REPORT 993

AN INTRODUCTION TO THE PHYSICAL ASPECTS OF HELICOPTER STABILITY

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SUMMARY

In order to provide engineers interested in rotating-wing aircraft but with no specialized training in stability theory some understanding of the factors that influence the flying qualities of the helicopter, an explanation is made of both the static stability and the stick-fixed oscillation in hovering and forward flight in terms of fundamental physical quantities. Three significant stability factors—static stability with angle of attack, static stability with speed, and damping due to a pitching or rolling velocity—are explained in detail.

INTRODUCTION

Most of the published literature on helicopter stability is written for the specialist in stability theory and is somewhat difficult for the average engineer to understand. An explanation of the fundamental ideas underlying helicopter stability in terms of the basic physical parameters involved rather than in specialized mathematics therefore appears desirable.

The subject is introduced by discussions of the means of helicopter control and the origin of the forces and moments which act on the helicopter as a result of deviations from trimmed flight conditions. These fundamental ideas are then applied to the hovering helicopter and to the helicopter in forward flight. An understanding of the stability of the helicopter in both of these conditions is aided by analogy with the stability of the fixed-wing airplane. This comparison is made possible in hovering because of the fact that the zero lateral velocity of the trimmed airplane in forward flight is analogous to zero translational velocity of the hovering helicopter. In forward flight, helicopter longitudinal stability may be directly compared with the corresponding motions of the airplane.

The handling qualities of an aircraft are those stability and control characteristics that affect the ease and safety of flying the aircraft. This report is primarily restricted to a study of helicopter stability which, aside from its direct effect on handling qualities, must also be studied in order to understand control characteristics. The first phase of stability considered is static stability, which has an obvious influence on the handling qualities of the helicopter. In the second phase, a detailed discussion of the period of the control-fixed oscillation of the helicopter is given, not because the period necessarily affects the pilot's opinion of the handling qualities (see reference 1) but because the factors that affect the period are thought to influence the pilot's opinion of the handling qualities. A study of the period is thus felt to be a convenient way to gain an understanding of these factors, which in turn is considered to be of value in evaluating and improving helicopter handling qualities.

In this report, only the single-rotor helicopter with fully articulated blades, flapping hinges on the rotor shaft, and a conventional control system is considered as it is the fundamental configuration.

SYMBOLS

W	gross weight of helicopter or airplane, pounds
T	rotor thrust, pounds
L	airplane or helicopter lift, pounds
V	true airspeed of helicopter or airplane along flight path, feet per second
R	blade radius, feet
b	airplane wing span, feet
S	rotor disk area or airplane wing area, square feet
ρ	mass density of air, slugs per cubic foot
Ω	rotational velocity of rotor, radians per second
C_T	thrust coefficient $\left(\frac{T}{\pi R^2 \rho (\Omega R)^2}\right)$
C_L	airplane or helicopter lift coefficient $\left(\frac{L}{\frac{1}{2}\rho V^2 S}\right)$
μ	tip-speed ratio $\left(\text{approx.}\ \frac{V}{\Omega R}\right)$
ω	angular velocity of helicopter (pitching or rolling), radians per second
γ	mass constant of rotor blades; expresses ratio of air forces to mass forces (γ is inversely proportional to blade moment of inertia about flapping hinge)
δ	angular displacement of rotor cone due to angular velocity of helicopter, radians
P	period of oscillation, seconds
g	acceleration due to gravity, feet per second per second
M	pitching moment, foot-pounds
M_ω	damping in pitch or roll (rate of change of pitching or rolling moment with pitching or rolling velocity), foot-pounds per radian per second
M_V	stability with speed (rate of change of moment with translational velocity), foot-pounds per foot per second

M_α static stability with angle of attack (rate of change of moment with angle of attack), foot-pounds per radian

T_α rate of change of thrust with angle of attack, pounds per radian

C_m pitching-moment coefficient $\left(\frac{M}{\frac{1}{2}\rho V^2 SR}\right.$ for helicopter; $\frac{M}{\frac{1}{2}\rho V^2 S\bar{c}}$ for airplane where $\bar{c}$ is mean aerodynamic chord$\left.\right)$

α helicopter or airplane fuselage angle of attack, degrees

e distance between rotor shaft and helicopter center of gravity; positive when center of gravity is rearward, feet

h height of rotor hub above helicopter center of gravity, feet

U_T component at blade element of resultant velocity perpendicular to blade-span axis and to rotor shaft, feet per second

U_P component at blade element of resultant velocity perpendicular both to blade-span axis and U_T, feet per second

α_r blade-element angle of attack measured from line of zero lift, radians

Helicopter nose-up moments, angular displacements, and angular velocities are assumed to be positive. For lateral motions from hovering, moments, angular displacements, and angular velocities which tend to raise the advancing side of the fuselage are positive. Changes in translational velocities in the direction of increasing velocity, as well as upward forces, are also positive.

STABILITY DEFINITIONS

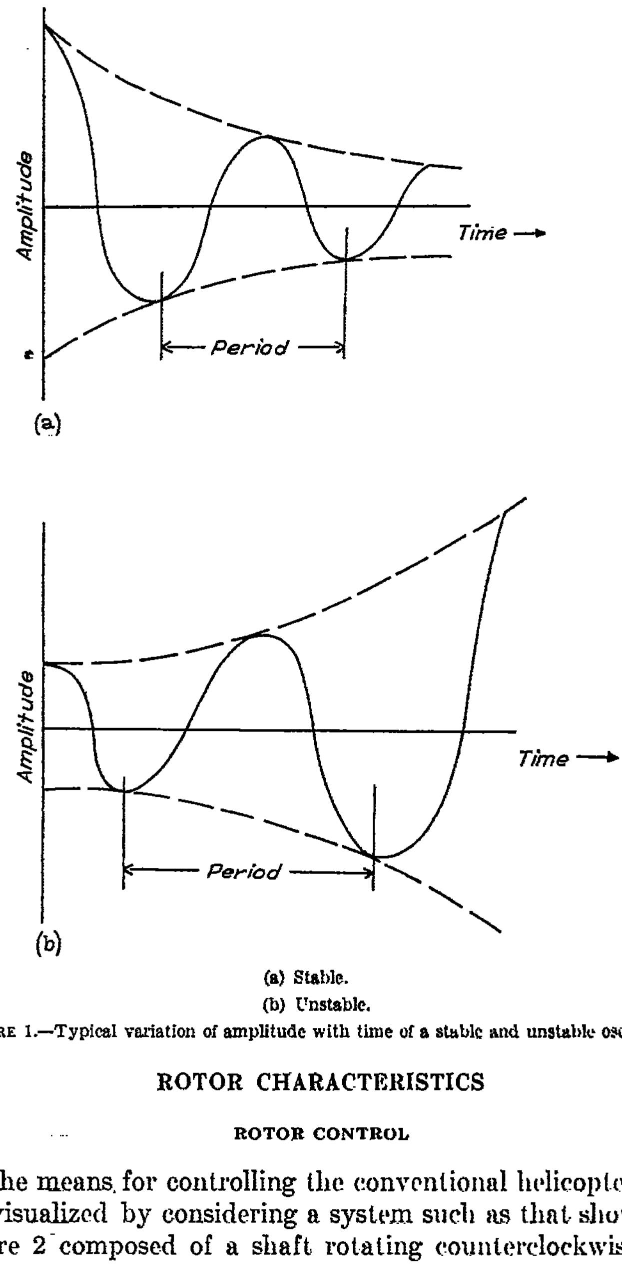
The following stability definitions are given for terms used herein:

Trim—An aircraft is trimmed in steady flight when the resultant force and moment on the aircraft are equal to zero.

Aircraft stability—Stability is related to the behavior of an aircraft after it is disturbed slightly from the trimmed condition.

Static stability—An aircraft is statically stable if there is an initial tendency for it to return to its trim condition after an angular displacement or after a change in translational velocity from that condition; it is unstable if it tends to diverge from trim after being displaced. An aircraft is neutrally stable if it tends to remain in the condition to which it has been displaced.

Dynamic stability—The dynamic stability of an aircraft deals with the oscillation of the aircraft about its trim position following a disturbance from trim. Figure 1 illustrates a typical variation of amplitude of two oscillations with time. The period of these oscillations, which is defined as the time required for the oscillation to go through one cycle, is shown in this figure. If the envelope of the oscillation (dash line) decreases in magnitude with time, the oscillation is dynamically stable; if it increases with time, the oscillation is dynamically unstable. The time to double or half the amplitude of the oscillation is defined as the time necessary for the amplitude of the envelope to double or half. This quantity is a measure of the degree of stability or instability of the oscillation in that a small time to half the amplitude indicates a rapidly convergent or highly stable oscillation; whereas, a small time to double amplitude indicates a rapidly divergent or highly unstable oscillation.



