REPORT No. 864

EFFECTS OF TEMPERATURE DISTRIBUTION AND ELASTIC PROPERTIES OF MATERIALS ON GAS-TURBINE-DISK STRESSES

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SUMMARY

Calculations were made to determine the influence of changes in temperature distribution and in elastic material properties on calculated elastic stresses for a typical gas-turbine disk.

Severe temperature gradients caused thermal stresses of sufficient magnitude to reduce the operating safety of the disk. Small temperature gradients were found to be desirable because they produced thermal stresses that subtracted from the centrifugal stresses in the region of the rim. The thermal gradients produced a tendency for a severe stress condition to exist near the rim but this stress condition could be shifted away from the region of blade attachment by altering the temperature distribution.

The investigation of elastic material properties showed that centrifugal stresses are slightly affected by changes in modulus of elasticity, but that thermal stresses are approximately proportional to modulus of elasticity and to coefficient of thermal expansion. Where thermal stresses are significant, use of materials with low moduli of elasticity and low coefficients of expansion would be desirable. Design calculations for disks to be operated at elevated temperatures would require accurate data on the values of these properties at all temperatures encountered in order that accurate calculations could be made. Centrifugal tangential stresses at the rim were decreased by increases in Poisson's ratio but the effect of changes in Poisson's ratio on the total stresses was small. Elevated-temperature measurements of Poisson's ratio would not, therefore, be essential for disk-stress calculations.

INTRODUCTION

Gas-turbine disks are normally operated at such high temperatures that the materials used are at a low-strength level. The hot gases contact the blades and the rim of the turbine rotor and thus maintain the rim at a high temperature. Various cooling methods have been used to reduce the temperature of the disk, but because the rim is always in contact with hot gases, it remains at a high temperature, whereas cooling decreases the temperature of the central portion of the rotor and thus increases the temperature gradients. These gradients are sources of thermal stresses that cause the stress distributions in gas-turbine disks to differ widely from those encountered in steam turbines, as steam turbines are mainly subjected to centrifugal stresses with small temperature gradients. An analysis of the types of stress to be expected in aircraft gas turbines requires a consideration both of the centrifugal stresses and of the stresses resulting from temperature gradients.

In order to investigate the net effects of superposed thermal and centrifugal stresses, a number of arbitrarily selected cooling conditions were assumed to exist in a typical disk and the corresponding thermal and centrifugal stresses were calculated. The margin of safety was determined by algebraically adding the centrifugal and thermal stresses present at any point and comparing the resulting stress state with the rupture strength of the material at the temperature of the point. This comparison of the strength of the material with stress conditions was considered to afford an indication of the relative desirability of various temperature distributions.

Elastic properties of materials such as Poisson's ratio, modulus of elasticity, and coefficient of thermal expansion vary with temperature. Aircraft gas turbines are usually operated with a radial variation of temperature in the disk. Accurate stress calculations would therefore require accurate data on the elastic properties of materials at all operating temperatures and a method of stress calculations would have to be used that would account for the point-to-point variation of material properties in a disk.

In order to determine which properties significantly affect the stresses in gas-turbine disks and to determine the types of effect that result from varying these properties, calculations of centrifugal and thermal stresses were made for a wide variation of values of elastic constants in the fall of 1946 at the NACA Cleveland laboratory. These calculations were facilitated by a method (reference 1) that is especially adapted to disks in which there are point-to-point variations of profile, temperature, and material properties.

ASSUMPTIONS AND METHOD OF ANALYSIS

Centrifugal stresses were calculated for a rotative speed of 11,000 rpm. The disk profile used in all calculations is shown in figures that are the results of stress computations. In the calculations of the influence of changes in radial temperature distribution, the assumption was made that no axial stresses or temperature gradients are present. (Axial stresses and temperature gradients would be expected in disks that are cooled on one side.)

It was assumed that the radial temperature distribution could be represented by a function of the form

$$T=ar^n+b$$

where

T temperature at radius r
a, b, n constants determined by assumed temperature distribution
r radius

This functional relation between temperature and radius was chosen as a convenient method of representing anticipated temperature distributions. Temperature measurements could prove other types of function to be more appropriate to particular turbines.

Centrifugal stresses were calculated on the assumption that the mean radial stress at the rim due to blade loading is a tensile stress of 8500 pounds per square inch. This value was obtained by dividing the total centrifugal force at the root of the blades by the total rim peripheral area. Temperature distribution in the blades was assumed to have no influence on the disk thermal stresses and the radial thermal stress was assumed to be zero at the rim.

The method of stress calculation used is given in reference 1. It is a finite-difference method that permits calculation of the stresses present in disks in which temperature, thickness, modulus of elasticity, Poisson's ratio, and coefficient of expansion are varied in an arbitrary manner. In order to calculate only centrifugal stresses, the coefficient of thermal expansion was set equal to zero; in order to calculate only thermal stresses, the angular velocity was set equal to zero. The total stresses were found by algebraically adding the centrifugal and thermal stresses.

