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AN ANALYSIS OF THE DISSIPATION OF HEAT IN CONDITIONS OF ICING FROM A SECTION OF THE WING OF THE C-46 AIRPLANE

By J. K. HARDY

1945

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AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbreviation	Unit	Abbreviation
Length	l	meter	m	foot (or mile)	ft (or mi)
Time	t	second	s	second (or hour)	sec (or hr)
Force	F	weight of 1 kilogram	kg	weight of 1 pound	lb
Power	P	horsepower (metric)		horsepower	hp
Speed	V	kilometers per hour	kph	miles per hour	mph
		meters per second	mps	feet per second	fps

2. GENERAL SYMBOLS

W Weight $=mg$

g Standard acceleration of gravity $=9.80665$ m/s^2 or 32.1740 ft/sec^2

m Mass $=\frac{W}{g}$

I Moment of inertia $=mk^2$. (Indicate axis of radius of gyration k by proper subscript.)

μ Coefficient of viscosity

ν Kinematic viscosity

ρ Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m^{-4}-s^2 at 15° C and 760 mm; or 0.002378 lb-ft^{-4} sec^2

Specific weight of "standard" air, 1.2255 kg/m^3 or 0.07651 lb/cu ft

3. AERODYNAMIC SYMBOLS

S Area

S_w Area of wing

G Gap

b Span

c Chord

A Aspect ratio, $\frac{b^2}{S}$

V True air speed

q Dynamic pressure, $\frac{1}{2}\rho V^2$

L Lift, absolute coefficient $C_L=\frac{L}{qS}$

D Drag, absolute coefficient $C_D=\frac{D}{qS}$

D_0 Profile drag, absolute coefficient $C_{D_0}=\frac{D_0}{qS}$

D_i Induced drag, absolute coefficient $C_{D_i}=\frac{D_i}{qS}$

D_p Parasite drag, absolute coefficient $C_{D_p}=\frac{D_p}{qS}$

C Cross-wind force, absolute coefficient $C_C=\frac{C}{qS}$

i_w Angle of setting of wings (relative to thrust line)

i_t Angle of stabilizer setting (relative to thrust line)

Q Resultant moment

Ω Resultant angular velocity

R Reynolds number, $\rho\frac{Vl}{\mu}$ where l is a linear dimension (e.g., for an airfoil of 1.0 ft chord, 100 mph, standard pressure at 15° C, the corresponding Reynolds number is 935,400; or for an airfoil of 1.0 m chord, 100 mps, the corresponding Reynolds number is 6,865,000)

α Angle of attack

ϵ Angle of downwash

α_0 Angle of attack, infinite aspect ratio

α_i Angle of attack, induced

α_a Angle of attack, absolute (measured from zero-lift position)

γ Flight-path angle

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(Staff Member, Royal Aircraft Establishment)

Ames Aeronautical Laboratory
Moffett Field, Calif.

National Advisory Committee for Aeronautics

Headquarters, 1500 New Hampshire Avenue NW., Washington 25, D. C.

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AN ANALYSIS OF THE DISSIPATION OF HEAT IN CONDITIONS OF ICING FROM A SECTION OF THE WING OF THE C–46 AIRPLANE

By J. K. Hardy [1]

(Staff Member, Royal Aircraft Establishment)

SUMMARY

A method is given for calculating the temperature that a surface, heated internally by air, will assume in specified conditions of icing. The method can be applied generally to predict the performance, under conditions of icing, of the thermal system for protecting aircraft. Calculations have been made for a section of the wing of the C–46 airplane, and the results agree closely with the temperatures measured. The limit of protection, when the temperature of the surface reaches 32° F, has been predicted for the leading edge. The temperature of the surface in conditions of icing with air at 0° F also has been calculated. The effect of kinetic heating and the effect of the concentration of free water and size of droplet in the cloud are demonstrated.

INTRODUCTION

It has been established, by extensive testing in flight, that it is practicable by means of heat to protect aircraft against ice. It is the object now to establish a method by which the system of protection can be designed both with accuracy and with economy of means to meet specified conditions of icing.

The performance of the heated wing of the C–46 airplane (reference 1), in conditions of icing, has been analyzed. The object of the analysis was to find if the temperature of the surfaces of the wing could be predicted by calculation and, if this was successful, to find the limiting conditions of icing which the particular design will withstand, and to suggest how the design might be improved. The primary object has been realized, since, as will be shown, the temperature of the surface can be predicted with considerable exactitude.

The method of calculating the dissipation of heat, both by convection and by evaporation, from a heated surface exposed to conditions of icing is given in reference 2. The method was applied to calculate the minimum rate of heating required to prevent the formation of ice, and the results were compared with the rate actually found by experiment to be necessary. These experiments were made in a wind tunnel with small airfoils and cylinders; these were heated electrically so that both the rate of heating and the distribution of heat were known accurately. The more difficult case of the air-heated wing, in which the distribution of heat is uncontrolled once the design is fixed, is a subject of recent tests on a model scale. An analysis of these tests is given in reference 3. The tests show that the temperature of the heated surface can be predicted closely by calculation.

The failure to apply the method of analysis earlier to the heated surfaces of an airplane, in tests under natural conditions of icing, can be attributed to two things. First is that the physical characteristics of the conditions of icing were not known. Second is that in clear air, when calculation is relatively simple, there was a considerable discrepancy between the temperature of the surface as calculated and that observed. In some of the later tests of the C–46 airplane in conditions of icing, the physical characteristics of these conditions have been measured. The results are reported in reference 4.

The discrepancy between the calculated and observed temperatures of the surface in clear air is owing to incorrectness in the method of calculating the rate of transfer of heat from the hot air to the inner surface of the wing. This is shown by the measurements of the coefficient of transfer recently made and reported in reference 5. The primary concern of the present analysis is with the outer surface of the wing. The difficulty with the internal coefficient of transfer of heat has been avoided by using a value, to calculate the temperature in conditions of icing, found to give the correct temperature in clear air.

The analysis is confined to one station of the outer wing of the C–46 airplane, and to the lower surface only. Surface-type thermocouples were installed at this station for the purpose of measuring, as nearly as possible, the actual temperature of the outer surface of the wing.

The wing of the C–46 airplane is heated by air delivered from a D-shape duct in the nose to the space between the outer skin and a corrugated inner skin which extends approximately to 10 percent chord. Details of the design may be found in reference 1.

The hot air, after discharge from the corrugations, flows at random through the interior of the wing. The indefiniteness of the flow, together with the complications introduced by the structure of the wing, makes it impossible to extend the analysis behind the position where the corrugations end.

The temperature of the surface of the wing has been calculated by a step-by-step method.

[1] This report was prepared by Mr. Hardy in collaboration with the staff of the Ames Aeronautical Laboratory during a period of active participation by Mr. Hardy in the NACA icing research program.

