of independent variables, the plan form of rudder and fin was made rectangular, the rudder hinge was located at the rear end of the fuselage, the bottom of the rudder was set at the bottom of the fuselage, and the top edge of the rudder was placed in line with the top edge of the fin. In addition, an attempt was made to keep the ratio of the area of the rudder to the area of the fin constant and equal to 2.0 and the ratio of the chord of the rudder to the chord of the fin constant and equal to 1.14, these values being selected after a study of the designs of present-day airplanes.

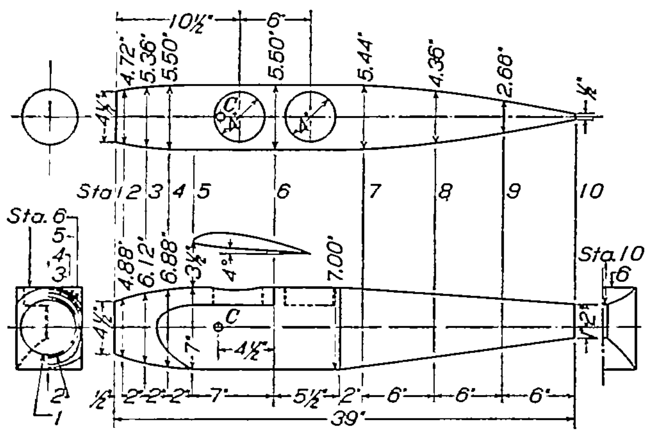


FIGURE 2.—Open flat deck fuselage

The airfoil sections of the several rudder and fin groups were also maintained geometrically similar so far as possible. The section adopted is an arbitrary one, having a thickness ratio of approximately 0.050 and built up of flat surfaces with a rounded nose. The radius of curvature of the nose is approximately one-fourth the maximum thickness of 0.0125 times the chord. The maximum thickness is reached at one-third the chord. The forward part of the section is wedge-shaped; from one-third the chord to the rudder hinge, the thickness is constant; while the rudder itself is wedge-shaped, tapering to a sharp trailing edge. The surfaces were made of wood to an accuracy of 0.01 inch.

The primary variables were intended to be the area of the rudder, which was to vary from 2 to 6 per cent of the wing area, and the aspect ratio of the fin and rudder group, which was to vary from 0.75 to 2.5. In accordance with the most recently approved definition (reference 2) the area of the vertical fin contained within the fuselage is not included in the fin area. Seven models were originally selected, divided into two groups, a constant area group and a constant aspect ratio group. Later an eighth model was interpolated to study the effect of variation of the ratio of the chord of the rudder to the chord of the fin.

The dimensions of the eight rudders are given in Figure 5 and certain additional data is assembled in Table I. Aspect ratio is defined as the ratio of the square of the span (dimension B) to the area of rudder and fin (excluding the part within the fuselage). Area ratio when used without qualification is understood to be the ratio of the area of rudder and fin to the wing area.

TABLE I

DIMENSIONS AND RELATED DATA ON THE MODEL RUDDERS

Rudder No.	Span of rudder inches	Chord of rudder inches	Chord of fin inches	Area of rudder square inches	Area of fin square inches	Area of rudder and fin square inches	Aspect ratio	Area ratio = $\frac{\text{Area of rudder and fin}}{\text{Wing area}}$	$\frac{\text{Area of rudder}}{\text{Wing area}}$	$\frac{\text{Area of rudder}}{\text{Area of fin}}$	$\frac{\text{Chord of rudder}}{\text{Chord of fin}}$
1	4.74	4.50	5.00	21.33	7.81	29.14	0.77	0.0486	0.0356	2.73	0.900
2	6.12	3.33	3.08	20.38	9.24	29.62	1.26	.0494	.0340	2.21	1.081
3	7.25	2.76	2.42	20.00	10.10	30.10	1.75	.0502	.0333	1.98	1.140
4	8.66	2.25	1.86	19.49	10.37	29.86	2.51	.0498	.0325	1.88	1.200
4A	8.66	2.25	2.50	19.49	13.94	33.43	2.24	.0555	.0325	1.40	.900
5	5.62	2.14	1.88	12.03	4.80	16.83	1.88	.0280	.0201	2.51	1.138
6	8.57	3.26	2.87	27.92	15.73	43.65	1.68	.0728	.0465	1.77	1.135
7	9.73	3.70	3.25	36.00	19.50	57.50	1.65	.0958	.0600	1.85	1.138

It will be observed that because of conflicting requirements it was not possible to secure an exact constancy of all variables but one. In the variable aspect ratio group including rudders 1, 2, 3, and 4, the area ratio varies from 0.0486 to 0.0502, the ratio of rudder area to fin area from 1.88 to 2.73, the ratio of rudder chord to fin chord from 0.900 to 1.209. In rudder 4A, the ratio of rudder chord to fin chord was made 0.900 to agree with that for rudder 1, but the area ratio was necessarily increased to 0.0555 and the ratio of rudder area to fin area was reduced to 1.40. In the variable area group, rudders 3, 5, 6, and 7, the aspect ratio varies from 1.65 to 1.88, the ratio of rudder area to fin area from 1.77 to 2.51, and the ratio of rudder chord to fin chord from 1.135 to 1.140. In both groups there are small variations in the airfoil section, the thickness ratio varying from 0.046 to 0.051 and the position of the maximum ordinate from 32 to 39 per cent of the chord. Except for rudders 1 and 4A, the variations are negligible, and are within the accuracy of construction of the models. The models selected therefore represent compromises, and the effects of the shape of the airfoil section, the ratio of rudder area to fin area, and of rudder chord to fin chord are not completely eliminated.

The rudders were hinged to the fins as shown in Figure 6. A brass plate on the end of the vertical fin and cut to the shape of the fin cross section carried a steel pivot which engaged a socket in a brass plate fastened to the rudder. The gap between the rudder and fin was sealed with a thin layer of petroleum jelly. Wires running from the trailing edge of the rudder were fastened to the horizontal tail surface to hold the rudder at any desired angle. The settings to the desired angles were made by the aid of metal templates. These settings were made with respect to the fixed surface, which had been aligned with the axis of the model. The axis of the model was set at the desired angle of pitch and at 0° yaw with respect to the wind by means of reference lines determined in previous tests by the usual methods, i. e., from results obtained on airfoils in the normal and inverted positions and from yawmeter readings.

