REPORT No. 764

A METHOD FOR CALCULATING HEAT TRANSFER IN THE LAMINAR FLOW REGION OF BODIES

By H. JULIAN ALLEN and BONNE C. LOOK

SUMMARY

This report has been prepared to provide a practical method far determining the chordwise distribution of the rate of heat transfer from the surface of a wing or body of revolution to air. The method is limited in use to the determination of heat transfer from the forward section of such bodies when the flow is laminar. A comparison of the calculated average heat-transfer coefficient for the nose section of the wing of a Lockheed 12–A airplane with that experimentally determined shows a satisfactory agreement. A sample calculation is appended.

INTRODUCTION

With the advent of thermal ice-prevention equipment for aircraft, the problem of determining the heat which may be transferred from the surface of aerodynamic bodies has become of considerable interest. Experimental investigations (reference 1) have indicated that adequate heating of the forward 10 to 20 percent of an airfoil will prevent ice formation on the entire airfoil. Moreover, experience has shown that in the cruising flight condition in the absence of icing conditions if sufficient heat can be supplied to this section of the airfoil to raise its surface temperature from 70° to 100° F. above the temperature of the ambient air, ice will not collect on the airfoil.

To determine whether this surface temperature rise can be obtained in the design of a particular installation, it is necessary to determine the rate of heat transfer from the airfoil surface to the atmosphere. In the usual cruising-flight condition the nose section of an airfoil experiences laminar flow. In the present report a method for calculating the rate of heat transfer from an airfoil section subject to laminar flow is developed. The method is made general to include laminar flows occurring over surfaces subjected to positive or negative pressure gradients and may be applied to two-dimensional bodies as well as bodies of revolution.

THEORY

The following development of an expression for the heat transfer between solids and fluids has been based upon Reynolds analogy. This analogy was presented by Reynolds in an early paper (reference 2) in which he suggested that, in a fluid, momentum and heat are transferred in the same way, and concluded that in geometrically similar systems a simple proportionality relation exists between heat transfer and fluid friction.

The analogy may be applied in the case of the heat transfer between solids and fluids when the fluid flow is laminar, provided the Prandtl number for the fluid is unity. The Prandtl number is the dimensionless parameter

$$Pr=\frac{c_p\mu}{k}$$

where

- c_p specific heat of the fluid at constant pressure
- μ absolute viscosity of the fluid
- k thermal conductivity of the fluid

For air the value of the Prandtl number is 0.73 rather than unity but, as is discussed later in this report, experimental investigations have shown that heat transfer is only slightly affected by variation of the Prandtl number so that for most practical purposes the Reynolds analogy may be applied. The analogy leads to the equation

$$h=\frac{1}{2}c_f\rho c_p V_1 \tag{1}$$

where

- h heat transfer coefficient defined as the heat transferred per unit time from a unit surface area for 1° difference of temperature between the surface and the fluid "outside" the boundary layer
- ρ fluid density
- V_1 stream velocity just outside the boundary layer
- c_f surface friction coefficient defined as the frictional drag per unit surface area in terms of the local fluid dynamic pressure, q_1

The surface friction coefficient may be expressed as

$$c_f=\frac{\tau}{q_1}=\frac{2\tau}{\rho V_1^2} \tag{2a}$$

where τ is the frictional force per unit area.

For laminar flow the frictional force τ is the product of the absolute viscosity and the velocity gradient in the fluid boundary layer at the solid surface. For any given boundary-layer velocity profile, this velocity gradient is directly proportional to the velocity V_1 and inversely proportional to the boundary-layer thickness. It follows that

$$c_f=\frac{2\lambda\mu}{\rho V_1\delta} \tag{2b}$$

where δ is the boundary-layer thickness and λ is a constant dependent on the shape of the boundary-layer velocity profile and on the definition of δ. It has become customary (reference 3) to define δ for laminar boundary layers as the distance from the solid surface to a point in the boundary layer where the dynamic pressure is one-half its value outside the boundary layer.

Inserting the value of c_f from equation (2b) into equation (1) the heat-transfer coefficient becomes

$$h=\lambda\left(\frac{c_p\mu}{\delta}\right)$$

Rearranging and dividing both sides by the thermal conductivity of the fluid k this becomes

$$\frac{h\delta}{k}=\lambda\left(\frac{c_p\mu}{k}\right)=\lambda Pr=\lambda \tag{3}$$

since the Prandtl number must be assumed to be unity by the analogy. The parameter $(h\delta)/k$ is nondimensional and might be properly termed "the boundary-layer Nusselt number."

