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REPORT No. 491

VIBRATION RESPONSE OF AIRPLANE STRUCTURES

By T. THEODORSEN and A. G. GELALLES

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AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbreviation	Unit	Abbreviation
Length	l	meter	m	foot (or mile)	ft. (or mi.)
Time	t	second	s	second (or hour)	sec. (or hr.)
Force	F	weight of 1 kilogram	kg	weight of 1 pound	lb.
Power	P	horsepower (metric)		horsepower	hp.
Speed	V	kilometers per hour	k.p.h.	miles per hour	m.p.h.
		meters per second	m.p.s.	feet per second	f.p.s.

2. GENERAL SYMBOLS

W, Weight $= mg$

g, Standard acceleration of gravity $= 9.80665$ m/s^2 or 32.1740 ft./sec.^{2}

m, Mass $= \frac{W}{g}$

I, Moment of inertia $= mk^2$. (Indicate axis of radius of gyration k by proper subscript.)

μ, Coefficient of viscosity

ν, Kinematic viscosity

ρ, Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m^{-4}-s^2 at 15° C. and 760 mm; or 0.002378 lb.-ft.$^{-4}$ sec.^{2}

Specific weight of "standard" air, 1.2255 kg/m^3 or 0.07651 lb./cu.ft.

3. AERODYNAMIC SYMBOLS

S, Area

S_w, Area of wing

G, Gap

b, Span

c, Chord

$\frac{b^2}{S}$, Aspect ratio

V, True air speed

q, Dynamic pressure $= \frac{1}{2}\rho V^2$

L, Lift, absolute coefficient $C_L = \frac{L}{qS}$

D, Drag, absolute coefficient $C_D = \frac{D}{qS}$

D_o, Profile drag, absolute coefficient $C_{D_o} = \frac{D_o}{qS}$

D_i, Induced drag, absolute coefficient $C_{D_i} = \frac{D_i}{qS}$

D_p, Parasite drag, absolute coefficient $C_{D_p} = \frac{D_p}{qS}$

C, Cross-wind force, absolute coefficient $C_C = \frac{C}{qS}$

R, Resultant force

i_w, Angle of setting of wings (relative to thrust line)

i_t, Angle of stabilizer setting (relative to thrust line)

Q, Resultant moment

Ω, Resultant angular velocity

$\rho\frac{Vl}{\mu}$, Reynolds Number, where l is a linear dimension (e.g., for a model airfoil 3 in. chord, 100 m.p.h. normal pressure at 15° C., the corresponding number is 234,000; or for a model of 10 cm chord, 40 m.p.s. the corresponding number is 274,000)

C_p, Center-of-pressure coefficient (ratio of distance of $c.p.$ from leading edge to chord length)

α, Angle of attack

ϵ, Angle of downwash

α_o, Angle of attack, infinite aspect ratio

α_i, Angle of attack, induced

α_a, Angle of attack, absolute (measured from zero-lift position)

γ, Flight-path angle

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By T. THEODORSEN and A. G. GELALLES
Langley Memorial Aeronautical Laboratory

70089—34——1

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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VIBRATION RESPONSE OF AIRPLANE STRUCTURES

By T. Theodorsen and A. G. Gelalles

SUMMARY

This report presents test results of experiments on the vibration-response characteristics of airplane structures on the ground and in flight. It also gives details regarding the construction and operation of vibration instruments developed by the National Advisory Committee for Aeronautics.

In the ground tests a study was made of the vibration response of the fuselage, wings, and tail by applying sinusoidal forces and couples at different parts of the fuselages of two airplanes. The amplitudes of vibration along the fuselage and wings at various frequencies were measured and plotted, and the important natural modes of vibration were determined.

In the flight tests vibration records were taken in the cockpits and the tails of two airplanes. The vibrograms obtained in flight tests were analyzed and the amplitudes of the fundamental frequencies and the most important harmonics were plotted.

INTRODUCTION

Important contributions to knowledge on the subject of vibration in aircraft have been made by Constant (references 1 and 2). In his papers an exposition is given of the physiological effect of the vibration as dependent on amplitude and frequency, and consideration is given to the importance of the various sources of vibration and their effects on the aircraft structure. Of interest is his establishment of a curve of amplitude against frequency that defines the limits beyond which an unpleasant sensation is experienced. The amplitude limit varies from about 0.003 inch at a frequency of 5,000 per minute to about 0.006 inch at a frequency of 1,000 per minute, with permissible amplitude increasing sharply toward the lower range.

The sources of vibration in an airplane are the engine, the propeller, and the aerodynamic effects. Vibrations originating in either the engine or the propeller have frequencies that are related to the engine speed. The frequencies of vibrations of aerodynamic origin, in general, bear no relation to the engine speed.

Vibrations having their origin in the engine may be due to unbalance of rotating and reciprocating parts or to fluctuations in the torque. In the conventional aviation engines, inertia and torque resultants higher than the second order of the engine speed are usually of small magnitude, as far as causing vibration in the fuselage and wing structure, and are therefore of no consequence. No inertia unbalance of the second order or lower should be present in multicylinder engines except in 4-cylinder in-line and in radial engines. Computations by a method given by Tanaka (reference 3) show the secondary unbalance of some American single-bank radial engines as varying from about 200 pounds in a 5-cylinder engine to 800 pounds or more in a 9-cylinder radial engine at rated speeds. Torque resultants lower than the second order are present chiefly because of the unequal contribution in mean torque by each cylinder. In some instances torque resultants of the 1/2 and the first orders may vary from a few hundred to several thousand pound-inches.

Vibrations in the aircraft attributed to the propeller are usually of the first and second orders of the engine speed. Large unbalanced gyroscopic and aerodynamic couples (that may cause large amplitudes of vibration of the structure) may be induced by rapid changing of the direction of flight—as in turning or looping—especially in airplanes with 2-bladed propellers.

The aerodynamic disturbances, flutter and those resulting from tail buffeting, are sources of dangerous vibration. Flutter may be described as an unstable flight condition. Wings, tail planes, and propellers are susceptible to flutter. The speed at which flutter occurs is dependent on a large number of factors, including the frequencies of the responding parts in the various degrees of freedom. Buffeting of the airplane tail is often considered to be the result of vortices originating at the wings and impinging on the horizontal tail surfaces. Buffeting is more prevalent in low-wing monoplanes than in high-wing monoplanes or in biplanes. Treatments of aerodynamic disturbances will be found in references 4, 5, 6, and elsewhere.

In dealing with the vibration problem, the aircraft designer is required (1) to eliminate as far as practicable the sources of vibration; (2) to reduce the transmissibility to the structure by isolating the sources; and (3) to avoid resonant response of the structure.

The undesirable effects on the structure of unbalanced forces or couples in the engines or propellers are well recognized by designers. Their elimination is carried out as far as possible or practicable, however, recourse must often be had to the other two alternatives. It is possible that the isolation of the engine-propeller unit from the aircraft structure in many cases is very helpful; but good results would be obtained only if the problem were sufficiently well understood.

The purpose of this report is to present the vibration-response characteristics of fuselages, wings, and tails of airplanes so as to convey some idea of what frequencies might be expected in the general case or, at least, in related designs. Obviously a great deal is to be gained by avoiding resonant responses of the fuselage-tail unit or the wing structure. If the engine is running at 1,800 revolutions per minute, it is evidently not desirable to have the main response of the fuselage at 900 vibrations per minute or to have a critical response of the wing tips at 1,800 vibrations per minute. A summary of the tests conducted, i.e., airplanes tested, type of tests, and response measured, are given in table I. The tests conducted herein were made at Langley Field, Va., during 1932 and 1933.

N.A.C.A. VIBRATION INSTRUMENTS

At the beginning of this study the necessity arose for developing instruments capable of conveniently and accurately recording the large number of vibration characteristics desired. Three instruments, using the seismograph principle of operation, were developed. One of these instruments, the vibrograph, records flexural vibrations in any given direction; another, the torsiograph, records torsional vibrations about any given axis; the third instrument, the vibration indicator, indicates effective amplitudes of vibration in any given direction and is used for a rapid survey of magnitudes of vibration. In order to permit recording of vibrations in all parts of the aircraft in flight, the additional feature of remote operation was incorporated in all three instruments.

VIBROGRAPH

A sectional view of the vibrograph is shown in figure 1. This instrument consists chiefly of a short piece of shafting A attached to the vibrating body, and a casing B with a film drum C mounted on the shaft through a helical spring. Floating bearings permit a sliding motion between the shaft and the casing. The casing with film drum and accessories constitutes the suspended weight.

An optical method is used for recording the vibration and for timing. A mirror D, held between the shaft and the casing by a spring E, is rocked along the shaft axis as the shaft vibrates. A light beam directed onto the mirror through a lens and then reflected back to a film drum records the rocking motion of the mirror; only vibrations parallel to the shaft are recorded. Similarly, a light beam, from a lamp not shown, is directed onto a mirror F, which is mounted on a high-frequency vibrating reed and records the timing along the lower edge of the film. The frequency of vibration of the reed is controlled by an independent timer which makes and breaks the circuit of the magnetic coil G at definite intervals.

Damping is obtained by means of the dashpot H. Liquids of varying viscosities are used in the dashpot, depending on the temperature at which the tests are conducted. A small shaft I, with a roller properly fitted in a slot in the casing, limits the circumferential and the maximum axial motion of the suspended casing. A locking device (not shown), which is simply a spring-loaded latch, locks the casing rigidly to the shaft when the instrument is not in use.

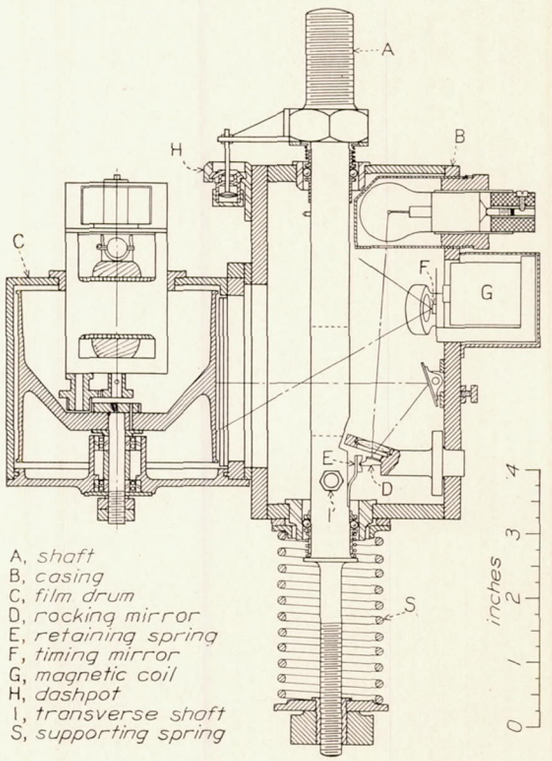


FIGURE 1.—Diagrammatic sketch of N.A.C.A. vibrograph.

The vibrograph, like all instruments of its type, has a definite frequency range of applicability. The lower limit of its reliability depends upon the natural frequency of the suspended weight, and on the amount and type of damping. The upper limit is determined only by the restoring force of the spring E, which holds the mirror in place. The calibration curve for the vibrograph is given in figure 2. Amplitudes of vibration at frequencies below about 600 vibrations per minute need a correction. Frequencies as low as 350 vibrations per minute are, however, obtained with accuracy. The instrument was calibrated and found to be accurate at frequencies up to 4,000 per minute. Computations indicate that the range extends to frequencies up to 15,000 per minute.

The sensitivity of the vibrograph depends mainly on the distance between the knife-edge pivots of mirror D. For the tests presented in this report 2 mirrors having 2 different widths between knife-edges were used. The sensitivity with the smaller mirror

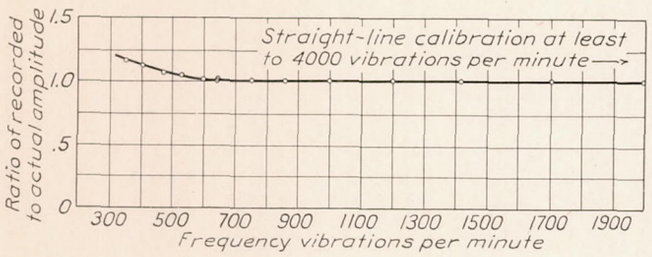


FIGURE 2.—Calibration curve of N.A.C.A. vibrograph.

was of the order of 0.50-inch amplitude recorded for 0.01-inch actual amplitude. With the larger mirror the ratio was 0.27 to 0.01. This sensitivity was constant over the whole range of amplitudes.

