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FREQUENCY OF OCCURRENCE OF ATMOSPHERIC GUSTS AND OF RELATED LOADS ON AIRPLANE STRUCTURES

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SUMMARY

A number of samples of flight acceleration data taken by the National Advisory Committee for Aeronautics under a variety of operating conditions were evaluated to determine the total frequencies and the frequency distribution of atmospheric gusts. The samples include 1748 hours of operation by several airplanes of the domestic airlines of the United States, a Martin M-130 airplane of the Pacific Division of Pan American Airways System, and the Boeing B-15 airplane of the Army Air Forces. These data are supplemented by V-G records, so that more than 9,000,000 miles of operation are represented. Samples taken on an Aeronca C-2 airplane at low altitude in the turbulent air of the earth's boundary layer are compared with similar samples taken on the Lockheed XC-35 airplane at high altitude within cumulus-congestus and cumulonimbus clouds.

Similar data of German origin have been reanalyzed and included for comparison.

It was concluded that the distribution of gusts within turbulent regions of the earth's atmosphere follows a substantially fixed pattern regardless of the source of the turbulence. The total frequencies are therefore governed by the total length of flight path in rough air, and operating conditions determine the total frequencies only by affecting the ratio of the length of flight path in rough air to total length of the path. Gust-load frequencies were found to be inversely proportional to airplane size.

It was further concluded that the gust frequencies can be applied with small error to the estimation of stress frequencies in the primary structures of airplanes. The results of the analysis are applicable to the fatigue

testing of the primary structure of the airframe and to the estimation of the probability of encountering gusts of excessive intensity within any stated period of operation.

INTRODUCTION

The trend in airplane design toward higher wing loading, higher speed, and larger size - and consequently toward higher mean stresses and greater severity of loads on the structure - has resulted in a growing appreciation by designers of the potential importance of fatigue in the primary structure and of the necessity for designing on the basis of fatigue strength for limited "life expectancy." Reference 1, for example, displays a great deal of concern about the fatigue life of airplane structures.

Life expectancy is governed not only by fatigue but also by the probability of occurrence of single quasi-static loads of such high magnitude as might endanger the structure directly. This problem has been made more acute by the overloading of airplanes due to wartime traffic demands.

An obvious prerequisite for control of fatigue strength and for the determination of the probability of single large loads is flight data that show the frequency of occurrence of loads or stresses in the structure correlated with the many factors that influence the frequencies. In the flight operations of transport-type airplanes the principal source of structural loads and stresses is atmospheric turbulence, and most of the required flight data applicable to transport airplanes may be obtained by measurements of the loads or stresses during cruising flight in rough air.

Kaul (reference 2) and Freise (reference 3) have presented data on the wing-load histories experienced by a number of airplanes both under special test conditions in rough air and in some 600 hours of cruising flight on several branches of the Deutsche Lufthansa. Kaul obtained results by means of an accelerometer located near the center of gravity of the airplane and Freise, by means of a strain gage mounted on a chord member of a wing spar near the wing root. The results were expressed in references 2 and 3 in terms of applied wing load.

The NACA has from time to time collected data similar to those presented by Kaul and Freise. These data include acceleration measurements from 1320 hours of the early operations of the domestic airlines of the United States, 313 hours of miscellaneous cross-country flying by the Boeing B-15 airplane, a 115-hour round-trip flight between Alameda, Calif., and Hong Kong, China, by a Martin M-130 airplane of Pan American Airways System, and two special gust investigations in the vicinity of Langley Field, Va. Data taken with the NACA V-G recorder (reference 4) during some 8,500,000 miles of airline operations are also included to take into consideration the rare gusts of great intensity that are not normally encountered during the taking of samples of limited scope. In the present paper these data are analyzed and compared with the German data of references 2 and 3 to establish a broader basis for the determination of the frequency of loads resulting from atmospheric gusts.

SYMBOLS AND NOMENCLATURE

Δn	acceleration increment normal to chord of wing, g units
W	weight of airplane
S	wing area
a	slope of lift curve
ρ_o	mass density of air at sea level
$V\sigma^{1/2}$	equivalent airspeed
U_e	effective gust velocity
K	relative alleviation factor
$\bar{c}$	mean wing chord
F	total frequency, total number of occurrences of a phenomenon in a sample
f	frequency, number of occurrences of a phenomenon within a class interval

f_r relative frequency (f/F)

λ_{av} average gust interval, average distance along flight path in turbulent air between significant gusts

L path of operation, total length of flight path for any considered scope of operation

R path ratio, ratio of length of flight path in turbulent air to path of operation

The class interval is the range between two values of a measured quantity within which measurements of like value are grouped (or classed) for the purpose of tabulation offrequencies. The class mark is the definitive value, or midvalue, of a class.

EFFECTIVE GUST VELOCITY AS BASIC ATTRIBUTE

In most investigations of atmospheric turbulence conducted by the NACA, the acceleration response of airplanes to the gusts has been utilized in the measurement of atmospheric turbulence. Although much of the philosophy underlying the concepts involved in the use of acceleration response in the measurement of turbulence has not been published, some basic considerations are discussed in references 4 to 6. These considerations lead to the relatively simple concept of an "effective gust velocity," which has been selected as the basic attribute or independent variable to which the statistical analysis best applies. The effective gust velocity is defined by the relation

$$\Delta n = \frac{\rho_o a K U_e V \sigma^{1/2} S}{2W} \tag{1}$$

The relative alleviation factor K allows for the velocity of the airplane normal to the flight path caused by application of acceleration during the finite time of action of the gust. The factor K is given as a function of the wing loading in figure 1. The derivation of this curve, which takes into consideration the lag in transient development of lift and the gust gradient, is attributable to the authors but has not been published. The curve in figure 1 is part of the American design requirements and has been published as figure 11(a) in reference 7. Although derived at a relatively early date when little information on gust gradients was available, the relationship described by the curve has remained in excellent agreement with subsequently obtained flight data and with advances in the theory of unsteady lift.

SCOPE OF MEASUREMENTS

Extent of Operations

Domestic airlines.- Acceleration records for 1320 hours, or about 145,000 miles, of flight were obtained during the early days of transport operations on the domestic airlines of the United States. The data were taken during routine scheduled operations over a period of about 2 years. The average operating altitude was about 4000 feet above sea level. The airplanes on which the measurements were made included the following types: Ford 5-AT, Fokker F-10-A, Boeing 40-B, and Boeing 80-A. The routes flown covered most sections of the United States and represent all types of climate and topography in this country. The data from these early domestic-airline operations are referred to subsequently as "sample 1." The characteristics of the airplanes and a summary of the operating conditions for all the samples are given in tables I and II, respectively.

A large number of acceleration records were obtained later on the domestic airlines. These records represent 42,105 hours, or about 7,000,000 miles, of routine transport operations by Boeing B-247, Douglas DC-2, and Douglas DC-3 airplanes on several airlines covering most sections of the United States. The data from these later domestic operations are called samples 2, 3, and 4 for the B-247, DC-2, and DC-3 airplanes, respectively. (See tables I and II.)

Alameda to Hong Kong.- Records were taken with a number of instruments during a round-trip flight in June 1938 from Alameda, Calif. to Hong Kong, China by a Martin M-130 airplane of Pan American Airways System. The average altitude was about 10,000 feet and the flying time was 115 hours, corresponding to 17,000 miles of flight. The data from this flight are called sample 5.

Records of acceleration covering 12,232 hours, or about 1,520,000 miles, of routine operations with Martin M-130 and Boeing B-314 airplanes are included in the analysis for the route from Alameda to Hong Kong. The data from these operations are called sample 6.

Boeing B-15 airplane.- Records of acceleration were taken on the B-15 airplane during 313 hours, or about 48,000 miles, of miscellaneous flying including a number of cross-country flights over various sections of the United States and one round trip to the Panama Canal Zone. These flights were made between November 1938 and June 1940. The average altitude of the operations was about 5000 feet. The data are subsequently called sample 7.

XC-35 airplane.- The Army Lockheed XC-35 airplane was flown in the vicinity of Langley Field, Va. during an investigation of atmospheric turbulence in the summers of 1941 and 1942. Measurements of acceleration and airspeed were taken only during flight through rough air, mostly within cumulus-congestus and cumulo-nimbus clouds. The surveys were made at various altitudes up to 34,000 feet. Only two samples from these surveys are included in the analysis. One of these samples (sample 8) was selected at random from the several sets of data; the other sample (sample 9) represents the roughest flight.

Aeronca C-2 airplane.- An Aeronca C-2 airplane was flown during an investigation in 1937 of turbulence at very low altitudes in the earth's boundary layer. A sample (sample 10) was selected at random from the complete data and is included here for analysis.

Apparatus and Limitations

Domestic airlines (early operations).- In the early transport operations only acceleration records were obtained. The records were made with commercial vibration recorders that had been rebuilt into accelerometers by the NACA. These accelerometers recorded against time on a waxed-paper disk about 4 inches in diameter. The instruments were arranged to make one revolution of the disk in several hours. The time scale was therefore cramped and only the moderate and the large values of acceleration could be counted.

As the airspeed was not recorded, effective gust velocities were evaluated on the basis of the known cruising speeds of the airplanes.

Although the slopes of the lift curves were known from available data, the wing loadings of the airplanes as flown were not usually known. Effective gust velocities were, therefore, evaluated on the basis of the assumption that the airplanes were flown at normal gross weight. This assumption leads to somewhat conservative values, as the airplanes were usually flown at less than normal gross weight.

