NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT 932

EFFECT OF REYNOLDS NUMBER IN TURBULENT-FLOW RANGE ON FLAME SPEEDS OF BUNSEN BURNER FLAMES

By LOWELL M. BOLLINGER and DAVID T. WILLIAMS

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AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbreviation	Unit	Abbreviation
Length	l	meter	m	foot (or mile)	ft (or mi)
Time	t	second	s	second (or hour)	sec (or hr)
Force	F	weight of 1 kilogram	kg	weight of 1 pound	lb
Power	P	horsepower (metric)		horsepower	hp
Speed	V	kilometers per hour	kph	miles per hour	mph
		meters per second	mps	feet per second	fps

2. GENERAL SYMBOLS

W Weight$=mg$

g Standard acceleration of gravity$=9.80665$ m/s^2 or 32.1740 ft/sec^2

m Mass$=\frac{W}{g}$

I Moment of inertia$=mk^2$. (Indicate axis of radius of gyration k by proper subscript.)

μ Coefficient of viscosity

ν Kinematic viscosity

ρ Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m^{-4}-s^2 at 15° C and 760 mm; or 0.002378 lb-ft^{-4} sec^2

Specific weight of "standard" air, 1.2255 kg/m^3 or 0.07651 lb/cu ft

3. AERODYNAMIC SYMBOLS

S Area

S_w Area of wing

G Gap

b Span

c Chord

A Aspect ratio, $\frac{b^2}{S}$

V True air speed

q Dynamic pressure, $\frac{1}{2}\rho V^2$

L Lift, absolute coefficient $C_L=\frac{L}{qS}$

D Drag, absolute coefficient $C_D=\frac{D}{qS}$

D_0 Profile drag, absolute coefficient $C_{D_0}=\frac{D_0}{qS}$

D_i Induced drag, absolute coefficient $C_{D_i}=\frac{D_i}{qS}$

D_p Parasite drag, absolute coefficient $C_{D_p}=\frac{D_p}{qS}$

C Cross-wind force, absolute coefficient $C_C=\frac{C}{qS}$

i_w Angle of setting of wings (relative to thrust line)

i_t Angle of stabilizer setting (relative to thrust line)

Q Resultant moment

Ω Resultant angular velocity

R Reynolds number, $\rho\frac{Vl}{\mu}$ where l is a linear dimension (e.g., for an airfoil of 1.0 ft chord, 100 mph, standard pressure at 15° C, the corresponding Reynolds number is 935,400; or for an airfoil of 1.0 m chord, 100 mps, the corresponding Reynolds number is 6,865,000)

α Angle of attack

ϵ Angle of downwash

α_0 Angle of attack, infinite aspect ratio

α_i Angle of attack, induced

α_a Angle of attack, absolute (measured from zero-lift position)

γ Flight-path angle

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EFFECT OF REYNOLDS NUMBER IN TURBULENT-FLOW RANGE ON FLAME SPEEDS OF BUNSEN BURNER FLAMES

By Lowell M. Bollinger and David T. Williams

SUMMARY

The effect of flow conditions on the geometry of the turbulent Bunsen flame was investigated. Turbulent flame speed is defined in terms of flame geometry and data are presented showing the effect of Reynolds number of flow in the range of 3000 to 35,000 on flame speed for burner diameters from ¼ to 1⅛ inches and three fuels—acetylene, ethylene, and propane.

The normal flame speed of an explosive mixture was shown to be an important factor in determining its turbulent flame speed, and it was deduced from the data that turbulent flame speed is a function of both the Reynolds number of the turbulent flow in the burner tube and of the tube diameter.

INTRODUCTION

A flame advancing through an explosive mixture at rest or in laminar flow assumes, at least initially, a smooth combustion front. The speed normal to the front and relative to the adjacent unburned mixture with which this front advances is known as the normal flame speed of the mixture (or transformation velocity). This speed is constant for a given set of physical and chemical conditions.

In turbulent flow, the combustion front is not smooth because it is disturbed by the fluctuating components of velocity. Yet if a surface sufficiently large in comparison with the scale of turbulence is considered, the average position of the combustion front advances with some definite speed normally to itself. This speed will be called the turbulent flame speed and its value will probably depend upon the aerodynamic as well as the physical and chemical conditions of the explosive mixture.

Because turbulence can greatly increase the rate of burning of fuel-air mixtures, an exact knowledge of the influence of turbulence on combustion is highly important to practical applications. Some information has been gathered from tests on internal-combustion engines (reference 1). In these investigations, no attempt was made to relate the rate of burning to any fundamental measurable property of the turbulence. The degree of turbulence of the charge in the engine cylinder was varied by changing engine speed or cylinder geometry. Two investigators, Damköhler (reference 2) and Shelkin (reference 3), each have proposed a theory to relate flame speed to turbulence. These theories require evaluation with experimental data obtained under well-defined conditions.

A possible method of investigating the relation between turbulence and rate of burning is by measurement of rate of flame propagation in a steady-flow Bunsen type burner. It is possible in such an apparatus to create forms of turbulence that have been quantitatively investigated. Furthermore, it is common to determine normal flame speed as the volume rate of flow of explosive mixture divided by the surface area of the inner cone of a laminar Bunsen burner flame. (See reference 4 for a discussion of the method.) This technique for measuring flame speed can also be used in the range of turbulent flow. Damköhler (reference 2) carried out some measurements in this manner, but his data are insufficient to establish conclusively a theory.

In experiments conducted at the NACA Cleveland laboratory during 1945, flame speeds were experimentally determined by the Bunsen burner technique with fully developed turbulent flow in the burner tubes. Data were obtained for three fuels, for burner diameters from ¼ to 1⅛ inches, and for Reynolds numbers in the range from 3000 to 35,000. In order to show the effect of turbulence on flame speed, these turbulent flame speeds were correlated with normal flame speed, tube diameter, and Reynolds number. In addition, the results were compared with the theories of references 2 and 3.

APPARATUS AND PROCEDURE

EQUIPMENT

The apparatus used in this investigation is diagrammatically shown in figure 1. Fuel and air were metered separately, then thoroughly mixed, and conducted into the Bunsen burner tube, which was long enough for fully developed turbulent flow to result at the burner outlet. The burners used were a series of four smooth seamless steel tubes, which had the following dimensions:

Nominal diameter (in.)	Inside diameter, d (cm)	Length, L (cm)	Ratio of length to inside diameter, L/d
¼	0.626	125	200
⅜	.943	215	228
⅝	1.579	215	136
1⅛	2.843	215	76

The tubes were cooled to room temperature by water jackets.