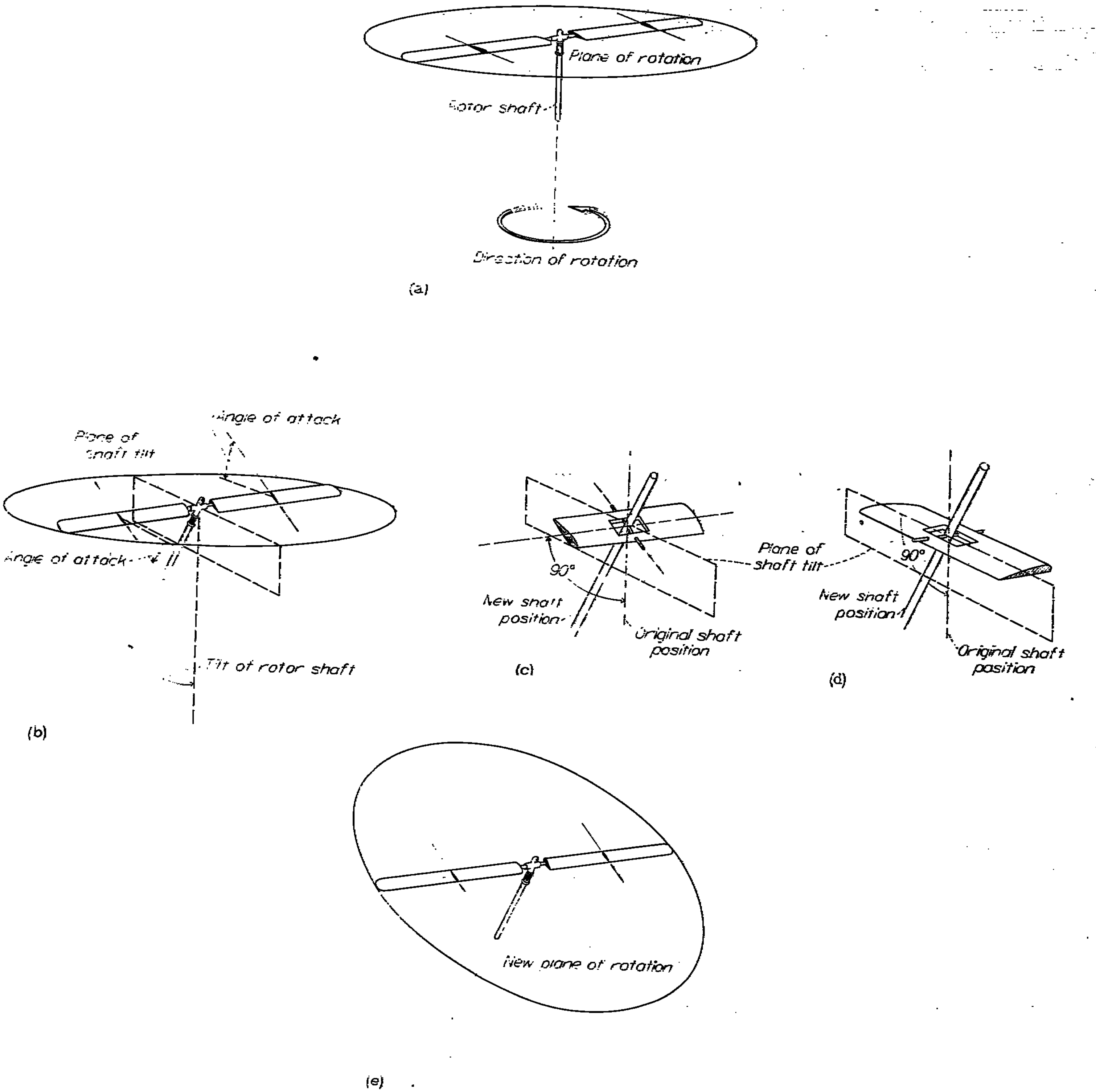
(a) Stable.
(b) Unstable.

FIGURE 1.—Typical variation of amplitude with time of a stable and unstable oscillation.

ROTOR CHARACTERISTICS

ROTOR CONTROL

The means for controlling the conventional helicopter can be visualized by considering a system such as that shown in figure 2 composed of a shaft rotating counterclockwise (as viewed from above) and to which are attached two blades which are free to flap about a chordwise axis perpendicular to the shaft.



(a) Original equilibrium position.

(b) Angles of attack produced by rotor-shaft tilt.

(c) Hub detail of figure 2 (b). (Same blade azimuth position.)

(d) Hub detail of figure 2 (b). (Blades rotated one-quarter revolution.)

(e) Final equilibrium position.

FIGURE 2.—The effect of rotor-shaft tilt on the plane of rotation.

If the shaft is suddenly tilted to the position shown in figure 2(b), the plane of rotation will, for an instant, remain unchanged because the blades are hinged. If the rotor were located in a vacuum, the plane of rotation would continue to remain in its original position because no forces normal to the plane of rotation are produced. Examination of the schematic detailed views of the rotor hub in figures 2(c) and 2(d) shows that this condition of no plane-of-rotation tilt is mechanically possible. Under actual operating conditions, however, the plane of rotation will change because of the air forces that are produced as a result of the shaft tilt. As can be seen in figure 2(b), the tilt of the shaft causes the angle of attack of the blades to change cyclically. Thus, the blade moving to the left has an increased lift and moves up to a maximum positive displacement one-quarter revolution after the position of maximum lift. The blade moving to the right has a decreased lift and moves down to a maximum negative displacement one-quarter revolution after the position of maximum negative lift. Therefore, a short time later, the plane of rotation is again perpendicular to the rotor shaft as shown in figure 2(e). Thus, although by tilting the shaft it was impossible to force physically the hinged blades to aline themselves with the shaft, the tilt produced a cyclic change in blade angle of attack such that the air forces brought the blades into proper alinement. This idea can be applied directly to a helicopter in that, if the rotor shaft is tilted, the rotor will quickly realine itself with respect to the shaft. A movement of the control stick of a conventional helicopter is equivalent to tilting the shaft with respect to the fuselage. The resulting tilt of the rotor with respect to the fuselage will produce a moment about the helicopter center of gravity, because the rotor thrust acts approximately perpendicular to the plane of rotation and the center of gravity lies on the trim line of thrust. (In the present report the rotor thrust is assumed to act at right angles to the plane of rotation. This assumption is sufficiently exact for a preliminary qualitative understanding of helicopter stability and control.)

DAMPING IN PITCH OR ROLL

The foregoing discussion points out that some delay exists between a rapid shaft tilt and the realinement of the rotor with the shaft. Thus, if the shaft continues to tilt, the plane of rotation will continue to lag behind the rotor shaft. Also pointed out was the fact that, when the rotor plane is displaced from its perpendicular position relative to the shaft, air forces are produced. It follows, therefore, that, although no moments can be transmitted directly from the shaft to the hinged rotating blades during steady pitching or rolling, the aerodynamic forces produced when the rotor is displaced from the shaft supply the moment necessary to overcome continuously the flapping inertia of the rotor.

A simple derivation given in reference 2 yields the following result for the angular displacement of the rotor plane with respect to the shaft per unit tilting velocity of the shaft:

$$\frac{\delta}{\omega}=\frac{16}{\gamma\Omega} \tag{1}$$

The dimensions of the quantities of either side of equation (1) will be noted to be the units of time. The quantity $16/\gamma\Omega$ can be interpreted physically as follows: If the rotor shaft is tilting at any constant angular velocity, the thrust vector reaches a given attitude in space $16/\gamma\Omega$ seconds after the rotor shaft has reached that attitude.

If a helicopter is tilted at an angular velocity ω, as shown in figure 3, the ensuing lag of the rotor plane displaces the thrust vector and thus produces a moment about the center of gravity. This moment due to tilting velocity is known as "damping in pitch" or "damping in roll," depending upon the axis about which the tilting occurs, and can be expressed mathematically as $\Delta M/\Delta\omega$ or M_ω. Because this moment is always opposite to the tilting velocity for the conventional rotor, M_ω is always stabilizing and according to the convention, negative in sign. (Inasmuch as the effects of lateral and longitudinal motions from hovering are similar, descriptions of either motion are applicable to the other.)

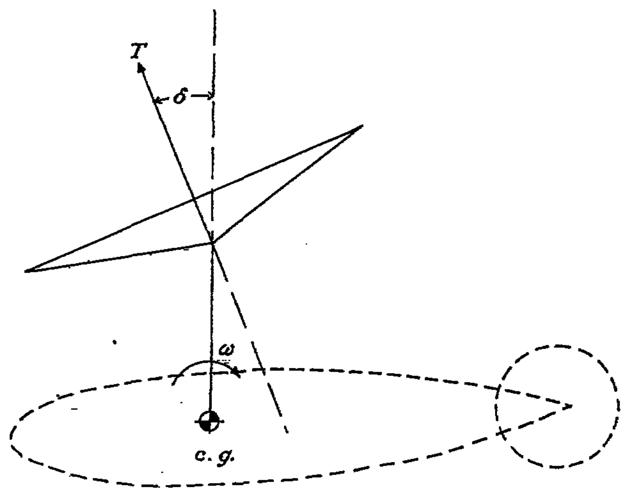


FIGURE 3.—Source of helicopter damping in pitch.