Because aircraft power plants are required to operate under severe conditions for only short periods of time (such as during starting and take-off), the 2-hour rupture strength was arbitrarily chosen as a criterion of the capacity of a material to withstand the stresses of aircraft gas-turbine disks. The 2-hour rupture strength is the value of uniaxial tensile stress that will cause failure of a tensile test specimen in 2 hours at a definite temperature. The high-temperature portion of figure 1 is a plot of 2-hour stress-rupture values taken from figure 46 of reference 2; the low-temperature portion of the figure was obtained by passing a smooth curve through the elevated-temperature points and a room-temperature point (107,000 lb/sq in.), which was taken from the tensile-strength curve of figure 44 of reference 2. The disk stresses are biaxial; hence, comparison of the severity of stresses at a point in a disk with the stress-rupture value for the corresponding temperature requires the use of a single quantity that represents the biaxial stress state. For this purpose, an "equivalent tensile stress," which is defined by the derivation given in the appendix, was calculated from the maximum-shear-strain-energy theory of failure. The excess of a stress-rupture value over the equivalent tensile stress is called the margin of safety. This quantity, which was calculated on the assumption that the disk remains perfectly elastic, provides a means of comparing the stress conditions at various points of the disk and also provides a qualitative means of studying the net effect of changes in temperature distribution. Calculation of the margin of safety using elastic stresses is not a method of estimating failure conditions because a disk, which is locally overstressed, will yield plastically and so relieve the high stresses caused by stress concentration.

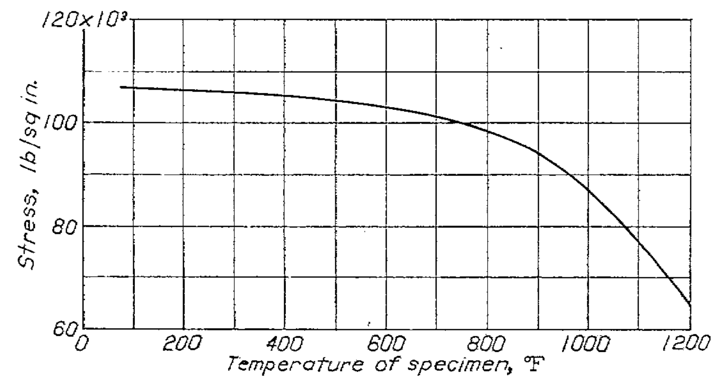
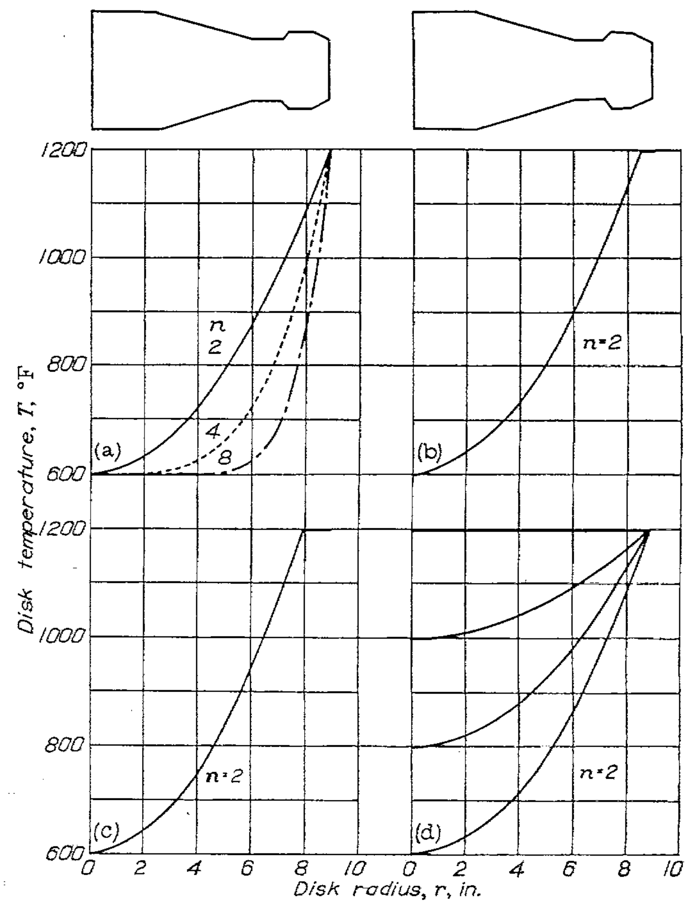


FIGURE 1.—Two-hour stress-rupture curve for solution-quenched 16-25-6 alloy. (Data from reference 2.)



(a) No uniform temperature region at rim. (b) Small uniform temperature region at rim
(c) Large uniform temperature region at rim. (d) Variable center temperature.