SYMBOLS

c	chord of wing, feet
c_p	specific heat of air, Btu per pound, °F
d	diameter of cylinder equivalent to leading edge, feet
D	diffusivity of water vapor, feet squared per hour
e	vapor pressure, millimeters of mercury
h	coefficient of transfer of heat, Btu per hour, square foot, °F
H	rate of transfer of heat per unit area, Btu per hour, square foot
k	thermal conductivity of air, Btu per hour, square foot, °F per foot
k_h	coefficient of transfer of heat $\left[k_h=\frac{H}{(\rho)'Vc_p(t-t_o)}\right]$
k_w	coefficient of evaporation of water $\left[k_w=\frac{W}{(\rho)'V(n_s-n_o)}\right]$
L	latent heat of vaporization of water, Btu per pound
$(m)'$	effective concentration of free water in cloud, pounds per cubic foot
M	rate of catch of water per unit area, pounds per hour, square foot
n	concentration of water vapor, pounds per pound of air
p	barometric pressure locally, millimeters of mercury
Pr	Prandtl's number $(\mu c_p/k)$
s	distance from stagnation point measured over surface, feet
t	temperature, °F
Tr	Taylor's number $[\mu/D(\rho)']$
V	velocity in free stream, feet per hour
w	rate of weight flow of hot air per unit span, pounds per hour, foot
W	rate of evaporation of water per unit area, pounds per hour, square foot
y	percentage catch of water
μ	viscosity of air, pounds per hour, foot
$(\rho)'$	density of air, pounds per cubic foot

Subscripts

a	hot air inside the wing
o	ambient air at kinetic temperature
s	surface of wing

THEORETICAL ANALYSIS

CLEAR AIR

In clear air, the equation which gives the balance of the flow of heat to and from the surface is

$$h_a(t_a-t_s)=h_o(t_s-t_o) \tag{1}$$

The equation which gives the change in temperature of the hot air t_a as it flows over the inner surface is

$$\Delta t_a=-\frac{h_a(t_a-t_s)}{wc_p}\Delta s \tag{2}$$

Values of t_s and t_a, which satisfy both equations, are found by trial. These are the mean values for a strip of width Δs in a chordwise direction and unit length spanwise. Equation (2) is inexact, since the air flowing through the space between inner and outer skins receives heat through the back of the corrugations from the air as it flows from the main duct to the entry to the corrugations. A more exact equation is

$$\Delta t_a=[h_{a_o}(t_{a_o}-t_a)-h_o(t_s-t_o)]\frac{\Delta s}{wc_p} \tag{3}$$

h_{a_o} is the over-all coefficient of transfer of heat from the hot air behind to the hot air within the corrugations and t_{a_o} is the temperature of the air before entering the corrugations.

In the present analysis, equation (2) has been used throughout. A trial calculation has been made using equation (3); the results of this calculation will be discussed later.

CONDITIONS OF ICING

The condition of icing with which the analysis is concerned is from a cloud of supercooled droplets of water. In considering the effect of this condition on a heated surface, there are three stages which are dependent upon the amount of free water in the cloud. In the first, the droplets of water are evaporated entirely by the action of kinetic heating before they strike the surface. At the limit of this stage the temperature of the heated surface has decreased from that in clear air, at the same static temperature, only by the small difference between the kinetic temperature in clear and in wet air (reference 6). The reality of this effect and its significance in reducing the severity of icing may be seen from the data given in reference 4. In the second stage, with the greater concentration of water than in the first, water reaches the surface and is dissipated completely by evaporation so that the surface remains dry. This stage is characterized by a sharp reduction in temperature of the surface, since the heat of vaporization is being taken from the surface. The limit of this condition is reached when the surface just becomes wet, showing that the rate of evaporation has reached a maximum. In the third stage, the condition of the surface may be described as fully wet. The second stage is transitory, since only a very small quantity of water is required to wet the surface, as will be shown later. The equations which follow are for the third stage when the surface is fully wet.

Equations of heat flow.—In conditions of icing, droplets of water strike the surface in an area about the stagnation point. The rate of dissipation of heat from the area on which the droplets strike is given by the equation

$$H=[Mc_{p_w}+k_h(\rho)'Vc_p](t_s-t_o)+Lk_w(\rho)'V(n_s-n_o) \tag{4}$$

where c_{p_w} is the specific heat of water. This assumes that the temperature of the water caught is raised from that of the air to that of the surface. The temperature of the air t_o is the kinetic temperature in wet air as derived in reference 4. The nondimensional coefficient of transfer of heat k_h is used, since it is related to the coefficient of evaporation k_w. It is related to the more familiar coefficient of transfer h_o by the equation

$$h_o=k_h(\rho)'Vc_p \tag{5}$$

Equation (4) may be put into a form which is more convenient for computation by substitution from equation (5) and rearrangement.

Thus

$$H=Mc_{p_w}(t_s-t_o)+h_o(t_s-t_o)\left[1+\frac{Lk_w(n_s-n_o)}{k_h c_p(t_s-t_o)}\right]$$

Since

$$H=h_a(t_a-t_s)$$

and

$$c_{p_w}=1$$

the balance of flow of heat to and from the surface is given by the equation

$$h_a(t_a-t_s)=(M+h_o x)\ (t_s-t_o) \tag{6}$$

in which

$$x=1+\frac{Lk_w(n_s-n_o)}{k_h c_p(t_s-t_o)} \tag{7}$$

Equation (6) in conjunction with equation (2) is used to calculate the temperature of the surface on which the droplets strike.

From the area behind that on which the droplets strike, the dissipation of heat is given by equation (4) with M omitted, so that the equation corresponding to (6) is

$$h_a(t_a-t_s)=h_o x(t_s-t_o) \tag{8}$$

The value of x in equations (6) and (8) may be found from equation (7). It is convenient to substitute vapor pressure for mass concentration of water from the equation $n=0.622\frac{e}{p}$. Equation (7) then becomes

$$x=1+\frac{k_w}{k_h}\frac{(e_s-e_o)}{(t_s-t_o)}\frac{0.622L}{p\ c_p}$$

The vapor pressures e_s and e_o are those at saturation at temperatures t_s and t_o, respectively. In the present calculation the ratio of k_w to k_h has been taken as unity so that the equation becomes

$$x=1+3.75\ \frac{(e_s-e_o)}{(t_s-t_o)}\frac{p_o}{p} \tag{9}$$

p_o is standard barometric pressure at ground level (760-mm Hg) and p, the barometric pressure locally. Vapor pressures are in millimeters of mercury and temperatures in degrees Fahrenheit.

In making a calculation of surface temperature it is convenient, after the temperature of the air and the height have been chosen, to prepare a table of values of x for the values of t_s likely to occur in the calculation. The values used in the calculations of this report are given in table I. The values of x in this table show by how much from its value in clear air the rate of transfer of heat is multiplied through the effect of evaporation of water.

