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THE RELATION BETWEEN THE TENSILE STRENGTH AND THE HARDNESS OF METALS

By O. Schwarz

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THE RELATION BETWEEN THE TENSILE STRENGTH AND THE HARDNESS OF METALS.*

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Summary

The practical importance of the indentation test made with a ball. In the case of nonferrous metals there exists, even for metals of the same group, aluminum and its alloys excepted, no linear relation between Brinell hardness and tensile strength. The conversion factors depend on the degree of cold working and may vary between 0.3 and 0.6. Effect and dependence of the hardness numbers on the strain-hardening. Relations between the hardness numbers and the ordinary stress-strain diagrams and tensile strength. Procedure for finding the Brinell strength. In the case of high yield-point ratios, the coefficient 0.36 is sufficiently accurate for practically all metals. Reference to conditions at higher temperatures and for cast metals.

When we have to judge a material, we generally determine the tensile strength first of all. In many cases valuable information is thus obtained, but difficulties are encountered in the

*"Zugfestigkeit und Härte bei Metallen" from Zeitschrift des Vereines deutscher Ingenieure (V.D.I., June 8, 1929, pp. 792-797. Extract from No. 313 of Forschungsarbeiten auf dem Gebeite des Ingenieurwesens, published by the V.D.I., Berlin, 1929.

determination of this quantity, when there are only small quantities of the material available, as is often the case with parts which have failed in use or when the material must not be destroyed. In the latter case we resort to the hardness test, made by impressing a ball on the material, and we consider the hardness to be measured, as proposed by Brinell, by the ratio of the load on the ball to the area of the spherical impression. Experience has shown that, for steel, under the standard conditions assumed for the size of the ball and its load the thus-determined hardness number H_n bears the relation to the tensile strength K_z shown by the formula

$$K_z = 0.36\ H_n\ (\text{kg/mm}^2)$$

This relation was so valuable that, despite the physical inadequacy of this hardness number, we were very glad to be able to determine the tensile strength by a short approximate method. In fact, the reliability of the tensile strength as calculated from the hardness number, which, for the sake of brevity, we shall call the Brinell strength, was so surprisingly great in most cases that the error was seldom greater than ±5%. It is also known, however, that for steel, the limiting values for the conversion factors are 0.3 and 0.4, and hence the usual ratio may yield deviations of ±10%. Even if we had no explanation of its underlying principles, the practical importance of the Brinell method would not be affected because, in many cases, even an approximate determination of the strength is of inestimable practical value, provided only that it can be quickly and economically made.

Brinell Hardness, Tensile Strength and Yield Point of Nonferrous Metals

There has been no lack of attempts to find a relation between tensile strength and hardness, even for nonferrous metals. They have led to no useful result, however, because the conversion factors for one and the same material may vary between 0.3 and 0.6, according to the amount of cold working. Hence the simple conversion of Brinell hardness into strength failed and the only remaining possibility was to investigate as to how the conversion factors vary with cold working for the different metals.

Figure 1 shows this relation for copper, the hardness and strength properties of which were increased as much as possible by cold rolling (up to 90%). In order to increase the practical value of this relation, the results for other forms of copper, in so far as they could be found in the literature on the subject, were also plotted. From these it is obvious that no linear relation exists. From this figure we might determine the tensile strength corresponding to certain hardness numbers, but even in these cases we find variations in strength for the same hardness numbers, when we compare the different forms of copper. The same is true of pure nickel.

With brass there is an additional difficulty, in that, even in the annealed state, considerable differences in hardness appear according to the alloy. Different alloys show similar curves, but the same hardness numbers of different alloys show quite different strengths. Thus, for example, for a Brinell hardness of $H_{10/1000} = 80$ kg/mm², we have strengths of 26.5 to 37.5 kg/mm². Hence even this method fails for brass, because its composition must be regarded as unknown and, even for the same composition, the structure affects the mechanical properties (Fig. 2). A relation given by Guillet for copper, brass, bronze, and aluminum bronze, according to which we can put
$K_z = 0.55\ H_{10/1000}$ for the annealed condition and
$K_z = 0.405\ H_{10/1000}$ for the cold-worked condition, cannot be generally adopted, because the smaller conversion factor is applicable only to a certain definite degree of cold working which cannot be determined by the Brinell test. Moreover, the value 0.55 for the annealed condition can result in variations of 20%, entirely aside from the fact that it is difficult to tell positively in all cases whether the metal is really in this condition.

For aluminum, cold working does not affect the conversion factors in the same degree as for the above-named metals, so that, according to Figure 3, the mean value can be put as

$$K_z = 0.33 \text{ to } 0.36\ H_{2.5D^2}.$$

A still better agreement can be attained, if we consider that the coefficient can reach 0.4 for soft-annealed and for spring-hard rolled aluminum.

Duralumin has a very good linear relation (Fig. 4):

Annealed duralumin	$K_z = 0.36\ H_{10}D^2$
Heat-treated "	$K_z = 0.34$ to $0.36\ H_{10}D^2$
Heat-treated skleron	$K_z = 0.35$ " $0.36\ H_{10}D^2$

For the aluminum group, the problem may therefore be considered as satisfactorily solved, and we shall not go wrong in extending the mean value of $K_z = 0.35$ to include the other heat-treatable aluminum alloys like construktal, lautal, aeron, etc.

The relations for the yield point can be expressed in the form $\sigma_s = BH - C$. However, since the constants B and C differ for different metals and depend, for like metals, on the manner of the cold working, the practical value of such relations is only slight.

For nonferrous metals, then, excepting the aluminum group, even if the individual metal groups are considered separately, no such simple ratio exists as for steel. This circumstance is due primarily to the fact that the Brinell hardness, as already shown by the necessary standardization of the test pressure, does not represent a constant and consequently is not directly comparable with the tensile strength, or only in special cases.