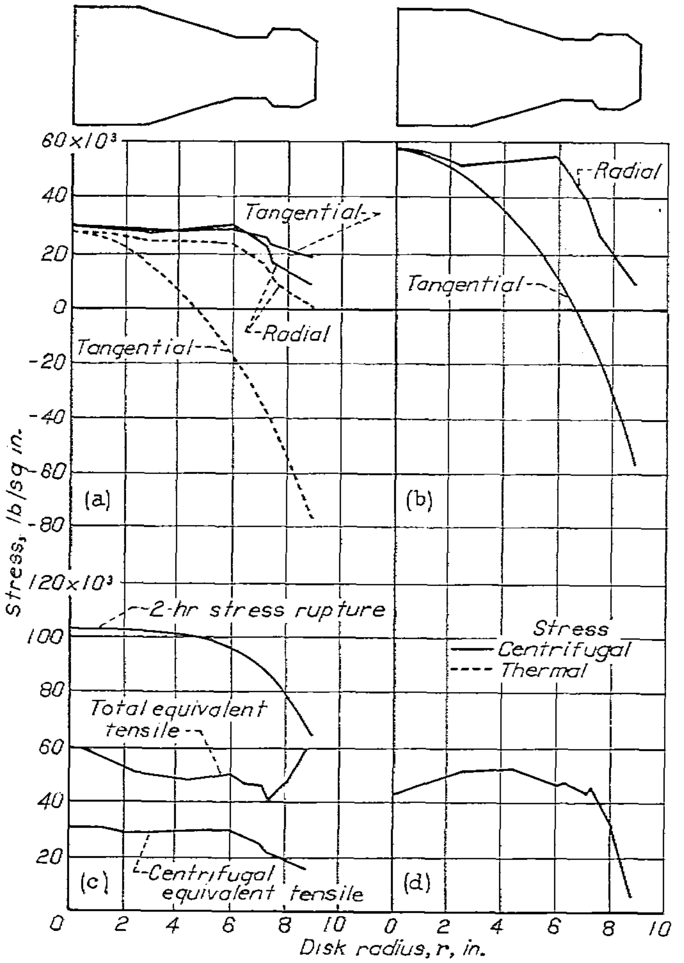
FIGURE 2.—Assumed radial temperature distribution relative to turbine-disk profile as given by $T=ar^n+b$.

EFFECTS OF TEMPERATURE DISTRIBUTION

In order to investigate the manner in which stresses depend on temperature distribution, a variety of arbitrarily selected temperature distributions (fig. 2) was assumed and the corresponding stresses were calculated.

TYPICAL STRESS DISTRIBUTION

A qualitative presentation of the radial variation of components of centrifugal and thermal stresses, total stresses, equivalent tensile stresses, 2-hour rupture strength, and margin of safety is shown for a typical disk in figure 3. The data for this illustration of the manner in which the



(a) Centrifugal and thermal stresses. (b) Total stresses.
(c) Two-hour stress-rupture curve and equivalent tensile stresses. (d) Margin of safety.

FIGURE 3.—Typical stress distribution for profile shown. Speed, 11,000 rpm; bucket loading at rim, 8500 pounds per square inch; center temperature, 600° F; rim temperature, 1200° F; temperature distribution given by $T=ar^n+b$; $n=2$.

components of stress combine were calculated for the temperature distribution of figure 2 (a) with n equal to 2. The convention of considering tensile stresses positive and compressive stresses negative was followed. Figure 3 (a) shows that the centrifugal stresses are all tensile but the thermal tangential stress takes on large compressive values. Because the tangential thermal and centrifugal stresses at the rim are of opposite sign, the total tangential stress at the rim is smaller than the thermal stress alone. The two stress systems have been added in figure 3 (b) and it is seen that a net tangential compressive stress exists in the region of the rim. The equivalent tensile stress for the centrifugal stresses and for the total-stress system are shown in figure 3 (c). The 2-hour stress-rupture values corresponding to the assumed temperatures are also shown. Figure 3 (d) shows the corresponding margin of safety. The low margin of safety at the rim is due to the presence of resultant compressive stresses. Excessive values of such stresses often cause plastic flow of the disk rim during engine operation and when the disk cools a system of tensile stresses is set up that can cause rim cracking. Because such cracks usually progress rather slowly, serious trouble from this source can, in most cases, be forestalled by removal of the wheel from service. The thermal and centrifugal stresses are both tensile at the center; in the event that these stresses become excessive, a sudden rupture of the rotor would probably occur. Disks are therefore usually designed to have a larger margin of safety at the center than at the rim.