Domestic airlines (recent operations).- In the more recent domestic transport operations, both acceleration and airspeed were recorded by means of NACA V-G recorders, which are described in reference 4. These instruments do not record against time; the accelerations are registered vertically on a small smoked-glass plate while the values of airspeed are recorded horizontally. The record is an envelope of the maximum and minimum values of acceleration against a scale of airspeed. The small accelerations are illegible within the envelope and only the larger values of acceleration that project beyond the envelope of the small values can be counted.

No assumption as to airspeed is required with the NACA V-G recorder, as the instantaneous value of airspeed associated with any observed acceleration is given by the record.

As in the case of the early transports, the wing loadings of the more recent transport airplanes as flown were not known exactly. It was determined, however, that a reasonable approximation of the average operation weight was 85 percent of the normal gross weight; this value was used in the evaluation of effective gust velocities.

Alameda to Hong Kong.- During the round-trip flight between Alameda and Hong Kong of the M-130, the airplane was equipped with an NACA V-G recorder, an NACA recording accelerometer, an NACA airspeed recorder, and several

NACA scratch-recording strain gages. Both the accelerometer and the airspeed recorder recorded the measured quantities against time with a scale sufficiently open to permit detailed evaluation of the records. The strain gages also recorded against time, but the motion was of an intermittent character so that all the strain peaks could not be counted. Only one strain gage operated satisfactorily throughout the flight. Many of the strain values could, however, be correlated with the acceleration measurements.

During the flight an observer operated the instruments and a complete log of time spent in rough air, total time, airplane weight, and other pertinent detail was kept. The records therefore permit a complete and accurate evaluation of the frequencies of effective gust velocities.

Except for the records taken on this round-trip flight, all records of acceleration and airspeed taken on the Alameda-Hong Kong route were made with NACA V-G recorders.

B-15 airplane.- The B-15 airplane was equipped with an NACA recording accelerometer and an NACA airspeed recorder having the time scales sufficiently open to permit detailed evaluation of the records. A number of NACA and DVL type scratch-recording strain gages were installed on shear and chord members of a wing spar at two stations along the span. The DVL type gages recorded continuously against time, and a count of the strain peaks is possible although such a count has not been made. As in the case of the round-trip flight to Hong Kong by the M-130 airplane, the strain records are used herein only to show the relationship between a number of measured strains and accelerations.

During the flights of the B-15 airplane, an observer operated the instruments and kept a complete log of time spent in rough air, total time, airplane weight, and other pertinent quantities. The records from these flights therefore permit a complete and accurate evaluation of the frequencies of effective gust velocities.

XC-35 airplane.- The XC-35 airplane was equipped with an NACA recording accelerometer and an NACA airspeed recorder set to give an open time scale. The records obtained are amenable to detailed evaluation. The operating weights for all flights are known, and effective gust velocities can be completely and accurately evaluated.

Aeronca C-2 airplane.- The Aeronca C-2 airplane was also fitted with an NACA recording accelerometer and an NACA airspeed recorder, and the operating weights are accurately known. Detailed evaluation of effective gust velocities is possible from the records.

EVALUATION OF FREQUENCY DISTRIBUTIONS AND TOTAL FREQUENCIES

Method of Count

The method of counting frequencies used herein was dictated largely by the type of record available for analysis and by the quality of the records. Only the records from the NACA accelerometer permitted detailed examination, but even with those records it was necessary for practical reasons to confine the count to single maximums and minimums, or peaks, between any two consecutive intersections of the record line with the 1g reference level. This method of count neglects the minor oscillations superimposed on those counted. Kaul (reference 2) employed a similar method of count, and in this respect the German and the American data are comparable.

From the records for sample 1, in which the time scales were cramped, and from the records taken with NACA V-G recorders it was not possible to determine whether the acceleration returned to or crossed the 1g reference level after the attainment of a maximum or minimum value. In these cases, therefore, the evaluation was made by counting the acceleration peaks standing out from the envelopes of the small accelerations.

Since, except for the V-G data, it was considerably more convenient to count accelerations directly than to convert accelerations to effective gust velocities prior to the count, the conversion was made for relatively short sections of each sample on the basis of mean airspeeds for these sections. In this way large errors in airspeed were avoided and the small deviations of the airspeed from the selected means were of no great significance.

Class Intervals

The intervals for the classification of frequencies were chosen at about the smallest values consistent with the accuracy of the several acceleration measurements - namely, about 0.1g. For a number of reasons the intervals were not always quite the same. This fact is of no consequence for, in any event, since the acceleration values were conveniently converted to effective gust velocities after the count was made, the class intervals expressed in terms of effective gust velocity would not remain equal for the various samples because of differences in airplane characteristics and airspeed. The class intervals, expressed in terms of gust velocity, corresponding to the actual evaluation are given in table III.

Threshold Values of Acceleration

and Effective Gust Velocity

In counting the frequencies in the lowest class (that is, the class containing the smallest values of acceleration), the result depends upon the minimum values that can be observed. On the records from the NACA accelerometer, variations in acceleration attributable to gusts as small as 0.02g can be conveniently observed, and all greater values can therefore be counted. This limit of acceleration for which the count can be made is termed herein the "threshold value" of the acceleration.

On the V-G records and the records from the converted commercial recorders used in obtaining sample 1, the threshold values of acceleration were rather high because of the limitations of the instruments previously described.

The threshold values for the samples are given in terms of effective gust velocity in table III.

Relative-Frequency Distribution

The frequencies f and the total frequencies F of the gusts for the 10 samples are given in table III as counted within the selected class intervals and to the threshold values of effective gust velocity.

In order to arrive at the broadest and most rational view of gust-frequency distribution, all data were plotted in the form of relative-frequency polygons (reference 8). The polygon of relative gust frequencies is a graph of the ratios $f/F = f_r$ for the different classes plotted at the respective class marks on a scale of effective gust velocity. Since the shape of such a polygon is dependent upon the size of the class interval and upon the class mark of the lowest class within which the count is made, polygons for the different samples can be compared only when plotted for a common class interval and for a common lowest class. In order to place all the data on a comparable basis, a common class interval of 4.5 feet per second, the largest of the class intervals for which count was made, was chosen.

Since sample 5 and samples 7 to 10 have about the same small threshold value falling within class 1, relative-frequency polygons for these samples can be plotted immediately after conversion to the common class interval. The polygons for samples 5 and 7 are shown in figure 2; the polygons for samples 8 and 10, in figure 3; and the polygon for sample 9, in figure 4. A reference polygon, "relative distribution A," is shown in these figures to facilitate comparisons.

In constructing polygons from the remaining data, samples representing generally similar operations were combined. The combination of these samples, which include the V-G data, was performed in such manner as to bring the relative frequencies of the rarer large gusts into a proper relationship with the other data. The basic assumption involved in the process was that, for data covering a large scope of operations, the relative-frequency distribution follows a single pattern. The validity of this assumption is discussed in a later section.

In the case of samples 1 to 4, all of which represent domestic transport operations, none of the data extended to low values of effective gust velocity for reasons previously given. The total frequencies for these samples are, therefore, relatively smaller than the total frequencies for the more refined samples because of the omission of the frequent low-value gusts. In order to bring the relative-frequency polygon for the combined samples 1 to 4 into proper relationship with the polygons for the more complete samples, it was necessary first to estimate the frequencies of the missing low-value gusts and the corresponding total frequencies. For this purpose a mean relative-frequency distribution from samples 5, 7, 8, and 10 was assumed to represent the missing low-value gusts of sample 1, which, of the combined samples 1 to 4, had the lowest threshold value. With this assumption, the total frequency of sample 1, including the frequencies of the lower classes, was estimated to be 1,600,000 gusts for the 1320 hours of operation.

The frequencies of sample 2 were then reduced by the ratio of the path of operations of sample 1 to the path of operations of sample 2 (table IV). Similarly, the frequencies of samples 3 and 4 were reduced to correspond to the path of operations of sample 1. The sum of the reduced frequencies within each class of samples 2, 3, and 4 was then added to sample 1 to obtain the polygon for the combined samples 1 to 4.

In combining samples 1 to 4 a precaution was necessary in regard to class 6 because of the following considerations. After conversion of sample 1 to class interval 4.5, the highest class in which data fell was class 6. This class is the lowest in which data from the V-G records fell. Thus, frequencies were available from all samples of the combination only in this class. In arriving at a combined frequency for class 6, two possible methods could have been used; namely, either the reduced frequencies from samples 2, 3, and 4 could have been averaged with the frequency of sample 1, or the most reliable sample could have been used without inclusion of the less reliable samples. The second method was actually used and the frequency for class 6 was taken from sample 1 since the obscuration of some class 6 acceleration peaks within the V-G envelopes of samples 2, 3, and 4 made these data less reliable for this class.

The frequencies for samples 5 and 6 were combined in a manner similar to that in which samples 1 to 4 were combined. In this case, however, it was unnecessary to estimate a total frequency for sample 5, as the threshold value was comparable to the threshold values of the other complete samples. Also, inasmuch as the highest gust-induced acceleration for both samples was recorded within the rather limited scope of sample 5, this one value was assigned a frequency of unity for the combined samples.

Polygons for the combined samples 1, 2, 3, and 4 and for the combined samples 5 and 6 are shown in figure 2.