Hardness and Tensile Stress-Strain Curves

An idea of the nature of the Brinell hardness is obtained when the relation of the test pressure P to the impression diameter d is considered. Then the relation $P = a\,d^n$ holds, with n as a constant. Since the value a depends on the size of the ball used as well as on the material, it is expedient to base the considerations on the value of a which corresponds to the standard ball D = 10 mm (about 3/8 inch). With this assumption, a also can be regarded as a constant. The conversion of a to other ball sizes makes the value $aD^{n-2} = A$ independent of the ball diameter. The Brinell hardness number for any impression diameter can be calculated from the hardness coefficients a and n (Fig. 5). As the more nearly physically correct hardness number we must regard the mean specific pressure (Meyer hardness), which is obtained from the simple formula $H_m = \frac{4\,a\,d^{n-2}}{\pi}$. From Figure 6 it is obvious that these hardness numbers constantly increase with the impression diameter and all the more, the more n differs from 2.

The increase in the hardness numbers, however, is nothing but the appearance of strain-hardening which, with respect to the hardness conception, leads to the knowledge that the hardness, as impression resistance, cannot represent a constant, because this resistance increases with the depth of the impression. Accordingly, the relation of the tensile strength and hardness can be constant only when the strain-hardening is equivalent in both processes. The deformation processes in the impression made by a ball can, therefore, be expressed not by a single hardness number, but rather by the whole hardness curve which is determined by the hardness coefficients a and n. Thereby the Brinell hardness curve is eliminated, however, because the strain-hardening process is obscured in the attempt to secure a nearly constant hardness number. The latter, moreover, occurs only when n lies between 2.1 and 2.2. The peculiar increase in the area of the impression yields a maximum value (maximum hardness number), which is of a more geometrical nature, however, and neither justifies nor furnishes a comparison with the maximum stress of the tensile stress-strain curve.

The exponent n, for steel and aluminum alloys in the annealed condition, lies between 2.1 and 2.3; for the rest of the nonferrous metals, however, between 2.3 and 2.6. Through cold working, n diminishes to 2 for both metal groups. This happens when the yield point and tensile strength coincide and the material can no longer be strain-hardened in the sense that the yield-point ratio increases (Fig. 7). In harmony with this, is the fact that, for $n = 2$, the mean specific pressure is independent of the size of the impression. This and the further fact that, for metals with exponents between 2 and 2.3, a slightly variable ratio exists between the tensile strength and the Brinell hardness, indicate that the exponent n must be related to the simple stress-strain curves.

These relations can be determined numerically, if we express the stress-strain curve by an equation of the form $\epsilon = (\alpha\ \sigma_o)^m$. Figures 8-9 show, in the nearly straight course of the customary tensile stress-strain curves in the logarithmic diagram, that such a relation (naturally not for all metals, but still, as the experiments show, in very many instances) enables a very clear representation, in which even the tensile stress-strain curves are characterized by two constants. On determining these constants, we find relations between a and α, as likewise between the exponents m and n (Fig. 10). Hence it follows that the significance of n is no longer confined alone to the Brinell test, but also (without reference to the condition of the metal) characterizes the rise of the stress-strain curve or, in other words, the capacity for strain-hardening.

Metals with high exponents n have steep stress-strain curves and therefore strain-harden rapidly. Metals with small exponents can be strain-hardened to a smaller degree, and the stress-strain curves ascend gradually. Strain-hardening is not possible when n = 2. This condition occurs in metals which are so far strain-hardened that they no longer deform uniformly, also in the plastic state of steel and finally at high temperatures when the strain-hardening is again removed by softening. In the latter case the uniform elongation may be considerable. If n is less than 2, as, for example, for lead, the material softens and m must be negative; i.e., the stress-strain curve descends, which, in fact, has been observed for lead. For such metals, however, the relations are easily disturbed by the influence of the speed of deformation. The fact that, with the ball impact test,* the dynamic exponent n is always smaller than the statically determined exponent and that, in the dynamic tensile test, the stress-strain curve ascends more gradually than in the static test,** must be regarded as a confirmation of this relation.

If n characterizes the rise of the stress-strain curve, then the ratio of the yield point to the tensile strength must be determined by n under the assumption that the uniform elongation must be regarded as constant. Since, however, in most cases, metals with high exponents n also have a large uniform elongation and the stress-strain curves in the region of the maximum load often run horizontally for a long distance, the effect of the uniform elongation is partially neutralized so that, as Figure 11 shows, n expresses the yield-point ratio with very close approximation. Thus the exponent n acquires special practical importance, and its determination should not be delayed, when it is desired to obtain from a hardness test the most information possible regarding the properties of the material.

*Class, "Der Kugelschlaghärteprufer" No. 296, of the Forschungsarbeiten auf dem Gebiete des Ingenieurwesens. V.D.I., 1927, p.1680.

**Meyer, "Zugversuch bei raschem Zerreissen," No. 295, of the Forschungsarbeiten auf dem Gebeite des Ingenieurwesens.

The fact that the hardness coefficients a and n express the same resistance to deformation which the tensile stress-strain curve represents, enables a clear comparison of both phenomena, if the hardness and tensile stress-strain curves are superposed and the stresses in the tensile stress-strain diagram are compared with the mean specific pressures. I arrived at this conclusion independently of Ludwik,* who compared the Brinell hardness numbers in a similar way with the stresses for the tensile stress-strain curve. According to Figure 12, a certain portion of the tensile stress-strain curve corresponds to the increase in the hardness numbers from the value a (d = 1 mm) to the value $A = a\,D^{n-2}$ (d = D = 10 mm).