EFFECT OF CHANGING TEMPERATURE DISTRIBUTION

The margins of safety that correspond to the several temperature distributions of figure 2 (a) are shown in figure 4. The margin of safety at the center is large and increases as n is increased but, as the temperature gradients in the region of the rim are increased by increasing n, the margin of safety at the rim is reduced. Figure 5 shows curves of the equivalent tensile stresses and of the margins of safety that correspond to the temperature distributions of figure 2 (d).

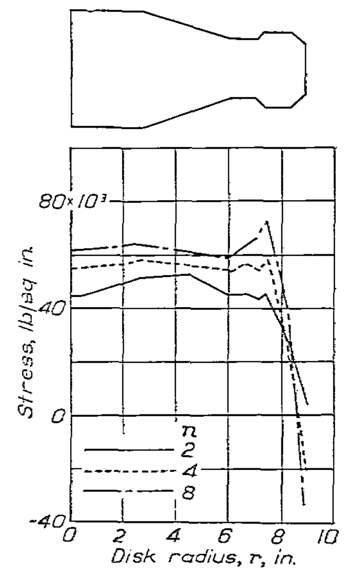
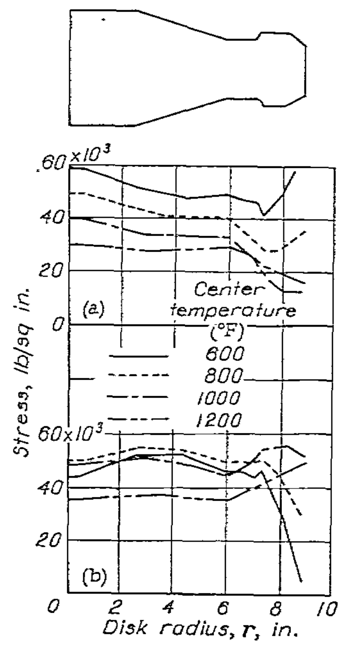


FIGURE 4.—Distribution of margin of safety for profile shown. Speed, 11,000 rpm; blade loading at rim, 8500 pounds per square inch; center temperature, 600° F; rim temperature, 1200° F; temperature distribution given by $T=ar^n+b$.



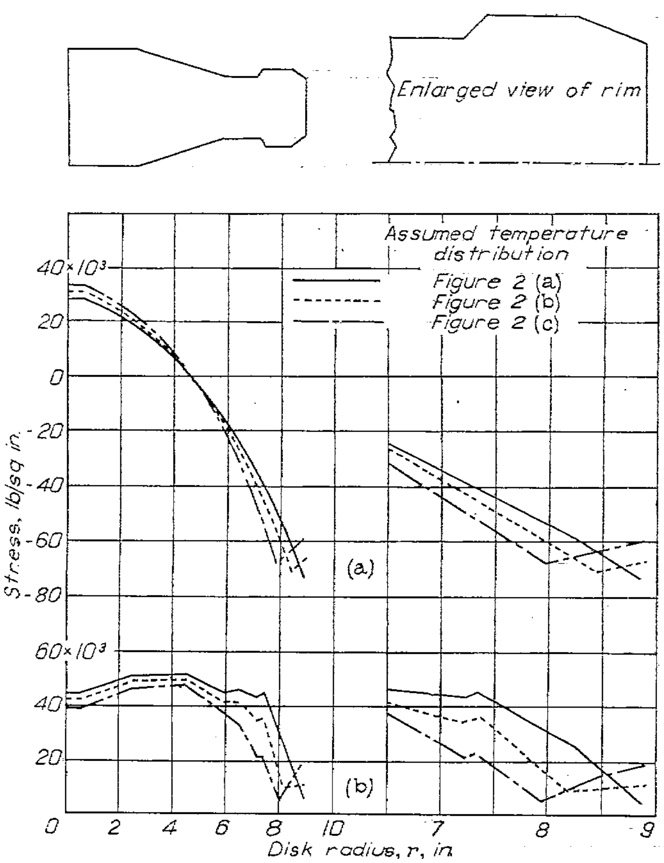
(a) Equivalent tensile stress.
(b) Margin of safety.

FIGURE 5.—Variation in equivalent tensile stress and in margin of safety for profile shown as temperature distribution is varied according to figure 2 (d). Speed, 11,000 rpm; blade loading at rim, 8500 pounds per square inch; rim temperature, 1200° F; temperature distribution given by $T=ar^n+b$; $n=2$.

In general, the reduced thermal stresses decrease the equivalent tensile stresses as the temperature of the disk center is raised; however, the stress at the rim is higher for a center temperature of 1200° F than for a center temperature of 1000° F. The tangential centrifugal and thermal stresses are of opposite sign and, for the temperature distribution corresponding to a center temperature of 1200° F, there is no thermal stress to reduce the effect of the centrifugal stress; therefore, the equivalent tensile stress at the rim for a center temperature of 1000° F is lower than that for a center temperature of 1200° F. Figure 5 (b) shows a different order of desirability of center temperatures from that which would be expected from figure 5 (a) when the center of the wheel is considered. This difference exists because low center temperatures raise the material strength at the center and thus influence the margin of safety. These results show that the margin of safety at any point is affected by temperature distribution in two ways; that is, the thermal stresses and the strength of the material are affected by changes in temperature.