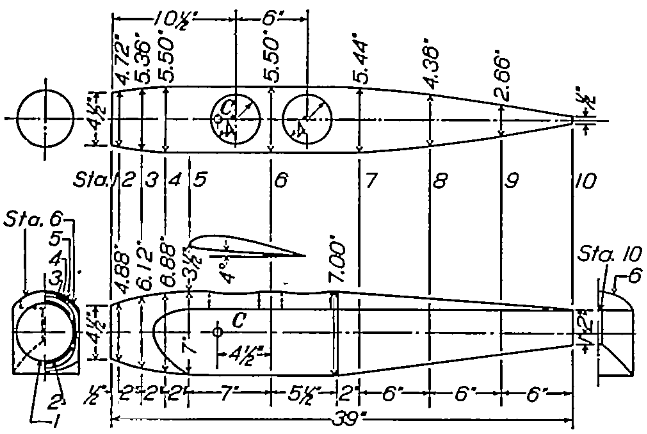


FIGURE 3.—Open round deck fuselage

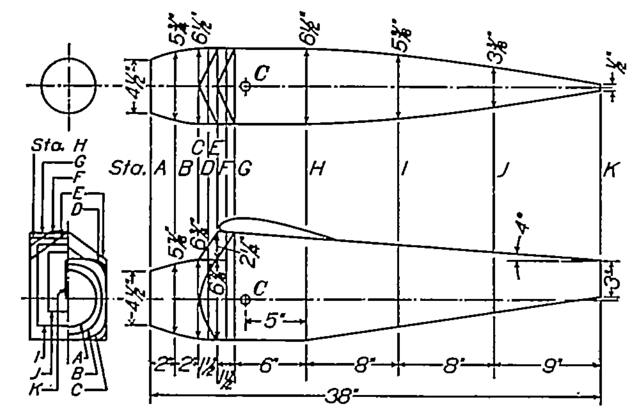
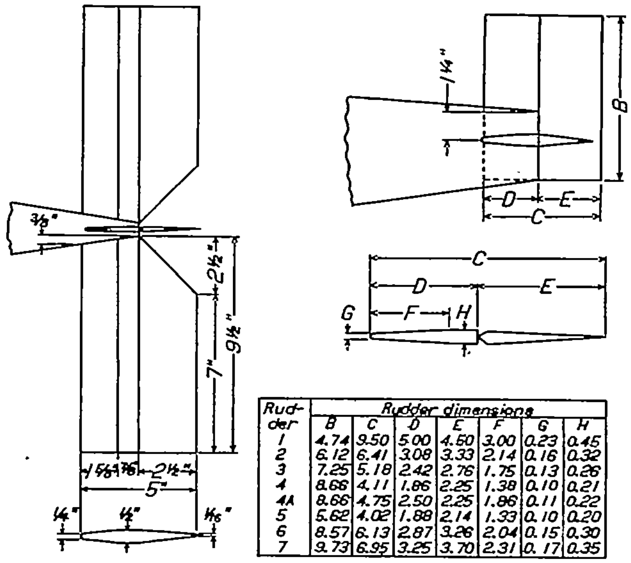


FIGURE 4.—Cabin fuselage

Wind tunnel and balance arrangement.—The measurements were made in the 10-foot wind tunnel of the Bureau of Standards, which has been described in reference 1. The model was mounted with the leading edge of the wing vertical on a mast extending from the tunnel wall as shown in Figure 7. On the end of the mast and contained within a housing inserted in the fuselage at the intended center of gravity position was a joint designed to give free motion in yaw about an axis perpendicular to the axis of the fuselage and in the plane of symmetry of the model, i. e., about a "body" axis. No motion in roll was permitted, but the angle of pitch could be varied and the model locked at any desired angle of pitch. The yawing moment was balanced by the tension in a vertical wire running from a point on the fuselage near the tail to a balance of the pendulum type. A counterweight was used when necessary to maintain tension in the wire. The drag force on the wire tended to bow the wire and thus change the angle of yaw. This effect and the similar change caused by motion of the pan of the balance was eliminated by adjusting a turnbuckle in the wire to keep the tail of the model in a fixed position, determined by sighting a mark on the tail through a fixed telescope.



Rudder	Rudder dimensions						
	B	C	D	E	F	G	H
1	4.74	9.50	5.00	4.60	3.00	0.23	0.45
2	6.12	6.41	3.08	3.33	2.14	0.16	0.32
3	7.25	5.18	2.42	2.76	1.75	0.13	0.26
4	8.66	4.11	1.86	2.25	1.38	0.10	0.21
4A	8.66	4.75	2.50	2.25	1.86	0.11	0.22
5	5.62	4.02	1.88	2.14	1.33	0.10	0.20
6	8.57	6.13	2.87	3.26	2.04	0.15	0.30
7	9.73	6.95	3.25	3.70	2.31	0.17	0.35

FIGURE 5.—Dimensions and arrangement of tail surfaces

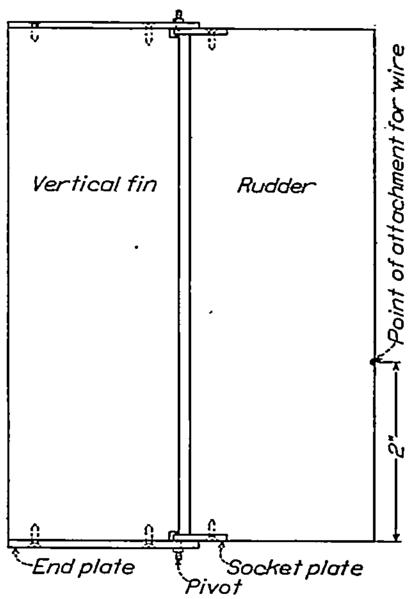


FIGURE 6.—Method of hinging rudders

REDUCTION OF OBSERVATIONS

Observations were made of the balance readings with each rudder set at 0°, 8°, 16°, 24°, 32°, and 44° to the left and also to the right, the model remaining

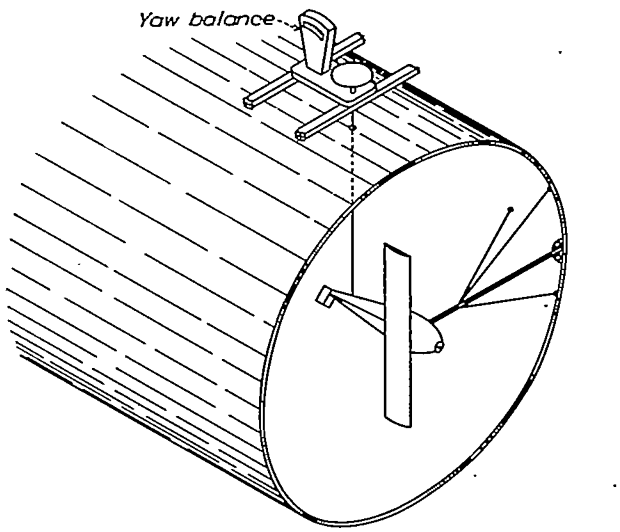


FIGURE 7.—Balance arrangement for measurements of yawing moment

at 0° yaw, for wind speeds of 40, 58.7, and 80 feet per second (27.3, 40, and 54.5 miles per hour) and for angles of pitch of 0°, 8°, 12°, 20°, 30°, and 40°. The mean of the readings with rudder to the right and to