In order to prevent blow-off of the Bunsen flame at high flows, it was necessary to surround it by an auxiliary flame.

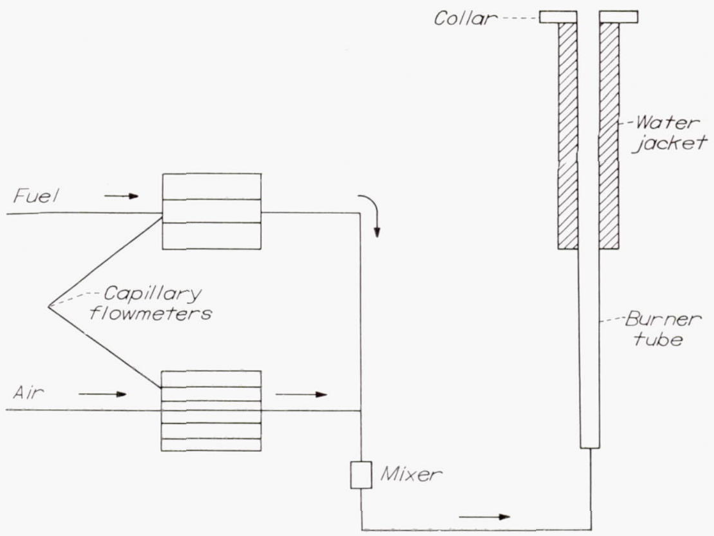


FIGURE 1.—Diagram of burner air-fuel system.

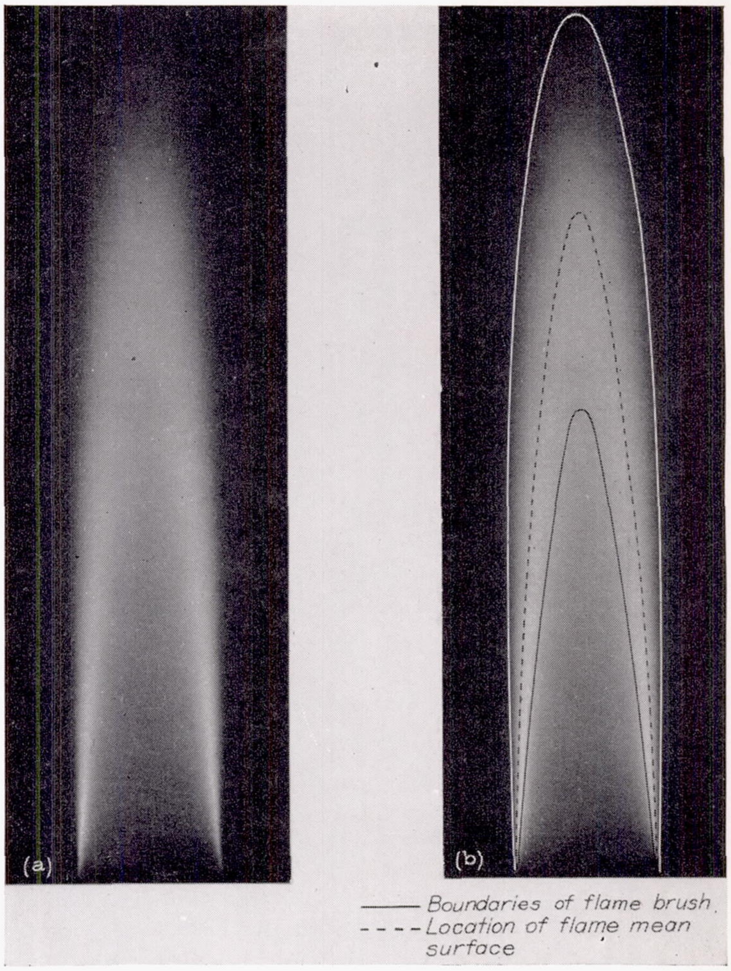
An annular-shaped burner was formed around the lip of the Bunsen burner by surrounding it with a tube of somewhat larger diameter. A fuel-air mixture at low velocity flowed through this auxiliary burner; the fuel in the auxiliary burner was the same as that in the main burner. This mixture burned at the outlet of the burner and kept the inner Bunsen flame seated to the lip. The auxiliary flame was always kept as low as possible so as to reduce to a minimum the errors caused by this flame. The auxiliary flame seemed to have no appreciable effect on the main body of the Bunsen flame in that, where flow conditions permitted, no change could be noticed in the Bunsen flame when the auxiliary burner was suddenly shut off.

FUELS

Measurements were made using three commercial-grade fuels—acetylene, ethylene, and propane. Because acetylene is dissolved in acetone in the tank, an appreciable amount of acetone is present in the gaseous fuel. Most of this acetone was removed by bubbling the acetylene-acetone gaseous mixture through flowing water. The combustion air used came from the laboratory service air supply, which, when expanded, had a relative humidity of approximately 15 percent. No temperature control was used; very little variation of temperature, however, was observed.

METHOD OF MEASURING FLAME SPEED

The combustion zone of a laminar Bunsen flame is thin and clear-cut, but that of a turbulent Bunsen flame consists of a "brush" of flame having a roughly conical shape but apparently formed of a rapidly fluctuating, much folded surface (fig. 2(a)). Nevertheless, certain surfaces can be identified in the turbulent flame; for example, in figure 2(b) an inner and outer envelope of the flame and a mean surface are indicated. A flame speed corresponding to each of these surfaces may be determined. Although Damköhler (reference 2) believed that the inner and outer envelopes are related to the turbulent and normal flame speed, respectively, it is considered herein that the flame front has some average position around which it fluctuates; the degree of fluctuation determines the position of the inner and outer



(a) Flame.

(b) Flame showing brush and flame mean surface outlined.

FIGURE 2.—Turbulent Bunsen burner flame. Nominal diameter of burner, ⅜ inch; ethylene gas in air.

envelopes. The turbulent flame speed should then be that flame speed corresponding to the surface that is the average position of the flame front, or

$$u_t = Q/S \tag{1}$$

where

u_t turbulent flame speed

Q volume rate of flow

S surface area of mean position of burning

The volume rate of flow was obtained from the flowmeter readings. The surface area of any given turbulent flame was determined in the following manner: The flame was photographed on 5- by 7-inch film with an exposure time of about 2 seconds. An average flame surface was drawn on the negative of the photograph and an attempt was made to draw the surface halfway between the inner and outer envelopes of the flame brush (fig. 2(b)). The surface area was then determined by the approximate equation for cone-like surfaces of revolution.

$$S = \pi A \frac{L}{h}$$

where

A area of longitudinal cross section of flame as photographed

L length of generating curve, excluding base, of cone-like surface

h height

This method has been used by others for laminar flames (reference 5).

