REPORT No. 711

ANALYSIS AND PREDICTION OF LONGITUDINAL STABILITY OF AIRPLANES

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SUMMARY

An analysis has been made of the longitudinal stability characteristics of 15 airplanes as determined in flight. In the correlation of satisfactory and unsatisfactory characteristics with determined values, the derivative $d\delta_e/d\alpha$ that expresses the ratio of static-restoring moments to elevator-control moments was found to represent most nearly the stability characteristic appreciated by the pilots. In the derivative, δ_e is the elevator angle and α is the angle of attack. This derivative may be readily determined in flight or in the wind tunnel by measuring the elevator angles required for trim throughout the angle-of-attack range. It affords, moreover, a means of comparing airplane characteristics because the stick movement and the type of stick-force gradient are dependent on the magnitude of $d\delta_e/d\alpha$.

The analysis was extended to study the effects of various design features on the observed stability characteristics. In this connection an expression was derived by means of which $d\delta_e/d\alpha$ may be computed on the basis of general airplane dimensions for the propeller-off condition. Comparison of computed values, or comparable wind-tunnel determinations with engine-idling flight conditions, shows a powerful destabilizing effect of the idling propeller. An empirical expression based on the propeller dimensions brought the values obtained in the propeller-off tests and the computations into good agreement with the flight values. General power-on effects observed in the various airplanes tested are discussed.

Design charts and data are included that show the effects on longitudinal stability of relative positions of wing and tail, fuselage size and location, engine nacelles, and horizontal-tail arrangements.

Also included is a discussion of desirable numerical values of $d\delta_e/d\alpha$. For design a value of $d\delta_e/d\alpha$ of 0.5 is suggested.

INTRODUCTION

A few years ago the National Advisory Committee for Aeronautics instituted a program for the study of the flying qualities of airplanes. The primary purpose of the research program was to determine quantitatively what constitutes satisfactory flying qualities and the stability and control requirements that an airplane can be expected to fulfill. Accordingly, the investigation has consisted mainly in determining these characteristics in flight tests of various airplanes. These airplanes were made available largely by the Army and more recently by private companies in accordance with requests by the Civil Aeronautics Authority.

Drawings of the airplanes tested in flight are given in figure 1 and pertinent dimensions are listed in table I.

Stability and control characteristics have now been determined for 15 airplanes of varied types. The data obtained have not only shown what constitutes satisfactory flying qualities but also show, by a proper analysis, how various design features influence the observed flying qualities.

The present report presents the results of such an analysis as regards the longitudinal-stability and control characteristics of the various airplanes tested.

DEFINITION OF LONGITUDINAL STABILITY

The longitudinal-stability characteristics of an airplane are conveyed to the pilot as the variation of stick force and stick position with air speed or angle of attack. The pilot's opinion of the stability is therefore governed not only by the static pitching-moment characteristics $dC_m/d\alpha$ of the airplane but also by the power of the elevators $dC_m/d\delta_e$ and their hinge-moment characteristics. Accordingly, a logical criterion for longitudinal stability is defined as the ratio of static-restoring moment to elevator-control moment $d\delta_e/d\alpha$ because both the stick movement that the airplane experiences with a changing angle of attack and the stick-force gradient depend on this derivative.

