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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT 944

HEAT TRANSFER TO BODIES TRAVELING AT HIGH SPEED IN THE UPPER ATMOSPHERE

By JACKSON R. STALDER and DAVID JUKOFF

1949

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AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbreviation	Unit	Abbreviation
Length	l	meter	m	foot (or mile)	ft (or mi)
Time	t	second	s	second (or hour)	sec (or hr)
Force	F	weight of 1 kilogram	kg	weight of 1 pound	lb
Power	P	horsepower (metric)		horsepower	hp
Speed	V	kilometers per hour meters per second	kph mps	miles per hour feet per second	mph fps

2. GENERAL SYMBOLS

W	Weight $=mg$
g	Standard acceleration of gravity $=9.80665$ m/s^2 or 32.1740 ft/sec^2
m	Mass $=\frac{W}{g}$
I	Moment of inertia $=mk^2$. (Indicate axis of radius of gyration k by proper subscript.)
μ	Coefficient of viscosity
ν	Kinematic viscosity
ρ	Density (mass per unit volume)
	Standard density of dry air, 0.12497 kg-m^{-4}-s^2 at 15° C and 760 mm; or 0.002378 lb-ft^{-4} sec^2
	Specific weight of "standard" air, 1.2255 kg/m^3 or 0.07651 lb/cu ft

3. AERODYNAMIC SYMBOLS

S	Area
S_w	Area of wing
G	Gap
b	Span
c	Chord
A	Aspect ratio, $\frac{b^2}{S}$
V	True air speed
q	Dynamic pressure, $\frac{1}{2}\rho V^2$
L	Lift, absolute coefficient $C_L=\frac{L}{qS}$
D	Drag, absolute coefficient $C_D=\frac{D}{qS}$
D_0	Profile drag, absolute coefficient $C_{D_0}=\frac{D_0}{qS}$
D_i	Induced drag, absolute coefficient $C_{D_i}=\frac{D_i}{qS}$
D_p	Parasite drag, absolute coefficient $C_{D_p}=\frac{D_p}{qS}$
C	Cross-wind force, absolute coefficient $C_C=\frac{C}{qS}$
i_w	Angle of setting of wings (relative to thrust line)
i_t	Angle of stabilizer setting (relative to thrust line)
Q	Resultant moment
Ω	Resultant angular velocity
R	Reynolds number, $\rho\frac{Vl}{\mu}$ where l is a linear dimension (e.g., for an airfoil of 1.0 ft chord, 100 mph, standard pressure at 15° C, the corresponding Reynolds number is 935,400; or for an airfoil of 1.0 m chord, 100 mps, the corresponding Reynolds number is 6,865,000)
α	Angle of attack
ϵ	Angle of downwash
α_0	Angle of attack, infinite aspect ratio
α_i	Angle of attack, induced
α_a	Angle of attack, absolute (measured from zero-lift position)
γ	Flight-path angle

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Ames Aeronautical Laboratory
Moffett Field, Calif.

National Advisory Committee for Aeronautics

Headquarters, 1724 F Street NW., Washington 25, D. C.

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SUMMARY

A general method has been developed, using the methods of kinetic theory, whereby the surface temperatures of bodies can be calculated for steady flight at any speed in a rarefied gas. The particular solution was made for a flat plate; however, the calculations can be easily extended to bodies of arbitrary shape.

It was found that the aerodynamic heating problem in the absence of solar radiation, that is, for the case of nocturnal flight, becomes of negligible importance at altitudes of 125 miles and higher and up to steady flight speeds of 36,000 feet per second. The effect of solar radiation, for the case of daytime flight, becomes increasingly important as the flight altitude is increased. At an altitude of 150 miles and higher, solar radiation is the predominating factor that determines skin temperature. Owing to the strong effect of solar radiation on skin temperatures at high altitudes, the desirability of nocturnal flight is indicated in order to minimize skin temperatures.

In order to maintain low skin temperatures, it was found that the angle of inclination of the body with respect to the flight path should be kept as small as possible. This may be accomplished in practice by designing the body to be finely tapered and by flying the body at small angles of attack.

It is pointed out that skin temperatures may be reduced by insuring thermal contact between portions of the skin inclined at positive and negative angles with respect to the flight path. As much surface as possible should be inclined at negative angles. Practically, this may be accomplished by boattailing the body.

In the event that an internal skin-cooling system is employed, it is shown that the rate of internal cooling must be of the same order of magnitude or greater than the rate at which heat is lost naturally by emitted radiation. If the cooling rate is below the natural radiation rate, cooling has little effect upon skin temperatures.

It is shown that, in the case of a missile designed to fly over a wide range of altitudes and speeds, it is desirable to make the emissivity of the skin as high as possible. This conclusion, however, is based upon a skin surface for which the emissivity is independent of the wave length of the emitted and absorbed radiant energy.

A possible method of reducing surface temperatures is indicated by the decrease in skin temperature which accompanies a decrease in thermal accommodation coefficient. This phenomenon may be used to advantage if it is possible to decrease the accommodation coefficient by altering the surface characteristics of the skin.

INTRODUCTION

Missile or aircraft flight at extremely high altitudes introduces many new problems to the designer. One of the more important problems that arises is due to the skin temperatures that are attained at very high velocities. The severity of the problem is commonly exemplified by consideration of the disintegration by combustion of most of the meteors that enter the earth's atmosphere at high speed.

In reference 1, Tsien has summarized the meager amount of research that has been done in the field of rarefied gas kinetics as applied to aeronautics. It is shown that the general field of aeronautics may be divided into three major regimes, and that the division lines, although poorly defined at present due to lack of experimental data, are marked by the ratio of the mean free molecular path length to a characteristic body dimension.

The three main regimes of aerodynamics, or gas kinetics, may be described very briefly as:

1. The region of conventional aerodynamics in which the mean free path length is negligible in comparison with the body size.
2. The region of free-molecule phenomena in which the mean free path is large with respect to the body size.
3. The so-called "slip flow" regime, intermediate between the conventional aerodynamics and free-molecule regimes.

In conventional aerodynamic theory, the first basic assumption that is made is that the fluid under consideration may be treated as a homogenous continuous medium. This assumption certainly should become invalid at very high altitudes owing to the extreme rarefaction of the atmosphere. A measure of the atmospheric density is furnished by the average length of path (mean free molecular path) that is traveled by a molecule between successive collisions. The mean free molecular path has been computed by Johnson and Slack in reference 2 for altitudes up to 150 miles. The result of their computations is shown in figure 1. The existence of a large mean free path should not be construed as an indication that the number of molecules per unit volume is small, in the ordinary sense of the word. For example, at a reduced density at which the mean free path is 10 feet, there are still 10^{13} molecules contained in a cubic inch.

The work of this report will be confined to a study of the heat-transfer processes occurring in the region of free molecule flow. The lower limit of the ratio of mean free path to a characteristic body dimension at which the derived theory becomes invalid cannot be stated at this time for

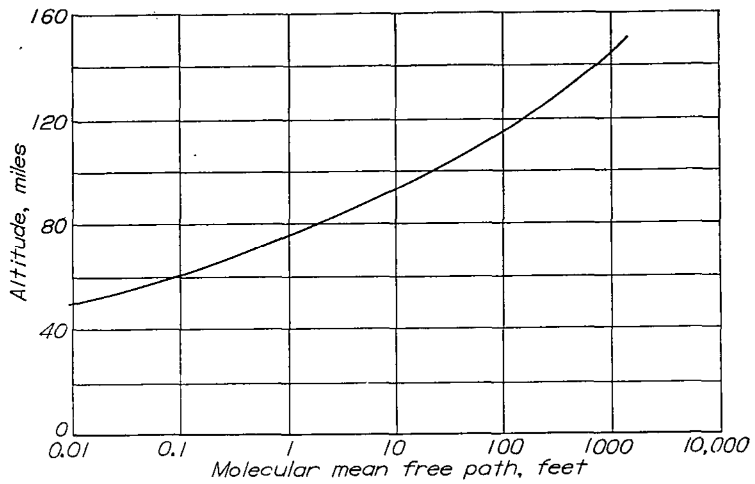

FIGURE 1.—Variation of the molecular mean free path of air with altitude.

lack of experimental evidence. The numerical computations, however, will be confined to altitudes of 75 miles and higher. The estimated mean free path at 75 miles altitude is 1 foot.

The specific purpose of the present report was to develop a method whereby the surface temperatures of bodies traveling at any speed in a rarefied gas could be determined. It should be pointed out that, to the authors' knowledge, no experimental work has been done in high-speed rarefied gas streams. Consequently, the numerical values obtained should be regarded with some reservation until experimental evidence is accumulated concerning the validity of the assumptions that are made in the analysis.