DISCUSSION

Relative-Frequency Distribution

Significance of various samples.- The relative-frequency distribution for any sample of data does not necessarily represent general average conditions. For instance, the frequency distribution of sample 5 is not representative of average conditions because of the occurrence in sample 5 of one of the most severe gusts ever experienced on the Pacific Division of the Pan American Airways System. Even without other samples for comparison, this fact might have been suspected from the form of the relative-frequency polygon for sample 5 in figure 2, which shows a sudden break to large values of U_e. Sample 9 is another case that is not representative of average conditions, because this sample was obtained during the roughest of a considerable number of flights made during a special investigation of turbulence within cumulus-congestus and cumulo-nimbus clouds. For sample 9, as can be observed from a comparison of the polygon in figure 4 with the other polygons in figures 2 and 3, the frequency distribution indicates relatively high proportion of gusts of high intensity.

In contrast to the "fullness" of the frequency distributions for samples 5 and 9, the frequency distribution for sample 7 shows relatively low proportion of gusts of high intensity. This result is in line with the conditions of operation, according to which regions of high turbulence were avoided as far as possible so that greater weight was given the frequencies of the smaller gusts.

Since the conditions governing samples 5, 7, and 9 are known to give rise to more or less extreme frequency distributions, a sample representative of average conditions applicable to large scope of operations would be expected to lie somewhere between the extremes. Probably the most representative of the samples containing detailed data in the lowest classes are samples 8 and 10, which were selected at random from a considerable mass of data. The relative-frequency polygons for these samples (fig. 3) may be observed by comparison with figures 2 and 4 to lie between the polygons for samples 7 and 9 and inside the end point of the polygon for sample 5.

The combination of samples 1 to 4 and of samples 5 and 6 in the manner described greatly extends the scope of the data applicable to the respective operating conditions represented. The combined samples are thus more true than any single small sample in the sense that the influence of accidental occurrences, such as the encountering of an unusually strong gust in sample 5, is submerged in the mass of data; that is, accidental occurrences of this sort occur in sufficiently large number within a sample of large scope that they become more truly representative of the average conditions. Figure 2 shows this effect clearly; the combined sample 5 and 6 and the combined sample 1 to 4 have relatively uniform distributions lying between the extreme distributions of samples 7 and 9.

For comparison with the samples presented herein, distribution polygons of U_e have been constructed from Kaul's data with a class interval of 4.5. It may be seen from figure 2, which shows the envelopes of the polygons for Kaul's data, that the German and the American results are in very good agreement.

Influence of airplane characteristics and source of turbulence.- It is evident from the preceding discussion that the major discrepancies between the frequency distributions for the various samples can be accounted for largely by accidental occurrences during the operations. When the scope of the samples is sufficiently increased to be representative of average operating conditions, these accidental influences are not so strong and the frequency distributions tend to fall into the same pattern regardless of the source of the data. The results therefore indicate that individual gusts in turbulent regions of the atmosphere are distributed on the whole in a fixed manner irrespective of the location of the turbulent regions and of the source of the turbulence.

Figure 3 further illustrates the similarity of distribution for different samples. Sample 8 was obtained at high altitude within cumulo-nimbus and cumulus-congestus clouds and represents turbulence having its origin in thermal convective processes. Sample 10, on the contrary, was obtained at very low altitude in the absence of thermal effects and the turbulence arose from the shearing of the wind in the earth's boundary layer. Notwithstanding these considerable differences in the aerological conditions, the frequency distributions are nearly the same and they are also in close agreement with those from other sources.

Another point, most clearly evident from samples 8 and 10 but also evident from the other data, is that the distribution of turbulence as measured is largely independent of airplane size and other airplane characteristics. The close similarity of the distributions for sample 8 (obtained with the Lockheed XC-35 airplane), sample 10 (obtained with the Aeronca C-2 airplane), and the samples from the airline operations indicates that the basic assumptions and concepts underlying the gust-load formula (equation (1)) are correct.

Influence of disturbed motion of airplane in continued severe turbulence.- Although the foregoing remarks about the influence of the airplane characteristics apply on the average, in continued severe turbulence the frequency distribution may appear to contain abnormal frequencies in the higher classes unless precautions are taken to eliminate the effect of disturbed and controlled motions of the airplane. In the flight from which sample 9 was derived, which was the roughest of a large number of flights through cumulo-nimbus clouds, the airplane motion was considerably disturbed from the desired straight path, so that the gyroscope of one of the flight instruments was at times put out of action (reference 9). Under these circumstances the airplane was subject to moderate acceleration fluctuations of long period upon which the short-period accelerations due to the turbulence were superimposed. When the count was made in the described manner chosen for the general analysis, abnormally high values of effective gust velocity were ascribed to the various frequencies and the polygon appeared full (fig. 4). When the count was made with respect to the variable datum caused by the disturbed motion rather than with respect to the 1 g datum, the frequency distribution conformed more nearly to the distributions of the other samples. The corrected polygon retained a certain degree of fullness, however, which may be ascribed to actual greater frequency of the more severe gusts.

Differences between two polygons like those shown in figure 4 provide means of evaluating the effect of the disturbed motion on the frequency of applied loads. The data given here apply specifically to the characteristics of the XC-35 airplane and cannot be safely applied to other cases. This fact is of small concern, because large disturbed motions are rarely encountered in normal operations, so that such effects as are shown in figure 4 would hardly be noticeable in a sample representing large scope of operations.

Factors Governing Estimation of Total Frequencies

Average and standard gust intervals.- The fact that the frequency distribution follows a fixed pattern for samples of large scope indicates that the total frequency is proportional to the distance flown within turbulent regions. Conversely, the average spacing between gusts is inversely proportional to the distance flown. In order to provide a useful basis for estimating the total frequencies of significant gusts (namely, those causing measurable acceleration of an airplane), the term "average gust interval" λ_{av} is introduced. This term is defined as the average distance along a flight path in turbulent air between significant gusts. Numerical values of λ_{av} have been derived from the total frequencies of samples 5, 7, 8, 9, and 10 and are given in table IV. In evaluating λ_{av} the actual path lengths in rough air, which are also given in table IV, were divided by the total frequencies.

The average gust interval λ_{av} is plotted against mean wing chord in figure 5. The dependence of λ_{av} on airplane size is evident, although the exact nature of the relationship is not entirely clear from the figure. The average gust interval for the four samples shown in figure 5 is 11 chord lengths. This value may be used to estimate total frequency when the path length in turbulent air and the airplane size are known. Although the points on figure 5 do not fall on a straight line, they could probably be made to do so by suitable correction. Figure 6 of reference 10, for example, shows a marked tendency for average gust interval to increase with gust intensity; corrections for this effect would raise the point for sample 7 and lower the point for samples 8 and 9.

Path ratio.- In order to estimate the total frequencies for actual operating conditions over a long period of operations, it is necessary to know something about the percentage of the total flight path that falls within regions of turbulence or about the actual total frequencies that occur within total paths of operation of large scope. Information on the relative period of operation within turbulent regions is given in table IV for samples 5 and 7 in terms of the path ratio R. The total frequencies are

$$F = 5280 \frac{RL}{\lambda_{av}}$$

or

$$F \approx 5280 \frac{RL}{11\bar{c}} \qquad (2)$$

when L is in miles, λ_{av} is in feet, and $\bar{c}$ is in feet.

Although the path ratio is not known for the other samples to which such a ratio is applicable, the total frequency of sample 1 is estimated at 1,600,000 gusts to a threshold value of $U_e = 0.3$ foot per second in the manner previously explained. Because this total frequency applies to a path of operations of 145,000 miles and because the mean chord was about 10.5 feet, R is approximately 0.24 from equation (2).

Operating conditions.- The path ratio and therefore the total gust frequency for any path of operations manifestly will depend on the operating conditions. A feeder-line transport operating overland at low altitude, for example, would be expected to encounter a greater percentage of turbulent air than an airplane operating at high altitude above the mechanical turbulence near the ground and above most of the convective clouds. Although the operating conditions are important in defining total frequencies, the data available at this time are too sketchy to permit correlations between total frequencies and the factors composing the operating conditions.

In order to permit estimations of total frequencies, all available pertinent data including those from German sources have been assembled in table V. The first four sets of German data in table V have been based on the data of reference 3. Owing to the fact that Freise presented frequencies for noncontiguous classes, the total frequencies given were obtained by multiplication of the frequencies counted by Freise by 2.5, which is the ratio of the interval between class marks to the interval within which the original count was made. The path ratios from the German data were estimated by application of equation (2).

In applying the data of table V to the estimation of total frequencies, some judgment will have to be used to ensure that values of path ratio most nearly representing the operating conditions are used. It will be noted that path ratios range from about 0.006 to 0.24, with an average value of about 0.1.

APPLICATION OF GUST FREQUENCIES TO ESTIMATION OF STRESS FREQUENCIES

Choice of Gust-Frequency Distribution

The relative-frequency polygons representing the available data permit some latitude in the selection of a frequency distribution to be applied in a design problem. Choice of a conservative gust-frequency distribution for use in estimations of stress frequency depends upon the relative significance of the small and large stresses in the problem under analysis. If the problem is to determine the probability of occurrence of large stresses in excess of the strength of the structure at the design limit load, a more conservative estimate will result from the selection of a frequency distribution having relatively high frequencies at the higher values of effective gust velocity. For other purposes, the selection of a distribution having the higher frequencies at the low effective gust velocities may give a more conservative estimate. Two limiting relative-frequency polygons, A and B, representing the approximate limits of the data are shown in figure 6. Polygon A has previously been used as "relative distribution A" to facilitate comparison of the data shown in figures 2 to 4. For some purposes summation curves, or ogives (reference 8), are more convenient representations of frequency distributions than frequency polygons. Unit summation curves corresponding to polygons A and B of figure 6 are therefore given in figure 7.