TORSIOGRAPH

Sectional views of the torsiograph are shown in figure 3. With this instrument torsional vibrations

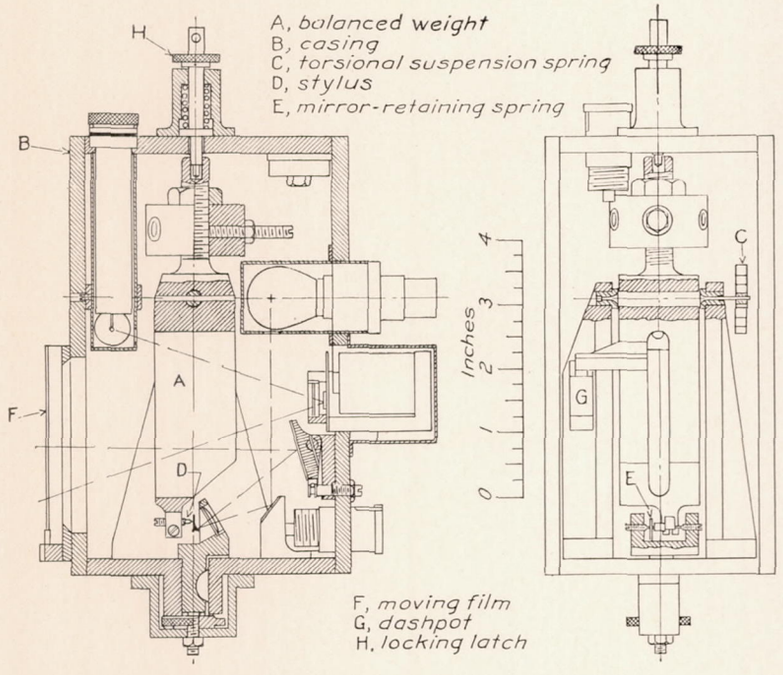


FIGURE 3.—Diagrammatic sketch of N.A.C.A. torsiograph.

in any given plane may be recorded. Like the vibrograph, it consists primarily of a suspended weight A and a casing B, which in this case is attached to the vibrating body. The weight A is suspended as a perfectly balanced flywheel and is held in its zero setting by the spiral spring C. For the useful range of torsional-vibration frequencies the casing vibrates with the body, while the suspended weight or flywheel remains stationary in its center position. Only torsional relative motion is possible between the weight and the casing. A stylus D at the lower end of the suspended weight rocks a mirror mounted on pivots. A spiral spring E retains the mirror in continuous contact with the stylus. The recording of the torsional motion and timing is accomplished optically in the same manner as that of the vibrograph. The motion recorded on the moving film F is proportional to the angular twist of the casing with respect to the stationary flywheel. Liquid damping is obtained by means of the dashpot G. A spring-loaded latch H holds the suspended weight rigidly to the casing when the instrument is not in use and limits the angular motion when it is operating.

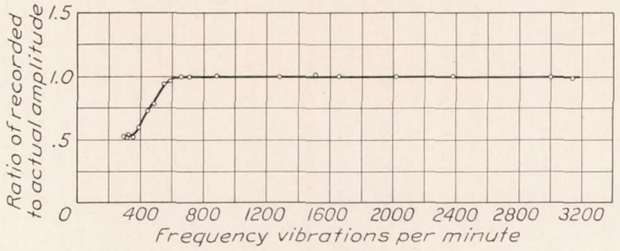


FIGURE 4.—Calibration curve of N.A.C.A. torsiograph.

The lower limit of reliability of the torsiograph, as for the vibrograph, depends largely on the natural frequency in torsion of the suspended weight, or flywheel, and on the type and amount of damping. The upper limit of this instrument is reached when the restoring acceleration of the mirror spring E is exceeded by that of the stylus D, i.e., when the mirror spring is no longer able to retain the mirror in continuous contact with the stylus.

A calibration curve of the torsiograph is shown in figure 4. From this curve it is seen that for frequencies below about 600 vibrations per minute corrections must be applied to the amplitude recorded by the torsiograph. Frequencies as low as 400 per minute may be recorded if proper corrections are applied to the amplitudes. No vibration tests were conducted to determine the upper limit of reliability of the torsiograph, as the calibration table in use was not suitable for very high frequencies. The natural frequency of the mirror-staff unit, which is the limiting factor for the upper range of the instrument, was determined by tests to be about 9,000 per minute.

The sensitivity of the torsiograph depends on the distance at which the stylus D acts from the pivot axis of the mirror staff. This distance can be varied. The sensitivity is constant for the whole amplitude range of the instrument for any given setting of this distance. Magnifications of 40, 55, and 70 were used for the tests of this investigation.

VIBRATION INDICATOR

A photograph of the vibration indicator is given in figure 5, and figure 6 is a sketch of the "feeler" unit of the indicator and a diagram of the wiring. With this instrument a quick survey of the effective amplitude of vibration in any part of the structure may be made. In common with the other two instruments, the underlying principle of its operation is the seismograph; but the hot-wire principle is utilized for recording the vibration.



FIGURE 5.—N.A.C.A. vibration indicator.

The vibration indicator consists of two separate units—the feeler unit, which may be held with a slight pressure or clamped to any part of the aircraft; and the recording unit, in which the observer may read the effective magnitude of vibration. As shown in figure 6 (a), the feeler unit consists of a weight A suspended on a spring B, a piston C connected rigidly to the weight, a nozzle D, and two hot-wire elements E and F of the same electrical resistance. In the recording unit are housed electrical resistances, flashlight batteries, a voltmeter, and a milliammeter.

The method of operation is indicated in figure 6 (b). The hot-wire element E and a similar element F constitute 2 arms of a Wheatstone bridge, and a fixed resistance G and a variable resistance H constitute the other 2 arms of the bridge. The variable resistance I controls the voltage across the bridge; the voltage is held constant for any desired calibration. When the feeler unit is held onto the vibrating body, its casing vibrates with the body. The weight A and the piston C, by virtue of their inertia, remain practically stationary for the range of frequencies for which the instrument is designed. As a result of this relative motion an air blast hits the element E, or air is drawn in through the nozzle D. This cooling effect changes the resistance of the element, the balance of the Wheatstone bridge is disturbed, and the effect is indicated by the milliammeter.

The principal problem in the early development of this instrument was to make the indicated amplitudes independent of the frequency and to obtain a calibration curve of satisfactory form. These difficulties were overcome by a systematic study of the functioning of all elements, and a final design was evolved. The development proceeded along a line similar to that of the development of a microphone of corresponding characteristics. The error in determining the effective amplitudes with this instrument at any frequency in the range does not exceed ± 3 percent. In figure 7 are shown the calibration curves of the instrument used in the tests of this report.

In the calibration of this instrument, frequencies as high as 10,000 vibrations per minute were used and the independence of amplitude and frequency was found to hold true. The lower frequency limit of the reliability of this instrument also depends largely on the natural frequency of the suspended weight. From the amplitude-frequency curve of figure 7 it is seen that the range of this particular instrument extends down to about 600 per minute; at this point the curve drops off sharply. The range of usefulness can easily be extended to lower frequencies by using a larger suspended weight or a more flexible spring.

The sensitivity of the instrument depends also on the dimensions of the various parts of the feeler unit and, in particular, on the resistances employed. The sensitivity in the lower range of the indicated amplitudes in this investigation amounts to about 1 milliampere per 0.01 inch amplitude, the actual ratio of the travel of the pointer to the vibration amplitude being of the order of 200:1. For larger amplitudes the sensitivity is gradually reduced. This latter property is very useful and was obtained only after considerable work. The absence of a response peak, common to all other vibration instruments at the natural fre-

quency of the suspended mass, is a very desirable attribute. The instrument exhibits a very sharp cut-off in the fashion of a high-pass filter—a property that is inherent in the method.

engine block the forces applied, varying as the square of the frequency, were 28 pounds at a frequency of 600 per minute and 770 pounds at 3,200 vibrations per minute. These values are comparable with the

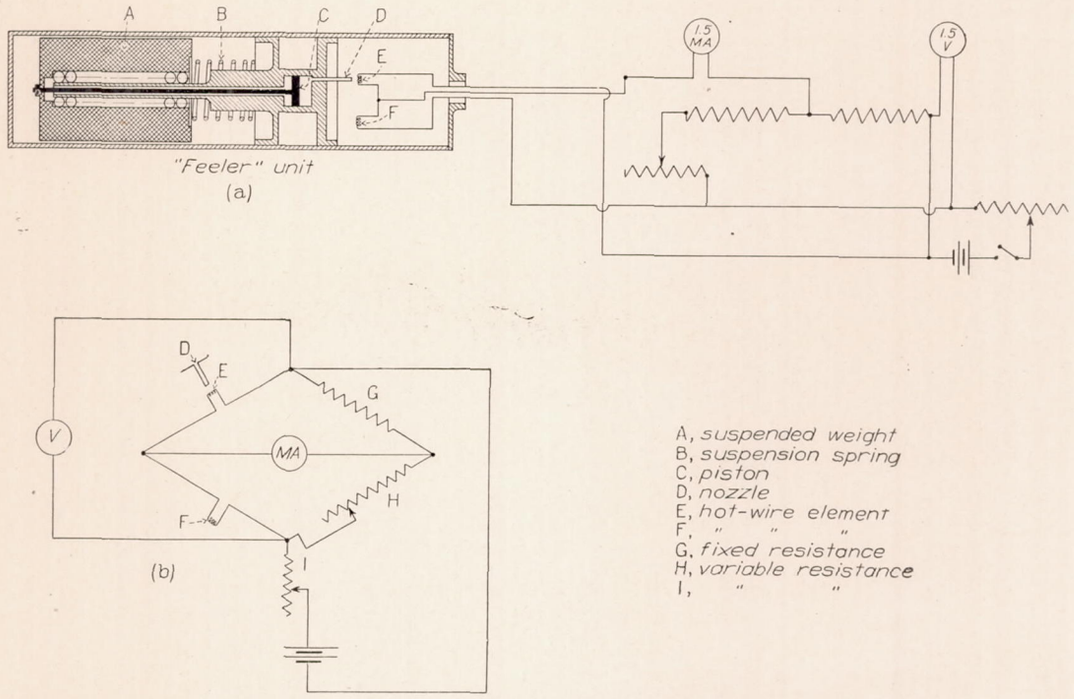


FIGURE 6.—Diagrammatic detailed sketch of N.A.C.A. vibration indicator.

VIBRATION TESTS ON THE RESPONSE OF AIRPLANE STRUCTURES

METHODS AND EQUIPMENT

The vibration response of the fuselage and wings of an airplane were determined by mounting the airplane on springs to obtain a floating suspension that would approximate conditions obtaining in flight. Forced vibrations of magnitude and frequencies comparable to those existing in aircraft were applied at different parts of the fuselage and the response was recorded with either the vibration indicator or the vibrograph. Tests were conducted on the Boeing PW–9, an Army pursuit airplane, and on the N2Y–1, a Navy training airplane.

Figure 8 is a photograph of the PW–9 mounted for the vibration-response tests. The weight of this airplane is approximately 3,100 pounds. Automobile springs were substituted for the landing wheels and the tail skid, as shown. The frequency of free vibrations of the airplane on the springs in the vertical direction was about 100 vibrations per minute.

A vibrator externally driven through a cog-belt by a motor on a separate stand was mounted successively at the top of the engine block, at the center of gravity of the airplane, and at the fuselage near the tail. With this vibrator sinusoidal forces and couples of varying magnitude and frequency could be impressed on the fuselage. With the vibrator on the secondary unbalanced inertia forces existing in present-day radial engines. Because of the much larger response with the vibrator mounted at the center of gravity and at the tail, smaller forces had to be used there.

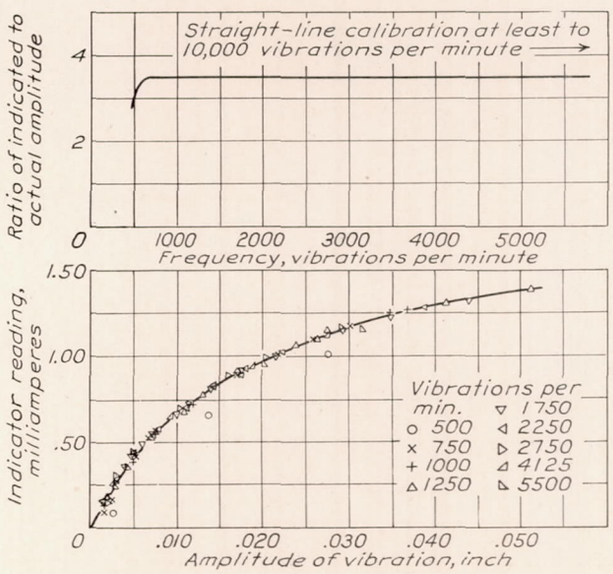


FIGURE 7.—Calibration curves for the N.A.C.A. vibration indicator.

For nearly all the vibration-response tests the hot-wire indicator was used in connection with an "auto-

matic observer" to take simultaneous pictures of the readings of the vibration indicator and of a tachometer indicating the frequency of the imposed vibrations. This automatic observer consists of a light-tight box with the tachometer and vibration indicator mounted on one side and a motor-driven motion-picture camera mounted on the opposite side. Electric lamps within the camera box are timed to switch on and off at regular intervals and provide the illumination necessary for taking records of the instrument readings. Readings of the amplitude [1] of vibration were taken along the length of the fuselage and wing tests did not deviate from the drawn curves more than about 0.0003 inch, the amplitude response at any one position was observed to vary as much as 10 percent for duplicate test conditions. The form of the elastic curve, however, was identical under all duplicate test conditions.