Specific values of this derivative desirable for the various types of airplane are still unestablished. Flight tests indicate definitely, however, that the derivative should never be negative or of so low a value that a reversal of either the stick movement or the stick force occurs. Preliminary analysis of the stick-force data shows that, for conventional tail arrangements, a value of $d\delta_e/d\alpha$ of approximately 0.2 is required to insure stick-free stability because of normal elevator-floating tendencies with angle-of-attack changes. Stick movement is, of course, very nearly proportional to $d\delta_e/d\alpha$. On airplanes intended for high maneuverability, large values of $d\delta_e/d\alpha$ have been found to be desirable provided that they are obtainable without heavy control forces. For airplanes of this category, $d\delta_e/d\alpha$ should be of such a value that at least a 4-inch movement of the top of the stick is required to change from a low to a high angle of attack in accelerated maneuvers.
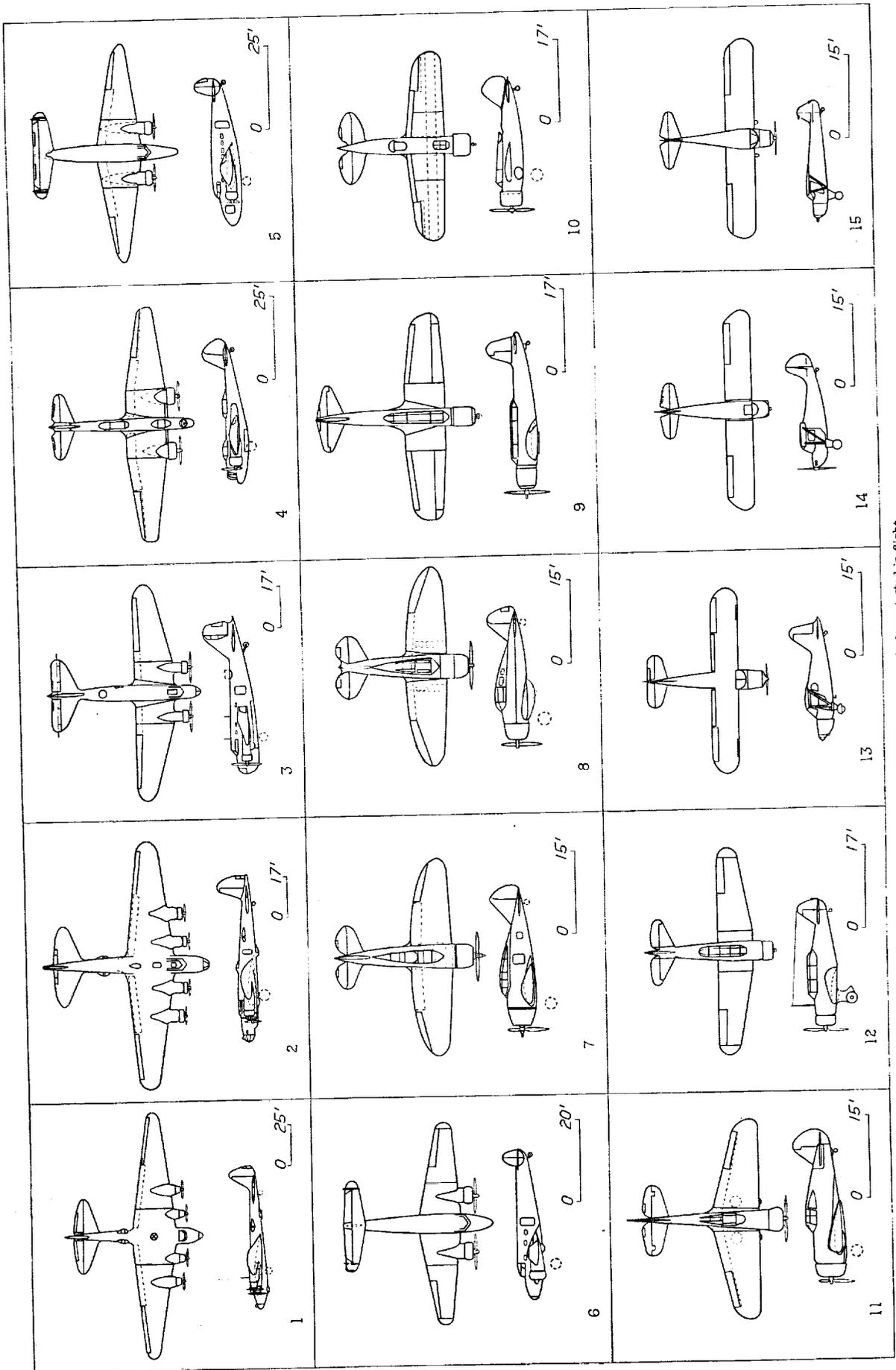
FIGURE 1.—Line drawings of airplanes tested in flight.

Airplanes of appreciably lower stick travel have been found to be very susceptible of inadvertent stalling in maneuvers.

No upper limit for $d\delta_e/d\alpha$ has been indicated except by considerations of maximum control requirements, such as in landing with flaps extended. It may also be true that for airplanes of the transport and the heavy-bomber type a sufficient hinge-moment reduction cannot be obtained to allow appreciably greater values of $d\delta_e/d\alpha$ than are representative at the present time. For reasons such as these, specific values cannot be recommended with assurance although all flight data indicate that large values of $d\delta_e/d\alpha$ are in themselves desirable provided that they are not obtained at the expense of acceptable control-force characteristics.

Additional discussion concerning numerical values of $d\delta_e/d\alpha$ is included later in this report in connection with suggested design practice.

STABILITY EQUATION

Preliminary attempts to correlate the stability characteristics determined in flight for the engine-idling condition with those computed on a basis of wing-and-tail theory alone indicated the necessity of considering the destabilizing effects of the fuselage, the nacelles, and the idling propeller. In the initial development of the stability equation the effect of the idling propellers is ignored.

Consideration of the forces and the moments acting on an airplane without propeller leads to the following expression for $d\delta_e/d\alpha$:

$$\frac{d\delta_e}{d\alpha}=\frac{1}{\tau}\left[\left(1-\frac{d\epsilon}{d\alpha}\right)+\frac{S_w d\frac{dC_L}{d\alpha_w}-K_f w_f^2 L_f-K_f N w_n^2 L_n}{\frac{q_t}{q_0}l_t S_t\frac{dC_{N_t}}{d\alpha_t}}\right]$$

where

τ elevator effectiveness factor $\left(\dfrac{dC_{N_t}/d\delta_e}{dC_{N_t}/d\alpha_t}\right)$

$\dfrac{d\epsilon}{d\alpha}$ rate of change of downwash over horizontal tail with angle of attack

S_w wing area, including section through fuselage and ailerons, square feet

d horizontal distance from aerodynamic center of wing to airplane center of gravity, feet

$\dfrac{dC_L}{d\alpha_w}$ slope of wing lift-coefficient curve, per radian

K_f fuselage and engine nacelle-moment factor

w_f maximum fuselage width, feet

L_f over-all fuselage length, feet

N number of engine nacelles in a multiengine airplane

w_n maximum engine nacelle width, feet

L_n over-all length of engine nacelle, estimated to be streamline body, feet

$\dfrac{q_t}{q_0}$ ratio of dynamic pressure over horizontal tail to free-stream dynamic pressure (0.9)

l_t horizontal distance from airplane center of gravity to elevator hinge line, feet

S_t total horizontal tail area including section through fuselage, square feet

$\dfrac{dC_{N_t}}{d\alpha_t}$ slope of curve of normal-force coefficient for horizontal tail, per radian

and

A aspect ratio

r factor in expression for slope of normal-force curve for tail surfaces with end plates

The defined symbols that occur in equation (1), the figures that are to be used for their evaluation, and the references from which the data for the figures were taken are listed in table II.