The appearance of Pr in equation (3) might suggest that its value for air (0.73) should be substituted to obtain a better approximation of the value of the boundary-layer Nusselt number. However, the meager experimental evidence available (reference 4, p. 249) indicates that for laminar flows, the Nusselt number is proportional to $(Pr)^{\frac{1}{3}}$ so that using unity for the value of Pr (equivalent to $(Pr)^0$) would appear to be preferable.

In order to determine h from equation (3) it is necessary to know the value of λ which is a function only of the shape of the boundary-layer velocity profile. Numerous experiments with airfoils have shown that the velocity profile of the laminar boundary layer in the presence of favorable pressure gradient (i. e., where the surface pressure gradient is negative proceeding in the downstream direction) is closely approximated by the Blasius distribution for the flat plate. For the Blasius type velocity profile the value of λ is 0.765.

At points on the surface of a body downstream of the minimum-pressure point a laminar boundary layer exhibits a tendency to separate. Since the velocity gradient at the surface decreases as separation develops the value of λ must diminish until at the separation point its value is zero. It is considered that reducing λ linearly from the minimum pressure point to the separation point will satisfactorily approximate the actual case, particularly since this region is of little importance from the viewpoint of heat transfer. The location of the laminar separation point may be calculated by the method of reference 5.

With the known variation of λ along the surface, the heat-transfer coefficient may be calculated when the boundary-layer thickness has been determined. The value of δ may be found for the region from the stagnation point to the minimum pressure point by the method of reference 3. Rearranged in a form more convenient for calculation, for airfoils the equation is

$$\delta^2=\frac{5.3c^2}{R_c}\left(\frac{s_1/c}{V_1/V_0}\right)\left[\frac{\int_0^{s_1/c}\left(\frac{V}{V_0}\right)^{8.17}d\left(\frac{s}{c}\right)}{\left(\frac{V_1}{V_0}\right)^{8.17}\left(\frac{s_1}{c}\right)}\right] \tag{4}$$

where

- c airfoil chord length
- s distance along the surface from the stagnation point
- s_1 distance to the point for which the boundary-layer thickness is being computed
- V_0 free-stream velocity
- V velocity outside the boundary layer at s/c
- V_1 velocity outside the boundary layer at s_1/c
- R_c body Reynolds number$=\frac{cV_0}{\nu}$

For bodies of revolution the corresponding equation is

$$\delta^2=\frac{5.3L^2}{R_L}\left(\frac{s_1/L}{V_1/V_0}\right)\left[\frac{\int_0^{s_1/L}\left(\frac{V}{V_0}\right)^{8.17}\left(\frac{r}{L}\right)^2 d(s/L)}{\left(\frac{V_1}{V_0}\right)^{8.17}\left(\frac{r_1}{L}\right)^2\left(\frac{s_1}{L}\right)}\right] \tag{5}$$

where, in addition

- L body length
- r radius of revolution at s
- r_1 radius of revolution at s_1
- R_L body Reynolds number$=\frac{LV_0}{\nu}$

The value of δ is indeterminate at the stagnation point. It is not necessarily zero as may be shown by the following approximate analysis. The initial flow near the stagnation point on an airfoil will be approximately that over a circular cylinder having a radius equal to the radius of curvature at stagnation $\bar{r}$. On the surface of a circular cylinder the velocity at θ, the angular coordinate measured from stagnation, is

$$\frac{V}{V_0}=2\sin\theta$$

For small values of θ this becomes

$$\frac{V}{V_0}=2\theta=\frac{2s}{\bar{r}}=\frac{2}{(\bar{r}/c)}\left(\frac{s}{c}\right)$$