Previous to the tests on the PW–9, a series of tests were conducted of the vibration-response characteristics of the fuselage of the Consolidated N2Y–1, a 5-cylinder Navy training airplane weighing about 1,500 pounds. This airplane was also mounted on leaf springs. The vibration indicator not having been

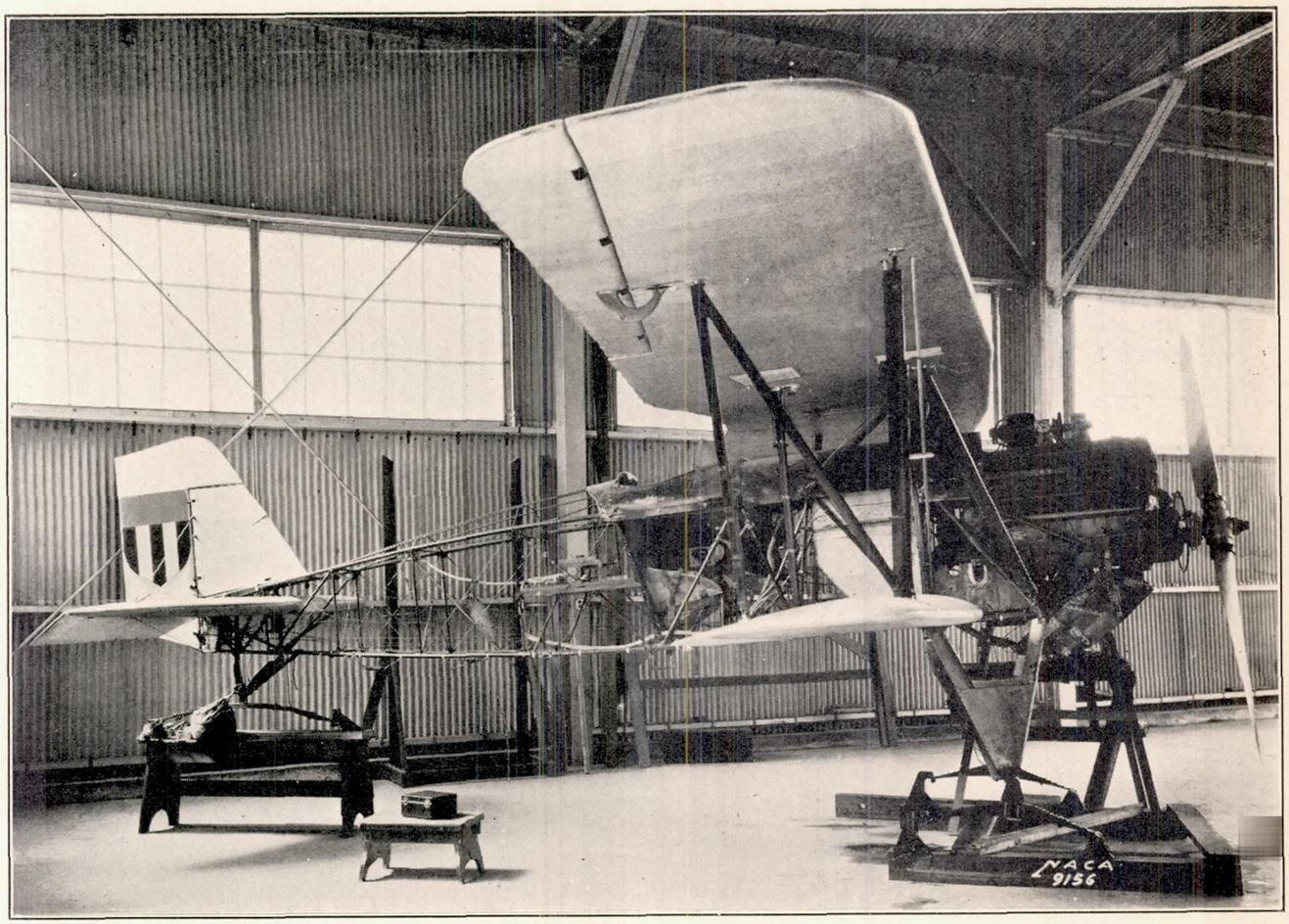


FIGURE 8.—Flexible mounting of PW–9 airplane for vibration-response tests.

every 6 to 12 inches, and a complete record of the vibration in any one position was obtained for the frequency range of 500 to 3,200 vibrations per minute in about 15 to 20 minutes. Check tests were made to verify the positions of the nodal points.

Because of the multiplicity of test points and the number of curves plotted together, the points have been omitted from the experimental curves for the convenience of easier reading. Although the amplitude of vibration as determined from these particular developed at the time, the vibrograph was used in recording the vibration. The vibrator was mounted on the top cylinder in the place of the removed cylinder head.

In the flight tests two Navy biplanes were used—an N2Y–1, similar to that used in the response tests, and an NY–2. The N2Y–1 is powered with a 5-cylinder Kinner radial engine, and the NY–2 with a 9-cylinder Wright J–5 radial engine. The framework of each of the airplanes tested was of welded tubular steel. The engines of these two airplanes and that of the PW–9 used in the vibrator tests were either

[1] Amplitude throughout this paper refers to the single amplitude, i.e., the distance from the mean position to the extreme.

rigidly mounted, or very nearly so, with only a thin layer of cushioning material between the clamps and the fuselage, chiefly used to prevent abrasion of the structural members. All three types of engines were equipped with 2-bladed direct-driven propellers.

When vibration records were taken at the front or rear cockpits, the vibrograph was mounted on the upper longeron of the fuselage on, or close to, the bulkheads. The torsiograph was mounted along the thrust axis of the airplane. Particular care had to be taken to mount the instruments rigidly and in such a manner that no local vibrations of the mounting lasted from one-third to one-half second. This timing was found satisfactory for the tests in the cockpit, as the vibrations there were found to be exact multiples of the engine speed. An independent chronometric timer is more convenient, however, when vibrations of frequencies other than multiples of engine speed are recorded, especially when records are taken during maneuvers and when the engine speed is continually changing.

Fourier's analysis was applied to separate the harmonic components of the vibrograph and torsiograph flight records. By the use of Runge's method

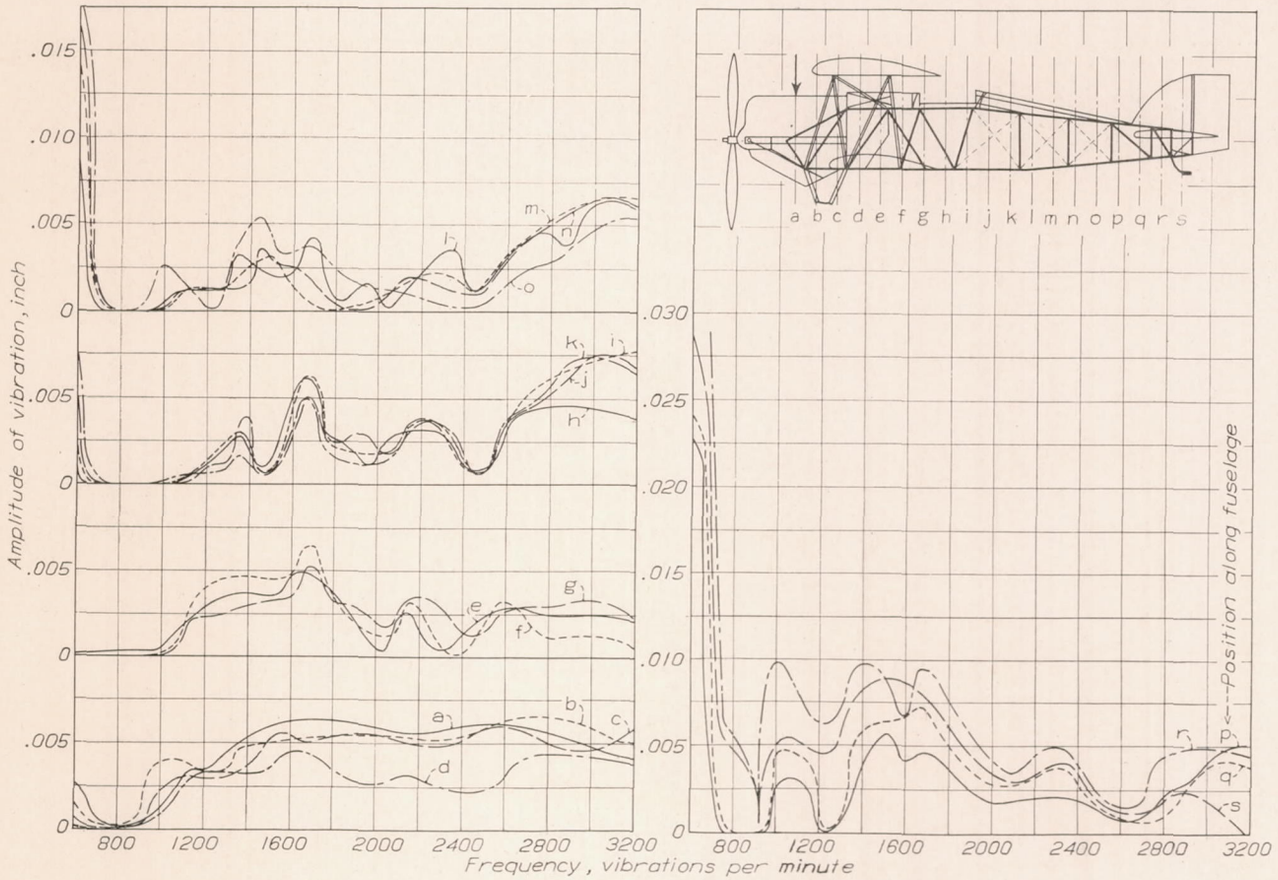


FIGURE 9.—Vibration response along the lower longeron of the PW–9 airplane.

interfered with the actual vibration at the point. In the tests at the tail of the airplane the instruments were mounted on the upper longerons, slightly back of the tail skid. Electric connections and a wire cable for operating and for locking and unlocking the instruments permitted remote operation from the cockpit.

Timing was obtained directly from the engine. A unit consisting of a commutator and a brush was attached to the camshaft connection of the tachometer drive. By means of this device, together with a magnetic coil and a high-frequency reed, four timing marks were made on the recording film for each revolution of the camshaft simultaneously with the recording of the vibration. The pilot observed the engine speed during the interval the record was taken, which of tabulation on prepared printed standard forms, an analysis could be made in less than 3 hours per record.

TEST RESULTS AND DISCUSSION

Vibration response of PW–9 fuselage.—Figures 9 and 10 show the response of the fuselage of the PW–9 to a vertical sinusoidal force applied on the engine block with the airplane mounted on springs as shown in figure 8. In figure 9 the amplitude in the vertical direction as measured at points a to s along the lower longeron is plotted against the frequency of vibration, and in figure 10 the amplitude is plotted against position along the fuselage or distance from the plane of rotation of the propeller. These curves show peaks at approximate frequencies of 600, 1,400 1,680, 2,250, and 3,100 per minute, the largest response occurring

at the frequencies of 600 and 3,100 vibrations per minute.

From subsequent experimental observations, the disturbance at the frequency of 1,400 vibrations per minute was traced to a resonant vibration of the trailing edge of the lower wing, that at 1,670 to one of the wing spans, and that at 2,200 to the bench supporting the rear spring. The disturbance due to the supporting bench was eliminated in all subsequent tests. The two main disturbances, that at 600 and that at 3,100 per minute, were identified as the resonant vibrations of the fuselage.

Because limitations of the vibrator unit did not permit a lengthy running at higher speeds, no record

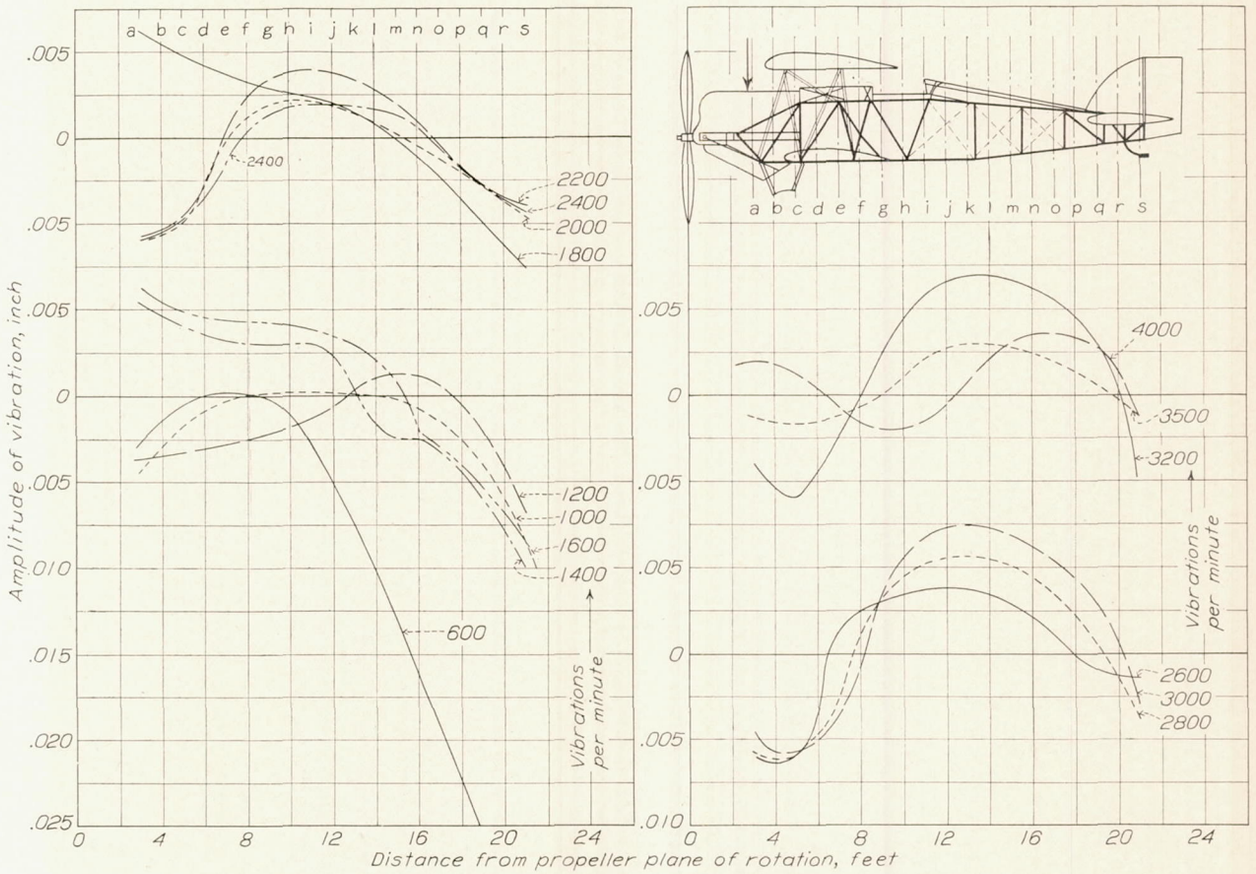


FIGURE 10.—Elastic curves of the lower longeron of the PW-9 airplane at different frequencies.

readings were taken with the automatic observer at the frequencies above 3,200 per minute. The elastic curves of the fuselage at any one frequency, however, could be determined approximately by observing amplitudes at different positions. The elastic curve at 4,000 per minute is shown in figure 10. No other amplitude peaks, small or large, were observed at the range of frequencies between 3,200 and 4,500 per minute.

