REPORT No. 501

RELATIVE LOADING ON BIPLANE WINGS OF UNEQUAL CHORDS

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SUMMARY

It is shown that the lift distribution for a biplane with unequal chords may be calculated by the method developed in N.A.C.A. Technical Report No. 458 if corrections are made for the inequality in chord lengths. The method is applied to four cases in which the upper chord was greater than the lower and good agreement is obtained between observed and calculated lift coefficients.

INTRODUCTION

In reference 1 it was shown that for conventional biplane arrangements the lift coefficient for the upper wing is given by

$$C_{L_U} = C_L \pm \Delta C_{L_U} \tag{1}$$

and the lift coefficient for the lower wing by

$$C_{L_L} = C_L \pm \Delta C_{L_L} \tag{2}$$

where C_L is the biplane lift coefficient and ΔC_{L_U} and ΔC_{L_L} are lift coefficient increments for the upper and lower wings, respectively. It was also shown that ΔC_{L_U} and ΔC_{L_L} are connected by the relation

$$\Delta C_{L_L} = -\Delta C_{L_U} \times \frac{S_U}{S_L} \tag{3}$$

where S_U and S_L are the areas of the upper and lower wings, respectively.

ΔC_{L_U} is given by an equation of the form

$$\Delta C_{L_U} = K_1 + K_2\, C_L \tag{4}$$

where the constant K_1 is a function of gap, chord, wing thickness, stagger, decalage, and overhang and the constant K_2 is a function of stagger, gap, chord, span, decalage, and overhang. Equations and charts in reference 1 enable the determination of K_1 and K_2 for any biplane with equal chords. Application of this method to biplanes with extreme differences in chords and spans has indicated considerable discrepancies between the calculated and observed values. A further study of the problem in the light of some rather limited test data indicates that a simple correction for the ratio of the wing chords will bring the calculated and experimental values into excellent agreement and that a chord correction should therefore be incorporated as an integral part of the general method.

In the discussion that follows the symbols used will be the same as in reference 1.

THE EFFECT OF WING CHORD ON K_1

When there is no stagger, decalage, or overhang the value of K_1 in equation (4) is a function of the ratio of wing thickness to gap. This basic value of K_1 may be designated K_{10}. It is due principally to the restriction in area which increases the velocity and decreases the static pressure between the wings. The curve of K_{10} against the ratio of wing thickness to gap given in figure 1 is the same as figure 9 of reference 1. This

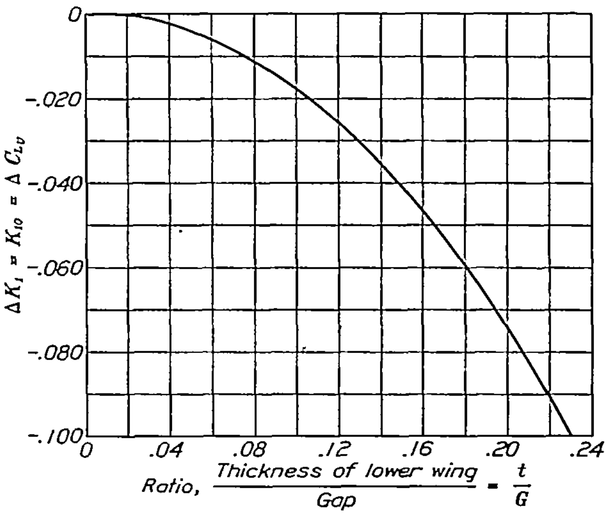

FIGURE 1.—Effect of wing thickness and gap on K_1 at zero lift and zero stagger for equal chords.

curve was based on biplanes of equal chords but it will apply to any biplane if the thickness of the lower wing is used in determining the ratio t/G and if the necessary correction is made to transfer the coefficient to the upper wing.