Using this value in equation (4), gives

$$\delta^2=\frac{2.65c^2}{R_c}\left(\frac{\bar{r}}{c}\right)\left[\frac{\left(\frac{s}{c}\right)^{9.17}\Big|_0^{s_1/c}}{9.17\left(\frac{s_1}{c}\right)^{9.17}}\right]$$

so that for small values we obtain the approximate value

$$\delta^2_{stag}=\frac{0.28c^2}{R_c}\frac{\bar{r}}{c}$$

A better approximation is to consider the nose of the airfoil elliptical in form. By such an approximation, if the stagnation point is located at the leading edge, the calculated value of δ_{stag} lies between the value obtained for the cylindrical nose and half this value, depending on the thickness ratio. Neither the order of approximation nor the importance of the value of δ_{stag} warrants much refinement of this calculation. It is recommended that the approximation

$$\delta^2_{stag}=\frac{c^2}{5R_c}\frac{\bar{r}}{c} \tag{6}$$

be used for two-dimensional bodies, where $\bar{r}$ is the radius of curvature at the stagnation point which is not necessarily the leading-edge radius.

For bodies of revolution, by a similar treatment it may be shown that equation (6) is, with R_C replaced by R_L, a fair approximation if $\bar{r}$ is the least radius of curvature at the stagnation point.

As has been noted, the equations (4) and (5) were devised on the assumption that the velocity profile of the boundary layer was of the Blasius type. In the region behind the minimum pressure point the velocity profile changes to a separated profile so that if these equations were used to determine δ in this region, it would be expected that the calculated would exceed the actual boundary-layer thickness. A comparison of the experimentally determined boundary-layer thickness for the NACA 0012 airfoil (reference 6) with that calculated by this equation and by the more exact but laborious method of reference (7) shows (see fig. 1) that this equation appears to yield satisfactory results over the separating region up to the separation point.

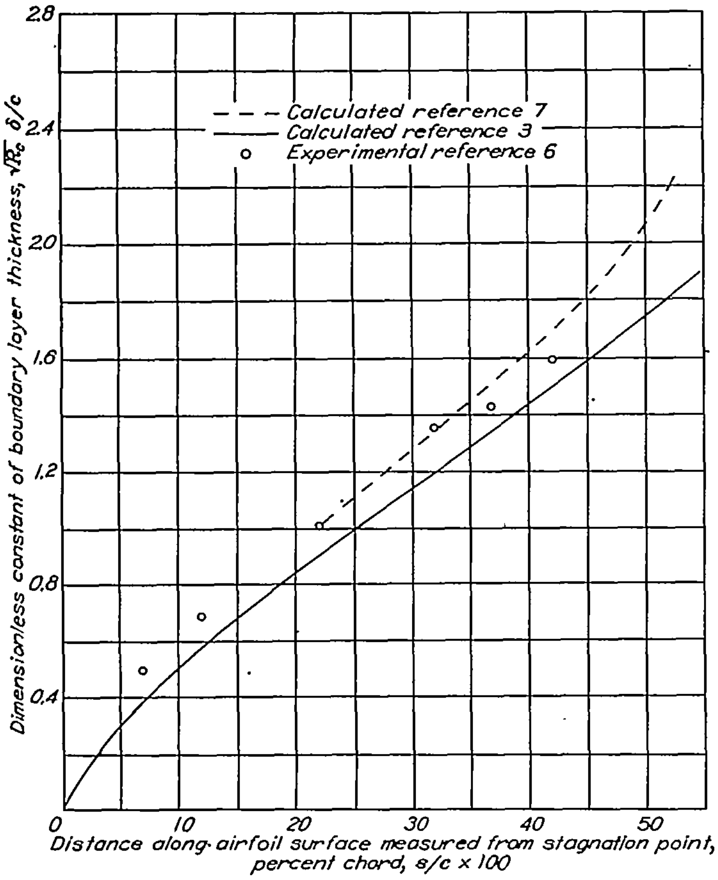


FIGURE 1.—The calculated and measured boundary-layer thickness on the surface of an NACA 0012 airfoil at zero lift.

In the preceding theory the effect of the temperature distribution within the heated boundary layer on the boundary-layer thickness and velocity profile has been neglected. This effect is known to be small for the temperature differences necessary for thermal ice-prevention and need not be considered for such practical applications.

DISCUSSION AND CONCLUSIONS

Recently an experimental investigation of a heat de-icer installation on a Lockheed 12–A airplane wing in flight was conducted. In the course of this investigation the average heat-transfer coefficient from the forward 10 percent of the airfoil surface to the air was determined.