Vibration response of the fuselage with the wings removed, and with the vibrator mounted at the center of gravity of the airplane.—The vibration tests on the PW-9 fuselage were continued with the vibrator mounted on the upper longerons immediately above the center of gravity of the airplane, with the wings and stabilizer replaced by equivalent weights located at their respective centers of gravity. Because of the very large response of the fuselage at the frequency of 2,500 vibrations per minute when the unbalanced weights used at the engine block were applied at this position, weights of less than one-third the magnitude were used in these tests. The results are given in figures 11 and 12. No vibration was observed at frequencies below about 1,800 and only small response immediately above 3,000 per minute. A pronounced maximum response was obtained at 2,500 per minute.

Only by using the larger weights was it possible to obtain even a small response at 600 per minute of the form given in figure 10, because the forces were applied immediately above the nodal point of the elastic curve at this frequency.

The unmistakable response at a frequency of 2,500 vibrations per minute was similar to that expected from a comparatively clean undamped structure. The form of the elastic curve at this critical frequency corresponds to that of a frequency of 3,100 vibrations per minute in the vibration results with the wings on. (See fig. 10.) The lowering in frequency of this mode of vibration from 3,100 with the wings on to 2,500 vibrations per minute without the wings may be attributed to the loss of the stiffening effect of the wing bracing. A smaller response at the frequency of about

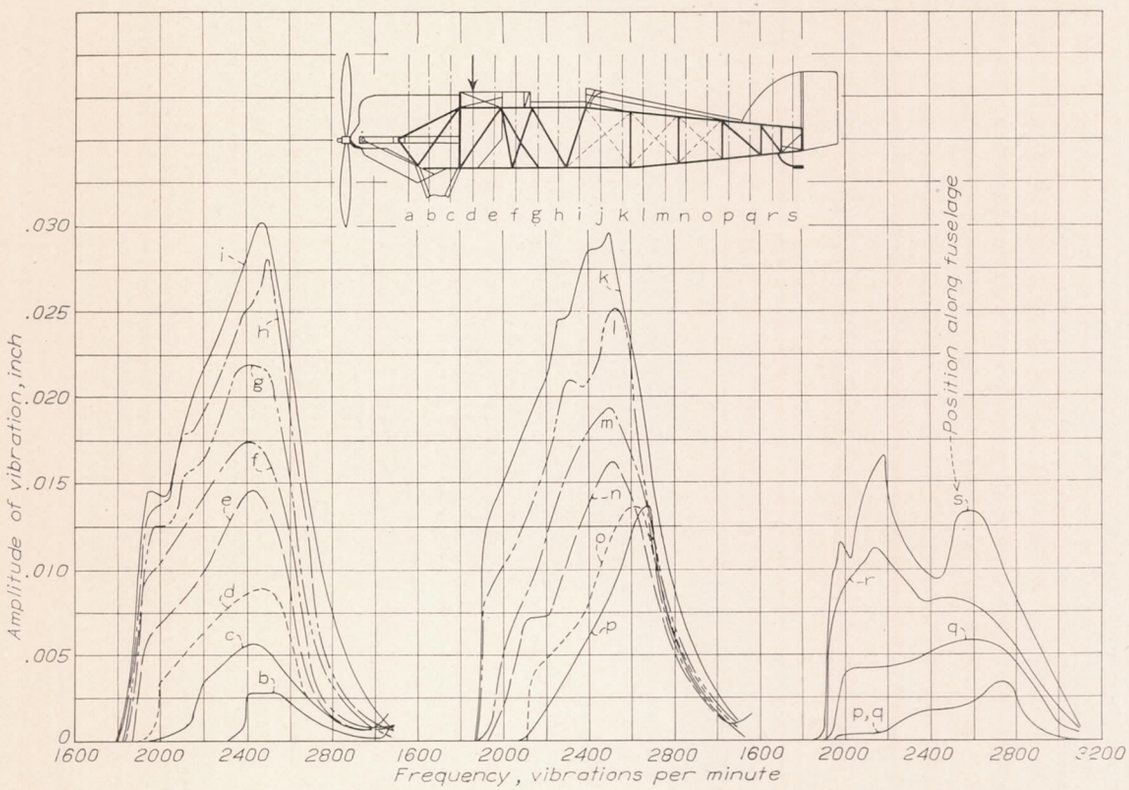


FIGURE 11.—Vibration response along the lower longeron of the PW-9 airplane. No wings nor stabilizer. Vibrator mounted at the center of gravity of the airplane.

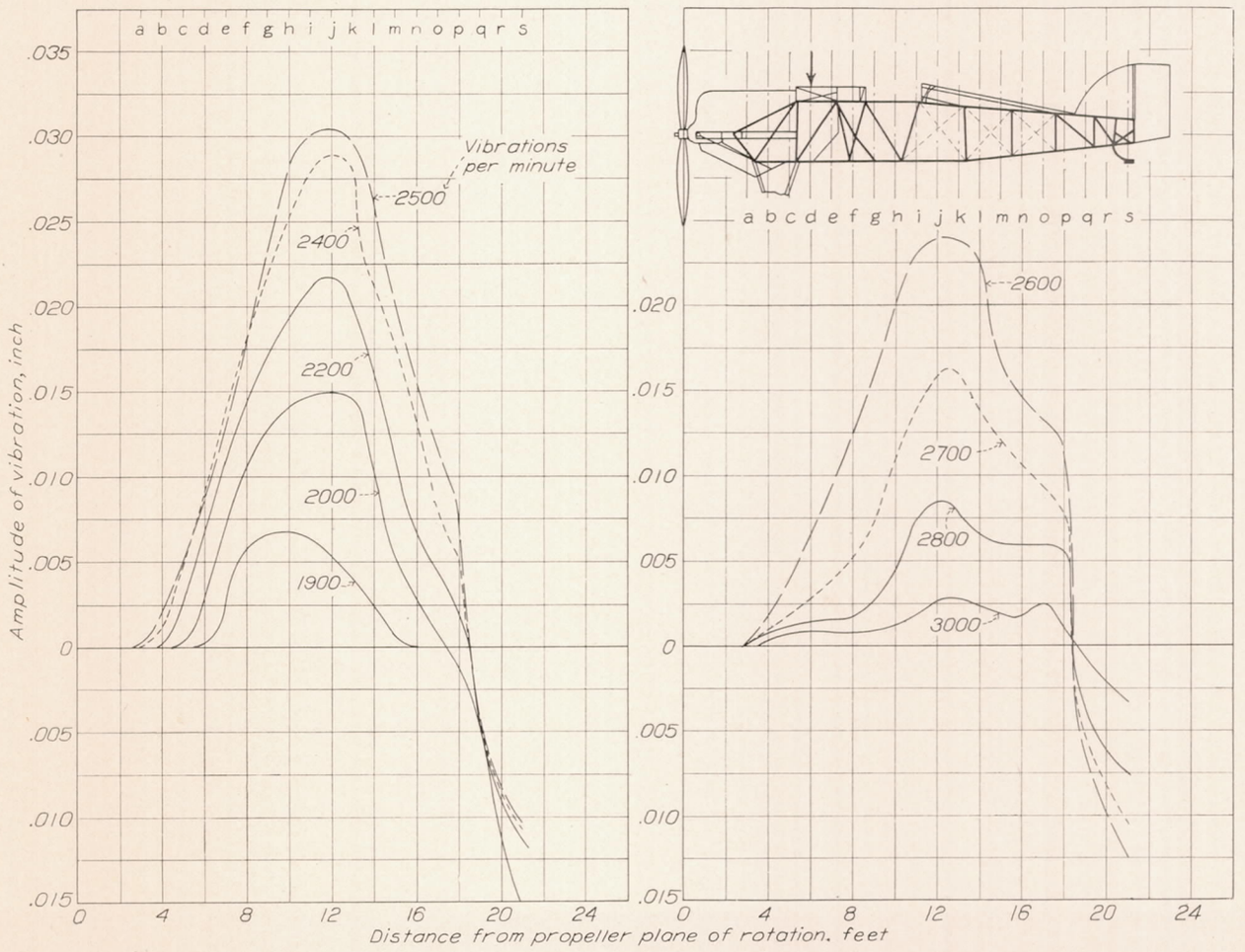


FIGURE 12.—Elastic curves of the lower longeron of the PW-9 airplane at different frequencies. No wings nor stabilizer. Vibrator at center of gravity of airplane.

2,100 per minute, localized at the span of the upper longerons on which the vibrator was mounted, was also observed. This response was found to be a resonant vibration of this span brought within the range of recording due to the added weight of the vibrator. The effect of this local disturbance is noticeable in the shape of the curves of figure 12. No other resonant vibrations were observed up to 4,000 vibrations per minute. The amplitude of vibration along the fuselage remained below about 0.0025 inch at these higher speeds, even though the force applied increased as the square of the speed.

Additional tests for the determination of the fundamental mode of the PW–9 fuselage.—The results of figures 9 to 12 have shown two critical frequencies of the fuselage, one at 600 vibrations per minute and the other at 3,100 with the wings attached, or at 2,500 with the wings replaced by an equivalent weight. Theoretical considerations show that the airplane being flexibly mounted, the fuselage may be considered as a free-end beam of nonuniform elasticity and weight distribution. The elastic curve of a free, uniform beam gives for the mode of lowest frequency very nearly three-fourths of a full wave corresponding to $3\pi/2$ radians, the second mode $5\pi/2$, the third $7\pi/2$, etc. The frequencies of these 2-, 3-, 4-node, etc., vibrations bear the relation to each other of 1, $(5/3)^2$, $(7/3)^2$, etc., or 1, 2.77, 5.44, etc. Beams having stiffness and weight distributions comparable to those of an airplane fuselage are expected to give larger frequency ratios between the fundamental and the higher principal modes of vibration. The ratio of the lower to the higher critical frequency for the fuselage of the PW–9 (figs. 9 to 12) was found to be 5.1 with the wings on and 4.1 without the wings. Further tests on other airplanes supplementing these results are considered desirable.

In order to obtain an unquestionable evidence whether the disturbance at 600 vibrations per minute was the fundamental mode of vibration of the fuselage, tests were conducted with a wooden-beam model of proportional elasticity and weight distribution. In figure 13 the forms of vibration obtained at different conditions are given. Weights were mounted on the forward part to represent the power-plant unit and wings, and a small weight on the other extreme end to represent the tail unit. With the amount and location of fixed weights, given in the figure, a fundamental mode of vibration of two nodes was obtained at the frequency of 460 per minute, and a second mode of three nodes at 1,550 per minute. These values give the ratio of the fundamental to the second mode as 3.4. One of the nodes of the fundamental frequency was located near the center of gravity of the forward weights, and the other one about two-thirds the distance toward the tail. As the ratio of the weights of the forward part to those at the tail was increased, however, the node nearer the tail of the fundamental mode moved forward and approached the second. Simultaneously, the amplitude of vibration under the larger weights diminished considerably. This form of vibration duplicated approximately the form that was obtained with the fuselage of the PW–9 at 600 per minute.

Supplementary static tests were made by supporting the fuselage under the center of gravity and applying weights at both ends and at only the tail end of the PW–9 fuselage. Computations made from the observed deflections gave the frequency of the fundamental mode as being within 50 cycles of 600 vibrations per minute.