SYMBOLS

A	constant of integration in Maxwell's equation
C	total velocity, feet per second
C_x C_y C_z	components of total velocity, feet per second
e	base of natural logarithms, 2.7183 . . .
E_i	energy of incident molecules, foot-pounds per square foot, second
E_p	energy of molecules at plate temperature, foot-pounds per square foot, second
E_r	energy of re-emitted molecules, foot-pounds per square foot, second
E_R	energy of rotation, foot-pounds per molecule
E_t	translational energy of molecules striking front of plate, foot-pounds per square foot, second
E_t'	translational energy of molecules striking rear of plate, foot-pounds per square foot, second
E_v	vibrational energy, foot-pounds per molecule
$f(V)$ $f'(C)$	Maxwell's distribution function, dimensionless
g	gravitational acceleration, assumed constant at 32.2, feet per second squared
j	number of degrees of freedom of motion
J_o	normal solar radiation at top of atmosphere, 93.4 foot-pounds per square foot, second
k	Boltzmann constant, 5.66×10^{-24} foot-pounds per degree F per molecule
m	mass of one molecule, slugs per molecule
M	molecular weight, pounds per pound-mole
n	number of molecules striking front of plate, molecules per square foot per second
n'	number of molecules striking rear of plate, molecules per square foot per second
N	number of molecules per unit volume of gas
N_o	Avogadro's number, 2.73×10^{26} molecules per pound-mole
Q	heat removed from skin by an internal cooling system, foot-pounds per square foot, second
R	Universal gas constant, 1544 foot-pounds per pound-mole, °F absolute
R_i	incident radiant energy, foot-pounds per square foot, second
R_r	re-emitted radiant energy, foot-pounds per square foot, second
s	molecular speed ratio $\left(\frac{U}{V_m}\right)$
S	surface area, square feet
T	absolute temperature, °F absolute
T_i	temperature of incident molecules, °F absolute
T_p	temperature of body skin, °F absolute
T_r	temperature of re-emitted molecules, °F absolute
U	mass velocity, feet per second
U_x U_y U_z	components of mass velocity, feet per second
V	thermal velocity, feet per second
V_m	most probable molecular speed, feet per second
V_x V_y V_z	components of thermal velocity, feet per second
x y z	space coordinates
α	accommodation coefficient, dimensionless
β	reciprocal of most probable molecular speed, seconds per foot
ϵ	emissivity, dimensionless
θ	angle of incidence, degrees
σ	Stefan-Boltzmann constant, 3.74×10^{-10} foot-pounds per square foot, second, °F ^{4} absolute
$\Phi(a)$	error function, $\frac{2}{\sqrt{\pi}}\int_0^a e^{-x^2}\,dx$, dimensionless
χ	dimensionless group defined by equation (7)
χ'	dimensionless group defined by equation (9)
ψ	dimensionless group defined by equation (14)
ψ'	dimensionless group defined by equation (19)

ANALYSIS

In order to determine the skin temperature of a body moving at constant speed through a rarefied gas, it is necessary, fundamentally, to make an energy balance on a section of the skin. Energy may be added to or subtracted from the skin by three distinct processes:

(*a*) Molecular energy transport to and from the skin.

(*b*) Radiant energy transport to and from the skin.

(*c*) Energy added to or removed from the skin by processes occurring within the body.

Thus, we may write a simple energy balance on a section of the skin as

$$E_i+R_i=E_r+R_r+Q \tag{1}$$

Each term in this equation can be considered separately.

MOLECULAR ENERGY TRANSPORT TO THE BODY

It is a matter of common knowledge that any body surrounded by a gas is constantly bombarded by molecules of the gas. It is this molecular bombardment, in fact, that causes a pressure to be exerted on a body or on the walls of a container in which the gas is confined. It is evident that the molecules possess, by virture of their mass and velocity, a certain kinetic energy. Thus, a body immersed in a gas is constantly receiving and emitting energy from the bombardment of impinging molecules and the re-emission of the same molecules from the body surface. If it were considered that the molecules carried no energy except that due to translational velocity, then, we should, by examination of their incident and re-emitted velocities, be able to determine the net increase or decrease of the kinetic energy of the re-emitted gas and, hence, determine the molecular energy added to or subtracted from the body. In the ordinary case, however, the impacts of the molecules with the body become obscured by the collisions of the molecules among themselves and the problem becomes exceedingly complex if treated on a microscopic basis. On the other hand, if we consider a body immersed in a gas that is of low enough density so that collisions among molecules in the vicinity of the body are rare in comparison with collisons with the body, the problem becomes susceptible to analysis by the conventional methods of kinetic theory.

If it is assumed that the gas through which the body is moving is so rarefied that the motion of the impinging molecules is essentially unaltered by collisions with reflected molecules, then it follows that the velocity distribution of the impinging molecules will be that of a gas in thermal equilibrium, the Maxwell distribution. The Maxwell distribution law for a gas at rest states that of a total of N molecules per unit volume, the fraction that has velocity components lying between the values, V_x to V_x+dV_x, V_y to V_y+dV_y, V_z to V_z+dV_z, is given by

$$f(V_xV_yV_z)dV_xdV_ydV_z=Ae^{-\beta^2(V_x{}^2+V_y{}^2+V_z{}^2)}dV_xdV_ydV_z \tag{2}$$

where V_x, V_y, V_z are the random thermal velocities with respect to a coordinate system x, y, z fixed in space.

To determine the energy transported to the body by the impinging molecules, it is necessary first to determine the number of molecules that strike the body. This is done as follows:

Assume that the body is stationary and that the gas has a macroscopic velocity U with components U_x U_y, U_z with respect to the fixed coordinates. Assume the body to be a flat plate and fix the plate at the origin of x, y, z, so that the x axis is normal to the plate. The mass velocity U makes an angle θ with the plate. Figure 2 is a diagram of the coordinate system.

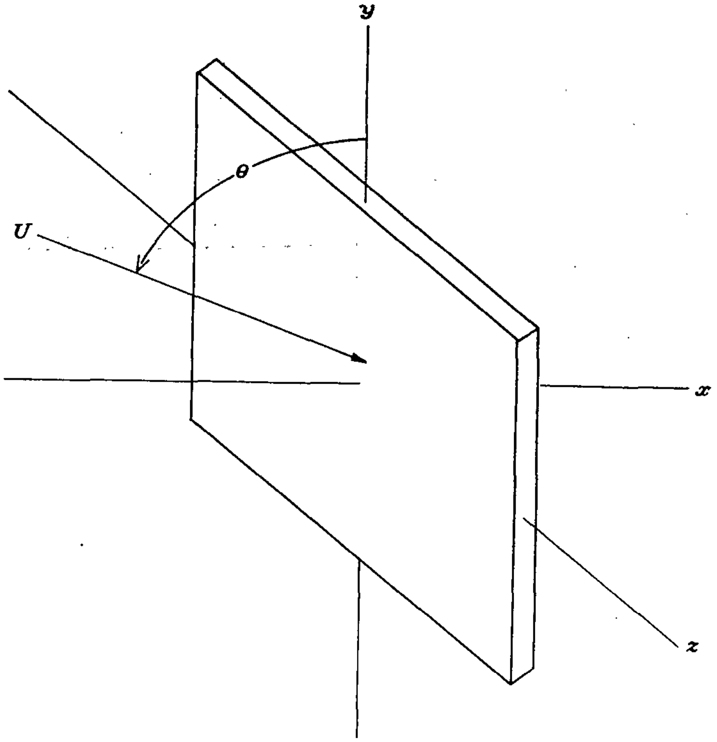

FIGURE 2.—Coordinate system used in analysis.

The total velocities may be written as the sum of the thermal and mass velocities, thus, $C_x=U_x+V_x$, $C_y=U_y+V_y$, and $C_z=U_z+V_z$. Consider an elemental area dS of the plate. All of the molecules that strike dS during a time interval dt that have velocities lying in the range C_x to C_x+dC_x, C_y to C_y+dC_y, and C_z to C_z+dC_z will lie in a cylinder having dS as a base and with a length $C\,dt$ in the direction of C, where $C=\sqrt{C_x{}^2+C_y{}^2+C_z{}^2}$. The volume of the cylinder is $C_x\,dS\,dt$. The number of molecules of this kind contained in the cylinder is then f' (C_x, C_y, C_z) $NC_x\,dS\,dt\,dC_x\,dC_y\,dC_z$ where f' (C_x, C_y, C_z) is Maxwell's distribution function for the total velocities. Then, to find the total number of molecules striking dS per second, we must integrate f' (C_x, C_y, C_z) $NC_x\,dS\,dC_x\,dC_y\,dC_z$ between the limits $C_x=0$ to ∞, $C_y=-\infty$ to ∞, and $C_z=-\infty$ to ∞. Molecules having a negative velocity in the x direction cannot strike the front side of the plate. Then,

$$n=N\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}f'(C_x,C_y,C_z)\,C_xdC_xdC_ydC_z \tag{3}$$

The distribution function for the thermal velocities with a superimposed mass velocity may be written as (cf., reference 3, p. 45)

$$f'(C_x,C_y,C_z)=Ae^{-\beta^2[(C_x-U_x)^2+(C_y-U_y)^2+(C_z-U_z)^2]}$$

Thus for a unit area,

$$n=NA\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}e^{-\beta^2[(C_x-U_x)^2+(C_y-U_y)^2+(C_z-U_z)^2]}\,C_xdC_xdC_ydC_z \tag{4}$$

This equation can be integrated; making use of the relation $A=\frac{\beta^3}{\pi^{3/2}}$ the result is

$$n=N\left\{\frac{e^{-\beta^2U_x{}^2}}{2\sqrt{\pi}\beta}+\frac{U_x}{2}[1+\Phi(\beta U_x)\right\} \tag{5}$$

The factor β is related to the most probable molecular speed V_m as $\beta=1/V_m$, while V_m is given by $V_m=\sqrt{2gRT/M}$. Also U_x may be written as $U \sin\theta$. Equation (5) may then be rewritten as

$$\nu=\frac{NV_m}{2\sqrt{\pi}}\{e^{-s^2\sin^2\theta}+\sqrt{\pi}\, s \sin\theta\,[1+\Phi(s\sin\theta)]\} \tag{6}$$

This is the result obtained by Sänger in reference 4 and rederived by Tsien in reference 1. A plot of the dimensionless group χ, defined as

$$\chi=\{e^{-s^2\sin^2\theta}+\sqrt{\pi}\, s\sin\theta\,[1+\Phi(s\sin\theta)]\} \tag{7}$$

is shown in figure 3.