The first condition is met by using the gap-chord ratio referred to the chord of the lower wing, so that

$$\frac{t}{G} = \left(\frac{t}{c}\right) \div \left(\frac{G}{c_L}\right).$$

The transfer on the coefficient basis requires division by the ratio of areas, lower to upper (S_L/S_U) since by definition the C_L for the cellule is so adjusted between the individual values. This means, however, that the correction must be made on the basis of the relative chords since the value of K_{10} assumes no overhang. Consequently, to find the value of K_{10} for a biplane having upper and lower chords of c_U and c_L, read the

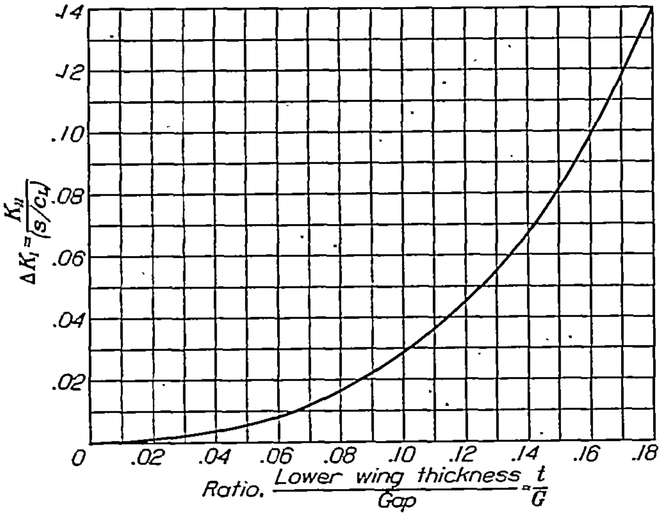

FIGURE 2.—Effect of stagger on K_1 for equal chords.

value of ΔK_1 from figure 1 and correct according to the ratio of the chords, or

$$K_{10} = \Delta K_1 \times \left(\frac{c_L}{c_U}\right) \tag{5}$$

The effect of stagger on K_1 may be designated K_{11} and it is given in figure 2 by the curve of $\Delta K_1/s$ as a

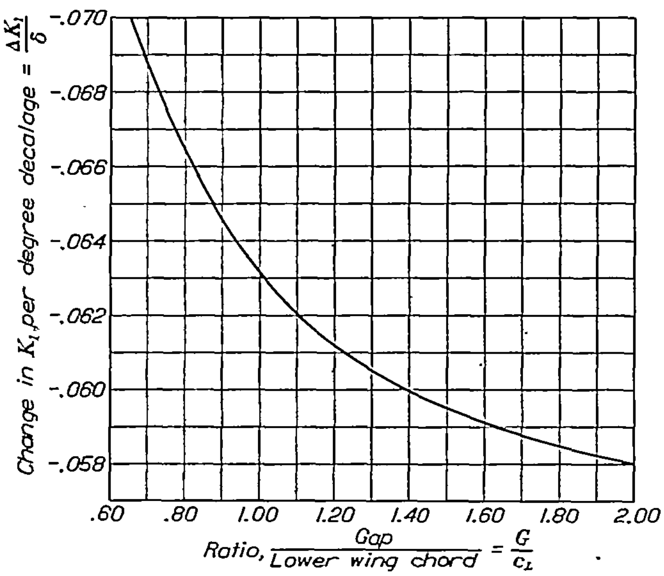

FIGURE 3.—Effect of decalage on K_1 for equal chords.

function of t/G or the ratio of thickness to gap. The curve of figure 2 is the same as that of figure 10 in reference 1 and is based on biplanes of equal chord. It may be applied to any biplane if the value of t/G is based on lower wing thickness and a chord correction is made as in the calculation of K_{10}. The stagger should be measured between the ¼ chord points at zero lift and referred to the chord of the lower wing.

The value of K_{11} is then given by

$$K_{11} = \frac{\Delta K_1}{(s)} \times s \times \left(\frac{c_L}{c_U}\right) \tag{6}$$

where s is the stagger in terms of the chord of the lower wing.