Relation between Effective Gust Velocity and Stress in the Structure

Direct application of the gust-frequency distribution and the total frequency by means of equation (1) with the usual design assumption of static load will yield approximately correct values of stress frequency. There are, however, several phenomena that modify the actual stress frequencies from the stress frequencies estimated in this simple manner. These phenomena include:

(1) Superposition of uncounted small gusts on the larger gusts counted

(2) Distribution of gust velocity across the span

(3) Dynamic response of the structure

Uncounted superimposed gusts.- As previously mentioned, the minor peaks in the acceleration records were not ordinarily counted unless they occurred as single phenomena between two consecutive intersections with the 1g datum. A special total count of these neglected peaks was made in one case from a clean-cut record without reference to the exact magnitudes of the acceleration increments or to the acceleration level at which they occurred. It was found that the number of these small superimposed peaks was about twice the total frequency counted in the manner adopted for the general analysis. These superimposed peaks were irregular in shape, sequence, and time or place of occurrence. The magnitudes of the superimposed acceleration peaks with respect to the adjacent acceleration levels were small and did not in any case exceed a value corresponding to ΔU_e = 4.5 feet per second. The great majority of these peaks were near the threshold value of 0.3 foot per second.

Discussion of the reason for the consistently small magnitude of the superimposed peaks is beyond the scope of this paper, as the question of the relationship between gust intensity and gust dimensions and the question of the probability of superposition of randomly distributed gusts are involved.

Kaul (reference 2) reports a similar count of superimposed peaks from a record of wing-tip deflection. Kaul implied that the acceleration records did not contain such peaks and that the extra peaks counted were due to damped vibration of the wing structure after disturbance by the individual gusts. The ratio of the number of extra peaks to the number counted with respect to the 1g datum was, however, about 2 - a result that is in agreement with the authors' count of the extra acceleration peaks. It seems probable, therefore, that some additional acceleration peaks due to superimposed gusts and some acceleration peaks due to vibration response of the wing-fuselage system were actually counted in both cases.

So far as the mere question of gust frequency is concerned, without regard to superposition, these additional small peaks may be placed in class 1. The inclusion of such small peaks in a fatigue test, however, cannot properly be effected on the basis of this simple classification. If the superposition of the additional small peaks is felt to influence the fatigue strength to an important degree, the phenomenon of superposition must be taken into account. The superposition may perhaps be pictured sufficiently well for application to fatigue tests by imagining the periods of the various stress cycles to be proportional to the amplitude. Further, assume the cycles corresponding to the basic gust frequency distribution to be applied without superposition. Finally, superimpose the additional small cycles on the basic cycles of class 2 and of the higher classes, distributing the additional small peaks uniformly along the time scale to determine the numbers to be superimposed on each basic cycle.

The actual application of superimposed cycles in fatigue testing is a difficult matter and requires either the construction and use of a family of summation curves with mean stress as a parameter or the construction of a complex fatigue machine with which the small cycles can be superimposed on the larger cycles. The derivation of the summation curves would require that the basic stress cycles be considered as square waves for the purpose of establishing a finite number of mean stress values, and the actual testing would involve the difficulty of occasionally holding the mean stress levels at very high values while the small cycles were being applied.

Distribution of gust velocity along span.- The distribution of gust velocity along the span of a wing is not always uniform, so that the usual assumption of uniform distribution leads to some error in estimation of stress frequencies from the gust frequencies. The results of the gust investigation with the XC-35 airplane indicate the various typical spanwise distributions that actually occur and the frequency of each type. If desired, further refinement of the stress frequencies can be made from these data, which are reported in reference 11.

Dynamic response of the structure.- Owing to the flexibility of wing structures, accelerations caused by gusts will not be the same at all points along the span. The accelerations at the wing tips will be somewhat greater than and out of phase with those at the fuselage. Some calculations pertaining to two typical large airplanes (reference 12) and tests in the Langley gust tunnel indicated that the maximum tip acceleration at about 200 miles per hour was about twice the acceleration at the fuselage and occurred earlier than the fuselage acceleration. The wing oscillation in these cases damped out in 1 to 2 cycles. The effect of such dynamic action is to cause, at the outer portions of the wing primary structure, superimposed stress cycles with a maximum amplitude about 10 percent of the static stress for the uniformly distributed gust.

Because the natural period of wings increases almost in direct proportion to the wing linear dimensions and because the size of gusts to which airplanes will respond also increases as the airplane size, the ratio of natural period to period of application of load remains about constant for constant flight speed. The dynamic response of the structure would, therefore, appear not to increase with airplane size.

If desired, the additional frequencies of the small dynamic stresses at the outer portions of the wings can be included in the same manner as the uncounted superimposed gust frequencies.

Experimental evidence.- Some test results from the stress and acceleration measurements on the M-130 and the B-15 airplanes are shown in figures 8 to 10. Comparative stress frequencies cannot be shown, but the figures illustrate the degree of agreement between peak stresses as measured and as would be calculated by the usual assumption of static load for the corresponding measured accelerations.

For the M-130 airplane (fig. 8) a datum stress increment corresponding to application of a load factor of 1 was determined by taking the difference between stress while in level flight in smooth air and stress while at rest on the water. Correction was made for wing weight. The plot therefore indicates the agreement between gust-induced stresses as measured and gust-induced stresses as determined by multiplication of the datum stress by the measured acceleration. The distribution of the points along a line of 45° slope indicates excellent agreement; this result and the lack of scatter beyond the limits of error denote lack of serious dynamic response of the structure.

The results shown for the B-15 airplane in figures 9 and 10 are given simply as plots of measured stress against measured acceleration because a datum stress increment was not measured. The stress-load relationships shown are, however, substantially linear; this fact, together with virtual absence of scatter beyond the limits of error, shows absence of serious dynamic response.

These results indicate that, with the exception of the small uncounted superimposed stress peaks, the stress frequencies of the primary wing structure will be given with sufficient exactness, for all practical purposes, by application of the gust frequencies through equation (1) and the usual assumption of static load.

Application to tail surfaces.- The gust-frequency data given herein are not directly applicable to tail surfaces. Some unpublished flight data on the relative magnitudes of effective gust velocities on wings and tail surfaces indicate, however, that a rough approximation of the tail-load frequencies might be obtained by utilizing the gust frequencies given here and by multiplying the values of effective gust velocity by 1.6 for the vertical tail surfaces and by 0.5 for the horizontal tail surfaces.

CONCLUDING REMARKS

Available flight data are sufficient to indicate that the distribution of gusts within turbulent regions of the atmosphere follows a substantially fixed pattern which is independent of the source or cause of the turbulence. The average interval between gusts causing measurable airplane response is about 11 chord lengths, and the total frequency of significant gusts in any stretch of rough air is therefore the length of the flight path in rough air divided by 11 times the mean wing chord.

The total gust frequency to be expected during the operating life of an airplane depends upon the operating conditions, which determine the ratio of path length in rough air to the total path of operations. Information on the path ratio as a function of operating conditions is sketchy at this time and should be supplemented by further measurements. From the available information, the average path ratio for a variety of operating conditions is about 0.1, although individual values vary between about 0.006 and 0.24.

The available data on gust frequencies permit approximate determination of stress frequencies in the primary structures of airplanes due to gusts. These frequencies appear to describe adequately, for many design purposes, the stress conditions for transport-type airplanes in flight. Supplementary information on stresses in secondary members of the structure and on the additional frequencies of small stresses in the primary structure resulting from dynamic structural response and nonlinear lateral gust distribution is desirable. This information will have to be obtained by stress measurements correlated with airplane size, dead-weight distribution, and other factors.

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TABLE I

CHARACTERISTICS OF AIRPLANES FROM WHICH SAMPLES

OF GUST-FREQUENCY DATA WERE OBTAINED

Sample	Airplane type	Weight in flight (lb)	Wing area (sq ft)	Wing loading (lb/sq ft)	Span (ft)	Mean chord (ft)	Relative alleviation factor, K	Slope of lift curve, a
1	Ford 5-AT	[a]13,500	835	16.16	77.8	10.73	1.000	4.76
	Fokker F-10-A	[a]10,500	728	15.80	71.2	10.22	0.995	4.76
	Boeing 40-B	[a]6,080	545	11.1	44.2	6.6	0.905	3.9
	Boeing 80-A	16,000	1220	13.1	80	9.0	0.955	4.0
2	Boeing B-247	11,100	836	13.3	74	11.0	0.955	4.52
3	Douglas DC-2	15,500	938	16.5	85	11.0	1.005	4.65
4	Douglas DC-3	21,000	988	20.4	95	10.40	1.05	4.79
5	Martin M-130	40,000	2270	17.60	130	16.70	1.020	4.66
6	Martin M-130	40,000	2270	17.60	130	16.70	1.020	4.66
	Boeing B-314	71,500	2868	24.90	152	17.70	1.085	4.69
7	Boeing B-15	55,000	2780	19.8	149	18.65	1.044	4.76
8	Lockheed XC-35	10,500	458	22.90	55	9.23	1.070	3.95
9	Lockheed XC-35	10,500	458	22.90	55	9.23	1.070	3.95
10	Aeronca C-2	782	144	5.43	36	4.0	0.772	4.73

[a]Gross weight.