STABILITY WITH SPEED

Consider the rotor of figure 2(a) mounted on a helicopter which is subjected to a translational velocity. The effect of this translational velocity is to tilt the plane of rotation in a direction away from the velocity of translation as shown in figure 4. This tilting of the rotor plane is a result of blade flapping which arises from differences in lift on the advancing and retreating blades brought about by differences in velocity. Blade flapping, which cyclically varies the blade angle of attack, tends to equalize these differences in lift.

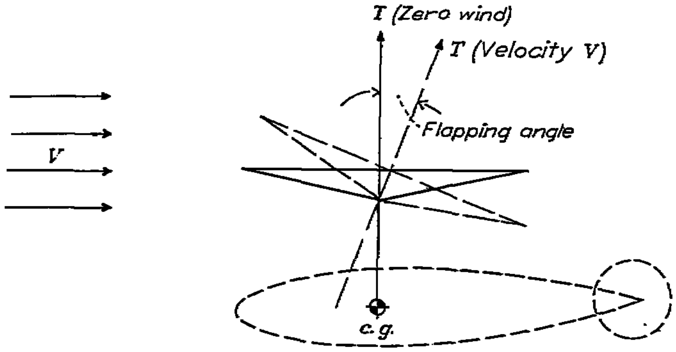
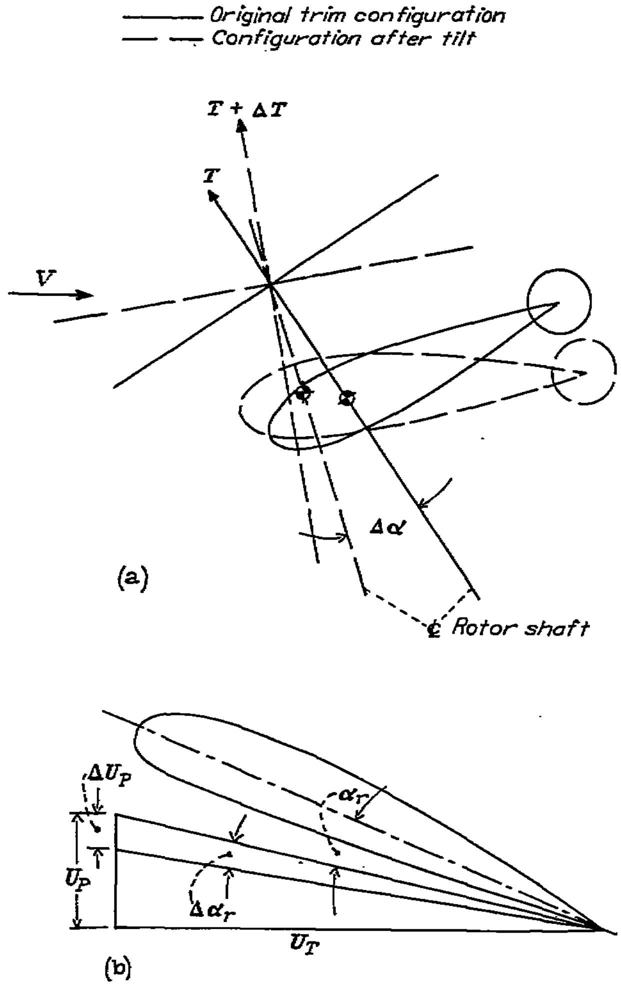


FIGURE 4.—Effect of translational velocity on attitude of rotor plane of rotation.

The rotor plane will tilt farther backwards (that is, flapping will increase) with increasing translational speeds, inasmuch as the velocity of the advancing blades becomes increasingly greater than the velocity of the retreating blades. Figure 4 indicates that this tilt of the rotor plane due to translational velocity will produce a moment about the helicopter center of gravity. The moment will be nose-up with increasing speed and nose-down with decreasing speed. The variation of moments due to changes in translational velocity is a measure of stability with speed, which can be expressed mathematically as $\Delta M/\Delta V$ or M_V. Inasmuch as nose-up moments are considered positive, M_V is always positive for the conventional helicopter rotor.

VARIATION OF ROTOR MOMENT AND FORCE WITH FUSELAGE ANGLE OF ATTACK

As shown in figure 2, a change in attitude of the hovering helicopter (which is prevented from translating) results in an equal tilt of the rotor plane with the result that no rotor moment or change in thrust occurs. In forward flight, however, a change in longitudinal attitude (fuselage angle of attack) will produce a rotor moment and a thrust change. This moment due to a change in fuselage angle of attack at constant velocity arises from the change in flapping (tilt of the rotor plane relative to the fuselage) and can be understood by an examination of figure 5. Consider a nose-up change in fuselage angle of attack α from the trim value as shown in figure 5(a). The changes in relative velocities and angle of attack of a typical blade element, which result from this change in fuselage angle, are shown in figure 5(b) where U_P, U_T, and α_r represent trimmed values. The change in blade section angle of attack $\Delta\alpha_r$ is equal to $\Delta U_P/U_T$ (for the usual assumption of small angles included in helicopter analyses), and the change in lift at this section, which is proportional to $\Delta\alpha_r U_T^2$, is therefore proportional to $\Delta U_P U_T$. Inasmuch as ΔU_P is constant over the rotor disk (the component of flight velocity through the disk is constant over the disk), the change in lift due to the change in fuselage angle of attack is greater on the advancing blade where U_T is highest. This unequal increase in lift between the advancing



(a) Side view of helicopter.
(b) Velocity diagram of typical blade element.
FIGURE 5.—Effect of change in fuselage angle of attack on resultant rotor force.

and retreating blades is compensated for by increased flapping or a backward tilt of the rotor cone with respect to the fuselage. At the same time, the increased lift at all sections results in an increase in the magnitude of the rotor thrust. Figure 5(a) shows that this tilt of the thrust vector with respect to the fuselage, which results from the nose-up change in fuselage angle, produces a nose-up moment about the fuselage center of gravity which is accentuated by the increased magnitude of the rotor thrust. If a nose-down change in fuselage angle had been considered, the result would have been a forward tilt of the rotor cone relative to the fuselage and a reduction in thrust. Inasmuch as a change in angle results in a change in magnitude as well as a tilt of the thrust vector, doubling a nose-up change in angle more than doubles the nose-up moment. Conversely, doubling a nose-down change in angle results in less than a doubled nose-down moment but nevertheless a nose-down moment.

The preceding discussion shows that the variation of moment about the center of gravity with angle of attack at

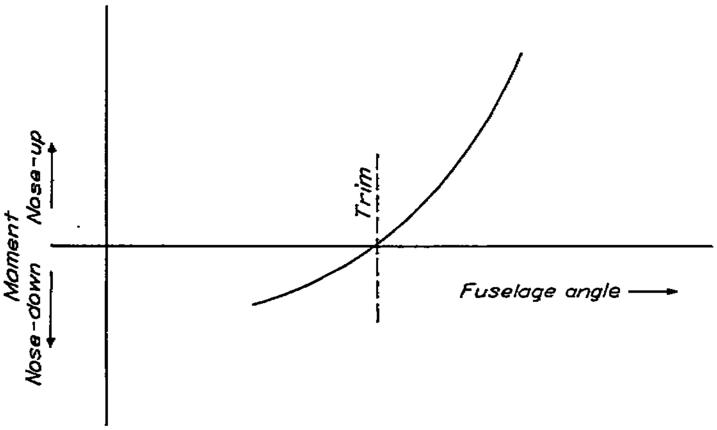


FIGURE 6.—Pitching moment contributed by rotor resultant force about center of gravity as a function of fuselage angle of attack.

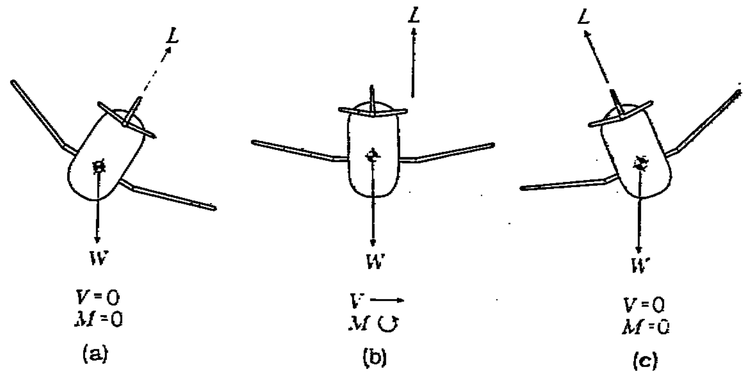


FIGURE 7.—The effect of dihedral and sideslip on the lateral motion of a fixed-wing airplane (prevented from rotating about its vertical axis) following a displacement in roll.

constant speed for the helicopter rotor would be as shown in figure 6. This figure shows that the rotor is unstable with fuselage angle of attack and that a given change in angle of attack from trim produces a greater moment change in the nose-up direction than in the nose-down direction. This figure also shows that the instability with angle of attack becomes greater with larger nose-up angle-of-attack changes and smaller with larger nose-down angle-of-attack changes.