TABLE I.—VALUE OF x

t_s (°F)	x	
	t_o=5° F	t_o=32° F
32	1.51	----
50	1.75	2.11
68	2.12	2.57
86	2.64	3.20
95	2.97	3.60
104	3.37	4.06
113	3.84	4.62
122	4.47	5.26

Coefficient of evaporation.—There is some uncertainty as to the correct value of the ratio of k_w to k_h owing to uncertainty in the value of the coefficient of diffusion of water vapor in air. When the flow is laminar

$$\frac{k_w}{k_h}=\left(\frac{Pr}{Tr}\right)^{2/3} \tag{10}$$

When the flow is turbulent, the ratio is more nearly unity; the value may be found by substitution in Kármán's equation (reference 7). Taylor's number Tr is the ratio of the diffusivity of momentum to that of any substance or heat. Prandtl's number Pr is Taylor's number in the particular case of heat, when the diffusivity is $k/(\rho)'c_p$.

Values for the diffusivity of water vapor in air are few and scattered. The most reliable is considered to be 0.22 centimeter squared per second at 0° C. (reference 8). With this value, equation (10) gives $k_w/k_h=1.12$.

Furthermore, the value of k_w/k_h may be found from the standard psychrometric equation used in determining humidity from wet- and dry-bulb temperatures. The equation from which the extensive tables, prepared by the Prussian Meteorological Institute, are calculated is

$$e'=e-\frac{1}{2}\ (t'-t)\ \frac{p}{755} \tag{11}$$

in which t and t' are the temperatures of the dry- and wet-bulb thermometers, respectively, in degrees centigrade; e is the atmospheric water vapor pressure, and e' is the saturation vapor pressure at the temperature t', in millimeters of mercury. The origin of this equation, it appears, is entirely empirical. It may be derived quite simply by equating the heat received by the wet bulb from the air by convection to that lost by evaporation of water from the surface. Thus,

$$k_h(\rho)'Vc_p(t-t')=Lk_w(\rho)'V(e'-e)\ \frac{0.622}{p}$$

from which

$$e'=e-\frac{k_h}{k_w}\frac{755c_p}{0.622L}\ (t'-t)\ \frac{p}{755} \tag{12}$$

where t and t' are in degrees Fahrenheit. It appears from equations (11) and (12) that the value of k_w/k_h should be 0.98. It is probable that this value is more reliable than the value of 1.12 given earlier, and in the circumstances, it appears reasonable to take the value as 1 for the purpose of evaluating x.

Rate of catch of water.—The value of M, the rate at which water from the cloud strikes the leading edge of the wing, is calculated by the method given in reference 9. The wing is considered as equivalent to a cylinder having the same radius as the radius of curvature of the leading edge. It is necessary to assume a size of droplet in the cloud, since this was not measured in the tests of the C–46 airplane. With an assumed size of drop, the percentage catch y is found from figure 1 of reference 9. The total rate of catch per foot span is

$$\frac{y}{100}\ (m)'Vd$$

The value of $(m)'$, the effective concentration of water in pounds per cubic foot, is found by deducting the amount of water evaporated by kinetic heating from the concentration specified for the particular conditions of icing. The method of calculating this is shown by the example given in reference 4.

The convention adopted with regard to the distribution of the catch of water on the surface of the wing is to assume that the water is caught in the area between $s/c=\pm 1.25$ percent, and that it is equally distributed over this area except at the stagnation point. At the stagnation point, the rate of catch, it is assumed, is given by the equation

$$M=\frac{y}{100}\,(m)'V \tag{13}$$

Between $s/c=0$ and $s/c=1.25$ percent on either the upper or the lower surface of the wing, the rate of catch for unit span is given by

$$\frac{y}{100}\,\frac{1.25}{100}\,\frac{(m)'Vd}{2}\,c$$

There is an important qualification which must be made to the method of calculating the rate of catch of water. In calculating the effective concentration of free water, it is assumed that the droplets respond fully to the change in temperature produced by kinetic heating as they approach the surface. This is consistent with the limited observations which have been made on the C–46 airplane. If, however, the droplets are large and the speed of flight is high, the droplets may not respond fully to the effect of kinetic heating. In consequence, the effective concentration of free water will be greater than that calculated. The response will depend on the size of wing as well as on the size of droplet. The extent to which the severity of icing may be reduced by the effect of kinetic heating at high speeds is not known.

Blow-off of water.—It is evident from equation (8) that no account has been taken of the heat carried by water from the area of catch to the surface behind this area. The reason for the omission is that the water runs back in large drops which cover only a very small area of the surface, and which are blown off the surface early in their travel.

As far as can be judged by eye, the water which strikes the surface collects near the stagnation point until a condition of instability is reached, when a portion of it is blown back as a large drop. This tends to lift from the surface, since its path is curved. It is restrained from lifting by the surface tension of the water, and is subject to deformation by the force of the air. It is probable, under the influence of these forces, that a second state of instability is reached; this results in the disruption of the drop and its dispersal into the air, except for a small residue of water. This residue which continues to flow back over the surface as a small drop, must be dispersed by evaporation before its temperature falls to 32° F, if the formation of ice is to be avoided.

Rate of evaporation.—The calculation of the rate of evaporation of water from the surface, when the surface is completely wetted, presents no difficulty. The rate of evaporation is given either by the equation

$$W=k_w(\rho)'V(n_s-n_o) \tag{14}$$

or by the equation

$$W=\frac{h_o}{L}\,(x-1)(t_s-t_o) \tag{15}$$

When the surface is just wet, the condition reached at the termination of stage 2, the value of M in equation (6) is equal to that of W. In this state, the value of t_s, which must be such as to satisfy this identity, can be found by trial.

Behind the point at which water is blown off the surface, there is a formidable difficulty. If it is assumed, as stated in the preceding section, that the residue of water is dispersed by evaporation, then the surface must be partially wetted only. This requires that equation (8) must be modified to suit this condition. This is not possible without knowing the degree of wetness of the surface. Alternatively, if it is assumed that sufficient water is carried to the surface through the boundary layer to satisfy the requirement of full evaporation, thereby making equation (8) valid, then the residue of water will not evaporate. The absence of ice from runback, in many of the tests of the C–46 airplane in conditions of icing, suggests that evaporation must occur. It is evident that a detailed investigation is necessary before an analysis can be made of the important process by which water leaves the surface of the wing. In the analysis of this report it is assumed that the surface is fully wetted.

DATA AND CONDITIONS FOR CALCULATIONS

EXTERNAL COEFFICIENTS OF TRANSFER OF HEAT

The values, used in the analysis, of the coefficient of transfer of heat h_o from the outer surface of the wing are those calculated by Squire (reference 10) for an NACA 2415 airfoil section at a value of C_L equal to 0.24. These have been used in preference to the values, calculated from measurements of distribution of pressure, which were used in the design of the thermal system of the C–46 airplane (reference 11) for the following reasons:

1. Squire has made calculations for both the laminar and the turbulent regions of the boundary layer, and it has been necessary to extend the analysis into the turbulent region.

2. The method of calculating coefficients of transfer from distribution of pressure, given in reference 12, is only for the laminar region. In the immediate vicinity of the stagnation point, the method gives coefficients of transfer considerably in excess, it appears, of those which actually obtain. This is evident from calculation of the coefficients for a cylinder, equivalent to the leading edge of the wing, from the equation given in reference 13. Squire's calculation gives values which are consistent with those for the cylinder, except for the small discrepancy which is noted in his report.