TABLE II.—REFERENCES FOR EVALUATION OF SYMBOLS USED IN EXPRESSION FOR $d\delta_e/d\alpha$

Symbol	Figure number for evaluation	Source of data, reference number
L_f	2	
w_n	2	
L_n	2	
l_t	2	
d	2	
w_f	2	
τ	3	1
$\dfrac{d\epsilon}{d\alpha}$	4, 5, 6	2
$\dfrac{dC_L}{d\alpha_w}$	7	1
$\dfrac{dC_{N_t}}{d\alpha_t}$	[a] 7	1
K_f	8	3

[a] For horizontal tails with vertical tails mounted as end plates, see figure 9.
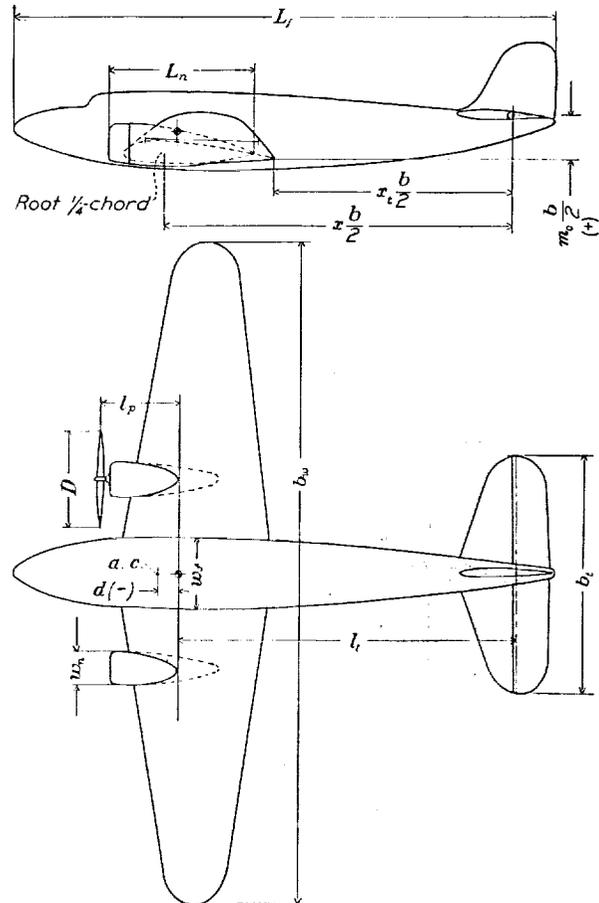
FIGURE 2.—Dimensions used in stability calculations. Distances measured at angle of zero lift.

Most of the quantities in equation (1) may be computed by the use of the figures listed in table II. Some explanation of the use of the charts in figures 4, 5, and 6 is, however, necessary for the evaluation of $d\epsilon/d\alpha$.

The procedure to be followed in the determination of $d\epsilon/d\alpha$ may be summarized in the following steps:

FIGURE 3.—Variation of elevator effectiveness factor τ with ratio of elevator area back of hinge line to horizontal tail area. Data taken from reference 1.

(1) Determine which group of the four groups given in figure 4 most nearly approximates the design taper ratio and aspect ratio. This step is not critical because corrections must later be applied for differences in aspect ratio and taper ratio.

(2) In the selected group, using values of m_0, x, and x_t as defined in figure 2, with linear interpolations for x and x_t wherever necessary, determine $d\epsilon/d\alpha$.

(3) In figure 5 (a) plot $d\epsilon/d\alpha$ as found in step (2) against the group taper ratio and from this point follow the contour line to the design taper ratio and read off a new value of $d\epsilon/d\alpha$.

(4) In figure 5 (b) determine the correction to $d\epsilon/d\alpha$ for the difference in the aspect ratio between the group value and the design value at the design taper ratio. Add this correction $\Delta(d\epsilon/d\alpha)$ to the value of $d\epsilon/d\alpha$ determined in step (3) if the design value of the aspect ratio is lower than the group value; subtract this correction if the design value of the aspect ratio is higher than the group value.

(5) From figure 6 determine the correction factor to be multiplied by the total value of $d\epsilon/d\alpha$ to convert it from the absolute value at the center line to an average value over the horizontal tail span.

With the following additional dimensions an example of the computation of $d\epsilon/d\alpha$ for airplane 1 (fig. 1) is indicated in figures 4, 5, and 6.