The effect of decalage on K_1 varies with gap-chord as shown by figure 3, which is the same as figure 17 of reference 1. This curve is based on biplanes with equal chords but it may be applied to any biplane if the chord correction is used. As before, a chord correction is equivalent to an area correction since the

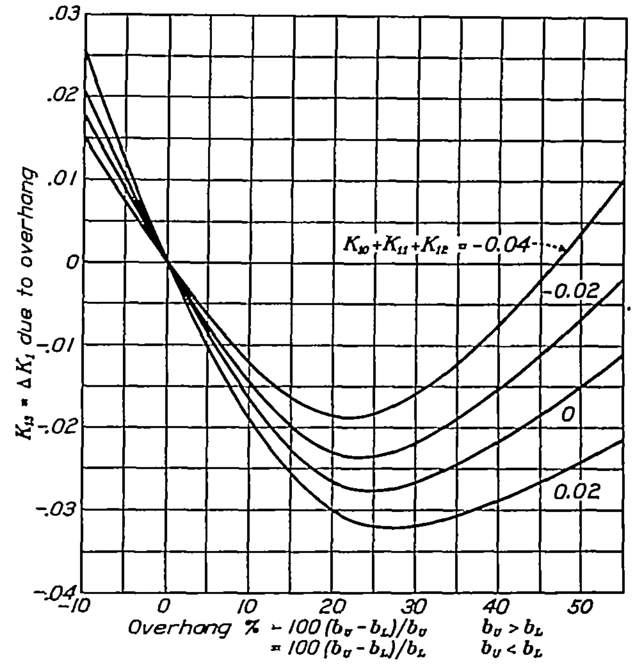

FIGURE 4.—Effect of overhang on K_1 for biplanes with equal chords.

effect of overhang is separated. Denoting the effect of decalage on K_1 by K_{12} it is found by

$$K_{12} = \frac{\Delta K_1}{\delta} \times \delta^\circ \times \left(\frac{c_L}{c_U}\right) \tag{7}$$

where $\frac{\Delta K_1}{\delta}$ is read from figure 3.

The effect of overhang on K_1 may be denoted by K_{13}. In figure 21 of reference 1, contour curves were given of K_1 against overhang. These curves were to be used by entering at zero overhang with the value of K_1 obtained by adding $K_{10}+K_{11}+K_{12}$ and passing along the appropriate contour to the desired overhang. In this manner the value of K_{13} was not determined directly. Since K_{13} is subjected to the same chord correction as the preceding factors, it is desirable to replot the data as in figure 4, giving the value of K_{13} directly. For any biplane the value of K_{13} is then obtained from

$$K_{13} = \Delta K_1 \times \left[\frac{c_L}{c_U}\right] \tag{8}$$

The final value of K_1 is now obtained by addition of the four factors

$$K_1 = K_{10} + K_{11} + K_{12} + K_{13} \qquad (9)$$

THE EFFECT OF WING CHORD ON K_2

The basic value of K_2 in equation (4) is determined by stagger. For biplanes with individual wings of aspect ratio 6 and equal chords, zero decalage and no overhang it was shown in reference 1 that

$$K_{20} = 0.050 + 0.17\left(\frac{s}{c}\right) \qquad (10)$$

The influence of aspect ratio and gap-chord ratio is combined in a factor F_2 which may be read from figure 5. Figure 5 is the same as figure 12 in reference 1. In finding F_2 the gap-chord ratio should be based on the lower wing.

The effect of decalage on K_2 may be denoted by K_{21}. In reference 1 it was shown that for the equal chord biplane

$$K_{21} = +0.0186\ \delta^\circ \qquad (12)$$

where δ is the angle between the zero lift lines of the wings, considered positive when these intersect forward of the leading edge. When the chord lengths differ K_{21} should be corrected accordingly, to give

$$K_{21} = 0.0186\ \delta^\circ \times \left[\frac{c_L}{c_U}\right] \qquad (13)$$