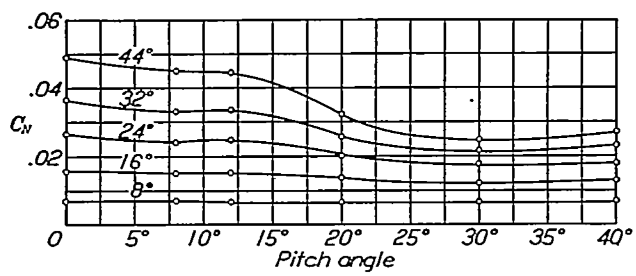


FIGURE 8.—Yawing moment coefficients for rudder No. 1 (aspect ratio 0.77, area ratio 0.0486) on open flat deck fuselage. (Angles marked on curves are rudder angles)

the left was used to compute the mean yawing moment produced by the given rudder displacement, a procedure which greatly reduces errors due to lack of symmetry and faulty alignment with the wind. The

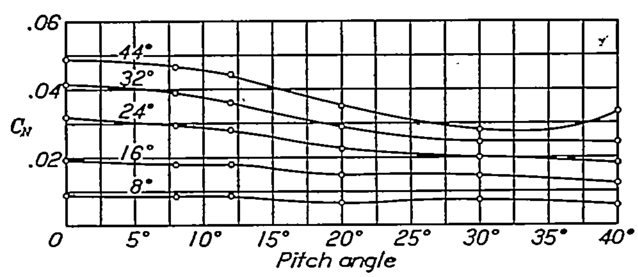


FIGURE 9.—Yawing moment coefficients for rudder No. 2 (aspect ratio 1.26, area ratio 0.0494) on open flat deck fuselage. (Angles marked on curves are rudder angles)

results were then reduced to the usual dimensionless coefficients in accordance with the relation

$$C_N=\frac{N}{q\,f\,S}\ ^{*}$$

where

C_N—absolute yawing moment coefficient

N—yawing moment due to a given rudder displacement in pounds-feet

q—velocity pressure $=\frac{1}{2}\rho V^2=0.001189\ V^2$

V—wind speed in feet per second

ρ—the density of air, i. e., 0.002378 slugs per cubic foot at 15° C. and 760 mm pressure

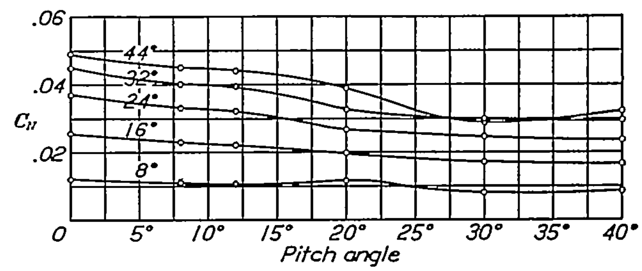


FIGURE 10.—Yawing moment coefficients for rudder No. 3 (aspect ratio 1.75, area ratio 0.0502) on open flat deck fuselage. (Angles marked on curves are rudder angles)

S—wing area in square feet (chord length × span)

f—the distance in feet from the center of rotation of the model to the hinge line of the rudder (30 inches for the models used. See figs. 2, 3, and 4).

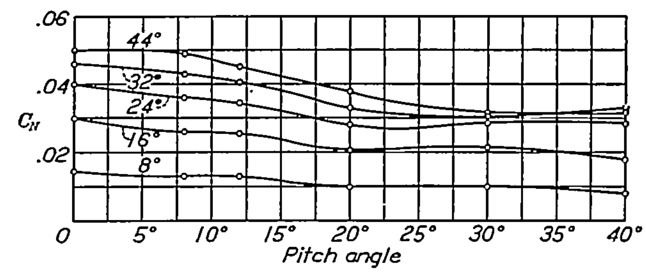


FIGURE 11.—Yawing moment coefficients for rudder No. 4 (aspect ratio 2.61, area ratio 0.0493) on open flat deck fuselage. (Angles marked on curves are rudder angles)

Moments tending to move the right wing tip to the rear as viewed from the pilot's seat are positive. Displacement of the rear edge of the rudder toward the right wing tip gives positive moments. The

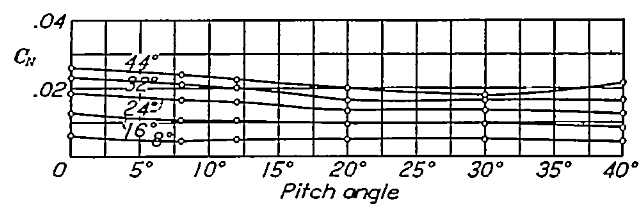


FIGURE 12.—Yawing moment coefficients for rudder No. 5 (aspect ratio 1.88, area ratio 0.0280) on open flat deck fuselage. (Angles marked on curves are rudder angles)

values given are averages for right and left displacements.

* Note that f is used instead of the span, b. The coefficient, C_n recently adopted by the N. A. C. A. is equal for this model to ½ C_N.

Since the scale effect was small as shown by the absence of systematic variation of C_N with speed, the values for the three speeds were plotted and a faired curve drawn through all the points. Values from the faired curves are plotted in Figures 8 to 29 inclusive.

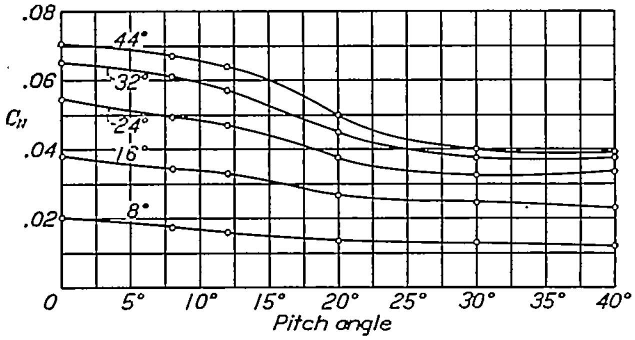


FIGURE 13.—Yawing moment coefficients for rudder No. 6 (aspect ratio 1.68, area ratio 0.0728) on open flat deck fuselage. (Angles marked on curves are rudder angles)

A consideration of the sensitivity of the balance and the steadiness of its reading fixes the final precision of C_N as about ± 0.001. Since the curves may

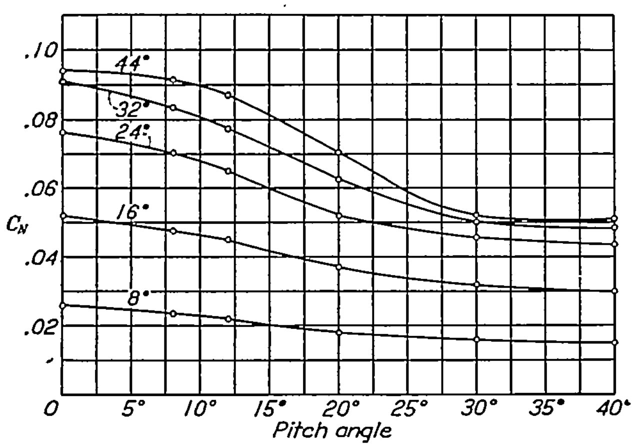


FIGURE 14.—Yawing moment coefficients for rudder No. 7 (aspect ratio 1.65, area ratio 0.0958) on open flat deck fuselage. (Angles marked on curves are rudder angles)

easily be read to this precision, tabular values are not given.