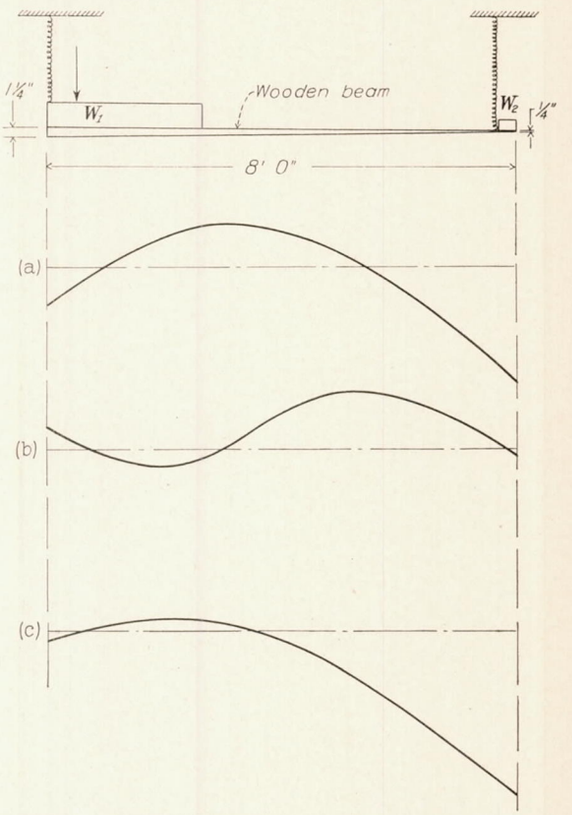


FIGURE 13.—Test results with a model beam.
a. Fundamental mode of vibration—W_1=70 pounds, W_2=2.5 pounds. Frequency=460 vibrations per minute.
b. Second principal mode—W_1=70 pounds, W_2=2.5 pounds. Frequency=1,550 vibrations per minute.
c. Fundamental mode—W_1=150 pounds, W_2=1 pound. Frequency=650 vibrations per minute.

The model and static tests proved beyond doubt that the 2-noded fundamental mode of vibration of this particular fuselage is at 600 per minute. The elastic curves of figure 10 indicate that the 3-noded second principal mode of vibration must lie in the range of frequencies between 2,500 and 4,500 per minute. Since no critical frequencies other than that at 3,100 per minute with the wings and 2,500 without the wings were observed up to frequencies of 4,500 per minute, it is logical to conclude that these frequencies are the second principal modes of vibration of the

fuselage, although some doubt still exists as to the location of their nodes.

Vibration response with the vibrator mounted near the tail of the PW–9 airplane.—The studies of the fuselage vibration were further pursued by mounting the vibrator on the fuselage immediately forward of the fin. Both forces and moments were applied. Different methods of suspension were employed to eliminate any possible effects of the supports and in particular to

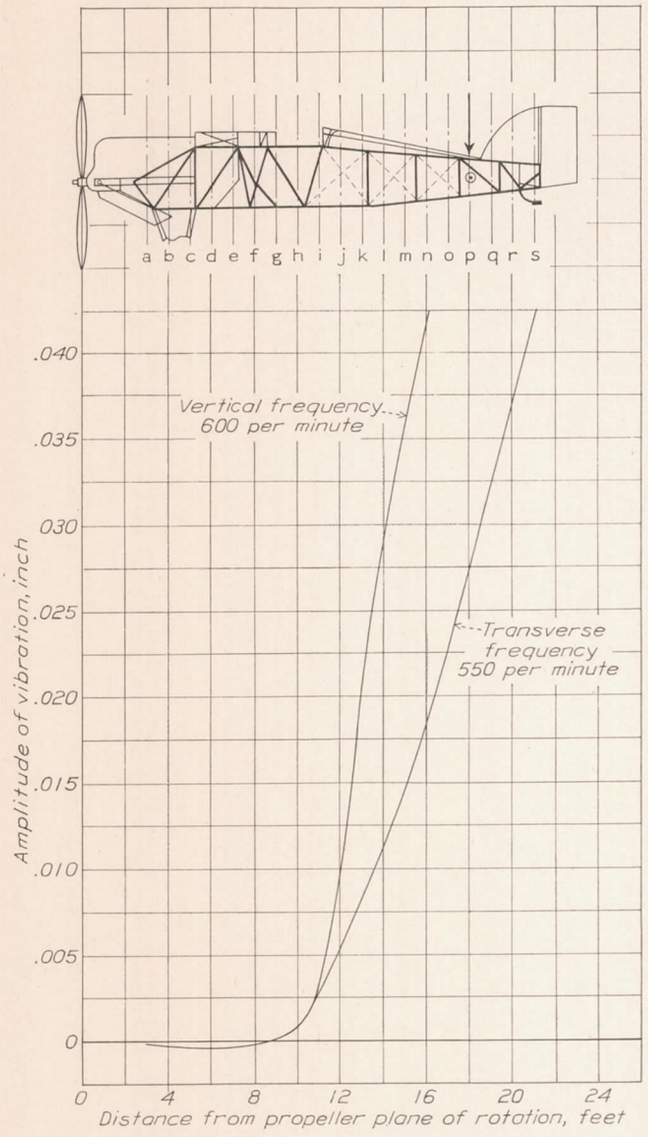


FIGURE 14.—Resonant response of the fuselage of the PW-9 airplane. Vibrator on tail of airplane.

insure full freedom in torsion. The principal results were:

1. A resonant vibration of the fuselage in flexure in the vertical direction was found to exist at a frequency of about 600 per minute.

2. A resonant vibration of the fuselage in flexure in the transverse direction was found to exist at about 550 per minute.

3. A resonant vibration of the fuselage in torsion was found to exist at a frequency slightly under 500 per minute.

4. A resonant vibration of the stabilizer in torsion about an axis transverse to the fore-and-aft axis at about 1,900 per minute.

5. A resonant torsional vibration of the fuselage alone with the fin and rudder and without the stabilizer at the frequency of about 1,700 per minute. Because of the play in the controlling gear between the stabilizer and the fuselage, the fuselage executed a torsional vibration of its own as though the stabilizer did not exist. The vibration, although damped by the large inertia of the stabilizer, was relatively noticeable. It was definitely identified only after the tests were conducted with the stabilizer removed.

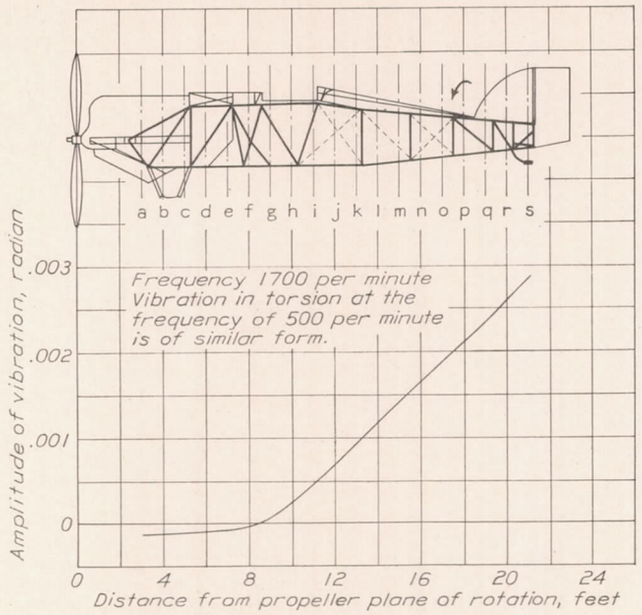


FIGURE 15.—Resonant response in torsion of the fuselage of the PW-9 airplane. Vibrator on tail of airplane.

The resonant vibrations of the fuselage in flexure and in torsion at the frequencies of 600, 550, and 1,700 per minute are shown in figures 14 and 15. The elastic curve in torsion at 500 per minute was similar to that at 1,700 per minute shown in figure 15. The mode of vibration having a frequency of 600 cycles per minute is undoubtedly the fundamental mode of the fuselage in flexure in the vertical direction that was determined in the previous tests; that at 550 is a flexure in the transverse direction; and that at 500 a torsion. It was practically impossible to distinguish the nodes of vibration forward of about 9 feet from the propeller axis, the vibration amplitudes being very small and within the limits of experimental error. The large inertia of the engine block, accessories, and wings damped any normal response of the fuselage structure.

Vibration response of N2Y–1 fuselage.—Another set of curves of flexural vibration of the lower longeron in the vertical direction, which were obtained with the vibrograph in its early stages of development, is shown in figure 16. The N2Y–1, a Naval training airplane weighing approximately 1,600 pounds, was used in these tests, and was mounted in a manner similar to the PW–9. Owing to the high magnification and the relatively high error in the instrument at the time, some difficulty was experienced in recording the response curves at the critical frequencies and in determining the exact mode of vibration of the structure. In general, however, the modes of vibration

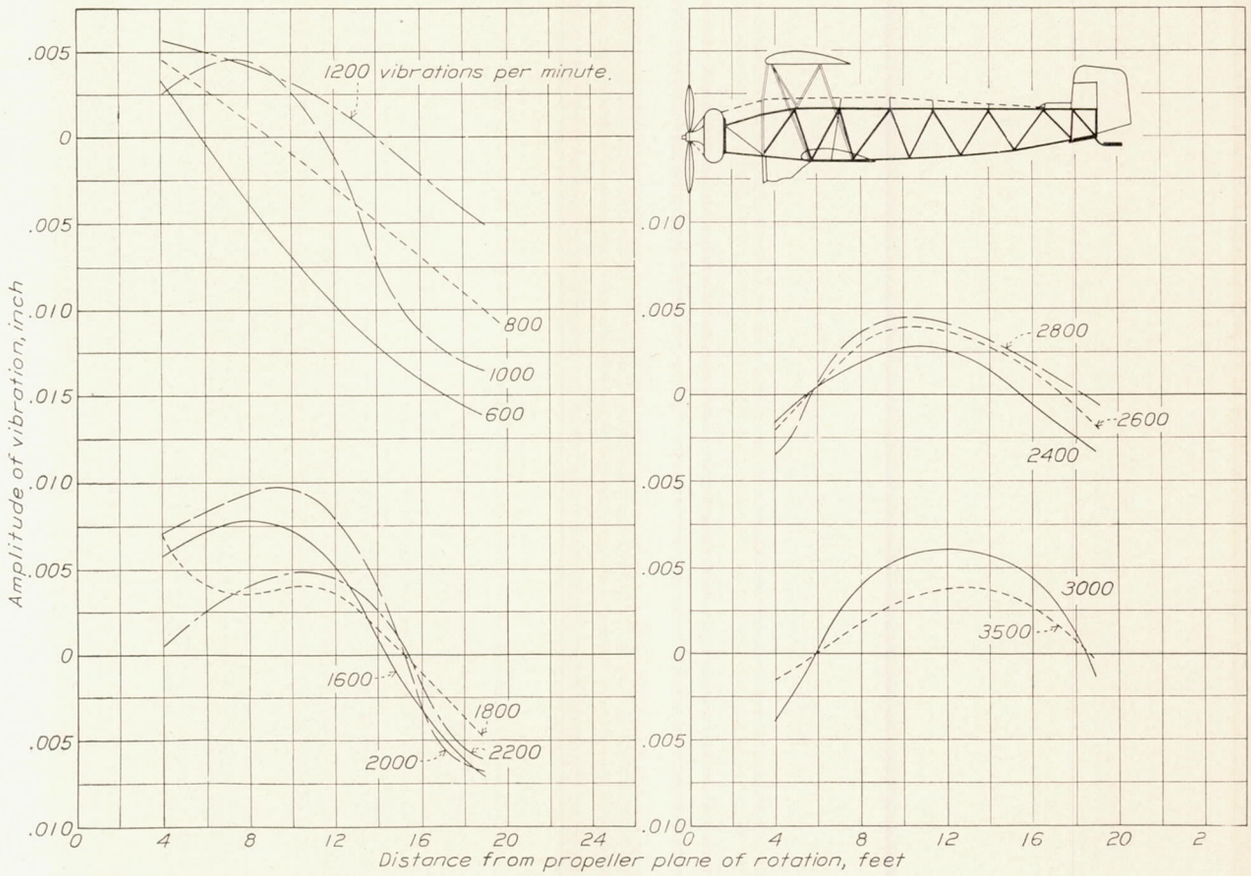


FIGURE 16.—Elastic curves of the lower longeron of N2Y–1 airplane.

are identical with those obtained with the PW–9. From these instrumental and also from visual observations, principal modes of vibration were noted at the approximate frequencies of 750, 1,100, 1,900, and 2,900 vibrations per minute. The mode at 750, judging from the results obtained with the PW–9 tests, is the fundamental mode of vibration of the fuselage in flexure in the vertical direction. The disturbances at 1,100 and 1,900 per minute were found to be resonant vibrations of the upper wing tips and what appeared to be a response of the inner bay spans of both wings, respectively. The resonant vibration at 2,900 per minute is undoubtedly the second principal mode of vibration of the fuselage in the vertical direction, and corresponds to that at 3,100 per minute for the PW–9 fuselage.

Vibration response of upper and lower wings of the PW–9.—In figures 17 to 20, the vibration responses of the upper and lower wings of the PW–9 airplane in the vertical direction are shown. A vertical force was applied at the engine block, as for the tests on the fuselage. The amplitudes were measured along the front spar of the right wings. Check tests on the left half of the wings indicated that there was no appreciable difference in the deflections at corresponding symmetrical positions. Particular attention was given to the location of nodal points. With the exception of frequencies above about 2,500 vibrations per minute, it was quite difficult to determine the exact location of the nodal points because of superimposed disturbances. At the nodal points shown in figure 18, the amplitudes actually did not fall below the values of 0.0005 to 0.0015 inch.

The principal amplitude peaks indicating resonant vibrations in the upper wing (fig. 17) are at the approximate frequencies of 800, 1,650, 2,350, and 2,850 vibrations per minute. In addition, there are smaller peaks at 1,050, 1,400, and 2,100 per minute. The principal amplitude peaks in the lower wing (figs.