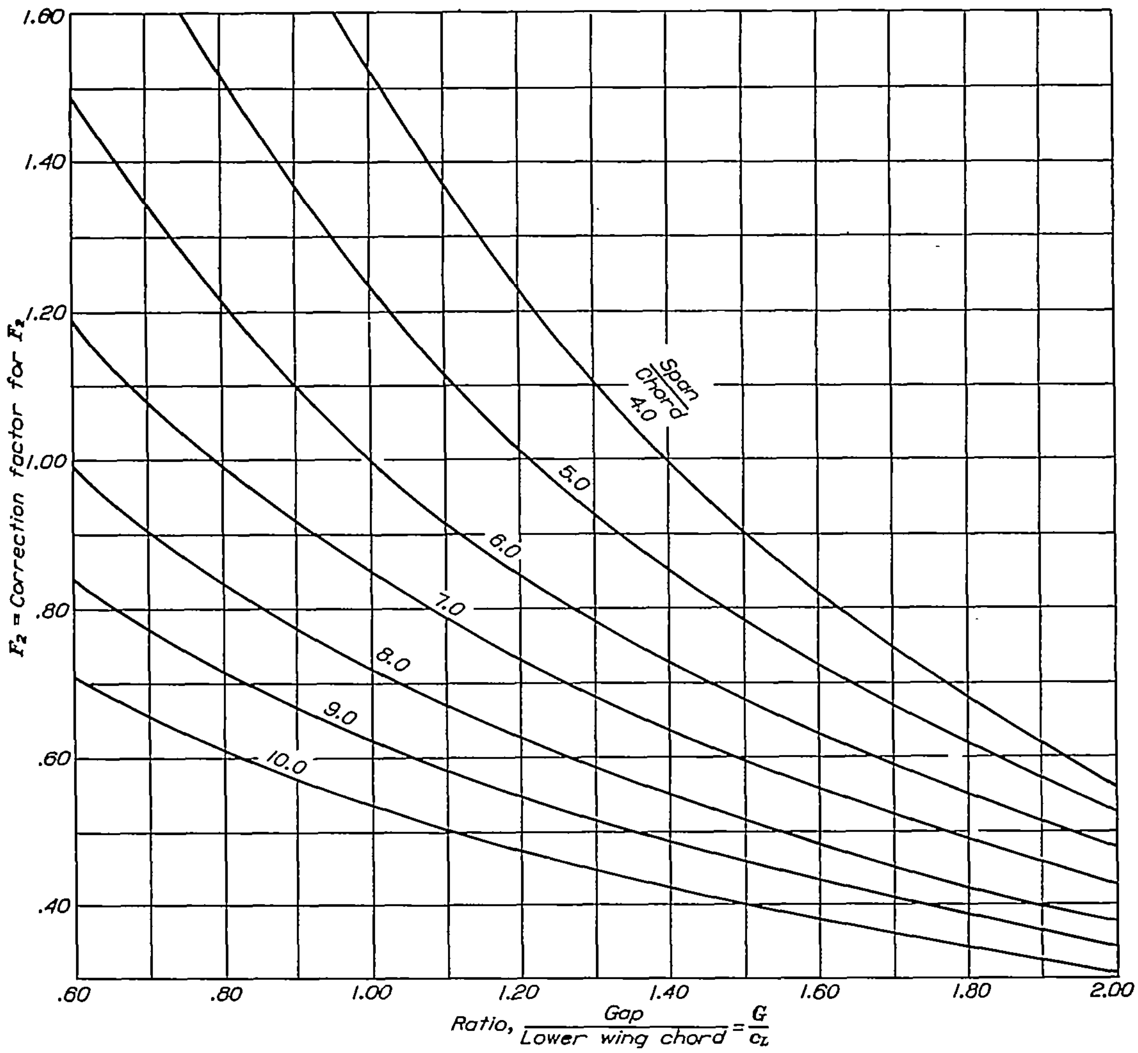

FIGURE 5.—Effect of $\frac{\text{gap}}{\text{chord}}$ and aspect ratio on K_2 for equal chords.

To apply this equation to any biplane the stagger should be measured between the ⅓ chord points at zero lift and referred to the chord of the lower wing c_L. The basic value of K_{20} should then be multiplied by the chord ratio or

$$K_{20} = \left[0.050 + 0.17\left(\frac{s}{c_L}\right)\right] \times \frac{c_L}{c_U} \qquad (11)$$

The effect of stagger on K_{20} varies with the aspect ratio of the individual wings and with the gap-chord ratio.