ANALYSIS OF THE RESULTS

Reference to Figures 8, 9, 10, and 11 shows that the variation of C_N with rudder angle is in all cases ap-

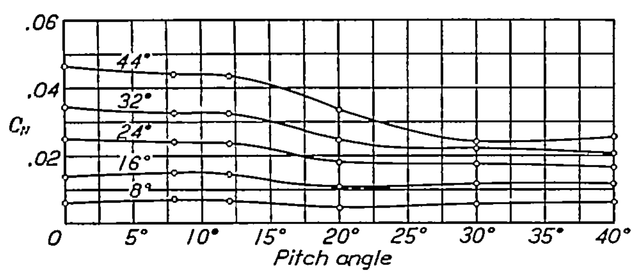


FIGURE 15.—Yawing moment coefficients for rudder No. 1 (aspect ratio 0.77, area ratio 0.0486) on open round deck fuselage. (Angles marked on curves are rudder angles)

proximately linear for angles up to 20° or 25°. The rate of variation increases with increase in aspect ratio, so that if rudder angles are limited to 20° or 25°, increased yawing moments can be obtained by increasing the aspect ratio without increase of area. However, it will be noted that for small aspect ratio (fig. 8) the yawing moment increases almost linearly with rudder angle up to the greatest angle used, 44°, whereas for large aspect ratio (fig. 11), the rate of

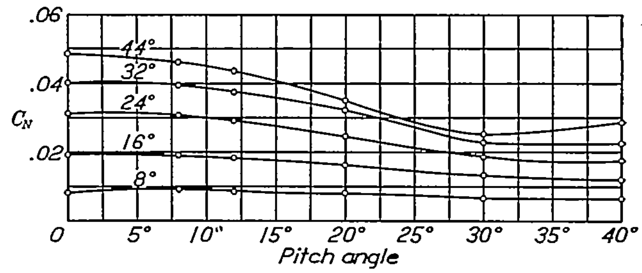


FIGURE 16.—Yawing moment coefficients for rudder No. 2 (aspect ratio 1.26, area ratio 0.0494) on open round deck fuselage. (Angles marked on curves are rudder angles)

increase falls off greatly above rudder angles of 25°. The values of the yawing moment for a rudder displacement of 44° show only a small variation with aspect ratio. Hence if the rudder travel may be as large as 44°, there is little advantage in increasing the aspect ratio.

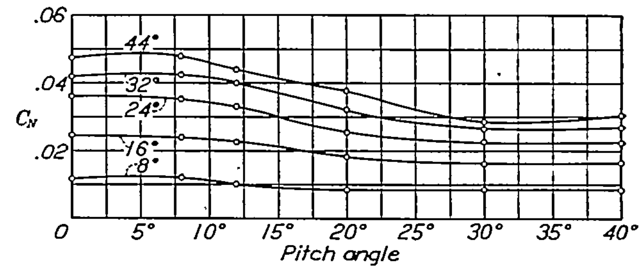


FIGURE 17.—Yawing moment coefficients for rudder No. 3 (aspect ratio 1.75, area ratio 0.0502) on open round deck fuselage. (Angles marked on curves are rudder angles)

The effect of increasing the angle of pitch from angles below the stalling angle of the wing to angles above the stall is to greatly decrease the yawing moment at a given rudder setting. However at large angles of pitch and large rudder angles, the yawing moment does not continue to decrease with increasing angle of

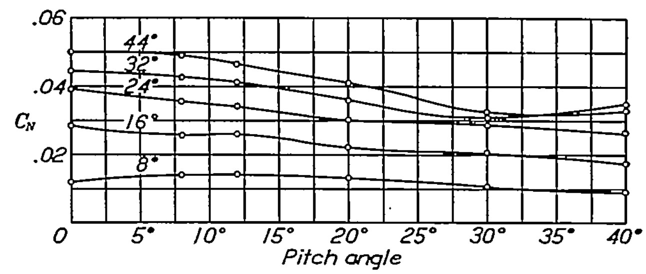


FIGURE 18.—Yawing moment coefficients for rudder No. 4 (aspect ratio 2.51, area ratio 0.0498) on open round deck fuselage. (Angles marked on curves are rudder angles)

pitch but remains nearly constant or even increases slightly. For rudder angles less than 25°, the decrease is greatest for rudders of large aspect ratio so that the effect of aspect ratio is relatively less important at high angles of pitch than at low angles. For large rudder

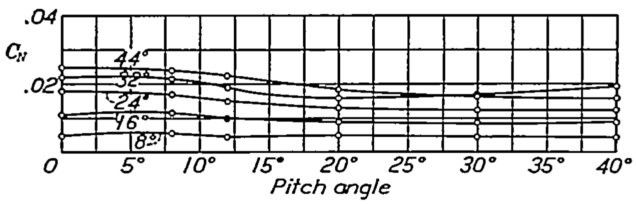


FIGURE 19.—Yawing moment coefficients for rudder No. 5 (aspect ratio 1.88, area ratio 0.0280) on open round deck fuselage. (Angles marked on curves are rudder angles)

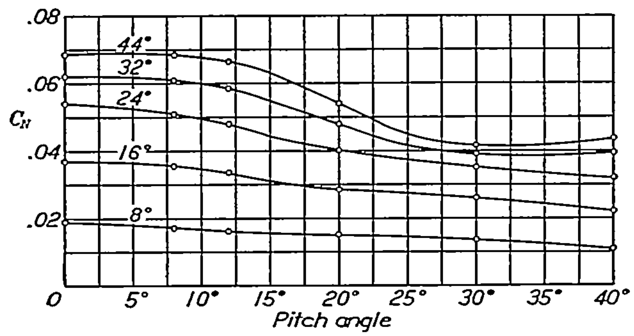


FIGURE 20.—Yawing moment coefficients for rudder No. 6 (aspect ratio 1.68, area ratio 0.0728) on open round deck fuselage. (Angles marked on curves are rudder angles)