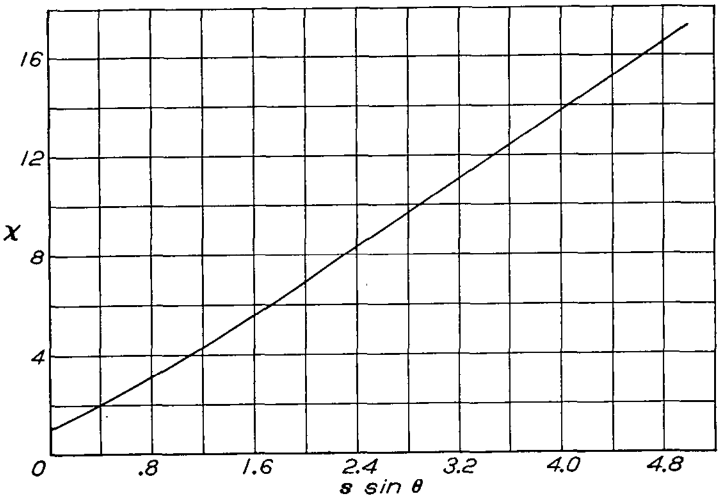

FIGURE 3.—The dimensionless quantity χ, as a function of $s \sin\theta$.

To determine the number of molecules striking the rear side of the plate n', equation (4) is again integrated except that the limits of C_x are changed to $C_x=-\infty$ to 0. The result is

$$n'=\frac{NV_m}{2\sqrt{\pi}}\{e^{-s^2\sin^2\theta}-\sqrt{\pi}\, s\sin\theta\,[1-\Phi(s\sin\theta)]\} \tag{8}$$

The group

$$\chi'=\{e^{-s^2\sin^2\theta}-\sqrt{\pi}\, s\sin\theta\,[1-\Phi(s\sin\theta)]\} \tag{9}$$

is plotted in figure 4.

To determine the kinetic energy incident upon the plate due to translation, Sänger in reference 4, and Tsien in reference 1, multiply the mass of gas striking the plate nm by the sum of the kinetic energy per unit mass due to the mass motion of the gas $U^2/2$ and the average kinetic energy due to thermal translation which may be shown, for a static gas, to have the value $\frac{3}{2}g\frac{RT}{M}$. This procedure, however, is an oversimplification of the problem as introduction of the term $\frac{3}{2}g\frac{RT}{M}$ assumes that the kinetic energy due to thermal velocity may be added to the kinetic energy due to mass velocity. That this is not permissible is explained by the fact that the total kinetic energy depends upon the square of the total velocity, hence the components of the kinetic energy due to the velocity components of the total velocity may not be added linearly. Symbolically, this may be expressed in simple form as

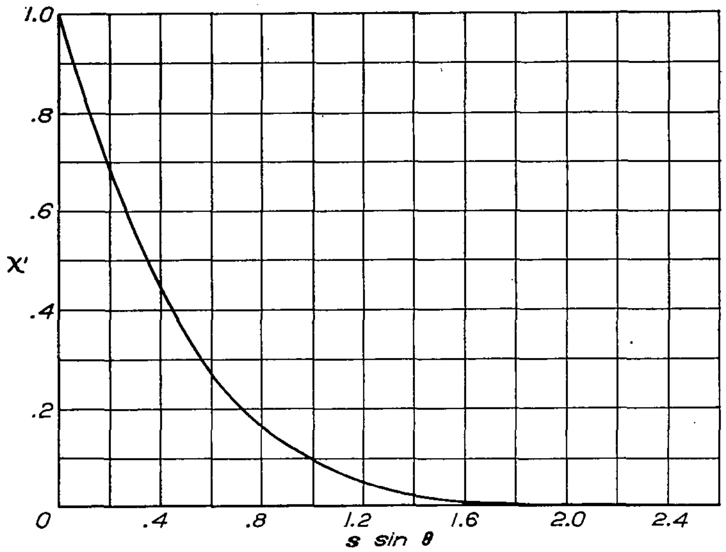

FIGURE 4.—The dimensionless quantity χ', as a function of $s \sin\theta$.

$$\frac{mV^2}{2}+\frac{mU^2}{2}\neq\frac{m(U+V)^2}{2}$$

A more exact calculation of the incident translational energy can be made as follows:

The kinetic energy per molecule contained in a cylinder placed on dS is $mC^2/2$. The kinetic energy of those molecules in the cylinder with velocity components between C_x to C_x+dC_x, Cy to $Cy+dCy$, C_z to C_z+dC_z is

$$\frac{m}{2}NC^2\,C_x f'(C_x,C_y,C_z)dS\,dt\,dC_x\,dC_y\,dC_z \tag{10}$$

The total kinetic energy of the molecules striking the front side of the plate is obtained by integrating equation (10) over the limits $C_x=0$ to ∞, $Cy=-\infty$ to ∞, and $C_z=-\infty$ to ∞. Then, for a unit area, per unit time,

$$E_t=\frac{m}{2}N\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}C^2C_x f'\,(C_x,C_y,C_z)\,dC_x\,dC_y\,dC_z \tag{11}$$

However, $C^2=C_x^2+C_y^2+C_z^2$, so that equation (11) may be written as

$$E_t=\frac{m}{2}NA\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}C_x(C_x^2+C_y^2+C_z^2)\,e^{-\beta^2[(C_x-U_x)^2+C({}_y-U_y)^2+(C_z-U_z)^2]}\,dC_x\,dC_y\,dC_z \tag{12}$$

The result of a tedious integration is

$$E_t=nm\left[\frac{U^2}{2}+\frac{1}{2\beta^2}\left\{1+\frac{1+\frac{3}{2}\sqrt{\pi}\,\beta U_x\,[1+\Phi(\beta U_x)]\,e^{\beta^2U_x^2}}{1+\sqrt{\pi}\,\beta U_x\,[1+\Phi(\beta U_x)]\,e^{\beta^2U_x^2}}\right\}\right] \tag{13}$$

where n is given by equation (6). Using the relations,

$$\beta^2=\frac{m}{2kT},\ \beta=\frac{1}{V_m},\ U_x=U\sin\theta,$$

and defining a dimensionless group ψ as

$$\psi=1+\frac{1+\frac{3}{2}\sqrt{\pi}\, s\sin\theta\,[1+\Phi(s\sin\theta)]\,e^{s^2\sin^2\theta}}{1+\sqrt{\pi}\, s\sin\theta\,[1+\Phi(s\sin\theta)]\,e^{s^2\sin^2\theta}} \tag{14}$$

Equation (13) may be written as,

$$E_t=n\left(\frac{mU^2}{2}+\psi\, kT\right) \tag{15}$$

When the mass velocity U is taken as zero, equation (15) reduces to

$$E_t=2nkT \tag{16}$$

which is the usual result obtained for the translatory kinetic energy crossing an imaginary plane drawn in a static gas. When $s \sin\theta$ is large, the equation approaches

$$E_t=n\left(\frac{mU^2}{2}+\frac{5}{2}kT\right) \tag{17}$$

A similar computation yields, for the incident translational energy on a unit area of the rear side of the plate,

$$E_t'=n'\left(\frac{mU^2}{2}+\psi' kT\right) \tag{18}$$

where

$$\psi'=1+\frac{1-\frac{3}{2}\sqrt{\pi}\, s\sin\theta[1-\Phi(s\sin\theta)]e^{s^2\sin^2\theta}}{1-\sqrt{\pi}\, s\sin\theta[1-\Phi(s\sin\theta)]e^{s^2\sin^2\theta}} \tag{19}$$

The value of E_t', of course, reduces to

$$E_t'=2n'kT=2nkT \tag{20}$$

for U equal to zero and approaches $E_t'=0$ for large values of $s \sin\theta$. The dimensionless groups ψ and ψ' are shown in figures 5 and 6, plotted as functions of $s \sin\theta$.

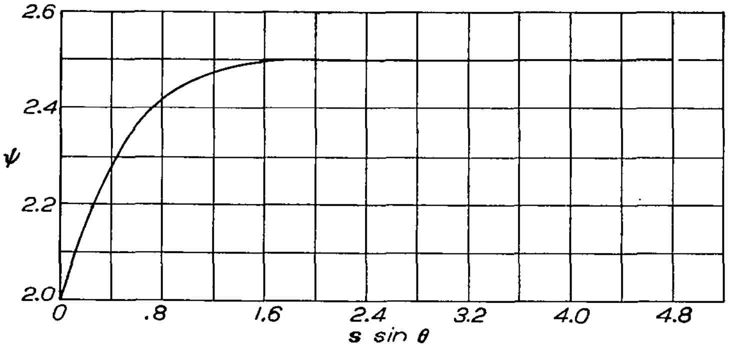

FIGURE 5.—The dimensionless quantity ψ, as a function of $s \sin\theta$.

If the gas is considered to be composed of hard-sphere molecules, then, for a monatomic gas, the total kinetic energy incident upon the plate is given by equations (15) and (18) for the front and rear sides, respectively. However, if the gas is composed of diatomic dumbbell molecules, each molecule may carry an additional amount of energy due to rotation and vibration. By the principle of the equipartition of energy, the maximum amount of energy carried by a molecule is $\frac{j}{2}kT$ where j is the number of degrees of freedom of motion. A rigid dumbbell molecule may rotate about two mutually perpendicular axes, hence the maximum average kinetic energy of rotation per molecule is