In reference 1 the effect of overhang on K_2 was given for the equal-chord biplane by figure 21 which consisted of a series of contour lines of K_2 plotted against overhang. To use these curves it was necessary to find $K_2 = (F_2 \times K_{20}) + K_{21}$ for zero overhang and passing along this contour to the desired overhang. The actual value of the effect of overhang which may be denoted by K_{22} was not directly determined. Since K_{22} is subjected to the same chord correction as the previous factors, it is desirable to replot figure 21 of reference 1 so that K_{22} can be read directly, as in

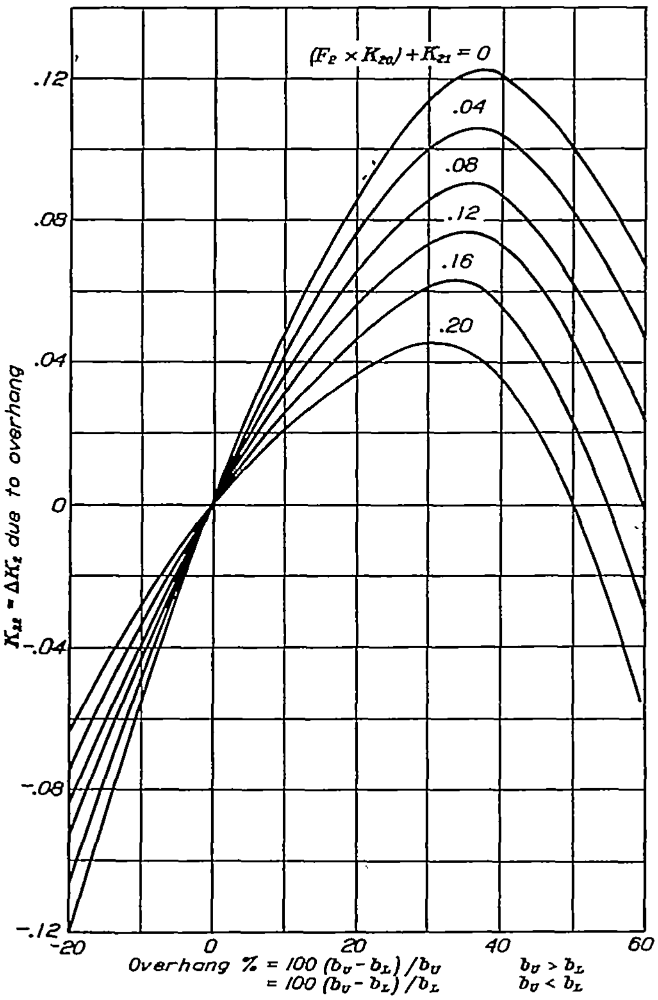

FIGURE 6.—Effect of overhang on K_2 for equal chords.

figure 6. For any biplane the value of K_{22} is obtained by

$$K_{22} = \Delta K_2 \times \left[\frac{c_L}{c_U}\right] \tag{14}$$

where ΔK_2 is the overhang correction for equal chords as read from figure 6.

The value of K_2 in equation (4) is then obtained by addition of the three corrected terms

$$K_2 = [F_2 \times K_{20}] + K_{21} + K_{22} \tag{15}$$

COMPARISON OF CALCULATED AND OBSERVED DATA

Available load distribution tests on biplanes with unequal chords are limited to four cases. Reference 2 reports tests on a biplane having the following characteristics:

Upper wing: span $b_U = 36$ inches, chord $c_U = 6$ inches

Lower wing: span $b_L = 24$ inches, chord $c_L = 4$ inches

Gap: $G = 4\frac{1}{2}$ inches, section R.A.F. 15

Stagger 20° on leading edge at $\alpha = 0°$, or 1.12 inches between $\frac{1}{3}$ chord point at zero lift.