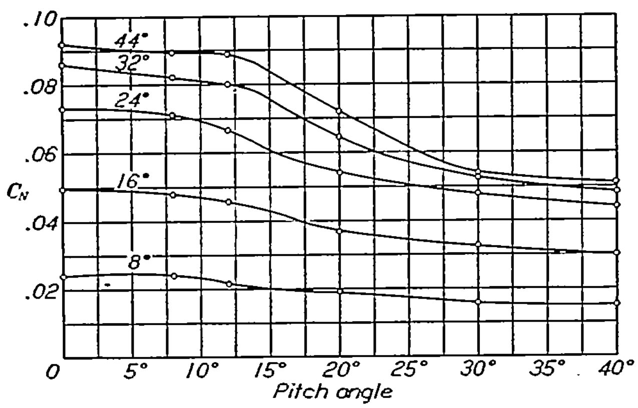


FIGURE 21.—Yawing moment coefficients for rudder No. 7 (aspect ratio 1.65, area ratio 0.0958) on open round deck fuselage. (Angles marked on curves are rudder angles)

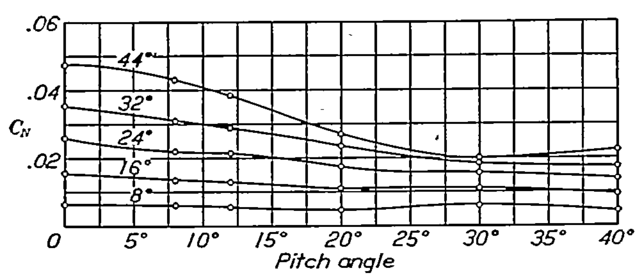


FIGURE 22.—Yawing moment coefficients for rudder No. 1 (aspect ratio 0.77, area ratio 0.0486) on cabin fuselage. (Angles marked on curves are rudder angles)

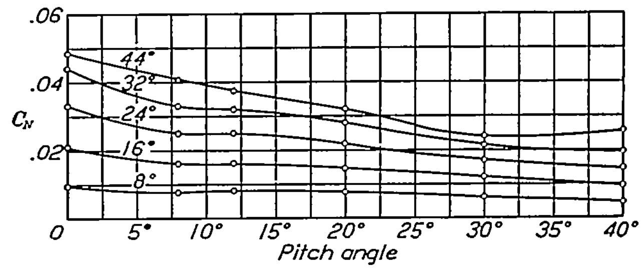


FIGURE 23.—Yawing moment coefficients for rudder No. 2 (aspect ratio 1.26, area ratio 0.0494) on cabin fuselage. (Angles marked on curves are rudder angles)

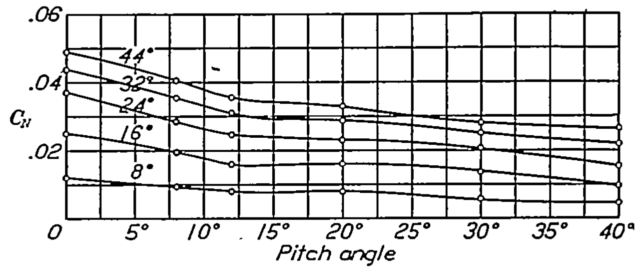


FIGURE 24.—Yawing moment coefficients for rudder No. 3 (aspect ratio 1.75, area ratio 0.0502) on cabin fuselage. (Angles marked on curves are rudder angles)

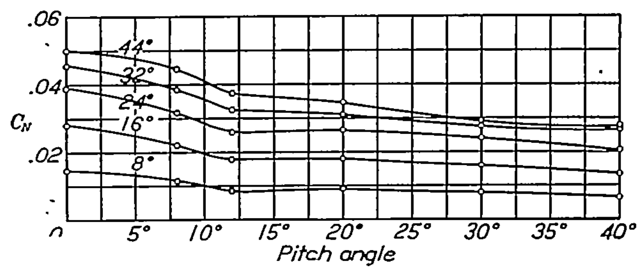


FIGURE 25.—Yawing moment coefficients for rudder No. 4 (aspect ratio 2.51, area ratio 0.0498) on cabin fuselage. (Angles marked on curves are rudder angles)

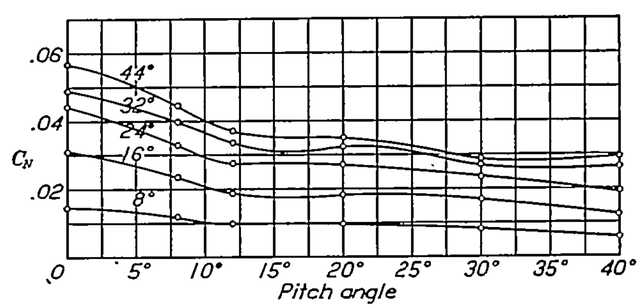


FIGURE 26.—Yawing moment coefficients for rudder No. 4A (aspect ratio 2.24, area ratio 0.0555) on cabin fuselage. (Angles marked on curves are rudder angles)

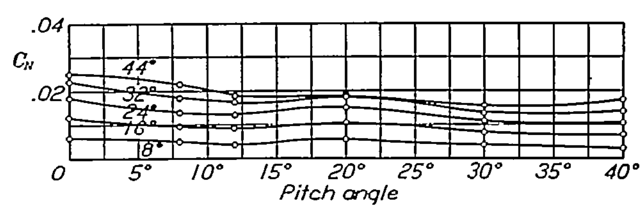


FIGURE 27.—Yawing moment coefficients for rudder No. 5 (aspect ratio 1.88, area ratio 0.0280) on cabin fuselage. (Angles marked on curves are rudder angles)

angles (44°), on the other hand, the decrease is greatest for rudders of small aspect ratio and the rudder moment falls off rapidly between angles of pitch of 10° to 30°. With rudders of larger aspect ratio, the decrease in the rudder moment is less. (Compare figs. 34 and 35, 36 and 37.)

The mass of data given in Figures 8 to 29 is of limited value unless some attempt at analysis is made. Figures 30, 31, 32, and 33 are cross plots, illustrating the effect of area-ratio, i. e., the ratio of the area of

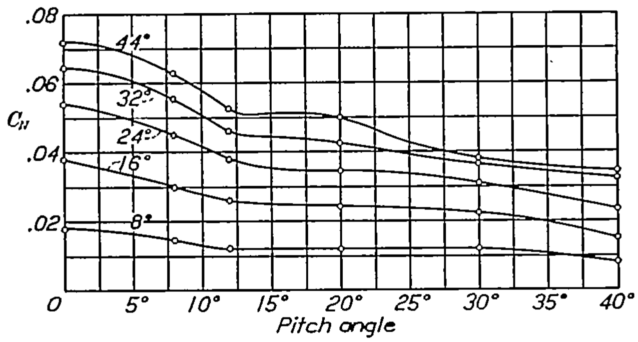


FIGURE 28.—Yawing moment coefficients for rudder No. 6 (aspect ratio 1.68, area ratio 0.0728) on cabin fuselage. (Angles marked on curves are rudder angles)

rudder and fin to the wing area. It will be noted that the variation is approximately linear, but the lines do not pass through the origin. If attention is confined to rudder angles less than 25°, it will be observed that the curves may be fitted reasonably well by lines intersecting the axis of abscissas at the point, area ratio =0.0075. Hence, within the limits, 0.03 to 0.10, for the area ratio, we may assume that the yawing moment is proportional to (area ratio − 0.0075).