The amount of work required to determine the surface area was shortened by plotting S/d^2 against h/d (where d is the burner-tube diameter). Once enough points had been plotted to form a curve, the surface area could be found simply by measuring the height of the flame cone. The height was measured directly on the flame by means of a horizontal sliding arm mounted on a scale with sights placed 18 inches apart on the arm. In this way, the whole flame could be kept within the field of view and fair reproducibility was possible.

Some doubt may exist as to the physical significance of a flame speed determined by the method outlined; especially, it may be that, although the idea of an average surface is sound, the position halfway between the inner and outer envelopes as seen on the photographs is not a true average surface. The data will therefore be presented first in terms of flame geometry and then in terms of flame speed.

The observations were made on fuel-air mixtures in each case having the maximum flame speed. Each measurement was made by use of a series of three or four observations in which the fuel-air ratio was varied in the neighborhood of that for maximum flame speed. The flame speed observed was taken as the maximum on a curve drawn through the data so taken. By this means any errors due to inaccuracy of flow-measuring devices were eliminated.

RESULTS

GEOMETRY OF TURBULENT BUNSEN FLAMES

Near the base a turbulent Bunsen flame is almost laminar in appearance. Higher up the flame front develops waviness; and still farther up individual peaks of flame are observed to shoot up from the interior of the inner cone. When these peaks of flame appear, the outer envelope begins to bend more sharply inward. These characteristics together with the three dimensionality of the flame make it difficult or impossible to locate the position of the inner envelope with certainty.

The ratio of mean flame surface area to the square of the burner inside diameter is plotted against the ratio of flame height to burner inside diameter in figure 3. The points for various burner sizes and a given fuel fall on a single curve when plotted in this manner. These curves indicate that the flame broadens out as the height becomes greater. For if, in becoming higher, all vertical dimensions of a flame were merely stretched, the curve of surface area against height would approximate a straight line through the origin, which is obviously not the case.

For any one fuel, the height of the average position of a flame is roughly a function of Reynolds number. (See fig. 4.) In every case, however, the curves of the smaller burners lie higher. The fuel-air ratio is in each case that corresponding to the maximum flame speed.

Some data of the height of the outer envelope of the flame brush were recorded. This height minus the height of the average surface is one-half the flame-brush width in the center of the tube, which has been plotted against Reynolds number

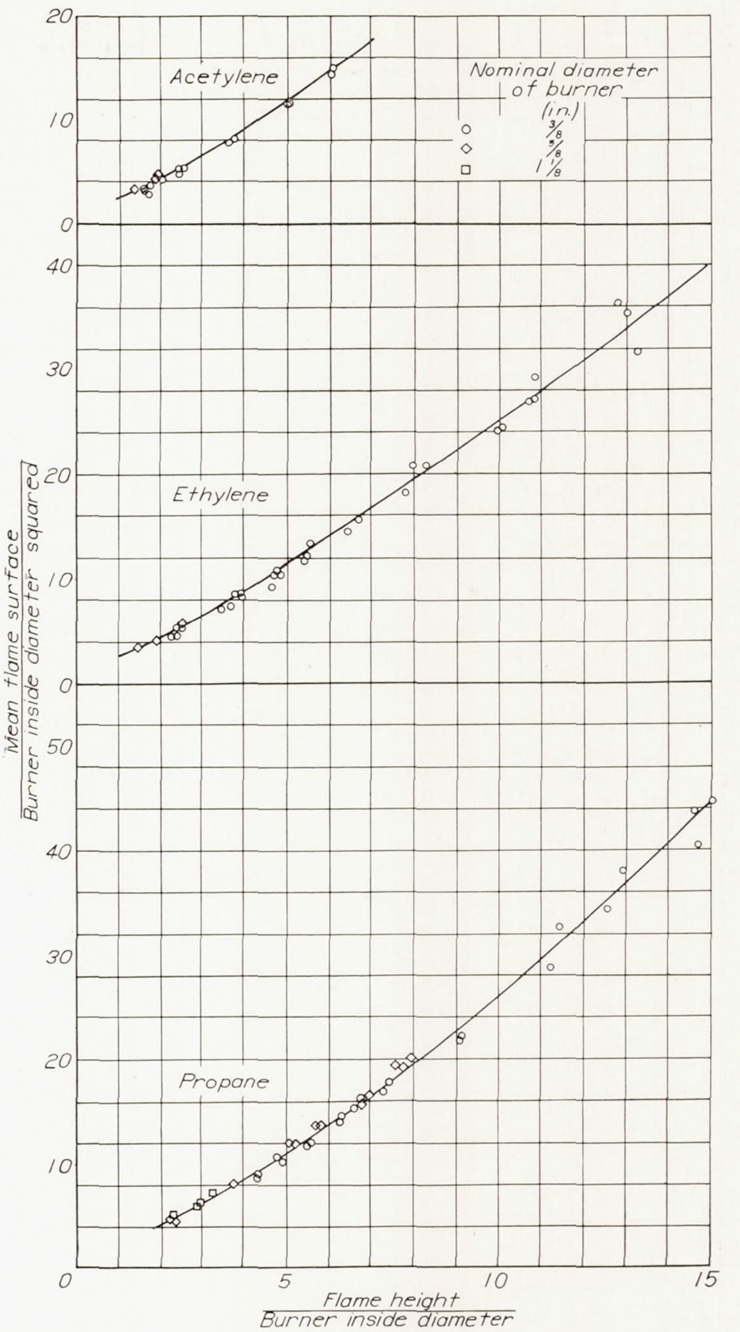


FIGURE 3.—Variation of ratio of mean flame height to burner inside diameter squared with ratio of flame height to burner inside diameter.

in figure 5. The large scatter is due to the uncertainty in locating the heights of the outer and inner envelopes.

TURBULENT FLAME SPEEDS

Examples of the variation of flame speed, as previously defined, with fuel concentration for various Reynolds numbers are shown in figure 6. The data shown were obtained by direct measurement of mean flame height. The fuel concentration for maximum flame speed is observed to be slightly richer for turbulent than for laminar flames. This effect is believed to be caused by the turbulent flame seeming to widen out slightly as it becomes richer so that, for the same mean surface area, the height of a rich flame is less than that of a lean flame.