3. The values of the coefficient used in the analysis for the laminar region, for $s/c=0.02$ and behind this position, are only 4 percent less than those calculated by the method of reference 12. The values calculated by the method of reference 12 were determined from pressures measured in flight at station 157 on the outer wing of the C–46 airplane. This station is 2 inches inboard of station 159, the station at which the temperatures were measured and calculated.

The values of h_o, used in the analysis, are plotted in figure 1. The form of the transition from the laminar to the turbulent regime which is shown in this figure is entirely arbitrary. It is known that transition, when it occurs in a forward position on a wing, will not be abrupt. The precise shape of the curve of transition is indeterminate.

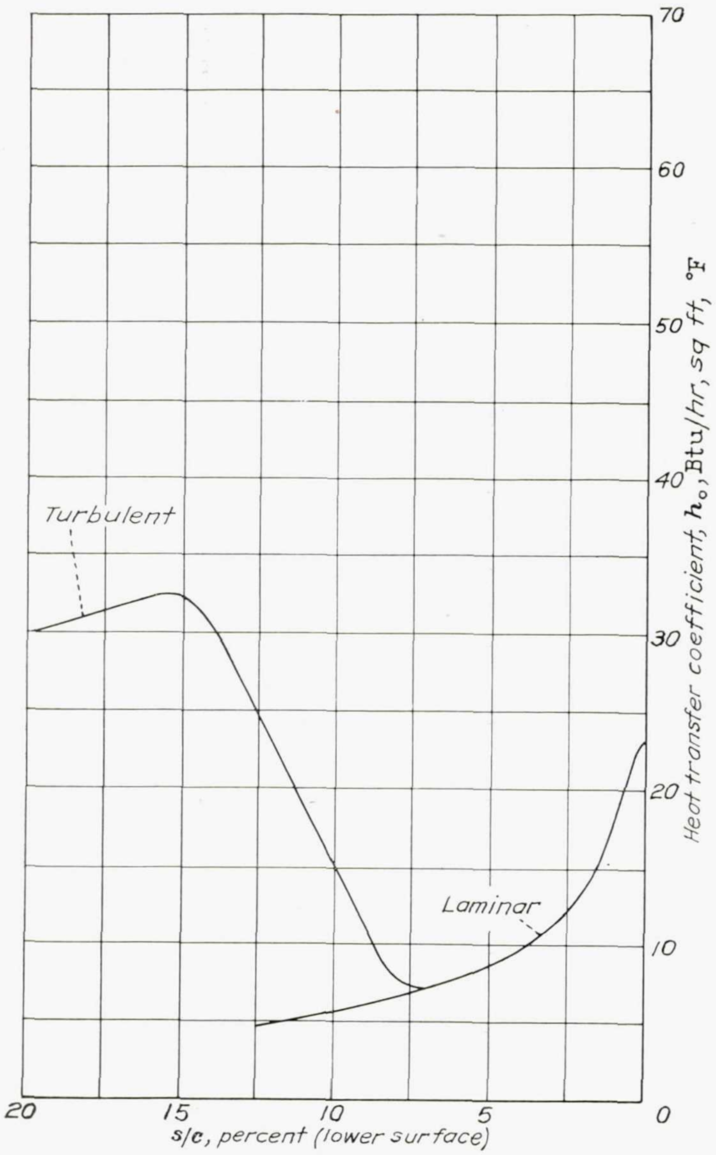

FIGURE 1.—Coefficients of heat transfer, outer surface of wing at station 159, C-46 airplane

INTERNAL COEFFICIENTS OF TRANSFER

The value, used in the analysis, for the coefficient of transfer of heat h_a to the inner surface of the wing has been deduced from the actual temperature of the surface, in clear air, measured in flight. Trial calculations were made, using equations (1) and (2) with the values of h_o from figure 1, to find the value of h_a which gave values of t_s in agreement with those measured. It was found that a value of 21 Btu per hour, square foot, ° F for h_a, constant for the whole extent of the double skin, appeared to be the most satisfactory. This value has been used for all other calculations.

The use of a value of h_a which is constant, irrespective of distance from entry to the corrugations, is at variance with the results of measurements given in reference 5. These measurements show that from entry the coefficient of transfer falls to a steady value, when, it appears, a condition of normal "pipe flow" is reached. This is described as the terminal value, and occurs at about 15 inches from entry to the corrugations. The value for the first 10 inches from entry is nearly constant, and is double the terminal value. The measurements were made on a small panel in the laboratory, and, as is remarked in the report, the results are not necessarily typical. They will depend on the character of the flow of air both over the back of the corrugations and at entry to the corrugations.

In the case of the wing of the C-46 airplane, the constancy in value of h_a may appear as a result of deficiencies in the method of calculation. At about the position when the reduction in value should be apparent ($s/c=7$ percent), there is an input of heat by direct conduction to the surface from the baffle which forms the rear wall of the D-duct. This has been omitted from the calculation. At about the same position there is transition, it appears, from laminar to turbulent flow in the boundary layer on the outer surface of the wing. The uncertainty as to the form of transition, discussed in the previous section, makes the value of h_a equally uncertain in this zone.

It is difficult to believe that the value of h_a, in the case of the C-46 wing, really is constant. The agreement, therefore, between the terminal value, 19.5 Btu per hour, square foot, ° F, calculated from the data of reference 5, and the value used in the calculations must be regarded as fortuitous.

DIMENSIONS AND CONDITIONS

The temperature of the surface of the wing has been calculated only for station 159 on the outer wing of the C-46 airplane. The chord at this station is 13.2 feet. The diameter of the equivalent cylinder, used for calculating the rate of catch of water, is 0.72 foot. It has been assumed that the stagnation point is coincident with the point at which the reference chord cuts the leading edge. The calculations have been made taking increments of surface 2½ percent chord wide, except at the leading edge in conditions of icing, when the width was reduced to 1¼ percent chord. The area of surface for a span of 1 foot corresponding to 2½ percent chord is 0.33 square foot.

All calculations have been made for a height of 4000 feet and a true speed of 180 miles per hour. This speed, in the C-46 airplane, is typical of normal cruise, at 1900 revolutions per minute, at this height. The values of h_o, the external coefficients of transfer in figure 1, are for this height and speed and for an air temperature of 26° F.

The rate of flow of hot air to the outer wing, on the average, is 6000 pounds per hour while cruising at a pressure altitude of 4000 feet. This value has been used in all calculations irrespective of the temperature of the air. The corresponding value of w, the rate of flow per foot span, is 84 pounds per hour. This is found, from the total area of cross section of the corrugations including the passages in the wing tip, by calculating the average rate of flow for one corrugation and multiplying by 12 since there are 12 corrugations per foot span.

Calculations for clear air have been made in terms of t_s-t_o, the elevation in temperature of the surface above the kinetic temperature of the air. The value of t_a at entry to the corrugations has been taken as $t_o+197°$ F which is consistent with the limited data for this condition.