FIGURE 4.—Variation of rate of change of downwash over horizontal tail with angle of attack $d\epsilon/d\alpha$ with location of horizontal tail m_0 for various wing aspect and taper ratios. Data taken from reference 2.
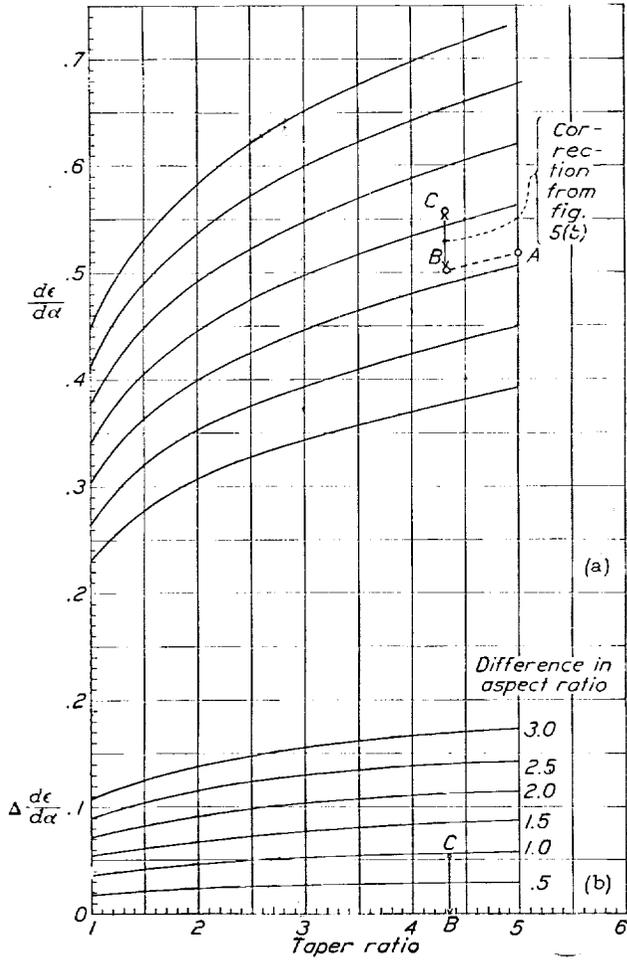
(a) Corrections to downwash for variations in taper ratio.
(b) Corrections to downwash for variations in aspect ratio.

FIGURE 5.—Corrections to downwash for variations in wing aspect and taper ratios. Data taken from reference 2.
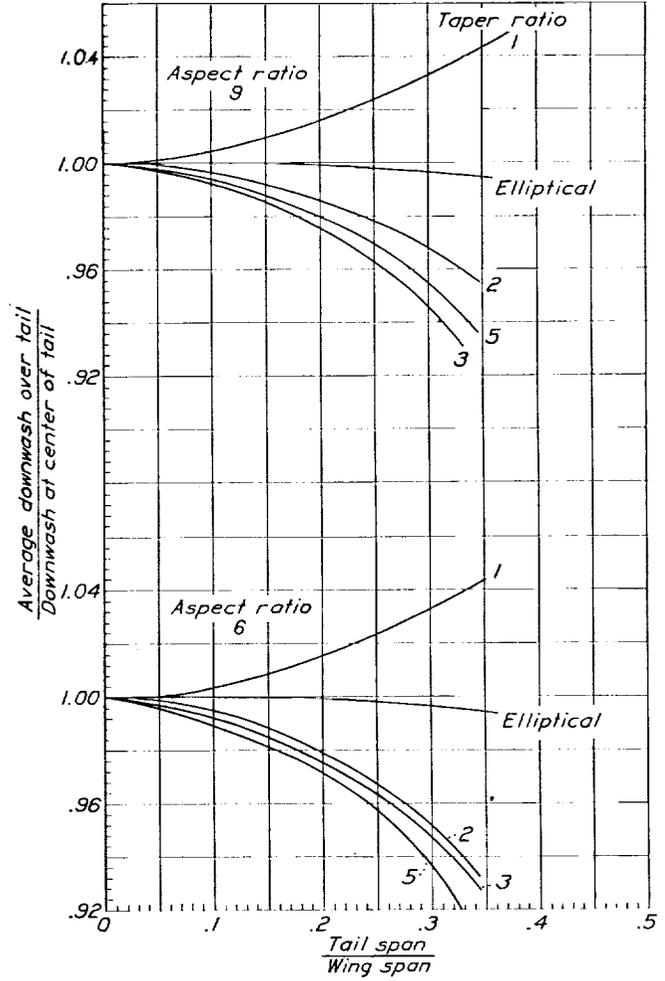
FIGURE 6.—Correction factor to convert downwash at center of horizontal tail to average downwash over tail. Data taken from reference 2.
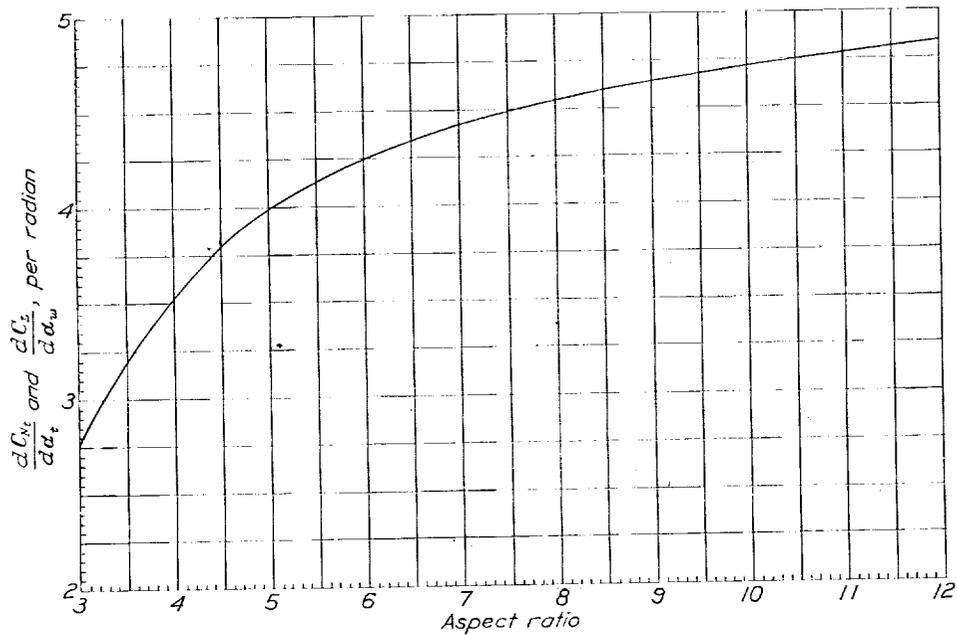
FIGURE 7.—Variation of $dC_{N_t}/d\alpha_t$ and $dC_L/d\alpha_w$ with aspect ratio for conventional sections. Data taken from reference 1.