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TABLE II

OPERATING CONDITIONS REPRESENTED BY SAMPLES

Sample	Airplane type	Route	Topography	Mean altitude above sea level (ft)	Flying time (hr)	Remarks
1	Ford 5-AT Fokker F-10-A Boeing 40-B Boeing 80-A	Various routes in U.S.	All types in U.S.	4,000	1,320	Early domestic transport operations
2	Boeing B-247	Various routes in U.S.	All types in U.S.	--------	12,247	Domestic transport operations
3	Douglas DC-2				10,534	
4	Douglas DC-3				19,324	
5	Martin M-130	Alameda to Hong Kong	Oceanic	10,000	115	Operations on Pacific Division of Pan American Airways System. Short stretch of extremely rough air experienced during operations
6	Martin M-130 Boeing B-314	Alameda to Hong Kong	Oceanic	10,000	12,232	Operations on Pacific Division of Pan American Airways System
7	Boeing B-15	Various routes in U.S. and one round trip from Langley Field, Va. to Canal Zone	All types in U.S. and oceanic	5,000	313	Miscellaneous Army peacetime operations. Turbulent regions, in general, avoided
8	Lockheed XC-35	Vicinity of Langley Field, Va.	Flat, wooded	--------	3.13	Flight selected at random from a number of gust surveys of cumulus-congestus and cumulo-nimbus clouds
9	Lockheed XC-35	Vicinity of Langley Field, Va.	Flat, wooded	--------	2.26	Roughest flight selected from gust surveys of cumulus-congestus and cumulo-nimbus clouds
10	Aeronca C-2	Vicinity of Langley Field, Va.	Flat, wooded	500	2.67	Flight selected at random from a number of gust surveys in earth's boundary layer

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TABLE III

TOTAL FREQUENCIES AND FREQUENCY DISTRIBUTIONS

Sample	Class interval (fps)	Threshold values of U_e (fps)	Total frequency	Sign of gust direction	Class											
					1	2	3	4	5	6	7	8	9	10	11	12
					Frequency											
1	3.00	9.0	---------	+	-------	---	----	1860	363	95	25	7	2	0	-	---
				-	-------	---	----	2708	797	182	50	15	3	0	-	---
2	4.50	20.0	---------	+	-------	---	----	----	-----	3	3	1	0	0	-	---
				-	-------	---	----	----	-----	6	1	1	0	0	-	---
3	4.50	20.0	---------	+	-------	---	----	----	-----	1	0	0	0	0	-	---
				-	-------	---	----	----	-----	3	0	0	0	0	-	---
4	4.50	20.0	---------	+	-------	---	----	----	-----	15	6	2	2	1	-	---
				-	-------	---	----	----	-----	9	3	6	1	1	-	---
a5	2.88	0.3	2,895	+	1,280.5	125	28.5	10	1.5	1	0.5	0	0	0	0	0.5
				-	1,280.5	125	28.5	10	1.5	1	0.5	0	0	0	0	0.5
a6	2.88	20.0	---------	+	-------	---	----	----	-----	---	----	1	2	0.5	0	0
				-	-------	---	----	----	-----	---	----	1	2	0.5	0	0
7	2.64	0.3	26,046	+	11,868	1002	114	17	16	1	2	2	0	1	-	---
				-	11,934	948	97	31	7	3	2	0	1	0	-	---
8	4.50	0.4	2,564	+	1,031	221	23	4	1	---	----	--	-	---	-	---
				-	1,030	238	16	0	0	---	----	--	-	---	-	---
a9	4.50	0.4	3,405	+	1,085.5	418	135	37	15	6	4	2	-	---	-	---
				-	1,085.5	418	135	37	15	6	4	2	-	---	-	---
10	3.00	0.2	5,361	+	1,740	735	162	38	4	2	----	--	-	---	-	---
				-	2,080	462	107	26	5	0	----	--	-	---	-	---

[a]Positive and negative accelerations not separately counted.

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TABLE IV

AVERAGE GUST INTERVALS AND RELATED DATA

Sample	Total flying time (hr)	Flying time in rough air (hr)	Average true airspeed (mph)	Path of operation, L (miles)	Path in rough air (miles)	Path ratio, R	Average gust interval, λ_{av} (ft)	Gusts per mile of path of operation, F/L
1	1,320	------------	110	145,000	-------	------	---------	------------
2	12,247	------------	147	1,800,000	-------	------	---------	------------
3	10,534	------------	180	1,900,000	-------	------	---------	------------
4	19,324	------------	180	3,480,000	-------	------	---------	------------
5	115	0.67	151	17,400	101	0.0058	180	0.166
6	12,232	------------	151	1,850,000	-------	------	---------	------------
7	313	4.84	153	47,800	741	.0155	150	.544
8	------	.35	170	---------	60	------	130	------------
9	------	.43	170	---------	73	------	[a]129	------------
10	------	.65	75	---------	49	------	48	------------

[a]Value based on special count from variable datum caused by disturbed motion of airplane.

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TABLE V

TOTAL FREQUENCIES, PATH RATIOS, AND OPERATING CONDITIONS

OF AMERICAN AND GERMAN TRANSPORT OPERATIONS

Route	Topography	Average flight altitude above sea level (ft)	Path of operation (miles)	Total frequency	Total frequency per mile of operations corrected to $\overline{c}$ = 10 ft	Path ratio, R	Remarks
			American data				
Various routes in U.S.	All types in U.S.	4,000	145,000	1,600,000	11.6	0.24	Early domestic transport data
Alameda to Hong Kong	Oceanic	10,000	17,400	2,895	0.278	0.0058	Cloud formations mostly avoided
Various routes in U.S. and one trip from Va. to Canal Zone	All types in U.S. and oceanic	5,000	47,800	26,046	1.02	0.0155	Turbulent air mostly avoided
			German data				
Berlin to Vienna	Low and flat to mountainous	2,700	7,400	50,800	7.05	0.147	Operations on Deutsche Lufthansa with early types of German transport airplane
Berlin to Königsberg	Low and flat	2,000	11,190	20,950	1.92	0.040	
Berlin to Paris	Hilly	2,600	7,960	65,010	8.38	0.174	
Stuttgart to Barcelona	Mountainous	4,700	40,260	395,500	8.84	0.184	
Vienna to Belgrade	Mountainous	------	---------	-------	4.77	0.0995	Modern transport
Azores to New York	Oceanic	Low	---------	-------	3.54	0.0738	Mail transport only
				Average	5.24	0.0993	

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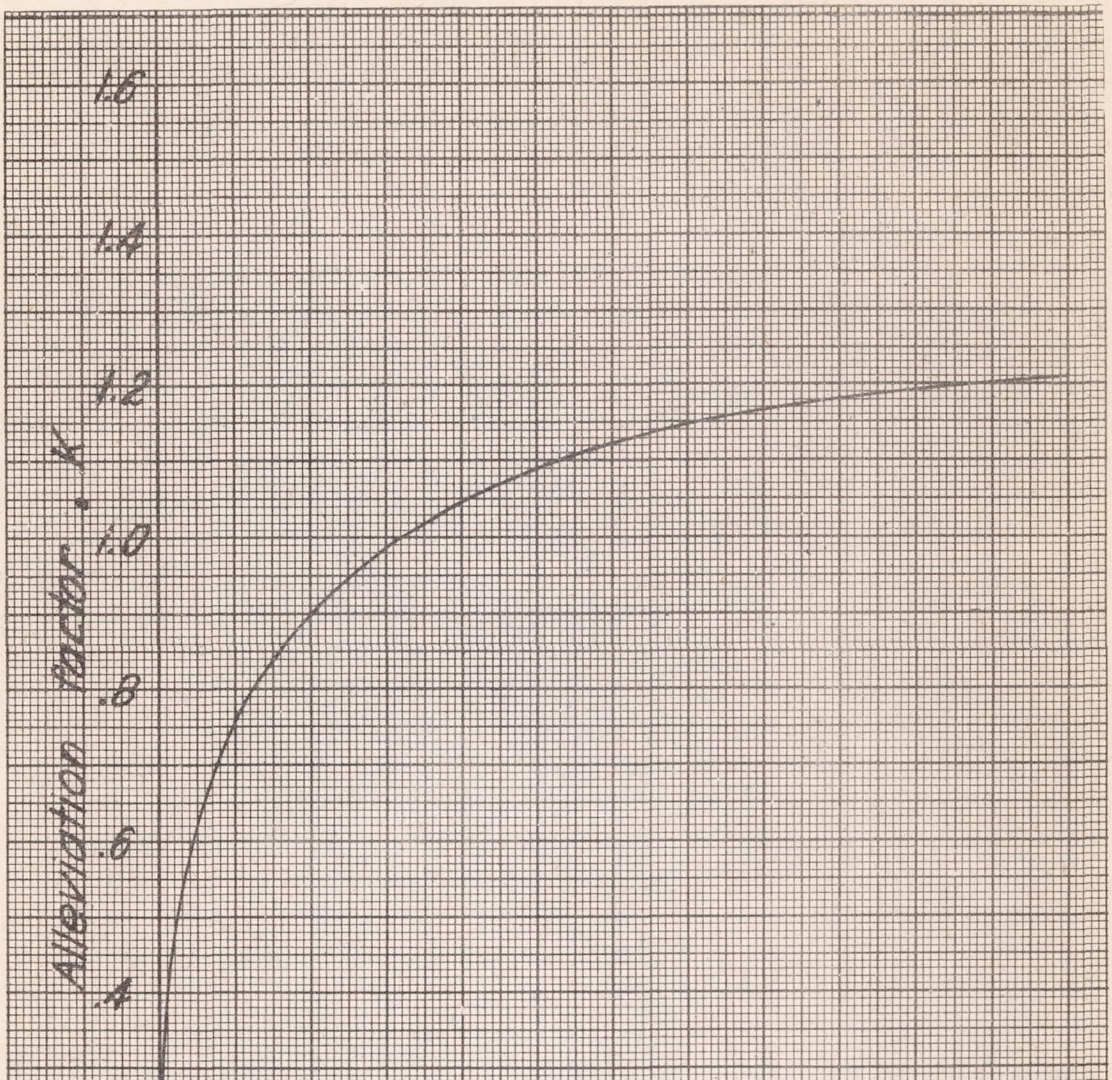

Figure 1.-Variation of alleviation factor with wing loading.