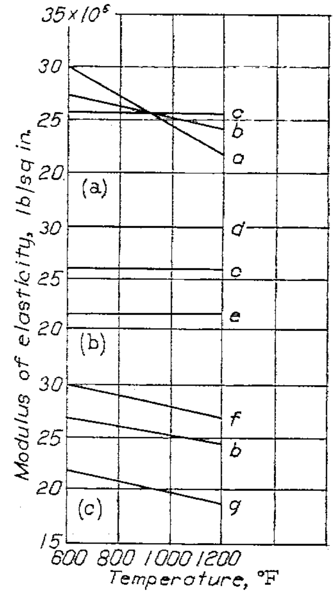
CONTROL OF STRESS DISTRIBUTION BY ADJUSTMENT OF TEMPERATURE DISTRIBUTION

The attachment of blades to the disk rim usually increases the local stresses. Centrifugal forces and thermal gradients that produce low stresses in the region of the rim are therefore desirable. If the turbine is so designed that hot gases are in contact with a large part of the rim, a uniform temperature



(a) Tangential thermal stresses.
(b) Margin of safety.

FIGURE 6.—Distribution of tangential thermal stress and margin of safety for several temperature distributions of profile shown. Speed, 11,000 rpm; blade loading at rim, 8500 pounds per square inch; center temperature, 600° F; rim temperature, 1200° F; temperature distribution given by $T=ar^n+b$; $n=2$.



(a) Various slopes.
(b) Zero slope.
(c) Constant, typical slope.

FIGURE 7.—Assumed relations between modulus of elasticity and temperature for calculations of thermal and centrifugal stresses.

region at the rim would be expected, as is illustrated by figures 2 (b) and 2 (c). In figure 6 (a), which shows the effect of a constant temperature near the rim, are plotted the tangential thermal stresses that accompany the temperature distributions of figures 2 (a), 2 (b), and 2 (c) with n equal to 2. Figure 6 (b), which is a plot of the corresponding margins of safety when all thermal and centrifugal stresses are considered, shows that the position of minimum margin of safety can be shifted by changing the temperature distribution. One method of changing the temperature distribution would be to relocate the seal between the cooling air and the hot gases. Another method would be to alter the supply of cooling air. In this manner, shifting the peak of stress due to temperature gradient away from the rim is possible. The stress-concentration factor caused by the method of blade attachment would then apply to lower nominal stresses.

INFLUENCE OF ELASTIC CONSTANTS

Calculations of centrifugal and thermal stresses were made for several assumed variations in elastic properties to indicate the changes that can be made in disk stresses by altering material properties and to show the effect on stress calculations of making the simplifying assumption that the elastic properties do not vary with temperature. Because the centrifugal-stress calculations were based on a blade stress at the rim of 8500 pounds per square inch, this value of radial centrifugal stress exists at the rim for all the calculations. In conformity with the assumed conditions, the radial thermal stress is zero at the rim for all variations in elastic properties. For a solid disk, the radial and tangential stresses are equal at the center. For all the calculations made to investigate the effects of changes in elastic properties, critical conditions were found to occur only at the rim and at the center, and the response of the stresses to changes in elastic properties may therefore be observed by consideration of only the tangential stresses at the center and at the rim. All calculations made to show the effects of change in elastic constants were based on the temperature distribution of figure 2 (a) with n equal to 2.