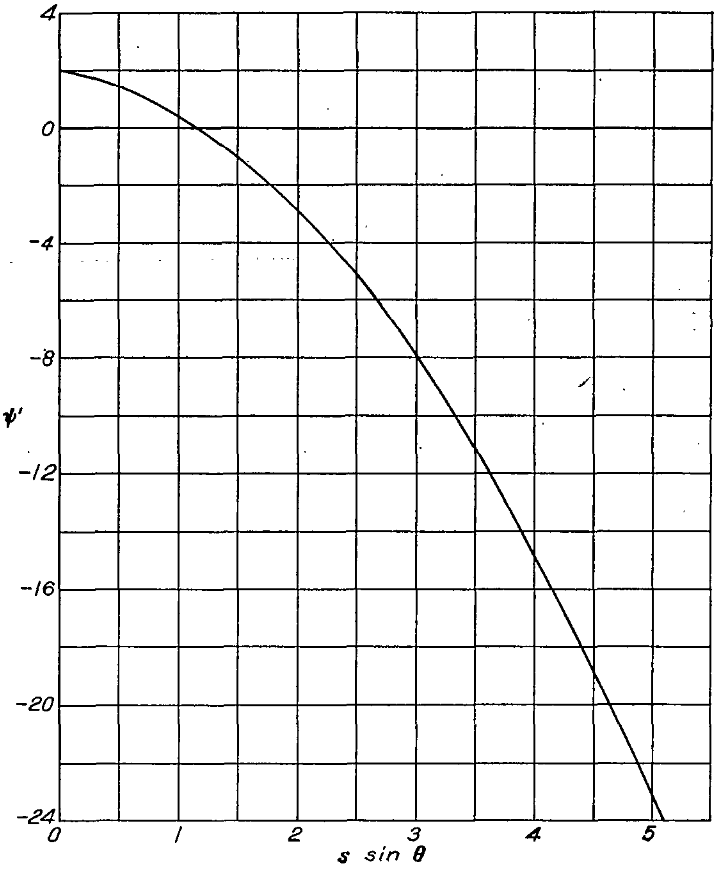

FIGURE 6.—The dimensionless quantity ψ', as a function of $s \sin\theta$.

$$E_R=kT \tag{21}$$

It is conceivable that collisions between dumbbell molecules could cause the atoms composing the molecule to vibrate or oscillate along the line joining their centers. In this case, the energy of vibration is composed of kinetic and potential energies and the vibrational energy is equally distributed between the kinetic and potential forms. Thus, the total energy due to vibration is

$$E_v=2\left(\frac{1}{2}kT\right)=kT \tag{22}$$

where the factor 2 accounts for the distribution of energy between the kinetic and potential forms.

Some question may arise as to the correct procedure to follow when considering a gas composed of two or more constituents. It can be shown that the results that have been obtained for a homogeneous gas, may be extended to mixtures of gases by calculating n, E_t, E_r, and E_v separately for each constituent. This is a result of the fact that, in a mixture of gases in equilibrium, each gas has the same velocity distribution that it would have if the other constituents were not present.

The result obtained in equations (15), (16), (18), and (20) shows that, in effect, both front and rear sides of the plate selectively collide with a group of molecules having an average thermal velocity in excess of the average thermal velocity of the molecules of the gas considered as a whole. This is true whether the plate is moving or stationary. As the plate velocity increases, the average thermal velocity of those molecules striking the front side approaches the average thermal velocity of the molecules of the gas considered as a whole, while the average thermal velocity of those molecules striking the rear side departs farther from the average thermal velocity of the molecules of the main mass of gas. Because of this selective interception, the plate is struck on both front and rear sides by molecules that possess higher-than-average translational energy associated with their higher-than-average thermal velocity. It is pertinent to inquire if this does not mean, also, that this nonaverage group of molecules carries nonaverage rotational and vibrational energies. It is pointed out in reference 5 that Maxwell's distribution law allows the assumption to be made that rotational and translational energies are completely independent, so that a molecule with a high translational energy is just as likely to have a low as a high rotational energy. In the case of vibrational energies, it is shown that those molecules that have higher translational velocities also carry higher vibrational energies. However, for reasons which will presently be discussed, the change in the vibrational energy is neglected in the energy balance equation, hence we need not be concerned with its magnitude in this simplified analysis.

MOLECULAR ENERGY TRANSPORT FROM THE BODY

In order to determine the energy carried from the skin by molecules returning to the main mass of gas, it is necessary to consider the mechanism of energy transfer and molecular reflection at the surface of the body. A complete discussion is beyond the scope of this paper; the reader is referred to references 5 and 6 for a detailed description of the phenomena involved. Nevertheless, the more important concepts are outlined below because an understanding of them is necessary in succeeding computations.

Two main types of reflection may occur when a molecule strikes a surface. The first and simpler type is specular reflection. In this case, a molecule leaves the surface with its normal velocity component reversed, retaining unchanged its tangential component. Any other type of rebound from a surface is known as diffuse reflection. It is possible to devise many subtypes of diffuse reflection; however, only a limited number are physically probable or even thermodynamically permissible. Insofar as direction of reflection or re-emission is concerned, it can be shown that the probability of a molecule leaving the surface at a specified angle is proportional to the cosine of the angle with respect to the normal. All directions about the normal are equally probable, regardless of the direction of impingement. Several hypotheses may be advanced concerning the speed at which diffusely reflected molecules are emitted. The most physically likely assumption is that the speeds are grouped in a Maxwellian distribution corresponding to a temperature which is not necessarily that of the surface.

It appears logical that diffuse reflection would predominate as, in reality, an impinging molecule collides, not with a smooth hard surface, but with an individual molecule or atom of the body material. It would seem likely that collisions of this type would obliterate, insofar as directions are concerned, any prior directional history of motion, especially if more than one collision occurred at the surface.

As yet, the temperature at which the molecules leave the surface has not been specified. In general, the molecules that leave the body will not possess an average velocity corresponding to the plate temperature. In the case with which we are concerned, namely, that of a cooled plate, the molecules will leave at some temperature which is higher than the plate surface temperature. This inequality of temperature (or energy levels) is defined by introduction of the accommodation coefficient, defined as

$$\alpha=\frac{E_i-E_r}{E_i-E_p} \tag{23}$$

The accommodation coefficient may be considered as a term that accounts for the inability of the impinging molecules to adjust themselves to the plate temperature during the time they are in contact with the plate. As pointed out by Loeb, in reference 5, it appears that the action of surface roughness is different for directional effects and energy or temperature effects. Apparently, one or two collisions with the molecules of the plate is sufficient to erase the directional history of a molecule; however, many collisions are required to attain temperature equilibrium with the plate. In the definition of the accommodation coefficient given by equation (23), the effects of specular and diffuse reflection on the energy exchange are included in the value of α.

The value of the accommodation coefficient has been measured for several static gases in contact with many different materials. Values reported range from less than 0.1 to 1.0, as shown in references 7 to 12. Reference 13 presents a method for correlating the effect of pressure upon accommodation coefficients. Apparently, no measurements have been made of the accommodation coefficient in a high-speed gas stream. It would appear probable, however, that the accommodation coefficient would depend upon both the speed of the gas stream and the angle of the surface with respect to the stream.

The accommodation coefficient, if defined rigorously, would have different values for translational, rotational, and vibrational energies. However, a series of experiments performed by Knudsen and described in reference 5 showed that the accommodation coefficients were numerically identical for rotational and translational energies. The vibrational components of the internal energy require much more time to adjust to new values than do the rotational and translational components. A discussion of the time of adjustment of vibrational energy for gases may be found in reference 14. In succeeding calculations, it will be assumed that the vibrational component of the internal energy is unchanged during the short time interval that a molecule is in contact with the body. By the same line of reasoning, dissociation of polyatomic molecules is neglected, since dissociation is a function of the intensity of the molecular vibrations. These assumptions are serious in that vibrational energy and the energy absorbed during dissociation would, if present, have a marked effect on the plate temperature. If the length of time a molecule is adsorbed on the plate is equal to or greater than the time required for fully developed vibration to be excited, then dissociation would probably occur at the high plate temperatures calculated for high-speed flight. A more complete analysis awaits experimental data on adsorption times and accommodation coefficients in a flowing gas.

To calculate the energy abstracted from the plate due to the return of molecules to the stream of gas, use is made of equation (23) rewritten as

$$E_r = E_i(1-\alpha) + \alpha E_p \tag{24}$$

If E_p is the energy of the molecules if they were at plate temperature, then equation (24) can be written for a monatomic gas that obeys the perfect gas laws as

$$E_r = E_t(1-\alpha) + 2\alpha nkT_p \tag{25}$$

and for a diatomic gas that obeys the perfect gas laws, neglecting vibrational energy changes, as

$$E_r = (E_t + nkT_i)(1-\alpha) + 3\alpha nkT_p \tag{26}$$

RADIANT ENERGY EXCHANGE WITH THE BODY

Any body is constantly receiving and emitting radiant energy. In the steady state, a balance is soon reached where the emitted energy is equal to the incident energy; the body temperature adjusts itself to a value such that this equilibrium is maintained.

In the present case, the body emits radiation at a rate given by

$$R_r = \epsilon\sigma T_p^4 \tag{27}$$

The body may absorb radiation by several processes. Probably the most important is that due to direct solar radiation. The mean value of the solar radiation at the top of the earth's atmosphere, the solar constant J_o is shown in reference 15 to have a maximum value of 93.4 foot-pounds per second, per square foot for a plate normal to the incident radiation. The body may also absorb radiation from constituents of the atmosphere and from the earth's surface. Most of the atmospheric gases do not emit appreciable radiation; however, water vapor, ozone and carbon dioxide emit strongly at certain wavelengths. In the interest of simplicity, radiation from the atmospheric constituents and from the earth's surface will be neglected and, where incident radiation is considered, it will be assumed to be due solely to solar radiation, having its previously mentioned maximum value of 93.4 foot-pounds per second, per square foot. The amount of this energy that is absorbed is then

$$R_i = \epsilon J_o \tag{28}$$

provided that gray body absorption is assumed; that is, the emissivity is constant for all wavelengths and temperatures.

ENERGY EXCHANGES OCCURRING WITHIN THE BODY

It is conceivable that, in a missile designed for sustained flight at very high speeds, a skin-cooling system could be used to maintain structure temperatures within specified limits. Disregarding a discussion of the possible cooling systems, we may simply assume that the cooling system removes heat at a rate Q foot-pounds per second, per square foot.

CALCULATIONS

Having outlined the processes by which energy is exchanged with the body, it is now possible to calculate the temperature of a plate moving at any speed through a rarefied atmosphere.