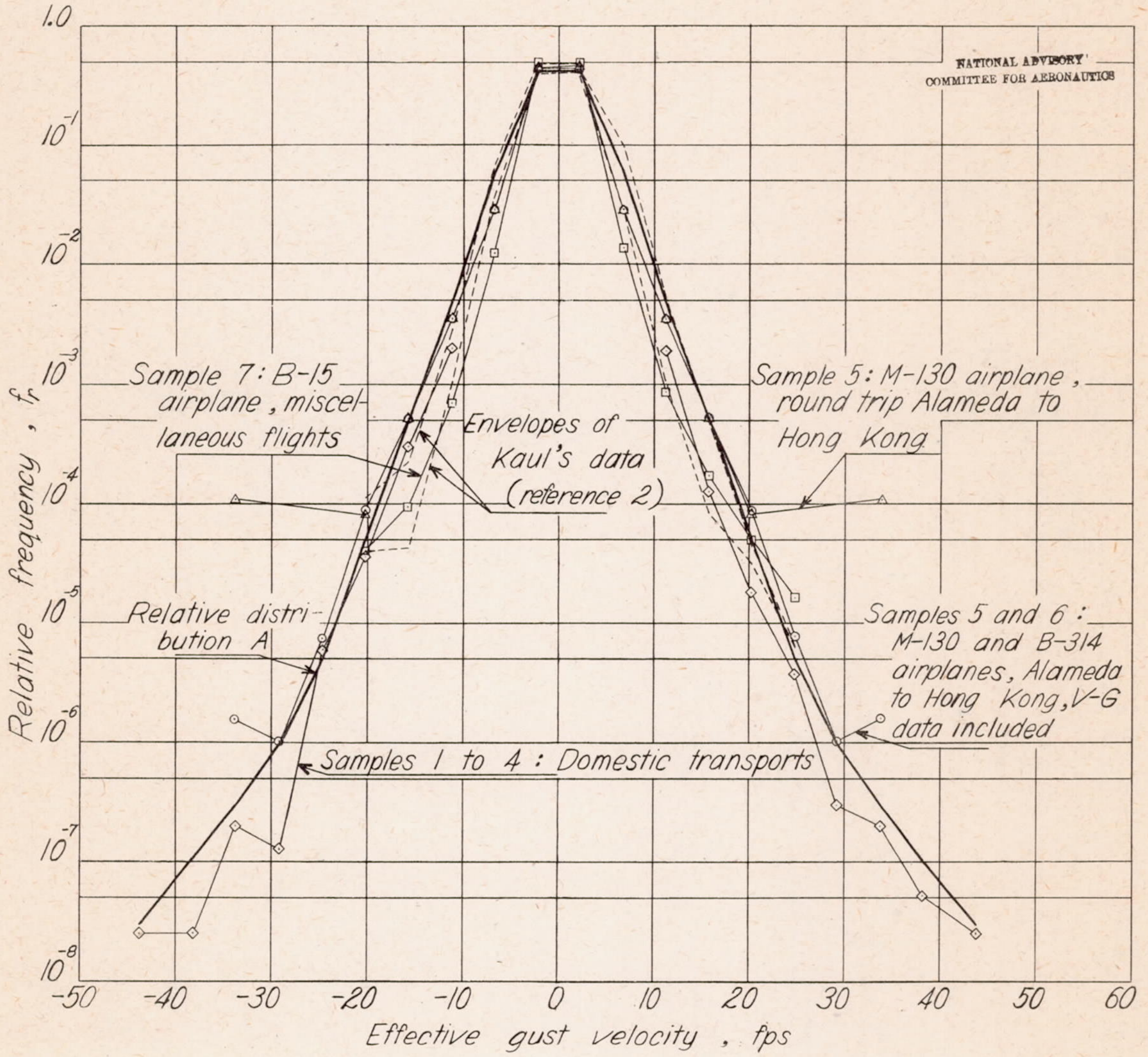

Figure 2.-Polygons of relative-frequency distribution of effective gust velocity

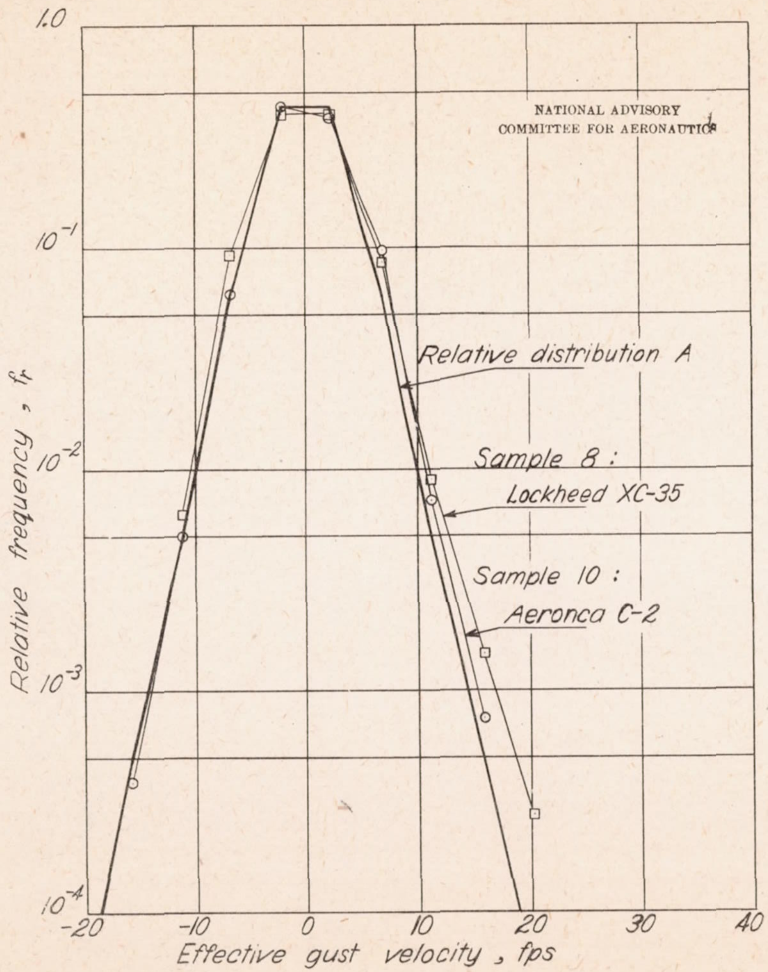

Figure 3.-Polygons of relative-frequency distribution for samples selected at random from turbulence surveys with Lockheed XC-35 and Aeronca C-2 airplanes.

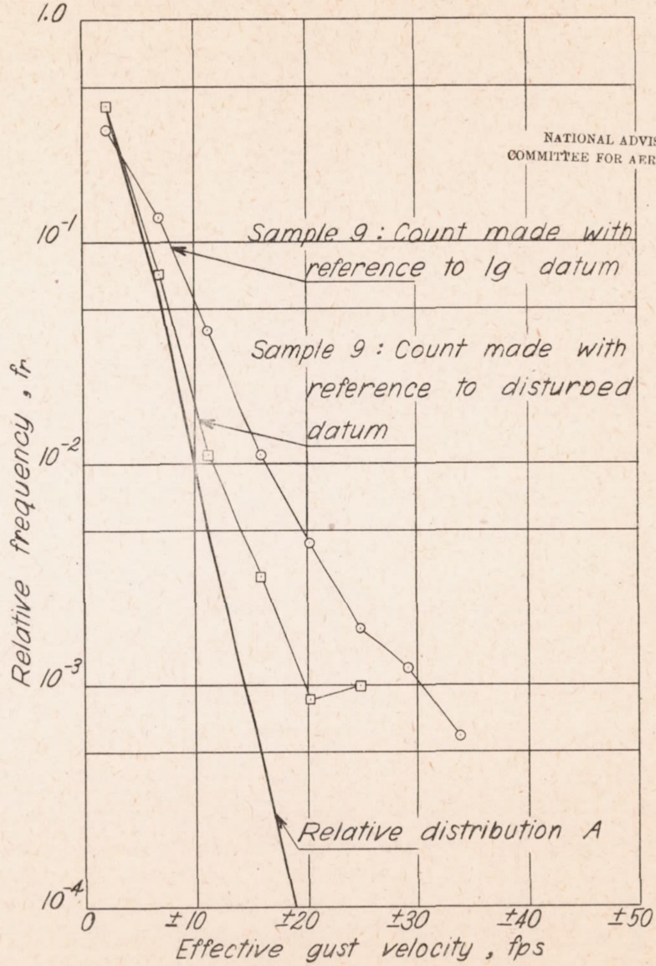

Figure 4.-Influence of disturbed motion of Lockheed XC-35 airplane on apparent gust frequency distribution in very rough air.