19 and 20) exist at the approximate frequencies of 1,650 and 2,350 per minute. In addition, there are small peaks at the frequencies of 1,400 and 2,800 per minute. It is difficult to attribute definitely any one of these resonant vibrations to any one of the wings, for they appear in both upper and lower wings. Close examination of the figure and visual observation, however, indicate a resonant vibration of the upper wing tip at a frequency of about 800 per minute, a resonance in the inner bay span at 2,350 per minute, and a resonance mode in mid-span at 2,800 per minute. In the lower wing the indications are that there is a resonant vibration of the wing tip at a frequency of 1,650 per minute and one in the trailing edge at 1,400 per minute. Although the effect of the vibration of the trailing edge of the lower wing is not marked by excessively high amplitudes at the leading edge, the observed amplitudes at the trailing edge at the frequency of 1,400 per minute exceeded the range of the available instruments. From figures 19 and 20 it is seen that the vibration of the entire lower wing appears to be largely influenced by vibrations in the upper wing and the fuselage. The tips of both wings are in a state of continual disturbance throughout almost the whole range of speeds.

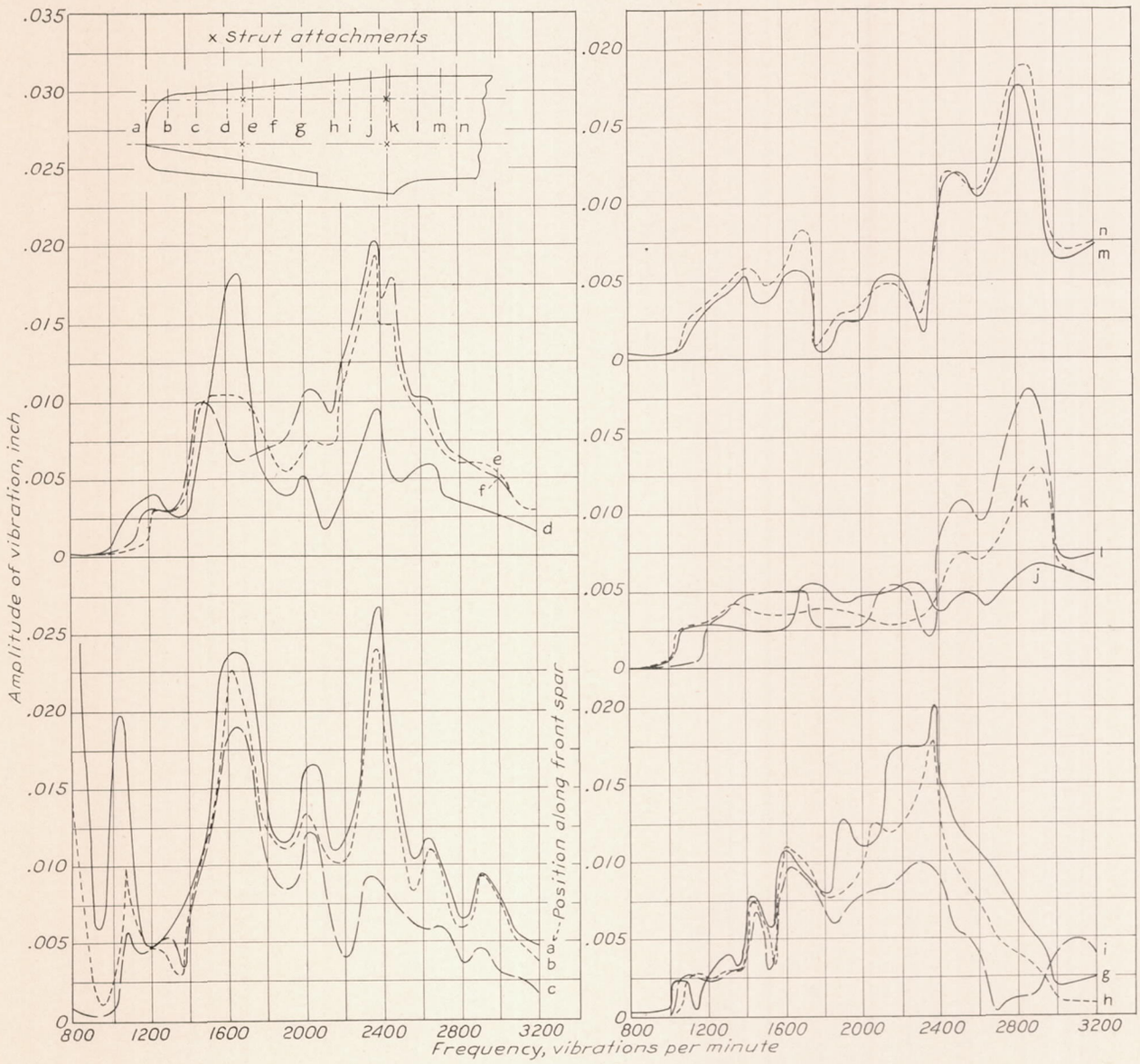


FIGURE 17.—Vibration response along the front spar of the upper wing of the PW–9 airplane.

General discussion of the vibrator tests.—A summary of the principal response of the PW–9 structure is given in table II. It may be concluded that the fuselage of an airplane usually has the fundamental modes of vibration in flexure and in torsion within the range of possible buffeting frequencies and possibly within the range of the 1/2 harmonic of the engine speed. Flight tests showed buffeting frequencies to vary from a few hundred to 1,200 vibrations per

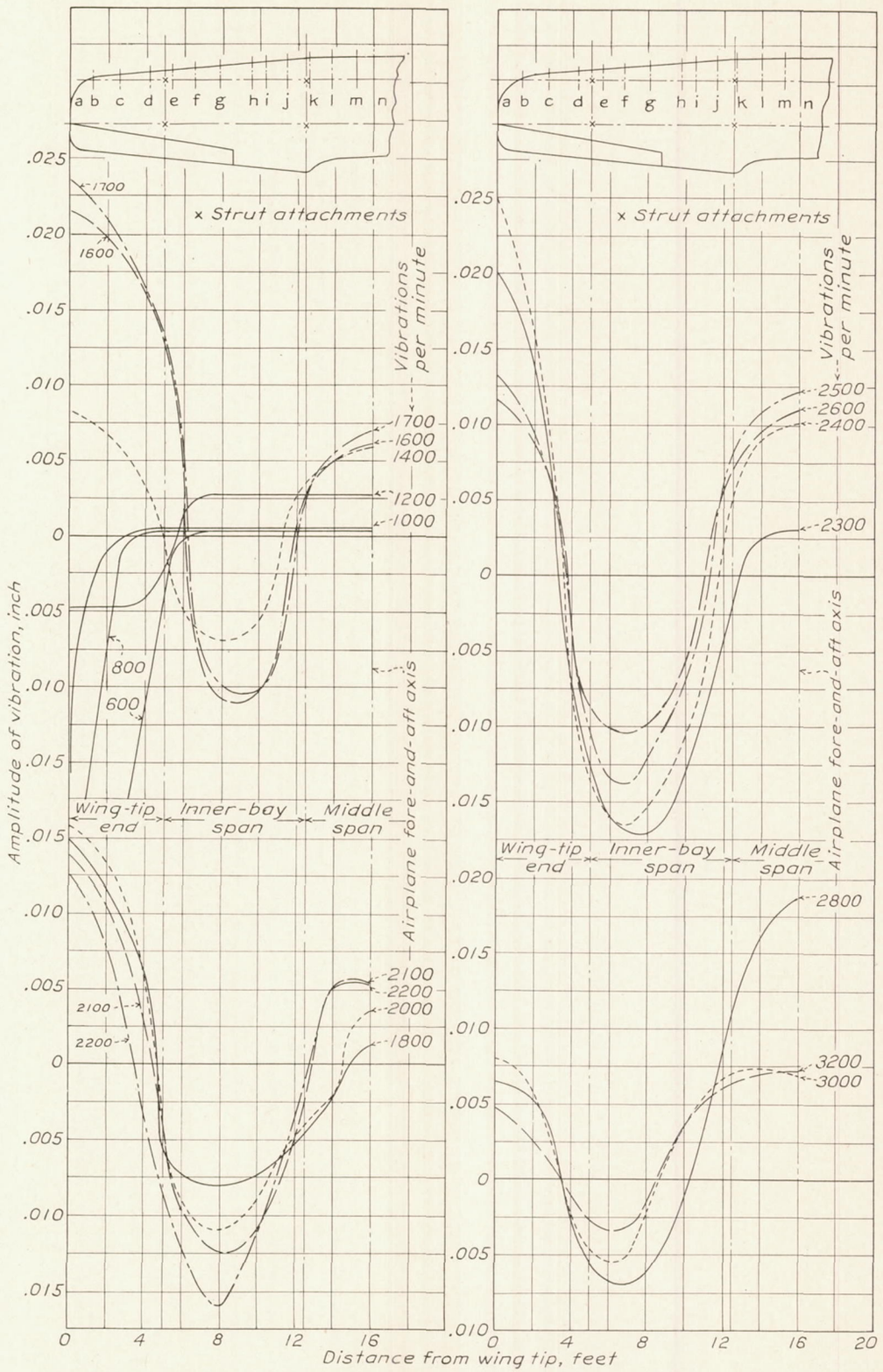


FIGURE 18.—Elastic curves of the front spar of the upper wing of the PW-9 airplane at different frequencies.

minute (reference 5). In order to avoid resonance, the fuselage, especially back of the cockpit, may have to be stiffened in order that its fundamental mode of vibration falls above these frequencies. If the operating range of the engine is between 1,500 and 1,900 revolutions per minute, the logical range of the fundamental mode of vibration of the fuselage is between 1,200 and 1,500 per minute. Since the other parts of the tail structure besides the fuselage are affected by buffeting impulses as well, these also must be designed to avoid resonance.

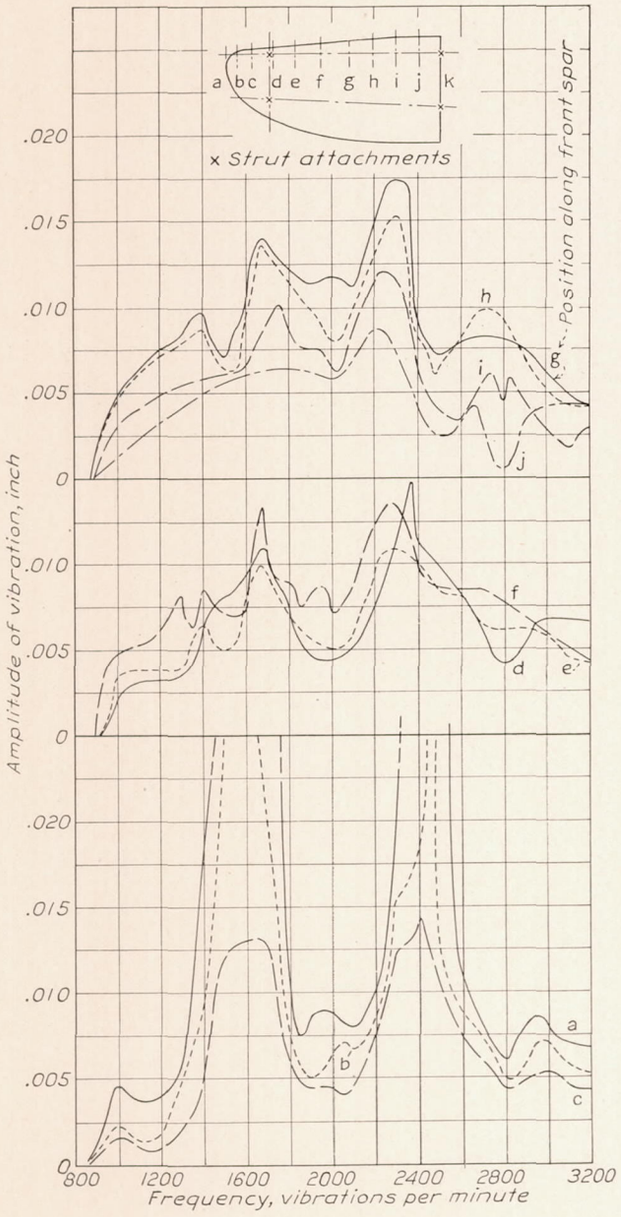


FIGURE 19.—Vibration response along the front spar of the lower wing of the PW–9 airplane.

The second principal modes of vibration of the fuselage in flexure or in torsion may fall at frequencies higher than the first order of the engine speed. For both airplanes tested, in which the frequency of the second principal mode of vibration of the fuselage in flexure in the vertical direction was found to be close to 3,000 per minute, the possibility exists that the fuselage may resonate either in flexure or in torsion with a poss ble exciting force of the second order of the inertia unbalance and the torque resultant of the same order from the engine or propeller.