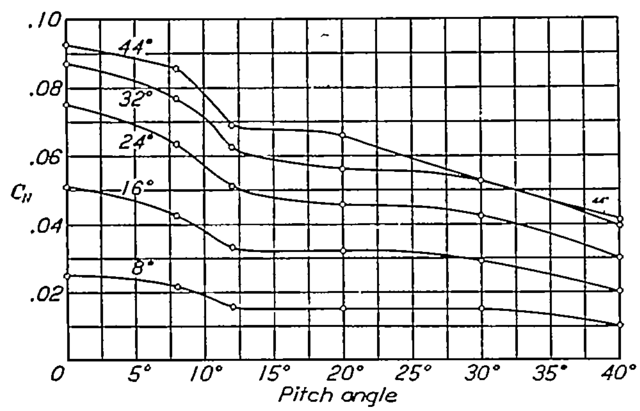


FIGURE 29.—Yawing moment coefficients for rudder No. 7 (aspect ratio 1.65, area ratio 0.0958) on cabin fuselage. (Angles marked on curves are rudder angles)

Figures 34, 35, 36, and 37 illustrate the effect of aspect ratio. The relations are here not so simple.

At 0° angle of pitch, the large rudder moments obtainable with large rudder angles (in the neighborhood of 44°) depend upon the area ratio rather than the aspect ratio. For rudder angles roughly 25° and less and for high angles of pitch in the neighborhood of 30°, the rudder moments increase with aspect ratio.

In order to give a general picture of the data in condensed form, an attempt was made to fit the data by means of empirical curves. In this analysis attention was centered on rudder angles less than 25°, and the first step was to compute the average slopes of the curves of yawing moment coefficient against rudder

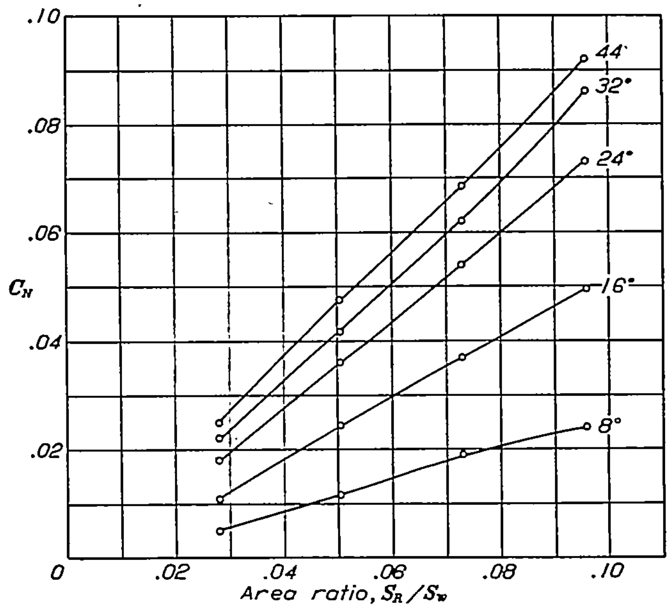


FIGURE 30.—Effect of area ratio (aspect ratio 1.65–1.88) on yawing moment coefficients of rudders on open round deck fuselage at 0° pitch. (Angles marked on curves are rudder angles)

angle. (The curves are not published.) The next step was to divide the average slopes by the quantity (area ratio − 0.0075). The numbers so obtained were plotted as a function of the aspect ratio for each fuselage and

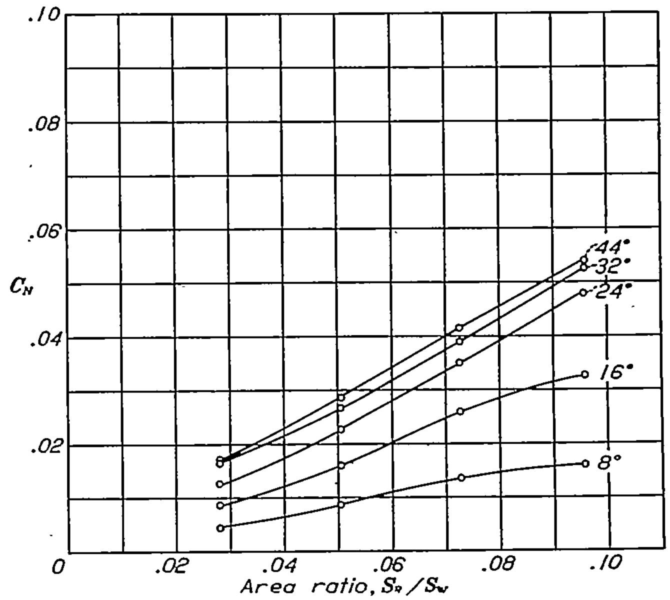


FIGURE 31.—Effect of area ratio (aspect ratio 1.65–1.88) on yawing moment coefficients of rudders on open round deck fuselage at 30° pitch. (Angles marked on curves are rudder angles)

angle of pitch and fitted by empirical curves. The final results are summarized in Tables II and III, in which δ = rudder angle, A_{ar} = area ratio, A_{as} = aspect ratio.

TABLE II

EMPIRICAL EQUATIONS REPRESENTING OBSERVED DATA ON YAWING MOMENTS DUE TO RUDDER DISPLACEMENT

C_N=Coefficient of yawing moment, δ=rudder angle ($<25°$), A_{ar}=area ratio, A_{as}=aspect ratio.

OPEN ROUND DECK FUSELAGE

Angle of pitch	
0°	$C_N=\delta(A_{ar}-0.0075)\ \left(0.0470-\frac{0.0200}{A_{as}}\right)$
8°	$C_N=\delta(A_{ar}-0.0075)\ \left(0.0446-\frac{0.0190}{A_{as}}\right)$
12°	$C_N=\delta(A_{ar}-0.0075)\ (0.0129+0.0106\ A_{as})$
20°	$C_N=\delta(A_{ar}-0.0075)\ (0.0107+0.0098\ A_{as})$
30°	$C_N=\delta(A_{ar}-0.0075)\ (0.0096+0.0081\ A_{as})$
40°	$C_N=\delta(A_{ar}-0.0075)\ (0.0092+0.0078\ A_{as})$