For purposes of comparison the chordwise distribution of the heat-transfer coefficient was calculated for the Lockheed wing at three spanwise stations and the average heat-transfer coefficient determined. The calculated value of 13 Btu per hour per square foot per degree Fahrenheit agrees well with the value of 11 Btu per hour per square foot per degree Fahrenheit as determined from flight tests.

In the past, for lack of appropriate experimental data, heat-transfer coefficients have been calculated on the assumption that the experimental data of reference 1 for the Clark Y airfoil at a given angle of attack may be considered to apply to airfoils of other shapes provided the results are corrected for the effect of scale found to apply over the limited range of test Reynolds numbers for the Clark Y airfoil. On this basis, the heat-transfer coefficient for the Lockheed 12–A airplane wing would be 18 Btu per hour per square foot per degree Fahrenheit. The agreement, such as it is, must be considered fortuitously close. The assumption that, at the same angle of attack and Reynolds number, the heat-transfer coefficient for the Lockheed wing (NACA 230–series sections) and the Clark Y will be identical is clearly unjustified. Moreover, there is no basis for the assumption that the heat-transfer characteristics at high Reynolds numbers may be determined by extrapolation of low Reynolds number test results using an extrapolation formula of the kind

$$h=a\frac{k}{c}R_c^n$$

where n and a are constants.

Since the heat-transfer coefficient is determined by the boundary-layer thickness which in turn is dependent on the pressure distribution, it is essential that the pressure distribution used in the determination of the heat-transfer coefficient by the method of this report, must be that corresponding to the proper wing Reynolds number. In the event that the required experimental pressure distribution is not available, the use of the methods of references 8 and 9 combined or the more laborious but more exact method of reference 10 as modified by the method of reference 11 is recommended. Without this modification, the method of reference 10 is unsatisfactory at high angles of attack as is indicated in reference 11.

In regard to the application of the calculated wing surface-to-air heat-transfer coefficient given by equation (3) to the determination of the rate of heat transfer from the airfoil surface, it should be noted that the temperature difference used in this calculation should be the surface temperature minus the air temperature as it is increased as a result of fluid friction. The air temperature rise due to fluid friction is approximately $1.7\left(\frac{V_0}{100}\right)^2$ in degrees Fahrenheit, where V_0 is the airplane airspeed in miles per hour so that the rate of heat transfer is

$$Q=hS\left[T_s-T_0-1.7\left(\frac{V_0}{100}\right)^2\right] \quad (7)$$

where

Q rate of heat transfer
S heated area
T_s surface temperature
T_0 ambient air temperature

and all variables are in consistent units. The correction due to aerodynamic heating is only important at very high speeds and so normally may be neglected.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif.

APPENDIX

Sample calculation:

In order to illustrate the method of determining the chordwise distribution of the heat-transfer coefficient developed in this report, calculations were made for the wing of the Lockheed 12–A airplane used in the experimental flight investigation of heat de-icing The wing section used for this illustration was taken at a spanwise station 123 inches from the center line of the fuselage.

For this calculation the following were assumed:

Air temperature (° F.)	30
Altitude (feet)	8,125
True airspeed (miles per hour)	173

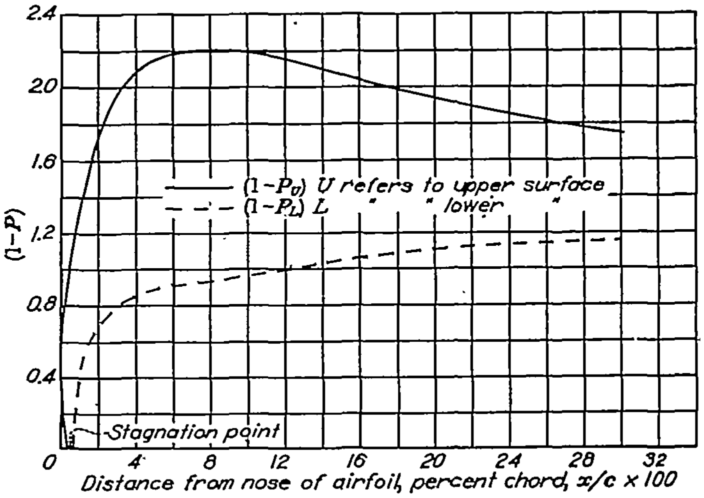
The wing-lift coefficient was 0.424. At spanwise station 123 the chord was 7.78 feet and the airfoil thickness was 14½ percent of the chord. Reynolds number, based upon the chord length, was

$$R_c=\frac{cV_0}{\nu}=10{,}330{,}000$$

where

c chord (7.78 ft.)
V_0 velocity of ambient stream (253 ft./sec.)
ν kinematic viscosity (0.0001906 ft.²/sec.)