Calculations for conditions of icing have been made for an air temperature (static) of 0° F and also a temperature of 28.5° F. The concentration of free water, for both temperatures, has been taken as 1.2 grams per cubic meter, and the size of droplet in the cloud as 10 microns diameter.[2] The calculations for 28.5° F were made to compare with a set of measurements taken in flight, so that for the temperature of the hot air at entry to the corrugations the measured value was used, namely, 236° F. For the calculations at 0° F, the temperature of the hot air at this point was taken as $t_o+217°$ F. This is the average value for station 159, from the records of 18 tests under conditions of icing. The discrepancy between this value and the value used in the calculations for clear air is owing to the limited data from which the latter was taken. There is a random variation of some 20° F in the temperature of the hot air.

In order to show in simple graphical form the effect of change in the concentration of free water, the temperature of the surface at the leading edge ($s/c=0$) has been calculated for a range of concentrations, both for air at 29° F and 0° F. These calculations have been extended to find the temperature of the air at the limit of protection against ice, namely, when t_s is 32° F. An effective concentration of free water of 0.5 gram per cubic meter has been assumed. For these calculations the value of t_a has been taken as $t_o+217°$ F.

In all the calculations for conditions of icing, it has been assumed that the kinetic temperature is that in wet air, and that full saturation of the air is maintained to the surface of the wing.

EXPERIMENTAL DATA

The experimental data consist of the temperature of the surface both of the wing and of the air, as measured in flight, and, in conditions of icing, the concentration of free water present in the cloud.

The measurement of concentration of free water in the cloud is the subject of a separate report (reference 4), which deals also with the measurement of the temperature of the air. The data of this report were taken in tests of the C–46 airplane in conditions of icing. These data have been used as necessary in the calculations of the present report.

The temperature of the surface of the wing was measured by a special type of thermocouple. This is a constantan-manganin couple rolled into a ribbon about 0.001 inch thick. This is cemented by cellulose lacquer to an undercoat of paint, with the ribbon running spanwise along the wing. By applying the finishing coat and rubbing down, the thermocouple is embedded just below the external surface of the paint with no deformation in the smoothness of the surface. These special thermocouples were installed only at station 159 on the undersurface of the wing of the C–46 airplane. The electrical potential at the thermocouple was measured with a potentiometer.

The general data taken in the tests of the C–46 airplane are given in reference 14. Information with regard to the effect of change in altitude or in engine power, and the method of measuring the rate of flow of hot air to the wing is given in this reference.

The analysis is based on the temperatures of the surface of the wing measured in clear air. It is a matter of importance, therefore, that these temperatures should be accurate and strictly representative of the condition of the airplane at the period when observations, against which the analysis is checked, were made in conditions of icing. It is important that the temperature of the surface should not be influenced by an input of heat from solar radiation. The values of t_s are taken from the curve in figure 2 in which

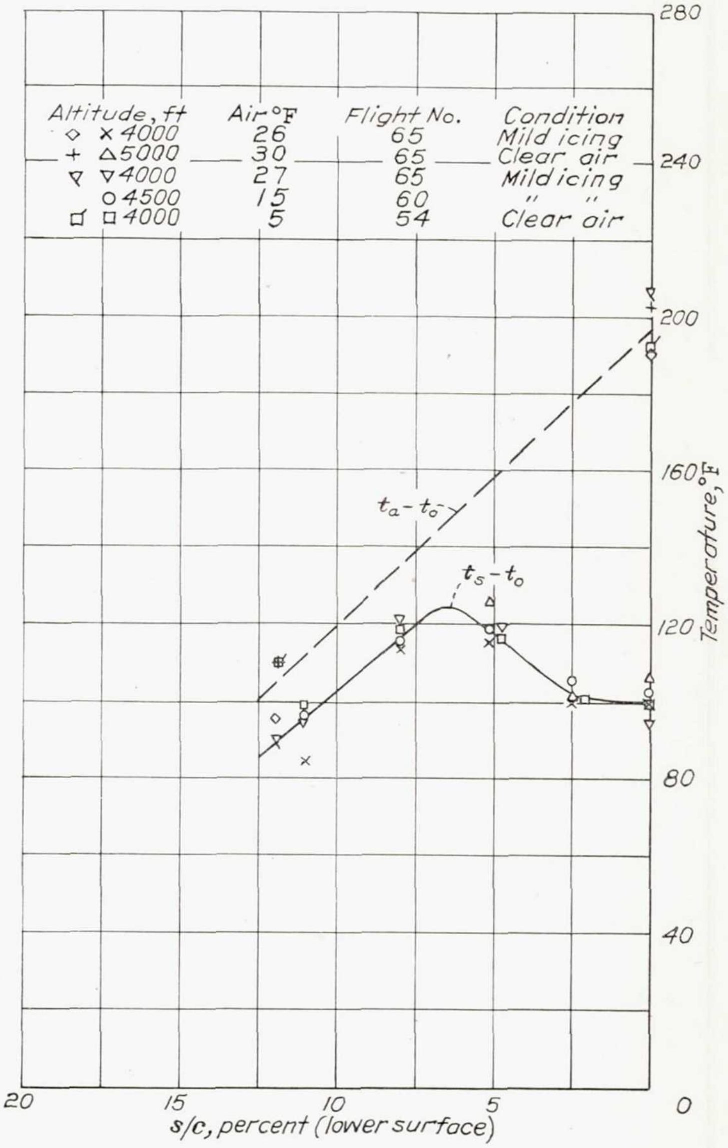

FIGURE 2.—Measured temperatures at wing station 159 during flight in C-46 airplane.

measured values are plotted. As will be noticed, the measurements were taken both in clear air and in mild conditions of icing; this will be referred to later.

For conditions of icing, the data selected are those taken

[2] 1 micron equals 10^{-6} meter.

at 3:25 hours on flight 65 (reference 4). Complete data for this flight are not presented in reference 14. This set is regarded as the best complete data available, and is believed to be representative.

RESULTS

The values calculated for the temperature both of the surface of the wing and of the hot air inside the wing are shown in figures 3 to 6. Measured values are shown as points on figures 3 and 4. There are no measured values for the condition of icing at 0° F.