OPEN FLAT DECK FUSELAGE

0°	$C_N=\delta(A_{ar}-0.0075)\ \left(0.510-\frac{0.0231}{A_{as}}\right)$
8°	$C_N=\delta(A_{ar}-0.0075)\ \left(0.0459-\frac{0.0209}{A_{as}}\right)$
12°	$C_N=\delta(A_{ar}-0.0075)\ (0.0150+0.0091\ A_{as})$
20°	$C_N=\delta(A_{ar}-0.0075)\ (0.0150+0.0060\ A_{as})$
30°	$C_N=\delta(A_{ar}-0.0075)\ (0.0150+0.0055\ A_{as})$
40°	$C_N=\delta(A_{ar}-0.0075)\ (0.0150+0.0047\ A_{as})$

CABIN FUSELAGE

0°	$C_N=\delta(A_{ar}-0.0075)\ \left(0.0505-\frac{0.0229}{A_{as}}\right)$
8°	$C_N=\delta(A_{ar}-0.0075)\ \left(0.0365-\frac{0.0136}{A_{as}}\right)$
12°	$C_N=\delta(A_{ar}-0.0075)\ (0.0147+0.0058\ A_{as})$
20°	$C_N=\delta(A_{ar}-0.0075)\ (0.0130+0.0060\ A_{as})$
30°	$C_N=\delta(A_{ar}-0.0075)\ (0.0110+0.0056\ A_{as})$
40°	$C_N=\delta(A_{ar}-0.0075)\ (0.0067+0.0048\ A_{as})$

From these equations, the value of $\frac{C_N}{\delta\ (A_{ar}-0.0075)}$ at certain values of aspect ratio and angle of pitch are computed and given in Table III.

TABLE III

VALUES OF $\frac{C_N}{\delta\ (A_{ar}-0.0075)}$ COMPUTED FROM EMPIRICAL EQUATIONS

OPEN ROUND DECK FUSELAGE

θ / A_{as}	0°	8°	12°	20°	30°	40°
1.0	0.0270	0.0256	0.0235	0.0205	0.0177	0.0170
1.5	.0336	.0319	.0278	.0254	.0217	.0209
2.0	.0370	.0351	.0341	.0303	.0258	.0248
2.5	.0390	.0370	.0394	.0352	.0298	.0287

OPEN FLAT DECK FUSELAGE

A_{as}	0°	8°	12°	20°	30°	40°
1.0	0.0269	0.0251	0.0241	0.0210	0.0205	0.0197
1.5	.0356	.0320	.0286	.0240	.0233	.0221
2.0	.0395	.0355	.0332	.0270	.0260	.0244
2.5	.0417	.0376	.0377	.0300	.0287	.0267

CABIN FUSELAGE

A_{as}	0°	8°	12°	20°	30°	40°
1.0	0.0276	0.0229	0.0205	0.0190	0.0166	0.0115
1.5	.0352	.0274	.0234	.0220	.0194	.0139
2.0	.0390	.0297	.0263	.0250	.0222	.0163
2.5	.0413	.0311	.0292	.0280	.0250	.0187

Before drawing any conclusions from these tables, or equations, some attention should be directed to the precision with which the empirical equations fit the observed data. It has already been stated that the precision of measurement of C_N is ± 0.001. When the deviations between the measurements and the empirical curves are computed, it is found that two-thirds of the deviations are less than 0.001 and nine-tenths are less than 0.002. There is some evidence of systematic deviation, especially in the case of rudder 1 of aspect ratio 0.77, but on the whole, the fit is considered satisfactory. The method of deriving the empirical equations is such that no fairing with angle of pitch

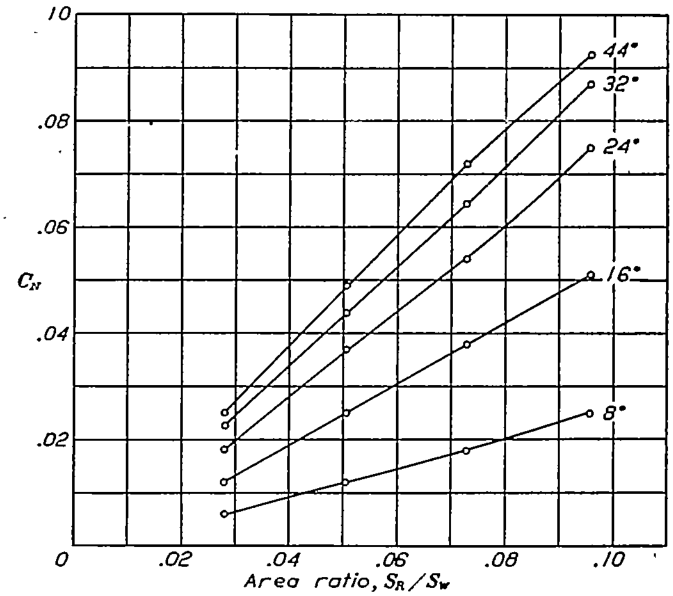


FIGURE 32.—Effect of area ratio (aspect ratio 1.65–1.88) on yawing moment coefficients of rudders on cabin fuselage at 0° pitch. (Angles marked on curves are rudder angles)

has been made. Thus some discrepancies become apparent on cross-plotting against angle of pitch.

The empirical equations are not suggested as applicable to design problems in general. For example, the

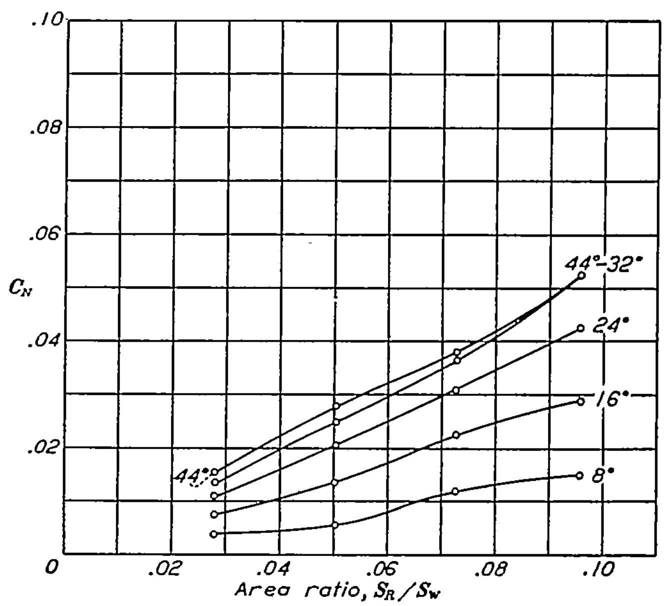


FIGURE 33.—Effect of area ratio (aspect ratio 1.65–1.88) on yawing moment coefficients of rudder on cabin fuselage at 30° pitch. (Angles marked on curves are rudder angles)

numerical values would have a limited significance in dealing with biplanes, and the use of triangular fins or vertical surfaces of more or less elliptical plan form would most certainly modify the numerical values. Data are not available to indicate the extent of the modification. Moreover, it is well known that the slipstream has an important effect on the rudder control. It is not believed that the presence of the slipstream would modify the conclusions as to the effects of area ratio, aspect ratio, and fuselage shape.