MODULUS OF ELASTICITY

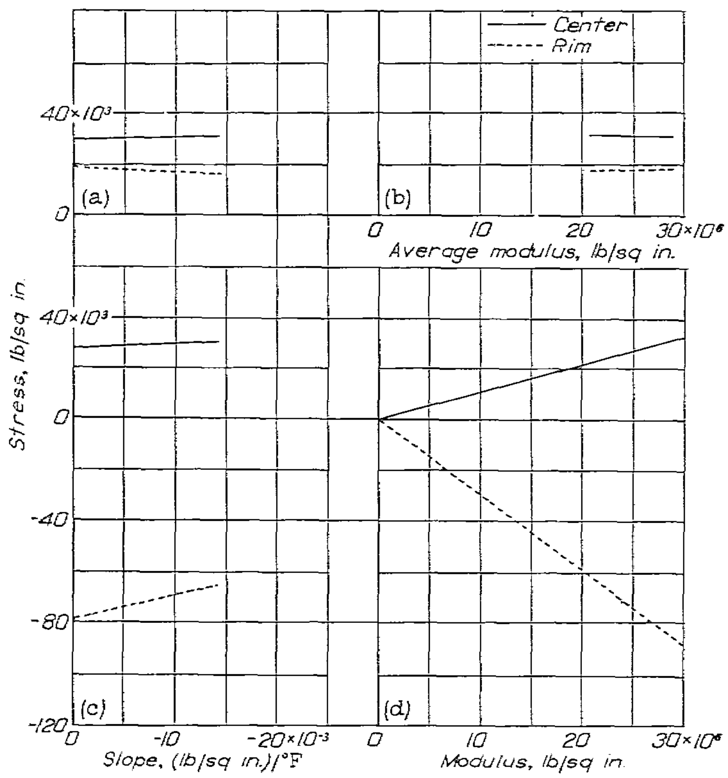
The influence of modulus of elasticity on centrifugal and thermal stresses was investigated for assumed relations between modulus of elasticity and temperature, as shown in figure 7. Line b approximates the actual variation of modulus of elasticity of a typical heat-resisting alloy in the temperature range shown. Centrifugal-stress calculations were made for the relations represented by lines a, b, and c of figure 7 (a) to show the effect of a change in slope of the modulus-temperature characteristic. The relation of the tangential centrifugal stress to the slope of these lines is shown by figure 8 (a). It is seen that a calculation which makes no allowance for the slope of the modulus of elasticity-temperature curve would yield rim stresses slightly higher than the correct values, but the center stresses would be approximately correct.

As shown in the appendix, any calculation of centrifugal stresses with the assumption that the modulus of elasticity does not vary with temperature gives results that are independent of the modulus of elasticity. Calculations of centrifugal stresses were made for the relations represented by lines b, f, and g of figure 7 (c) to show the effect of a shift in level of the modulus-temperature characteristic at a given slope. The effects on stresses are shown in figure 8 (b); for a given slope, the center and the rim centrifugal stresses are negligibly affected by the level of the modulus of elasticity.

Lines a, b, and c of figure 7 (a) show assumed relations between elastic modulus and temperature in which the average value of the modulus is constant but the slope of the curve is changed. The effect on thermal stresses of such change is shown by figure 8 (c); a thermal-stress calculation, which assumes a constant modulus equal to the average of the actual moduli across the disk, would yield correct center stresses but the magnitude of the rim stresses would be significantly increased. The influence of level of the modulus of elasticity on thermal stresses was investigated for the relations represented by lines c, d, and e of figure 7 (b). The result is shown in figure 8 (d) where the thermal stresses are directly proportional to the magnitude of elastic modulus.

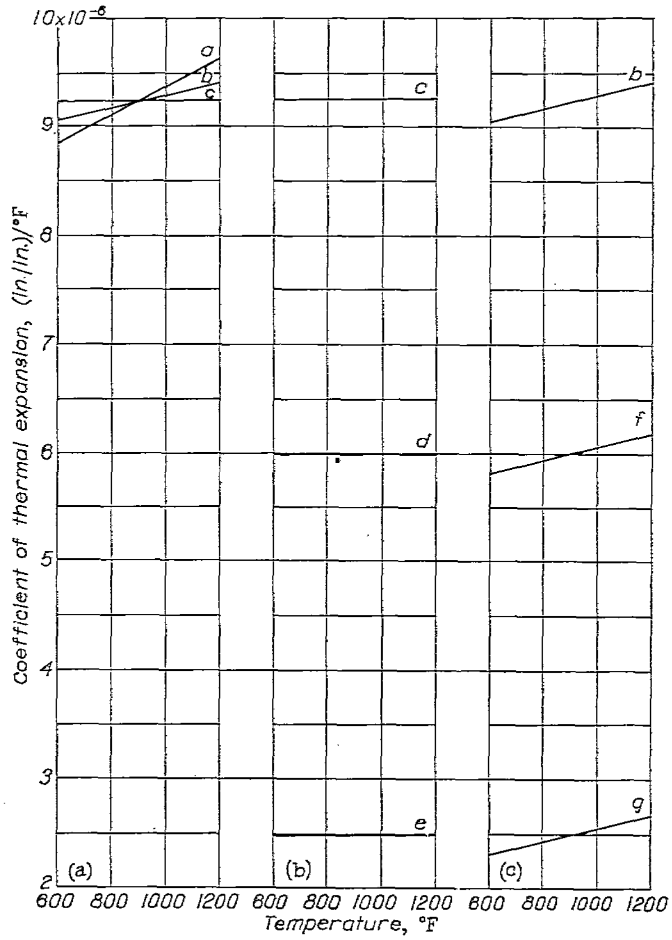
COEFFICIENT OF THERMAL EXPANSION

The assumed relations between coefficient of thermal expansion and temperature, which were used in investigating the influence of this coefficient on thermal stresses, are given in figure 9. Line b represents the actual values of a typical heat-resisting alloy. Lines a, b, and c of figure 9 (a) show assumed relations between coefficient and temperature in which the slope varies but the average value remains constant. The effect of this type of variation on the calculated stresses is shown in figure 10 (a), where it is seen that a