Horizontal distance from wing root trailing edge to elevator hinge line, $(x_t)(b/2)$ ------------feet------------30.5

Horizontal distance from wing root ¼-chord point to elevator hinge line, $(x)(b/2)$------------feet------------51.9

Vertical distance from wing root trailing edge to elevator hinge line, $(m_0)(b/2)$------------feet------------6.3

Then

$$m_0 = \frac{6.3}{74.5} = 0.084$$

$$x_t = \frac{30.5}{74.5} = 0.41$$

$$x = \frac{51.9}{74.5} = 0.696$$
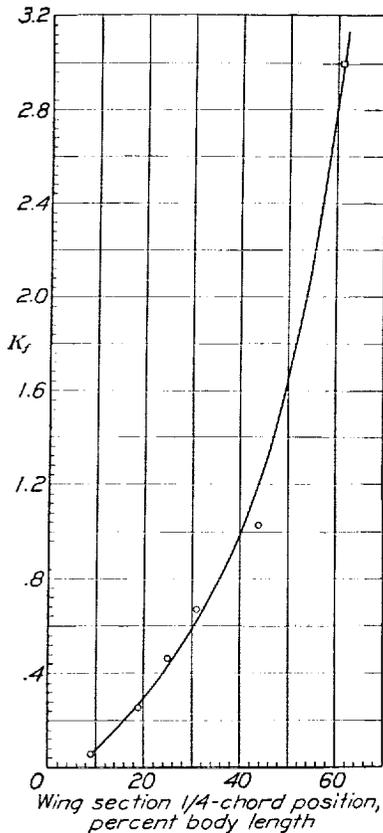
FIGURE 8.—Variation of fuselage and engine nacelle moment factor K_f with relative location of wing and body. Data taken from reference 3.

These values determine point A in figure 4 (step 2). If point A is transposed to figure 5 (a) and the contour lines are followed, as indicated by the dashed line, point B is located (step 3). The increment BC, transferred from figure 5 (b) to figure 5 (a), establishes a value of $d\epsilon/d\alpha$ of 0.56 (step 4).

Interpolation is necessary in figure 6 to determine the correction factor of 0.95, which, when multiplied by the value of $d\epsilon/d\alpha$ of 0.56, yields 0.53 (step 5).

The discontinuities evident in the curves of figure 4 arise from the fact that the empirical expression for $d\epsilon/d\alpha$ used in constructing the curves contained an absolute value of α for which a mean value was assumed. The discontinuities then occur for tail locations that bring the horizontal tail on the wing wake center at the assumed angle.

Actually, because the wake has a finite thickness and because other locations near those corresponding to the discontinuities will similarly affect the tail, the curves should be faired in some way. This fairing has not been attempted because the results would be misleading; if the tail traverses the wing wake at some portion of the angle-of-attack range, it will be operating at different values of $d\epsilon/d\alpha$ before and after passing through the wake. Strictly, then, no single faired value of $d\epsilon/d\alpha$ is applicable throughout the speed range for these tail locations.
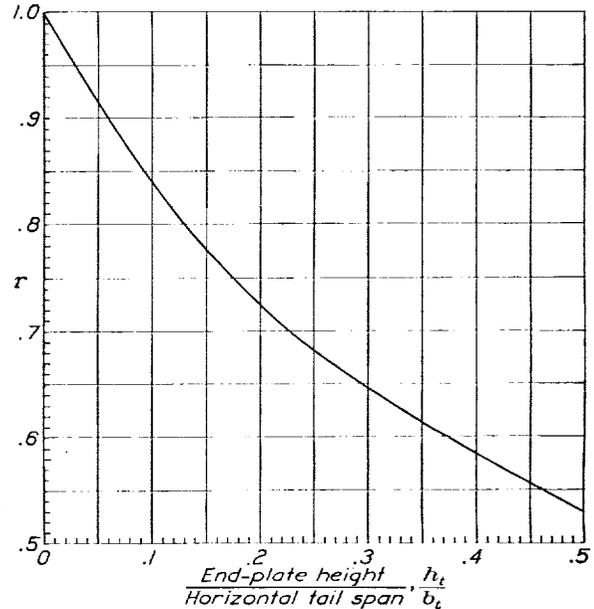
FIGURE 9.—Correction for $dC_{N_t}/d\alpha_t$ of horizontal tail with end plates.