$$\text{Overhang} = \frac{36-24}{36} = 0.33$$

From the above:

$$\frac{s}{c_L} = \frac{1.12}{4.0} = 0.28 \qquad \frac{G}{c_L} = \frac{4.5}{4.0} = 1.125$$

$$\frac{t}{c} = 0.070 \qquad \frac{t}{G} = \frac{0.070}{1.125} = 0.062$$

$$\frac{c_L}{c_U} = \frac{2}{3}$$

From figure 1 and equation (5)

$$K_{10} = -0.005 \times \frac{2}{3} = -0.0033$$

From figure 2 and equation (6)

$$\frac{\Delta K_1}{(s/c_L)} = 0.010 \qquad K_{11} = 0.010 \times 0.28 \times \frac{2}{3} = +0.0019$$

$$K_{12} = 0$$

From figure 4 and equation (8)

$$\Delta K_1 = -0.025 \qquad K_{13} = (-0.025) \times \frac{2}{3} = -0.0167$$

Hence $K_1 = -0.0033 + 0.0019 - 0.0167 = -0.018$

From equation (11)

$$K_{20} = [0.050 + (0.17 \times 0.28)] \times \frac{2}{3} = 0.065$$

From figure 5, $F_2 = 0.90$

$$F_2 \times K_{20} = 0.90 \times 0.065 = 0.058 \qquad K_{21} = 0$$

From figure 6, $\Delta K_2 = 0.096$

From equation (14), $K_{22} = 0.096 \times \frac{2}{3} = 0.062$

Hence, $K_2 = 0.058 + 0.062 = 0.120$

and

$$\Delta C_{L_U} = -0.018 + 0.120\ C_L \tag{16}$$

The test data are as follows:

Angle of attack						
α-----	−4.25°	−0.25°	3.75°	7.75°	11.75°	15.75°
Upper wing						
C_{N_U}---	−.122	+.178	.488	.756	1.016	1.160
Lower wing						
C_{N_L}---	−.076	+.140	.374	.550	.704	.900
Biplane						
C_N----	−.108	+.166	.453	.692	.920	1.080
ΔC_{N_U}----	−.014	+.012	.035	.064	.096	.080

These values of ΔC_{N_U} are plotted against C_N on figure 7. Two points calculated from equation (16) are given on figure 7 and it will be noted that the agreement is satisfactory. The equation of ΔC_{N_U} from the experimental data is $\Delta C_{N_U} = -0.007 + 0.106\ C_N$ which may be compared with equation (16).

Reference 3 reports tests on a biplane differing from the one preceding only in the overhang, the spans being equal in this case. The aspect ratio for the upper wing was 6, for the lower wing 9, average 7.5.

$K_{10} = -0.0033$ as for first arrangement.
$K_{11} = +0.0019$ as for first arrangement.
$K_{12} = 0$ (no decalage).
$K_{13} = 0$ (no overhang).
$\therefore K_1 = -0.0014$.
$K_{20} = 0.065$ as for first arrangement.

From figure 5, $F_2 = 0.73$.

$F_2 \times K_{20} = 0.73 \times 0.065 = 0.0475$.
$K_{21} = 0$ (no decalage).
$K_{22} = 0$ (no overhang).
$\therefore K_2 = 0.048$.

and

$$\Delta C_{L_U} = -0.0014 + 0.048\ C_L \tag{17}$$

The test data obtained are as follows:

Angle of attack α	$-4°$	$0°$	$4°$	$8°$	$12°$	$16°$
Upper wing C_{N_U}	−0.098	+0.174	0.438	0.696	0.968	1.056
Lower wing C_{N_L}	−.074	+.170	.412	.618	.818	.946
Biplane C_N	−.088	+.173	.428	.665	.908	1.012
ΔC_{N_U}	−.010	+.001	.010	.031	.060	.044

These values of ΔC_{N_U} are plotted against C_N on figure 8. The equation of the line through the test points is $\Delta C_{N_U} = -0.006 + 0.054\ C_N$ which should be compared with equation (17). The agreement is again satisfactory.