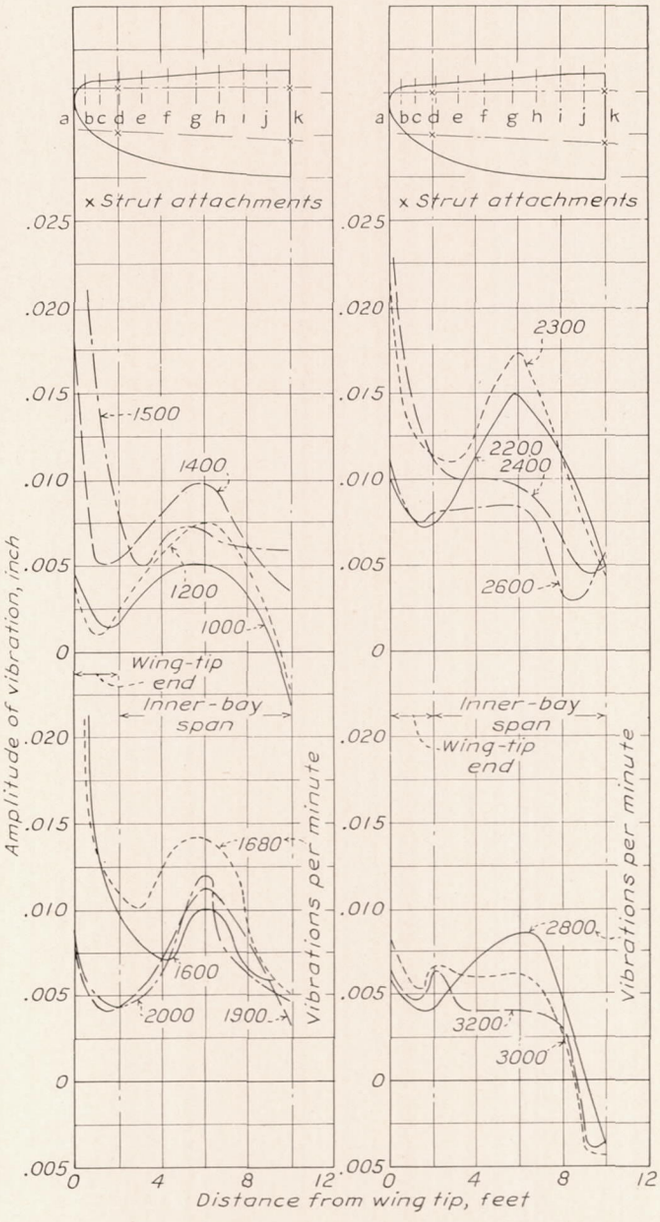


FIGURE 20.—Elastic curves of the front spar of the lower wing of the PW–9 airplane at different frequencies.

Although no tests were conducted to determine the second principal mode of vibration in flexure in the transverse direction, these are believed to be at slightly lower frequencies than that in the vertical direction, as the stiffness of the structure is usually made somewhat less in these directions.

Similar considerations apply to the design of the aircraft wings. It would seem from these tests that

the vibrations transmitted to the fuselage because of resonant responses in the wings are of sufficient magnitude to be a source of discomfort to passengers, although they may not endanger the structure. In biplanes there is a considerable likelihood that the principal modes of vibration of the several wing spans will be distributed over a wide range of frequencies. Considerable caution must therefore be exercised in designing the wings to avoid a resonant response by any of their spans with exciting sources in the engine or propeller.

In general, in the design of the aircraft structure, it is necessary first to design for strength and lightness,

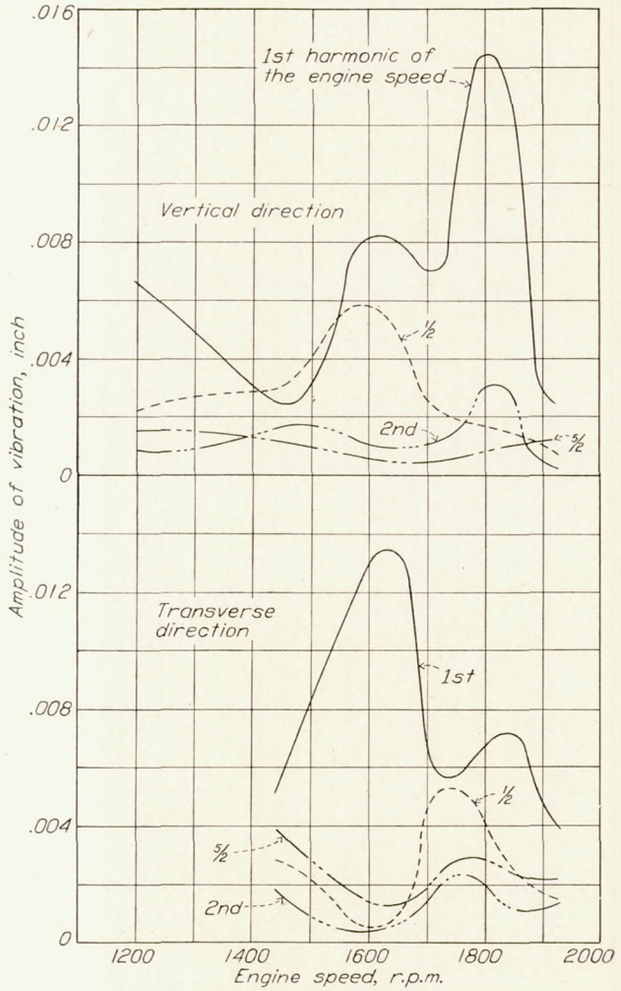


FIGURE 21.—Vibration in the rear cockpit of the N2Y–1 airplane in flight.

to obtain the minimum possible weight, and then to determine the natural modes of vibration of the various members of the structure by computation or by other means. If, in normal operating conditions of the aircraft, the frequency of any of these modes synchronizes with any of the exciting frequencies, the logical course is to change the design of the member, provided that the change does not conflict too much with other requirements.

Flight tests with vibration recorders in rear cockpit of N2Y–1.—The results of tests made with the N2Y–1 are shown in figures 21 and 22. This airplane was similar to the one that was used to determine the vibration response of the fuselage given in figure 16.

In figure 21 the vibration of the upper longeron in the rear cockpit in the vertical and transverse directions is given. The flights were made with the airplane in a level attitude. The principal amplitude peaks are at 1,630 and 1,800 revolutions per minute. Since the frequency of these peaks coincides with the engine speed, the cause of the excessive vibration at these speeds can confidently be attributed to the engine or propeller.

The amplitude peak at 1,800 revolutions per minute was determined by visual observations to be due to a flexure in the inner bay span of the wings in

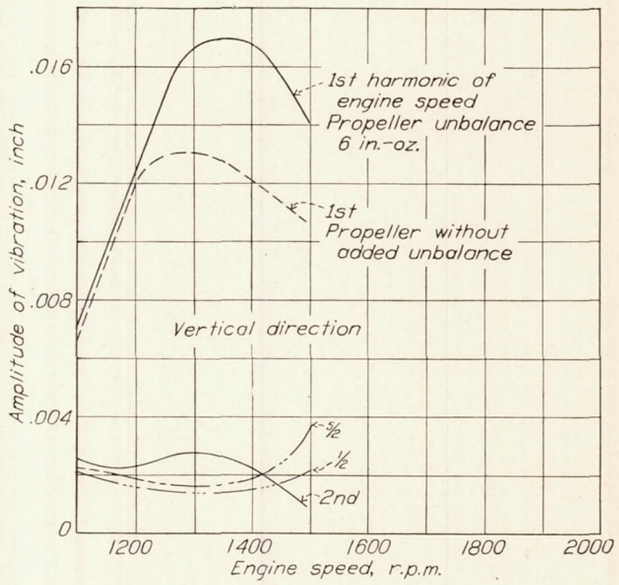


FIGURE 22.—Vibration of engine block of N2Y–1 airplane on the ground with propeller unbalance.

the vertical direction. In the description of the vibration-response tests for an N2Y–1 airplane it was explained that a resonant vibration existed in the wings at 1,950 vibrations per minute. In this airplane, which is very similar to the one used to obtain the results of figure 16, a corresponding mode of vibration cannot be far from the above frequency. The peak of the first harmonic at 1,630 is probably mostly due to a torsional disturbance, for the vibration is noticeable in both the vertical and transverse directions.

Figure 21 shows that in addition to the first, only the 1/2, 2, and 5/2 harmonics are of importance. The other amplitudes were much less than 0.001 inch. The 1/2 and 5/2 harmonics must be attributed to the engine torque alone, as it is not possible for inertia forces or other couples of these orders to exist in either the engine, the propeller, or any other source in level flight. A resultant couple in torsion of the 1/2 order

is usually caused by the unequal contribution of mean torque by each cylinder of the engine, which, in turn, is the result either of faulty ignition of one or more cylinders or of difference in the mixture strength. A resultant couple of the 5/2 order, inherent in the torque of this engine, is large for a 5-cylinder engine because the torque components of this order are in phase. The second harmonics can be attributed mostly to second-order inertia unbalance due to articulation, which is known to exist in all radial engines.

Neither the second nor the 5/2 harmonics cause any appreciable disturbance in the rear cockpit. The 1/2 harmonic shows amplitude peaks at engine speeds of 1,600 and 1,740 revolutions per minute in the vertical and transverse directions, respectively. The increase of vibration at these engine speeds is a possible indication that the 1/2-order resultant engine torque is in

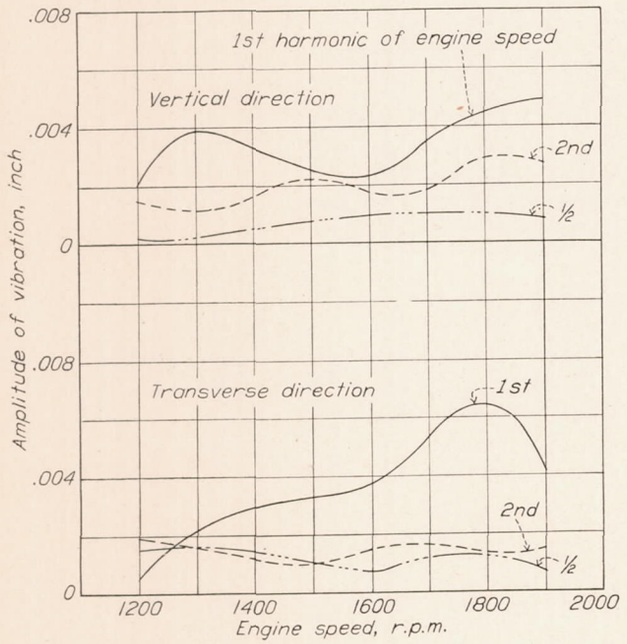


FIGURE 23.—Vibration forward of the front cockpit of the NY–2 airplane in flight.

or near resonance with a resonant vibration of some part of the aircraft structure in torsion—possibly the fuselage, of which the frequency of the fundamental mode of vibration is usually low, as seen from the PW–9 tests.

In figure 22 is shown the vibration of the engine block with added propeller static unbalance. The tests were run on the ground with vibrograph mounted on the engine block so as to record vibration in the vertical direction. There is a peak of vibration of the first harmonic, for both unbalances used, at about 1,300 revolutions per minute. Apparently, a mode of free vibrations of the engine mounting occurs at that frequency. The amplitudes of the other harmonics are not appreciable.

The magnitude of vibration over the entire operating range of this airplane is far beyond the 0.004-inch limit of amplitude for comfortable riding, set by the test results obtained by Constant (reference 1). The explanation of repeated complaints that the N2Y–1 airplane was "rough" and somewhat uncomfortable in the air becomes obvious upon examination of the results of figures 21 and 22.

Flight tests with vibration recorders in front and rear cockpits of NY–2.—In figures 23 and 24 are shown the results obtained with the vibrograph mounted in the front and rear cockpits, respectively, of the NY–2 airplane. The vibrations shown in these figures are those of the upper left longeron.

In flexure, the vibration is hardly noticeable at 1,800 revolutions per minute. The maximum amplitude of 0.006 inch is recorded in the front cockpit and is that of the first harmonic. There appears to be a peak in the amplitude of the first harmonic in the vertical direction at 1,900 revolutions per minute and in the transverse direction at 1,800 revolutions per

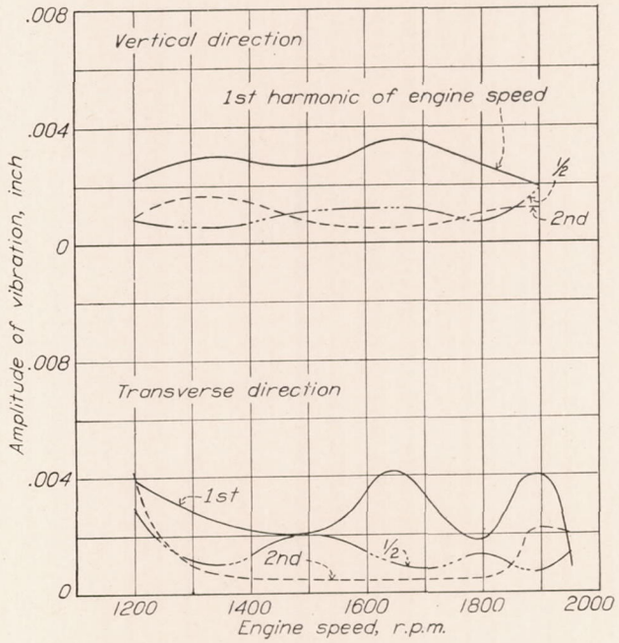


FIGURE 24.—Vibration in the rear cockpit of the NY–2 airplane in flight.

minute, but the vibration is well damped and no definite statement can be made as to whether a mode of vibration in any part of the airplane structure in flexure is of either of these frequencies.

In figure 25 are shown the results obtained with the torsiograph in the front and rear cockpits of the NY–2. The instrument was mounted on the bulkhead along the thrust axis of the airplane. There are amplitude peaks of the 1/2 and second harmonics at 1,800 revolutions per minute and of the first harmonic at 1,900 revolutions per minute or slightly above.

The pilot reported that the vibration in this airplane was noticeable at speeds between 1,800 and 1,900 revolutions per minute and became somewhat disagreeable as the airplane was nosed slightly down at engine speeds slightly above 1,900 revolutions per minute.