$$\frac{dC_{N_t}}{d\alpha_t} = \frac{5.7}{1+1.8\frac{r}{.1}}$$
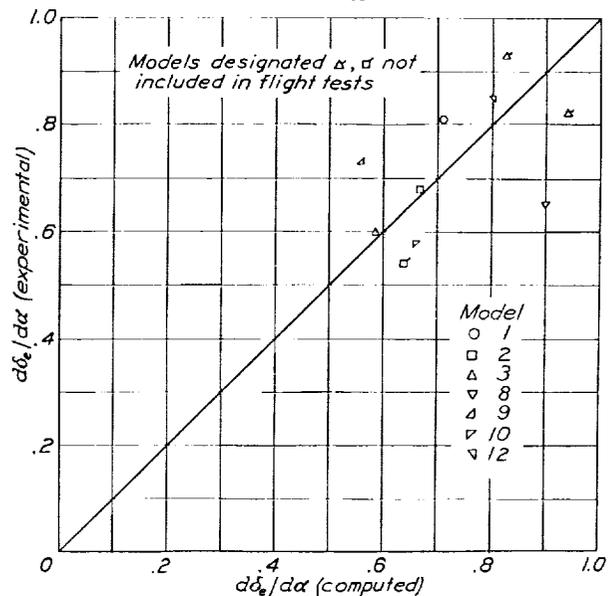
FIGURE 10.—Comparison of experimental and computed values of stability, propeller off.

The charts of figure 4 also indicate qualitatively the effects on $d\delta_e/d\alpha$ of wing dimensions and wing and tail arrangements. Thus the increased stability of a high-wing-monoplane arrangement over that of a low-wing monoplane may be partly attributed to the relative vertical location of the wing and the horizontal tail.

A comparison of propeller-off wind-tunnel results with computed values of $d\delta_e/d\alpha$ is given in figure 10. The feasibility of predicting propeller-off stability characteristics by means of the expression given is indicated by the relatively small deviations of the data from the line of perfect agreement.

Values of $d\delta_e/d\alpha$ for wind-tunnel models for which only curves of $dC_m/d\alpha$ are available may be calculated from the following expression:

$$\frac{d\delta_e}{d\alpha} = -\frac{S_w c_w}{\tau l_t S_t \frac{q_t}{q_0}\frac{dC_{Nt}}{d\alpha_t}}\frac{dC_m}{d\alpha}$$

where c_w is the mean aerodynamic chord of the wing.

CORRELATION WITH FLIGHT RESULTS

A comparison of the computed values of $d\delta_e/d\alpha$ with the values obtained in gliding flight with idling propellers is shown in figure 11. A destabilizing moment is clearly evident in the lack of agreement.
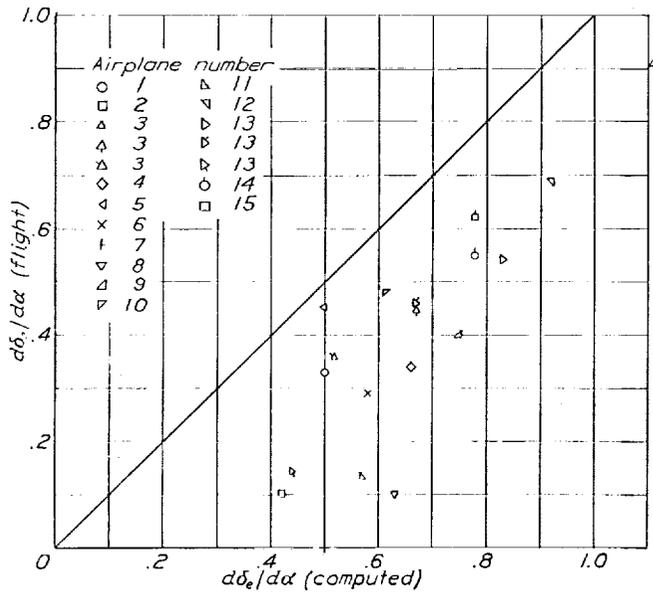
FIGURE 11.—Comparison of flight stability as determined with idling propellers with computed stability, propeller off.

It was found that the effect of the idling propeller could be represented by a term composed of the propeller dimensions with a constant empirical coefficient:

$$-\frac{N_p K_p D^2 l_p}{\tau \frac{q_t}{q_0} l_t S_t \frac{dC_{Nt}}{d\alpha_t}}$$

with previously undefined symbols listed below:

K_p empirical propeller coefficient (0.65)

N_p number of propellers

D diameter of propeller

l_p horizontal distance from center of gravity to propeller plane, feet

D and l_p are given in figure 2.

The value of $d\delta_e/d\alpha$ for gliding flight with idling propellers then becomes:

A comparison of the values of $d\delta_e/d\alpha$ as computed from this expression with those determined in flight is given in figure 12.
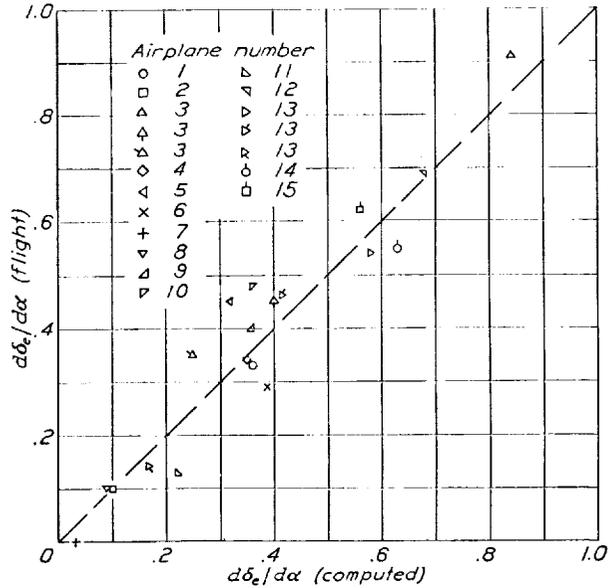
FIGURE 12.—Comparison of computed stability with flight stability as determined with idling propellers.