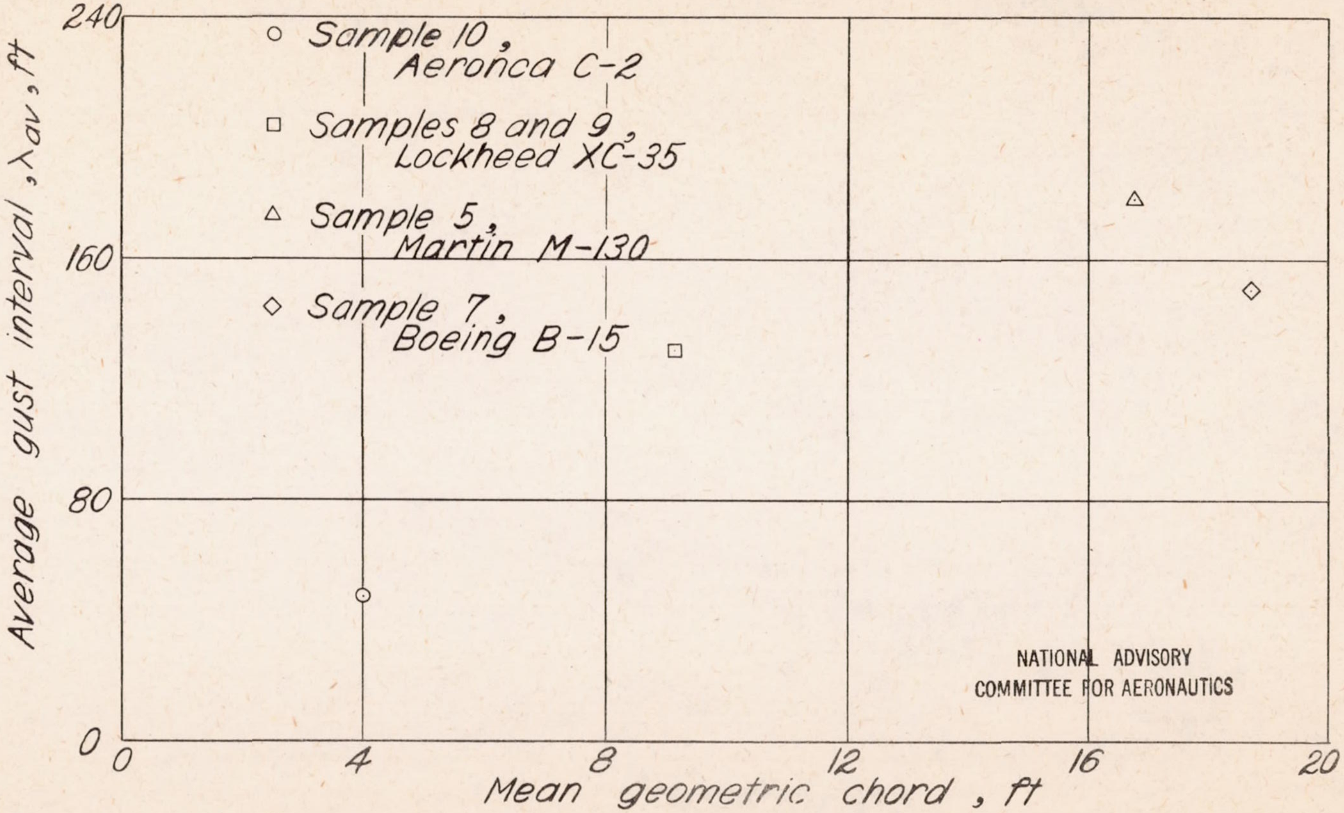

Figure 5.- Influence of airplane size on spacing of significant gusts in rough air.

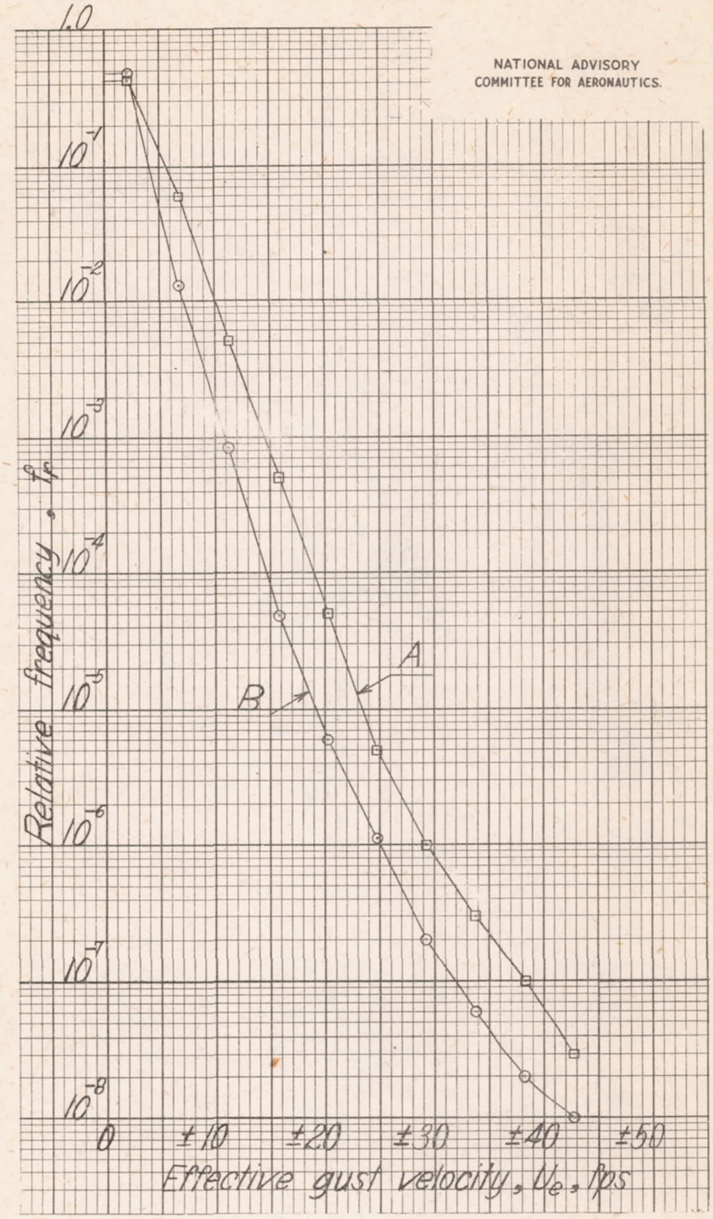

Figure 6.- Relative-frequency polygons defining approximate limits of samples.

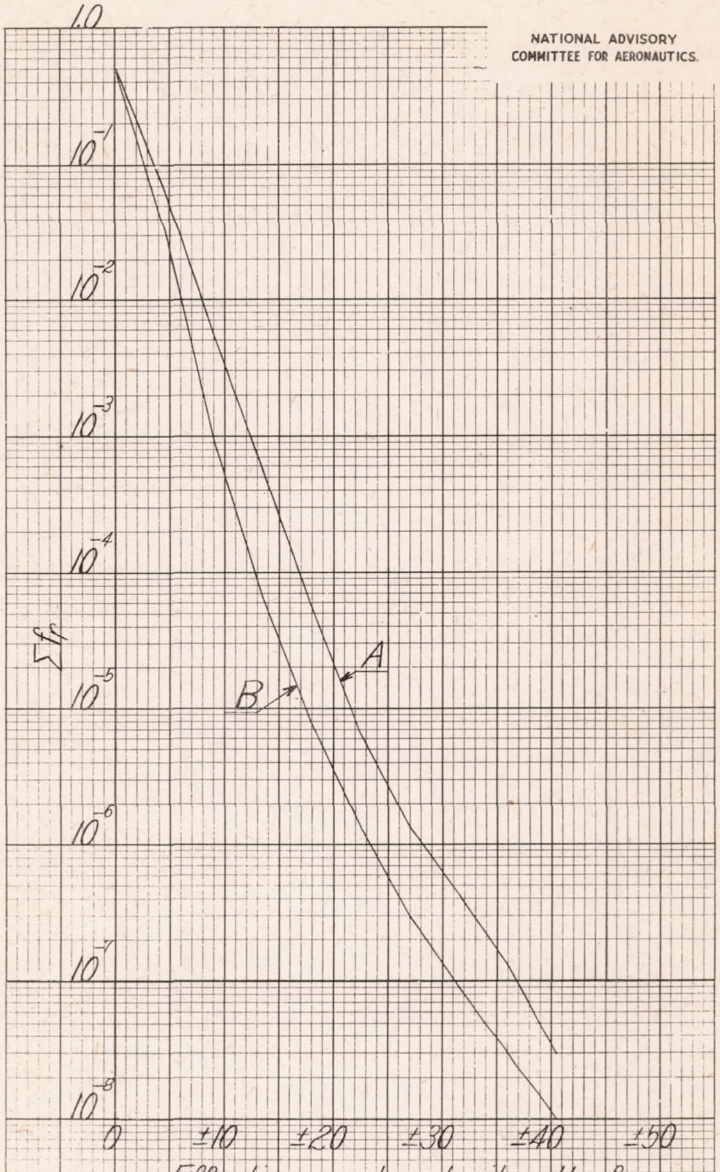

Figure 7.- Unit summation curves defining approximate limits of samples.