With respect to the ratio of the tensile strength to the hardness number, it can be deduced that, of all the hardness numbers for the most different metals in any condition, the value $a\,D^{n-2}$ has the least changeable ratio, and the hardness numbers stand in a slightly varying ratio to the tensile strength, only when the yield-point ratio (ratio of the yield point to the tensile strength) is large and the elongation and hardness curves slope gently, i.e., when n is small. Hardness numbers, which are determined from small impressions, are more closely related to the yield points. Conversely, deep impressions are required if the ratio of the tensile strength to the hardness is to be as constant as possible. It is also obvious that, for

*Ludwik, V.D.I., 1927, p.1532. The present work was finished near the end of 1927.

metals with high exponents n even with these hardness numbers, the degree of strain-hardening will not be attained which corresponds to the tensile strength, as is also shown in Figure 7 by the plotted ratio $K_z : A$.

Tensile Strength and Hardness Coefficients

In order to find a legitimate connection with respect to the ratio of the tensile strength to the hardness, Kokado's method was adopted.* Kokado succeeded in demonstrating mathematically that the spherical impression, which is defined by a and n, may be expressed in terms of the same quantities which represent the relation between the actual stress and elongation for the simple compressive stress-strain curve. The mathematical deductions yield the mutual relations $m = \frac{2}{n-2}$ and $\alpha = a\, f(n)$, when $\frac{\epsilon_d}{1 - \epsilon_d} = (\alpha\, \sigma)^m$ is adopted as the equation of the compressive stress-strain curve.

If we consider that the tensile stress-strain curve can be found for the compressive stress-strain curve by replacing the expression $\frac{\epsilon_d}{1 - \epsilon_d}$ by the tensile strain ϵ, we obtain, for the relation between the tensile strength and hardness numbers, the following expression

$$K_z = \frac{(4-n)^{\frac{4-n}{2}}\,(n-2)^{n-2}\,2^{\frac{n-2}{n}}}{(6-n)^{\frac{n-2}{2}}}\, a\, D^{n-2}$$

*Kokado, "Hardness and Hardness Measurement," Technical Reports, Tokyo Imperial University, Vol. VI (1927), No. 4. An abstract is contained in Forschungsheft 313, p. 3 ff.

Figure 13 shows the relation determined by this equation for $D = 10$ mm in terms of the exponent n. Even though this relation cannot be evaluated numerically, it nevertheless expresses the fact that, in the most common case, not a single hardness number, but the hardness coefficients a and n are related to the tensile strength. There is also the new viewpoint that the uniform elongation (to be mathematically considered as the abscissa corresponding to the tensile strength) is contained in this relation or in the function for n.

If we plot the ratio $K_z : a$ against n for the tested metals, we obtain Figure 14, in which the light lines enclose a region of ±5% deviations from the mean value. The effect of the uniform elongation almost entirely disappears because of the same relations as apply at the yield point, conditioned, however, upon the deviations due to the irregularities in the course of the stress-strain curves. The Brinell hardness can therefore be determined from the values a and n. This requires at least two impressions to be made with different test pressures.

For a given test pressure (e.g., a ratio of 1:2 or 1:3), n can be determined directly from the ratio of the impression diameters, but a must be determined logarithmically. In order to avoid this, the table in Figure 15 is so constructed as to render it possible to determine the Brinell strength from the ordinary Brinell hardness and the exponent n. If, for example, at $P_1 = 1000$ kg, we find $d_1 = 4$ mm, corresponding to

$H_{10/1000} = 76.6$ kg/mm², and, at $P_3 = 3000$ kg, $d_3 = 6.32$ mm, we then obtain $n = 2.4$ from $d_3 : d_1 = 1.581$. On the vertical line through $n = 2.4$ (Fig. 15) we find, at the point of intersection with $d = 4$ mm, the conversion factor 0.49. Hence $K_Z = 0.49 \times 76.6 =$ about 38 kg/mm². Furthermore, we can determine from Figure 11 that the yield-point ratio is about 0.3 for $n = 2.4$. If we find $n \leqq 2.2$, the conversion factor lies between 0.3 and 0.4, the same as for steel, and $K_Z = 0.36\ H_B$ is then sufficiently accurate for all metals.

The practical determination of the Brinell strength is greatly simplified when the metal is in the cold-worked state. In the case of rolled and drawn metals, it is customary to distinguish the hardness stages as quarter-hard, half-hard, three-quarters-hard, hard, and spring-hard.* Since n was found to be less than 2.2 for all the metals tested, the determination of n can be dispensed with in these cases, and $K_Z = 0.36\ H_B$ can be immediately written. The determination of n is necessary, therefore, only for annealed or very slightly worked metals. Thus the given method has demonstrated its practical utility, even in the intervening experiments with rolled bronze, durana metal, and elektron.

The determined relations have also been confirmed for copper at a higher temperature, when it was found, as was to be expected, that the coefficient, valid at ordinary temperatures, can

*According to a proposal, not yet definitely adopted, the strength in these cases would be 1.1, 1.2, 1.3, 1.4, and 1.8 of what it is in the annealed condition.

be transferred to high temperatures. These relations are of general importance only in so far as in the case of steel the ratio 0.36 also deviates upward and is affected by a and n, when n is greater than 2.3, i.e., when the yield-point ratio diminishes. This may be caused by the coarseness of the grain or by the addition of alloys (austenitic steels). A high yield-point ratio, such as that of chrome-nickel steel, for example, necessitates, on the contrary, smaller conversion factors (0.34).

Cast Metals

The determined relations are first valid only for tough metals, i.e., for metals which neck down in a tensile test. When, however, this condition is fulfilled by the cast metal, they apply to it also. It is to be noted that such metals are never entirely free from local defects which impair the strength more than the hardness, so that care must always be exercised in determining the strength of cast metals from the Brinell hardness. Due to the coarse structure, high values for n are generally found.

For brittle cast metals, we measure, with the tensile strength, no resistance to deformation, but resistance to separation. We can therefore decide only by purely empirical means whether the strength and the hardness number run parallel. For such metals, the conversion factor for $H_{10/1000}$ seems to be about 0.25.