The variation of moment due to changes in fuselage angle is a measure of static stability with angle of attack which may be expressed mathematically as $\Delta M/\Delta\alpha$ or M_α. For the statically unstable helicopter rotor, M_α is, according to the sign convention, always positive in sign. The variation in thrust with angle change is expressed mathematically as $\Delta T/\Delta\alpha$ or T_α. For the conventional helicopter rotor, T_α is positive.

STABILITY IN HOVERING FLIGHT

STATIC STABILITY

According to the definition of static stability, the hovering helicopter possesses neutral static stability with respect to angular displacements in that, if it is displaced in roll or pitch and prevented from translating, no moments will arise to tend to restore it to its original position. The concept can be understood by remembering that the resultant rotor thrust always passes through the helicopter center of gravity irrespective of the angular position of the helicopter. It might be pointed out that the conventional fixed-wing airplane in forward flight is also neutrally stable in roll in that no restoring or upsetting moments are produced when the airplane is displaced in roll and prevented from translating laterally.

Although no restoring moments will be produced by the angular displacement of the airplane, this displacement will result in a lateral velocity due to the unbalanced lateral component of lift force. Once the airplane is moving laterally, the dihedral of the wings, combined with the sideslip velocity, produces a moment tending to reduce its lateral velocity by tilting the airplane in a direction opposite to its initial tilt. This effect can be seen in figure 7. Thus, an airplane with sufficient wing dihedral is statically stable with regard to changes in lateral velocity.

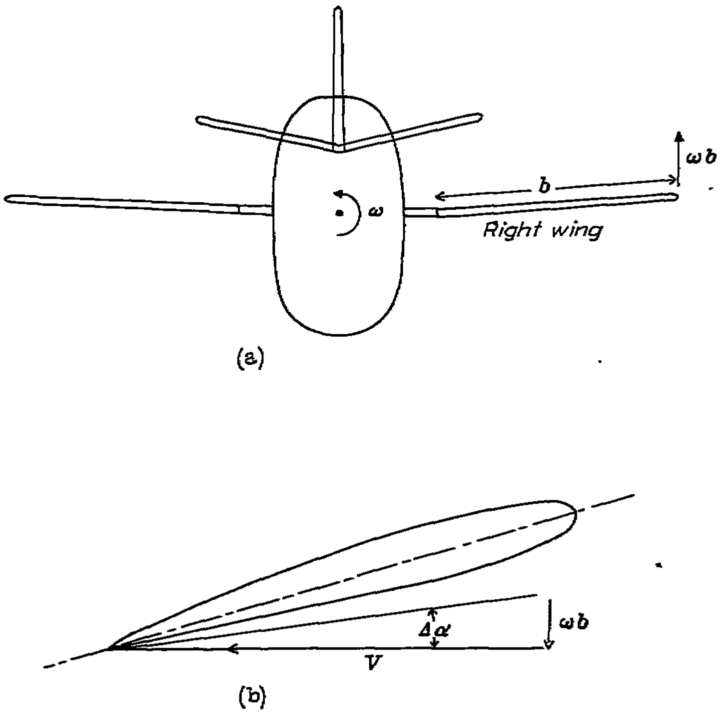
A similar situation exists for the hovering helicopter. An angular displacement of the helicopter, while directly producing no restoring moment, will result in a translational velocity due to the unbalanced horizontal component of the thrust force. As a result of stability with speed, a moment is produced which tilts the helicopter so that the horizontal component of the thrust vector acts to reduce the translational speed to its initial zero value. Thus, because of its positive stability with speed, the helicopter is statically stable with regard to changes in translational velocity. The moment produced by a translational velocity should be noted to be analogous to the moment produced by wing dihedral and sideslip velocity for the fixed-wing airplane in forward flight.

DYNAMIC STABILITY

The dynamic behavior of the hovering helicopter when upset in roll or pitch can best be explained by first examining the elements that influence the behavior of the fixed-wing airplane in forward flight when upset in roll, inasmuch as the behavior of both aircraft in these conditions are similar in many respects.

Analogy with the airplane.—In order to study the dynamic behavior of the airplane, a more detailed discussion of its behavior when displaced in roll is desirable. Consider again the airplane displaced in roll to the right as in figure 7(a). A resultant force to the right can be observed that causes the airplane to sideslip to the right. Once the airplane is moving laterally, the dihedral of the wings combined with the sideslip velocity produces a moment tending to restore the airplane to a level attitude as in figure 7(b). If the airplane is assumed to be restrained from yawing about its vertical axis so that no other effects are present, this moment will succeed in leveling the airplane. However, when the airplane reaches a level attitude, it still has a lateral velocity that causes it to continue to roll. The horizontal component of wing lift, now acting to the left, causes the airplane to lose its lateral velocity and to end up in the condition shown in figure 7(c), wherein the airplane has zero lateral velocity but is displaced in roll to the left. The resultant force to the left causes a movement to the left, and the cycle of events is repeated in the form of an oscillation. If the amplitude of the oscillation increases with time, the airplane is by definition dynamically unstable; if the motion decreases in amplitude with time, it is considered dynamically stable.

During the oscillation, the airplane has an angular velocity about its longitudinal axis. At the instant when the airplane is in the position shown in figure 7(b), for example, it is rolling to the left. The result of the rolling velocity is to reduce the angle of attack of the right wing. (See fig. 8.) Similarly, the angle of attack of the left wing is increased. Thus a clockwise moment is produced that tends to oppose the counterclockwise angular velocity of the airplane. The initial angular displacement of an airplane thus results in an oscillation during which the airplane is acted upon by two opposing moments: the first, a moment produced by the sideslip velocity; and the second, a damping moment produced by the angular velocity of the airplane.



(a) Rear view.
(b) Side view of right wing.

FIGURE 8.—Source of damping moment on a fixed-wing airplane as a result of rolling velocity.

Helicopter motion following a disturbance.—The motion following an initial angular displacement of a helicopter, as well as the moments acting on it during the oscillation, is analogous to the motion (and moments) just described. Just as for the airplane, it is desirable in the study of the dynamic behavior of the hovering helicopter to discuss the motion of the helicopter following an angular displacement in greater detail than was done in the section entitled "Static Stability."

If the hovering helicopter is displaced in roll to the right (fig. 9(a)), the resultant force to the right will cause the helicopter to move to the configuration shown in figure 9(b). The helicopter in moving from the position of figure 9(a) to that of figure 9(b) is subjected to a counterclockwise moment due to stability with speed. This moment rolls the helicopter until it reaches the configuration shown in figure 9(c). A horizontal force now tends to slow down the helicopter, so that it returns to zero horizontal velocity in the position of figure 9(d). Because a horizontal force to the left is now present, the helicopter starts to move to the left. By proceeding in the manner described for the first half of the cycle, the helicopter reaches the position shown in figure 9(a), at which time one cycle of the oscillation will have been completed, and the process repeats.

Just as is true of the fixed-wing airplane in a lateral oscillation, the helicopter has an angular velocity about its own axis during the oscillation which also results in a moment due to damping in roll. This moment has an important effect on the oscillation. Examine the position of the helicopter shown in figure 9(c). At this instant the helicopter has a counterclockwise angular velocity which causes a small clockwise tilt of the rotor cone from that shown with damping neglected. The actual configuration of the rotor, with damping considered, is as shown in figure 10. As can be seen from this figure, the angular velocity of the helicopter causes the rotor cone to lag behind the position it would have if no damping were present.

Thus far the separate effects resulting from an angular displacement in attitude of the helicopter have been examined. It has been seen that the result of the displacement

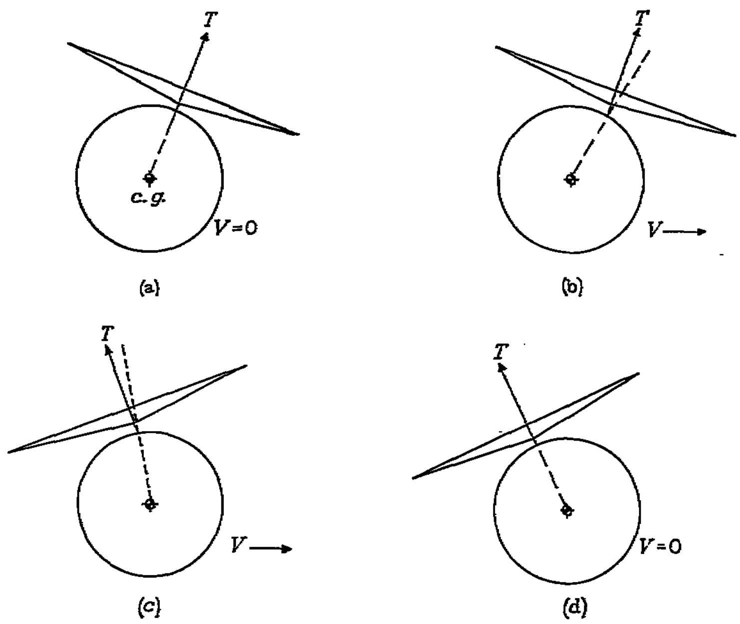


FIGURE 9.—Effect of stability with speed on the translational motion of a helicopter following a displacement in pitch (or roll) from hovering.