METHOD OF CALCULATION

We may substitute equations (25) or (26), (27), and (28) in equation (1) and obtain, for a monatomic gas, where $E_i = E_t$

$$\alpha E_t - 2\alpha nkT_p + \epsilon J_o - \epsilon\sigma T_p^4 - Q = 0 \tag{29}$$

and for a diatomic gas, where $E_i = E_t + nE_R$

$$\alpha(E_t + nkT_i) - 3\alpha nkT_p + \epsilon J_o - \epsilon\sigma T_p^4 - Q = 0 \tag{30}$$

In equations (29) and (30), the value of E_t may be taken from equation (15) and the value of n from equation (6) for the front side of the plate. For the rear side of the plate, equations (18) and (8) are substituted for values of E_t' and n', respectively.

For a mixture of monatomic and diatomic gases, the incident energies of the components of the gas are added together, and the re-emitted energies of the components of the gas are added together, so that the energy equation assumes the form

$$\Sigma E_i - \Sigma E_r + \epsilon J_o - \epsilon\sigma T_p^4 - Q = 0 \tag{31}$$

To determine the temperature of a plate with no cooling except that due to radiation, the factor Q in equations (30) and (31) is set equal to zero and the equations are solved for values of T_p.

To determine the effect of internal cooling on the plate temperature, arbitrary values of Q are substituted into equations (30) and (31) and the equations are again solved for values of T_p.

SCOPE OF CALCULATIONS

Calculations were made to determine the temperature of a plate traveling at velocities of from zero to 36,000 feet per second at altitudes of from 75 to 190 miles. A speed of 36,000 feet per second is approximately the minimum speed required to escape from the earth at these altitudes. The plate, in the first case, was assumed to be insulated between the front and rear surfaces so that the two surfaces could be treated independently. In the second case, the front and rear sides were assumed to be in perfect thermal contact; that is, the temperature of the front side of the plate was equal to that of the rear side of the plate. In these two sets of calculations, it was assumed that solar radiation was absent.

The above calculations were then repeated, for one altitude, and angle of incidence (or body angle) with arbitrarily assumed values of internal cooling and with solar radiation absent.

A fourth set of calculations was made for several altitudes, considering that the plate was uncooled and that solar radiation was present.

In the first four sets of calculations the values of the accommodation coefficient and the emissivity were assumed to be constant at unity. This assumption simplified the computations and, at the same time, reasonably approximated physical conditions so that the trends of skin temperature shown should be correct in order of magnitude.

A fifth and sixth set of calculations were made to show the effect of emissivity and accommodation coefficient on skin temperature at several altitudes and flight speeds.

The atmospheric properties used in the calculation were taken from references 2 and 16. Reference 16 was used to obtain the properties of the atmosphere at 75 miles; whereas reference 2 was used for the higher altitudes. There is some difference in the assumed atmospheric compositions between the tables of references 2 and 16. Warfield, in reference 16, assumed that the composition by weight at all altitudes was constant at its sea-level value, except for dissociation of oxygen. The amount of dissociation was assumed to increase with altitude, complete dissociation occurring above altitudes of 75 miles. On the other hand, Johnson and Slack assumed sea-level composition at all altitudes, neglecting dissociation entirely. In order to compare values of skin temperature at the several altitudes for which computations were made, it was arbitrarily decided to use the values of temperature and density given by Johnson and Slack and to assume complete dissociation of the assumed constant sea-level percentage of oxygen. The following table lists the values of atmospheric density and temperature that were used in the computations.

Altitude (miles)	Temperature (°F absolute)	Density (lb/ft^3)
75	680	2.55×10^{-8}
100	1025	1.05×10^{-9}
125	1385	1.00×10^{-10}
150	1735	1.75×10^{-11}
190	1990	1.61×10^{-14}

DISCUSSION

INCIDENT TRANSLATIONAL ENERGY

In the computation of the incident kinetic energy due to molecular translation, it was shown that both the number of molecules striking unit area of the body and the resultant kinetic energy incident upon the body were functions of the term $s \sin \theta$. An examination of the graphs of the dimensionless groups χ and χ', ψ and ψ', which govern the number of molecules striking a section of the body and the magnitude of the incident translational energy, reveal several interesting features. With respect to the number of molecules that strike a body on the front side, it can be seen from figure 3 that this number increases practically linearly with flight speed for a given plate or body angle. On the other hand, as shown in figure 4, the group χ' that determines the number of molecules that strike the rear side of a body, decreases rapidly with flight speed and is practically zero for values of $s \sin \theta$ greater than 2.

From equations (15) and (18), it may be seen that the total incident translatory kinetic energy term is composed of two parts: The first due to mass motion of the gas relative to the body and the second due to the apparent translatory molecular motions. In the case of the front side of the body, the apparent molecular energy term can be seen, from figure 5, to increase rapidly at first with increasing values of $s \sin \theta$ and then to approach a constant value of 5/2 at values of $s \sin \theta$ greater than 2. On the rear side of the same body, the molecular energy term continues to decrease rapidly as $s \sin \theta$ increases.

A simple physical interpretation of the trends noted above can be given. First, it is necessary to understand that with a Maxwellian type of velocity distribution, most molecules have speeds close to the average molecular speed. A very small fraction has speeds much greater than the average or much less than the average. For example, only about two percent of the molecules in a given volume of gas have speeds that exceed twice the average molecular speed. Thus, it should be apparent that, at high resultant speeds (the resultant speed being the component normal to the body $U \sin \theta$), the relatively small spread of molecular speeds predicted by the Maxwellian distribution become unimportant in comparison with the resultant speed. The dimensionless number, s, the ratio of flight speed to the most probable molecular speed, which appears in free-molecule flow calculations, may be shown to be related to the conventional Mach number by a constant factor. Its significance, however, is most logically interpreted as describing the relative importance of flight speed and molecular speed.

This diminishing importance of the spread of molecular velocities is the reason for the constancy of the second term of equation (15) for values of $s \sin \theta$ greater than 2. It is interesting to note that the term in equation (15), which accounts for the translatory molecular motions, has values ranging from $2kT$ at zero velocity to $(5/2)kT$ at values of $s \sin \theta$ of 2 and higher. This may be compared with the constant value of $(3/2)kT$ given in the simplified analysis of Sänger. The increased value at zero velocity in the present analysis is due to the fact that the faster-moving molecules are both more likely to collide with the body and also carry more translational energy than do the slower-speed molecules. The increase in the term with increasing values of the parameter $s \sin \theta$ is due, as previously inferred, to the contribution of the cross-product term of the thermal and mass velocities in equation (12), the unintegrated expression for the translational energy. The actual difference in the total kinetic energy term computed by means of the simplified theory and the more exact theory depends upon the relative magnitudes of the terms $mU^2/2$ and ψkT. For most of the cases computed in this report, the term $mU^2/2$ became large in comparison with the term ψkT at flight speeds above 8000 feet per second; consequently, little difference between the simplified and more-exact theory would be expected at flight speeds greater than 8000 feet per second.

Physically, it is clear why both the number of molecules striking the rear side of the body, and the translatory energy incident upon the rear side of the body, decrease with increasing values of $s \sin \theta$. The front of the body acts as a shield and prevents collisions of the rear side with all molecules except those that have absolute velocity components in the direction of flight with magnitude equal to the flight speed or higher. Consequently, as the flight speed is increased, we should expect that the incident energy on the rear of the plate would eventually approach zero.

SKIN TEMPERATURES IN THE ABSENCE OF SOLAR RADIATION

Figure 7 shows the magnitude of plate temperatures calculated for several altitudes in the absence of solar radiation.

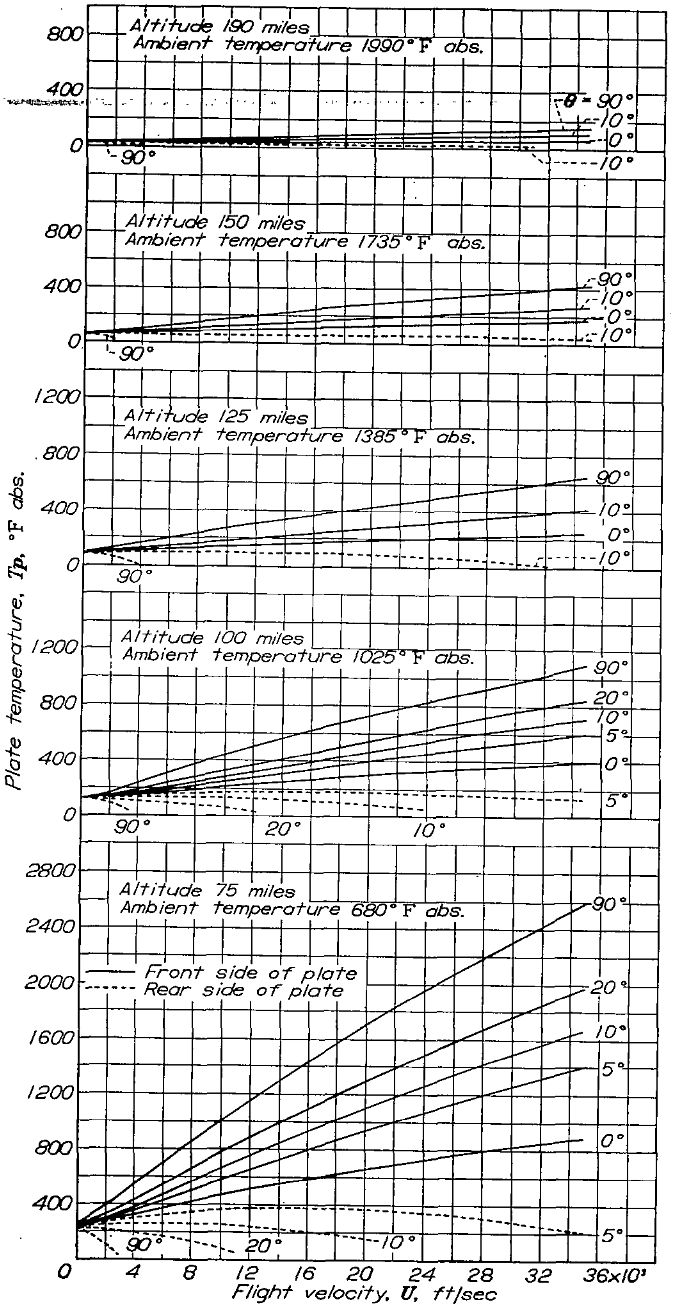

FIGURE 7.—The effect of flight velocity and angle of incidence on the temperature of the front and rear sides of an uncooled plate. Accommodation coefficient, 1.0; solar radiation absent; angle of incidence indicated; emissivity, 1.0; insulation between front and rear sides of plate.