In Figure 16, the relations for cast iron are compared with each other and with those for phonolite. Hence it follows that a simple ratio applies approximately only to cast iron with a ferritic-pearlitic base (K_z = 0.09 to 0.11 H_n). For cast iron with a pearlitic base, however, no general ratio can be given, because the strength, in opposition to the hardness, is affected by the quantity of carbon in the form of graphite more than by the metallic base.

Translation by Dwight M. Miner,
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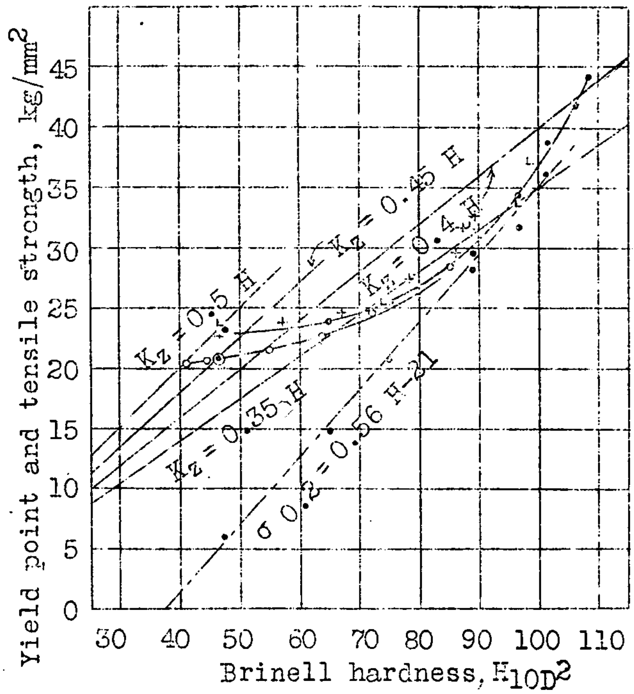
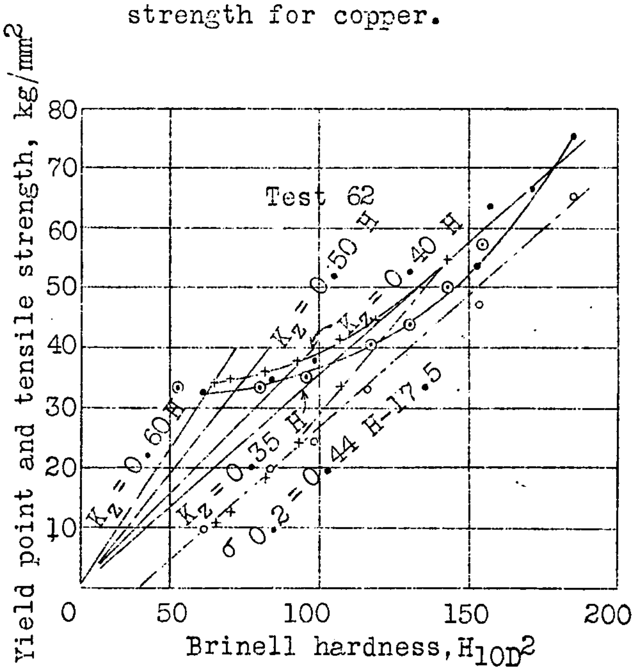


+ Drawn.
• Rolled.
L Pressed.
∘ Tests by Kürth (drawn) with chem. pure copper.
⊙ Annealed electrolytic copper.

$10D^2$ is the requisite test pressure P in kg, when D = diameter of ball in mm.

Fig.1 Relations of Brinell hardness No., yield point, and tensile strength for copper.



• Rolled.
+ Drawn.
⊙ Previous tests.

$10D^2$ is the requisite test pressure P in kg, when D = diameter of ball in mm.

Fig.2 Relations of Brinell hardness No., yield point, and tensile strength for brass. (test 62)

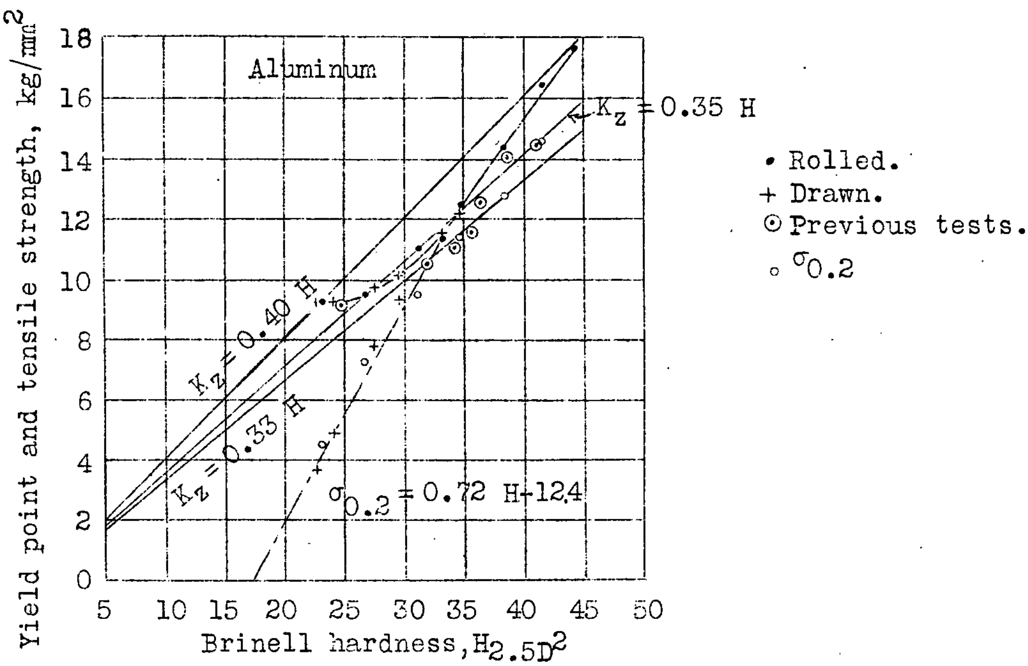


Fig.3 Relations of Brinell hardness No., yield point, and tensile strength for aluminum.