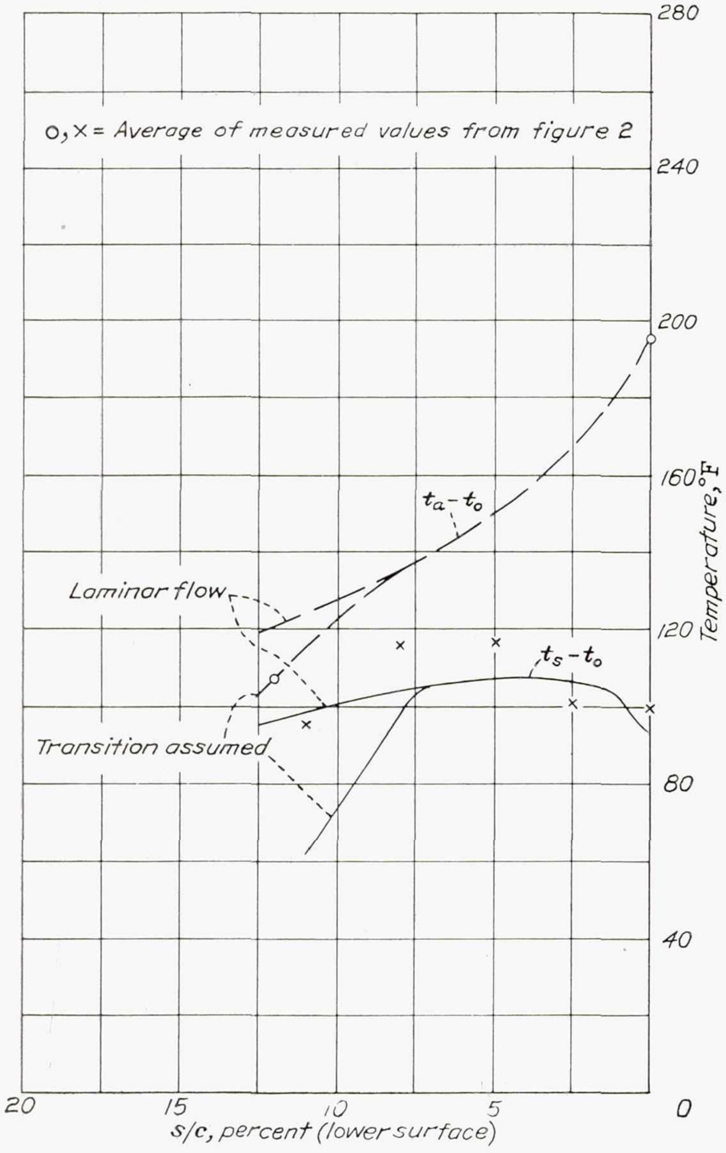

FIGURE 3.—Calculated temperatures in clear air for wing station 159 of the C-46 airplane.

The calculations for figures 3 and 4 have been made for laminar flow to $s/c=12\frac{1}{2}$ percent, and also on the assumption that transition occurred at the position which gives the best agreement with the run of measured values. It has been assumed that transition starts at $s/c=7$ percent and takes the form shown in figure 1. This form of transition has also been assumed in the calculations for icing at 0° F (fig. 5) and in calculating the rate of evaporation from the surface given in table II.

TABLE II.—RATE OF EVAPORATION FROM SURFACE OF WING AT STATION 159 FOR 1-FOOT SPAN

s/c (percent)	Rate of evaporation (lb/hr)	
	Air 0° F.	Air 28.5° F.
0–1¼	0.19	0.28
1¼–2½	.20	.24
2½–5	.30	.37
5–7½	.22	.27
7½–10	.17	.22
10–12½	.14	.19
Total 0–12½	1.22	1.57

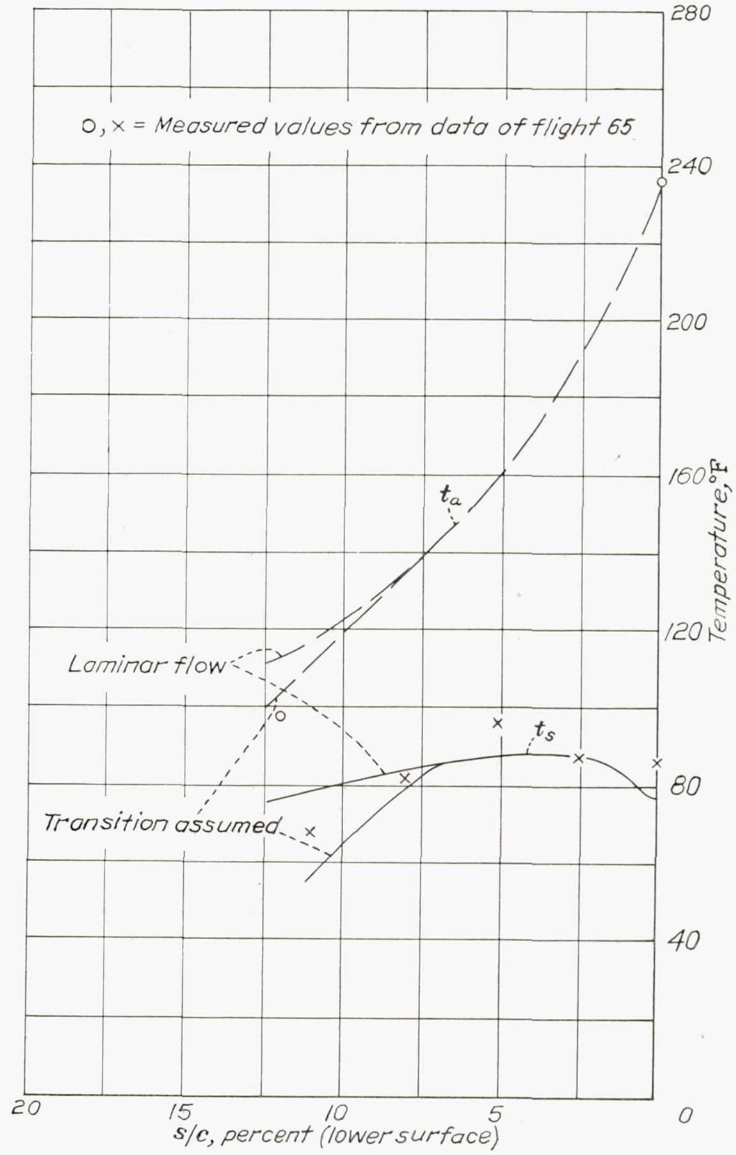

FIGURE 4.—Calculated temperatures in icing at 28.5° F; 1.2 grams per cubic meter; 10-micron droplet, for wing station 159 of the C-46 airplane.

The temperature of the leading edge ($s/c=0$), calculated for a range of conditions from dry air to a concentration of free water of 4 grams per cubic meter, is shown in figure 6. The calculated values may be compared, in a general way, with the measured values given in table I of reference 4.