The marked effect of emitted radiation from the body in lowering the plate temperature is striking. For a large range of flight speeds and small plate angles, the plate temperatures are actually lower than the ambient-air temperature. The curves of temperature of the rear side of the body show an eventual decrease with flight speed in accordance with the discussion of the preceding paragraph. The effect upon body temperature of perfect thermal contact between the front and rear sides of a flat plate is shown in figure 8. It will be noted that the plate temperatures are considerably lower than those calculated for the front side of an insulated plate, as shown in figure 7. It would appear that the cooling problem could be alleviated by insuring thermal contact between portions of the body inclined at positive and negative angles with respect to the flight path. Practically, this could be accomplished by boattailing the body and using a heavy skin of high thermal conductivity between the nose section and the boattailed section. An alternative scheme would consist of employing a circulating-type cooling system, abstracting heat from the nose section and rejecting this heat at the boattailed section. It is also apparent that, from a skin temperature standpoint, large positive body angles should be avoided; the body should preferably be finely tapered. Fortunately, this dictate coincides with aerodynamic requirements.

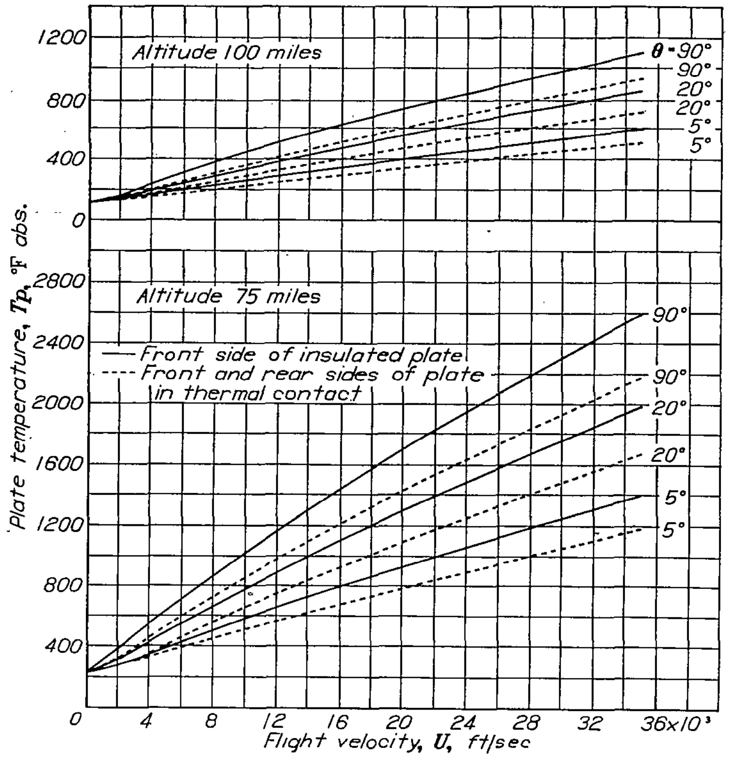

FIGURE 8.—The effect of thermal contact between front and rear sides on plate temperature. Accommodation coefficient, 1.0; angle of incidence indicated; emissivity, 1.0; solar radiation absent.

The effect of altitude on the temperature of the front side of an insulated plate is also indicated in figures 7 and 8. It is apparent that the problem of high skin temperatures becomes of negligible importance above altitudes of 125 miles for flight speeds up to 36,000 feet per second and with solar radiation absent. The condition of zero solar radiation corresponds, of course, to nocturnal flight.

INTERNAL COOLING

The effect upon body temperature of internal cooling is shown in figure 9 for several flight speeds and for a body angle of 10°. It can be seen that the body temperature decreases rapidly above a certain critical value of rate of internal cooling. This effect is due to the combined actions of emitted radiant energy and the rate of internal cooling.

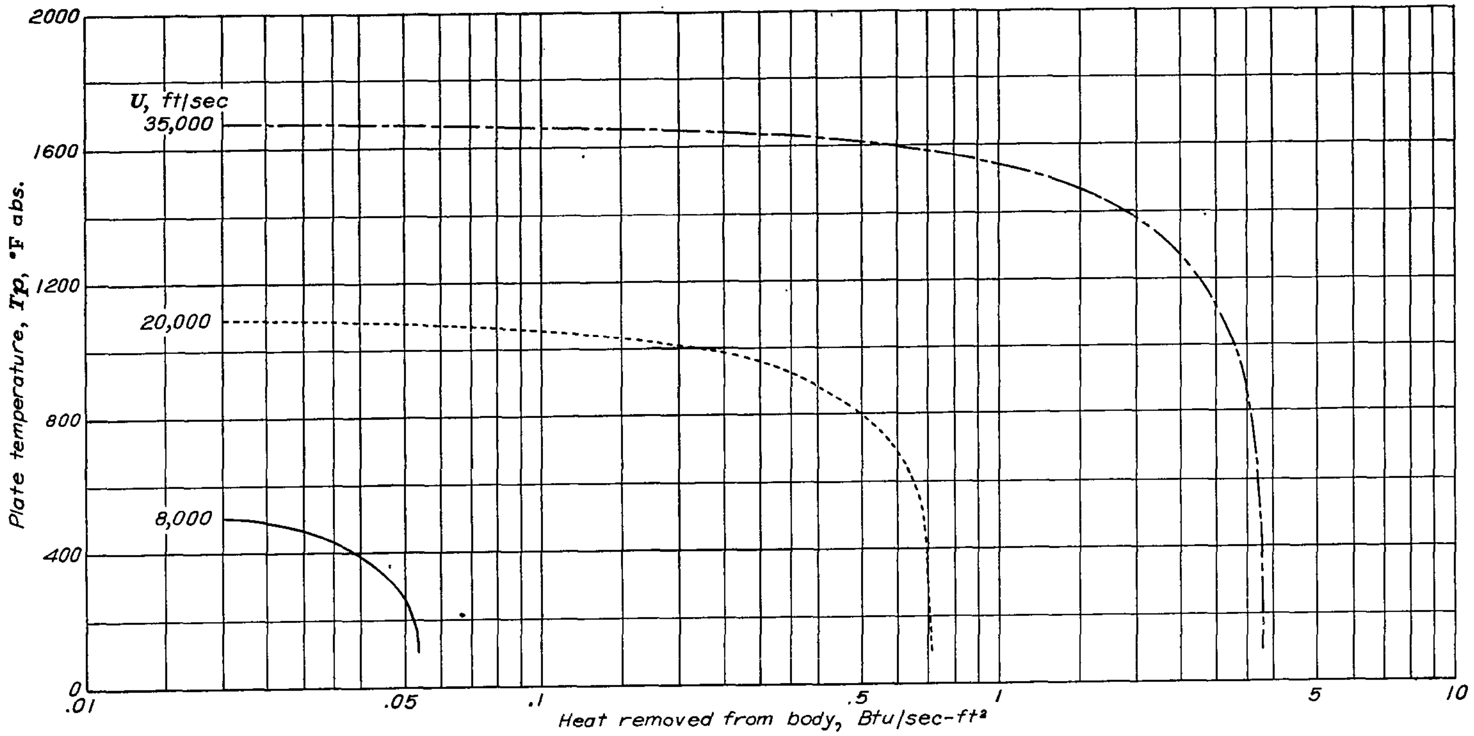

FIGURE 9.—The variation of plate temperature due to heat removal from the surface. Altitude, 75 miles; accommodation coefficient, 1.0; solar radiation absent; emissivity, 1.0; angle of incidence, 10°; rear side of plate insulated.

At the lower rates of internal heat removal, the emitted radiant energy comprises the major portion of the total energy leaving the surface, hence changes in the rate of internal cooling have little effect upon the surface temperature. As the rate of internal cooling increases, however. and the surface temperature decreases, internal cooling becomes the controlling factor, since the emitted radiant energy is proportional to the fourth power of the surface temperature. As the rate of internal cooling is increased, a point is finally reached where its magnitude approaches the value of the incident molecular energy and when this occurs, the surface temperature rapidly approaches zero.

THE EFFECT OF SOLAR RADIATION ON SKIN TEMPERATURE

The variation of plate surface temperatures with flight velocity in the presence of solar radiation is shown in figure 10. It can be seen that the effect of solar radiation on the surface temperatures becomes increasingly important as the flight altitude is increased and that the effect is strongest at the lower flight speeds. At altitudes greater than 100 miles, the presence of solar radiation markedly increases the plate surface temperature and this effect becomes practically independent of flight velocity. The implication is, of course, that heat transfer by molecular energy exchange is negligible. The strong effect of solar radiation on skin temperature at the higher altitudes suggests the desirability of nocturnal flight in order to prevent excessive skin temperatures.