The variation of flame speed with Reynolds number of pipe flow is shown in figure 7 for fuel concentrations of maxi-

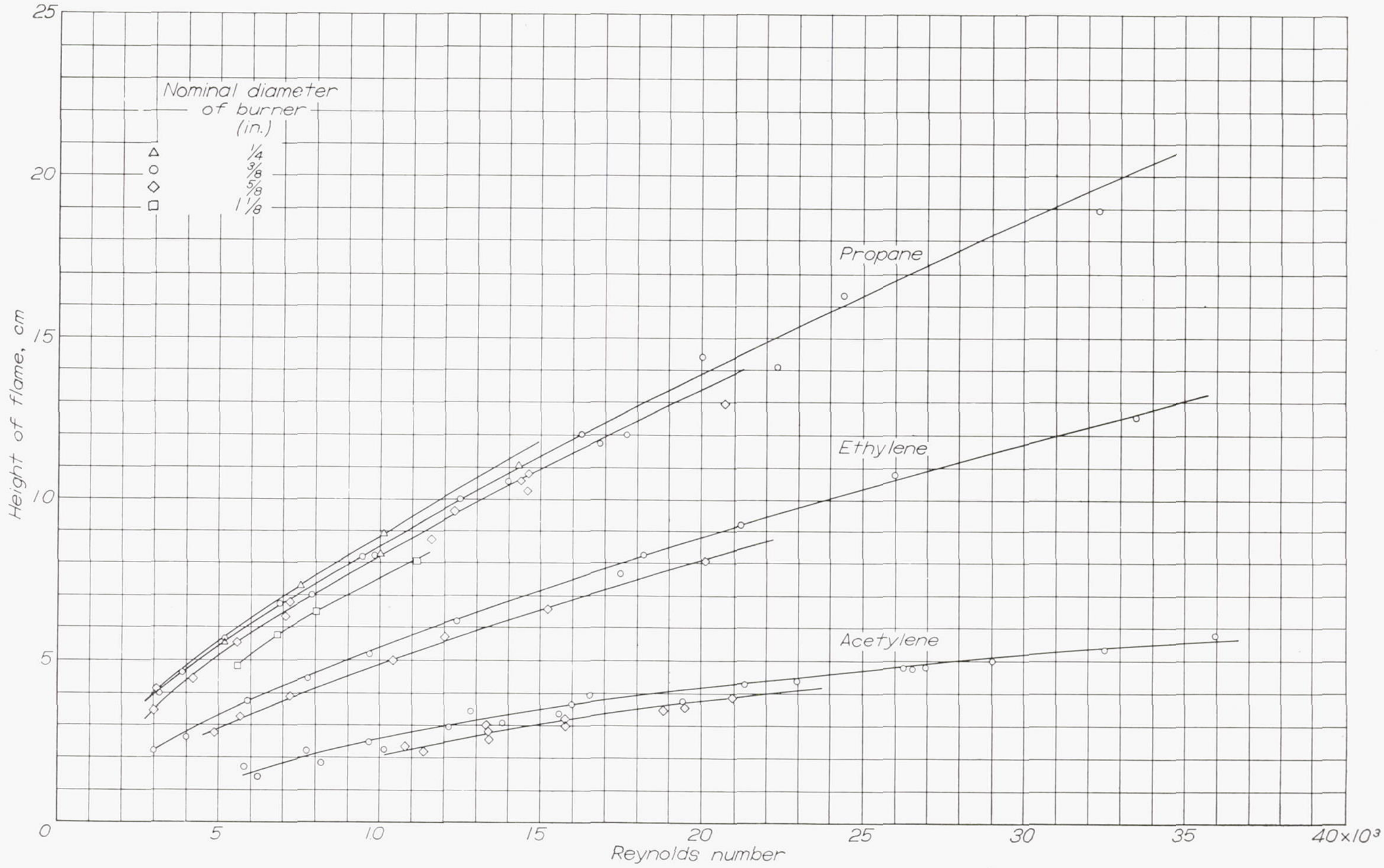


FIGURE 4.—Variation of mean flame height with Reynolds number.

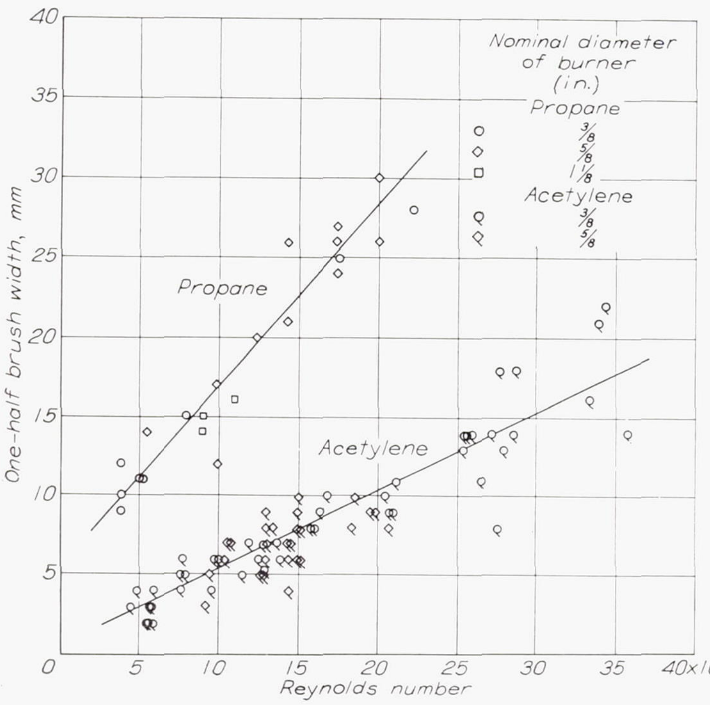


FIGURE 5.—Variation of flame-brush width with Reynolds number.