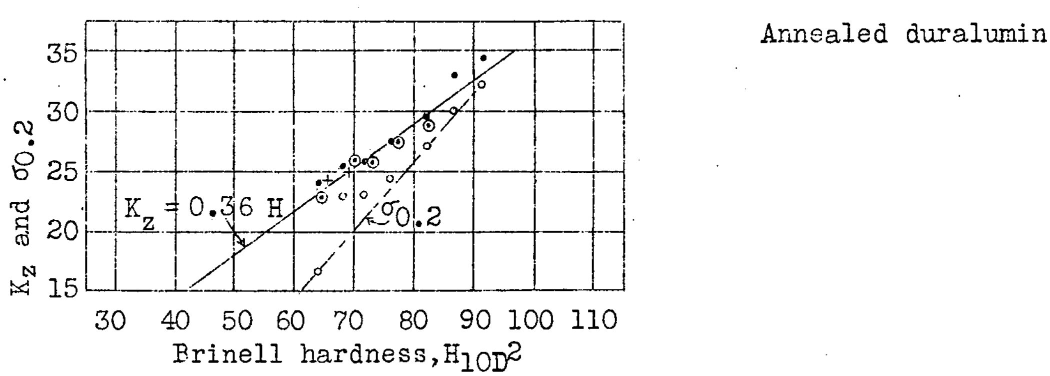


Fig.4 Relations of Brinell hardness No., yield point, and tensile strength for annealed duralumin.

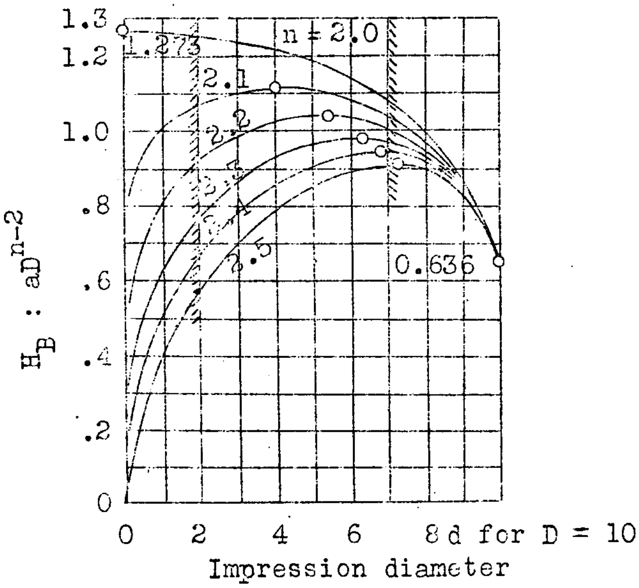


Fig.5 Brinell hardness plotted against impression diameter.

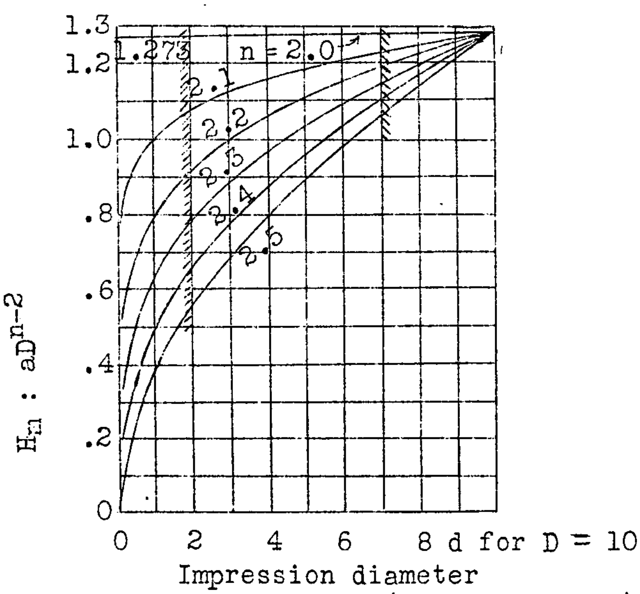


Fig.6 Mean specific pressure (Meyer hardness) plotted against impression diameter.

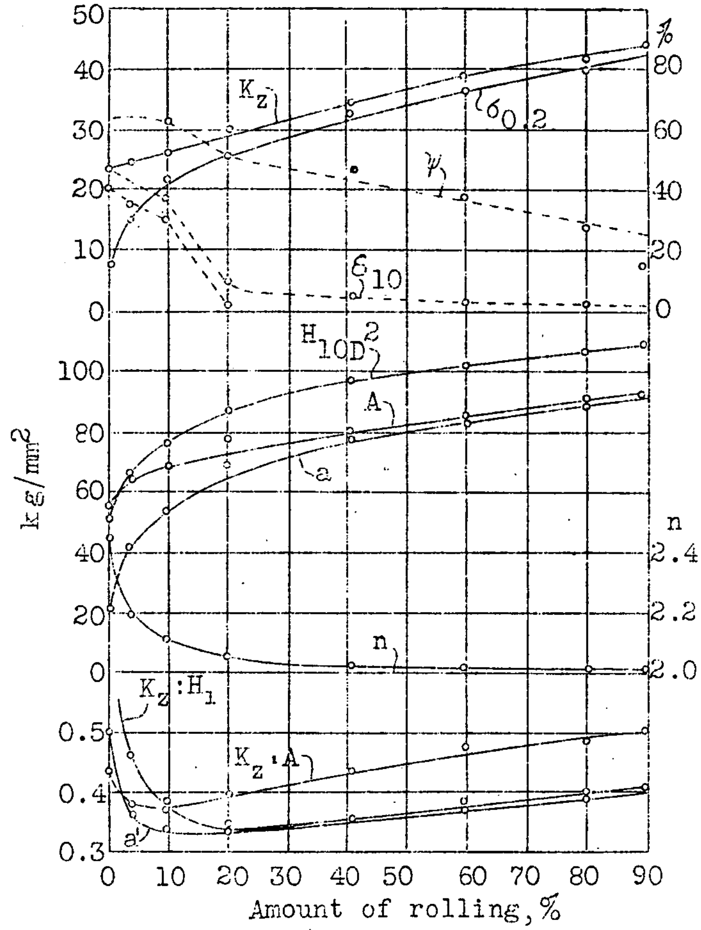


Fig.7 Increase in hardness and strength of copper with amount of rolling.