THE EFFECT OF EMISSIVITY ON SKIN TEMPERATURE

The effect of the emissivity of the plate material on the surface temperatures is shown in figure 11 for the condition of no solar radiation and in figure 12 for the condition of maximum solar radiation. It is apparent that it is desirable to

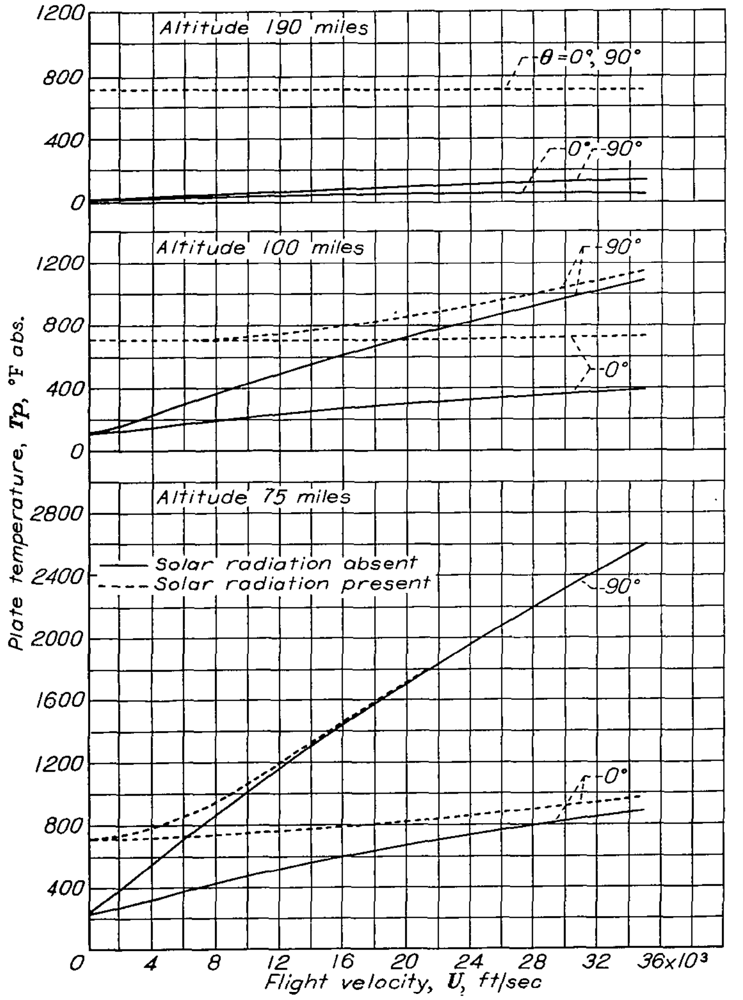

FIGURE 10.—The effect of solar radiation on the surface temperature of an uncooled plate. Accommodation coefficient, 1.0; angle of incidence indicated; emissivity, 1.0; rear side of plate insulated.

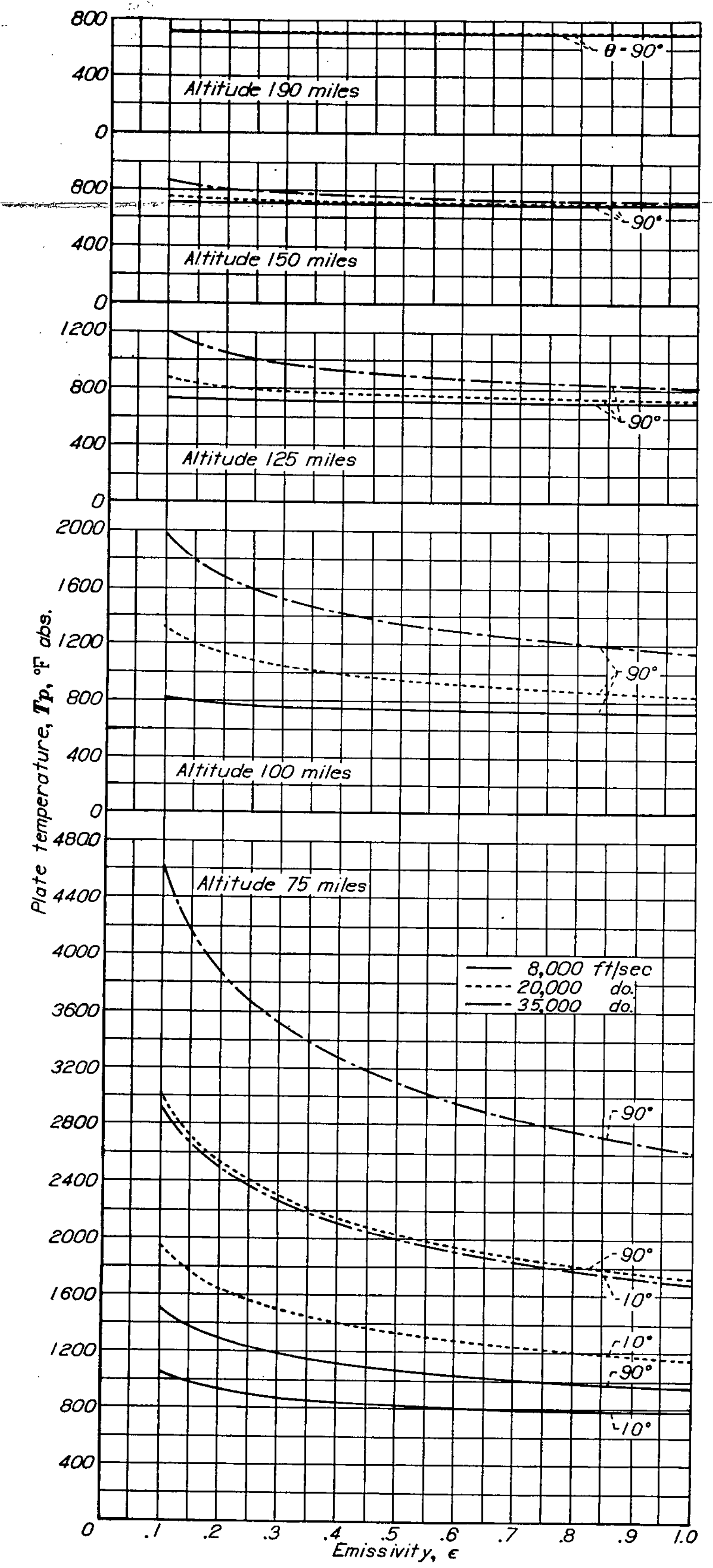

FIGURE 11.—Effect of emissivity on plate temperature assuming solar radiation absent. Accommodation coefficient, 1.0; angle of incidence indicated; rear side of plate insulated.

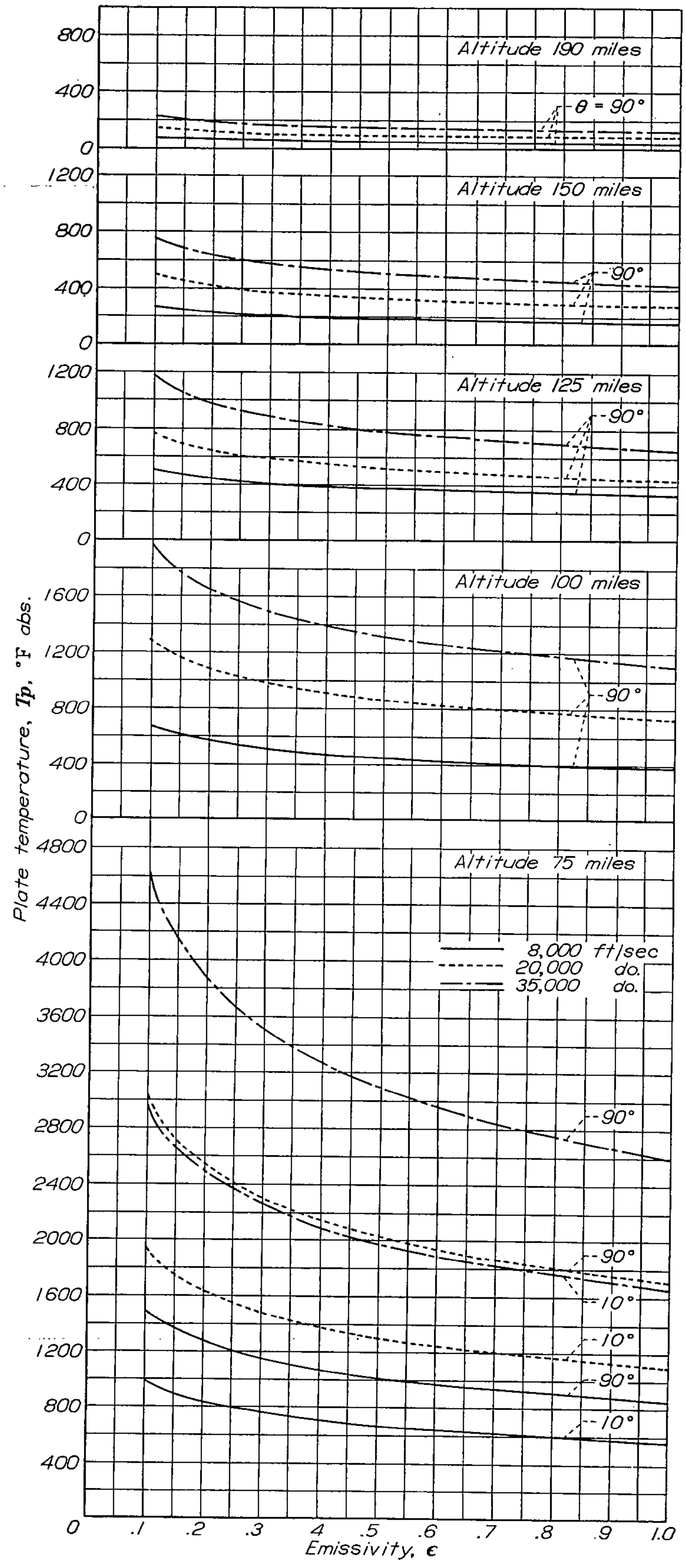

FIGURE 12.—Effect of emissivity on plate temperature assuming solar radiation present. Accommodation coefficient, 1.0; angle of incidence indicated; rear side of plate insulated.

utilize as high a value of skin emissivity as possible. This conclusion is reached because any practicable missile must fly over a wide range of altitude and speed and, under all conditions of flight speed and altitude, a high emissivity tends to reduce skin temperature. The effect of emissivity is most marked at the lower altitudes; at the higher altitudes the surface temperatures become almost independent of the emissivity. This somewhat surprising trend may be explained as follows: At altitudes of 100 miles and higher, the magnitude of the molecular energy interchange is small in comparison with the radiant energy exchange. Thus, neglecting all energy exchange except that due to radiation, an energy balance on the plate may be written as $\epsilon J_o = \epsilon \sigma T^4$. The emissivity ϵ is seen to cancel out and the plate temperature is thus dependent only upon the magnitude of the solar radiation J_o. It should be re-emphasized that these computations are valid only for gray-body radiation.