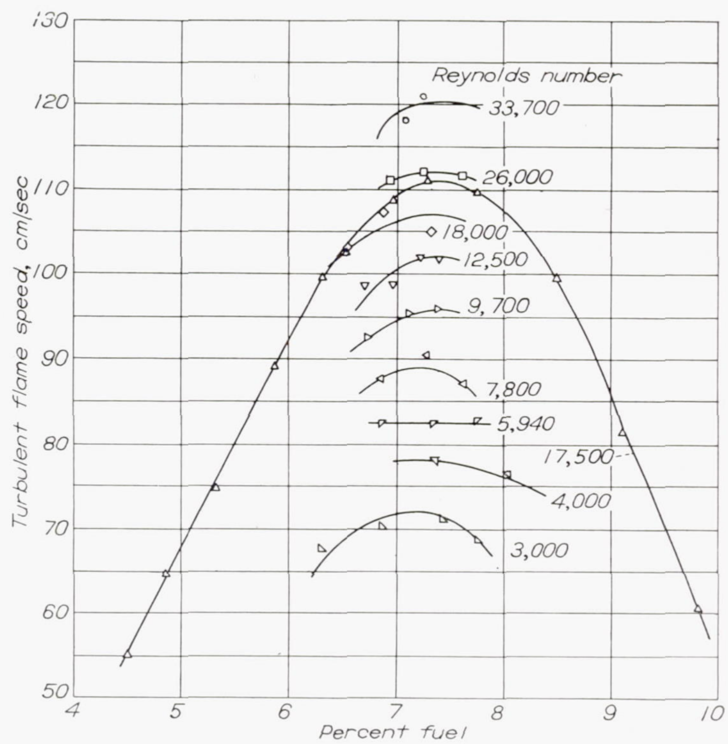


FIGURE 6.—Variation of flame speed with fuel concentration. Burner diameter, 3/8 inch; fuel, ethylene in air.

mum flame speed. Values of normal flame speed u_n from reference 4 are indicated for the three fuels by points on the ordinate. Reynolds number of flow is, however, not necessarily descriptive of turbulence at the flame front. The data of figure 7 were obtained both by photographing the flame and by direct measurement of flame height; in both cases the mean surface area was obtained from the curves in figure 3.

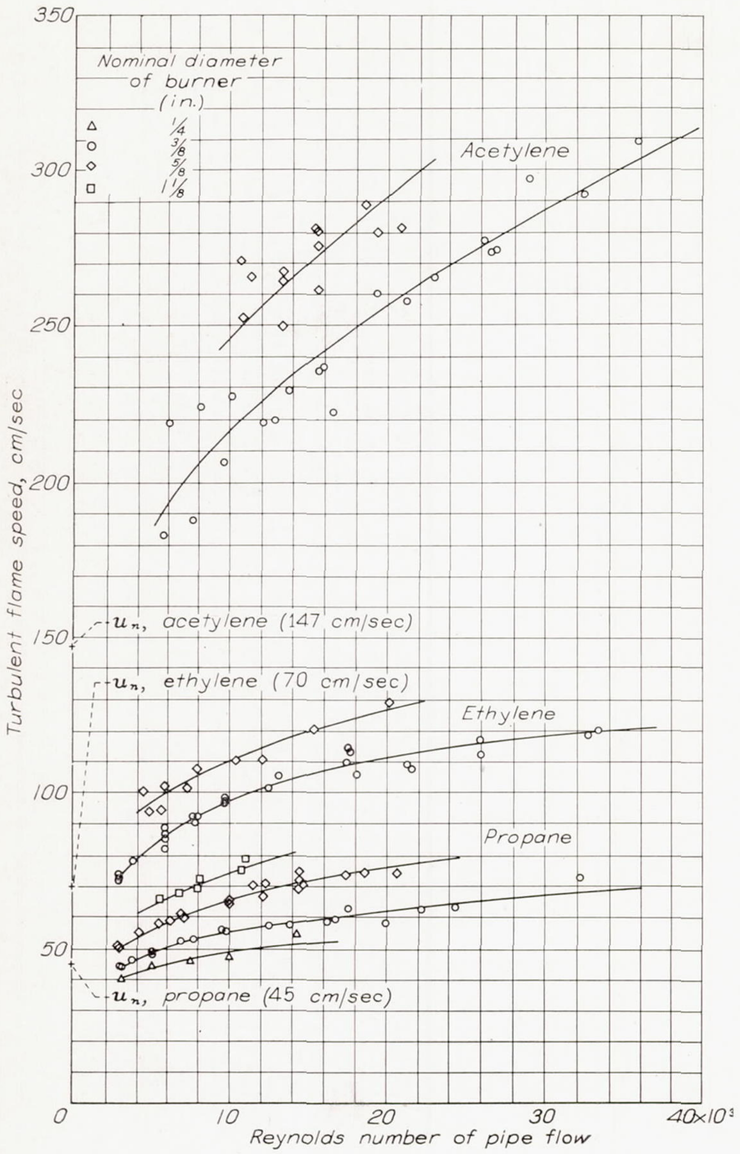


FIGURE 7.—Variation of flame speed with Reynolds number of flow. (Normal flame speed u_n from reference 4.)

The points representing data taken by direct measurement are the maximum flame speeds from curves like those of figure 6 and are therefore, in most cases, averages of several measurements. Most of the points representing photographic data are averages of two separate flame-speed determinations. The large scatter of the acetylene flame speeds for the 3/8-inch-diameter burner at low Reynolds numbers is believed due to the fact that, because the flame was small, a small absolute error in height measurement resulted in a large relative error.

The data of figure 7 show several consistent characteristics. For any one explosive mixture, the turbulent flame speeds are, in general, greater than the normal flame speed; for a given Reynolds number, the flame speed increases with increasing burner diameter. In general, flame speed increased with Reynolds number but at a decreasing rate, and at the larger values of Reynolds number, flame speed approached asymptotically a linear relation with Reynolds number.

DISCUSSION

NATURE OF TURBULENCE IN PIPE FLOW

The fact that the flame speed as defined in turbulent flow cannot be determined with very high accuracy is a circumstance that is implicit in the very nature of turbulence. The results of the measurements are principally an identification of certain trends in turbulent flame speed as summarized. It is thus desirable to identify the nature of the turbulence in which the measurements were made.

The theory of turbulent flow in general is reviewed in references 6 to 8. It is concluded in the theory, particularly in reference 8, that a scale factor and an intensity of turbulence are sufficient to characterize isotropic turbulence. These two factors have been measured in various ways. The turbulent intensity based on longitudinal turbulent motion is shown in dimensionless form in figure 8, as measured between two infinite planes (reference 9); the distribution is expected to be similar to that in a tube. It is notable that the turbulence in the tube is not completely isotropic so that the mean turbulent intensities are different in different directions.