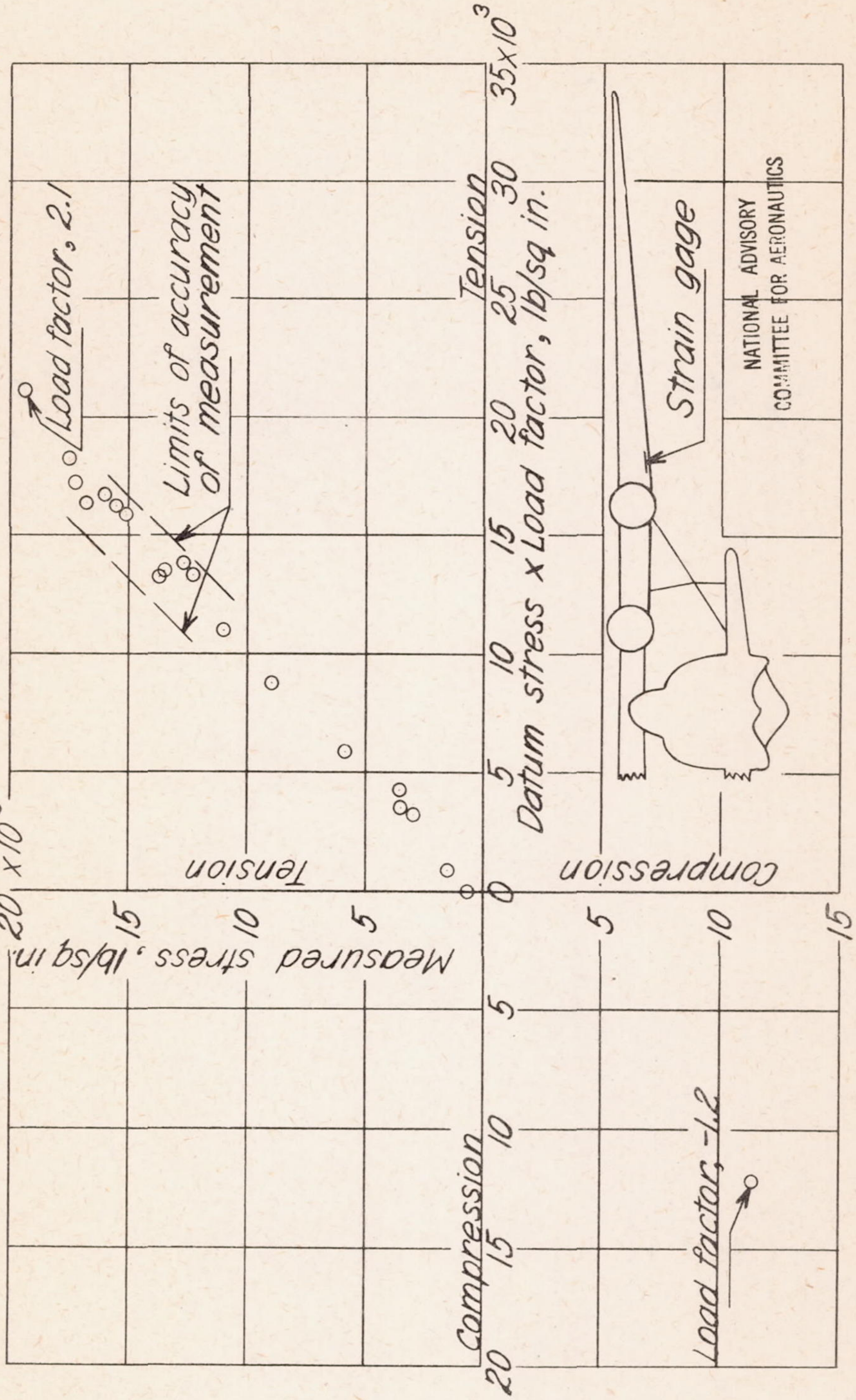

Figure 8.- Stress in wing of M-130 airplane during flight through gusts.

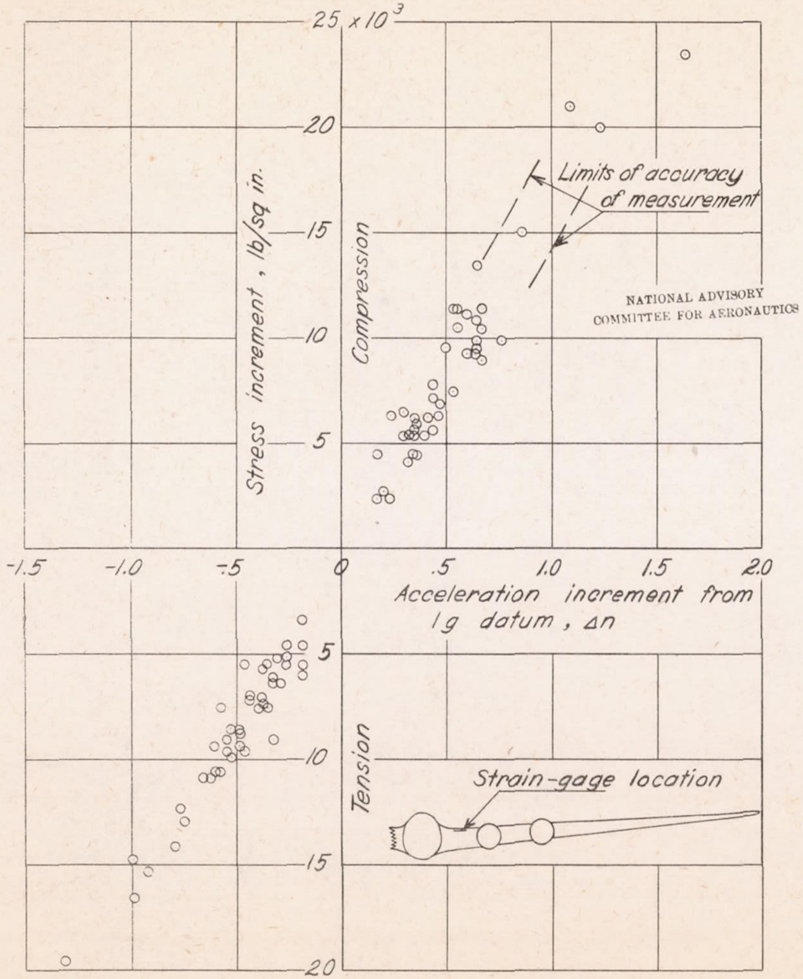

Figure 9.- Stress at inboard location in wing of B-15 airplane during flight through gusts.

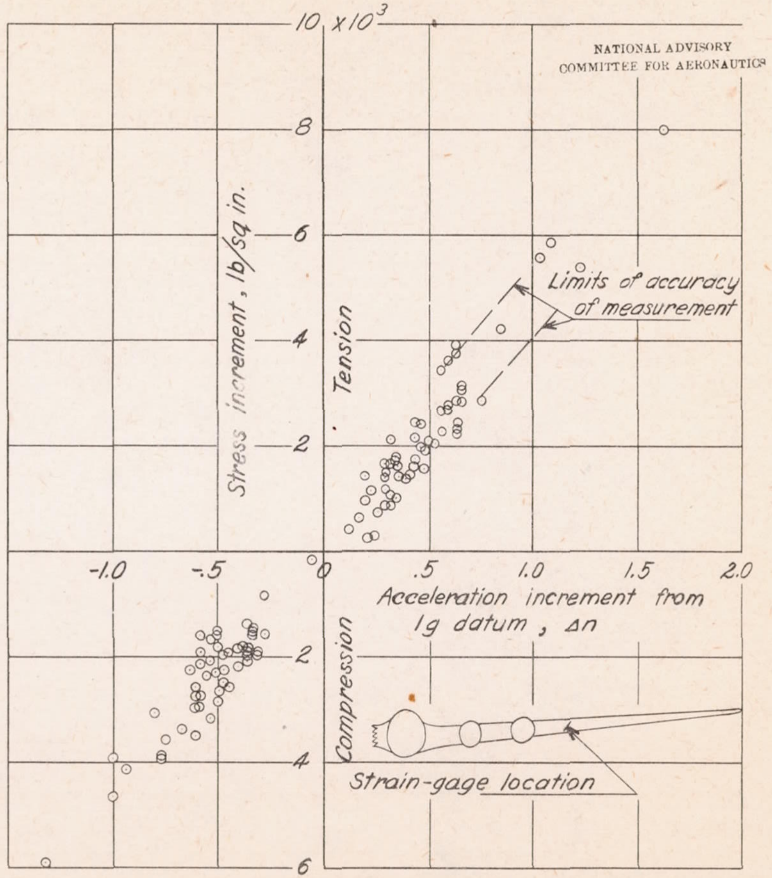

Figure 10.- Stress at outboard location in wing of B-15 airplane during flight through gusts.