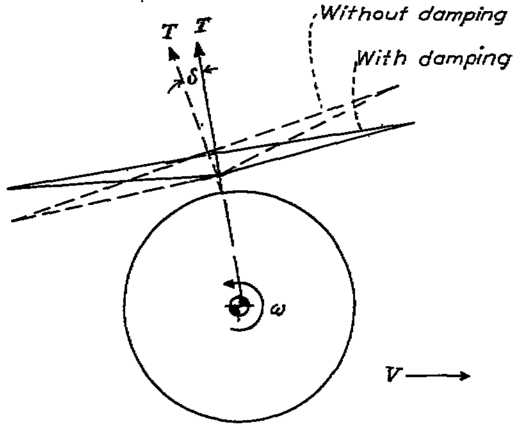


FIGURE 10.—Position of rotor cone with and without damping in roll for helicopter in figure 9(c).

is an oscillation, and it will now be shown that stability with speed and damping in pitch (or roll) are most important in influencing the period of the oscillation. (The factors that influence the divergence or convergence of the oscillation are indicated subsequently herein.)

In order to examine the combined effects of stability with speed and damping in pitch, the motion following an angular displacement of a hovering helicopter is examined in successive steps. For the sake of clarity, stability with speed and damping in pitch are assumed to act successively, although their effects actually occur simultaneously. Each of the following cycles of events should, therefore, be considered as occurring over a very short interval of time. Also, the moment of inertia of the fuselage is assumed to be negligible for the immediate discussion.

Consider a hovering helicopter displaced in roll (or in pitch) to an attitude shown in figure 11(a). Although no moment is produced about the center of gravity of the helicopter, a resulting force occurs to the right which will cause a velocity to the right, and the helicopter is displaced to the configuration of figure 11(b). In this configuration, the thrust vector has been inclined to the left and produces a counterclockwise moment about the center of gravity as a result of stability with speed. Inasmuch as the fuselage moment of inertia was assumed to be negligible, this moment in turn quickly produces a counterclockwise angular velocity so that in a short interval of time the helicopter is in the configuration of figure 11(c). Because of damping in pitch, the counterclockwise angular velocity has permitted the fuselage to overtake the rotor cone, so that, after a negligible interval of time, the rotor tilt originally produced by the stability with speed is neutralized. Inasmuch as a horizontal component of force to the right still exists, the helicopter continues to accelerate in that direction and the process is repeated—that is, the additional translational velocity causes an additional thrust vector tilt to the left which produces a counterclockwise moment and an increase in angular velocity. Because of the damping in pitch, this increased angular velocity permits the fuselage to aline itself with the thrust vector so that again, after a negligible time interval, the additional tilt produced by the stability with speed is neutralized.

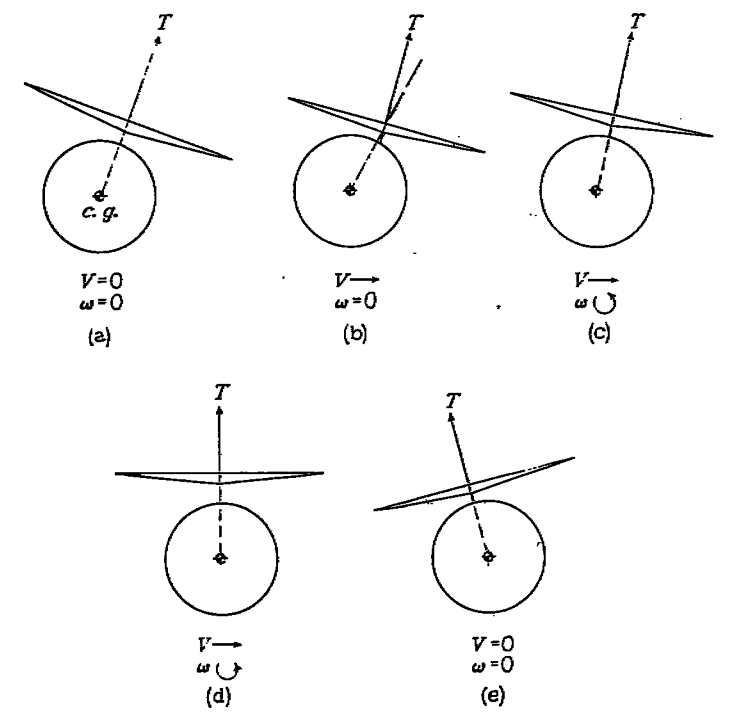


FIGURE 11.—Translational oscillation of a helicopter following an attitude displacement in hovering.

Because each cycle has thus far rotated the helicopter toward a level attitude, the helicopter soon attains a horizontal attitude as shown in figure 11(d). The previous cycles of events continue to occur in the same way except that from now on the thrust vector is tilted to the left, and the velocity of the helicopter is thus reduced until it reaches the position of figure 11(e) where it has zero angular and translational velocity. This position corresponds to that of figure 11(a). Because a horizontal component to the left is still present, the helicopter starts to move left, the process represented by figures 11(a) to 11(e) is repeated, and the helicopter continues to oscillate back and forth. The time required for the helicopter to move from the position shown in figure 11(a) to that of figure 11(e) is one-half the period of the oscillation.

In reference 2, a formula is derived for the period of the oscillation of a hovering helicopter having zero fuselage moment of inertia, which can be written as

$$P=\frac{2\pi}{\sqrt{g}}\sqrt{\frac{-M_\omega}{M_V}} \tag{2}$$

The formula for M_ω is approximately $-Th\frac{\delta}{\omega}$. The stability with speed M_V can be approximately calculated from an equation that represents the variation of longitudinal flapping with translational velocity.

From the preceding discussion, the effect of stability with speed and damping in pitch on the period can be physically interpreted. Consider the helicopter moving from the position shown in figure 11(a) to that shown in figure 11(b). The larger the stability with speed is, the greater the thrust vector tilt in figure 11(b). A larger angular velocity results and, therefore, the position of figure 11(c) is reached more quickly. An increase in stability with speed thus reduces the period of the oscillation. Equation (2) gives the same result inasmuch as the stability-with-speed term appears in the denominator. The effect of stability with speed on period appears to explain the experimentally observed difference noted in reference 1 between the period of the pitching and the rolling oscillation for the conventional single-rotor helicopter. If the tail rotor shaft, for example, is mounted above the center of gravity, the tail rotor will add to the helicopter's stability with speed during lateral motion, and thus the period will be decreased. This effect arises from the change in tail rotor thrust due to the change in inflow that occurs while the tail rotor is experiencing a lateral velocity.

The effect of damping in pitch can be seen by comparing figures 11(b) and 11(c). The larger the damping in pitch is, the smaller the angular velocity necessary to neutralize the thrust vector tilt that was produced by the stability with speed in figure 11(b). Slower changes in attitude result and thus the position of figure 11(e) is reached later than if less damping were present. An increase in damping in pitch thus increases the period of the oscillation. Equation (2) gives the same result inasmuch as the damping-in-pitch term appears in the numerator.

Of greater importance than its effect on the period of the oscillation is the effect of damping in pitch on helicopter handling qualities. Most single-rotor helicopters with conventional control systems, especially those of small size, tend to have insufficient damping in pitch, which, according to reference 1, is a serious handling-qualities deficiency. (The effect of increased damping in pitch on helicopter handling qualities has also been discussed in several other papers, among which are references 3 and 4.) Many present-day helicopters are equipped with additional devices that increase damping in pitch.

According to the mathematics of reference 2, the presence of a finite fuselage moment of inertia results in a higher period of the oscillation than that given by equation (2). The general effects, however, of stability with speed and damping in pitch are believed to be valid also for the case of finite moment of inertia.

Although a physical representation of the effect of the various parameters on the convergence or divergence of the hovering oscillation is difficult, their effects have been investigated theoretically. In reference 5, it was concluded that the dynamic instability of the conventional helicopter in hovering flight could be reduced by decreasing the moment of inertia of the helicopter fuselage, by increasing the moment of inertia of the rotor blades about their flapping hinges (which increases the damping in pitch), by increasing the vertical height of the rotor above the center of gravity of the helicopter, and by offsetting the flapping hinges from the center of the rotor.