The mixing length has also been measured and is shown in figure 8 as published in reference 6. It is evident that both

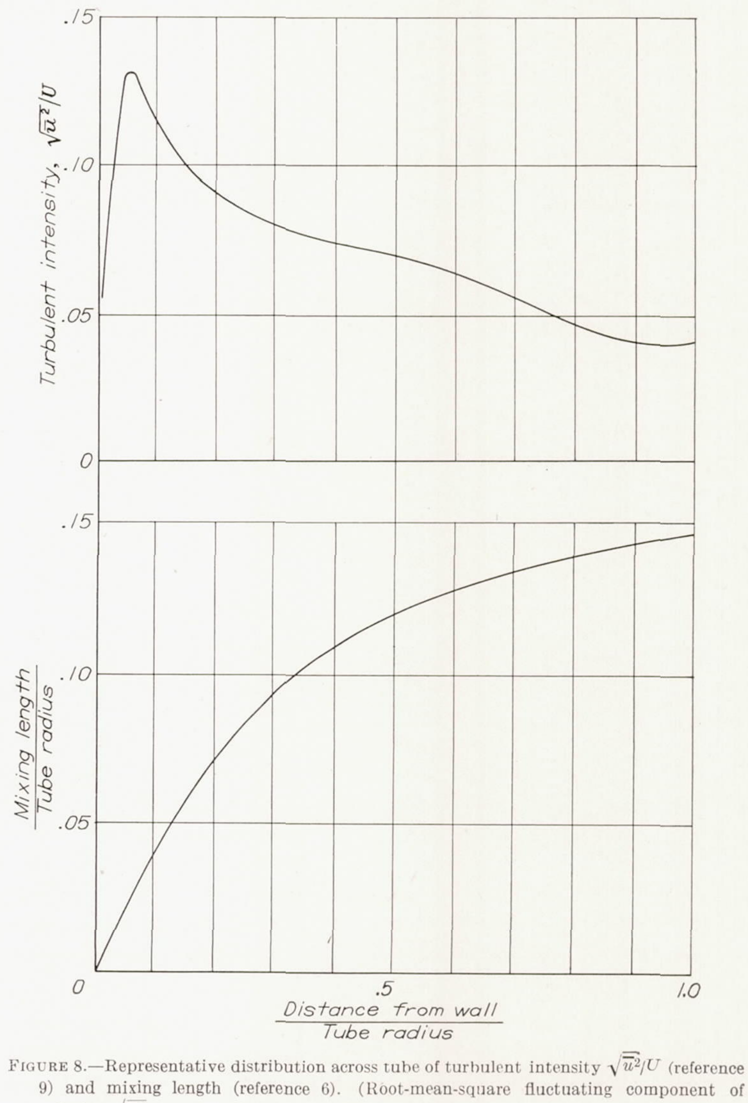


FIGURE 8.—Representative distribution across tube of turbulent intensity $\sqrt{\overline{u^2}}/U$ (reference 9) and mixing length (reference 6). (Root-mean-square fluctuating component of velocity $\sqrt{\overline{u^2}}$; mean gas-flow velocity U.)

mixing length and turbulence vary widely across the flow. It follows that, insofar as the turbulent flame speed is to be associated with the nature of the turbulence, it is a mean flame speed that is associated with an average type of turbulence in the gas issuing from a uniform tube.

In measurements of flame speed in turbulent flow, the scale of the turbulence in the tube is proportional to the size of the tube; the intensity component is proportional to the mean gas flow velocity, as follows from the general similarity relation.

THEORIES OF TURBULENT FLAME SPEED

Damköhler (reference 2) in an attempt to determine the effect of turbulence on flame speed presents some data obtained from a Bunsen propane-oxygen flame on a burner tube that was long enough to have fully developed turbulent flow. It was found that flame speeds corresponding to the outer envelopes were approximately equal to the normal flame speed of the mixture. The flame speed corresponding to the inner envelope was assumed to be the turbulent flame speed. The ratio of the flame speeds corresponding to the inner and outer envelopes was therefore considered equal to the ratio of turbulent flame speed to normal flame speed. Results from three burners of 1.385-, 2.180-, and 2.718-millimeter diameters were obtained. The flame-speed ratio was then shown by somewhat limited data to vary linearly with the square root of the Reynolds number $\sqrt{Re}$ for the two smaller tubes in the range $2300<Re<5000$ and linearly with Re for the two larger tubes in the range $5000<Re<18,000$.

In the discussion of results of reference 2, two conditions of turbulence are considered that depend on whether the scale of the turbulence is much greater or much less than the thickness of the laminar flame front. Fine-scale turbulence was considered to influence the rate of diffusion between the burned and unburned gas without roughening the flame front, and large-scale turbulence was considered only to roughen the flame front, thus increasing its surface area. The theory developed attempted to show that for fine-scale turbulence,

$$\frac{u_t}{u_n}=\sqrt{\frac{\epsilon}{\nu}} \tag{3}$$

and that for large-scale turbulence

$$\frac{u_t}{u_n}\simeq\epsilon \tag{4}$$

where

ϵ turbulent diffusion coefficient

ν kinematic viscosity of explosive mixture

The turbulent diffusion coefficient ϵ is an empirical factor that was devised to develop a theory for the turbulent-velocity distribution in a pipe or tube. By analogy with the molecular diffusion coefficient, it is written as

$$\epsilon=l\sqrt{\overline{u}^2} \tag{5}$$

where

l so-called mixing length, scale factor

$\sqrt{\overline{u}^2}$ root-mean-square fluctuating component of velocity

In the Kármán theory of pipe turbulence, the factors comprising ϵ enter into the theory as empirical parameters evaluated from the velocity distribution, but equivalent or analogous quantities are directly measurable. It has been shown by direct observation that in turbulent pipe flow, $\sqrt{\overline{u}^2}$ varies as the mean gas velocity, and that the scale, and therefore presumably l, varies as the pipe diameter. Both quantities are hardly approximately constant across the flow; but in predicting the results of any experiments dependent on ϵ, it is apparent that mean ϵ must vary as the Reynolds number of the pipe flow.

In reference 2, it is considered that the smallest burner used gave fine-scale turbulence; whereas the largest burner gave large-scale turbulence, the difference of which was believed to account for the difference in the dependence of flame-speed ratio on Reynolds number.

A theoretical analysis of the effect of turbulence on flame speed is presented in reference 3. The two cases of fine-scale and large-scale turbulence are again considered. Based upon the same considerations as in reference 2, the expression for turbulent flame speed u_t in fine-scale turbulence was derived

$$u_t=u_n\sqrt{1+\frac{\epsilon}{K}} \tag{6}$$

where K is the thermal conductivity due to molecular motion.

For the case of large-scale turbulence, the expression for u_t was derived to be

$$u_t=u_n\sqrt{1+B\,\frac{\overline{u}^2}{u_n{}^2}} \tag{7}$$

where B is a nondimensional coefficient and is approximately unity. It should be noted that flame speed for large-scale turbulence is here predicted to be independent of the scale of turbulence and to be independent of the normal flame speed if the mean square fluctuating component of velocity $\overline{u}^2$ becomes great compared to $u_n{}^2$.