LIMITATIONS OF METHOD

Some deviation from perfect agreement may be noted in figure 12. Several possible explanations for these irregularities are:

(1) The effect of the vertical center-of-gravity position has been neglected. This procedure makes the method somewhat conservative for high-wing monoplanes, particularly at high angles of attack. For low-wing and midwing monoplanes the vertical center of gravity is usually sufficiently close to the aerodynamic center to be neglected, at least for attitude changes of the magnitude experienced in power-off flight. For power-on flight, however, the vertical center-of-gravity position has a marked effect because of the large attitude changes experienced in going from low to high angles of attack.

(2) As calculated, the value of $d\epsilon/d\alpha$ is substantially independent of C_L. Under certain conditions, usually near the stall, a local flow breakdown may make the computed results inapplicable. In many airplanes this breakdown results in values of $d\delta_e/d\alpha$ near the stall that are much greater than the values calculated or existing over the rest of the angle-of-attack range.

(3) All of the effects of the idling propeller cannot be adequately expressed by the simplified empirical term used. In this connection it should be appreciated that the value of the coefficient for the propeller-moment term was obtained for conventional tractor arrangements and may not be applicable for pusher or unconventional arrangements.

$$\frac{d\delta_e}{d\alpha} = \frac{1}{\tau}\left[\left(1-\frac{d\epsilon}{d\alpha}\right)+\frac{S_w d\frac{dC_L}{d\alpha}-K_f w_f^2 L_f - K_f N w_n^2 L_n - K_p N_p D^2 l_p}{\frac{q_t}{q_0} l_t S_t \frac{dC_{Nt}}{d\alpha_t}}\right]$$

(4) Because few data are available for the determination of K_f at the more rearward positions of the wing with respect to the body, these values are subject to modifications. In addition, no allowance was made for the cross-sectional shape or plan form of the fuselage and the nacelles in the calculation of their moments.

(5) The flight values of $d\delta_e/d\alpha$, although substantially constant over the normal-flight range, were subject to some individual variations. In the present analysis minimum values were always taken.

POWER EFFECTS

For all the airplanes tested, the power-on and the engine-idling values of $d\delta_e/d\alpha$ were substantially the same at low angles of attack. Without exception, low-wing monoplanes progressively lost stability with power on as the angle of attack increased. A large part of this effect was merely an increase in the power of the elevators as a result of the increase in q_t/q_0 that occurred. This conclusion is indicated by the fact that the stability lost was a function of the original power-off stability. Low-wing monoplanes, however, that were neutrally stable with engine idling or with power on at high speeds also lost stability, a representative amount for high angles of attack being approximately $d\delta_e/d\alpha = -0.2$.

On the basis of present knowledge, placing the center of gravity below the aerodynamic center appears to be the most direct method of limiting the stability loss due to power. This effect can be easily computed from considerations of the power-on attitude changes. High-wing monoplanes tested in flight showed relatively little stability loss due to power; midwing monoplanes were generally intermediate in this respect.

SUGGESTED DESIGN PRACTICE

Observations indicate that a value of $d\delta_e/d\alpha$ of approximately 0.5 should be set as a lower limit of desirable stability in the gliding condition. Experience with several pursuit-type airplanes has shown that, with conventional ratios of stick to elevator movement, this value of propeller-idling stability results in approximately 4 inches of stick movement in power-on maneuvers. With the center of gravity well forward, this value of $d\delta_e/d\alpha$ may be obtained with a small tail volume. This procedure is not recommended, however, because it results in undue sensitivity to changes in the center-of-gravity location and makes more critical the control requirements for three-point landings. It is better to use a large tail volume with normal center-of-gravity locations.

For large airplanes not required to maneuver, in which either visual or instrument references are always available to the pilot, this stability limit does not appear as essential. It is always desirable, however, that the stability be sufficiently great that a reversal of stick travel or stick force does not occur.

The value of $d\delta_e/d\alpha$ of approximately 0.5 is dictated not alone by the fact that large values are in themselves desirable; in addition, the reduction in stability to be anticipated with power on at low speeds should not diminish $d\delta_e/d\alpha$ to undesirably low values.

The methods of predicting longitudinal stability described in this report do not include the effects of wing flaps. For all the airplanes tested, however, the flap-down engine-idling conditions showed improved stability over that obtained with the flap up although with the flap down the destabilizing effect of power was in some cases greater than with the flap up.

The requirements for stick-free stability are met when the airplane has only one trim speed for a given trim tab setting, above which speed push forces are required and below which, pull forces are required. A positive gradient of stick force against angle of attack of a magnitude sufficient to minimize the effects of control friction should exist throughout the speed range. Preliminary investigation of the flight data indicates that a value of $d\delta_e/d\alpha$ of about 0.2 must be exceeded to obtain stick-free stability.

Other considerations, chiefly the ability to lower the tail to the three-point attitude for landing with flaps down, determine the upper limit for $d\delta_e/d\alpha$, although it should be apparent that, where large values of $d\delta_e/d\alpha$ are obtained by increased tail volume with a conventional amount of elevator control, no difficulty should occur.