LONGITUDINAL STABILITY IN FORWARD FLIGHT

STATIC STABILITY

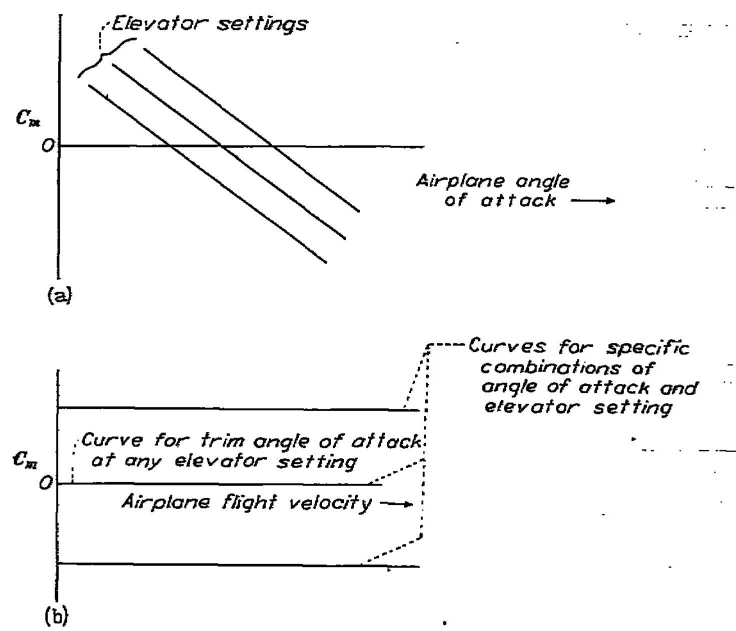
As was done in the study of stability of the helicopter in the hovering condition, some airplane stability concepts are used for the interpretation of the physical parameters affecting helicopter stability in forward flight.

Analogy with the airplane.—Inasmuch as an airplane can be displaced in pitch (angle-of-attack change) or by a change in forward speed, in general, two aspects of static stability exist because of the two sets of forces and moments produced by these two changes.

If an airplane is flying in a trimmed position and the angle of attack is increased while its speed is kept constant, the airplane is statically stable with respect to angle of attack if the resulting aerodynamic moment is a nose-down moment. The airplane static stability with angle of attack is dependent upon center-of-gravity position, inasmuch as variations in center-of-gravity position affect the moment arm of the lift forces on the wing and tail.

Consider now the static stability of an airplane with changes in speed and with angle of attack kept constant. If power and Mach number effects are neglected, which is justified for the present discussion, a variation in speed from trim speed while the angle of attack and flight path are kept constant (as could be done in a wind tunnel) produces no aerodynamic moment about the center of gravity. In other words, the airplane is neutrally statically stable with speed at constant angle of attack because no change is obtained in lift or moment coefficients with speed. A given speed change from trim merely changes all of the aerodynamic forces and moments acting on the airplane in the same proportion, and the airplane is thereby maintained in trim.

With these concepts in mind, the static stability of a given airplane with fixed center-of-gravity location can be expressed by the plots of moment coefficient C_m against angle of attack and speed of figure 12, data for which can be obtained from wind-tunnel tests. Because the moment coefficient at constant



(a) Plot of C_m against airplane angle of attack.
(b) Plot of C_m against airplane flight velocity.
FIGURE 12.—Basic static-stability curves of a typical fixed-wing airplane in gliding flight as obtained from wind-tunnel tests.

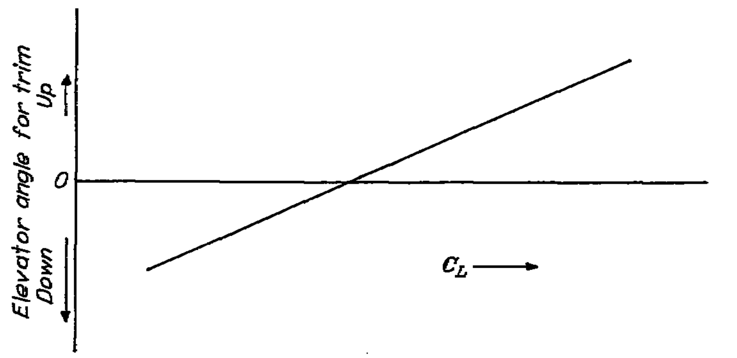


FIGURE 13.—Static-stability curve of a typical airplane obtained in gliding flight. This curve is independent of speed.

angle of attack and control deflection is independent of speed as shown in figure 12(b), the single static-stability curve of figure 13, which does not depend on speed, can be obtained from figure 12(a) alone. Figure 13 was obtained from figure 12(a) by picking off the elevator settings and their corresponding trim angles of attack, the trim angles of attack being readily converted to lift coefficients. This type of plot is the conventional way of representing the static stability of an airplane because it can be easily obtained from flight tests of an airplane trimmed in straight, steady flight (that is, $L \approx W$, $C_m=0$). A positive slope to the curve of figure 13 means that the airplane is stable stick fixed in that a forward movement of the control stick (or down elevator) is required for trim at a decreased airplane angle of attack (or C_L).

It should be emphasized that the single curve in figure 13 completely defines the static stability of an airplane (at fixed center-of-gravity position) only because the static stability of an airplane with speed at constant angle of attack is neutral. When the effect of propeller operation is considered, however, a single curve such as that given in figure 13 is no longer sufficient as the airplane is no longer neutrally stable with speed at constant angle of attack. Because the helicopter has positive and not neutral static stability with speed, it is therefore apparent that, like the airplane in the power-on condition, a single curve does not suffice.

Static stability of helicopter.—The static stability of the helicopter in forward flight depends upon the moments produced on the helicopter by a change in speed from trim during flight at a constant angle of attack as well as moments produced by a change in angle of attack from trim at constant speed. The moment contributed by the rotor as a result of either of these changes has already been discussed in the section entitled "Rotor Characteristics."

For the actual helicopter, the fuselage and stabilizing surfaces (if any) will also contribute aerodynamic moments which vary when either the speed or angle of attack is changed. These moments are brought about in three different ways:

(1) Effect of a variation of moment coefficient with angle of attack on angle-of-attack stability. The conventional helicopter fuselage has an unstable variation of moment with angle of attack which adds to the rotor angle-of-attack instability. A fixed tail surface would contribute a stabilizing variation of moment with angle of attack.

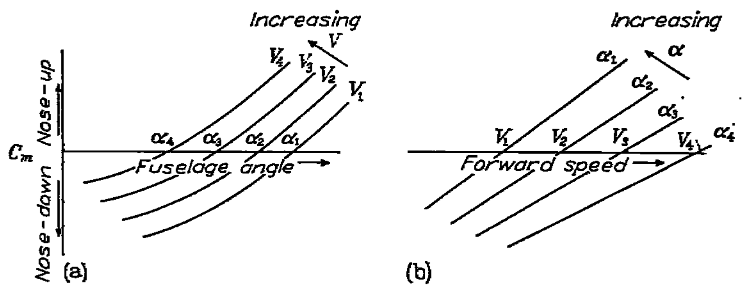
(2) Effects of a constant moment coefficient during steady flight on stability with speed. The conventional helicopter fuselage has a nose-down moment coefficient during steady flight. Thus, if the speed of the helicopter is varied from trim at constant angle of attack, the resulting variation in moment arising from the change in dynamic pressure is destabilizing. If stabilizing surfaces contribute a nose-up moment in steady flight, the resulting variation of moment with speed will be stabilizing.

(3) Effect of a thrust-axis offset due to a constant moment coefficient during steady flight on stability with angle of attack. The conventional helicopter fuselage has a nose-down moment in steady flight which is compensated for by the thrust vector being offset ahead of the helicopter center of gravity. This offset results in the rotor contributing an additional unstable moment variation with angle-of-attack change as can be understood by again examining figure 5(a). An increase in the fuselage angle of attack results in a nose-up moment greater by an amount equal to the product of the thrust increment and the initial center-of-gravity offset than the moment produced by the rotor with no center-of-gravity offset. Thus, nose-down fuselage moments, which require the thrust axis to be offset forward of the center of gravity, add to the angle-of-attack instability of the rotor. If stabilizing surfaces contribute a nose-up moment in steady flight, the resulting offset between the thrust vector and the helicopter center of gravity counteracts the rotor instability with angle of attack or, if the offset is great enough, will even make the helicopter rotor statically stable with angle of attack.

The two types of forward-flight static stability can be represented by the moment-coefficient curves of figures 14(a) and 14(b) which can be obtained from wind-tunnel tests. Figure 14(a) shows the variation of moment coefficient about the helicopter center of gravity with fuselage angle of attack at various speeds. Figure 14(b) shows the variation of moment coefficient with speed for each of the trim angles of attack shown in figure 14(a). (Figs. 14(b) to 17 are presented to show general trends but the shapes of the curves are arbitrary.)

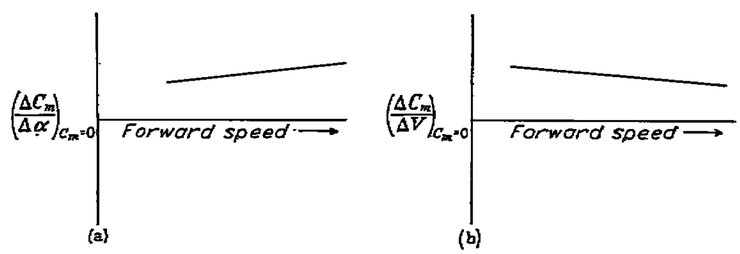
In figure 14(a), a separate curve is required for each speed; whereas, the static stability of the airplane requires only the single curve shown in figure 12(a). The reason for these separate curves arises from the moments produced by a change in speed from a trim point as can be seen in figure 14(b); thus, the trim point and curve of figure 14(a) are shifted.