COMPARISON WITH THEORIES OF EFFECT OF TURBULENCE ON FLAME SPEED

According to the theories of references 2 and 3, the flame speed in turbulent flow is predicted to depend on the turbulent diffusion coefficient ϵ for small-scale turbulence. It has been shown that $\epsilon=l\sqrt{\overline{u}^2}$ and that ϵ should vary as the Reynolds number of flow Re inside the tube. If the change of ϵ is neglected in the space between the tube outlet and the flame, it follows that the turbulent flame speed is predicted for small-scale turbulence (a) in reference 2 to vary as $u_n\sqrt{Re}$, and (b) in reference 3 to vary as $u_n\sqrt{1+kRe}$ where k is a constant. Actually the theories as presented were somewhat more specific in their predictions. The predictions of reference 3, for example, would seem to approximate those of reference 2 because the second term in the radical is much larger than 1.

Again for large-scale turbulence, the predictions are that flame speed should vary as u_nRe (reference 2) and as $u_n\sqrt{1+B\,\frac{\overline{u}^2}{u_n{}^2}}$ (reference 3), where B is a constant of approximately 1. For the range of the experiments, u_n is approximately 1 foot per second, and $\overline{u}^2$ varies in the range of magnitude of u_n^2.

Summarizing the results of the flame-speed measurements, figure 7 serves to give some fairly definite indications as to whether the theories mentioned are accurate.

First, according to the theory of turbulence in pipes or tubes, the mixing length of the turbulent flows used is approximately 0.1 inch, which is definitely greater than the 0.001 inch usually considered to be the approximate thickness of the laminar flame. The data show that: (*a*) The Reynolds number is inadequate alone to correlate the turbulent flame speed; and (*b*) in any case the variation of the turbulent flame speed with Reynolds number for a given tube is nonlinear in the range of large-scale turbulence. It is seen that the results of this method of investigation differ from the theory of reference 2 concerning the dependence of flame speed on Reynolds number for large-scale turbulence in a Bunsen burner flame.

The turbulent intensities used are certainly large enough to indicate whether the turbulent flame speeds tend to be independent of the nature of the fuel at large values of $\overline{u}^2$. That is, the curves for various fuels are predicted in reference 3 to converge at higher Reynolds numbers; however, they show no sign of converging in the range of this investigation.

NATURE OF VARIATION OF TURBULENT FLAME SPEED

The data of figure 7 obviously require more than one correlation variable. The fuel, the velocity, and the burner diameter are one possible set of parameters; the fuel, the Reynolds number, and the burner diameter are a possible alternative choice. Reynolds number rather than velocity is used herein because of the general use of Reynolds number in the analysis of fluid behavior and because previous proposed theories used it. In any case, the choice of variables with which to correlate the data is arbitrary.

The general appearance of the data suggests u_t/u_n as a possible variable to express the effect of the fuel.

A second trend is indicated by the fact that u_t at any value of Re is greater with larger tube diameter by a factor that varies proportionally with u_n.

A third phenomenon is indicated by the curvature of the various graphs, suggesting that u_t might vary as a power of Reynolds number different from 1, for given fuel and burner diameter.

The data of figure 7 are used to fix the manner of variation of u_t in the form

$$u_t = C u_n^a d^b Re^c \tag{8}$$

where

u_t turbulent flame speed, (cm/sec)
C constant, empirically determined from data
u_n normal flame speed, (cm/sec)
d burner diameter, (cm)
Re burner Reynolds number based on mean air speed and burner diameter
a, b, c exponents, empirically determined from data

The exponent of u_n is chosen by inspection as equal to 1. The exponent of the diameter d is fixed by cross-plotting the data of figure 7. The flame speeds u_t taken from the curves for the various tube sizes were compared for given Reynolds number and fuel. The comparison is shown in graphical form in figure 9. Relative flame speeds were compared for 11 different fuel—Reynolds number choices with relative diameter for the ⅝-inch-diameter tube. The log plot is seen to be fairly representable as a straight line, though strictly speaking a slight curvature is noted through the points. The line as shown has a slope equal to 0.26

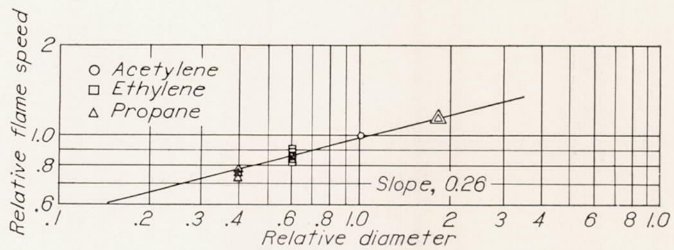


FIGURE 9.—Variation of relative flame speed with relative burner diameter, based on ⅝-inch burner.

so that the exponent b of the burner diameter in equation (8) is

$$b = 0.26$$

The value of the exponent c of Reynolds number in equation (8) is found by making log plots of the quantity

$$y = \frac{u_t}{u_n d^{0.26}} \tag{9}$$

as a function of the burner-tube Reynolds number Re. The data for acetylene, ethylene, and propane are plotted in figures 10(a), 10(b), and 10(c), respectively. The lines considered by inspection to represent the data most accurately are indicated on the curves. The three curves are shown on figure 10(d) with the curve obtained by averaging the three. The correlation represented by the average curve is suitable as approximating all the data. The scatter of experimental points is a little greater than would be anticipated from the methods used.