CONCLUDING REMARKS

As a result of the foregoing analysis of flight data of longitudinal stability, the following conclusions may be stated:

1. For many purposes the criterion of longitudinal stability is logically taken as $d\delta_e/d\alpha$, the rate of change of elevator angle with angle of attack, where δ_e is the elevator angle and α is the angle of attack.

2. From only a knowledge of the basic airplane dimensions, $d\delta_e/d\alpha$ may be predicted for both propeller-off and propeller-idling conditions for conventional airplane designs.

3. The empirical factor that expresses the destabilizing effect of the idling propeller may be used to correct propeller-off wind-tunnel data.

4. For all airplanes tested the stability with the propeller idling and with power on at cruising and high speed were very nearly the same. Low-wing monoplanes progressively lost stability as the angle of attack increased to the stall and high-wing monoplanes tended to retain their propeller-idling stability.

5. For design a value of $d\delta_e/d\alpha$, with the propeller idling, of 0.5 is suggested to insure power-on stability and to give adequate stick movement for airplanes intended for high maneuverability.

6. The suggested values of $d\delta_e/d\alpha$ should be obtained by an adjustment of the effective tail volume with the normal center-of-gravity positions and the conventional amounts of elevator control.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,
    NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
        LANGLEY FIELD, VA., *December 12, 1940.*

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2. Silverstein, Abe, and Katzoff, S.: Design Charts for Predicting Downwash Angles and Wake Characteristics behind Plain and Flapped Wings. Rep. No. 648, NACA, 1939.
3. Jacobs, Eastman N., and Ward, Kenneth E.: Interference of Wing and Fuselage from Tests of 209 Combinations in the N. A. C. A. Variable-Density Tunnel. Rep. No. 540, NACA, 1935.

TABLE I.—CHARACTERISTICS OF AIRPLANES TESTED

Airplane number	Wing area, S_w (sq ft)	Wing span, b_w (ft)	Distance from a. c. to c. g., d (ft)	Aspect ratio of wing, A	Taper ratio of wing, λ	Horizontal tail area, S_t (sq ft)	Horizontal tail span, b_t (ft)	Aspect ratio of tail, A_t	$\frac{S_e}{S_t}$	Distance from c. g. to elevator hinge, l_t (ft)	Fuselage length, L_f (ft)	Maximum fuselage width, w_f (ft)	Equivalent nacelle length, L_n (ft)	Maximum nacelle width, w_n (ft)	Propeller diameter, D (ft)	Distance from c. g. to propeller plane, l_p (ft)
1	2780	149	−0.45	8.0	4.33	505	45.0	4.0	0.36	49.5	87.6	10.4	26.0	6.4	11.5	13.4
2	1420	104	−.45	7.6	2.66	254	33.8	4.5	.37	38.8	68.0	8.5	20.0	4.3	11.5	13.9
3	965	89.5	.77	8.4	2.78	203	26.7	3.5	.30	37.9	56.0	6.3	15.4	4.3	11.5	11.6
3	965	89.5	−.29	8.4	2.78	203	26.7	3.5	.30	38.9	56.0	6.3	15.4	4.3	11.5	12.6
3	965	89.5	−.66	8.4	2.78	203	26.7	3.5	.30	39.4	56.0	6.3	15.4	4.3	11.5	13.1
4	678	70.5	−.21	7.3	2.78	98	20.2	4.2	.33	28.5	44.7	3.7	11.2	4.6	11.5	7.4
5	545	65.5	−.58	7.9	3.45	116	21.7	4.1	.34	28.5	43.7	5.5	16.0	4.0	10.5	8.0
6	352	49.5	−.37	7.0	3.00	87	18.3	3.9	.39	23.4	36.3	5.4	10.7	3.9	8.8	6.4
7	224	36	−.26	5.8	(a)	41	11.5	3.2	.33	17.2	25.6	4.5	----------	--------	10.5	6.5
8	220	36	−.14	5.9	(a)	41	11.5	3.2	.33	15.9	25.0	4.5	----------	--------	10.5	7.1
9	305	42	0	5.8	1.50	47	13.3	3.8	.40	21.7	34.0	3.8	----------	--------	11.0	8.9
10	258	39	−.28	5.9	1.50	60	14.8	3.6	.46	17.1	28.2	4.0	----------	--------	9.5	7.8
11	236	37	−.31	5.9	2.50	48	12.8	3.4	.33	17.9	29.0	4.4	----------	--------	10.0	7.0
12	248	42	.07	7.1	2.00	48	13.0	3.5	.39	16.6	27.6	3.9	----------	--------	9.0	6.4
13	155	34	−.08	7.5	1.00	28	9.3	3.1	.39	13.7	22.0	3.5	----------	--------	6.0	6.2
13	155	34	−.22	7.5	1.00	28	9.3	3.1	.39	13.6	22.0	3.5	----------	--------	6.0	6.3
13	155	34	−.40	7.5	1.00	28	9.3	3.1	.39	13.4	22.0	3.5	----------	--------	6.0	6.5
14	169	36	−.15	7.7	1.00	25	10.2	4.1	.46	13.6	18.5	3.3	----------	--------	5.8	4.7
15	180	36	−.13	7.2	1.00	26	10.0	3.9	.42	14.1	19.8	3.5	----------	--------	6.5	5.3

ª Elliptical.