The amount of static stability or instability of the helicopter is quantitatively defined by the curves of figure 15, which represent the slopes of the curves of figure 14 at the trim conditions. Specifically, the curve of figure 15(a) was obtained by picking off values of airspeed and $\Delta C_m/\Delta\alpha$ at



(a) Plot of C_m against α. (b) Plot of C_m against V.

FIGURE 14.—Basic static-stability curves for a typical tailless helicopter at a fixed operating condition.



(a) Plot of $\left(\frac{\Delta C_m}{\Delta_\alpha}\right)_{C_m=0}$ against V. (b) Plot of $\left(\frac{\Delta C_m}{\Delta V}\right)_{C_m=0}$ against V.

FIGURE 15.—Slopes of curves of figure 14 at $C_m=0$.

$C_m=0$ from the curves of figure 14(a). Similarly, the curve of figure 15(b) was obtained from the curves of figure 14(b). Methods of obtaining curves similar to those of figure 15 from flight tests have not yet been fully explored.

The curves of figure 15 represent a typical tailless helicopter (one with no horizontal tail surface) in power-on flight because it is unstable with angle of attack and stable with speed. According to reference 1, this instability with angle of attack is a principal stability deficiency of the conventional tailless helicopter in forward flight.

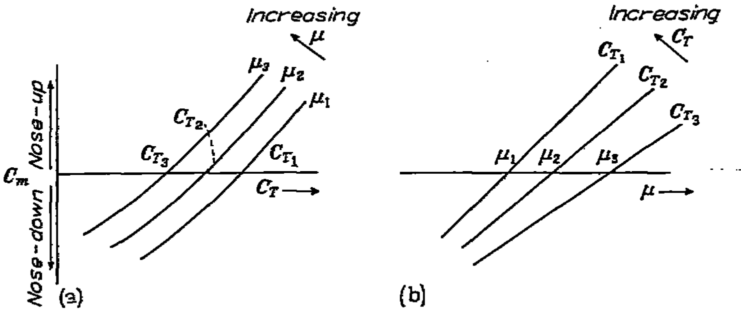
It should be emphasized that the curves of figures 14 and 15 represent the characteristics of a helicopter having given center-of-gravity and stick positions, gross weight, rotor speed, and collective pitch and flying at a given altitude. The effect of variations in gross weight, rotor speed, and altitude can be accounted for by plotting the stability data in nondimensional form. One possible method of plotting is shown in figures 16 and 17.

In order to account for a change in stick position, the contribution of the fuselage and tail surfaces (if any) to the total value of C_m must be known. The effect of a center-of-gravity change with fixed stick position can be effectively accounted for by correcting each value of C_m in figure 16 by an amount equal to $C_L\left(\frac{\Delta e}{R}\right)$. For the special case of no moment contribution by the fuselage or tail surface, a center-of-gravity change at a given flight condition results in a change in fuselage attitude which is compensated for by a change in stick position, and the stability of the aircraft is unaffected.[a] If, however, either the fuselage or a fixed tail surface do contribute moments that change with angle of attack, a center-of-gravity change, which tilts the fuselage, will change the fuselage moments and thus change the horizontal distance between the thrust vector and the center of gravity in trimmed flight. As discussed previously, this change in center-of-gravity offset during trimmed flight does affect the stability of the helicopter.

In order to take account of variations in collective pitch, curves similar to those in figures 16 and 17 would be needed for several pitch values.

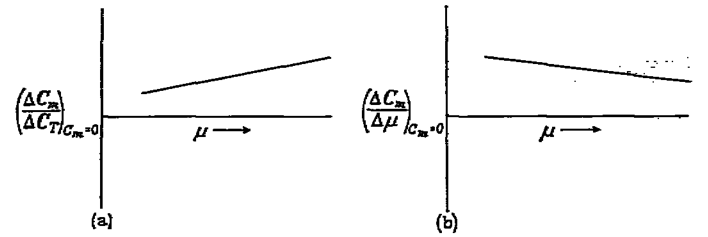
Curves similar to those in figure 16 not only take account of variations in the trim value of rotor speed but also variations in rotor speed which will normally occur during changes in fuselage angle of attack or forward speed. This variation in rotor speed affects the static stability of the helicopter. For example, the autorotating rotor has different stability characteristics from the powered rotor. The primary reason for this difference is the fact that the rotor speed of the autorotating rotor is not controlled by the engine but is free to vary with changes in forward speed or angle of attack. Reference 6 states that the effect of these variations in rotor speed is to make the autorotating rotor neutrally stable with changes in speed at constant angle of attack and positively stable with changes in angle of attack at constant speed. Thus, the power-speed characteristics of the helicopter engine affect the stability of the helicopter.

[a] For a given flight condition, the attitude of the rotor plane in space is fixed. Thus, in order to maintain a given flight condition when the center of gravity is shifted and a tilt of the fuselage and rotor plane results, the control stick must be moved to a position such that the rotor plane returns to its initial attitude.



(a) Plot of C_m against C_T. (b) Plot of C_m against μ.

FIGURE 16.—Basic nondimensional static-stability curves for a typical tailless helicopter at a given value of collective pitch, center of gravity, and stick position.



(a) Plot of $\left(\frac{\Delta C_m}{\Delta C_T}\right)_{C_m=0}$ against μ. (b) Plot of $\left(\frac{\Delta C_m}{\Delta \mu}\right)_{C_m=0}$ against μ.

FIGURE 17.—Slopes of curves of figure 16 at $C_m=0$.

DYNAMIC STABILITY

Many of the factors that influence the dynamic stability of the helicopter in forward flight can be understood from the information already presented about the dynamic behavior of the helicopter in hovering. If the helicopter is assumed to have neutral static stability with respect to changes in angle of attack (as it has in hovering oscillations as a result of near-zero airspeeds), then the period of the longitudinal oscillation in forward flight is primarily influenced by the same quantities as the hovering oscillation: namely, stability with speed and damping in pitch. This contention is borne out by an examination of the approximate equation in reference 6 for the period of the longitudinal oscillation of a helicopter in forward flight. This equation, which may be written as follows, neglects, among other things, the moment of inertia of the helicopter (moment of inertia is expected to increase the period):

$$P=2\pi\sqrt{\frac{-M_\omega-\frac{WV}{g}\frac{M_\alpha}{T_\alpha}}{M_V g}} \tag{3}$$

If M_α is assumed equal to zero, this formula reduces exactly to the formula for the period in hovering (equation (2)).

Helicopter motion following a disturbance.—The importance of stability with speed and damping in pitch can be shown physically by means of the following discussion. (The description of the oscillation which follows is only approximate, as secondary effects are ignored.) Consider a longitudinal oscillation of a helicopter having neutral stability with angle of attack. Assume the helicopter to be flying at a trimmed condition in level flight, at which time a disturbance causes it to nose down and start to descend as shown in figure 18(a). The component of weight along the flight path will accelerate the helicopter and increase its speed until the helicopter

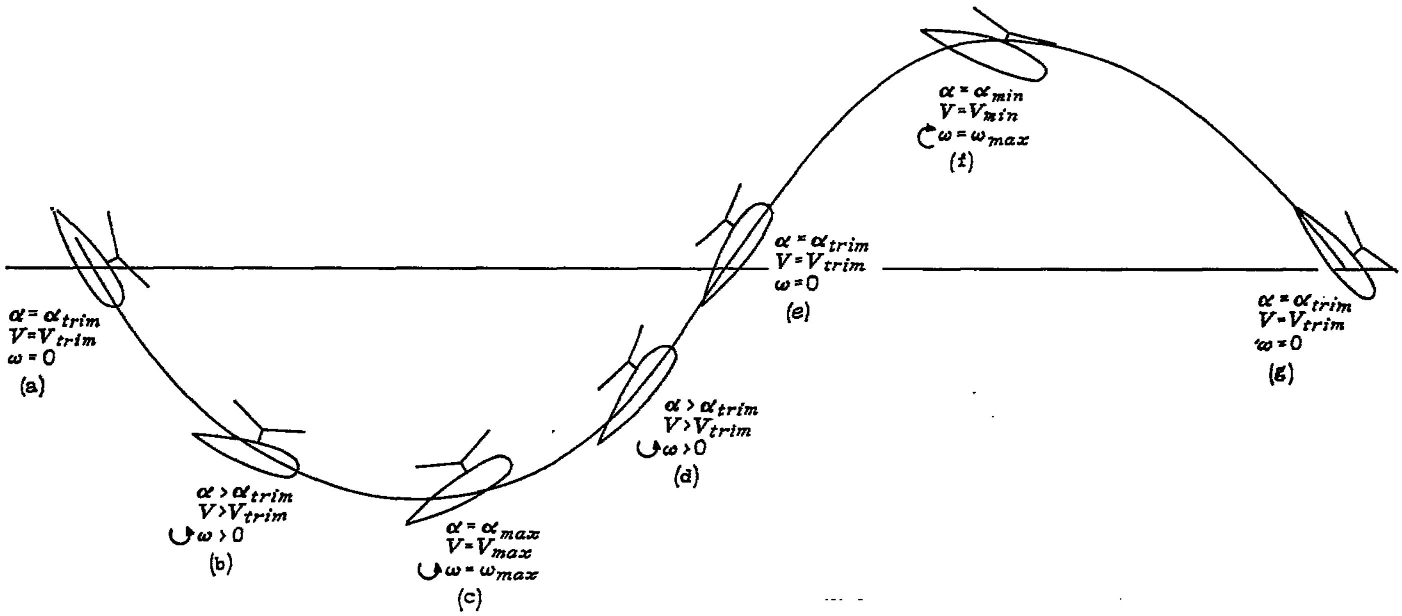


FIGURE 18.—Longitudinal oscillation of a typical helicopter in forward flight.

reaches the position shown in figure 18(b). Because of stability with speed, this increased velocity produces a backward tilt of the rotor plane and a nose-up moment, which in turn causes a nose-up angular acceleration.