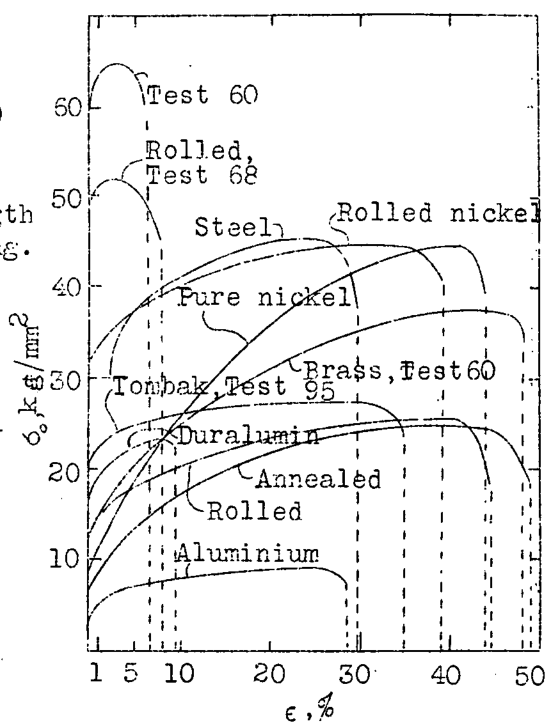


Fig.8 Stress-strain curves of different metals.

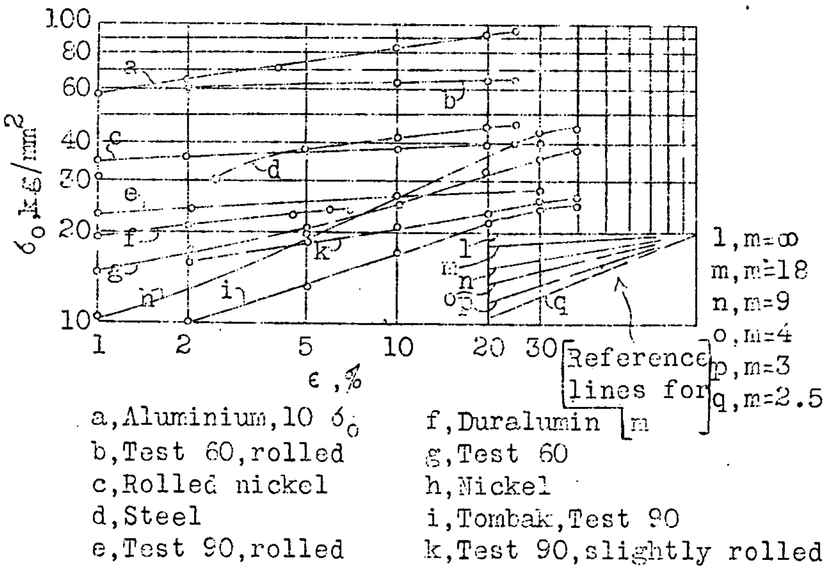


Fig.9 Stress-strain curves of different metals represented logarithmically. (Cf Fig.8)

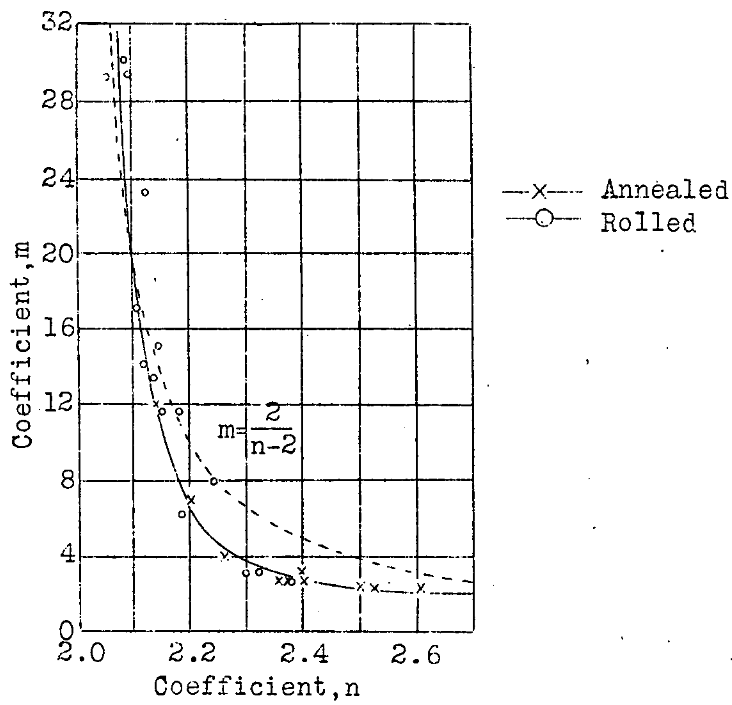


Fig.10 Coefficient $m[\epsilon=(\alpha\tau_0)^m]$ plotted against coefficient $n(P=ad^n)$