It was first necessary to calculate the chordwise pressure distribution because no data could be found which were applicable to this case. The section-lift coefficient was computed by the method given in reference 12. The chordwise normal-force distribution was then found, using the method given in reference 8. To find the chordwise pressure distribution on the upper and lower surfaces of the airfoil, and from this the corresponding velocity distribution, the method of reference 9 was used. These calculated values have been tabulated in the form $(1-P)$ and $\frac{V}{V_0}$, where V_0 is the velocity in ambient stream. (See table I.) A plot of the pressure distribution was made to determine the stagnation point (fig. 2). The position of the laminar-separation point on the upper surface was found by the method of reference 5. (See fig. 3.)



$P=\frac{p-p_0}{q}$ = pressure coefficient

p = static pressure on airfoil surface
p_0 = static pressure in ambient stream
q = dynamic pressure in ambient stream

Figure 2.—The chordwise pressure distribution at wing station 123 of the Lockheed 12A airplane at wing C_L=0.424 and R_c=10,330,000.

The heat-transfer coefficient h was calculated by equation (3) of this report

$$h=\frac{k\lambda}{\delta}$$

δ, the laminar boundary-layer thickness was computed by equation (4) of this report

$$\delta^2=\frac{5.3c^2}{R_c}\left(\frac{s_1/c}{V_1/V_0}\right)\left[\frac{\int_0^{s_1/c}\left(\frac{V}{V_0}\right)^{8.17}d\left(\frac{s}{c}\right)}{\left(\frac{V_1}{V_0}\right)^{8.17}\left(\frac{s_1}{c}\right)}\right]$$

Substituting value of c=7.78 feet and R_c=10,330,000 this becomes

$$\delta^2=0.000031\left(\frac{s_1/c}{V_1/V_0}\right)\left[\frac{\int_0^{s_1/c}\left(\frac{V}{V_0}\right)^{8.17}d\left(\frac{s}{c}\right)}{\left(\frac{V_1}{V_0}\right)^{8.17}\left(\frac{s_1}{c}\right)}\right]$$

The term

$$\left[\frac{\int_0^{s_1/c}\left(\frac{V}{V_0}\right)^{8.17}d\left(\frac{s}{c}\right)}{\left(\frac{V_1}{V_0}\right)^{8.17}\left(\frac{s_1}{c}\right)}\right]$$

was evaluated graphically, and is given in Table I.

Equation (6) was used to find δ near the stagnation point. In the calculation of h the value of λ used was 0.765, except between the minimum pressure point and the laminar-separation point on the upper surface of the airfoil. In this region λ was reduced linearly from 0.765 at the minimum pressure point to 0 at the laminar-separation point. This