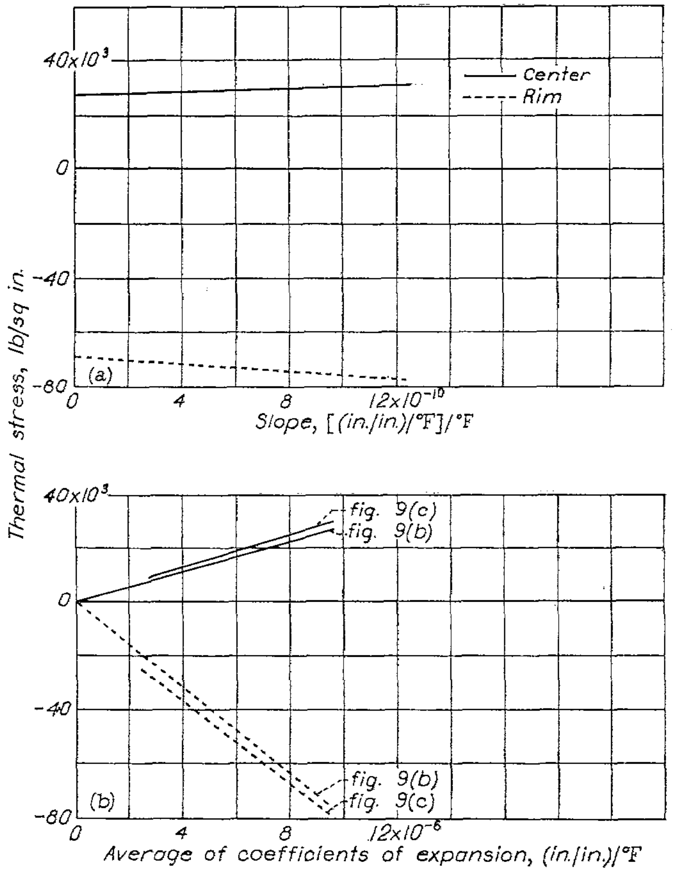
(a) Centrifugal stress for various slopes.
(b) Centrifugal stress for constant, typical slope.
(c) Thermal stress for various slopes.
(d) Thermal stress for constant moduli.

FIGURE 8.—Effect of varying moduli of elasticity as shown in figure 7 on tangential stresses at center and rim of turbine disk. Speed, 11,000 rpm; blade loading at rim, 8500 pounds per square inch; center temperature, 600° F; rim temperature, 1200° F; temperature distribution given by $T=ar^n+b$, $n=2$.



(a) Various slopes.
(b) Zero slope.
(c) Constant, typical slope.

FIGURE 9.—Assumed relations between coefficient of thermal expansion and temperature for calculations of thermal stresses.



(a) Thermal stress for various slopes.
(b) Thermal stress for two constant slopes.

FIGURE 10.—Effect of varying coefficients of expansion as shown in figure 9 on tangential stress at center and at rim of turbine disk. Speed, 11,000 rpm; blade loading at rim, 8500 pounds per square inch; center temperature, 600° F; rim temperature, 1200° F; temperature distribution given by $T=ar^n+b$; $n=2$.

calculation of thermal stress, which neglects the variation of coefficient of expansion with temperature, yields stresses slightly lower than the correct values. The sets of the values represented by lines c, d, and e of figure 9 (b) were used to determine the effect of magnitude of the coefficient when it does not vary with temperature. Figure 10 (b) shows that the thermal stresses are directly proportional to the level of coefficient. The sets of values represented by lines b, f, and g of figure 9 (c) were used to determine the effect of magnitude of coefficient of expansion when the coefficient of expansion-temperature characteristics have a given slope. Figure 10 (b) shows that a linear relation exists between the level of the coefficient and the stresses.

POISSON'S RATIO

Calculations of thermal and centrifugal stresses were also made for values of Poisson's ratio of 0.2, 0.3, and 0.5. Most heat-resisting alloys have a value of Poisson's ratio at room temperature of approximately 0.3. At elevated temperatures, a material approaches the plastic condition and, according to the theory of plasticity, Poisson's ratio for the fully plastic state would be 0.5 (reference 3, p. 79). Thus, the range of 0.2 to 0.5 covers all values of Poisson's ratio that may be expected to occur.

A change in the value of Poisson's ratio from 0.2 to 0.5 decreased the centrifugal tangential stress at the rim by 22 percent and the thermal stress at the rim by 1.4 percent and increased the net or total stress at the rim by 6 percent. For disks in which thermal stresses are predominant, the precise evaluation of Poisson's ratio is therefore unimportant.