Two special biplane tests have been made at Wright Field by the Army Air Corps and reported in

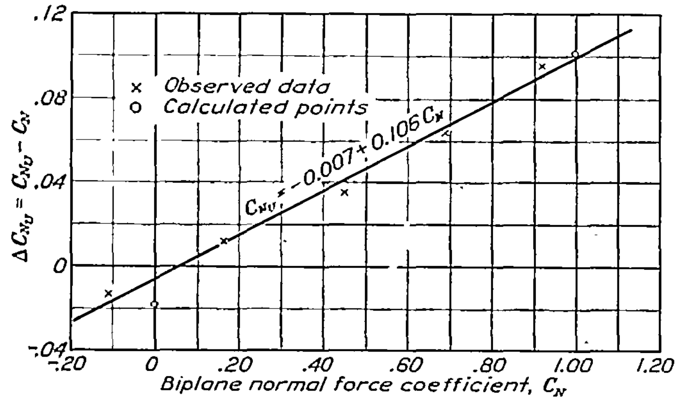

FIGURE 7.—Biplane test R. & M. 997, Br. A.R.C.

reference 4. The biplane used in the first test had the following characteristics:

Upper wing: span $b_U = 36$ inches, chord $c_U = 6$ inches.
Lower wing: span $b_L = 18$ inches, chord $c_L = 3$ inches.
Gap: $G = 4\frac{1}{2}$ inches, wing section Clark Y.
Stagger 3.63 inches measured on L.E. at $\alpha = 0°$ or 3.06 inches measured between $\frac{1}{3}$ chord points at zero lift.

$$\text{Overhang} = \frac{36-18}{36} = 0.50.$$

From the above data:

$$\frac{s}{c_L} = \frac{3.06}{3.0} = 1.02 \qquad \frac{G}{c_L} = \frac{4.5}{3.0} = 1.5$$

$$\frac{t}{c} = 0.117 \qquad \frac{t}{G} = \frac{0.117}{1.5} = 0.078$$

$$\frac{c_L}{c_U} = \frac{1}{2}$$

From figure 1 and equation (5)

$$K_{10} = -0.009 \times \tfrac{1}{2} = -0.0045$$

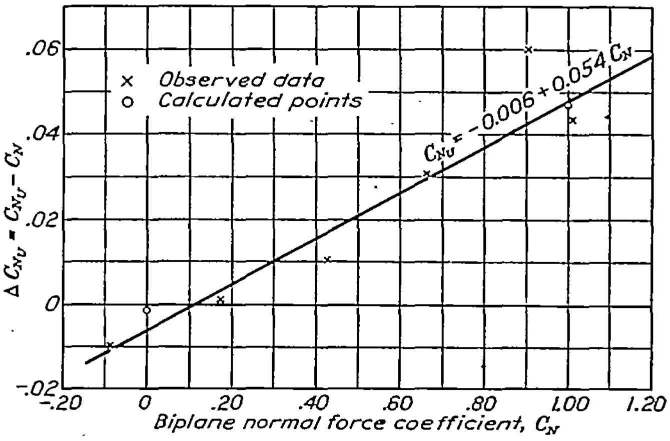

FIGURE 8.—Biplane test R. & M. 1098, Br. A.R.C.—no overhang.

From figure 2 and equation (6)

$$\frac{\Delta K_1}{\left(\frac{s}{c_L}\right)} = 0.017 \qquad K_{11} = 0.017 \times 1.02 \times \tfrac{1}{2} = +0.0087$$

$K_{12} = 0$ (no decalage)

From figure 4 and equation (8)

$$\Delta K_1 = -0.0170 \qquad K_{13} = -0.0170 \times \tfrac{1}{2} = -0.0085$$

Hence $K_1 = -0.0045 + 0.0087 - 0.0085 = -0.0043$

From equation (11)

$$K_{20} = [0.050 + (0.17 \times 1.02)] \times \tfrac{1}{2} = +0.112$$

From figure 5, $F_2 = 0.67$

$$F_2 \times K_{20} = 0.67 \times 0.112 = +0.075 \qquad K_{21} = 0$$

From figure 6, $\Delta K_2 = 0.064$

From equation (14) $K_{22} = 0.064 \times \frac{1}{2} = 0.031$

Hence, $K_2 = 0.075 + 0.031 = 0.106$

and

$$\Delta C_{L_U} = -0.0043 + 0.106\ C_L \tag{18}$$

From equation (3)

$$\Delta C_{L_L} = +0.0172 - 0.424\ C_L \tag{19}$$

The report tabulates the lift coefficients at two points only. These are compared with the calculated values below:

	From test		Calculated	
Upper wing C_{L_U}	1.096	0.135	1.102	0.131
Lower wing C_{L_L}	0.618	0.081	0.593	0.087
Biplane C_L	1.000	0.122	1.000	0.122
ΔC_{L_U}	0.096	0.013	0.102	0.009
Ratio $C = \frac{C_{L_U}}{C_{L_L}}$	1.77	1.67	1.86	1.50

The biplane used in the second test reported in reference 4 had the following characteristics:

Upper wing: span $b_U=36$ inches, chord $c_U=6$ inches

Lower wing: span $b_L=18$ inches, chord $c_L=3$ inches

Gap: $G=4\frac{1}{2}$ inches, wing section Clark Y

Stagger 0 measured on L.E. at $\alpha=0°, -0.47$ inch measured between $\frac{1}{3}$ chord points at zero lift.

$$\text{Overhang}=\frac{36-18}{36}=0.50$$

From the above data:

$$\frac{s}{c_L}=-\frac{0.47}{3.0}=-0.16 \qquad \frac{G}{c_L}=\frac{4.5}{3.0}=1.5$$

$$\frac{t}{c}=0.117 \qquad \frac{t}{G}=\frac{0.117}{1.5}=0.078 \qquad \frac{c_L}{c_U}=\frac{1}{2}$$

From figure 1 and equation (5)

$$K_{10}=-0.009\times\frac{1}{2}=-0.0045$$

From figure 2 and equation (6)

$$\frac{\Delta K_1}{\left(\frac{s}{c_L}\right)}=0.017 \qquad K_{11}=0.017\times(-0.16)\times\frac{1}{2}=-0.0014$$

$$K_{10}+K_{11}=-0.0059$$

$K_{12}=0$ (no decalage)

From figure 4 and equation (8)

$$\Delta K_1=-0.012 \qquad K_{13}=-0.012\times\frac{1}{2}=-0.006$$

Hence, $K_1=-0.0045-0.0014-0.006=-0.012$

From equation (11)

$$K_{20}=[0.050+0.176(-0.16)]\times\frac{1}{2}=+0.0114$$

From figure 5, $F_2=0.67$

$$F_2\times K_{20}=0.67\times0.0114=0.008$$

$$K_{21}=0 \text{ (no decalage)}$$

From figure 6 and equation (14)

$$\Delta K_2=0.100 \qquad K_{22}=0.100\times\frac{1}{2}=0.050$$

Hence,

$$K_2=0.008+0.050=0.058$$

and

$$\Delta C_{L_U}=-0.012+0.058\ C_L \tag{20}$$

From equation (3)

$$\Delta C_{L_L}=+0.048-0.232\ C_L \tag{21}$$

The report tabulates the lift coefficients at two points only. These are compared with the calculated values below:

	From test		Calculated	
Upper wing C_{L_U}	1.025	0.110	1.031	0.108
Lower wing C_{L_L}	0.776	0.126	0.805	0.135
Biplane C_L	0.985	0.113	0.985	0.113
ΔC_{L_U}	0.040	−0.003	0.046	−0.005
Ratio $C=\frac{C_{L_U}}{C_{L_L}}$	1.32	0.87	1.28	0.80

CONCLUSIONS

Based on the limited test data available, it is concluded that the relative loading of any biplane having equal or unequal chords, is given with satisfactory accuracy by the method outlined in this report. As applied, the method has considered only the normal case of the unequal chord biplane in which $c_U>c_L$. It would be desirable to have some lift-distribution data for cases in which $c_U<c_L$.

The greatest need, however, is for a series of tests to determine more exactly the effect of overhang as given on figures 4 and 6. While these curves have been prepared with care, they are based on four arrangements only and the extrapolation is subject to considerable error.

As pointed out in reference 1, it is also desirable that special tests be made to determine more accurately the curves of figures 1 and 2, giving the effect of wing thickness and stagger on the value of K_1 for an orthogonal biplane.

In any future tests the data should extend to maximum negative lift.

Bureau of Aeronautics,
Navy Department,
Washington, D.C., *May 18, 1934.*

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