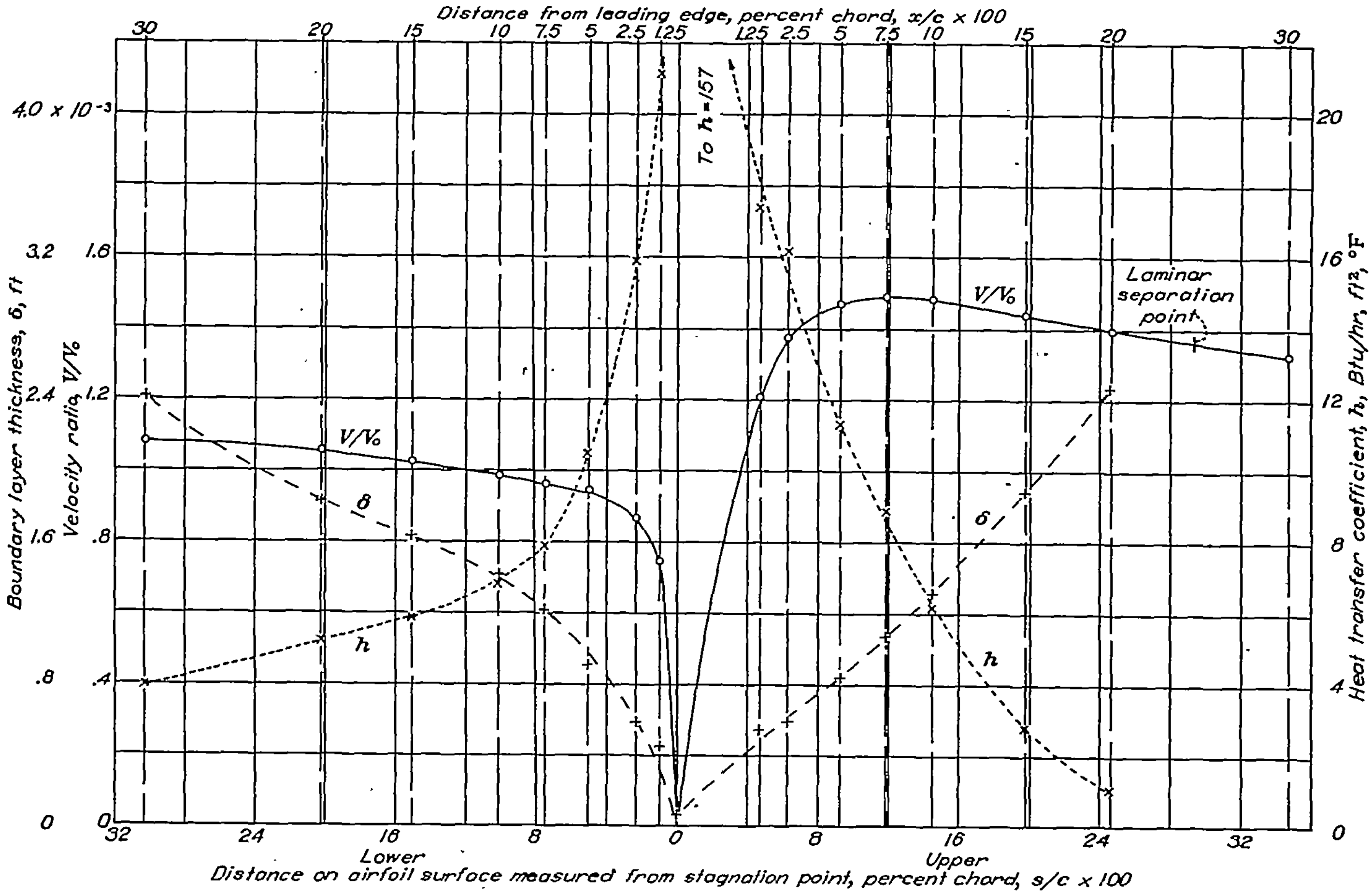


FIGURE 3.—Velocity boundary layer thickness and heat transfer coefficient distribution along the surface at wing station 123 of the Lockheed 12A airplane at wing $C_L=0.424$ and $R_e=10{,}330{,}000$.

variation of λ has been discussed in the section of the report giving the theoretical development of h.

For these calculations

$$h=\frac{1.24\times10^{-2}\lambda}{\delta}$$

Substituting for k = thermal conductivity of the air at 30° F. = 1.24×10^{-2} Btu per hour per square foot per degree Fahrenheit per foot.

Values of λ, δ, and h are given in table I.

The chordwise distribution of δ and h have been plotted in figure 3.