The angular acceleration leads to an angular velocity of such magnitude that the damping in pitch allows the fuselage to overtake the rotor thrust; thus, the vector tilt due to stability with speed is neutralized. As long as there is a component of weight along the flight path, the helicopter speed will continue to increase and the preceding steps will be repeated. The continually increasing angular velocity of the helicopter during these steps results in a continuously increasing fuselage angle of attack. In turn, the thrust will continuously increase until it levels off the glide path and the helicopter reaches the position shown in figure 18(c). In this position, the helicopter has approximately maximum forward speed, maximum nose-up angular velocity, and maximum fuselage angle of attack. Inasmuch as the thrust at this point is greater than the helicopter weight (because the angle of attack is greater than the trim value), the helicopter will start to climb. The component of weight along the flight path now opposes the forward motion and the helicopter begins to slow down, and the backward tilt caused by stability with speed is reduced. The resulting tilt of the rotor plane is forward, inasmuch as the forward tilt due to damping in pitch is now greater than the rearward tilt due to stability with speed. The nose-down moment in turn reduces the nose-up angular velocity of the helicopter to a value such that the damping in pitch again neutralizes the remaining backward tilt of the rotor plane from trim position which was brought about by stability with speed, and the helicopter is in the position shown in figure 18(d). The component of weight continues to slow down the helicopter and the preceding steps are repeated until the helicopter reaches the position of figure 18(e) where its velocity and angle of attack are equal to the trim values and it has zero angular velocity. Because the helicopter is now climbing, it will continue to decelerate and the cycle of events depicted by figures 18(a) to 18(e) will be repeated except that all changes will be in the opposite direction. Thus, as shown in figure 18(f), the helicopter will have approximately minimum forward speed, maximum nose-down angular velocity, and minimum fuselage angle of attack. When the helicopter reaches the position of figure 18(g), it is in the same flight condition as figure 18(a), and the cycle of events depicted in figures 18(a) to 18(e) is repeated.

Effect of stability with speed and damping in pitch on period of oscillation.—An increase in stability with speed will cause a larger nose-up moment for the increase in speed shown in figure 18(b). This moment will cause larger nose-up angular velocities than hitherto attained and the position shown in figure 18(c) will be reached sooner. Thus, an increase in stability with speed reduces the period. Equation (3) gives the same result inasmuch as M_V appears in the denominator. The larger the magnitude of the damping in pitch is, the smaller the angular velocity, produced by the nose-up moment of figure 18(b), required to neutralize the stability with speed. A longer time is thus necessary to reach the angle of attack required to level off the helicopter in the position shown in figure 18(c). Thus, an increase in damping in pitch increases the period. Equation (3) gives the same result in that M_α has been assumed to be equal to zero and $-M_\omega$ is a positive quantity in the numerator.

Effect of angle-of-attack static stability on period of oscillation.—Equation (3) shows that the effect of static stability with angle of attack M_α is to add to, or subtract from, the effect of damping in pitch M_ω. If a helicopter is statically unstable with angle of attack, M_α is positive and inasmuch as

T_α is positive, the term $\frac{WV}{g}\frac{M_\alpha}{T_\alpha}$ is positive. Thus, the magnitude of the numerator and, consequently, the period is reduced. Inasmuch as moments due to changes in angle of attack and angular velocity vary during the oscillation, they must be approximately in phase in order that they may be added algebraically. Figures 18(c) and 18(f) show that α and ω reach peak values simultaneously.

Physically, the effect of angle-of-attack stability M_α on the damping in pitch and thus on the period can be seen from a study of figure 18(c). When the helicopter is in this position, its nose-up angular velocity, which is a maximum, produces a maximum nose-down moment due to damping in pitch. At the same time, the angle of attack, which is also a maximum, results in a maximum nose-up moment in that the helicopter was assumed to be statically unstable with angle of attack. Thus, the effect of static instability with angle of attack is to reduce the effect of damping in pitch and, consequently, the period of the oscillation. It follows that, if a stabilizing device such as a tail surface is installed on a helicopter to make it statically stable with angle of attack, the period of the oscillation will be increased.

Influence of WV/g and T_α on period of oscillation.—As previously discussed, M_α, if stable, adds to, or if unstable, subtracts from the effect of M_ω. The relative contributions of these two quantities depend upon the relative magnitudes of the angle-of-attack change and the pitching velocity. The effects of WV/g and T_α are present because they determine the magnitude of the angle-of-attack change for a given pitching velocity. These two terms affect the maximum change in angle of attack for a given maximum pitching velocity as follows: At any point in the oscillation, the thrust force will differ from the weight of the helicopter by an amount equal to the centrifugal force produced by the curved flight path. A change in angle of attack is necessary to produce this change in thrust. If the change in thrust with angle of attack T_α is increased, a given increase in thrust can be obtained by a smaller change in angle of attack. Thus, the larger the value of T_α is, the smaller the effect of M_α. This conclusion is substantiated by equation (3) inasmuch as M_α is divided by T_α. The magnitude of the centrifugal force acting on the helicopter per unit of pitching velocity depends upon WV/g. Therefore, the larger the value of this quantity is, the greater the required change in thrust, the greater the change in angle of attack during the oscillation, and the greater the effect of M_α. Equation (3) gives the same result, inasmuch as M_α is multiplied by WV/g.

Effect of stability parameters on divergence of oscillation.—An example of the influence of the stability parameters that were previously discussed on helicopter handling qualities is their effect on the rate of divergence of an oscillation in forward flight. In practice, the rate of divergence may have an important effect on handling qualities, particularly if the divergence is so great that only a fraction of one cycle can be tolerated. (See reference 1.) According to an approximate formula in reference 6, a helicopter that is statically unstable with angle of attack will also be dynamically unstable, but a large amount of damping in pitch or a sacrifice in stability with speed will reduce the influence of a given amount of static instability. Thus, it appears desirable to incorporate in the helicopter some means of producing stability with angle of attack or a large amount of damping in pitch. The theory of reference 6 also indicates that the effect of fuselage moment of inertia is to increase the dynamic instability—that is, the moment of inertia of the fuselage causes the oscillation to diverge more rapidly.

CONCLUDING REMARKS

In order to impart an understanding of some of the factors that affect the handling characteristics of the helicopter, the physical aspects of both the static stability and the control-fixed oscillation of the helicopter in hovering and longitudinal forward flight have been explained. This explanation, which was made for a single-rotor helicopter with fully articulated blades, flapping hinges on the rotor shaft, and a conventional control system, indicates that:

(1) In hovering, the helicopter possesses neutral static stability with respect to attitude changes but has positive static stability with respect to changes in translational velocity.

(2) When disturbed from a hovering condition, the resulting motion of a helicopter is an oscillation, the period of which depends primarily upon two factors: namely, moments due to changes in speed (stability with speed) and moments due to the angular velocity of the helicopter (damping in pitch or roll).

(3) For the helicopter in forward flight, static stability with attitude change as well as static stability with speed change must be considered; whereas, for the low-speed fixed-wing airplane, only static stability with attitude change need be considered (with power effects neglected).

(4) In forward flight, the helicopter rotor is statically stable with speed and statically unstable with angle of attack. The instability with nose-up changes is greater than that with nose-down changes. Also, the instability with large nose-up changes is greater than the instability with small nose-up changes.

(5) The static stability of the helicopter in forward flight is unaffected by a center-of-gravity shift if no moments are contributed by components other than the rotor. If there are other moment contributions, as for example, from a fixed tail surface, the static stability is affected.

(6) If neutral angle-of-attack stability is assumed and if fuselage inertia effects are neglected, then the motion of a helicopter following a disturbance in forward flight is an oscillation, the period of which depends, as in the hovering condition, mainly upon stability with speed and damping in pitch. The presence of static instability of the helicopter with angle of attack causes the oscillation to decrease in period.

(7) According to an approximate theory of K. Hohenemser, dynamic instability in forward flight can be reduced by the addition of positive static stability with angle of attack, by increasing the damping in pitch, or by a sacrifice in stability with speed.

Means have been sought for the improvement of helicopter handling qualities by the use of devices which alter the magnitude of one or more of the pertinent stability factors. For example, several devices already in use either increase the damping in pitch or add positive static stability with angle of attack.

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *September 19, 1949.*

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