Table III shows that at 0° pitch, the shape of the fuselage has only a small effect on the rudder control. The open flat deck fuselage and open round deck fuselage are not greatly different at any point, but the

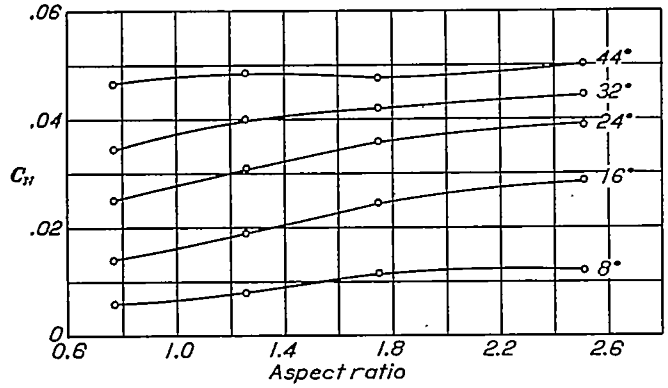


FIGURE 34.—Effect of aspect ratio (area ratio 0.0486–0.0502) on yawing moment coefficients of rudders on open round deck fuselage at 0° pitch. (Angles marked on curves are rudder angles)

effectiveness of high aspect-ratio rudders falls off somewhat more rapidly with increasing angle of pitch on the open flat deck fuselage. The effect of aspect ratio to sustain the yawing moment at high angles of pitch is slightly less for the flat deck than for the round deck fuselage.

The cabin fuselage has a much greater interference effect on the rudder moments at high angles of pitch, as would be expected. The decrease in rudder moment begins at smaller angles and persists with increas-

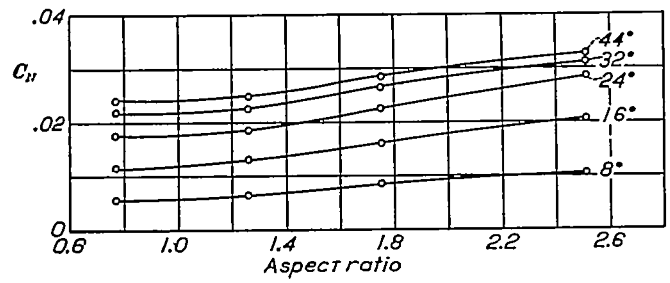


FIGURE 35.—Effect of aspect ratio (area ratio 0.0486–0.0502) on yawing moment coefficients of rudders on open round deck fuselage at 30° pitch. (Angles marked on curves are rudder angles)

ing angle of pitch throughout the range. The values at 40° are roughly two-thirds of those for the open cockpit fuselages.

The effect of a change in aspect ratio from 1 to 2 is to increase the rudder control (rudder angles not exceeding 25°) by some 30 to 45 per cent. For the average size rudder such an increase could be produced without a change of aspect ratio only by an increase in area of some 20 to 35 per cent. Thus the effect of aspect ratio appears to be of sufficient magnitude to warrant consideration in design.

CONCLUSION

The yawing moment produced by a rudder is approximately proportional to the angular displacement of the rudder for angles less than 25° and the yawing moment is larger when the aspect ratio is larger. The yawing moment continues to increase with increasing rudder angle at approximately the same rate for rudders of small aspect ratio (<1.2), but for rudders of

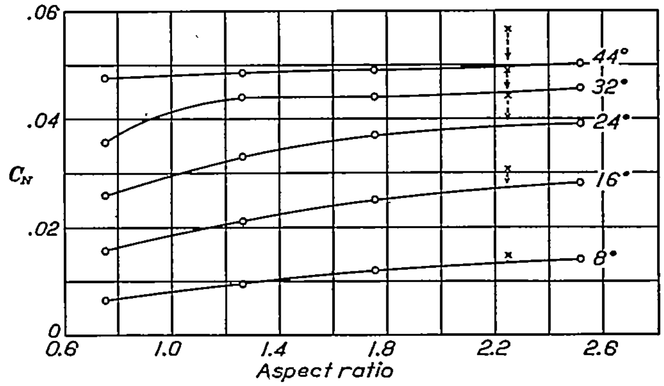


FIGURE 36.—Effect of aspect ratio (area ratio 0.0486–0.0502) on yawing moment coefficients of rudders on cabin fuselage at 0° pitch. (Angles marked on curves are rudder angles)

large aspect ratio, the rate of increase falls off rapidly above rudder angles of 25°. The value of the rudder moment for rudders of large aspect ratio is however never less than for rudders of the same area and smaller aspect ratio.

The effect of increasing the angle of pitch is greatly to decrease the yawing moment at a given rudder angle. The decrease is greatest for rudders of large aspect ratio, when the rudder angle is less than 25°; but when

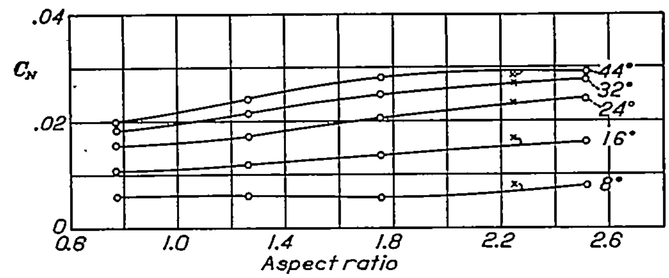


FIGURE 37.—Effect of aspect ratio (area ratio 0.0486–0.0502) on yawing moment coefficients of rudders on cabin fuselage at 30° pitch. (Angles marked on curves are rudder angles)

the rudder angle is large (44°), the decrease is greatest for rudders of small aspect ratio.

The effect of the shape of the fuselage is quite noticeable, being especially marked in the case of the cabin fuselage. The shielding effects are such for the cabin fuselage that the yawing moment due to a given rudder setting at an angle of pitch of 40° is about two-thirds of that for the open cockpit fuselages. This shielding is especially marked with rudders of small aspect ratio.

When the aspect ratio is maintained constant, the effectiveness of the rudder is linearly related to the area ratio, but increases somewhat faster than in direct proportion.

The effect of aspect ratio is sufficiently large to be considered in design. If rudder angles approaching 45° are permitted, the effect of increasing the aspect ratio is small and may be ignored for practical purposes unless the rudder is shielded by a large cabin fuselage. If, however, the rudder angle is restricted to 25° or less, an increase of rudder control of 30 to 45 per cent may be produced by increasing the aspect ratio from 1 to 2.

ACKNOWLEDGMENT

The authors wish to acknowledge the assistance of W. Hunter Boyd in the preparation of the models and in making the measurements.

BUREAU OF STANDARDS,
WASHINGTON, D. C., *May 16, 1932.*

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