REFERENCES

1. Theodorsen, Theodore, and Clay, William C.: Ice Prevention on Aircraft by Means of Engine Exhaust Heat and a Technical Study of Heat Transmission from a Clark Y Airfoil. Rep. No. 403, NACA, 1931.
2. Reynolds, O.: On the Extent and Action of the Heating Surface for Steam Boilers. Proc. of the Manchester Literary and Phil. Soc., vol. 14, 1874, p. 7.
3. Jacobs, E. N., and von Doenhoff, A. E.: Formulas for Use in Boundary-Layer Calculations on Low-Drag Wings. NACA A. C. R., Aug. 1941.
4. McAdams, William H.: Heat Transmission. McGraw-Hill Book Co., Inc., 1933.
5. von Doenhoff, Albert E.: A Method of Rapidly Estimating the Position of the Laminar Separation Point. T. N. No. 671, NACA, 1938.
6. von Doenhoff, Albert E.: Investigation of the Boundary Layer about a Symmetrical Airfoil in a Wind Tunnel of Low Turbulence. NACA A. C. R., Aug. 1940.
7. von Kármán, Th., and Millikan, C. B.: On the Theory of Laminar Boundary Layers Involving Separation. Rep. No. 504, NACA, 1934.
8. Jacobs, Eastman N., and Rhode, R. V.: Airfoil Section Characteristics as Applied to the Prediction of Air Forces and Their Distribution on Wings. Rep. No. 631, NACA, 1938.
9. Allen, H. Julian: A Simplified Method for the Calculation of Airfoil Pressure Distribution. T. N. No. 708, NACA, 1939.
10. Theodorsen, T., and Garrick, I. E.: General Potential Theory of Arbitrary Wing Sections. Rep. No. 452, NACA, 1933.
11. Pinkerton, Robert M.: Calculated and Measured Pressure Distributions Over the Midspan Section of the N. A. C. A. 4412 Airfoil. Rep. No. 563, NACA, 1936.
12. Anderson, Raymond F.: Determination of the Characteristics of Tapered Wings. Rep. No. 572, NACA, 1936.

APPENDIX

TABLE I.—CALCULATIONS OF THE HEAT-TRANSFER COEFFICIENT FOR WING STATION 123, LOCKHEED 12-A AIRPLANE

Upper surface of airfoil								
Station percent chord $x/c \times 100$	Station percent chord $\frac{s}{c} \times 100$	$(1-P_U)$	$\frac{V}{V_0}$	$0.000031 \frac{\frac{s_1}{c}}{\frac{V_1}{V_0}} \times 10^5$	$\frac{\int_0^{s_1/c} \left(\frac{V}{V_0}\right)^{2.17} d\left(\frac{s}{c}\right)}{\left(\frac{V_1}{V_0}\right)^{2.17} \left(\frac{s_1}{c}\right)}$	δ_U (ft. $\times 10^3$)	λ	h_U (Btu/hr. ft.^{2} °F.)
0.5	0	-----	-----	-----	-----	0.0605	0.765	157
0	.95	-----	-----	-----	-----	-----	.765	-----
1.25	4.75	1.447	1.202	1.188	0.251	.546	.765	17.40
2.5	6.30	1.879	1.371	1.432	.244	.591	.765	16.08
5.0	9.25	2.15	1.468	1.958	.359	.839	.765	11.31
7.5	11.90	2.21	1.486	2.49	.465	1.076	.765	8.82
10	14.50	2.20	1.483	3.04	.569	1.316	.651	6.14
15	19.80	2.07	1.438	4.23	.841	1.888	.425	2.80
20	24.70	1.942	1.393	5.50	1.103	2.46	.202	1.02
30	34.80	1.743	1.322	-----	-----	-----	----	----
Lower surface of airfoil								
Station percent chord $x/c \times 100$	Station percent chord $\frac{s}{c} \times 100$	$(1-P_L)$	$\frac{V}{V_0}$	$0.000031 \frac{\frac{s_1}{c}}{\frac{V_1}{V_0}} \times 10^6$	$\frac{\int_0^{s_1/c} \left(\frac{V}{V_0}\right)^{2.17} d\left(\frac{s}{c}\right)}{\left(\frac{V_1}{V_0}\right)^{2.17} \left(\frac{s_1}{c}\right)}$	δ_L (ft. $\times 10^3$)	λ	h_L (Btu/hr. ft.^{2} °F.)
0	-----	-----	-----	-----	-----	-----	0.765	-----
1.25	0.95	0.549	0.741	0.405	0.500	0.451	.765	21.1
2.5	2.35	.752	.867	.844	.423	.598	.765	15.88
5.0	5.0	.889	.943	1.640	.507	.912	.765	10.42
7.5	7.5	.925	.962	2.43	.601	1.208	.765	7.86
10	10.1	.962	.981	3.19	.620	1.407	.765	6.75
15	15.15	1.047	1.023	4.65	.581	1.627	.765	5.84
20	20.20	1.112	1.054	5.95	.562	1.829	.765	5.19
30	30.30	1.158	1.077	8.76	.661	2.41	.765	3.94