SUMMARY OF RESULTS

When perfect elasticity was assumed, the following qualitative results concerning the influence of temperature gradients and elastic properties on stress conditions in aircraft gas-turbine disks were obtained:

1. Severe temperature gradients caused thermal stresses of sufficient magnitude to reduce the operating safety of the disk. Attempts to increase the strength of the material by cooling the center of a disk might reduce the margin of safety as a result of the thermal stresses set up.

2. Small temperature gradients were desirable because they created thermal stresses that subtracted from the centrifugal stresses in the region of the rim. Center stresses were increased by the small temperature gradients.

3. Thermal gradients produced a tendency for the maximum stress to exist near the rim. By altering the temperature distribution, the position of this maximum stress could be shifted away from the rim. A stress-concentration factor caused by the method of blade attachment then applied to lower nominal stresses.

4. Centrifugal stresses were slightly affected by changes in the relation between modulus of elasticity and temperature. The thermal stresses were approximately proportional to the coefficient of expansion and to the modulus of elasticity. Because the effects of thermal stresses are likely to predominate, it is desirable that the modulus of elasticity and coefficient of thermal expansion be small. Design calculations require accurate data on elastic modulus and coefficient of thermal expansion over the temperature ranges encountered.

5. A change in Poisson's ratio from 0.2 to 0.5 decreased the centrifugal tangential stresses at the rim but increased the total stresses by a negligible amount. Elevated-temperature measurements of Poisson's ratio were therefore considered to be of small value for calculations of gas-turbine-disk stresses.

FLIGHT PROPULSION RESEARCH LABORATORY,
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APPENDIX

MATHEMATICAL ANALYSES

SYMBOLS

The following symbols are used in this analysis:

A	maximum shear-strain energy, in.-lb/cu in.
E	modulus of elasticity, lb/sq in.
G	modulus of rigidity, lb/sq in.
h	axial thickness of disk, in.
r	disk radius, in.
s	equivalent tensile stress, lb/sq in.
s_m	stress margin of safety, lb/sq in.
s_o	tensile test stress, lb/sq in.
s_1, s_2, s_3	principal stresses, lb/sq in.
ΔT	difference between temperature of disk and temperature at zero thermal stress, °F
α	mean coefficient of thermal expansion, (in./in.)/°F
μ	Poisson's ratio, 0.3
ρ	mass density of disk material, lb sec²/in.⁴
σ_r	total radial stress, lb/sq in.
σ_t	total tangential stress, lb/sq in.
ω	angular velocity, radians/sec

EQUIVALENT TENSILE STRESS

The maximum shear-strain energy is considered to be the index of the severity of the stress state in a high-temperature rotating disk. This quantity is given in reference 3 (p. 73) as

$$A=\frac{1}{12G}\left[(s_1-s_2)^2+(s_2-s_3)^2+(s_3-s_1)^2\right] \tag{1}$$

In a rotating disk, σ_r and σ_t are principal stresses whereas the axial stress is zero, so that at any point

$$A=\frac{1}{12G}\left[(\sigma_r-\sigma_t)^2+\sigma_t^2+\sigma_r^2\right] \tag{2}$$

The stress s in a tensile specimen is uniaxial; the substitution in equation (1) of $s=s_1$ and $s_2=s_3=0$ gives

$$A=\frac{1}{12G}(2s^2) \tag{3}$$

The uniaxial tensile stress, which is equivalent to the biaxial stress in a rotating disk, is now defined by equating these two values of maximum shear-strain energy. Thus

$$\frac{2s^2}{12G}=\frac{1}{12G}\left[(\sigma_r-\sigma_t)^2+\sigma_t^2+\sigma_r^2\right] \tag{4}$$

or

$$s=\sqrt{\sigma_r^2-\sigma_r\sigma_t+\sigma_t^2} \tag{5}$$

Then, inasmuch as s_o is the stress-rupture value of a disk material at a temperature corresponding to the temperature in a disk at a point where the state of stress is being evaluated, the margin of safety s_m is defined as

$$s_m=s_o-s \tag{6}$$

EFFECT OF CONSTANT MODULUS OF ELASTICITY ON CENTRIFUGAL STRESSES

The equilibrium equation (7) and the compatibility equation (8) are given in reference 1.

$$\frac{d}{dr}(rh\sigma_r)-h\sigma_t+\rho\omega^2hr^2=0 \tag{7}$$

$$\frac{d}{dr}\frac{\sigma_t}{E}-\frac{d}{dr}\frac{\mu\sigma_r}{E}+\frac{d}{dr}(\alpha\Delta T)-\frac{(1+\mu)(\sigma_r-\sigma_t)}{Er}=0 \tag{8}$$

Equation (7) contains no terms that involve the modulus of elasticity. In equation (8), the substitution of $\alpha=0$ and a constant value of modulus of elasticity establishes that the centrifugal stresses are not influenced by the magnitude of any constant modulus of elasticity because it cancels out of the compatibility equation and is not involved either in the equilibrium equation or in the boundary conditions.

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