The limit of protection against ice occurs when the temperature of the surface falls to 32° F. It is calculated that this condition is reached at the leading edge of wing station 159 on the C-46 airplane at an air temperature of −52° F

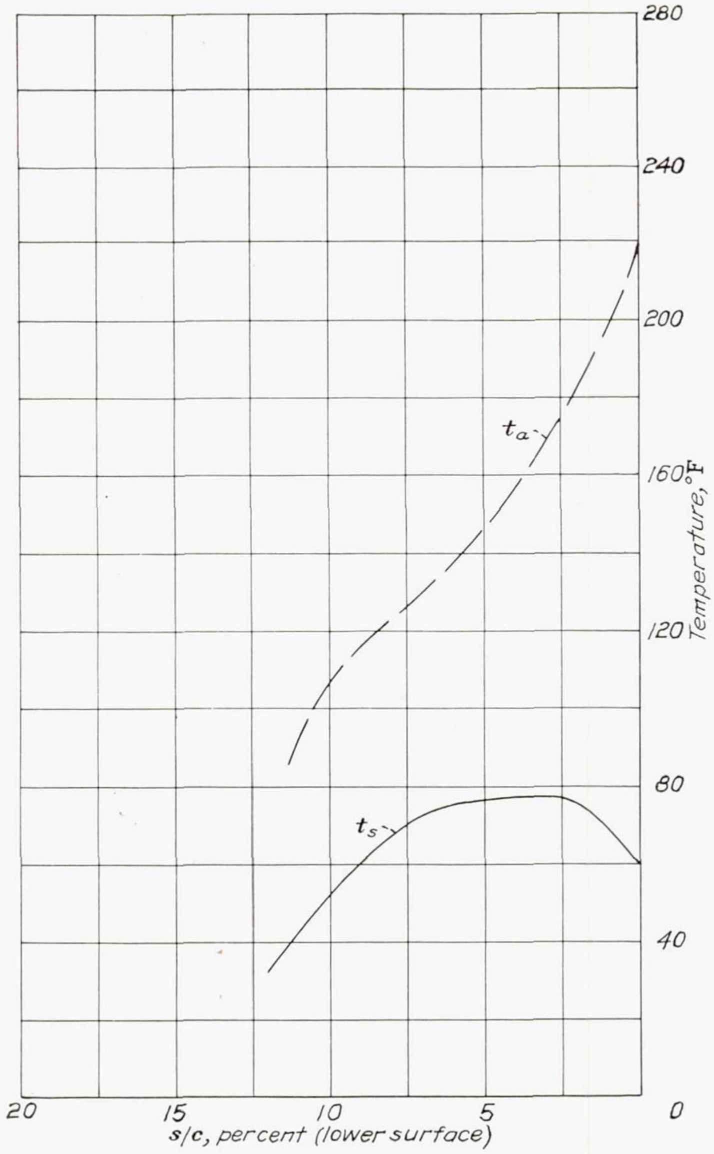

FIGURE 5.—Calculated temperatures in icing at 0° F; 1.2 grams per cubic meter; 10-micron droplet, for wing station 159 of the C-46 airplane.

(static). This is for the wing just wet. With an effective concentration of water of 0.5 gram per cubic meter and 10-micron droplets, the air temperature at the limit is −44° F (static).

DISCUSSION

The agreement between the measured and predicted temperatures of the surface in conditions of icing, as shown in figure 4, is satisfactory. At the leading edge some difference in value might be anticipated, both from the uncertainty in the actual rate of catch of water and from the thermal resistance of the water on the surface. The effect of the latter might be large. The rate of flow of heat through the surface at the leading edge in conditions of icing at 28.5° F is 3,340 Btu per hour, square foot. At this rate, the difference in temperature across a stagnant film of water is 0.8° F per thousandth of an inch thickness.

Behind the leading edge, the agreement, in conditions of icing, is better than might be anticipated from the divergence in clear air, between the calculated and observed temperatures. This divergence, it is believed, is caused primarily by inexactness in the method of calculating the rate of transfer of heat to the inner surface of the wing. Outside the wing, the uncertainty as to the true position of the stagnation point is not such as to explain the divergence.

The rate of flow of heat to the inner surface can be calculated with greater exactitude by the use of equation (3), which takes account of the heat transferred through the back of the corrugations. A trial calculation for clear air, using equation (3), showed an increase in value of t_s of 3° F at $s/c=6$ percent. The value chosen for h_{a_0}, the over-all coefficient of transfer, was 4 Btu per hour, square foot, °F. This is appropriate to a uniform flow of air in the gap between the back of the corrugations and the wall of the main duct, but the value probably is too low. The air flows from the main duct in the form of a jet, and the coefficient of transfer appropriate to this condition can only be guessed. It appears that heat is transferred also by direct conduction from the baffle which forms the rear wall of the main duct. This also is indeterminate.

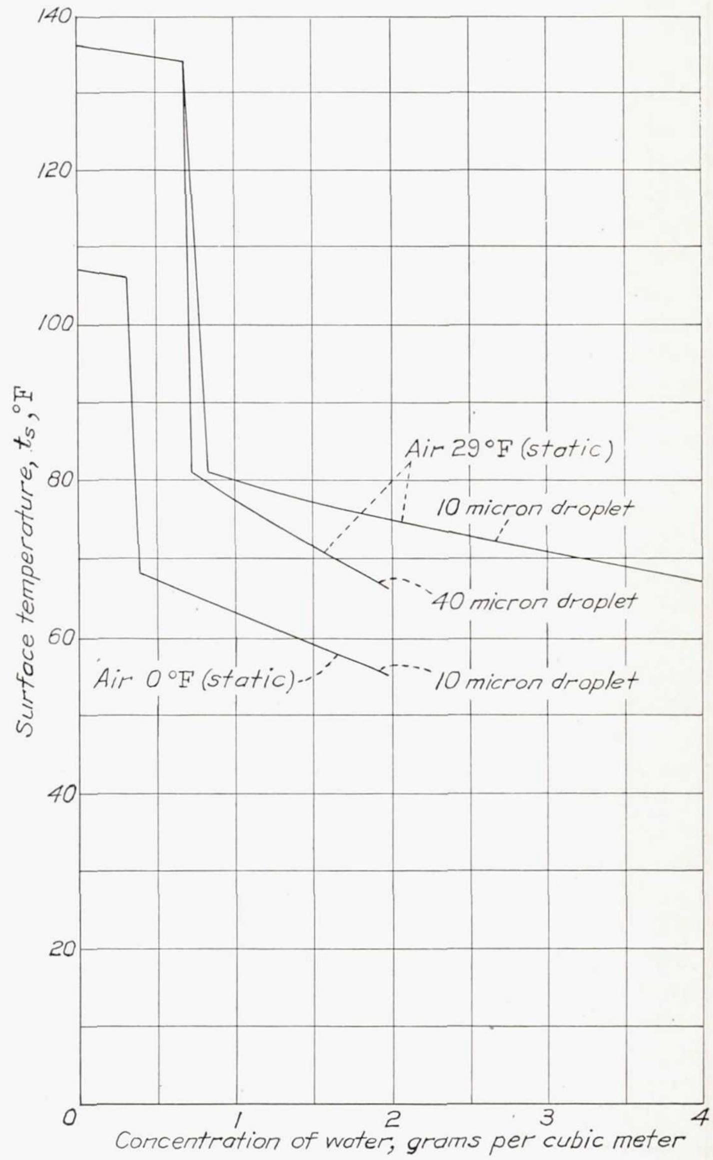

FIGURE 6.—Variations of calculated surface temperatures at the leading edge (s/c=0) of wing station 159, C-46 airplane, with concentration of free water.