The equation of the average curve (fig. 10(d)) is

$$\frac{u_t}{u_n d^{0.26}} = 0.18\, Re^{0.24}$$

or

$$u_t = 0.18\, u_n d^{0.26} Re^{0.24} \tag{10}$$

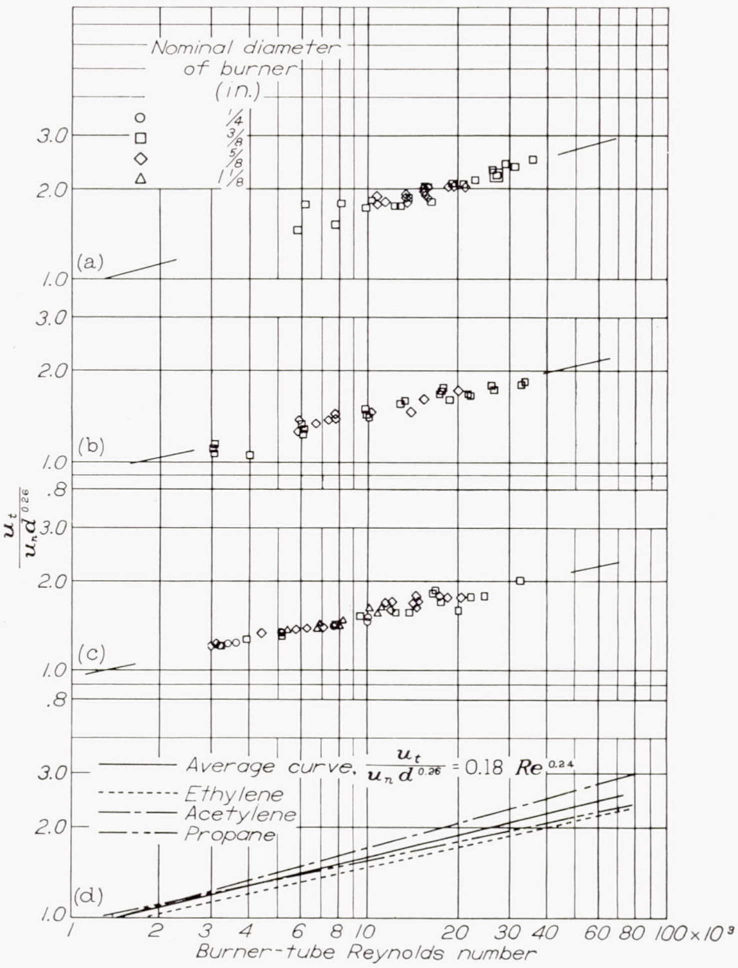
Any uncertainty as to the exact relation among the chosen variables might be removed by an extension of the experiments. It is possible, however, that the same kind of experimental uncertainty is to be anticipated in any study of turbulent flames.

The results here shown will suggest some approximate relations for use in future extensions of the theory. Thus, approximately

$$u_t \propto u_n$$

$$u_t \propto \sqrt{l}$$

$$u_t \propto \sqrt[8]{\overline{u^2}}$$



(a) Acetylene in air.
(b) Ethylene in air.
(c) Propane in air.
(d) Comparison of fuels.

FIGURE 10.—$\frac{u_t}{u_n d^{0.26}}$ as function of burner-tube Reynolds number. (Turbulent flame speed u_t, cm/sec; normal flame speed u_n, cm/sec; diam. d, cm)

The direct dependence of turbulent flame speed on laminar flame speed is in agreement with reference 2.

SUMMARY OF RESULTS

From an investigation to determine the effect of Reynolds number in the turbulent-flow range on flame speeds of Bunsen burner flames, the following results were obtained:

1. A correlation of the flame speed in a Bunsen burner with several variables has been empirically derived in the turbulent-flow range at sea-level atmospheric conditions in the form

$$u_t=0.18u_n d^{0.26}Re^{0.24}$$

where

u_t turbulent flame speed, (cm/sec)
u_n normal flame speed (transformation velocity), (cm/sec)
d burner diameter, (cm)
Re Reynolds number based on mean mixture speed and d

The experiments ranged from a Re value of 3000 to 35,000, tube diameters from 0.626 to 2.843 centimeters, and the fuels were acetylene, ethylene, and propane. The fuel-air mixtures having maximum u_n were used.

2. The results obtained, based on the mean flame surface, agreed with the theories of Damköhler on the effect of large-scale turbulence on the flame speed in a Bunsen burner flame as to the dependence on laminar flame speed, but differed as to the dependence on Reynolds number.

3. The data showed no tendency for turbulent flame speeds of different fuels to approach one another at high flow rates, as given by the theory of Shelkin.

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CLEVELAND, OHIO, *June 30, 1948*.

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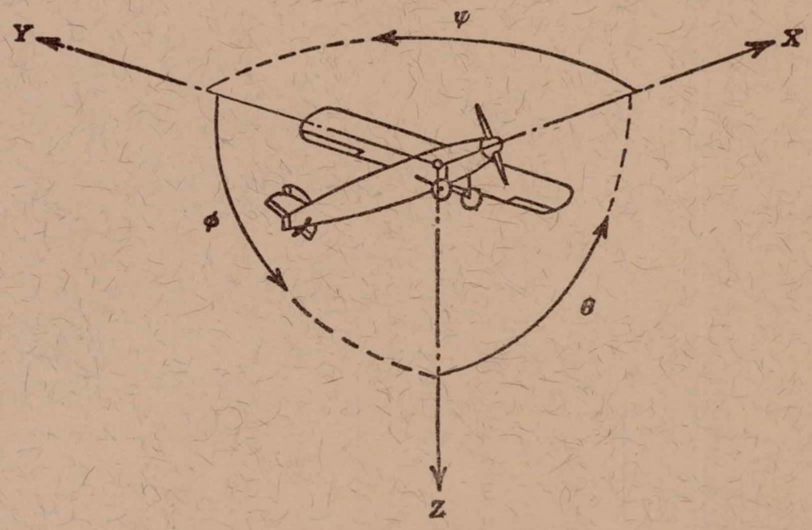

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	Rolling	L	$Y \longrightarrow Z$	Roll	ϕ	u	p
Lateral	Y	Y	Pitching	M	$Z \longrightarrow X$	Pitch	θ	v	q
Normal	Z	Z	Yawing	N	$X \longrightarrow Y$	Yaw	ψ	w	r

Absolute coefficients of moment

$C_l=\frac{L}{qbS}$ (rolling) $C_m=\frac{M}{qcS}$ (pitching) $C_n=\frac{N}{qbS}$ (yawing)

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D Diameter
p Geometric pitch
p/D Pitch ratio
V' Inflow velocity
V_s Slipstream velocity
T Thrust, absolute coefficient $C_T=\frac{T}{\rho n^2 D^4}$
Q Torque, absolute coefficient $C_Q=\frac{Q}{\rho n^2 D^5}$

P Power, absolute coefficient $C_P=\frac{P}{\rho n^3 D^5}$
C_s Speed-power coefficient$=\sqrt[5]{\frac{\rho V^5}{Pn^2}}$
η Efficiency
n Revolutions per second, rps
Φ Effective helix angle$=\tan^{-1}\left(\frac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS

1 hp=76.04 kg-m/s=550 ft-lb/sec
1 metric horsepower=0.9863 hp
1 mph=0.4470 mps
1 mps=2.2369 mph

1 lb=0.4536 kg
1 kg=2.2046 lb
1 mi=1,609.35 m=5,280 ft
1 m=3.2808 ft