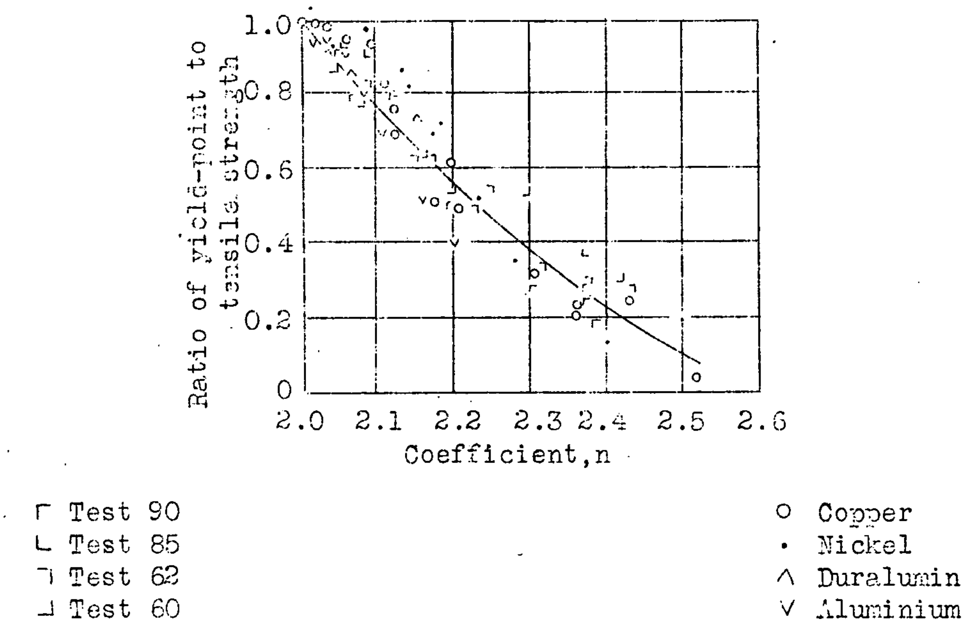


Fig.11 Yield-point ratio against coefficient,n

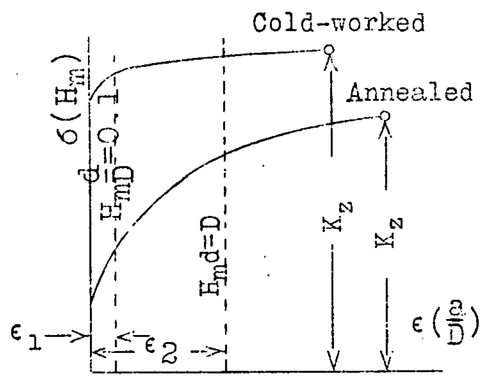


Fig.12 Superposing of hardness curve and tensile stress-strain curve.

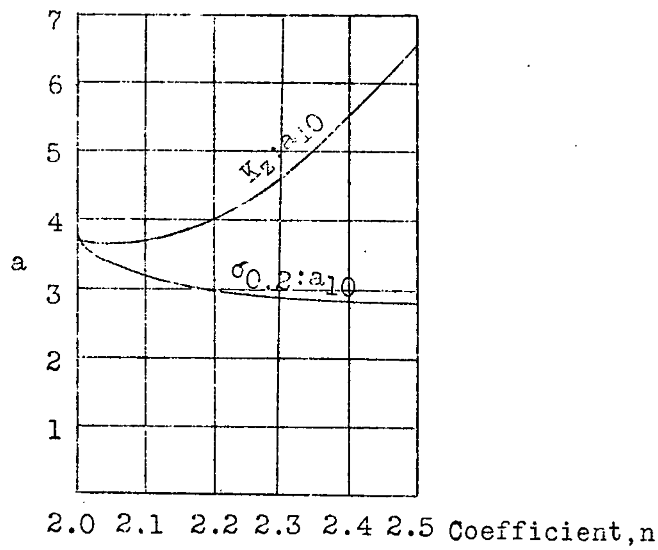


Fig.13 Tensile strength and yield point plotted against the hardness coefficients a and n on the basis of the deduced relations.