The three stages in condition of the outer surface referred to previously are shown clearly in figure 6. In stage 1, the small change in temperature is owing to the change in temperature rise from kinetic heating from that in dry air to that in wet air. The extent of stage 1 is determined both by the intensity of kinetic heating and, therefore, by speed of flight, and by the temperature of the air in its effect on the vapor pressure of water at saturation. The reality of stage 1 is shown by the identity of temperatures, measured both in clear air and in conditions of icing, shown in figure 2. It is shown also by the observations both of temperature of the surface and of concentration of water which are recorded in table I of reference 4. In stage 2 the temperature of the surface falls rapidly owing to the effect of evaporation of water from the surface until, at the commencement of stage 3, a condition of full evaporation is reached. The extent of stage 2, in terms of concentration of free water, depends upon the efficiency of catch of water. For the sizes of droplet assumed, 10 microns and 40 microns diameter, the efficiencies of catch are 26 percent and 75 percent, respectively, on the cylinder equivalent to the leading edge of the wing.

The effect of change in size of droplet may not, in fact, be quite as shown in figure 6. The assumption that droplets as large as 40 microns respond fully to the effect of kinetic heating may be incorrect. If so, stage 1 will be terminated at a concentration of water below that shown.

When the concentration of free water is high, it is evident that the greater part of the water which strikes the wing must be blown off, since the amount dissipated by evaporation is comparatively small. In air at 28.5° F, for instance, the rate of catch per foot of span for 10-micron droplets and a concentration of 2 grams per cubic meter is 14.4 pounds per hour. Of this, as shown in table II, 1.6 pounds per hour is dissipated by evaporation from the lower surface.

Severe conditions of icing, for the thermal system of protection, are a combination of low temperature of the air and a high concentration of water in the cloud together with a large size of droplet. In flying in severe conditions, the magnitude of the change in temperature of the surface from that in stage 3 to that in stage 1 is of considerable practical significance. Typically, in conditions of icing, there is variation in density of the cloud. This, it appears, often is sufficient to cause transition momentarily from stage 3 to stage 1, thus detaching ice which has formed from runback of water.

Ice will not form on the leading edge, it is predicted, until the temperature of the air is below −40° F. At a temperature of 0° F, as is shown in figure 5, ice from runback will form at $s/c=12$ percent, and this has been observed to occur in flight. It is more prejudicial to the aerodynamic efficiency of the wing that a ridge of ice should form on the surface behind the leading edge than on the leading edge itself It may be advantageous, therefore, to change the distribution of heat so as to favor the former position at the expense of the latter. The advantage of directing some of the hot air from the main duct to the surface behind the leading edge is shown by the tests, on a model scale, described in reference 3.

The effect of change in the rate of catch of water on the temperature of the surface behind the area of catch is small. For most purposes it is sufficient to calculate the temperature of the surface for the fully wet condition, and to make a separate calculation for the temperature of the leading edge for different concentrations of free water.

The position at which transition occurs in the boundary layer of the outer surface of the wing is the same, so far as can be ascertained, in conditions of icing as in clear air. In the case of this particular wing, transition occurs so far forward that the roughening effect of the drops of water on the surface is unlikely to be appreciable. The effect on an airfoil of the laminar-flow type requires investigation.

CONCLUSIONS

It has been shown that the temperature of the surface of the wing in conditions of icing can be predicted with considerable exactitude from the temperatures measured in clear air. It is possible, therefore, to calculate the thermal requirements of a wing which is subject to conditions of icing, if the coefficients of transfer of heat, both to the inner surface and from the outer surface in clear air, are known. For the outer surface, there is little doubt as to the correctness of the method by which the values of the coefficients of transfer are calculated. The uncertainties are the location of the point of transition and the form of the curve of transition from laminar to turbulent flow. There is, as well, the uncertainty as to the effect, on transition, of water on the surface. The rate of transfer of heat from the hot air to the inner surface cannot at present be calculated with any confidence. There is need for systematic experiment to establish the fundamental data required.

The analysis of this report is incomplete, since the point at which water is blown off the wing cannot be predicted, and, behind this point, the heat required to prevent ice is not known. In these circumstances it is not possible to find how far back from the leading edge the double skin should be carried. The optimum distribution of heat also is uncertain. Again, there is need for systematic experiment.

The lack of knowledge as to the physical characteristics of severe conditions of icing, as they occur in nature, is a severe handicap. With this knowledge, the specification for the design of thermal systems of protection can be given in terms of definite conditions of icing.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif.

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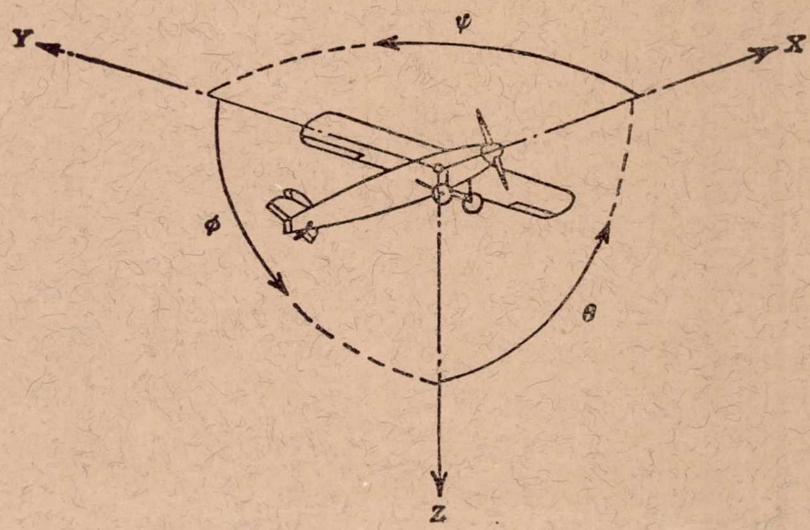

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	Rolling	L	$Y \longrightarrow Z$	Roll	ϕ	u	p
Lateral	Y	Y	Pitching	M	$Z \longrightarrow X$	Pitch	θ	v	q
Normal	Z	Z	Yawing	N	$X \longrightarrow Y$	Yaw	ψ	w	r

Absolute coefficients of moment

$C_l = \frac{L}{qbS}$ (rolling) $C_m = \frac{M}{qcS}$ (pitching) $C_n = \frac{N}{qbS}$ (yawing)

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D Diameter

p Geometric pitch

p/D Pitch ratio

V' Inflow velocity

V_s Slipstream velocity

T Thrust, absolute coefficient $C_T = \frac{T}{\rho n^2 D^4}$

Q Torque, absolute coefficient $C_Q = \frac{Q}{\rho n^2 D^5}$

P Power, absolute coefficient $C_P = \frac{P}{\rho n^3 D^5}$

C_s Speed-power coefficient $= \sqrt[5]{\frac{\rho V^5}{P n^2}}$

η Efficiency

n Revolutions per second, rps

Φ Effective helix angle $= \tan^{-1}\left(\frac{V}{2\pi r n}\right)$

5. NUMERICAL RELATIONS

1 hp = 76.04 kg-m/s = 550 ft-lb/sec

1 metric horsepower = 0.9863 hp

1 mph = 0.4470 mps

1 mps = 2.2369 mph

1 lb = 0.4536 kg

1 kg = 2.2046 lb

1 mi = 1,609.35 m = 5,280 ft

1 m = 3.2808 ft