Flight tests with vibration recorders on tail of NY–2.—A number of tests of a preliminary nature were made with the vibration recorders mounted on the tail of the airplane. The information obtained thus far is given in tables III and IV. The angles of attack were estimated from aerodynamic data with a probable error of less than 1°. A complete analysis of the records by Fourier series was found somewhat impracticable

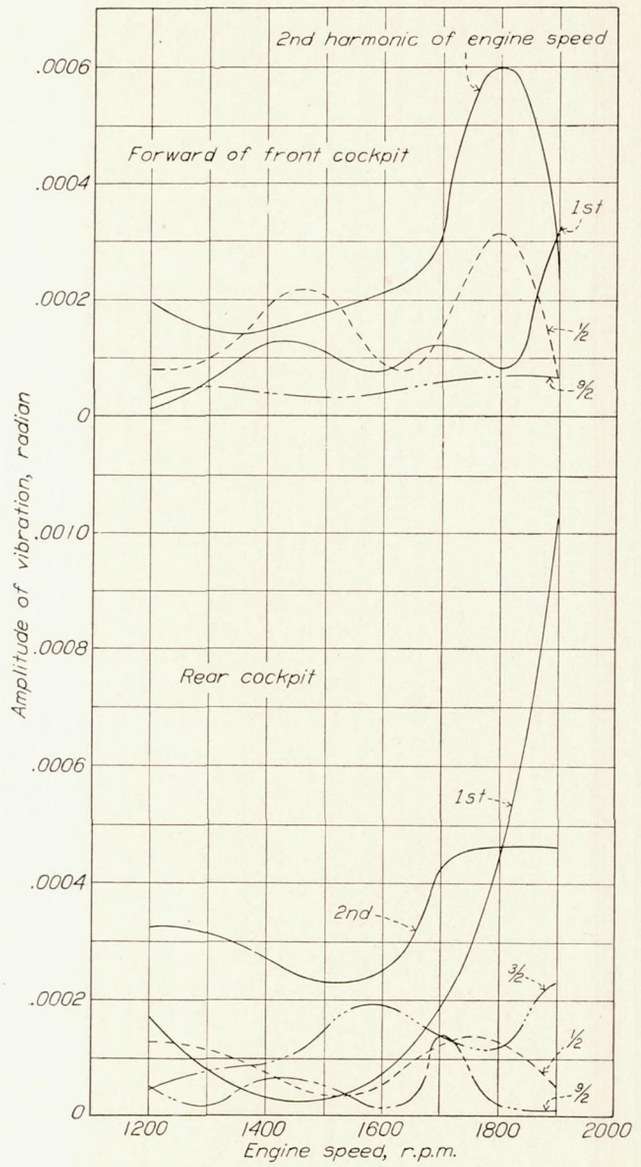


FIGURE 25.—Torsional vibration in the front and rear cockpits of the NY–2 airplane in flight.

and unnecessary in these tests in which the determination of the frequencies of vibration was of more importance. The few superimposed harmonics could be readily separated by an approximate graphical method. The frequencies could be determined to within an accuracy of 10 vibrations per minute. The amplitudes given contain a possible error of 25 percent.

An examination of the frequency column in both the tables shows that a frequency of about 670 vibrations per minute appears often in both flexure and torsion. This vibration made its appearance and was damped out within half a second, sometimes in less time. Since it appears in flexure as well as in torsion, it is reasonable to conclude that it is a torsional frequency of free vibrations of the horizontal tail plane or possibly the fundamental mode of vibration of the fuselage.

In the comparison of the frequencies, air speeds, and angles of attack it is noticed that, in addition to the vibrations whose frequencies are multiples of engine speed, forced vibrations of frequencies in the range of 400 to 650 vibrations per minute make their appearance. These vibrations can only be of aerodynamic origin. Supplementary tests have shown these low-frequency vibrations to be a maximum near the stern post, but decreasing rapidly forward of the tail; and they were hardly noticeable immediately back of the cockpit.

CONCLUSIONS

The results of the ground and flight tests with specially designed vibration instruments to determine the response of the several parts of the airplane structure to the forces encountered in flight may be summarized as follows:

1. In ground tests with a vibrator, the frequencies of the fundamental modes of vibration of the fuselages of the two biplanes tested were found to lie between 500 and 750 vibrations per minute for both flexure and torsion. The frequency of the second principal mode of vibration of the fuselages in flexure in the vertical direction was found to be close to 3,000 per minute. The wings vibrated approximately as multispan beams having the supports close to the strut attachments, and the natural frequencies of the several spans ranged from about 800 to 2,800 per minute.

2. In flight tests with airplanes equipped with radial engines, vibrations at, or forward of, the rear cockpit were found to be of frequencies that were multiples of the engine speed, a positive indication of their origin being in the engine or the propeller. In addition, vibrations of low frequencies were found to be present in the part of the fuselage back of the cockpits. These vibrations were of maximum intensity near the stern post. Some of these low frequencies were not multiples of the engine speed, apparently being of aerodynamic origin.

3. Ground and flight tests indicate that unless resonance occurs between a free mode of vibration of the airplane structure and the exciting source, the magnitude of vibration is usually of no practical consequence. It follows that considerable reduction of vibration may be obtained by avoiding a resonance of

a mode of free vibration of the structure and the exciting sources. Because of the large number of independent degrees of freedom of the several parts of the structure, however, the reduction can only be accomplished after close study of the response characteristics of the particular type of structure in connection with a knowledge of the exciting forces.

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *May 10, 1934.*

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TABLE I.—TYPES OF TESTS AND AIRPLANES USED

Airplane tested	Type of tests	Vibration measured
PW-9; pursuit biplane; 12-cylinder Curtiss engine.	Stationary tests with vibrator on engine block, *c.g.* of airplane, and tail.	Along lower longeron and front spar of both wings.
	Tests with wings on and without the wings	Along lower longeron.
N2Y-1; training biplane; 5-cylinder radial Kinner engine.	Stationary tests with vibrator on engine block	Do.
	Ground tests engine running	Of engine block.
NY-2; training biplane; 9-cylinder radial, Wright engine.	Flight tests	In rear cockpit.
	do	In front cockpit.
	do	In rear cockpit.
	do	On tail.

TABLE II.—SUMMARY OF PRINCIPAL RESPONSES OF THE PW–9 STRUCTURE AS DETERMINED IN THE VIBRATOR TESTS

Member responding	Type of response	Frequency vibrations per minute	Mode of vibration
Fuselage	Flexure in vertical direction	600	Fundamental.
Fuselage with wings	do	3, 100	Second principal mode.
Fuselage without wings	do	2, 500	Do.
Do	Flexure in transverse direction	550	Fundamental.
Do	Torsion about fore-and-aft axis	500	Do.
Upper wing:			
Wing tip or outer bay span	Flexure in vertical direction	800	Do.
Inner bay span	do	2, 350	Do.
Middle span	do	2, 800	Do.
Lower wing:			
Outer bay span	do	1, 650	Do.
Inner bay span	do	2, 300	Do.
Trailing edge	Torsion about axis along spar	1, 400	Do.
Stabilizer	Torsion about axis transversely to fore-and-aft axis	1, 850	Do.

TABLE III.—NY–2 TAIL VIBRATIONS IN THE VERTICAL DIRECTION VIBROGRAPH TEST IN FLIGHT

Record no.	Approximate angle of attack	Air speed	Engine speed	Frequency, vibrations per minute	Amplitude of vibrations	Record no.	Approximate angle of attack	Air speed	Engine speed	Frequency, vibrations per minute	Amplitude of vibrations
	°	*M.p.h.*	*R.p.m.*		*Inch*		°	*M.p.h.*	*R.p.m.*		*Inch*
1	0.0	75	1,650	1,650	0.005	11	11.4	40	1,200	2,400	0.003
				825	.014					1,200	.008
				670	.015					600	.012
2	−.5	80	1,700	1,700	.006	12	11.4	40	1,500	1,500	.005
				850	.010					750	.012
3	1.2	70	1,600	1,600	.008	13	10.2	43	1,300	2,600	.004
				800	.007					1,300	.005
4	−1.0	84	1,700	1,700	.008					650	.008
				850	.011	14	10.8	43	1,480	1,480	.007
				670	.017					740	.012
5	−1.7	92	1,800	1,800	.010	15	13.0	38	1,500	1,500	.003
				900	.013					750	.013
6	3.2	61	1,500	1,500	.005	16	[2] 16.0	37	1,600	1,600	.0025
				750	.008					800	.016
				670	.011					600	.028
7	[1] −2.6	107	1,900	1,900	.005					400	.027
				950	.012	17	[2] 16.0	37	1,600	800	.013
				670	.034					550	.044
8	[1] −2.7	109	1,900	1,900	.008	18	[3] 18.0	36	1,600	800	.010
				950	.012					640	.013
				640	.018	19	[4] −3.4	127	1,950	1,940	.009
				490	.024					970	.013
9	8.2	46	1,300	2,600	.004					550	.032
				1,300	.008	20	[4] −2.4	104	1,200	600	.008
				650	.013					480	.015
10	11.4	40	1,200	1,200	.008					390	.021
				600	.016	21	[4] −2.4	104	1,200	1,200	.011
				670	.024					600	.019
										400	.021

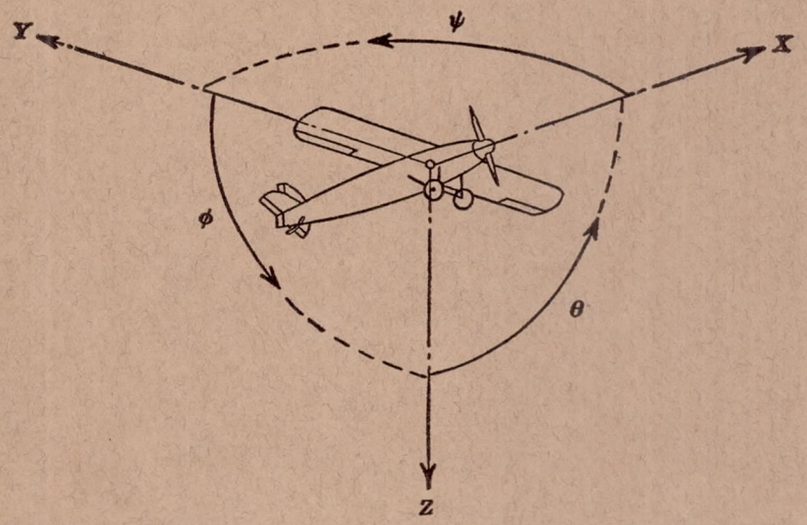
[1] Slight diving angle.
[2] Stall, or slightly above.
[3] Above stall.
[4] Dive.

TABLE IV.—TORSIONAL VIBRATIONS OF THE NY–2 TAIL[1] TORSIOGRAPH TEST IN FLIGHT

Record no.	Approximate angle of attack	Air speed	Engine speed	Frequency, vibrations per minute	Amplitude of vibrations	Record no.	Approximate angle of attack	Air speed	Engine speed	Frequency, vibrations per minute	Amplitude of vibrations
	°	*M.p.h.*	*R.p.m.*		*Radian*		°	*M.p.h.*	*R.p.m.*		*Radian*
1	0.6	73	1,600	670	0.00063	8	−3.0	115	1,810	600	0.00059
2	−1.0	84	1,700	680	.00091	9	−3.2	121	1,800	620	.00063
3	−1.6	92	1,800	680	.00031	10	−3.6	132	1,900	480	.00123
4	10.2	43	1,400	700	.00063	11	−3.4	127	1,600	560	.00086
5	13.0	38	1,510	640	.00118	12	−4.0	150	1,600	550	.00078
6	16.0	37	1,600	580	.00137					570	.00088
7	16.0	37	1,560	780	.00101	13	−4.0	150	1,800	600	.00080
				620	.00084					620	.00080
				480	.00096						

[1] Amplitudes of most frequency multiples of engine speed are not included because their low magnitudes were not easily determinable.

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Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	Rolling	L	$Y \longrightarrow Z$	Roll	ϕ	u	p
Lateral	Y	Y	Pitching	M	$Z \longrightarrow X$	Pitch	θ	v	q
Normal	Z	Z	Yawing	N	$X \longrightarrow Y$	Yaw	ψ	w	r

Absolute coefficients of moment

$C_l = \frac{L}{qbS}$ (rolling) $\quad C_m = \frac{M}{qcS}$ (pitching) $\quad C_n = \frac{N}{qbS}$ (yawing)

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D, Diameter

p, Geometric pitch

p/D, Pitch ratio

V', Inflow velocity

V_s, Slipstream velocity

T, Thrust, absolute coefficient $C_T = \frac{T}{\rho n^2 D^4}$

Q, Torque, absolute coefficient $C_Q = \frac{Q}{\rho n^2 D^5}$

P, Power, absolute coefficient $C_P = \frac{P}{\rho n^3 D^5}$

C_s, Speed-power coefficient $= \sqrt[5]{\frac{\rho V^5}{P n^2}}$

η, Efficiency

n, Revolutions per second, r.p.s.

Φ, Effective helix angle $= \tan^{-1}\left(\frac{V}{2\pi r n}\right)$

5. NUMERICAL RELATIONS

1 hp. = 76.04 kg-m/s = 550 ft-lb./sec.
1 metric horsepower = 1.0132 hp.
1 m.p.h. = 0.4470 m.p.s.
1 m.p.s. = 2.2369 m.p.h.

1 lb. = 0.4536 kg.
1 kg = 2.2046 lb.
1 mi. = 1,609.35 m = 5,280 ft.
1 m = 3.2808 ft.