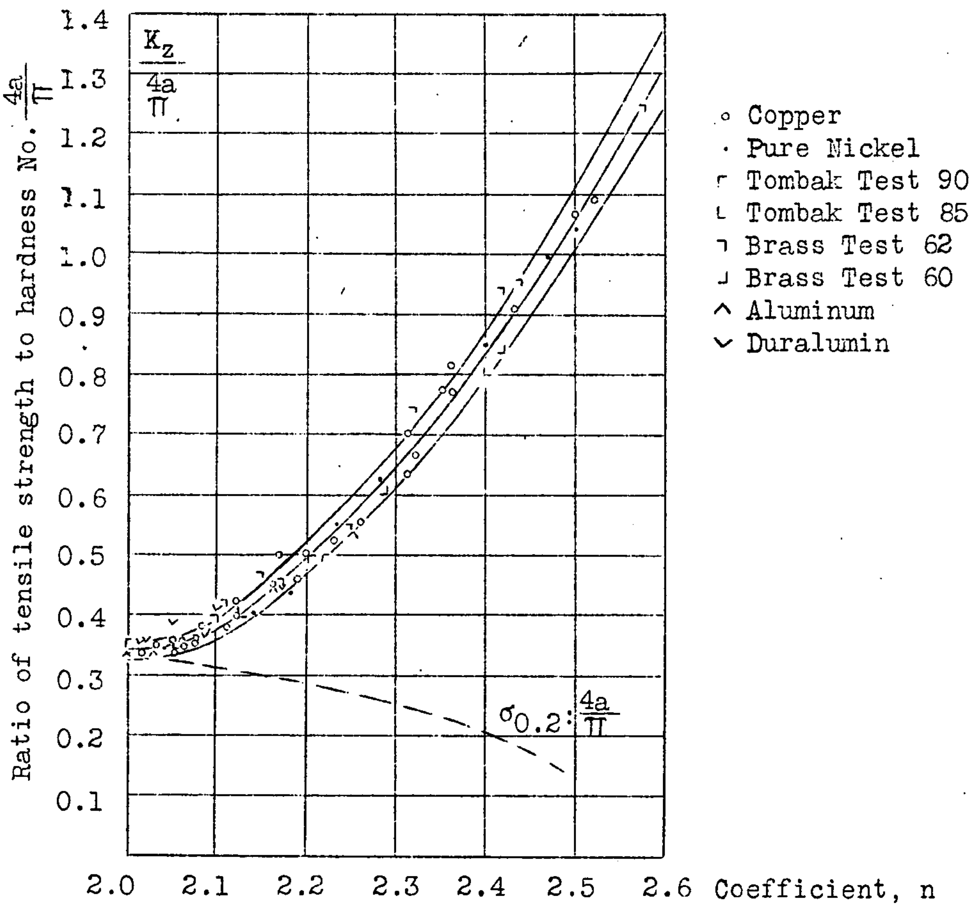


Fig.14 Tensile strength ratio plotted against hardness coefficients, a_{10} and n on basis of tests.

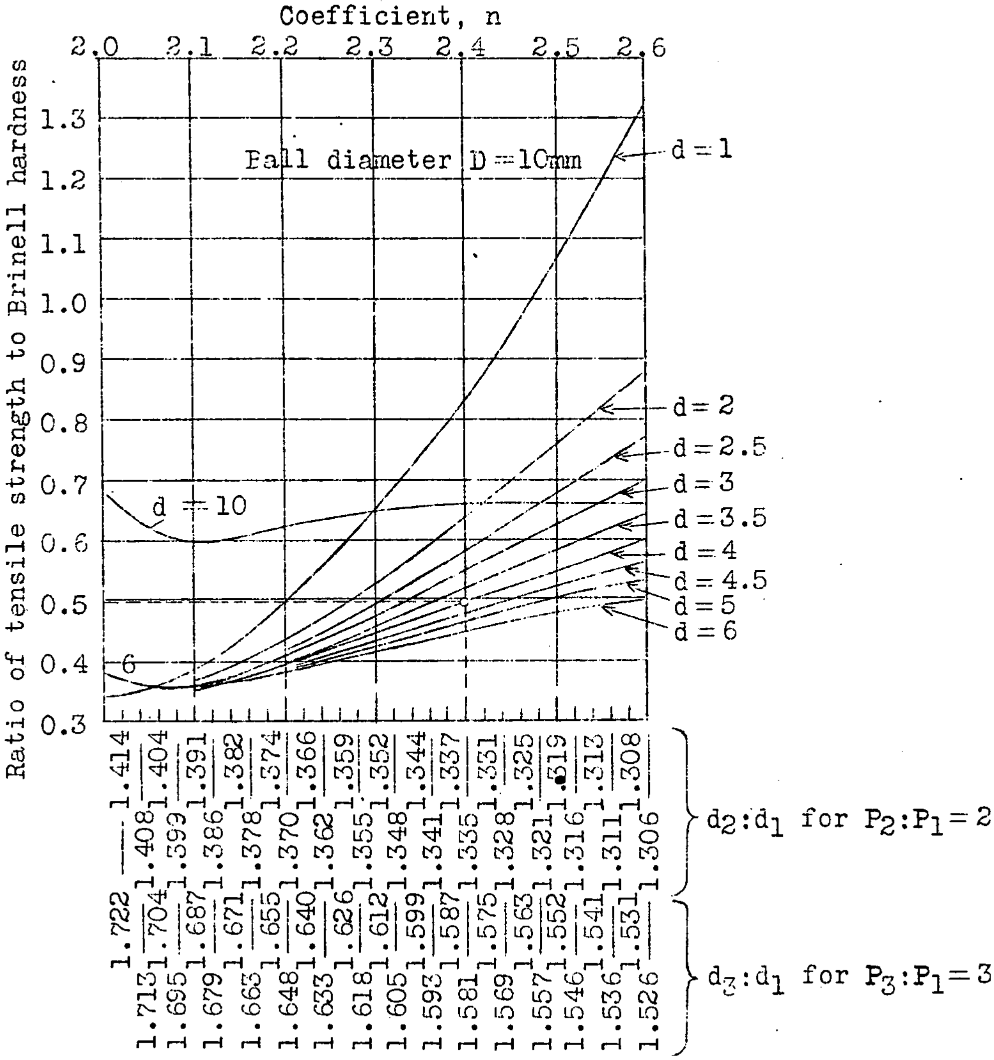


Fig.15 Table for finding conversion factors

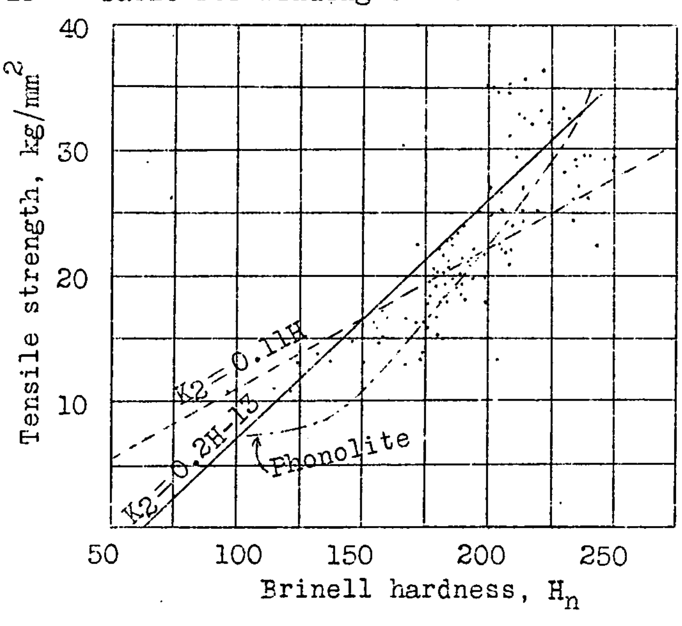


Fig.16
Tensile strength plotted against Brinell hardness for cast iron.