THE EFFECT OF ACCOMMODATION COEFFICIENT ON SKIN TEMPERATURE

The effect of the magnitude of the accommodation coefficient on the plate surface temperatures is shown in figure 13. It may be noted that, as the accommodation coefficient is lowered, implying increasing difficulty of molecular energy exchange, the plate temperature likewise decreases. This phenomenon may offer a method of cooling, as it may be possible to decrease the accommodation coefficient by altering the surface characteristics of the skin.

CONCLUSIONS

The following conclusions can be drawn from the trends shown in the preceding analysis:

1. With solar radiation absent, that is, for the condition of nocturnal flight, the skin cooling problem associated with high-speed missile flight becomes negligible at altitudes of 125 miles and higher up to steady flight speeds of 36,000 feet per second.

2. The effect of solar radiation on skin temperatures becomes increasingly important as the flight altitude is increased. At an altitude of 150 miles and higher, solar radiation is the predominating factor that determines skin temperature.

3. With solar radiation present, the effect of emissivity on skin temperatures becomes of decreasing importance as the altitude is increased. At altitudes of 125 miles and higher, the skin temperature at a given flight speed is practically independent of the emissivity and depends only on the intensity of the solar radiation.

4. The strong effect of solar radiation on skin temperatures at altitudes above 75 miles suggests the desirability of nocturnal flight in order to minimize skin temperatures.

5. In order to maintain low skin temperatures, the angle of inclination of the body with respect to the flight path should be kept as small as possible. This may be done by designing the body to be finely tapered and by flying at small angles of attack.

6. Skin temperatures may be reduced by insuring thermal contact between portions of the body inclined at positive and negative angles with respect to the flight path. As much surface as possible should be inclined at negative angles. Practically, this may be accomplished by boattailing the body.

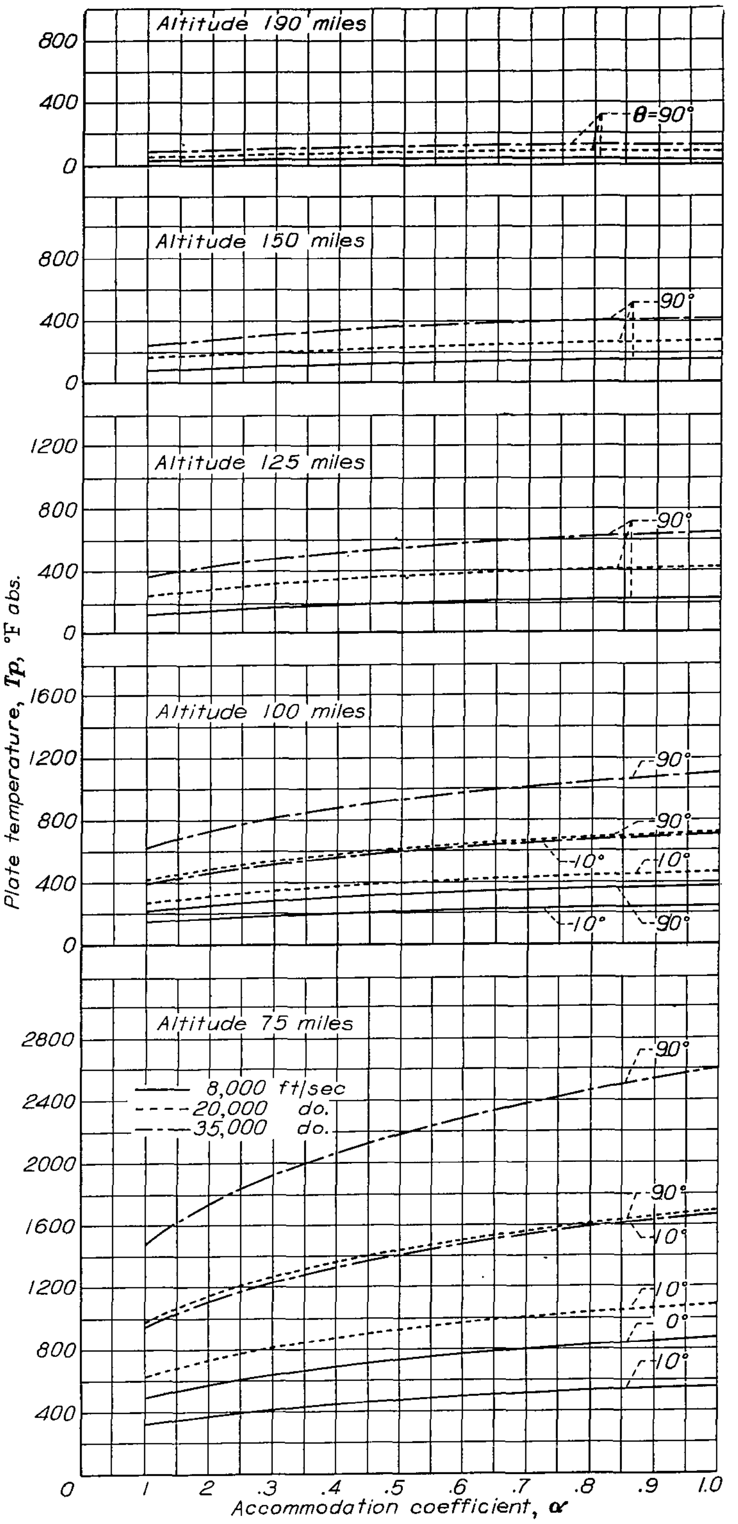

FIGURE 13.—Variation of plate temperature with accommodation coefficient. Solar radiation absent; angle of incidence indicated; emissivity, 1.0; rear side of plate insulated.

7. In order for a skin-cooling system to be effective, the rate of internal cooling must be of the same order of magnitude or greater than the rate at which heat is lost naturally by radiation from the body. Internal cooling rates below the natural radiation loss rate have little effect on skin temperatures.

8. For a practical missile designed to fly over a wide range of altitudes and speeds, it is desirable to make the emissivity of the skin as high as possible. This conclusion, however, is restricted to the case of skin surfaces for which the emissivity is independent of the wavelength of the emitted and absorbed radiant energy.

9. A decrease in accommodation coefficient results in an accompanying decrease in skin temperature. This phenomenon may offer a method of skin cooling if it is possible to lower the accommodation coefficient by altering the surface characteristics of the skin.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., *August, 1948.*

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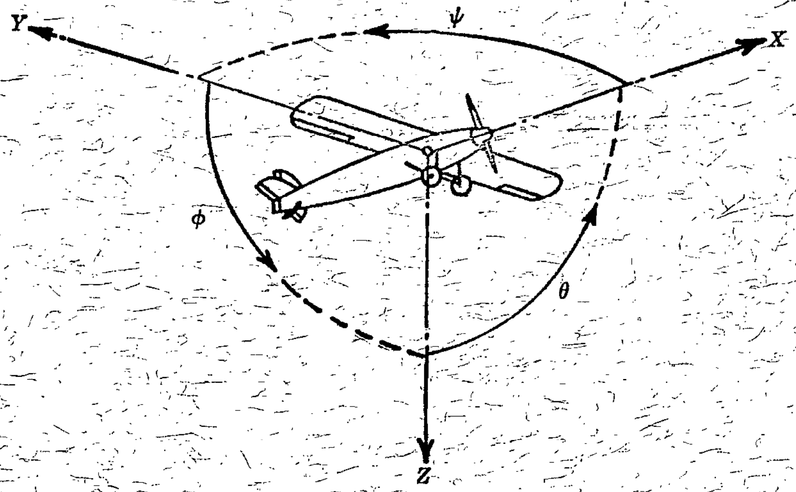

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	Rolling	L	$Y \longrightarrow Z$	Roll	ϕ	u	p
Lateral	Y	Y	Pitching	M	$Z \longrightarrow X$	Pitch	θ	v	q
Normal	Z	Z	Yawing	N	$X \longrightarrow Y$	Yaw	ψ	w	r

Absolute coefficients of moment

$C_l=\frac{L}{qbS}$ (rolling) $C_m=\frac{M}{qcS}$ (pitching) $C_n=\frac{N}{qbS}$ (yawing)

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D	Diameter
p	Geometric pitch
p/D	Pitch ratio
V'	Inflow velocity
V_s	Slipstream velocity
T	Thrust, absolute coefficient $C_T=\frac{T}{\rho n^2 D^4}$
Q	Torque, absolute coefficient $C_Q=\frac{Q}{\rho n^2 D^5}$
P	Power, absolute coefficient $C_P=\frac{P}{\rho n^3 D^5}$
C_s	Speed-power coefficient $=\sqrt[5]{\frac{\rho V^5}{P n^2}}$
η	Efficiency
n	Revolutions per second, rps
Φ	Effective helix angle $=\tan^{-1}\left(\frac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS

1 hp = 76.04 kg-m/s = 550 ft-lb/sec
1 metric horsepower = 0.9863 hp
1 mph = 0.4470 mps
1 mps = 2.2369 mph

1 lb = 0.4536 kg
1 kg = 2.2046 lb
1 mi = 1,609.35 m = 5,280 ft
1 m = 3.2808 ft